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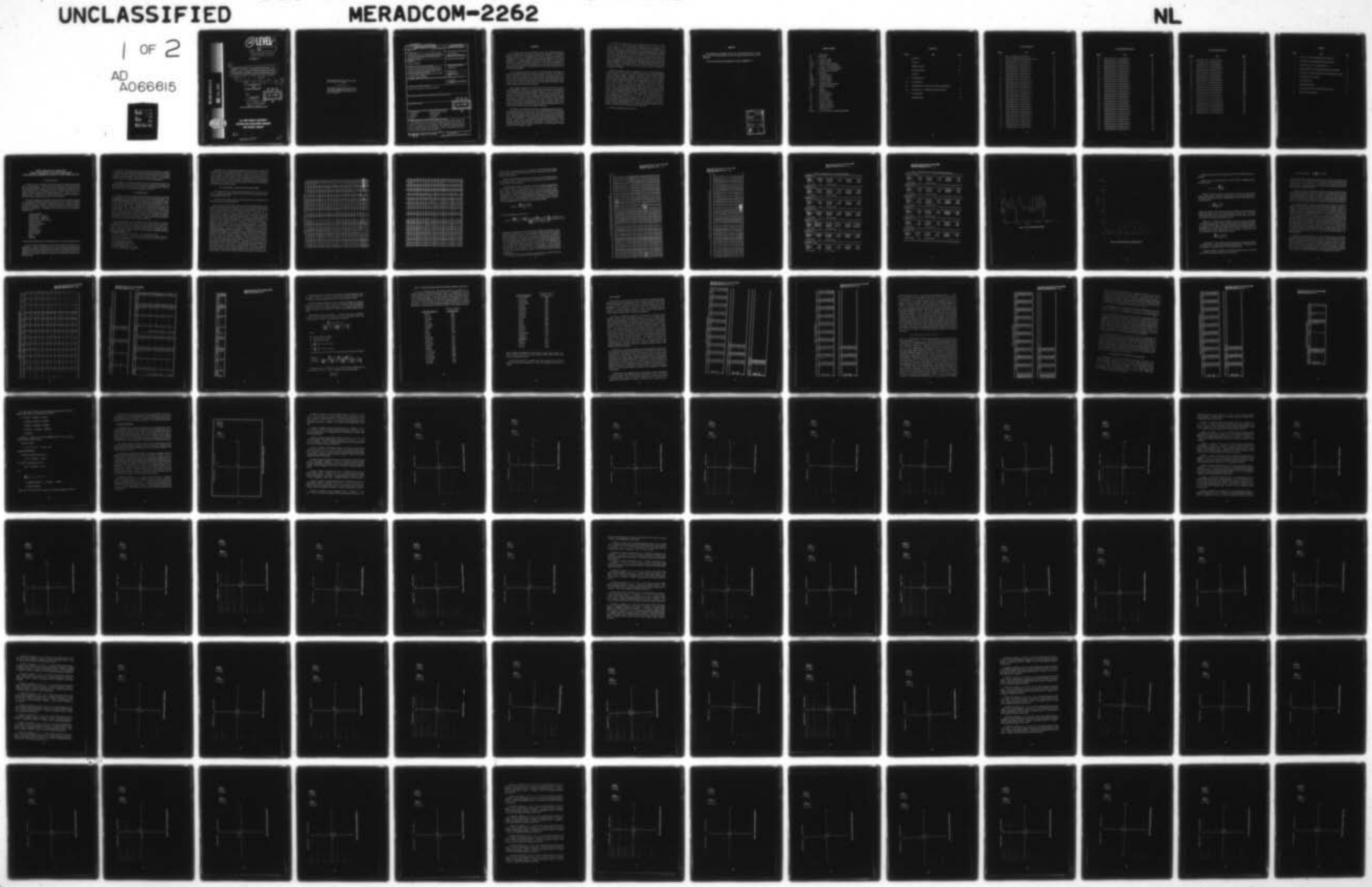

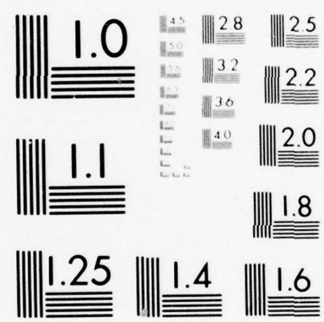

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CORRELATION STUDY OF LABORATORY PHYSICAL AND CHEMICAL DATA WITH DYNAMOMETER ENGINE SEQUENCE PERFORMANCE TESTING OF ENGINE LUBRICATING OILS.

(9) Technical rept. Sep 74 - Sep 78

by

(10) Maryland D. Kemp

and

Francis E. Council, Jr

(11) December 1978

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Statistics	Lubrication

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A statistical study has been made on laboratory physical and chemical data obtained on 80 lubricating oils meeting requirements of Military Specification MIL-L-2104C. The purpose of the study is to seek correlations between laboratory physical and chemical data and dynamometer and field performance. The initial study produced internal and group correlations among the data and suggests a means for relating laboratory-observed chemical and physical data to field performance.

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SUMMARY

In this study, many tests on lubricating oils have been considered collectively and not as separate tests, and comments are made on the significance of the data with particular reference to the correlations among the data. Normally, an assessment of the significance of a test is accomplished by examining the correlation of the test with performance, i.e., deciding whether a change in the performance of an oil will be predicted by a change in a laboratory test result. In this report, the significance of a test is assessed by considering its relationship with other tests and not from a consideration of performance data. Performance data of the quality and quantity desired are not presently available.

The tests considered include some of the common inspection tests usually employed to characterize physical and chemical properties of the oils. The data was retrieved from the Laboratory files. Twenty-six pieces of analytical data were collected but because of the lack of data on all the samples, 13 items were finally chosen for the study. The items included analytical results for gravity; flash point; viscosity at 210°F, 100°F, and 0°F; viscosity index; pour point; total acid number; carbon; sulphur; sulphated ash; phosphorus; and calcium.

Three statistical techniques have been used to analyze the data and to examine the interdependence of these tests. The first task of the analysis is to determine the basic distributional characteristics (descriptive statistics) of each of the variables to be used in the subsequent statistical analysis. Information on the distribution, variability, and central tendencies of the variables provides necessary information required for selection of subsequent statistical techniques and constitutes a basic computer reference document for the entire data file. The mean, for example, provides a measure of central tendency; standard deviation and variance indicate the degree of dispersion around the mean, and measures such as skewness and Kurtosis allow one to more precisely define the shape of the variables' distribution.

The analysis of the descriptive data shows a considerable amount of variance – the data does *not* cluster around the mean. The standard deviation shows large values for the standard deviation. The skewness, which takes a value of zero when distribution is a symmetric bell-shaped curve, and a plot of the data for gravity show departure from a normal distribution. The Kurtosis further shows departure from a normal distribution. A positive value for kurtosis shows a peaked (narrow) distribution. A negative value shows a flatter distribution. Both positive and negative values exist for Kurtosis with positive values (narrow) predominating. Conclusive deductions cannot be made from the descriptive statistics.

The distinctive characteristic of factor analysis is its data-reduction capability. Once an array of correlation coefficients is obtained for a set of variables, factor analysis enables one to see whether some underlying pattern of relationships exist such that the data may be arranged or reduced to a smaller set of factors or components that may be taken as source variables accounting for the observed interrelations in the data. The results of the factor analysis show that, of the total variance in the data, 85% of the total variance registered for the 13 elemental pieces of data is accounted for by the first four factors, G, FL, Viscosity (210°F, 10°F, 0°F, VI), and TAN. This suggests that a relationship between these four factors will define performance parameters of the lubrication oils. Although individual parameters such as CRAM and SAH (.7709), CRAM and CA (.2167), and SAH and CA (.8679) show good dual correlations, their effects on the total variance are small (a correlation of 1.0 is perfect correlation).

