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THE MOTION OF A PLANE-PARALLEL HEAVY LIQUID IN A CHANNEL WITH A--ETC(U)

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THE MOTION OF A PLANE-PARALLEL HEAVY LIQUID IN A CHANNEL WITH A BOTTOM WHICH HAS A STEP

By

G. Abduvaliyev, Ch. Dzhanybekov

FOREIGN TECHNOLOGY DIVISION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	ɑ	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	ϵ	Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	T	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ κ	ϰ	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

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THE MOTION OF A PLANE-PARALLEL HEAVY LIQUID IN A CHANNEL WITH A BOTTOM WHICH HAS A STEP

G. Abduvaliyev, Ch. Dzhanybekov

The problem of motion of heavy liquid in a channel with a bottom which has a step is examined. Such a problem was first studied by N. E. Kochin [1] in a linear setup and was resolved for the case of a step of negligible height.

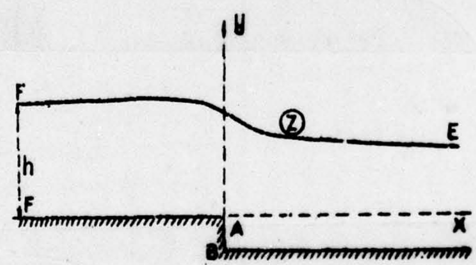

Fig. 1.

Assume that a heavy incompressible liquid moves at a speed of V_0 (Fig. 1). The presence of the step at the bottom results in a disturbance of the liquid. Our objective is to determine the form of free surface EF in the region near the step.

Employing the theory of functions of a complex variable, we formulate a system of integral-differential equations, the solution of which will determine the form of free surface EF.

Let us assume that in the case of a weightless liquid we have a complex potential of $w = \varphi + i\psi$ and a Zhukovskiy function of

$$F_0 = \ln \frac{V_0\, dz}{d W_0} = i\theta_0 + \ln \frac{V_0}{V}.$$

For determining function $W = \varphi + i\psi$ let us plot a band of width q on plane W onto the upper half plane of plane $t = \xi + i\eta$. At the same time the solid walls FABE and free surface EF are plotted onto half lines FA, ABE and segment EF of the true axis, respectively (Fig. 2).

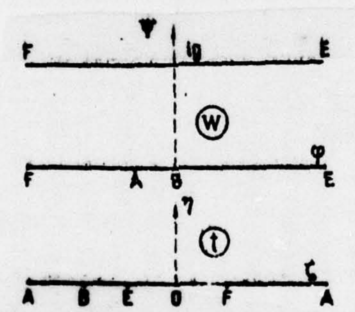

Fig. 2.

It is not difficult to verify that this representation is given by the function

$$W(t) = \frac{q}{\pi} \ln \frac{t-1}{t+1} + iq. \qquad (1)$$

where q is the flow of the liquid per second.

The function

$$\Phi(t) = \frac{F(t) - F_0(t)}{\sqrt{1-t^2}}, \qquad (2)$$

comes into consideration

where

$$F(t) = \ln \frac{V_0 dz}{dW}, \quad F_0(t) = \ln \frac{V_0 dz}{dW_0} \qquad (2.1)$$

is the Zhukovskiy function for weighable and weightless liquid of the scheme being considered, V_0 is the speed of the weighable liquid at infinity upstream. The limiting values for determining functions $\Phi(t)$ and $F_0(t):\sqrt{1-t^2}$ are given in Table 1.

Table 1.

$t = \xi$	$F_0(t):\sqrt{1-t^2}$	$\Phi(t)$
$-\infty < \xi < -b$	$Re[F_0(\xi):\sqrt{1-\xi^2}] = \frac{\alpha \pi}{\sqrt{\xi^2-1}}$	$Re\,\Phi(\xi) = 0$
$-b < \xi < -1$	$Re[F_0(\xi):\sqrt{1-\xi^2}] = 0$	$Re\,\Phi(\xi) = \ln \frac{V_0}{V}:\sqrt{1-\xi^2}$
$-1 < \xi < 1$	$Re[F_0(\xi):\sqrt{1-\xi^2}] = 0$	$Re\,\Phi(\xi) = 0$
$1 < \xi < \infty$	$R[F_0(\xi):\sqrt{1-\xi^2}] = 0$	$Re\,\Phi(\xi) = 0$

