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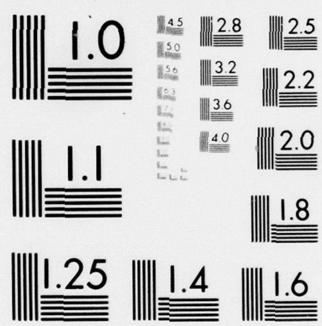

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By

N. F. Vorob'yev

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TIP EFFECT OF SUPERSONIC FLOW PAST DIHEDRAL ANGLE

N. F. Vorob'yev

Examined here in linear formulation is a supersonic flow of gas about a dihedral angle formed by intersecting wings. In the case where the leading edges are entirely supersonic, the flows in the interior angle γ and exterior angle $2\pi - \gamma$ can be studied individually. For this case we solve a model problem on the flow inside a dihedral angle at any value of γ [1]. For wings consisting of angles $\gamma = \pi/n$ ($n = 1, 2, 3, \ldots$), where the value of the potential inside the angle is written in explicit form, the solution to the problem of the tip effect reduces to solving the Abel equation [2]. In the case of a flow past a dihedral angle, where one of the angles - either exterior or interior - must be greater than π, we must find the solution in this angle with diffraction on the angle considered. In the present study we find the solution for a flow in dihedral angle $\gamma = [m/n]\pi$. On the basis of solving this model problem we solve the problem of the effect of tips on a dihedral angle in the case where the dihedral angle is $\gamma = \pi/n$. The problem is reduced to solving generalized Abel equations.

FLOW INSIDE DIHEDRAL ANGLE $\gamma = [\pi/n]$.

The flow of a supersonic stream past slightly curved intersecting wings is studied in linear formulation. The conditions on the wing surfaces are transferred to planes which are parallel to the oncoming flow. The dihedral angle is formed by the intersection of these planes. As our coordinate system we use a left-hand rectangular system. Axis ox takes the direction of the velocity of the oncoming flow. Plane xoz coincides with plane Σ_1, to which the boundary conditions of one of the wings have been transferred. Plane Σ_2, to which the boundary conditions of the second wing have been transferred, constitutes with plane xoz the angle g. Here the edge of the dihedral angle coincides with axis ox (Fig. 1). The potential of the disturbances satisfies the wave equation, which, by transformation of the coordinates, can be reduced to a form which is independent of the number M:

$$\varphi_{xx} - \varphi_{yy} - \varphi_{zz} = 0. \quad (1)$$

The solution to equation (1) in the Volterra form gives us the expression for the velocity potential in terms of the value of conormal derivatives and the potential on the wing surface. In linear formulation on the wing surface the conormal derivatives coincide with the normal and represent known functions. In the case of dihedral angles $\gamma = \pi/n$, terms containing unknown values of the velocity potential on the wing surface must be excluded from the Volterra formula. The velocity potential in this case is selected in quadratures through the normal components on the sides of the angle [1, 2].

Dihedral angle $\gamma = [m/n]\pi$ ca be broken down into $l \leqslant m$ dihedral angles, each of which equals $\gamma_k = \frac{\pi}{k}$, where k is a certain whole number $k \leqslant n$. The method is illustrated by the case where $l = 2$, $\gamma = \gamma_1 + \gamma_2$. In examining the tip effect later, the case $l = 3$ will be used. When $l = 2$ in the case of $\gamma > \pi$, where $\gamma = \pi + \pi/k$, we can assume k = 3 without limiting generality. The case k = 2 leads to a particular form of the generalized Abel equation and simplifies solution to the problem. As angle γ_1 we select angle $\Sigma_1 O \Sigma_3$, where the half-plane Σ_3 represents a continuation of half-plane Σ_1. The side of half-plane Σ_3, which is turned toward the edge of Σ_1, is designated as Σ_{3+}. The side of half-plane Σ_3, which is turned toward the edge of Σ_2, is designated as Σ_{3-}.