Principal component analysis was employed to study the overall relationship of groups of tests, and some use was made of correlation coefficients and Pearson's product moment correlations. Principal component analysis is used to evaluate components which are linear combinations of the original tests, and the analysis shows how many independent properties are being measured by the test considered. All the solutions employed previously extract orthogonal factors for the matrix in order of their importance. The first factor, say gravity, greatly affects (loads) on all other factors. In the principal-component matrix, the variables are in turn rotated and the data is presented in graphical form. Each possible pair of factors, including all the 13 items of data considered, is orthogonally rotated. From the rotation, one obtains overall relationship of groups of data. For example, employing gravity 2 as the horizontal factor and flash point 2 as the vertical factor, elements 8, 9, and 12 form a separate group to elements 2, 10, 7, 13, and 3. That is, flash point, sulphur, total acid number, pour point, and viscosity (2, 10, 7, 13, 3) all react to one property of the lubricating oil while carbon, sulphurated ash, and calcium (8, 9, 12) react to another property. Similar applications can be made with other plots, and when dynamometer and field performance data are included, information relating the three (lab data, dynamometer data, performance data) should evolve.

Further study along these lines is indicated.

PREFACE

The inspiration and motivation for this work came from Mr. Maurice E. LePera, Chief, Fuels & Lubricants Division, under whose general supervision the tests were performed.

The work was performed under Mission Account No. A8H20EL0221.

ABBREVIATIONS

G	=	Gravity, API
FL	=	Flash Point
V_{210}	=	Viscosity @ 210°F
V_{100}	=	Viscosity @ 100°F
V_0AP	=	Viscosity @ 0°F (Apparent)
V_0EX	=	Viscosity @ 0°F (Extrapolated)
V_{-20}	=	Viscosity @ -20°F (Extrapolated)
VI	=	Viscosity Index (Calculated)
pp	=	Pour Point
stpp	=	Stable Pour Point
TBN	=	Total Base Number
TBNN	=	Total Base Number
TAN	=	Total Acid Number
CRAM	=	Ramsbottom Carbon Residue
SAH	=	Sulphated Ash Residue
S	=	Sulphur
P	=	Phosphorus
CA	=	Calcium (Additive)
Ba	=	Barium (Additive)
Zn	=	Zinc (Additive)
NA	=	Sodium (Additive)
K	=	Potassium (Additive)
N	=	Nitrogen (Additive)
Mg	=	Magnesium (Additive)
B	=	Boron (Additive)
Other	=	To include wear and contaminant materials.

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Cont'd)

ILLUSTRATIONS (Cont'd)

TABLES

CORRELATION STUDY OF LABORATORY PHYSICAL AND CHEMICAL DATA WITH DYNAMOMETER ENGINE SEQUENCE PERFORMANCE TESTING OF ENGINE LUBRICATING OIL

I. INTRODUCTION

A lubricant serves a multitude of functions in many mechanical systems among which are the following: (1) prevention of friction and wear, (2) removal of heat from the lubricant-engine system, (3) removal of contaminants and debris from the lubricant-engine system, (4) transmission of power in hydraulic systems, and (5) prevention of metallic corrosion in the lubricant-engine system. At the same time, the lubricant must be compatible with a variety of gaskets and elastomers employed to seal lubricant-engine systems to protect them from the elements.

It has been established and adequately documented that the lubricants required by the sophisticated mechanical system and power plants of today cannot be formulated by simple base stocks and additives. Lubricants used today are complex mixtures of base stocks incorporating a combination of chemical additives to achieve the desired performance characteristics. In addition to base stocks, lubricants contain additives designed to effect:

oxidation inhibition
cleansing action – detergents
dispersing action – dispersants
counteraction of extreme pressure
corrosion inhibition
depression of pour point
improved viscosity
rust inhibition
chatter prevention
squawk prevention
foam prevention
gelling
thickening

At the same time, lubricants must remain homogeneous over broad temperature ranges.

In order to insure that lubricating oils meet these stringent requirements as are appropriate, a number of laboratory physical and chemical tests are performed for characterization, quality control purposes, and evaluation. The laboratory tests include measurements of viscosity, specific gravity, flash point, calcium, sodium, barium, zinc, potassium, carbon, nitrogen, magnesium, sulphur, ash, base number/acid number, pour point, and boron.

In addition to the laboratory physical and chemical tests performed on candidate lubricating oils, engine dynamometer, multicylinder sequence tests are also performed to evaluate engine oils. These tests are designed to evaluate tendencies toward rusting, wearing, oxidation (stability), ring sticking, corrosive tendency, and accumulation of deposits both in the crankcase and on the surface of the mechanical parts.

The purpose of the work reported herein is to seek correlations between the laboratory physical and chemical test data and the performance of lubricating oils in the dynamometer sequence tests, to examine the interdependence of the tests, and from the results to comment on the significance of the tests.

II. BACKGROUND

A number of studies have been made on lubricating oil base stocks, and meaningful relationships have been established between certain measurable properties. The relationship between thermal properties and inspection tests was reviewed by Cragoe[1] in 1929. Physical properties and chemical composition were summarized by Van Nes and Van Westen[2] and Waterman.[3] Van Nes and Van Westen showed that $\%C_A$, $\%C_N$, $\%C_P$, R_T, R_A, and R_N[4] could be calculated from the refractive index, density, and molecular weight. These chemical properties have originally been determined using elemental chemical analyses, thus illustrating the interdependence of the chemical analyses and the inspection tests. Waterman described a similar method of carbon-type analysis using measurements of viscosity, refractive index, and density and reported correlations of ultrasonic viscosity and surface tension, Faraday effect, and parachor with other physical constants and the chemical composition.

When large numbers of tests are available for the characterization of a material, it is generally realized that many of the tests are interrelated, and some of these relationships may be known. The relationships among specific groups of tests can be readily established by using the statistical technique known as regression analysis. What is not generally known is how many truly independent properties are being measured by any group of tests.

1 C. S. Cragoe, "Thermal Properties of Petroleum Products," N. B. S. Publication # 97 (1929).

2 K. Van Nes and H. A. Van Westen, "Aspects of the Constitution of Mineral Oils," Amsterdam; Elsevier (1951).

3 H. I. Waterman, "Correlation Between Physical Constants and Chemical Structure," Elsevier (1958), p. 11; Anal. Chem. Actor (1958), *18*, p.5.

4 $\%C_A$: Percentage carbon in aromatic ring.
$\%C_N$: Percentage carbon in naphthenic ring.
$\%C_P$: Percentage carbon not in ring structures.
R_T: Number of rings per molecule.
R_A: Number of aromatic rings per average molecule.
R_N: Number of naphthenic rings per average molecule.

This initial study of lubricating oils had several goals. One was to obtain some base-line values of any randomly selected group of oils such that any particular oil could be compared with the group as a whole. Another goal was to see if there were any relationships among the various chemical properties of oils which could be exploited in such a way as to achieve a better understanding of the properties of a lubricating oil which make for a better lubrication. A third goal was to see what relationships existed for the various oils such that, hopefully, some of the tests performed on oils could be determined as redundant.

III. EXPERIMENTAL AND ANALYTICAL PROCEDURES

The 80 lubricating oils studied included preparations from a variety of base stocks. In addition, the additives employed were from a variety of suppliers and were not individually characterized.

