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A TECHNIQUE FOR A MULTI-DOPPLER, QUADRATURE CORRELATION PROCESS--ETC(U)
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TRACOR Project No. 002-134
Document No. SD/69-008-U

TECHNICAL NOTE

A TECHNIQUE FOR A MULTI-DOPPLER, QUADRATURE CORRELATION PROCESSING SCHEME

by

Dr. D. B. Doan

Submitted to
Naval Undersea Research and Development Center
San Diego Division
San Diego, California 92132
Attn: Mr. Louis Strauss
Code 603

5 August 1969

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Contract No. N66001-69-C-1206
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MULTI-DOPPLER QUADRATURE CORRELATOR

The location of echoes resulting from transmitted waveforms exhibiting simultaneous Doppler and range resolution requires that a two-dimension search be undertaken. Techniques for range searching using correlation methods are well known. Doppler search using multiple references has been used, but this method is quite inefficient when simulated on a digital computer.

The method presented here makes use of the observation that if the Doppler shift is relatively small, it can be treated, approximately, as a frequency shift. It is a simulation of a method sometimes used in analog correlators in which the multiplier is followed by a bank of band-pass filters instead of a simple integrator. The difference in center frequency of the various filters produces a result which is identical to that produced by offsetting the frequency of the reference by the same amount. Thus, with a single reference, one can obtain several Doppler channels. The limit, which depends on the time-bandwidth product, βT, is the approximation of Doppler time compression by a simple frequency shift.

The simulation of this type of correlator is made practical by using the Fast Fourier Transform technique to simulate the comb filter bank. The method would not be practical were it not for the exceptional speed of the FFT algorithm. A further increase in efficiency is obtained by partially integrating the output of the correlation multiplier before it is introduced to the FFT filter. This has the effect of placing a low pass filter between the multiplier and the filter bank. As long as the cut-off point of the low pass filter is higher than the maximum Doppler offset, this has no effect on the output. It does permit lowering the sampling rate at the filter input, and thus speeds up the simulation. A block diagram is shown in Figure 1.

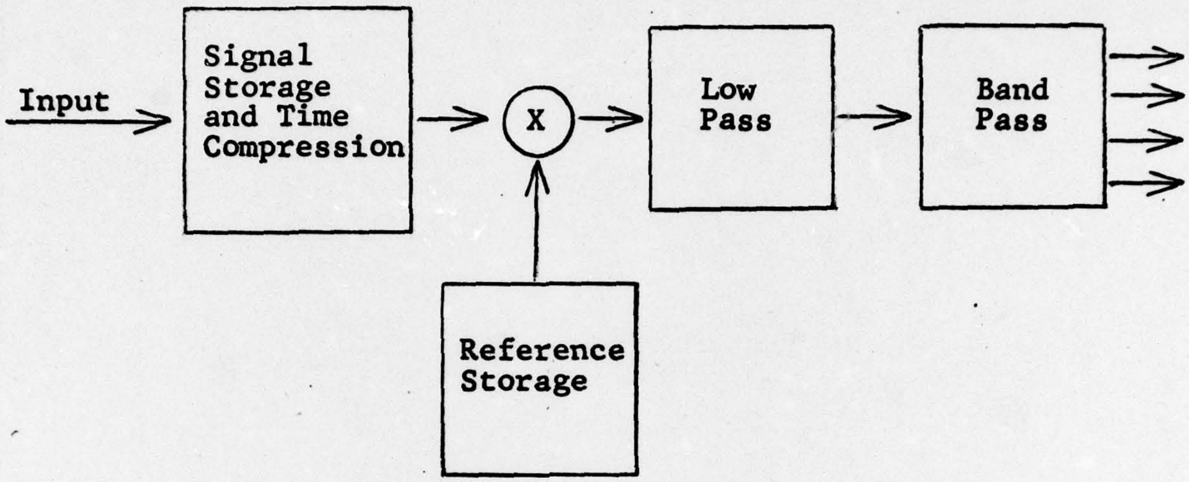

Figure 1

In order to make the most effective use of the FFT technique, the correlation is performed using a quadrature reference. This removes the ambiguity between positive and negative frequencies.