According to the limiting values given in Table 1, based on the Schwarz integral, for $\Im m\, t > 0$ we have:

$$\frac{F_0(t)}{\sqrt{1-t^2}} = \frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{Re[F_0(\xi):\sqrt{1-\xi^2}]d\xi}{\xi - t} =$$

$$= \frac{1}{\pi i} \left[\int_{-1}^{1} \frac{\ln V_0/V}{\sqrt{1-\xi^2}} \cdot \frac{d\xi}{\xi - t} - \int_{-\infty}^{b} \frac{\alpha}{\sqrt{\xi^2-1}} \cdot \frac{d\xi}{\xi - t} \right],$$

$$\Phi(t) = \frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{Re\,\Phi(\xi)}{\xi - t} d\xi = \frac{1}{\pi i} \int_{-1}^{1} \frac{\ln V_0/V}{\sqrt{1-\xi^2}} \cdot \frac{d\xi}{\xi - t}.$$

Here the unessential constants equate to zero. Considering the constancy of the speed magnitude vector on the free surface ($V_c = V$) and function (2), from the last equalities we have:

$$F_0(t) = \varkappa \ln \frac{1+Bt-\sqrt{t^2-1}\sqrt{B^2-1}}{(B+t)(t-\sqrt{t^2-1})} \quad (\operatorname{Im} t > 0), \tag{3}$$

$$F(t) = F_0(t) + \frac{\sqrt{1-t^2}}{\pi i}\int_{-1}^{1} \frac{\ln V_0/V}{\sqrt{1-\xi^2}} \cdot \frac{d\xi}{\xi - t}. \tag{4}$$

It is possible to determine that equalities (3) and (4) satisfy the boundary conditions of Table 1 and give the desired **Zhukovskiy** functions in the case of weightless liquid and also with $\ln V_0/V$ given for weighable liquid.

On the free surface the function $\ln V_0/V$, which is part of the expression under the integral sign from formula (4), satisfies the Bernoulli equation. Thus from

$$V^2 + 2gy = V_0^2 + 2gy_0$$

we find

$$\ln \frac{V_0}{V} = -\frac{1}{2}\ln[1+\varepsilon(y_0-y):h]. \tag{5}$$

Here

$$\varepsilon = \frac{1}{F} = \frac{2gy}{V_0^2}$$

of the Froude reciprocal, V_0, V, are the speed vector magnitudes of weighable liquid at points with ordinates y_0 and y respectively, g is the acceleration of gravity, h is the height of the free surface from the bottom at infinity upstream. From equality (4). proceeding to the limit with $t \to \xi$ (first designating the variable of integration by ξ_1, with $\operatorname{Im} t > 0$) and equating the imaginary parts, we obtain

$$\theta = \theta_0 + \frac{\sqrt{1-\xi^2}}{\pi}\int_{-1}^{1} \frac{\ln[1+\varepsilon(y_0-y):h]}{\sqrt{1-\xi_1^2}(\xi_1-\xi)}\, d\xi_1, \tag{6}$$

where

$$\theta_0 = \varkappa \operatorname{arctg} \frac{(1+B\xi-\xi\sqrt{B^2-1})\sqrt{1-\xi^2}}{(1+B\xi)\xi+(1-\xi^2)\sqrt{B^2-1}}$$

with $-1<\xi<1$, b is constant ($b>1$).
From the first equation of system (2.1) we have

$$\exp F(t)=\frac{V_0}{V}e^{i\theta}=V\frac{dz}{dw}$$

or with $t=\xi$, considering that $z=x+iy$ and equality

$$\frac{dw}{d\xi}=\frac{2q}{\pi(\xi^2-1)},$$

we obtain

$$\frac{dz}{d\xi}=\frac{2q}{\pi(\xi^2-1)}\,\frac{e^{i\theta}}{V},$$

from which we have finally

$$\begin{aligned}\frac{dx}{d\xi}&=-\frac{2q}{\pi(1-\xi^2)}\cdot\frac{\cos\theta}{V},\\ \frac{dy}{d\xi}&=-\frac{2q}{\pi(1-\xi^2)}\cdot\frac{\sin\theta}{V}.\end{aligned}\tag{7}$$

The parametric equation of the free surface and thereby its form are determined from set of equations (6) and (7) with $-1<\xi<1$.

