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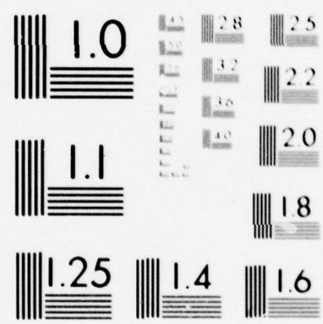
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A PROCEDURE FOR OBTAINING MORE ACCURATE TIMING INFORMATION FOR THE SOUND RANGING PROBLEM

FEBRUARY 1979

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Contract Monitor: BERNARD ENGEBOS

US Army Electronics Research and Development Command

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A model for the sound ranging problem is formulated and simplified. A procedure is then given for obtaining more accurate timing information from the received microphone signals. The procedure described consists of four basic parts: (1) a prefiltering operation, (2) a pair by pair correlation, (3) an elimination of inaccurate data, and (4) an overall least square time fit.

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CONTENTS

I. Introduction

The main problem of sound ranging is to determine the location of a sound source by examining the signals received at an array of microphones. For example consider the two dimensional problem with a sound source on the same plane as a linear array of six microphones as in Figure 1.

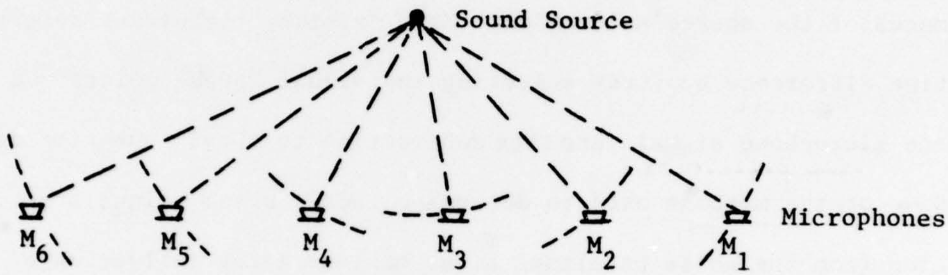

Figure 1. Linear Array Sound Ranging

The sound wave travels outward from the source having roughly a spherical wavefront. The time the wave takes to reach each microphone depends upon the temperature, atmosphere, wind profile and distance between the source and the microphone as well as the terrain if the array were positioned on the ground. In Figure 2 an example of the signals from the outputs of the configuration in Figure 1 for a transient source are given. These signals are typical of those received from a Howitzer or explosion excluding any ballistic waves.

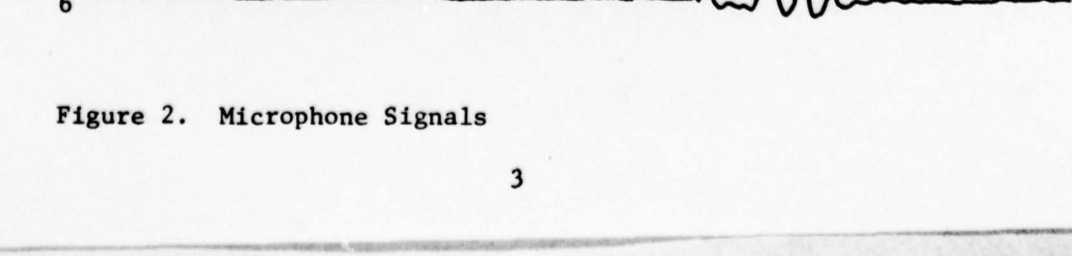

Figure 2. Microphone Signals

Most existing sound ranging solutions, for example Field Manual [1], Miller-Engebos [2], and Bangs-Schultheiss [3], use the time differences between the sound bursts and the existing temperature and wind information at the microphone sites as critical information. It is thought that better estimates of the time differences between bursts will provide better estimates of the source's location. Many existing techniques determine the time difference by first selecting individual "break points" on each of the microphone signals and then subtracting to obtain the time differences. Some of the methods used to determine these break points are: first inflection from the noise baseline, first maximum after inflection, or first zero cross over after inflection. In contrast to these methods the general approach presented in this paper is one of estimation of time differences between signals by using the normalized cross correlation functions between filtered versions of the signals. In this way the entire structure of the signals is used rather than a sometimes vague "break-point" and error caused by noise can be minimized. After these time differences have been obtained the location of the sound source can be determined by any of the existing algorithms, some of which are compared in Engebos and Miller [4].