To reduce the problem of the flow past a dihedral angle $\gamma = [m/n]\pi$ to the problem of a flow past dihedral angle $\gamma = \pi/k$, which has its solution in quadratures, we must find the value of the normal derivative θ_3 in plane Σ_3. To find the value of θ_3, we write the expression for the velocity potential at point M_0, which lies on one side of plane Σ_3, in terms of the values of the normal derivatives on planes Σ_1 and Σ_{3+}. As values of normal derivatives on planes Σ_2 and Σ_{3-} we write the expression for the velocity potential at this point on the other side of plane Σ_3.

$$\varphi_{3+} = -\frac{1}{\pi}\left\{\iint_{\sigma_{30}} \frac{\theta_{3+}\,ds}{\sqrt{(x-\xi)^2 - r_{30}^2}} + \iint_{S_{10}} \frac{\alpha_1\,ds}{\sqrt{(x-\xi)^2 - r_{10}^2}}\right\},$$

$$\varphi_{3-} = -\frac{1}{\pi}\left\{\iint_{\sigma_{30}} \frac{\theta_{3-}\,ds}{\sqrt{(x-\xi)^2 - r_{30}^2}} + 2\iint_{\sigma_{31}} \frac{\theta_{3-}}{\sqrt{(x-\xi)^2 - r_{31}^2}} + \right.$$

$$\left. + 2\iint_{S_{20}} \frac{\alpha_2\,ds}{\sqrt{(x-\xi)^2 - r_{20}^2}} + \iint_{S_{22}} \frac{\alpha_2\,ds}{\sqrt{(x-\xi)^2 - r_{22}^2}}\right\}, \qquad (2)$$

where S_{ij}, σ_{ij} are the areas on plane Σ_i $(i=1, 2, 3)$ cut off by the characteristic cone, whose tip is at point M_j (the area on plane Σ_i is designated as S_i where the value of the normal derivative α_i is known and as σ_i where the value of the normal θ_i is unknown). Quantity r_{ij} represents the distances from point M_j to the points of integration located on plane Σ_i when $x = \xi$; M_1 - the point symmetrical to point M_0 relative to plane Σ_2; M_2 - the point symmetrical to point M_2 in relation to plane Σ_3.

From equation (2), considering the fact that on plane Σ_3 the velocity potential and normal derivatives are continuous functions $\varphi_{3+} = \varphi_{3-}$, $\theta_{3+} = -\theta_{3-}$, we get an integral equation for determining normal derivative $\theta_3 = \theta_{3+}$.

$$\iint_{\sigma_{30}} \frac{\theta_3\,ds}{\sqrt{(x-\xi)^2 - r_{30}^2}} - \iint_{\sigma_{31}} \frac{\theta_3\,ds}{\sqrt{(x-\xi)^2 - r_{31}^2}} + \frac{1}{2} \iint_{S_{10}} \frac{a_1\,ds}{\sqrt{(x-\xi)^2 - r_{10}^2}} - \iint_{S_{20}} \frac{a_2\,ds}{\sqrt{(x-\xi)^2 - r_{20}^2}} - \frac{1}{2} \iint_{S_{22}} \frac{a_2\,ds}{\sqrt{(x-\xi)^2 - r_{22}^2}} = 0. \quad (3)$$

The first term of equation (3) represents integral Abel operator $A(\theta_3)$; the second term - integral operator $B(\theta_3)$ of the Volterra type with integrable feature r^{-1}; the last three terms - integral operator $F(a_1, a_2)$ from known functions a_1, a_2. Equation (3) is written in the form of

$$A(\theta_3) + B(\theta_3) + F(a_1, a_2) = 0. \quad (4)$$

Equation (4) is a generalized Abel equation and can be solved by the method of successive approximations of [3].

