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U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

(6) PENNSYLVANIA STATE VIBRATION AND VIBRATION DAMPING SEMINAR

by

(10) Howard N. Phelps, Jr

USL Technical Memorandum No. 933-429-64

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INTRODUCTION

The writer attended a one-week, 19-24 July 1964, seminar at the Pennsylvania State University. The seminar, "Vibrations and Vibration Damping," was conducted by the Pennsylvania State University Ordnance Research Laboratory.

This memorandum covers the material that was presented by several lecturers, who are members of the Ordnance Research Laboratory, Pennsylvania State University.

(9) Technical memo

Speakers at the seminar included:

Mr. G. P. Haddle, Research Associate in Engineering Research;
Mr. R. H. Koebke, Research Assistant in Engineering Research;
Dr. J. M. Lawther, Associate Professor of Engineering Research;
Dr. V. H. Neubert, Associate Professor of Engineering Mechanics;
Dr. Maurice Sevik, Assistant Professor of Aeronautical Engineering;
Dr. E. J. Skudrzyk, Professor of Physics; and
Dr. J. C. Snowdon, Assistant Professor of Engineering Research.

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POINT MASS-SPRING VIBRATOR

A review was made of important principles in the theory of vibrations, Fourier Analysis, the theory of functions of a complex variable and rotating vectors.

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The similarity of the differential equations of a tuned electrical circuit, of the fundamental electrical vibrator and the point mass spring system, and of the fundamental mechanical vibrator were discussed.

For the electrical oscillator, the differential equation is:

$$L\frac{di}{dt} + Ri + \frac{1}{C}q = u$$

where: L = inductance (henrys)

i = current (amperes)

R = Resistance (ohms)

C = capacitance (farads)

u = periodic voltage (volts)

q = charge (coulombs)

t = time (seconds)

For the mechanical vibrator, the differential equation is:

$$M\frac{dv}{dt} + Rv + \frac{1}{K}\xi = f$$

where: M = mass (lb. sec^2/inch)

V = velocity (inches/second)

R = damping constant (lb. -sec/inch)

K = stiffness or spring constant (lbs/inch)

ξ = displacement (inches)

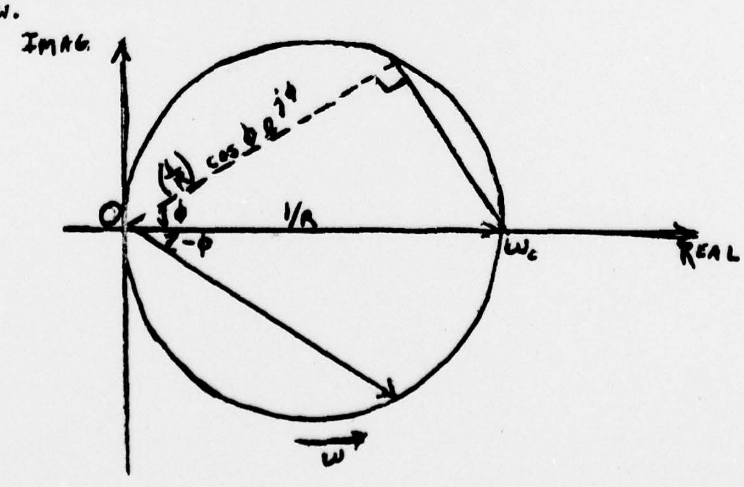
f = periodic force (lbs)

It was shown that these two equations are of the same form, and that mathematically there is no need to distinguish between the electrical and mechanical systems. The complex solution for these equations was found for the subcritically damped decaying vibration, for the supercritically damped decaying vibration, and for the forced vibrations. The real solution was obtained by expressing the complex solution in its polar form and disregarding the imaginary part of the exponential. The advantage of finding the complex impedance is that it gives the amplitude of the velocity for a periodic force and the phase retardation of the velocity with respect to the phase of the force. Other topics, such as the locus of the impedance in the complex plane, bandwidth, decay factor, and logarithmic decrement, were discussed during this lecture. Very briefly, the locus of impedance in the complex plane, when the frequency is varied from zero to infinity, is defined as a line parallel to the imaginary axis at a distance R from it that extends from minus infinity to plus infinity. The impedance is represented by the vector that starts at the origin and ends at the line and at a point that is determined by the frequency. At the resonant frequency, the imaginary part of the impedance is zero and Z = R. The imaginary part becomes minus infinity as the frequency approaches zero and plus infinity as the frequency becomes infinite.