II. Mathematical Model

In order to estimate the time differences a mathematical frame work must be established.

The general model to be used in this paper is shown in Figure 3. The sound source is assumed to generate a compression wave that is defined as $p(t)$. As the wave travels through the atmosphere or along or through the ground it is changed as if it has passed through a filter of some kind. Although in general this filtering action is time varying and distributive we will assume a linear time invariant filter with frequency response $A_i(\omega)$ followed by a time delay t_i. The subscript indicates filtering of the wave before it reaches the i^{th} microphone. Each microphone itself has a frequency response $M_i(\omega)$. The microphones when positioned sufficiently far apart each receive independent background noise, wind noise, and other extraneous noise.

It is assumed that these noise sources can be collectively modeled as wide sense stationary independent random processes $\eta_i(t)$ with power spectral densities $\Phi_{ii}(\omega)$. The basic problem then is to estimate the times t_i after receiving the signals $x_i(t)$: $i=1,2,\ldots,6$ over the time interval from t_r to t_f.

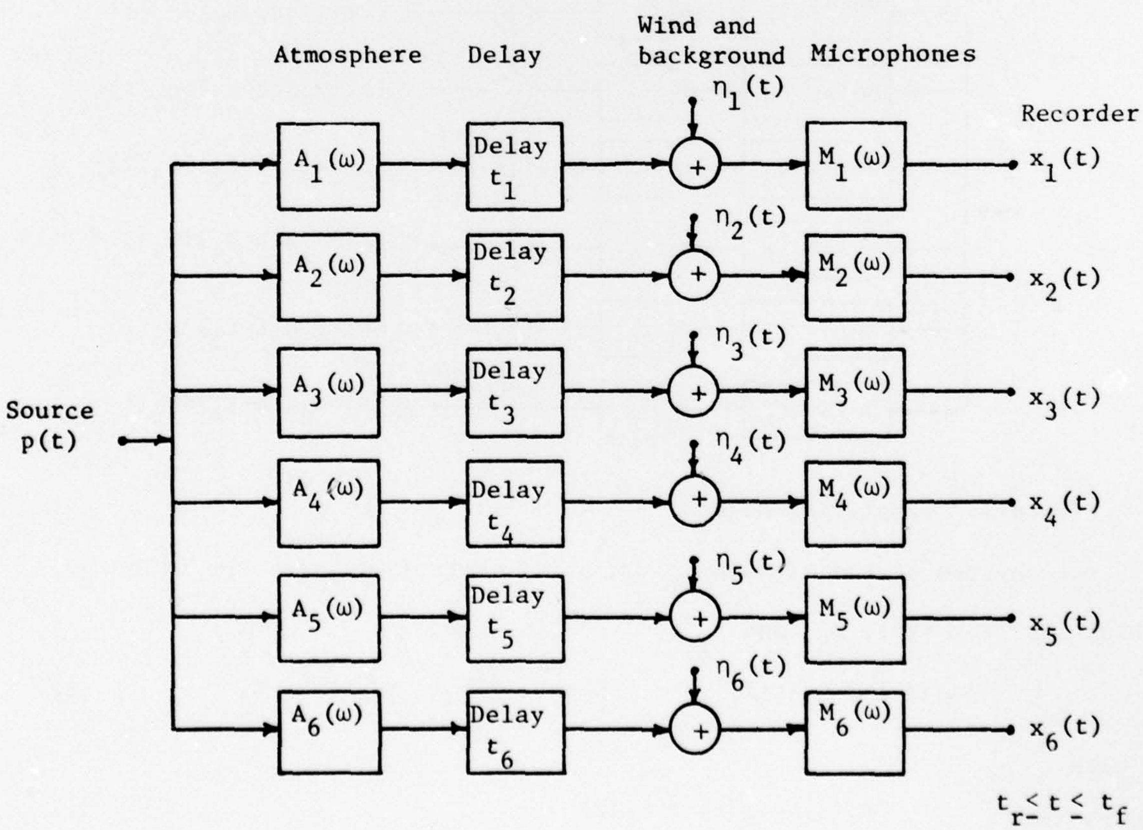

Figure 3. General Model

The general model can be simplified to that shown in Figure 4 if we assume:

(1) $A_i(\omega) = \alpha_i A(\omega)$, that is we have identical filtering in all paths but a different attenuation α_i due to the distance traveled from the source to each microphone.