The results of the chemical analysis of 80 lubricating oils were obtained from the files of the Fuels and Lubricating Division.

The properties of the 80 samples taken for the study are shown in Table 1. The samples were identified simply by number, and the test results were taken randomly from oil samples submitted for approval under Specification MIL-L-2104. The test results were taken from an variety of laboratories certified to perform the tests. A tabulation of the physical and chemical properties of an oil can assist the user and the oil refiner in defining a consistently uniform product. While the physical and chemical properties of an oil (discussed in this report) do not in themselves define oil performance, these individual oil properties are meaningful and are related to the ability of the oil to fulfill its function as a lubricant. Crude petroleum oils have as principal components three basic types of hydrocarbon molecules, i.e., paraffinic, naphthenic, and aromatic. The types of molecules that predominate are a basis for the classification of oils. Crude oils are typified as belonging to one of four classes: paraffinic, naphthenic, asphaltic, and mixed base. In the paraffinic type, paraffinic hydrocarbons predominate; in the naphthenic type, naphthenic hydrocarbons predominate; in the asphaltic type, naphthenic and aromatic hydrocarbons exist together; and in the mixed type, paraffinic, naphthenic, and aromatic types exist together. Crude oils as they come from the ground can be mixtures of gaseous products, gasolines, diesel fuels, lubricating oil stocks, asphalt, etc. The various classes of products are separated primarily through distillation. Precipitation of the heaviest viscous fractions using a solvent is also practiced. The lubricating oil fractions resulting therefrom provide a series of base stocks of varying volatility and viscosity. These base stocks are referred to as neutral fractions and bright stock. These fractions generally require further refining plus the addition of additives to make them suitable for engine oil applications. The complete chemical and physical data collected on engine oils is

Table 1. Properties of 80 Lubricating Oils Selected for Study

G	FL	V_{210}	V_{100}	V_o EX	VI	TAN	CRAM	SAH	S	P	CA	PP
25.70	450.0	12.67	135.10	20300.0	93.0	1.6	1.20	1.57	.18	.960	.090	-5.00
27.10	460.0	12.64	125.60	5400.0	100.0	2.9	.90	.97	.52	.090	.140	-5.00
26.50	440.0	12.44	118.40	12000.0	104.0	1.7	1.45	1.52	.88	.065	.390	-5.00
26.80	430.0	11.28	98.91	9200.0	110.0	2.3	1.31	1.75	1.44	.080	.410	-15.00
27.90	455.0	12.03	113.90	10600.0	104.0	3.4	.95	1.12	.48	.100	.040	-10.00
26.50	465.0	11.90	112.40	12200.0	100.0	2.0	1.69	1.32	.99	.120	.360	-5.00
25.80	490.0	12.24	122.60	15900.0	98.0	2.6	1.20	1.49	1.17	.070	.380	-5.00
27.40	462.0	11.70	111.40	11100.0	102.0	2.9	1.09	1.35	1.26	.100	.340	0.00
26.14	435.0	11.62	113.30	12000.0	100.0	3.5	1.20	1.20	1.24	.100	.040	0.00
25.90	495.0	12.29	123.70	15000.0	95.0	1.7	1.20	1.57	1.28	.054	.447	0.00
27.00	480.0	10.09	87.61	8000.0	104.0	1.8	1.14	1.11	1.03	.082	.295	0.00
28.10	470.0	12.22	121.62	17000.0	99.0	2.0	1.49	1.50	.63	.091	.406	-15.00
28.10	490.0	12.10	118.20	13600.0	99.0	2.7	1.21	1.10	.43	.093	.280	0.00
25.72	460.0	17.34	112.60	20100.0	95.0	3.5	1.09	1.24	1.05	.100	.040	0.00
28.20	440.0	10.50	94.10	9000.0	102.0	1.9	1.02	.96	1.00	.106	.242	0.00
26.10	485.0	12.70	125.00	14800.0	101.0	2.3	1.40	1.36	1.10	.092	.357	0.00
26.10	465.0	12.33	120.47	14500.0	101.0	2.1	1.28	1.25	1.41	.086	.340	0.00
25.80	465.0	12.14	118.14	14000.0	101.0	3.2	2.31	2.37	1.14	.121	.650	0.00
26.80	465.0	12.16	118.68	13000.0	100.0	3.6	1.88	1.60	1.02	.123	.429	-10.00
26.70	460.0	12.18	120.90	15300.0	99.0	2.5	.75	1.07	.42	.083	.215	-5.00
25.00	445.0	12.40	133.95	27500.0	90.0	2.4	1.10	.91	.30	.065	.150	0.00
23.00	425.0	12.80	155.50	99999.9	72.0	2.8	1.70	1.82	.71	.080	.460	-5.00
27.00	480.0	10.09	87.61	8000.0	104.0	1.8	1.14	1.11	1.03	.082	.295	0.00
27.50	460.0	11.84	113.28	14000.0	96.0	2.0	1.00	1.35	.47	.084	.374	-10.00
27.40	500.0	12.20	117.20	13000.0	102.0	2.5	1.10	.93	.29	.065	.018	-5.00
27.60	485.0	12.21	117.10	13000.0	101.0	1.5	1.00	.99	.26	.060	.250	0.00
27.70	455.0	11.36	113.80	15300.0	94.0	2.8	1.10	.97	.33	.080	.150	-5.00
28.60	471.0	12.18	112.46	11000.0	107.0	1.9	1.27	1.35	.32	.090	.360	-9.00
28.00	460.0	12.36	113.61	12000.0	109.0	2.4	1.66	2.04	.37	.092	.580	0.00
28.00	470.0	12.60	120.40	15000.0	101.0	2.2	1.54	1.54	.41	.090	.400	-5.00
26.20	480.0	10.83	100.23	11000.0	100.0	2.0	1.35	1.38	1.20	.071	.370	0.00
27.20	500.0	11.46	112.63	10500.0	93.0	2.7	1.30	1.18	.88	.100	.430	0.00
26.70	440.0	12.62	116.10	11200.0	110.0	1.6	1.60	1.60	.89	.100	.430	-25.00
26.60	480.0	12.12	120.30	15000.0	98.0	2.3	1.38	1.52	.80	.066	.380	-10.00
26.70	475.0	11.23	118.83	15000.0	94.0	1.9	1.23	1.23	.81	.072	.333	0.00
25.50	465.0	11.80	112.23	12500.0	101.0	1.9	1.94	2.09	1.15	.089	.560	0.00
26.70	460.0	12.18	120.90	15300.0	99.0	2.5	.07	1.07	.42	.083	.215	-5.00
26.60	478.0	12.36	121.00	[illegible]	100.0	1.8	1.55	1.39	.57	.060	.384	-10.00
25.60	498.0	11.97	137.96	10000.0	109.0	2.0	1.55	1.58	.46	.085	[illegible]	-5.00
25.50	470.0	12.22	123.00	19500.0	92.0	1.6	1.75	1.70	.45	.090	.410	-5.00
26.00	450.0	12.36	120.36	14100.0	101.0	1.8	1.54	1.56	1.20	.130	.440	-15.00
25.80	450.0	12.37	135.40	18000.0	96.0	2.1	1.25	.97	.46	.170	.230	-20.00