The velocity is proportional to the admittance $\bar{Y}$, which is the reciprocal of the impedance

$$\bar{Y} = 1/Z = \frac{1}{Z} \cdot \frac{R}{R} = \frac{1}{R}\ \frac{R}{Ze^{j\phi_Z}} = \left(\frac{1}{R} \cos \phi_z\right) e^{-j\phi Z}$$

where R/Z is the cos ϕ_z (angle between the impedance and the real axis.) The angle ϕ_z varies from $-\pi/2$ to $\pi/2$ as the frequency varies from zero to plus infinity. If plotted as a function of ϕ_z, the magnitude (1/R) cos ϕ_z describes the circumference of a circle. The end point of the admittance vector describes a similar circle, but because of the factor $e^{-j\phi}Z$, the locus of its points is the mirror image of the first circle with respect to the real axis. As the frequency is increased, the head of the admittance vector moves clockwise along this circle. See the figure below.



The frequency curve of a complex mechanical vibrator usually exhibits many resonance peaks. The bandwidth is based on the half energy points. The half energy bandwidth is the width of the resonance peak above a height of $1/\sqrt{2}$ times that of the maximum energy height. Since the vibrational energy at the limits of the bandwidth of a linear system is proportional to the square of the velocity amplitude, the velocity amplitude decreases to one-half of its value at the resonant frequency and the vibrational energy is one half of that at the resonant frequency. The bandwidth is therefore a measure of the damping of the system.

The decay constant is another measure of the energy absorption of the system. It describes the rate of decay of the vibration amplitude of the system after the force has been removed.

The logarithmic decrement is defined as the logarithm of the ratio of successive maxima. A discussion of the measurements techniques involving the decay constant and the logarithmic decrement is given in reference (a).

The mechanical parallel resonant circuit was also discussed. The only difference between a mechanical series resonant circuit and a mechanical parallel resonant circuit is the manner in which it is driven by the external force. In the series resonant circuit, the force drives the mass; in the anti-resonant or parallel resonant circuit, the force drives the spring. A mathematical analysis of such a circuit was undertaken.

A mathematical analysis was made on the energy and the dissipation of energy by a point-mass spring vibrator.

MATERIAL DAMPING AND VIBRATION ISOLATION

The mechanical properties of rubber-like materials were the main topic discussed during this lecture. A theoretical analysis was undertaken on the frequency dependence of the elastic moduli of the material and their associated damping factors. The materials that were discussed are neoprene rubber, SBR rubber, filled natural rubber (50 parts by weight of HAF carbon black), plasticized polyvinyl butyral resin, natural rubber, Thiokol RD, plasticized polyvinyl acetate, and filled butyl rubber (40 parts by weight of MPC carbon black).

The concept of a complex modulus was also discussed. It was pointed out that the strain induced in a linear viscoelastic material may be expressed as a function of the applied stress experienced by the material. However, the two fundamental types of deformation (shear and volume) are not related to the applied stress by a simple constant of proportionality (the elastic modulus G or B). Rather, the relation between stress and strain is most generally expressed by a linear partial differential equation of arbitrary order:

$$\left[a_0 + a_1\left(\frac{\partial}{\partial t}\right) + a_2\left(\frac{\partial^2}{\partial t^2}\right) + \ldots + a_n\left(\frac{\partial^n}{\partial t^n}\right) \ldots\right]\sigma$$
$$= \left[b_0 + b_1\left(\frac{\partial}{\partial t}\right) + b_2\left(\frac{\partial^2}{\partial t^2}\right) + \ldots + b_n\left(\frac{\partial^n}{\partial t^n}\right) \ldots\right]\epsilon$$

where: σ = stress (lbs/square inch)

ϵ = strain (inches/inch)

t = time (seconds)

a_n = constant

b_n = constant

Several examples were discussed of simple systems that duplicate the mechanical behavior of the material under strain.

A. Spring of stiffness K

$$\sigma = \kappa K_0 \epsilon$$

a_o and b_o are finite; all other values of a_n and b_n are zero.

k is a constant with dimensions of $(\text{length})^{-1}$.
This equation follows directly from Hooke's Law.

B. Dashpot of viscosity n_o.

$$\sigma = \kappa \eta_0 \left(\frac{\partial}{\partial t}\right)\epsilon$$

a_o and b_1 are finite; all other values of a and b are zero. This equation follows directly from Newton's Law.

C. One spring and one dashpot

$$\sigma = \kappa\left[K_0 + \eta_0\left(\frac{\partial}{\partial t}\right)\right]\epsilon$$

a_o, b_o, and b_1 are finite; all other values of a and b are zero.

