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ON THE CHOICE OF BEAM SPACING IN  
ROTATIONAL DIRECTIONAL TRANSMISSION

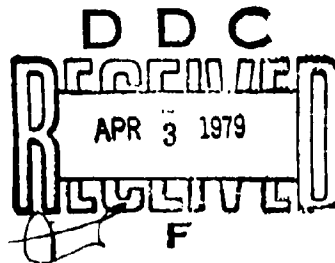
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**ON THE CHOICE OF BEAM SPACING IN  
ROTATIONAL DIRECTIONAL TRANSMISSION**

by

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*11*  
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Contract NObsr-93023

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## SECTION I

## INTRODUCTION

A possible method of insonifying a large sector with a Conformal/Planar (henceforth denoted by C/P) sonar is the use of rotational directional transmission. This technique consists of sequentially transmitting in contiguous beams. An alternate method of insonification consists of defocusing the beams and transmitting long coded pulses. (The use of coding is necessary to maintain the desired transmitted bandwidth.) This latter method was not judged as desirable as the first because of the possible signal processing losses due to the medium and own ship's motion, as well as possible difficulties associated with driving the elements. Furthermore, the increased system costs associated with the additional signal processing equipment decrease the attractiveness of the beam broadening approach.

This paper considers the quantitative determination of the average echo-to-reverberation ratio for two transmission beam spacings; in particular, beam crossover points of -3.9 and -0.9 db were investigated. The first spacing is the classical Rayleigh separation based on the resolution of two points with equal illumination; the second spacing essentially doubles the number of transmitted beams by forming all of the previous ones plus the beams half-way in between. The results clearly show that the sparser spacing yields significantly higher echo-to-reverberation ratios.

SECTION II  
DISCUSSION

The basic question studied was the effect of RDT step size on the average echo-to-reverberation ratio. (The spatial insonification assumed using RDT is illustrated in Figure 1.) Two beam spacings were investigated: Rayleigh, and one-half Rayleigh. In addition, the effect of receiving beam placement relative to the transmitting beams was also investigated.

The reverberation level in a given receiving beam was determined as follows. First, a reverberation intensity in the receiving beam is calculated for each of the pulses transmitted in the neighboring transmitting beams. These intensities are then summed to obtain the total reverberation intensity in that beam. Differences in travel times between pulses due to transmitted pulse length and finite stepping times were assumed negligible. This is a valid assumption for short pulse length systems or cases where the travel time is large compared to the insonification time.

The results obtained in this study are not a sensitive function of the particular sonar parameters assumed. (The beam spacings are proportional to the beam widths, and these are proportional to the dimensions of the sonar under consideration. The only requirement for this scaling is that the beamwidths between nulls be less than about twenty degrees.) Calculations were performed for both near-axis and off-axis reverberation; the relative differences in reverberation levels were the same and these differences are shown in Table 1.

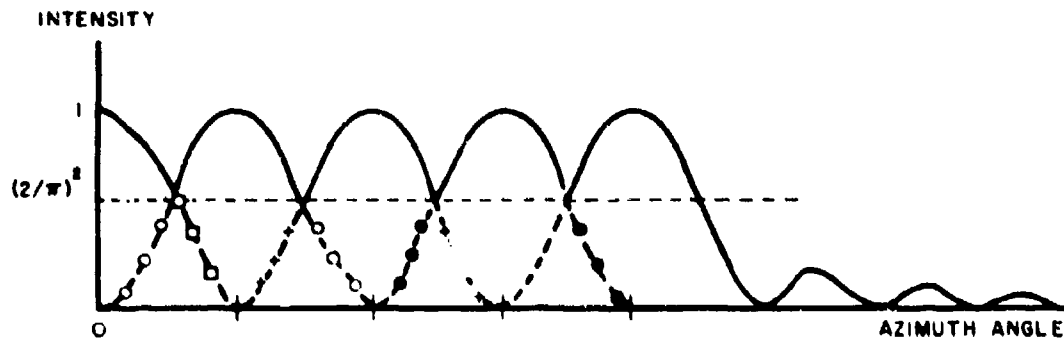


Figure 1. Illustration of RDT Insonification Using Rayleigh Beam Spacing. (Side lobes are shown only for right hand beam. Drawing not to scale )

Table 1. Summary of Calculations. Reverberation level differences in db versus transmitting and receiving beam.

Transmitting Angle, deg		Relative Reverberation Level, db		
		Receiving Angle, deg		
Rayleigh	Intermediate	90.0°	89.50°	89.75°
90.0		0.0	-2.6	-0.6
	89.5	-2.6		
89.08		-8.2	-1.7	-4.2
	88.6	-11.8		-10.7
88.16		-14.2	-15.5	-12.4
	87.7	-16.2		-15.6
87.24		-17.7	-16.2	-16.8
	86.8	-19.0		-18.5
86.32		-20.1	-19.1	-19.5
	85.9	-21.2		-20.8
85.40		-22.2	-21.2	-21.5
	84.9	-23.0		-22.7
84.48		-23.7	-22.9	-23.2
	84.0	-24.4		



These levels were computed by numerically evaluating the reverberation integral, which is proportional to the following:

$$I \propto \int_0^{2\pi} V_t(\theta_o, \phi_{ot}, \theta, \phi) V_r(\theta_o, \phi_{or}, \theta, \phi) d\phi$$

where

$V$  is the intensity pattern function for a rectangular<sup>1</sup> aperture of point sources in an infinite rigid baffle,

$\theta_o$  is the steering depression angle,

$\phi_o$  is the azimuthal steering angle,

$\theta$  is the depression angle of the reverberation ray path, and

$\phi$  is the azimuthal angle variable.