25.60	470.0	12.83	132.00	17800.0	91.0	2.2	.95	.95	.40	.095	.220	-5.00
25.80	430.0	12.45	127.30	19000.0	96.0	6.7	1.28	1.00	.55	.130	.240	-5.00
25.80	425.0	12.21	125.60	17500.0	95.0	2.5	1.29	1.00	.45	.120	.240	-20.00
25.60	465.0	12.45	139.01	24000.0	86.0	1.6	1.47	1.00	.47	.100	.250	-25.00
27.00	465.0	12.38	124.50	14000.0	98.0	2.2	1.05	1.48	.43	.099	.410	-5.00
27.00	445.0	12.40	127.10	99999.9	96.0	2.6	1.10	.96	.40	.065	.150	0.00
25.00	445.0	12.40	133.95	21500.0	90.0	2.4	1.10	.91	.30	.065	.150	0.00
24.90	450.0	12.43	146.30	31010.0	80.0	2.7	1.46	1.34	.54	.123	.321	0.00
26.00	450.0	12.45	125.60	15094.0	100.0	3.1	1.24	.98	.48	.093	.260	0.00
26.00	450.0	12.45	123.60	15094.0	100.0	3.1	1.24	.98	.48	.093	.260	0.00
25.20	470.0	11.34	117.91	18000.0	89.0	2.0	1.56	1.60	1.25	.078	.440	0.00
26.10	460.0	12.35	123.00	17000.0	99.0	2.1	1.49	1.55	.88	.100	.420	-5.00
28.20	440.0	10.50	94.10	9000.0	102.0	1.9	1.02	.96	1.00	.106	.292	0.00
24.80	445.0	11.53	115.80	16300.0	93.0	3.1	1.46	1.62	.93	.090	.320	-5.00
25.30	465.0	11.67	122.43	99999.9	90.0	1.9	1.17	.93	.34	.086	.233	-5.00
25.00	445.0	12.40	133.95	21500.9	90.0	2.4	1.10	.91	.30	.065	.150	0.00
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26.70	460.0	12.18	120.90	15380.0	99.0	2.5	.75	1.07	.42	.083	.215	-5.00
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25.00	445.0	12.40	133.95	21500.0	90.0	2.4	1.10	.90	.30	.065	.150	0.00
27.70	455.0	11.36	113.80	15300.0	94.0	2.8	1.10	.97	.33	.080	.150	-5.00
27.90	450.0	11.76	111.32	14100.0	102.0	2.3	1.14	.91	.40	.090	.242	-5.00
27.70	455.0	11.36	113.80	15300.0	94.0	2.8	1.10	.97	.33	.080	.150	-5.00
25.60	475.0	12.69	130.19	17500.0	94.0	2.0	1.49	1.51	1.00	.085	.439	-10.00
28.50	450.0	12.30	120.00	14000.0	101.0	2.4	1.72	1.68	.44	.090	.449	0.00
28.50	480.0	12.30	121.20	14500.0	100.0	1.8	1.10	.95	.34	.070	.240	0.00
26.70	460.0	12.18	120.90	15380.0	99.0	2.5	.75	1.07	.42	.083	.215	-5.00
25.80	480.0	12.65	123.80	17500.0	97.0	2.7	1.10	.98	.43	.090	.140	0.00
25.80	490.0	12.75	127.53	15000.0	100.0	2.2	1.47	1.51	.84	.079	.410	0.00
28.30	475.0	12.22	120.40	14700.0	100.0	1.6	1.12	.96	1.09	.075	.150	-5.00
27.60	480.0	12.20	122.20	15500.0	97.0	2.5	1.17	1.50	.42	.090	.400	0.00
23.50	444.0	12.57	130.00	17000.0	95.0	10.0	1.50	1.65	.91	.094	.420	0.00
26.50	455.0	11.85	117.00	12000.0	96.6	2.8	2.00	1.74	.40	.070	.420	-5.00
27.70	455.0	11.35	113.80	15300.0	94.0	2.8	1.10	.97	.33	.080	.150	-5.00
26.40	505.0	12.30	127.00	18450.0	95.0	2.4	1.55	1.60	.35	.090	.310	-5.00
26.90	450.0	11.92	125.60	18000.0	91.0	2.3	1.10	1.00	.39	.090	.240	0.00
25.80	480.0	12.88	138.70	21140.0	93.0	2.2	1.15	.96	.41	.090	.130	-5.00

shown in Table 2 and includes 26 pieces of information. Due to the lack of complete results on all 80 samples, 13 items of information have been selected for these studies. They are shown in Table 3.

The initiation of this study includes a plot of a typical set of test results (Gravity, G) as shown in Figure 1 and a presentation of the gravity test measurement in distribution form as shown in Figure 2.

An initial statistical survey of these oils, those with a sufficiently large data set, was performed using the *Statistical Package for the Social Sciences (SPSS)*[5] and a CDC computer. These are summarized in Table 3. They include the mean, variance, range, standard error, Kurtosis, maximum, minimum, skewness, and standard deviation where Kurtosis = measure of relative peakedness or flatness of curve defined by the distribution of cases. (A normal distribution will have a Kurtosis of zero.) When the Kurtosis is positive, then the distribution is more peaked (narrow) than it would be for a normal distribution while a negative value means that it is flatter:

$$\text{Kurtosis} = \frac{\sum_{i=1}^{N} [(X_i - \bar{X})/S]^4}{N}$$

The computing formula used by *SPSS* is:

$$\text{Kurtosis} = \frac{\left\{\left[\sum_{i=1}^{N} X_i^4 - 4\bar{X}\left(\sum_{i=1}^{N} X_i^3\right) + 6\bar{X}^2 / \sum_{i=1}^{N} X_i^2 - 4\bar{X}^3 \left(\sum_{i=1}^{N} X_i\right)\right] /N\right\} + \bar{X}^4 - 3}{\left\{\left[\left(\sum_{i=1}^{N} X_i^2\right) - N\bar{X}^2\right] /(N-1)\right\}^2}$$

In looking at the Kurtosis of the data as it related to individual parameters, it is seen that the data for individual sets do not fall together. Kurtosis measures the degree of peakedness exhibited in individual sets of data, and a Kurtosis of zero corresponds to a normal distribution. Large, positive values indicate more peakedness (narrow) than for a normal distribution. Negative values mean a curve flatter than normal distribution. In the data collected in these tests, it is seen that for G, the API gravity; FL, the flash point; SAH, sulphated ash residue; and CA, calcium, the value of Kurtosis is approaching that of a normal distribution – a value less than 1. Values less than 5 and more than 1 can be assigned to only four additionally measured parameters: pp, pour point; CRAM, Ramsbottom Carbon; VI, Viscosity Index; and V 100, Viscosity 100°F.

[5] Norman H. Nir, C. Hadlai Hull, Jean G. Jenkins, Karin Steinbrenner, and Dale H. Bent, *Statistical Package for the Social Sciences*, 2nd Edition, McGraw Hill Book Company, New York.