D. <u>Two springs and one dashpot</u>

$$\left[K_1 + \eta_1\left(\frac{\partial}{\partial t}\right)\right]\sigma = \kappa\left[K_0 K_1 + (K_0 + K_1)\,\eta_1\left(\frac{\partial}{\partial t}\right)\right]\epsilon$$

a_o, a_1, b_o, and b_1 are finite; all other values of a and b are zero.

E. <u>Three Springs and three dashpots</u>

$$\left[K_1 K_2 + (K_1\eta_2 + K_2\eta_1)\left(\frac{\partial}{\partial t}\right) + \eta_1\eta_2\left(\frac{\partial^2}{\partial t^2}\right)\right]\sigma = \kappa\Big[K_0 K_1 K_2 +$$

$$\left[K_1 K_2(\eta_0 + \eta_1 + \eta_2) + K_0(K_1\eta_2 + K_2\eta_1)\right]\left(\frac{\partial}{\partial t}\right) +$$

$$\left[\eta_1\eta_2(K_0 + K_1 + K_2) + \eta_0(K_1\eta_2 + K_2\eta_1)\right]\left(\frac{\partial^2}{\partial t^2}\right) + \eta_0\eta_1\eta_2\left(\frac{\partial^3}{\partial t^3}\right)\Big]\epsilon$$

a_o, a_1, a_2, b_o, b_1, b_2, b_3 are finite; all other values of a and b are zero.

For the above equations, K = spring constant (lbs/in);
κ = constant $(\text{length})^{-1}$; η = viscosity (lb-sec^2/inch)
ϵ = strain (in/in); σ = stress (psi)

Such systems can be used to represent the mechanical behavior of rubberlike materials under strain.

Included in this lecture was the analysis of the vibration of a linear one-degree-of-freedom system. An element of mass supported by a linear rubberlike material on a foundation that vibrates sinusoidally was discussed. The general transmissibility and phase equations were developed for such a system. Next, the same equations were developed for dual force excitation or for a simple system that is excited by two forces that are out of phase. The discussion continued into the application of the general transmissibility and phase equations and the assumptions that can be made concerning the properties of rubberlike materials for simplification of these equations.

A detailed discussion of rubberlike-material mechanical properties of low and high damping was presented. Summarizing, for low damping materials, the dynamic modulus and the damping factor vary only slowly with frequency; therefore, they may be considered to be constants through the range of frequencies normally of concern in vibration problems. Typically, the damping factor has a value of about 0.1. For high damping materials, the dynamic modulus increases very rapidly with frequency and, for some rubbers, the transition frequencies that fall within the range of frequencies normally of concern in vibration problems may increase at a maximum rate that is essentially proportional to frequency. The damping factor is large, but again varies only slowly with frequency; typically, they may be considered to have a constant value of about 1.0. Discussion also concerned the vibration of a linear two-degree-of-freedom system. Vibration isolation, the general transmissibility and phase equations, and solid and viscous damping were the topics covered for the compound system.

Methods by which vibration may be isolated from structures that possess finite mechanical impedance were described. General expressions were derived from which the performance of various mounting systems may be determined when the driving-point impedance of the non-rigid structure that supports them is known. Mount and foundation damping were shown to be effective in suppressing the peak values of response ratio, although the increase in stiffness of high-damping rubber mounts with frequency has the detrimental effect of increasing the response ratio at high frequencies. A mounting comprised of low and high-damping rubbers in parallel was demonstrated to effectively suppress the peak values of response ratio and yet to provide significantly smaller values of response ratio than that of the constituent high-damping rubber at other frequencies.

Two methods were discussed by which the over-all level of the response ratio of a mounting system may be reduced. Both methods introduce a mass that is employed either to form a compound mounting or to load the foundation that supports the mounting system. It was shown to be desirable, in each case, that the additional mass be as large a fraction of the mass of the mounted item M as possible. When the foundation is mass loaded and when both the ratio M/M_b and the loading mass are large, the response ratio is reduced significantly and approaches the level predicted by the transmissibility curve of the mounting system at frequencies above the fundamental resonant frequency of the mass-loaded foundation. In these circumstances, therefore, low-damping rubbers such as natural rubber are the most suitable mount materials.