In order to solve the obtained set of nonlinear singular kernel integral-differential equations (6) and (7) we employ the Newton-Kantorovich method [2]. As the initial approximation of $x_0=x_0(\xi)$ and $y_0=y_0(\xi)$ let us take the solution of the corresponding problem for weightless liquid and for arriving at the first approximation let us proceed as follows.

Assuming the zero approximation is known, in the vicinity of point (θ_0,y_0) let us replace the nonlinear terms from expressions (6) and (7) with the linear terms relative to unknowns θ_1,y_1:

$$\begin{aligned}\left.\frac{\sin\theta}{V/V_0}\right|_{\theta_1,y_1}&=\frac{1}{[1+\varepsilon(y_1-y_0):h]^{1/2}}\Big[\sin\theta_0+(\theta_1-\theta_0)\cos\theta_0+\\ &+\frac{\varepsilon}{2}\,\frac{\sin\theta_0}{h+\varepsilon(y_1-y_0)}(y_1-y_0)-\frac{\varepsilon\cos\theta_0\sqrt{1-\xi^2}}{2\pi}\times\\ &\times\int_{-1}^{1}\frac{d\xi_1}{[1+\varepsilon(y_1-y_0):h](\xi_1-\xi)\sqrt{1-\xi_1^2}}(y_1-y_0)\Big].\end{aligned}\tag{8}$$

$$\ln[1+\varepsilon(y_0-y):h]\Big|_{y_1}=\ln[1+\varepsilon(y_0-y_0):h]-\frac{\varepsilon(y_1-y_0)}{h+\varepsilon(y_0-y_0)}. \tag{9}$$

Then on the basis of equation (9), formula (6) is presented as

$$\theta_1-\theta_0=F_0(\xi)-\frac{\varepsilon\sqrt{1-\xi^2}}{2\pi h}\int_{-1}^{1}\frac{[y_1(\xi_1)-y_0(\xi_1)]\,d\xi_1}{\sqrt{1-\xi_1^2}(\xi_1-\xi)[1+\varepsilon(y_0-y_0):h]}. \tag{10}$$

here

$$F_0(\xi)=\frac{\sqrt{1-\xi^2}}{2\pi}\int_{-1}^{1}\frac{\ln[1+\varepsilon(y_0-y_0):h]}{\sqrt{1-\xi_1^2}(\xi_1-\xi)}\,d\xi_1.$$

From the second equation of set (7), in conformance with formulas (8) and (9), we obtain

$$\frac{dy_1}{d\xi}=-\frac{2h}{\pi(1-\xi^2)[1+\varepsilon(y_0-y_0):h]^{1/2}}\Big\{\sin\theta_0+\cos\theta_0\Big[F_0(\xi)-$$

$$-\frac{\varepsilon\sqrt{1-\xi^2}}{2\pi h}\int_{-1}^{1}\frac{y_1(\xi)-y_0(\xi)}{\sqrt{1-\xi_1^2}(\xi_1-\xi)[1+\varepsilon(y_0-y_0):h]}\,d\xi\Big]+$$

$$+\frac{\varepsilon}{2}\Big[\frac{\sin\theta_0}{1+\varepsilon(y_0-y_0):h}-\frac{\cos\theta_0\sqrt{1-\xi^2}}{\pi}\int_{-1}^{1}\frac{d\xi_1}{(1+\varepsilon u_0)(\xi_1-\xi)\sqrt{1-\xi^2}}\Big](y_1-y_0)\Big\}$$

or

$$\frac{d\tilde{u}_0}{d\xi}+\frac{\varepsilon}{\pi(1-\xi^2)(1+\varepsilon u_0)^{1/2}}\Big[\frac{\sin\theta_0}{1+\varepsilon u_0}-\frac{\cos\theta_0\sqrt{1-\xi^2}}{\pi}\times$$