Table 2. Complete Chemical and Physical Data on Engine Oils

IDENT	G*	FL	V_{210}	V_{100}	V_0AP	V_0EX	VIS_{20}	VI	PP	STPP	TBN	TBNN	TAN	CRAM	SAH	S	P	CA	BA	ZN	NA	K	N	MG	B	OTHER
MC0459	25.70	450.0	12.67	135.10	99999.9	20300.0	9999.9	93.0	-5.00	99.9	99.0	99.9	1.60	1.20	1.57	.250	.050	.410	.0250	.040	99.9	99.9	.080	9.93	9.99	
MC0423	27.10	460.0	12.64	125.60	99999.9	5400.0	9999.9	100.0	-5.00	99.9	99.0	99.9	2.90	.90	.97	.420	.090	.140	.1800	.090	99.9	99.9	.090	9.99	9.99	
MC0301	26.50	440.0	12.44	118.40	99999.9	12000.0	9999.9	104.0	-5.00	99.9	99.0	99.9	1.70	1.45	1.52	.800	.065	.390	.0200	.060	99.9	99.9	.060	9.99	9.99	
MC0403	26.80	430.0	11.28	38.91	99999.9	9200.0	9999.9	113.0	-15.00	99.9	99.0	99.9	2.27	1.31	1.75	1.440	.080	.410	.1400	.040	99.9	99.9	.040	9.99	9.99	
MC0213	27.90	455.0	12.03	113.00	99999.9	10600.0	9999.9	104.0	-10.00	99.9	99.0	99.9	3.35	.95	1.12	.480	.100	.0400	.0000	.070	0.0	0.0	.070	.17	9.99	
MC0006	25.50	465.0	11.90	112.40	99999.9	12200.0	9999.9	107.0	-5.00	99.9	99.0	99.9	2.00	1.69	1.32	.990	.120	.360	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0120	25.80	490.0	12.24	122.60	99999.9	15900.0	9999.9	98.0	-5.00	99.9	99.0	99.9	2.60	1.20	1.49	1.170	.070	.380	.0260	.080	99.9	99.9	.080	9.99	9.99	
MC0121	27.40	482.0	11.70	111.40	99999.9	11100.0	9999.9	102.0	0.00	99.9	99.0	99.9	2.30	1.09	1.35	1.240	.100	.340	.9999	.040	99.9	99.9	.040	9.99	9.99	
MC0207	26.14	435.0	11.62	113.30	99999.9	12000.0	9999.9	100.0	0.00	99.9	99.0	99.9	3.50	1.20	1.20	1.240	.100	.0400	.0000	.070	0.0	0.0	.070	.17	9.99	
MC0268	25.90	495.0	12.29	123.70	99999.9	16000.0	9999.9	98.0	0.00	99.9	99.0	99.9	1.70	1.20	1.57	1.280	.064	.647	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0284	27.00	480.0	10.09	87.61	99999.9	8000.0	9999.9	104.0	0.00	99.9	99.0	6.7	1.79	1.14	1.11	1.030	.082	.295	.9999	.010	99.9	99.9	.010	9.99	9.99	
MC0172	28.10	470.0	12.22	121.62	99999.9	17000.0	9999.9	93.0	-15.00	99.9	99.0	99.9	1.96	1.49	1.50	.630	.091	.436	.9999	.041	99.9	99.9	.041	9.99	9.99	
MC0289	28.10	490.0	12.10	118.20	99999.9	13600.0	9999.9	93.0	0.00	99.9	99.0	99.9	2.70	1.21	1.10	.430	.093	.280	.9999	.042	99.9	99.9	.042	9.99	9.99	
MC0138	25.72	460.0	12.34	112.60	99999.9	20100.0	9999.9	95.0	0.00	99.9	99.0	99.9	3.45	1.09	1.24	1.050	.100	.0400	.0000	.070	0.0	0.0	.070	.17	9.99	
MC0203	28.20	440.0	10.50	94.10	99999.9	9000.0	9999.9	102.0	0.00	99.9	99.0	99.9	1.90	1.02	.96	1.000	.106	.2420	.0000	.080	0.0	0.0	.080	9.99	9.99	
MC0256	26.10	485.0	12.70	126.00	99999.9	14800.0	9999.9	101.0	0.00	99.9	99.0	68.7	2.34	1.40	1.36	1.100	.092	.3570	.0000	.070	0.0	0.0	.070	0.03	9.99	
MC0276	26.10	465.0	12.33	120.47	99999.9	14500.0	9999.9	101.0	0.00	99.9	99.0	99.9	2.05	1.28	1.25	1.410	.486	.340	.9999	.030	99.9	99.9	.030	9.99	9.99	
MC0458	25.80	465.0	12.14	118.14	15000.0	14000.0	9999.9	101.0	0.00	99.9	99.0	99.9	3.17	2.31	2.37	1.140	.121	.650	.9999	.038	99.9	99.9	.038	9.99	9.99	
MC0156	26.80	465.0	12.16	118.68	99999.9	13000.0	9999.9	101.0	-10.00	99.9	99.0	99.9	3.57	1.88	1.60	1.020	.123	.429	.9999	.040	99.9	99.9	.040	9.99	9.99	
MC0294	26.70	460.0	12.18	120.90	99999.9	15380.0	9999.9	93.0	-5.00	99.9	99.0	4.1	2.50	.75	1.07	.420	.083	.215	.1450	.048	99.9	99.9	.048	9.99	9.99	
MC0153	25.00	445.0	12.40	132.95	99999.9	27500.0	9999.9	90.0	0.00	99.9	99.0	95.9	2.40	1.10	.91	.300	.065	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0269	23.00	425.0	12.80	156.50	99999.9	99999.9	9999.9	78.0	-5.00	99.9	99.0	99.9	2.75	1.70	1.82	.710	.080	.460	.1500	.040	99.9	99.9	.040	9.99	9.99	
MC0278	27.00	480.0	10.09	87.61	99999.9	8000.0	9999.9	104.0	0.00	99.9	99.0	6.7	1.79	1.14	1.11	1.030	.082	.295	.9999	.010	99.9	99.9	.010	9.99	9.99	
MC0111	27.50	460.0	11.84	118.28	99999.9	14000.0	9999.9	95.0	-10.00	99.9	99.0	99.9	2.00	1.00	1.35	.467	.084	.374	.9999	.007	99.9	99.9	.007	9.99	9.99	