(2) $M_i(\omega) = M(\omega)$, that is all microphones have the same frequency response.

(3) $n_i(t) \triangleq \eta_i(t) * m(t)$ represents the background noise $\eta_i(t)$ reflected through the microphone where m(t) is the impulsive response of the microphones and * means convolution in the time domain.

(4) $q(t) \triangleq p(t) * a(t) * m(t)$ represents the undelayed signal version including the filtering action of the atmosphere and microphone. The a(t) is the impulsive response of this filtering action, i.e., $a(t) = \mathcal{F}^{-1}[A(\omega)]$.

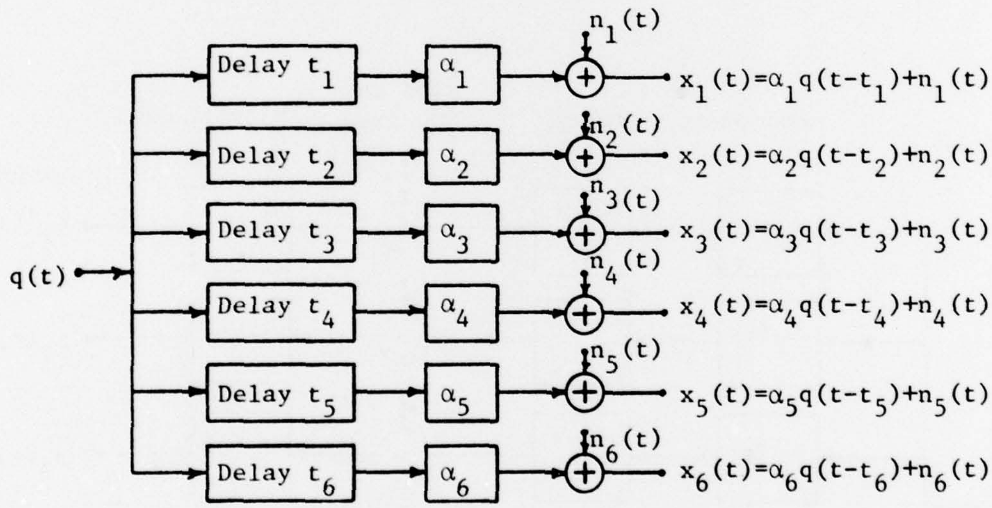

Figure 4. Simplified Model

For any two of the signals $x_i(t)$, $x_j(t)$ where $i \neq j$, and the following definitions for s(t), B_{ij} and τ_{ij}

$$s(t) \triangleq \alpha_i q\,(t-t_i), \qquad \beta_{ij} \triangleq \frac{\alpha_i}{\alpha_j}, \qquad \tau_{ij} \triangleq t_i - t_j \tag{1}$$

we have

$$x_i(t) = s(t) + n_i(t) \tag{2}$$

$$x_j(t) = \beta_{ij}\; s(t-\tau_{ij}) + n_j(t)$$

The problem now becomes the estimation of τ_{ij} from the received signals $x_i(t)$ and $x_j(t)$. This simplified model will be used as the basic model for the paper. Hannan and Thompson [5] have developed a maximum likelihood estimator for τ_{ij} and Knapp and Carter [6] present several other estimator forms for various performance characteristics. The basic structure of these estimators has been shown to be a combination of prefiltering and cross-correlation as given in Figure 5. The time argument $\hat{\tau}_{ij}$ at which the correlator reaches a maximum γ_{ij} is the delay constant. The $\hat{\tau}_{ij}$ and γ_{ij} for all i and j thus determined can then be used in any of the existing algorithms to estimate the location of the sound source.