VIBRATIONS OF RODS AND BEAMS

In this lecture consideration was given to substituting the complex modulus of elasticity for the real modulus of elasticity in equations describing the forced vibrations of a simple lumped-mass system and the vibration of simple distributed systems such as a rod excited in its longitudinal modes or as bars excited in their transverse or bending modes. However, it was found unnecessary to derive equations describing the response to vibration of damped systems from first principles. It was shown that solutions already available for describing the response to vibration of systems with negligible damping could be adopted simply by replacing the real modulus of elasticity with the complex modulus.

Even though it is possible to seek either a progressive-wave solution or a standing-wave solution to the wave equations that describe the vibration of distributed systems, it was decided to place emphasis only on the standing-wave solution. This was done because in the frequency range of interest in vibration problems and for the dimensions and damping normally possessed by structures of engineering interest, pure wave propagation is not often observed. Wave equations were solved for longitudinal vibrations of a damped rod and the transverse vibrations of damped beams.

A theoretical discussion also took place on the manner in which the driving-point and transfer impedance of a free-free beam depend upon the position of the driving force and upon the position along the beam at which the velocity is monitored. Also considered was the effect of one or more arbitrarily situated forces upon the driving-point impedance that is presented to a primary force at a separate location.

COMPLEX VIBRATORS AND SOUND ACTIVATION

The first topic that was discussed in this lecture was the differential equation for a system with continuously distributed mass and compliance. Previous lectures demonstrated the theoretical treatment of simple systems such as vibrating strings or rods having distributed mass and compliance. In this lecture, these derivations were generalized for more complex homogeneous systems or three-dimensional vibrators. It was pointed out that mode parameters depend on the nature of the driving force and its point of attack, the natural functions of the system, and the coordinates of the point of observation. The mode parameters are k_v, q_v, M^*_v, R^*_v, and K^*_v. k_v is the excitation constant and is equal to the ratio of the mode amplitude at the force point to that at the point of observation. If the force point coincides with the point of observation, $k_v = \xi_v(F)/\xi_v(A) = 1$ regardless of the complexity of the system. The mode constant q_v is defined as the average square of the velocity of the system divided by the square of the velocity at the point of observation. If the vibrator is driven at its center or at its free end or edge, the mode parameter is usually $\frac{1}{2}$ for one-dimensional vibrations and $\frac{1}{4}$ for two-dimensional vibrations. The mode mass, M_v, are either one half or one quarter of the total mass of the system, for one-dimensional vibrations and two-dimensional vibrations, respectively.

$$M^*_v(A) = M_v(A)/k_v(A).$$

$$R^*_v(A) = (w_v^2/w)nM^*_v(A). \quad (w = \text{frequency in rad/sec})$$

$$K^*_v(A) = (1/w_v^2)M^*_v(A)$$

M is the total mass and $M^*_v(A)$, $R^*_v(A)$, and $K^*_v(A)$ represent the effective mass, resistance, and compliance of the system for the v^{th} mode of vibration, referred to the motion of an artitrary point A and to the total driving force.

The frequency resonance curve for the velocity amplitude of a homogeneous system; the locus of the mechanical impedance of a system with continuously distributed mass and compliance; the driving-point and transfer impedance at frequencies much higher than the fundamental resonance; the mean square of velocity over the system and effective impedance at high frequencies; the dissipation resistance referred to space-average square of velocity; and inhomogeneous vibrating systems were discussed in some detail.

REACTION BETWEEN A STRUCTURE AND THE FLUID MEDIUM

The first general topic that was discussed was hydrodynamic excitation due to wake instabilities. Severe oscillations of bridges, smoke stacks, radar antenna members, missiles on launching pads, and electric power lines were included in this topic. The nature of the flow around a circular cylinder was discussed as the general case. The Strouhal number, which depends on the Reynolds number, was derived by dimensional analysis and discussed in detail. The three-dimensional character and randomness of vortex wakes were also discussed as well as the correlations and spectral densities of lift and drag forces. Under this topic, such items as cross-correlation function, cross-correlation coefficient, spectral correlation, and cross-spectral density were discussed. Typical power spectra of lift and drag forces, as well as co and quad-spectral densities were shown. The effects of vibration by the body shedding the wake and the shedding frequencies of other bodies such as airfoils were also discussed. Finally, the dynamic response of linear structures to random excitatations was discussed.

VIBRATION MEASUREMENTS

The three most important ways of carrying out vibration tests are the shock test, the random motion test, and the frequency sweep test. The several types of vibration shakers were mentioned; only the electrodynamic shaker and the hydraulic shaker were discussed in detail. The four vibrational quantities of displacement, velocity, acceleration, and jerk were derived. Acceleration levels and velocity levels are expressed in decibels with respect to a reference. The reference values generally used are 10^{-6} cm/sec for velocity and 10^{-3} cm/sec^2 for acceleration.