MC0130	27.40	500.0	12.20	117.20	99999.9	13000.0	9999.9	102.0	-5.00	99.9	99.0	99.9	2.50	1.10	.93	.290	.065	.018	.1900	.080	99.9	99.9	.080	9.99	9.99	
MC0176	27.60	485.0	12.21	117.10	99999.9	13000.0	9999.9	101.0	0.00	99.9	99.0	99.9	1.50	1.00	.99	.260	.060	.250	.9999	0.000	0.0	0.0	0.000	0.00	0.00	
MC0185	27.70	455.0	11.36	113.80	99999.9	15300.0	9999.9	94.0	-5.00	99.9	99.0	95.4	2.80	1.10	.97	.330	.080	.150	.1700	.080	0.0	0.0	.080	9.99	0.00	
MC0470	28.60	471.0	12.18	112.40	99999.9	11000.0	9999.9	107.0	-9.00	99.9	99.0	98.0	1.90	1.27	1.35	.320	.090	.3600	.0000	.050	0.0	0.0	.050	0.00	0.00	
MC0131	28.00	460.0	12.36	113.61	99999.9	12000.0	9999.9	103.0	0.00	99.9	99.0	99.9	2.44	1.66	2.04	.374	.092	.580	.9999	.240	99.9	99.9	.240	9.99	9.90	
MC0180	28.00	470.0	12.60	120.40	99999.9	15000.0	9999.9	101.0	-5.00	99.9	99.0	10.5	2.20	1.54	1.54	.410	.090	.400	.0400	.060	0.0	0.0	.060	0.03	0.00	
MC0305	26.20	480.0	10.83	100.23	99999.9	11000.0	9999.9	100.0	0.00	99.9	99.0	99.9	2.00	1.35	1.38	1.200	.071	.370	.9999	.010	99.9	99.9	.010	9.99	9.90	
MC0112	27.2	500.0	11.46	110.63	99999.9	11500.0	9999.9	99.0	0.00	99.9	99.0	99.9	2.71	1.[illegible]	1.88	.880	.090	.430	.1400	.040	99.9	99.9	.040	9.99	9.90	
MC0137	28.70	440.0	12.62	116.10	99999.9	11200.0	9999.9	[illegible]	-25.00	99.9	99.0	99.9	[illegible]	[illegible]	1.60	.890	.100	.430	.9999	.020	99.9	99.9	.020	9.99	9.90	
MC0177	26.60	480.0	12.12	120.30	99999.9	15000.0	9999.9	98.0	-10.00	99.9	99.0	99.9	2.30	1.30	1.52	.800	.066	.380	.0210	.064	99.9	99.9	.064	9.99	9.90	
MC0212	26.70	475.0	11.23	118.83	99999.9	15000.0	9999.9	94.0	0.00	99.9	99.0	99.9	1.87	1.23	1.23	.606	.072	.333	.9999	.004	99.9	99.9	.004	9.99	9.90	
MC0239	25.50	465.0	11.80	113.23	99999.9	12500.0	9999.9	101.0	0.00	99.9	99.0	15.0	1.93	1.94	2.09	1.150	.089	.560	.9999	.010	99.9	99.9	.010	9.99	9.90	
MC0292	26.70	460.0	12.18	120.90	99999.9	15380.0	9999.9	99.0	-5.00	99.9	99.0	4.1	2.50	.75	1.07	.420	.083	.215	.1450	.048	99.9	99.9	.048	9.99	9.90	
MC0210	26.60	470.0	12.36	121.99	99999.9	15000.0	9999.9	100.0	-10.00	99.9	99.0	99.9	1.77	1.45	1.39	.569	.060	.384	.9999	.013	99.9	99.9	.013	9.99	9.90	
MC0135	26.60	460.0	11.97	107.96	99999.9	10000.0	9999.9	103.0	-5.00	99.9	99.0	99.9	2.77	1.65	1.58	.460	.085	.450	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0037	25.50	470.0	12.22	129.00	99999.9	19500.0	9999.9	92.0	-5.00	99.9	99.0	11.4	1.60	1.75	1.70	.450	.090	.410	.3500	.060	99.9	99.9	.060	9.99	9.99	
MC0447	26.00	450.0	12.36	120.36	99999.9	14100.0	9999.9	101.0	-15.00	99.9	99.0	99.9	1.81	1.56	1.56	1.200	.130	.440	.9999	.020	99.9	99.9	.020	9.99	9.99	
MC0479	25.80	450.0	12.37	125.40	99999.9	18000.0	9999.9	96.0	-20.00	99.9	99.0	99.9	2.05	1.25	.97	.460	.170	.2300	.0000	.052	0.0	0.0	.052	0.00	0.00	
MC0250	25.60	470.0	12.83	132.00	99999.9	17800.0	9999.9	91.0	-5.00	99.9	99.0	99.9	2.20	.95	.95	.400	.095	.220	.9999	.070	99.9	99.9	.070	9.99	9.99	
MC0306	25.80	430.0	12.45	127.30	99999.9	19000.0	9999.9	95.0	-5.00	99.9	99.0	4.2	6.73	1.28	1.00	.550	.130	.240	.9999	.060	99.9	99.9	.060	9.99	9.99	
MC0318	25.80	425.0	12.21	125.60	99999.9	17500.0	9999.9	95.0	-20.00	99.9	99.0	99.9	2.50	1.29	1.00	.450	.120	.2400	.0000	.051	0.0	0.0	.051	0.03	0.00	
MC0316	25.60	465.0	12.45	139.01	99999.9	24000.0	9999.9	85.0	-25.00	99.9	99.0	5.9	1.60	1.47	1.00	.470	.100	.250	.9999	.022	99.9	99.9	.022	9.99	9.99	
MC0031	27.00	465.0	12.38	124.50	99999.9	14000.0	9999.9	98.0	-5.00	99.9	99.0	99.9	2.20	1.05	1.48	.430	.099	.410	.9999	.075	99.9	99.9	.075	9.99	9.99	
MC0113	27.00	445.0	12.40	127.10	99999.9	99999.9	9999.9	96.0	0.00	99.9	99.0	99.9	2.60	1.10	.96	.400	.065	.150	.1800	.080	99.9	99.9	.080	9.99	9.99	
MC0183	25.00	445.0	12.40	133.95	99999.9	21500.0	9999.9	93.0	0.00	99.9	99.0	95.9	2.40	1.10	.91	.300	.065	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0220	26.90	450.0	12.43	146.30	99999.9	31010.0	9999.9	83.0	0.00	99.9	99.0	9.7	2.70	1.46	1.34	.540	.123	.3210	.0000	.097	0.0	0.0	.097	0.03	9.99	