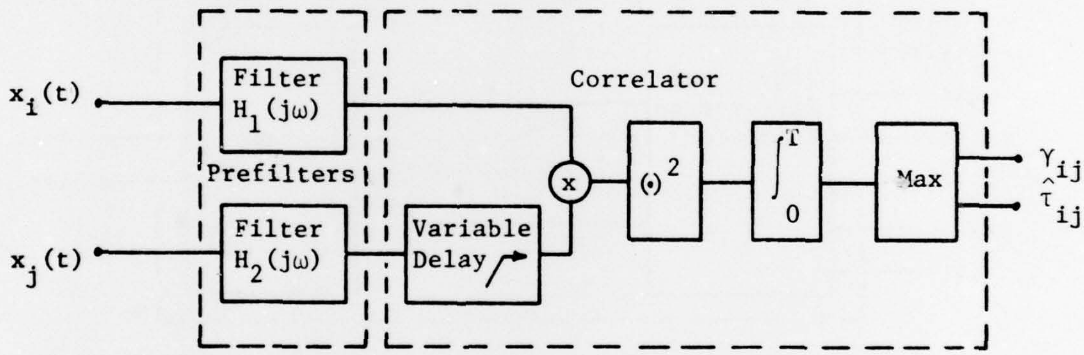

Figure 5. Structure of Estimator for a Pair of Signals

The prefilters are determined by the type of estimator used and assumptions on the noise source and sound source; examples are given by Carter and Knapp [6]. In the usual case the actual frequency content of the signals may be unknown as well as the constants α_i and the noise power spectral densities. Forms of filters $H_1(j\omega)$ and $H_2(j\omega)$ might then be formed using on line estimates of these parameters. Estimates of the noise spectral densities of each signal can be easily obtained prior to the signal epoch if enough lead time is given by using an FFT algorithm. This noise energy level together with the signal pulse noise energy could be used to estimate α and a filter $H_1(j\omega)$ and $H_2(j\omega)$ formed from these estimates.

III. Proposed Method

From the six microphone signals the location of a sound source is desired. The method proposed for determining the sound source location is a digital processor composed of two basic parts. The purpose of the first is to estimate the appropriate time differences between the arrival of sound source signals, while the purpose of the second is to use these estimates to obtain an estimate of the source location. The first section is the main subject of this report while details of the implementation of the second on a desk computer (HP 9825) are given by Miller and Engebos [7].

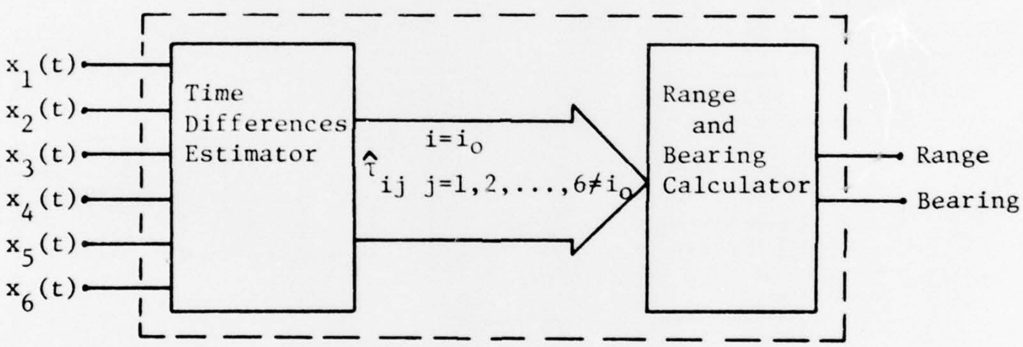

Figure 6. Digital Processor Structure

The time differences estimator used is a combination of optimal estimator structures (Figure 5) for all possible pairs of signals followed by selection of one of the signals as a reference (call it $i_0 \,\varepsilon[1,2,\ldots,6]$. The overall structure of the time differences estimator is given in Figure 7 and the details are presented in the following section.