The mathematical development of the technique is as follows: let the received signal be x(t), the replica of the transmitted signal be y(t), and let q(t) be identical to y(t), but phase shifted by 90^o. The quadrature representation of y(t) is the complex function

$$r(t) = y(t) + jq(t). \qquad (1)$$

The structure of r(t) assures that only positive frequencies are present. Thus, the frequency shifted references can be obtained by simple multiplication and expressed as $r'(t) = r(t)e^{-i\omega t}$. Note that ω may be either positive or negative and that $r'(t)$ will contain only sum frequencies.

The correlation function is then

$$\rho(\tau,\omega) = \left| \int_0^{T_x} x(t+\tau)r(t)e^{-j\omega t}dt \right| . \qquad (2)$$

The use of the quadrature reference and the absolute value operation removes the carrier term from $\rho(\tau)$. The result is the same as that obtained by passing the output of a conventional correlator through an ideal envelope detector.

Equation (2) is the basic equation for a quadrature reference correlator with ω being a frequency shift parameter to be used for Doppler search. The integral is to be evaluated for each value of τ and ω for which it is desired to test for the presence of an echo. It is an approximation only in the sense that the frequency shift is not exactly equivalent to a Doppler (time compression) shift. The limits placed on ω are examined later in this report.

As it stands, Equation (2) is very time consumming to evaluate on a general purpose digital computer. In order to improve the computational efficiency, Equation (2) is rewritten as follows:

$$\rho(\tau,\omega) = \left| \sum_{n=0}^{N-1} \int_{\frac{Tn}{N}}^{\frac{T(n+1)}{N}} x(t+\tau)r(t)e^{-j\omega t}dt \right| . \qquad (3)$$

If $\frac{T\omega}{N}$ is sufficiently small, the exponential term can be removed from the integral and replaced by its average value. Thus, we have approximately,

$$\rho(\tau,\omega) \approx \left| \sum_{n=0}^{N-1} \left\{ \int_{\frac{Tn}{N}}^{\frac{T(n+1)}{N}} x(t+\tau)r(t)dt \right\} e^{-j\left[\frac{\omega T(n+\frac{1}{2})}{N}\right]} \right| . \qquad (4)$$

If we let

$$I_n(\tau) = \int_{\frac{Tn}{N}}^{\frac{T(n+1)}{N}} x(t+\tau)r(t)dt , \qquad (5)$$

we may write

$$\rho(\tau,\omega) \approx \left| \sum_{n=0}^{N-1} \left\{ I_n(\tau) e^{-j\left[\frac{\omega T n}{N}\right]} \right\} e^{\frac{-j\omega T}{2N}} \right| \quad . \tag{6}$$

Equation (5) is seen to be a series of samples obtained by partially integrating the output of the correlation multiplier. Equation (6) is the discrete Fourier transform of the function, $I_n(\tau)$. The transform serves the function of a comb filter bank. Since $I_n(\tau)$ is obtained by multiplying a complex function, $r(t)$, which contains only positive frequencies by a real function and then integrating (low pass filtering), only those components wiich result from the difference in frequencies in $x(t)$ and $r(t)$ remain. Thus, $I_n(\tau)$ is a discrete complex function of the integer n, which has either a positive or negative frequency but not both. In Equation (6), ω may be either positive or negative, providing a method for Doppler search in either direction without ambiguity. The term $e^{\frac{-j\omega T}{2N}}$ does not alter the absolute value of $\rho(\tau,\omega)$ and thus may be eliminated from the expression.