Table 2. Complete Chemical and Physical Data on Engine Oils (Cont'd)

IDENT	G*	FL	V_{210}	V_{100}	V_0AP	V_0EX	VIS_{20}	VI	PP	STPP	TBN	TBNN	TAN	CRAM	SAH	S	P	CA	BA	ZN	NA	K	N	MG	B	OTHER
MC0182	26.00	450.0	12.45	126.60	99999.9	15094.0	9999.9	100.0	0.00	99.9	99.0	9.9	3.10	1.24	.98	.480	.093	.260	.9999	.014	0.0	0.0	.014	0.00	0.00	
MC0604	25.00	450.0	12.45	123.60	99999.9	15094.0	9999.9	100.0	0.00	99.9	99.0	9.9	3.10	1.24	.98	.480	.093	.250	.9999	.014	0.0	0.0	.014	0.00	0.00	
MC0181	25.20	470.0	11.34	117.91	99999.9	18000.0	9999.9	89.0	0.00	99.9	99.0	9.9	2.04	1.56	1.60	1.250	.070	.440	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0136	25.10	460.0	12.35	123.00	99999.9	17000.0	9999.9	99.0	-5.00	99.9	99.0	9.9	2.07	1.49	1.55	.880	.100	.420	.9999	.020	99.9	99.9	.020	9.99	9.99	
MC0199	28.20	440.0	10.50	94.10	99999.9	9000.0	9999.9	102.0	0.00	99.9	99.0	9.9	1.90	1.02	.96	1.000	.106	.292	0.0000	.020	0.0	0.0	.020	0.00	0.00	
MC0022	24.80	445.0	11.53	116.80	99999.9	16300.0	9999.9	93.0	-5.00	99.9	99.0	9.9	3.10	1.46	1.62	.930	.090	.320	.1900	.070	99.9	99.9	.070	9.99	9.99	
MC0102	25.30	465.0	11.67	122.43	99999.9	99999.9	9999.9	90.0	-5.00	99.9	99.0	9.9	1.90	1.17	.93	.342	.086	.233	.9999	.007	99.9	99.9	.007	9.99	9.99	
MC0161	25.00	445.0	12.40	133.95	99999.9	21500.9	9999.9	90.0	0.00	99.9	99.0	95.9	2.40	1.10	.91	.300	.065	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0319	25.60	460.0	12.12	121.57	99999.9	15000.0	9999.9	97.0	-5.00	99.9	99.0	99.9	2.23	1.19	.85	7.859	.083	.243	.9999	.008	99.9	99.9	.008	9.99	9.99	
MC0277	26.10	465.0	12.33	120.47	99999.9	14500.0	9999.9	101.0	0.00	99.9	99.0	99.9	2.05	1.28	1.25	1.410	.086	.340	.9999	.030	99.9	99.9	.030	9.99	9.99	
MC0140	25.70	460.0	12.18	120.90	99999.9	15380.0	9999.9	99.0	-5.00	99.9	99.0	99.9	2.50	.75	1.07	.420	.083	.215	.1450	.048	99.9	99.9	.048	9.99	9.99	
MC0221	25.50	450.0	11.60	111.00	99999.9	12740.0	9999.9	100.0	-5.00	99.9	99.0	99.9	1.60	1.47	.95	.350	.060	.150	.1700	.070	99.9	99.9	.070	9.99	9.99	
MC0245	25.00	445.0	12.40	133.95	99999.9	21500.0	9999.9	90.0	0.00	99.9	99.0	5.9	2.40	1.10	.90	.300	.065	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0246	27.70	455.0	11.36	113.80	99999.9	15300.0	9999.9	94.0	-5.00	99.9	99.0	5.4	2.80	1.10	.97	.330	.080	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0117	27.90	450.0	11.76	111.32	99999.9	14100.0	9999.9	102.0	-5.00	99.9	99.0	99.9	2.30	1.14	.91	.400	.090	.242	.9999	.002	99.9	99.9	.002	9.99	.02	
MC0172	27.70	455.0	11.36	113.80	99999.9	15300.0	9999.9	94.0	-5.00	99.9	99.0	95.4	2.80	1.10	.97	.330	.080	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0273	25.60	475.0	12.69	130.19	99999.9	17500.0	9999.9	96.0	-10.00	99.9	99.0	9.9	1.90	1.49	1.51	1.000	.085	.439	.9999	.003	99.9	99.9	.003	9.99	9.99	
MC0178	28.50	450.0	12.30	120.00	99999.9	14000.0	9999.9	101.0	0.00	99.9	99.0	9.9	2.40	1.72	1.68	.440	.090	.443	.9999	.020	99.9	99.9	.020	9.99	.01	
MC0215	28.00	480.0	12.30	121.20	99999.9	14500.0	9999.9	100.0	0.00	99.9	99.0	6.3	1.80	1.10	.95	.340	.070	.240	.9999	.012	99.9	99.9	.012	9.99	9.99	
MC0291	25.70	460.0	12.18	120.90	99999.9	15380.0	9999.9	99.0	-5.00	99.9	99.0	4.1	2.50	.75	1.07	.420	.083	.215	.1450	.048	99.9	99.9	.048	9.99	9.99	
MC0297	25.80	480.0	12.66	129.80	99999.9	17500.0	9999.9	97.0	0.00	99.9	99.0	3.0	2.60	1.10	.90	.430	.090	.140	.1600	.080	99.9	99.9	.080	9.99	9.99	
MC0331	25.80	490.0	12.75	127.53	99999.9	15000.0	9999.9	100.0	0.00	99.9	99.0	99.9	2.24	1.47	1.51	.840	.079	.410	.9999	.999	99.9	99.9	.999	9.99	9.99	
MC0321	28.30	475.0	12.22	120.40	99999.9	14700.0	9999.9	100.0	-5.00	99.9	99.0	5.0	1.60	1.12	.96	1.090	.075	.150	.1700	.120	99.9	99.9	.120	9.99	9.99	
MC0063	27.60	480.0	12.21	122.20	99999.9	15500.0	9999.9	97.0	0.00	99.9	99.0	11.5	2.50	1.17	1.50	.420	.090	.400	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0139	23.50	444.0	12.57	130.00	99999.9	17000.0	9999.9	95.0	0.00	99.9	99.0	13.0	2.60	1.50	1.65	.910	.094	.420	.9999	.057	99.9	99.9	.057	9.99	9.99	
MC0610	25.50	455.0	11.85	117.00	99999.9	12000.0	9999.9	95.0	-5.00	99.9	99.0	12.0	2.60	2.00	1.74	.400	.070	.420	.1500	.030	99.9	99.9	.030	9.99	9.99	
MC0171	27.70	455.0	11.36	113.80	99999.9	15300.0	9999.9	94.0	-5.00	99.9	99.0	5.4	2.80	1.10	.97	.330	.080	.150	.1700	.080	99.9	99.9	.080	9.99	9.99	
MC0094	26.40	505.0	12.30	127.00	99999.9	18450.0	9999.9	95.0	-5.00	99.9	99.0	99.9	[illegible]	[illegible]	[illegible]	.350	.090	.310	.1900	.060	99.9	99.9	.060	9.99	9.99	
MC0209	26.90	450.0	11.92	125.60	99999.9	18000.0	9999.9	91.0	0.00	99.9	99.0	6.4	[illegible]	[illegible]	1.00	.390	.090	.240	.9999	.050	99.9	99.9	.050	9.99	9.99	
MC0267	25.80	480.0	12.38	138.70	99999.9	21140.0	9999.9	93.0	-5.00	99.9	99.0	5.1	[illegible]	[illegible]	[illegible]	.410	.090	.130	.1800	.090	99.9	99.9	.090	9.99	9.99	

Table 3. Statistical Survey of 13 Engine Oil Samples

VARIABLE G

MEAN	26.523	STD ERR	.127	STD DEV	1.134
VARIANCE	1.287	KURTOSIS	.270	SKEWNESS	-.311
RANGE	5.600	MINIMUM	23.000	MAXIMUM	28.600
VALID CASES	80	MISSING CASES	0		

VARIABLE FL

MEAN	462.150	STD ERR	1.972	STD DEV	17.634
VARIANCE	310.965	KURTOSIS	-.239	SKEWNESS	.203
RANGE	80.000	MINIMUM	425.000	MAXIMUM	505.000
VALID CASES	80	MISSING CASES	0		

VARIABLE V_{210}

MEAN	12.121	STD ERR	.093	STD DEV	.833
VARIANCE	.694	KURTOSIS	19.488	SKEWNESS	2.539
RANGE	7.250	MINIMUM	10.090	MAXIMUM	17.340
VALID CASES	80	MISSING CASES	0		

VARIABLE V_{100}

MEAN	120.172	STD ERR	1.263	STD DEV	11.295
VARIANCE	127.567	KURTOSIS	2.193	SKEWNESS	-.273
RANGE	68.890	MINIMUM	87.610	MAXIMUM	156.500
VALID CASES	80	MISSING CASES	0		

VARIABLE VoEX

MEAN	18320.607	STD ERR	1871.409	STD DEV	16738.393
VARIANCE	.2801E+09	KURTOSIS	20.140	SKEWNESS	4.497
RANGE	94599.900	MINIMUM	5400.000	MAXIMUM	99999.900
VALID CASES	80	MISSING CASES	0		

VARIABLE VI

MEAN	97.695	STD ERR	.641	STD DEV	5.732
VARIANCE	32.855	KURTOSIS	1.866	SKEWNESS	-.683
RANGE	32.000	MINIMUM	78.000	MAXIMUM	110.000
VALID CASES	80	MISSING CASES	0		

VARIABLE TAN

MEAN	2.396	STD ERR	.077	STD DEV	.693
VARIANCE	.480	KURTOSIS	17.966	SKEWNESS	3.171
RANGE	5.200	MINIMUM	1.500	MAXIMUM	6.700
VALID CASES	80	MISSING CASES	0		

Table 3. Statistical Survey of 13 Engine Oil Samples (Cont'd)

VARIABLE CRAM

MEAN	1.290	STD ERR	.033	STD DEV	.293
VARIANCE	.086	KURTOSIS	1.125	SKEWNESS	.767
RANGE	1.560	MINIMUM	.750	MAXIMUM	2.310
VALID CASES	80	MISSING CASES	0		

VARIABLE SA

MEAN	1.281	STD ERR	.038	STD DEV	.336
VARIANCE	.113	KURTOSIS	.114	SKEWNESS	.772
RANGE	1.520	MINIMUM	.850	MAXIMUM	2.370
VALID CASES	80	MISSING CASES	0		

VARIABLE S

MEAN	.760	STD ERR	.098	STD DEV	.877
VARIANCE	.768	KURTOSIS	55.711	SKEWNESS	6.889
RANGE	7.610	MINIMUM	.250	MAXIMUM	7.860
VALID CASES	80	MISSING CASES	0		