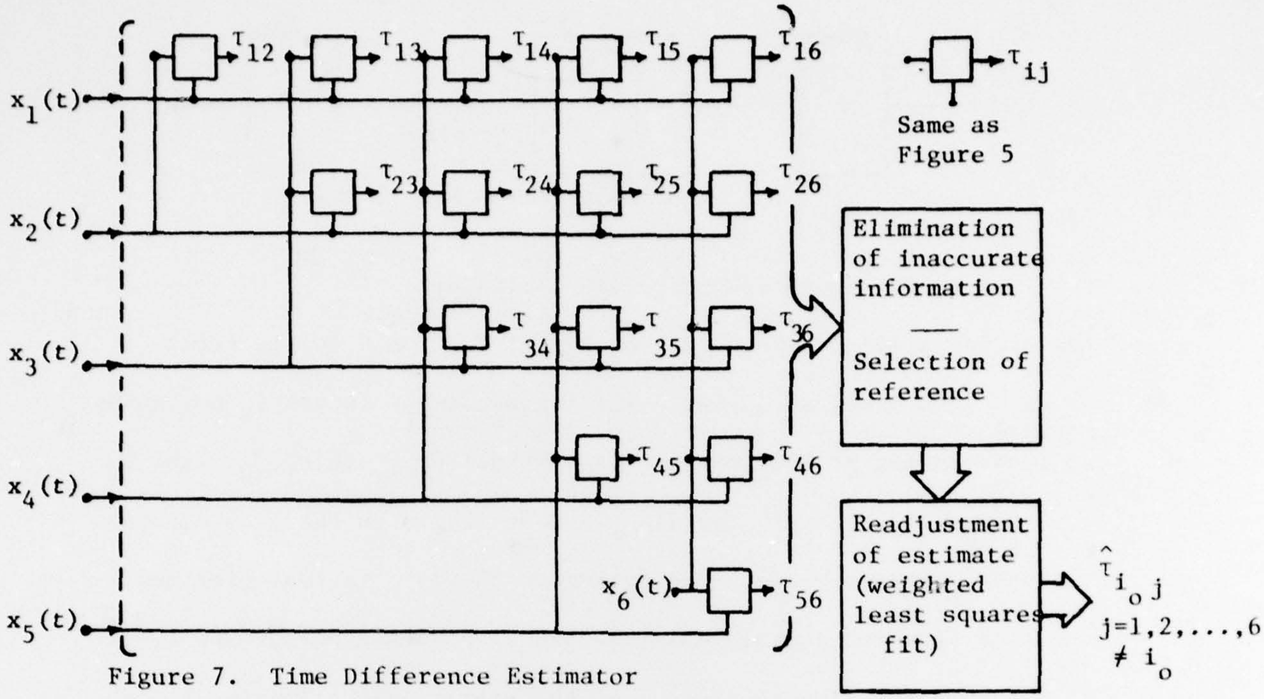

Figure 7. Time Difference Estimator

IV. Time Difference Estimator

The time difference estimator can be broken into three subsections :(A) Estimation of rough time differences and measure of correlation for all pairs of signals (B) Elimination of unreliable time difference information (C) Re-adjustment of time difference estimates for overall best fit. The details of each of these subsections follow.

A. Rough Estimation of Time Differences

Each of the received signals $x_i(t)$ has been received over the time interval t_r to t_f. In many cases the signal duration that is being searched for is a small percentage of this total interval. The structure shown in Figure 5 could be used throughout the interval from t_r to t_f but the actual correlation done, either direct or with the Fast Fourier Transform, may take an exorbitant amount of time. Since the signal to be located is a transient type it is more efficient to determine rough starting times for the source signals by using an energy detector and then use the cross correlation as a vernier. The overall block diagram for the estimation of rough arrival times t_i is given in Figure 8.

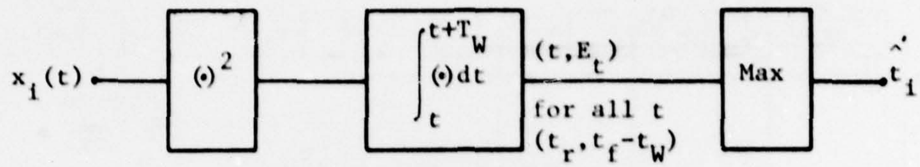

Figure 8. Rough Starting Time Estimator

The signal $x_i(t)$ or the output of Filter F_1 from $x_i(t)$ is first squared then integrated over a sliding window of length T_W beginning at t and ending at $t+T_W$ where t is varied from t_r to t_f-T_W. The T_W is selected to be approximately the same length as the s(t) expected. The energy detector can be operating on the data in real time and the time at which a maximum is reached gives a rough estimate $\hat{t}_i$ of the relative time of arrival of the signal we are looking for at each of the microphones.