THE EFFECT

The tip effect is examined for the case where the interior dihedral angle is $\gamma = \pi/n$ (n = 1, 2, 3 ...). Then, exterior angle $2\pi - \gamma = [2n - 1/n]\pi$ represents the angle $2\pi - \gamma = [m/n]\pi$, in which the description of the flow is reduced to solving generalized Abel equation (4). Without limiting generality we examine the case where interior angle $\gamma = \pi/3$. The case $n = 2$ leads to a particular form of the integral equation and simplifies solution to the problem. When $n = 3$ the exterior angle can be represented in the form of $2\pi - \gamma = \pi + \pi/3 + \pi/3$. This breakdown is shown in Fig. 2, which represents a section of the dihedral angle with plane $\xi = x$. Periphery Γ (section of cone of disturbances with tip at point of intersection of the leading edge of the wings Σ_1 and Σ_2) is the region where the mutual effect of both wings is realized.

Let the effect of the flows between the interior and exterior dihedral angles occur through the tip of the wing which lies in plane Σ_1. In the part of plane Σ_1, which lies outside of the wing itself (σ_1) the value of horizontal derivative θ_1 is an unknown quantity. The problem of the flow past the dihedral angle will be solved if the values of θ_l on planes Σ_4, Σ_3, Σ_1 (σ_1) are found. Then the potential at any point inside the region of disturbances of the dihedral angle can be represented in quadratures through normal derivatives in planes Σ_1, Σ_2, Σ_3, Σ_4. To find quantities θ_l, we write the expressions for the velocity potential at the same point on the positive and negative sides of planes Σ_4, Σ_3, σ_1. For the point which lies in plane σ_1, the value of the potential is written in angle $\Sigma_2 0 \Sigma_1$ (φ_{1+}) and in angle $\Sigma_1 0 \Sigma_4 (\varphi_{1-})$; for the point lying in plane Σ_4, - in angle $\Sigma_3 0 \Sigma_4 (\varphi_{4+})$ and in angle $\Sigma_4 0 \Sigma_1 (\varphi_{4-})$; for the point in plane Σ_3, - in angle $\Sigma_2 0 \Sigma_3 (\varphi_{3+})$ and in angle $\Sigma_4 0 \Sigma_3 (\varphi_{3-})$. From the condition of continuity of the velocity potential and normal derivatives in planes σ_2, Σ_4, Σ_3 we get the integral equations for determining quantities θ_l.

The value of θ_l depends on the position of the point in the leading cone of disturbances (Fig. 3). To determine the value of θ_{10} in zone σ_{10}, wherre the second edge has no influence, we solve the problem of the tip effect of an isolated wing. Here we get the integral Abel equation

$$A(\theta_{10}) + F(\alpha_{1+}, \alpha_{1-}) = 0. \tag{5}$$

To determine the value of θ_{41} in zone σ_{41}, where the wing tip of Σ_1, has no influence, we solve the diffraction problem on angle $2\pi - \gamma$ in the case where angle $[m/n]\pi$ is broken down into three angles γ_k $(l-3)$. Here we get a system of generalized Abel equations

$$\begin{aligned} A(\theta_{41}) + B_1(\theta_{41}) + C_1(\theta_{31}) + F_1(\alpha_{1-}) = 0, \\ A(\theta_{31}) + B_2(\theta_{31}) + C_2(\theta_{41}) + F_2(\alpha_{2-}) = 0. \end{aligned} \tag{6}$$

Quantity θ_{31} is the value of θ_3 in plane Σ_3, where the wing tip of Σ_1; has no effect; operators B_1, B_2, C_1, C_2 are integral operators of the

Volterra type with integrable feature r^{-1}.

To determine the value of θ_{11} in zone e_{11}, where we have the tip effect of wing Σ_1 and diffraction on angle $2\pi - \gamma$ without the wing tip effect, we again get an Abel integral equation

$$A(\theta_{11})+F(\alpha_{1-}, \alpha_{1+}, \alpha_{2+}, \theta_{10}, \theta_{41})=0. \quad (7)$$

The value of θ_{11} is determined through the known values of α_1 and α_2 and the values of θ_{10} and θ_{41} found on the preceding zones.