$$\times\int_{-1}^{1}\frac{d\xi_1}{(1+\varepsilon u_0)(\xi_1-\xi)\sqrt{1-\xi^2}}\Big]\tilde{u}_0=R(\xi)+\frac{\varepsilon\cos\theta_0}{\pi\sqrt{1-\xi^2}(1+\varepsilon u_0)^{1/2}}\times \tag{11}$$

$$\times\frac{1}{\pi}\int_{-1}^{1}\frac{\tilde{u}_0(\xi_1)\,d\xi_1}{\sqrt{1-\xi_1^2}(\xi_1-\xi)(1+\varepsilon u_0)},$$

where

$$R(\xi)=-\frac{2}{\pi(1-\xi^2)(1+\varepsilon u_0)^{1/2}}\Big[\sin\theta_0+\cos\theta_0(\theta_0-\theta_1+$$

$$+\frac{\sqrt{1-\xi^2}}{2\pi}\int_{-1}^{1}\frac{\ln[1+\varepsilon u_0(\xi_1)]}{\sqrt{1-\xi_1^2}(\xi_1-\xi)}\,d\xi_1)\Big]-\frac{dy_0(\xi)}{h\,d\xi}.$$

Let us introduce the notation

$$\frac{\tilde{u}_0(\xi)}{1+\varepsilon u_0(\xi)}=V_0(\xi). \tag{12}$$

Then equation (11) has the form

$$(1-\xi^2)\frac{dV_0}{d\xi}+A_0(\xi)V_0(\xi)+\frac{B_0(\xi)}{\pi}\int_{-1}^{1}\frac{V_0(\xi_1)d\xi_1}{\sqrt{1-\xi_1^2}(\xi_1-\xi)}=f(\xi). \tag{13}$$

Here

$$A_0(\xi)=\frac{\varepsilon}{\pi(1+\varepsilon u_0)^{3/2}}\left[\frac{\sin\theta_0}{1+\varepsilon u_0}-\frac{\cos\theta_0\sqrt{1-\xi^2}}{\pi}\cdot\right.$$
$$\left.\cdot\int_{-1}^{1}\frac{d\xi_1}{(1+\varepsilon u_0)(\xi_1-\xi)\sqrt{1-\xi_1^2}}\right]-\frac{\varepsilon(1-\xi^2)}{1+\varepsilon U_0}\cdot\frac{dy_0}{hd\xi}, \tag{13,1}$$

$$B_0(\xi)=-\frac{\varepsilon}{\pi}\cdot\frac{\cos\theta_0\sqrt{1-\xi^2}}{(1+\varepsilon u_0(\xi))^{3/2}}, \tag{13.2}$$

$$f_0(\xi)=-\frac{2}{\pi(1+\varepsilon u_0)^{3/2}}\left[\sin\theta_0+\cos\theta_0\left(\theta_0-\theta_1+\right.\right.$$
$$\left.\left.+\int_{-1}^{1}\frac{\ln(1+\varepsilon u_0)}{\sqrt{1-\xi^2}(\xi_1-\xi)}d\xi_1\right)\right]-\frac{1-\xi^2}{1+\varepsilon u_0(\xi)}\cdot\frac{dy_0}{hd\xi}. \tag{13.3}$$

The obtained linear singular kernel integral-differential equation is solved under the secondary condition

$$V_0(1-\delta)=0 \tag{14}$$

at interval $(-1+\delta, 1-\delta)$. ($\delta>0$ is a fairly small number).

Solving equation (13) taking condition (14) into account, we find $V_0(\xi)$, and $y_1(\xi)$ is determined in conformance with formula (12)

$$\frac{y_1(\xi)}{h}=\frac{y_0(\xi)}{h}+\left[1+\varepsilon u_0(\xi)\right]V_0(\xi), \tag{15}$$

$\theta_1(\xi)$ is found from expression (10) by means of the computation of singular integrals

$$\theta_1(\xi)=\theta_0(\xi)+F_0(\xi)-\frac{\varepsilon\sqrt{1-\xi^2}}{2\pi}\int_{-1}^{1}\frac{V_0'(\xi_1)}{\sqrt{1-\xi_1^2}}\cdot\frac{d\xi_1}{\xi_1-\xi}. \tag{10.1}$$