Operating on all of the six signals in this fashion produces a set of rough arrival times $\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_6$. In this way we can narrow the cross correlation operation window and reduce the calculation time for estimating the rough time differences. The normalized cross correlation for a pair of signals $x_i(t)$ and $x_j(t)$ is then given by

$$\gamma_{ij}(\tau) = \int_{\hat{t}_i - T_W/4}^{\hat{t}_i + 5T_W/4} x_i(t)\, x_j(t+\tau)dt/\sqrt{E_i E_j} \tag{3}$$

$$\text{where } E_i = \int_{\hat{t}_i - T_W/2}^{\hat{t}_i + 3T_W/2} x_i^2(t)dt$$

The total time of the integration is selected as $3/2\ T_W$. This makes sure the entire signal will be within the cross correlation window provided the time of arrival is roughly guessed to within one quarter of its duration in either direction.

If $\hat{t}_j<\hat{t}_i$ then τ is varied from $\hat{t}_i-\hat{t}_j - T_W/2$ to $\hat{t}_i-\hat{t}_j + T_W/2$ while if $\hat{t}_j>\hat{t}_i$, τ is varied from $\hat{t}_j-\hat{t}_i - T_W/2$ to $\hat{t}_j-\hat{t}_i + T_W/2$. The value $\hat{\tau}_{ij}$ for which $\gamma_{ij}(\tau)$ reaches its maximum γ_{ij} is a rough estimate of the time difference between the signals $x_i(t)$ and $x_j(t)$, while the γ_{ij} gives a measure of the correlation between the signals. The $\gamma_{ij}(\tau)$ can be computed directly by time integration for various values of τ or by the use of the FFT.

B. Elimination of Unrealiable Time Difference Information

In some cases there may be very weak correlation between the signals $x_i(t)$ and $x_j(t)$. This weak correlation could be caused by signal s(t) being very small compared to the additive noise or by the fact that s(t) has been severely distorted in transmission shedding doubt on the simplified model of Figure 4. In either case the rough time difference estimate is inaccurate, sometimes to the point of being totally unreliable. If these unreliable time differences were used in the final refinement or readjustment explained in part C, severe errors in the refinement may result and consequently be the cause of large errors in the desired range and bearing estimates.

If γ_{ij} is small compared to one it means that the signals $x_i(t)$ and $x_j(t)$ are weakly linearly related and the time difference $\hat{\tau}_{ij}$ is unreliable while if γ_{ij} is approximately one, the signals are strongly linearly related and the time difference estimate is reliable. Define a threshold T_0 in such a way that $\gamma_{ij} < T_0$ would imply the time difference estimate is unreliable and $\gamma_{ij} \geq T_0$ implies a reliable estimate. In this way a signal $x_i(t)$, i=1,2,..,6 for which $\gamma_{ij} < T_0$ for all j=1,2,...6 would be eliminated from consideration. If $\hat{\gamma}_{ij} < T_0$ for just some j's, but not all of them, those τ_{ij}'s should be eliminated from consideration but not the entire signal.

C. Readjustment of Time Difference Estimates

After the unrealiable time differences have been eliminated a best overall fit may be obtained by a least square procedure. The following presentation is for when none are eliminated. If elimination occurs, a renumbering must take place. By selecting one of the signals, say $x_1(t)$ as a reference (if another is chosen simply renumber) the time difference of signal $x_i(t)$ relative to $x_1(t)$ for $i = 2,3,\ldots,6$ can be calculated as (see appendix for derivation)

$$\begin{aligned}
\hat{t}_2 &= \frac{1}{6}\,[T + \tau_{12} - \tau_{23} - \tau_{24} - \tau_{25} - \tau_{26}] \\
\hat{t}_3 &= \frac{1}{6}\,[T + \tau_{13} + \tau_{23} - \tau_{34} - \tau_{35} - \tau_{36}] \\
\hat{t}_4 &= \frac{1}{6}\,[T + \tau_{14} + \tau_{24} + \tau_{34} - \tau_{45} - \tau_{46}] \qquad (5) \\
\hat{t}_5 &= \frac{1}{6}\,[T + \tau_{15} + \tau_{25} + \tau_{35} + \tau_{45} - \tau_{56}] \\
\hat{t}_6 &= \frac{1}{6}\,[T + \tau_{16} + \tau_{26} + \tau_{36} + \tau_{46} + \tau_{56}]
\end{aligned}$$

where

$$T = \tau_{12} + \tau_{13} + \tau_{14} + \tau_{15} + \tau_{16} \qquad (6)$$

The reference signal $x_R(t)$ chosen should be the one for which the sum of the γ_{Rj} for all $j \neq R$ is a maximum. If some of the data is unreliable we may proceed as follows. Suppose γ_{Ij} is less than T_0 thus giving rise to an unreliable $\hat{\tau}_{Ij}$. If we go ahead and use this unreliable estimation of the above formulas we adversely affect our estimates. Nonetheless we must still substitute something into equations for τ_{Ij}. If we can find a k such that γ_{Ik} and $\gamma_{jk} > T_0$ then replace τ_{Ij} in formulas (6) by

$$\tau_{Ij} = \tau_{Ik} + \tau_{jk}. \qquad (7)$$

If no such k is available then a least squares method can be used only with the times available yielding formulas different from equations (5).

