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THEORETICAL PREDICTION OF BUBBLE PATHS,(U)

JUN 66 J LURYE, R MORGAN NOBSR-93023

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(6) # THEORETICAL PREDICTION OF BUBBLE PATHS,

by

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THEORETICAL PREDICTION OF BUBBLE PATHS

Air bubbles in the water can degrade the performance of a ship-borne sonar by generating noise or by diffracting or absorbing acoustical signals. A program of full-scale experimental work is under way to discover which portions of a ship's hull are likely to encounter bubbles and to what extent. It is in support of this experimental program that we have begun to study theoretical methods of predicting bubble paths, and in particular, of predicting the deviation between bubble paths and the paths of the fluid particles.

A reasonably precise model of the bubble's motion leads to an extremely difficult boundary value problem, viz., the solution of the time-dependent Navier-Stokes equations in the presence of a moving and deformable surface (the bubble surface) whose shape is time-dependent and not known in advance and whose interior is filled with a compressible fluid - in this case, air. The stress on this surface has both tangential and normal components due to the fluids on either side, as well as an effective normal component arising from surface tension.

Fortunately, certain facts lead to the hope that a simplified analysis would predict the bubble paths with sufficient accuracy, at least in certain ranges of speeds and bubble sizes. These are:

1) Experiments by Rosenberg [1] at DTMB indicate that for Reynolds numbers less than 70 (based on bubble diameter) the bubbles behave like rigid spheres. The velocity entering into this Reynolds number is the bubble velocity relative to the fluid.

2) The bubbles of interest tend to have diameters in the millimeter range and are usually small compared to the distance over which the underlying flow - this is the flow about the ship in the absence of the bubbles - departs sensibly from uniform flow. Accordingly, one is led to try for a perturbation solution based on the smallness of the bubble diameter.

Guided by these considerations, our first effort has been to derive the equations of motion of a small rigid sphere moving through a steady but arbitrary non-uniform underlying flow of an incompressible inviscid fluid. The neglect of viscosity in this first attempt was justified on the grounds that its inclusion would tend to reduce the discrepancy between the paths of the bubbles and those of the fluid particles, so that without viscosity, we would obtain an upper bound on this discrepancy.

The problem was attacked by expanding the force on the bubble in powers of the bubble radius, a, and retaining in the equations of motion only the leading term in the expansion, which turned out to be of order a^3. This leading term is itself composed of four terms, each having a readily understood meaning. These are:

1) The ordinary inertial reaction term, equal to minus the mass of the bubble times its acceleration.

2) The ordinary added-mass term, equal to minus one half the mass of the fluid displaced by the bubble (for spherical bubbles) times the acceleration of the bubble.

3) A "dynamic buoyancy" term, equal to minus $\frac{3}{2}$ the volume of the bubble times the dynamic pressure gradient associated with the underlying flow. This underlying pressure gradient is calculated

with the bubble absent, but is evaluated at the point instantaneously occupied by the center of the bubble.

The coefficient $\frac{3}{2}$ in this term arises from the fact that since the underlying pressure field is dynamically determined, it is altered by the presence of the bubble. We have shown that the effect of this alteration on the force on a sphere is, to order a^3, correctly taken account of by the factor $\frac{3}{2}$. It should be remarked, however, that this observation has also been made by G.I. Taylor[2].

4) The final term in our equation is the ordinary buoyant force on the bubble.

We then have for the equation of motion of a small rigid sphere of radius a moving in the underlying steady flow of an incompressible inviscid fluid:

$$1) \quad (M + \frac{1}{2}\rho V)\frac{d\bar{U}}{dt} = -\frac{3}{2} V \operatorname{grad} p + \rho g V \hat{z}$$

where M is the mass of the sphere, ρ the fluid density, $\bar{U}$ the velocity of the center of the sphere, V the volume of the sphere, p the underlying dynamic pressure, g the acceleration of gravity, and $\hat{z}$ a unit vector in the vertical direction (positive upwards).

Although the individual terms in this equation have simple interpretations, the rigorous proof that this equation is correct to order a^3 is quite elaborate. The equation was derived in two ways. In the first derivation, which was the more straightforward but also the more laborious of the two, the time-dependent form of Bernoulli's equation was used to calculate the fluid pressure, which was then integrated over the surface of the sphere to give the desired force. The fluid velocity and velocity-potential entering into the Bernoulli equation were expanded in powers of the radius, a, as far as necessary to yield the lowest-order term for the force on the sphere. In the second derivation, we combined the aforementioned "dynamic buoyancy" result with a transformation to a coordinate system moving with the sphere to obtain the lowest-order force term more directly.

As yet, we have not attempted to solve Equation (1). This would be done numerically in most cases.

We are planning to continue work along the foregoing lines, but with viscosity taken into account. Several approaches suggest themselves.

1) We might simply add a viscuous drag term to Equation (1). Note that the bubble velocity entering into this term would be not the absolute velocity, $\bar{U}$, of Equation (1) but the velocity relative to the local fluid. A procedure of this kind has been followed by G.J. Franz.[3] However, our equation would differ from his in at least one term since he has -V grad p where we would have $-\frac{3}{2}$ V grad p. [See Equation (1).]

2) A more powerful attack would be to go back to the Navier-Stokes equations and attempt to deduce from them a rigorous perturbation equation of motion valid for small spheres, analogous to Equation (1) of the inviscid case. The word "small" must still be understood to mean small compared with the distance over which the underlying flow departs sensibly from uniformity. Spheres so small that the Reynolds number falls within the Stokes or creeping flow range would not be of great interest in our case. Some clues as to how to proceed may be available in papers such as that of Saffman[4].

Notice that the assumption of inviscid flow past a spherical bubble is not merely unrealistic, (because a sphere is a blunt body) but is in direct contradiction to the assumption of constant pressure inside the sphere.

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