Equation (6) may be rewritten in the form

$$\rho(\tau,\ell) \approx \left| \sum_{n=0}^{N-1} I_n(\tau) e^{-j\frac{2\pi \ell n}{N}} \right| \quad , \tag{7}$$

where $\ell = \frac{\omega T}{2\pi}$. The summation is now in the form which may be evaluated by the Fast Fourier Transform technique, provided ℓ is an integer. The frequency resolution is thus

$$\Delta f = \frac{1}{T} \quad . \tag{8}$$

This resolution is adequate, in principle, since it is equal to the Doppler resolution of the pulse. In practice, however, we should be over-sampled in the frequency domain. This is readily accomplished by writing Equation (7) as

$$\rho(\tau,\ell) \approx \left| \sum_{n=0}^{2N-1} I_n(\tau) e^{-j\frac{2\pi n \ell}{2N}} \right| \quad . \tag{9}$$

and defining $I_n(\tau) = 0$ for $N \le n \le 2N - 1$. The frequency resolution is now

$$\Delta f = \frac{1}{2T} . \tag{10}$$

Equations (5) and (9) provide the working equations for the Multi-Doppler Quadrature Correlator. Equation (5) can be rapidly evaluated by using the SCAT-CORR correlator which is part of the TIMFAX System. This technique involves the use of clipped references. However, it has been repeatedly shown that with approriate choices of the center frequency and the sampling frequency, clipping of the reference causes a negligible degradation of the performance.

It remains to determine workable limits for ω and N. Under the assumption that x(t) and r(t) are the same function except for a frequency difference, ω_o, the integral in Equation (4) reduces to

$$\int_{\frac{Tn}{N}}^{\frac{T(n+1)}{N}} e^{j\omega_o t} dt. \tag{11}$$

If we consider only the difference between the two limits, we may write

$$I = \frac{N}{T} \int_{\frac{-T}{2N}}^{\frac{T}{2N}} e^{j\omega_o t} dt = \frac{\sin \frac{\omega_o T}{2N}}{\frac{\omega_o T}{2N}} . \tag{12}$$

The factor $\frac{N}{T}$ normalizes the equation so that $\lim_{\omega \to o} \{I\} = 1$.

If we require that the result be no more than 1 dB down at ω_o, we have

$$20 \log \left[\frac{\sin \frac{\omega_o T}{2N}}{\frac{\omega_o T}{2N}} \right] = -1 . \tag{13}$$

This is readily solved numerically, yielding

$$\frac{\omega_o T}{2N} \approx \frac{\pi}{4} \quad , \tag{14}$$

or if $\omega_o = 2\pi f_{max}$,

$$N = 4 f_{max} T \quad , \tag{15}$$

The determination of f_{max} requires a study of the ambiguity function of the transmitted signal, r(t). Kramer* has shown that for LFM signals, Doppler can be approximated by a frequency shift if

$$\left|\frac{2V}{C}\right| < \frac{1.74}{\beta T} \quad , \tag{16}$$

where V is the radial velocity of the target, C is the velocity of sound, β is the signal bandwidth, and T is the pulse length. The limit is set by requiring that the correlation peak be degraded by not more than 3 dB.

We shall extend the analysis to include an arbitrary waveform having a rectangular power spectrum.

Let r(t) be the complex function given by

$$r(t) = \frac{1}{\sqrt{\omega_2 - \omega_1}} \int_{\omega_1}^{\omega_2} R(\omega) e^{-j\omega t} dt \quad , \tag{17}$$

where $|R(\omega)|$ is essentially equal to 1 and Arg $R(\omega)$ is arbitrary.

Consider the integral

$$I(k,\Delta) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} r[t(1+k)] r^*(t) e^{-j\Delta t} dt \quad . \tag{18}$$

* Kramer, S. A., "Doppler and Acceleration Tolerances of High-Gain, Wideband Linear FM Correlation Sonars, Proc IEEE, Vol. 55, No. 5, pp 627-636, May 1967.

This integral, when normalized, is the correlation peak with a doppler shifted (time compressed) signal and a frequency shifted reference. The Doppler shift factor is $k = \frac{2V}{C}$. The frequency shift factor for the reference is Δ.