V. Conclusions

A model for the sound ranging problem has been formulated and simplified. A procedure has been presented for obtaining more accurate timing information from the received microphone signals. The procedure described consisted of four basic parts: (1) a pre filtering operation, (2) a pair by pair correlation, (3) an elimination of inaccurate data, and (4) an overall least square time fit.

The overall procedure appears to be feasible for operation but not in real time. Even not operating in real time, the procedure could be implemented at least an order of magnitude faster than the present technique. Also, operation in an interactive mode would greatly minimize the possibility of presenting erroneous timing information. The prefiltering operation, elimination of inaccurate data and least square time fit parts can be easily implemented almost in real time. The most time consuming operation is the pair by pair correlation; for example, with six recorded signals we must evaluate fifteen different cross correlations. These correlations would probably be best done by using the product of Fourier Transforms and the inverse transform. Existing software and hardware could be used in a parallel structure to accomplish these cross correlations efficiently, thus making the overall procedure feasible in a practical sense.

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APPENDIX A: Refinement of time differences estimates

The problem discussed in this appendix is that of trying to obtain the best possible time difference estimates for a collection of time signals $\{s_1(t), s_2(t), \ldots, s_N(t)\}$. Using $s_1(t)$ as a reference we wish to select times $t_2, t_3, \ldots, t_N$ such that the signals $\{s_1(t), s_2(t-t_2), \ldots, s_N(t-t_N)\}$ provide the best collective correlation where best will be defined in terms of minimizing a performance measure e.

The general approach to this problem will be to find the τ_{ij}'s that maximize the cross correlation function for all pairs of signals $s_i(t)$ and $s_j(t)$ and use these to determine the t_i's. The cross correlation function $R_{ij}(\tau)$ for signals $s_i(t)$ and $s_j(t)$ is given by

$$R_{ij}(\tau) = \int_{-\infty}^{\infty} s_i(t)s_j(t-\tau)dt. \tag{A-1}$$

Let τ_{ij} be the value of τ for which $R_{ij}(\tau)$ is a maximum. This specifies a set of time values $\{\tau_{12}, \tau_{13}, \ldots, \tau_{1N}, \tau_{23}, \tau_{24}, \ldots, \tau_{2N}, \ldots, \tau_{N-1,N}\}$

Now consider the set of such time values τ'_{ij} for the following set of signals

$$\begin{aligned} s'_1(t) &= s_1(t) \\ s'_i(t) &= s_i(t-t_i) \qquad i = 2, \ldots, N \end{aligned} \tag{A-2}$$

The performance measure e that is desired to be minimized is defined by

$$\begin{aligned} e = (\tau'_{12})^2 &+ (\tau'_{13})^2 + \ldots + (\tau'_{1N})^2 \\ &+ (\tau'_{23})^2 + \ldots + (\tau'_{2N})^2 \\ &\qquad\vdots \\ &\qquad + (\tau'_{N-1,N})^2 \end{aligned} \tag{A-3}$$

where the τ'_{ij}'s are the values of τ that maximize $R^{(1)}_{ij}(\tau)$ given by

$$R^{(1)}_{ij}(\tau) = \int_{-\infty}^{\infty} s'_i(t)s'_j(t-\tau)dt. \tag{A-4}$$

The τ'_{ij}'s that maximize $R^{(1)}_{ij}(\tau)$ given in A-4 can be found in terms of the t'_{ij}'s and τ_{ij}'s by rewriting (A-4) in terms of (A-1). From (A-2) we can write (A-4) evaluated at τ'_{ij} as

$$R^{(1)}_{ij}(\tau'_{ij}) = \int_{-\infty}^{\infty} s_i(t-t_i)s(t-t_j-\tau'_{ij})dt \tag{A-5}$$