To determine the value of θ_{42} in zone e_{42}, where diffraction on the angle $2\pi - \gamma$ depends on the flow in the interior angle γ over the edge of the win Σ_1, which lies in zone e_{10}, we get the system of generalized Abel equations

$$\begin{aligned} A(\theta_{42})+B_1(\theta_{42})+C_1(\theta_{32})+F_1(\alpha_{1-}, \theta_{10}, \theta_{31}, \theta_{41})=0, \\ A(\theta_{32})+B_2(\theta_{32})+C_2(\theta_{42})+F_2(\alpha_{2-}, \theta_{31}, \theta_{41})=0, \end{aligned} \quad (8)$$

where θ_{32} is the value of θ_3 in the part of plane Σ_3, which corresponds to zone e_{42} of plane Σ_4. The values of θ_{42} and θ_{32} are determined from system (8) through the values of θ_{10}, θ_{31}, θ_{41}, determined on the preceding zones. Henceforth the values of the normal derivatives in subsequent zones through the values in the preceding are determined in plane e_1 from the Abel equation and in planes Σ_4, Σ_3 from a system of generalized Abel equations.

We are familiar with the generalization of the Abel equation [4]. The system of generalized Abel equations can be solved by the method of successive approximations in [3].

Note. In writing expression $\frac{\partial\varphi}{\partial N}\Big|_{\Sigma_3}$ in formula (3) of [1] an error was made. It should be

$$\frac{\partial\varphi}{\partial N}\Big|_{\Sigma_3} = -\frac{1}{\sqrt{1+k^2}}\left(\frac{\partial\varphi}{\partial y} - k\frac{\partial\varphi}{\partial z}\right)_{y=kz}.$$

Expressions $\frac{\partial\varphi}{\partial N}\Big|_{\Sigma_i}$ in formula (3) are written into formulas (19) and (20), which should not be done, since the values themselves $\frac{\partial\varphi}{\partial N}\Big|_{\Sigma_i}$ are assigned values (formula 2). We might get the impression that equations (19) and (20) represent integro-differential equations with unknowns $\varphi, \varphi_\xi, \varphi_\eta, \varphi_\zeta$, while in fact the unknowns are only φ and φ_ξ. It should also be considered that $ds|_{\Sigma_1} = d\xi\, d\zeta$, $ds|_{\Sigma_3} = \sqrt{1+k^2}\, d\xi\, d\zeta$.

Moreover, on page 8, in the first line after formula (18), Σ_j should be read in place of Σ_i.

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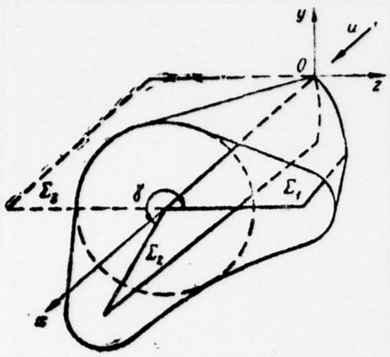

Fig. 1.

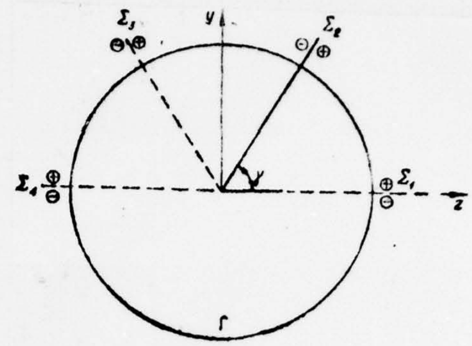

Fig. 2.

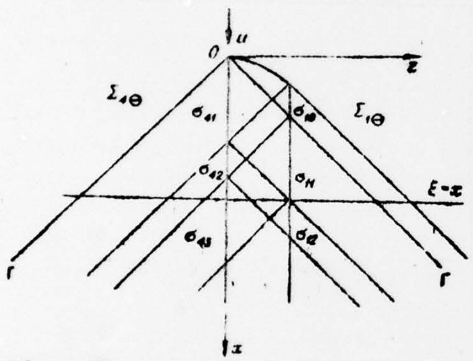

Fig. 3.

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О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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