Inserting (17) into (18), we have

$$I(k,\Delta) = \frac{1}{T(\omega_2 - \omega_1)} \int_{\frac{T}{2}}^{\frac{T}{2}} dt \int_{\omega_1}^{\omega_2} d\omega \int_{\omega_1}^{\omega_2} d\omega' R(\omega) e^{-j\omega t(1+k)}$$

$$\cdot R^*(\omega)\ e^{j\omega' t} e^{i\Delta t} \quad . \tag{19}$$

Integrating over t gives

$$I(k,\Delta) = \frac{1}{T(\omega_2 - \omega_1)} \int_{\omega_1}^{\omega_2} d\omega \int_{\omega_1}^{\omega_2} d\omega' R(\omega) R^*(\omega)$$

$$\cdot \frac{2 \sin\left\{[(1+k)\omega - \omega' + \Delta]\frac{T}{2}\right\}}{[(1+k)\omega - \omega' + \Delta]} . \tag{20}$$

Since Arg $R(\omega)$ is arbitrary, we may use the principle of stationary phase to assert that there will be a significant contribution to the integral only when

$$\omega = \omega' . \tag{21}$$

Thus, we write, approximately

$$I(k,\Delta) \propto \frac{1}{T(\omega_2 - \omega_1)} \int_{\omega_1}^{\omega_2} d\omega\ R(\omega) R^*(\omega)\ \frac{2 \sin[k\omega + \Delta]\frac{T}{2}}{k\omega + \Delta} \quad , \tag{22}$$

and since $R(\omega)R^*(\omega) \approx 1$, it can be removed from the integral.

If we let $x = (k\omega + \Delta)\frac{T}{2}$, we have

$$I(k,\Delta) = \frac{2}{kT(\omega_2 - \omega_1)} \int_{(k\omega_1 + \Delta)\frac{T}{2}}^{(k\omega_2 + \Delta)\frac{T}{2}} dx\ \frac{\sin x}{x} \quad , \tag{23}$$

where the integral has been renormalized so that $I(0,0) = 1$.

Maximizing $I(k,\Delta)$ with respect to Δ, we have

$$\frac{dI}{d\Delta} = \frac{2}{T(\omega_2 - \omega_1)} \left\{ \frac{\sin\left[(k\omega_2 + \Delta)\frac{T}{2}\right]}{(k\omega_2 + \Delta)\frac{T}{2}} - \frac{\sin\left[(k\omega_1 + \Delta)\frac{T}{2}\right]}{(k\omega_2 + \Delta)\frac{T}{2}} \right\} = 0 \qquad (24)$$

This equation has an infinite number of solutions. The largest maximum occurs when

$$k\omega_2 + \Delta = -(k\omega_1 + \Delta) , \qquad (25)$$

or

$$\Delta = -\frac{k(\omega_2 + \omega_1)}{2} . \qquad (26)$$

If we let $2\pi\beta = \omega_2 - \omega_1$ and also use Equation (26), Equation (23) becomes

$$I(k) = \frac{2}{2\pi\beta Tk} \int_{-\frac{k\pi\beta T}{2}}^{\frac{k\pi\beta T}{2}} \frac{\sin x}{x}\, dx , \qquad (27)$$

or

$$I(k) = \frac{2}{\pi\beta Tk} \int_0^{\frac{k\pi\beta T}{2}} \frac{\sin x}{x}\, dx . \qquad (28)$$

It is easily verified that $I(0) = 1$.

The 3 dB down point occurs when $I(k) = \frac{1}{\sqrt{2}}$. Equation (28) is readily solved for k by use of a table of Si (x) $= \int_0^x \frac{\sin V}{V}\, dV$. We find that for $I(k) = \frac{1}{\sqrt{2}}$

$$k\pi\beta T = 2.5 , \qquad (29)$$

or

$$k_{-3\ dB} = \frac{1.59}{\beta T} . \qquad (30)$$

Similarly, the 1 dB down point yields

$$k_{-1\ dB} = \frac{.89}{\beta T} \qquad (31)$$

The value for f_{max} is determined from k and the center frequency, f_o:

$$f_{max} = f_o\ k \qquad (32)$$

For the -3 dB criterion,

$$f_{max} = f_o \frac{1.59}{\beta T}\ , \qquad (33)$$

and therefore

$$N = f_o \frac{6.36}{\beta}\ , \qquad (34)$$

where β is the bandwidth and T is the pulse length.

The criterion derived in Equation (30) is somewhat more pessimistic than the result obtained by Kramer. Both methods involve the use of the method of stationary phase, which is an approximation. Equation (30) should be considered an approximation to be verified numerically by actual tests of the correlator.