After obtaining $\theta_1(\xi)$ and $y_1(\xi)$ the unknown $x_1(\xi)$ is computed, according to the first equation from (7), using the simple quadrature

$$\frac{dx_1}{h\,d\xi} = -\frac{2}{\pi(1-\xi^2)}\frac{\cos\theta_1}{(1+\varepsilon U_0)^{1/2}} \tag{16}$$

under the secondary condition

$$x_1(1-\delta) = x_n \tag{17}$$

Here

$$x_n = -\frac{2b_1 h}{\pi}\int_0^1 \left[\frac{\xi(\xi + b_1 b - \sqrt{b_1^2-\xi^2}\sqrt{b^2-1}}{(b\xi + b_1)(b_1 - \sqrt{b_1^2-\xi^2})}\right]^{1/2} \frac{d\xi}{b_1^2-\xi^2},$$

$b, b_1 > 1$ are the abscissae of point B and the point which lies between A and F on plane t, respectively. Calculation is made with B = 5 and $b_1 = 1{,}01$.

Let us look for the solution of the problem in question (13)-(14) in terms of

$$V_0(\xi) = \sum_{m=0}^{N} a_m T_m(\xi), \tag{19}$$

where $T_m(\xi)$ are Chebyshev polynomials of the first kind, a_m are the coefficients still unknown.

We find

$$\frac{dV_0}{d\xi} = \sum_{m=0}^{N} a_m T_m'(\xi) = \sum_{m=0}^{N} a_m m\, U_{m-1}(\xi), \tag{20}$$

where $U_{m-1}(\xi)$ are the Chebyshev polynomials of the second kind.

Substituting the relationships (19) and (20) into equation (13), considering the following equality [3]

$$U_{m-1}(\xi) = \frac{1}{\pi}\int_{-1}^{1}\frac{T_m(y)}{\sqrt{1-y^2}}\frac{dy}{y-\xi},$$

we obtain the transcendental equation

$$\sum_{m=0}^{N} a_m[m(1-\xi^2)U_{m-1}(\xi) + A_0(\xi)T_m(\xi) + B_0(\xi)U_{m-1}(\xi)] = f_0(\xi),$$

from which for determining the coefficients a_m, substituting $\xi = \xi_i$ ($\xi_i = -\cos\frac{i\pi}{N}$ are the colocation points), we arrive at the algebraic system

$$\sum_{m=0}^{N} a_m C_{nm} = b_n, \qquad (n = 0, 1, 2, \ldots, N-1), \tag{21}$$

where

$$\begin{aligned} C_{nm} &= A_0(\xi_n) T_m(\xi_n) + \left[m(1-\xi_n^2) + B_0(\xi_n) \right] U_{m-1}(\xi_n), \\ b_n &= f_0(\xi_n). \end{aligned} \tag{22}$$

Here $U_0(\xi) = 0$, $U_{-1}(\xi) = 0$, $T_0(\xi) = 1$. The coefficients $V_0(\xi)$ are determined from the set of algebraic equations (21).

The Newton-Kantorovich method used has a quadratic convergence, the convergence of the process depending essentially on how well the initial approximation has been selected. The solution of a specific problem for a weightless liquid was taken as the initial approximation, which is considered most appropriate. Calculations were conducted on a "Minsk-22" computer for $\varepsilon = 0{,}01$; $\varepsilon = 0{,}1$; $\varepsilon = 0{,}2$ with $x = 1:2$ and for $\varepsilon = 0{,}01$; $\varepsilon = 0{,}1$; $\varepsilon = 0{,}2$; $\varepsilon = 0{,}4$ with $x = 1:4$.

Numerical calculation shows that in the case $x = 1:2$ iteration converges for $\varepsilon = 0{,}1$ in the third approximation, and for $\varepsilon = 0{,}2$ – in the fourth approximation. But if $x = 1:4$, then the iteration converges for $\varepsilon = 0{,}1$ in the third approximation and for $\varepsilon = 0{,}2$ in the fifth approximation.

In our calculations we were guided by the following: the process of iteration was terminated when the following conditions had been met

$$|x_{i+1} - x_i| < 2 \cdot 10^{-4}, \quad |y_{i+1} - y_i| < 2 \cdot 10^{-4}, \quad i = 0, 1, 2, \ldots .$$

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