The various systems used to measure vibration may be entirely mechanical or a mixture of mechanical, electrical, and optical elements. These systems are described in reference (b). Displacement pickups, velocity pickups, and acceleration pickups are used to measure vibrations, The characteristics of the following sensing elements were also mentioned: namely, strain-sensitive resistance wire, resistive potentiometer, electromagnetic generating elements, variable inductance elements, variable capacitance elements, and piezoelectric elements. The characteristics and mounting of the three main classes of accelerometer, velocity, and displacement pickups were included.

VIBRATION OF LUMPED MASS SYSTEMS

The attenuation of structural vibrations, by the addition of masses or passive mechanical dynamic absorbers, was discussed. The one-degree-of-freedom system was presented first with examples given on the response after the addition of mass or dynamic absorbers. Work dealing with the steady response of a uniform clamped-free beam and with the behavior under additional mass loading was summarized. Results were shown on the attenuation of vibrations of a bar with solid internal damping using masses or tuned dynamic absorbers. Optimum tuning and damping curves for the absorbers were included. The results were presented in the form of mechanical mobility or impedance. Some useful concepts related to normal mode or series solutions for mechanical impedance were discussed.

Dynamic absorbers applied to points other than the drive point on a bar that has solid damping were discussed. The following is a summary of this topic. If one absorber is to be added and the bar damping is small, the absorber can be equally effective at points other than the drive points if the velocity ratios are unity between the drive point and the attachment point. When internal damping is high, the absorber position becomes more important. The addition of a second absorber has an advantage if that absorber is tuned to a separate frequency. When both absorbers are tuned to the same frequency and located at points of equal velocity, the effect is about the same as increasing the size of the single absorber.

If a tuned absorber is used, tuning will depend on the spacing of the modes and on the average or characteristic mobility. Displacement amplitude is tuned differently from velocity amplitude or mobility.

Finally, if the need for adding a practical absorber arises, consideration should be given to adding a lumped mass connected rigidly, since results show that in the frequency range above the absorber frequency, the mass may be more beneficial in attenuating vibrations than the dynamic absorber.

GEAR NOISE

The basic principles of gear mechanics were reviewed in detail. The presentation then went into a discussion of the state of the art of gear noise reduction. Transmitted shaft power and the rigidity of the transmission structure were discussed as well as lubrication and gear precision. Work that is being conducted at the Ordnance Research Laboratory in this field was also discussed. It was pointed out that it is too early to report on any conclusive test results on gear noise at this time.

ACTIVE DAMPERS

This presentation consisted mostly of the use of the Nichols Chart techniques for the analysis of the variation in structural response resulting from arbitrary point loading. The second part of the presentation involved the use of active feedback elements to produce damping of multi-resonant structures. A system was examined analytically and compared with experimental results obtained for a 48-inch long uniform beam damped at its midpoint. The analysis method involved an adaption of the conventional Nichols Chart technique and appeared to be quite reliable. By using commercially available components, a simultaneous reduction in resonant mobilities of resonances below 10Kc was predicted and demonstrated. Reductions are proportional to the mobility values prior to active damping application and also to the gain in the feedback path. The limitations on the amount of damping possible result from stability conditions of the feedback loop, which in turn depends primarily on the transfer function of the forcing and pickup components of the loop. The stability analysis for particular components used in the beam experiment was presented.

RANDOM VIBRATIONS

An intensive mathematical analysis in random vibrations was presented. Fourier spectrum of pressure, solutions for the pulse response of a system, energy power spectrum, correlation function of the forcing function, normalized correlation function, and the cross-spectral density were derived.

SUMMARY

The material that was presented at the Vibrations and Vibration Damping Seminar held by the Ordnance Research Laboratory at the Pennsylvania State University is considered to be essential to an understanding of complex vibration phenomena.

Because the sessions were conducted by highly capable authorities, who covered the subject quite comprehensively, it is suggested that other USL personnel who are involved in vibration problems attend the seminar that is to be given next year. In preparation for the seminar, one should review the subject of Advanced Engineering Mathematics, Theory of Functions of a Complex Variable, and Theory of Vibrations.

Howard N. Phelps, Jr.
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Mechanical Engineer

LIST OF REFERENCES

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(B) C. M. Harris and C. E. Crede, "Shock and Vibration Handbook," McGraw-Hill Book Company, Inc., New York, New York, 1961.

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