By letting $t - t_i = \alpha$ in (A-5) and using (A-1) we have

$$R^{(1)}_{ij}(\tau'_{ij}) = \int_{-\infty}^{\infty} s_i(\alpha)s(\alpha-(\tau'_{ij}+t_j-t_i))d\alpha \tag{A-6}$$

$$= R_{ij}(\tau'_{ij} + t_j-t_i)$$

Since τ_{ij} maximizes $R_{ij}(\tau)$ we easily see that

$$\tau_{ij} = \tau'_{ij} + t_j-t_i \quad \text{or} \quad \tau'_{ij} = \tau_{ij} + (t_i-t_j) \tag{A-7}$$

Substituting these values into (A-3) yields the following

$$\begin{aligned} e = {} & (\tau_{12}-t_2)^2 + (\tau_{13}-t_3)^2 + \dots + (\tau_{1N}-t_N)^2 \\ & + (\tau_{23}-(t_2-t_3))^2 + \dots + (\tau_{2N}-(t_2-t_N))^2 \\ & \vdots \\ & + (\tau_{N-1,N}+(t_{N-1}-t_N))^2 \end{aligned} \tag{A-8}$$

A necessary and in this case sufficient condition, since e is convex, is that

$$\frac{\partial e}{\partial t_i} = 0 \quad i = 2,3,\ldots,N \tag{A-9}$$

Taking the partial of e with respect to the t_i and simplifying gives

$$\begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_N \end{bmatrix} = \begin{bmatrix} (N-1) & -1 & -1 & \ldots & -1 \\ -1 & (N-1) & -1 & \ldots & -1 \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ -1 & -1 & \ldots & -1 & (N-1) \end{bmatrix} \begin{bmatrix} t_2 \\ t_3 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{bmatrix} = A_{N-1} \cdot \begin{bmatrix} t_2 \\ t_3 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{bmatrix} \tag{A-10}$$

where

$$\alpha_2 = [\tau_{12} - \tau_{23} - \tau_{24} - \ldots - \tau_{2N}]$$

$$\alpha_3 = [\tau_{13} + \tau_{23} - \tau_{34} - \tau_{35} - \ldots - \tau_{3N}]$$

$$\vdots$$

$$\alpha_N = [\tau_{1N} + \tau_{2N} + \tau_{N-1,N}]$$

To solve for the t_i's of (A-10) we simply need to find the inverse of the coefficient matrix A_{N-1}. It can be easily shown that

$$A_{N-1}^{-1} = \frac{1}{N} \begin{bmatrix} 2 & 1 & 1 & . & . & . & 1 \\ 1 & 2 & 1 & 1 & . & . & . \; 1 \\ & & & \cdot & & & \\ & & & \cdot & & & \\ & & & \cdot & & & \\ 1 & 1 & . & . & . & 1 & 2 \end{bmatrix} \tag{A-11}$$

thus giving the t_i vector that minimizes e as

$$\begin{bmatrix} t_2 \\ t_3 \\ \cdot \\ \cdot \\ \cdot \\ t_N \end{bmatrix} = A_{N-1}^{-1} \begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_N \end{bmatrix} \tag{A-12}$$

For the six microphone case the t_i's can easily be found as.

$$
\begin{aligned}
t_2 &= \frac{1}{6}\left[T + \tau_{12} - \tau_{23} - \tau_{24} - \tau_{25} - \tau_{26}\right] \\
t_3 &= \frac{1}{6}\left[T + \tau_{13} + \tau_{23} - \tau_{34} - \tau_{35} - \tau_{36}\right] \\
t_4 &= \frac{1}{6}\left[T + \tau_{14} + \tau_{24} + \tau_{34} - \tau_{45} - \tau_{46}\right] \\
t_5 &= \frac{1}{6}\left[T + \tau_{15} + \tau_{25} + \tau_{35} + \tau_{45} - \tau_{56}\right] \\
t_6 &= \frac{1}{6}\left[T + \tau_{16} + \tau_{26} + \tau_{36} + \tau_{46} + \tau_{56}\right]
\end{aligned}
\qquad \text{(A-13)}
$$

where

$$T = \tau_{12} + \tau_{13} + \tau_{14} + \tau_{15} + \tau_{16}$$

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