The subscripts  $t$  and  $r$  denote transmission and reception respectively. The actual numerical integration was restricted to the main lobe and approximately the first ten side lobes on either side of the main lobe of the receiving pattern. The total reverberation level was obtained by summing all of the individual reverberation intensities and taking  $10 \log_{10}$  of the sum. The pattern function is symmetrical about  $90^\circ$  azimuth and only the results for azimuthal steering angles less than or equal to  $90^\circ$  are shown.

The average azimuthal deviation losses to the target were also calculated; the derivation of the expression is given in the Appendix.

<sup>1</sup> See, for example, Schulkenoff and Friis, Antennas: Theory and Practice, John Wiley and Sons, Inc., New York, N.Y. 1952.

The conclusions presented here are average echo-to-reverberation ratios for a target on the axis of the receiving beam.\* Such factors as multiple echoes (from adjacent pulses) and/or echo overlap (for the case where the target echo is long compared to a pulse length) are not considered in this analysis.

The combinations of parameters investigated were:

- Case 1 - - Rayleigh spacing; receiving and transmitting beams coincident;
- Case 2 - - one-half the Rayleigh spacing; receiving and transmitting beams coincident.
- Case 3 - - Rayleigh spacing; receiving beams approximately midway between transmitting beams.
- Case 4 - - Rayleigh spacing; receiving beams displaced from transmitting beams by one-quarter of the beam spacing.

The results of the calculations are summarized below.

Table 2. Echo-to-reverberation factor, db

	Case			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
Average echo deviation loss, db	1.1	0.3	1.1	1.1
Relative reverberation level, db	1.6	4.5	1.4	1.7
Echo-to-reverberation factor, db	-2.7	-4.8	-2.5	-2.8

---

\* The object of this study is to compare these beam spacings in transmission. The spacing in reception is an independent parameter. It is clear from the results of this study that the sole effect of increasing the number of receiving beams will be to reduce the average azimuthal deviation loss to the target; any gain achieved here will probably be offset by the increases in the detection thresholds required to maintain a constant false alarm rate.

From the tabulated results, one may observe that the use of Rayleigh spacing over half-Rayleigh clearly results in a 2 db superiority in the echo-to-reverberation ratio. In addition, if one assumes that the increase in the detection threshold for Case 2 over the other cases is on the order of 1 db<sup>\*</sup>, then, in a reverberation limited environment, the effective degradation in performance for Case 2 approaches 3 db.

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\*For fixed false alarm ratio since there are twice the number of beams.

APPENDIX: CALCULATION OF AZIMUTHAL DEVIATION LOSS

Consider the azimuthal intensity pattern function below:

$$y = \left( \frac{\sin Mx}{M \sin x} \right)^2$$

(M is the number of columns in the array)

For large M, and in the region of the main lobe,

$$y \approx \left( \frac{\sin Mx}{Mx} \right)^2$$

The position of a target in a beam is a random event with a rectangular distribution, that is, there is no preferential position. To compute the average deviation loss, we proceed as follows. For a C/P array,

$$x = \frac{\pi a}{\lambda} (\cos\theta \cos\phi - \cos\theta_0 \cos\phi_0)$$

where

a is the spacing between columns,

$\lambda$  is the wavelength,

$\theta$  is the elevation angle,

$\phi$  is the azimuth angle, and

a subscript  $\underline{0}$  denotes a steering direction. For this analysis,

let  $\theta = \theta_0$  and let us use a two term Taylor expansion of  $\phi$

about  $\phi_0$ . We obtain

$$x = \frac{\pi a}{\lambda} \cos \theta_0 \sin \phi_0 (\phi_0 - \phi)$$

where we note that  $x$  is now linear in  $\phi$ . This expansion is valid as long as the next term in the series is small, that is, for azimuth angles away from endfire and for  $|\phi - \phi_0| < .1$

The average height of the pattern between beam cross over points is given by

$$\bar{y} = \frac{1}{2\Delta\phi_c} \int_{\phi_0 - \Delta\phi_c}^{\phi_0 + \Delta\phi_c} \left( \frac{\sin Mx}{Mx} \right)^2 d\phi.$$

For values of  $|Mx| \leq \frac{\pi}{2}$ , the following approximation is given in Reference 1, pg 73 (4.3.96)

$$\frac{\sin x}{x} = 1 - .16605x^2 + .00761x^4 + \epsilon(x)$$

where  $|\epsilon(x)| \leq 2 \times 10^{-4}$ .

The approximate expression for  $y$  is symmetrical about  $\phi_0$  so that the expression for  $\bar{y}$  may be rewritten after some transformation as

$$\bar{y} = \frac{1}{U_c} \int_0^{U_c} \left( \frac{\sin u}{u} \right)^2 du$$

where

$$U_c = \frac{M\pi a}{\lambda} \cos \theta_0 \sin \phi_0 \Delta\phi_c.$$

If one uses the Rayleigh cross-over criterion, the beams overlap at the 3.9 db down points (or equivalently,  $U_c = \frac{\pi}{2}$ ). Using the polynomial expansion for the integrand, one obtains

$$\bar{y} = 1 - c_1 U_c^2 + c_2 U_c^4 - c_3 U_c^6 + c_4 U_c^8$$

where

$$\begin{aligned} c_1 &= 0.1107 & c_3 &= .1805 \times 10^{-3} \\ c_2 &= 0.8559 \times 10^{-2} & c_4 &= .6435 \times 10^{-5} \end{aligned}$$

For  $U_c = \frac{\pi}{2}$ ,  $\bar{y} = 0.77$  (or - 1.1 db)

For one-half the Rayleigh spacing,  $U_c = \frac{\pi}{4}$  and  $\bar{y} = .935$   
(or - 0.3 db).