VARIABLE P

MEAN	.098	STD ERR	.010	STD DEV	.089
VARIANCE	.008	KURTOSIS	72.654	SKEWNESS	8.340
RANGE	.820	MINIMUM	.050	MAXIMUM	.870
VALID CASES	80	MISSING CASES	0		

VARIABLE CA

MEAN	.300	STD ERR	.015	STD DEV	.131
VARIANCE	.017	KURTOSIS	-.303	SKEWNESS	-.001
RANGE	.632	MINIMUM	.018	MAXIMUM	.650
VALID CASES	80	MISSING CASES	0		

VARIABLE PP

MEAN	-4.487	STD ERR	.638	STD DEV	5.704
VARIANCE	32.531	KURTOSIS	3.679	SKEWNESS	-1.830
RANGE	25.000	MINIMUM	-25.000	MAXIMUM	0
VALID CASES	80	MISSING CASES	0		

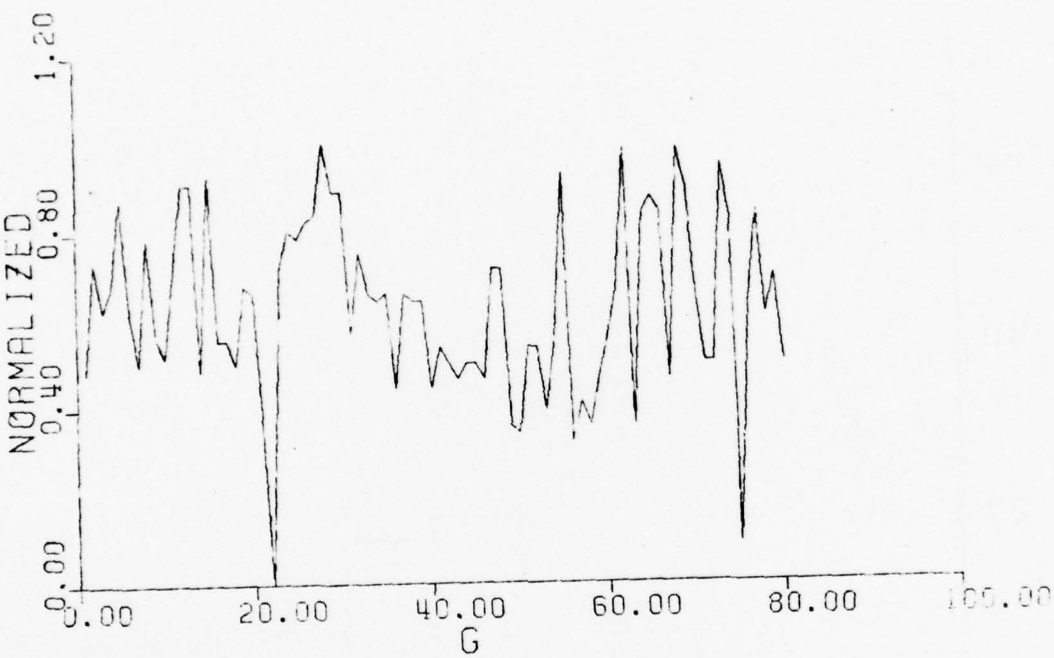

Figure 1. Plot of a typical set of test results.

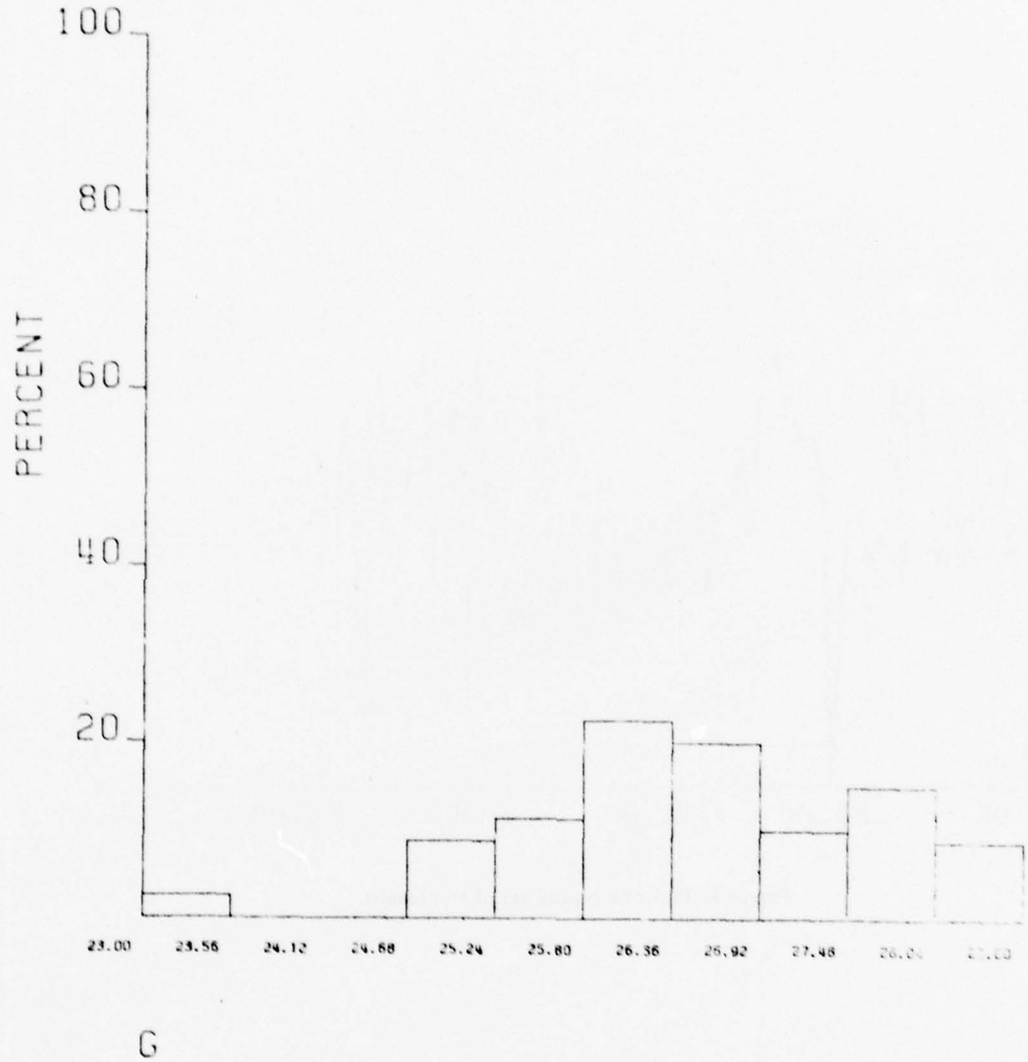

Figure 2. Gravity test measurement in distribution form.

The Kurtosis values measure the grouping of the values for individually measured properties:

"Mean" is the most common measure of central tendency for variables measured at the interval level:

$$\bar{X} = \text{mean} = \frac{\sum_{i=1}^{N} X_i}{N} .$$

"Variance" measures dispersion of data about the mean of the interval-level variable. This statistic is one way of measuring how closely the individual scores on the variable cluster around the mean. Mathematically, it is the average squared deviation from the mean:

$$s^2 = \frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N - 1} .$$

Squaring the deviations from the mean considers all differences from the mean both positive and negative, and it gives additional weight to extreme cases. The variance will be small when there is a great deal of homogeneity in the data; for, then, most cases will have very small deviations from the mean.

"Skewness" is a statistic used to determine the extent to which a distribution of cases fits or approximates a normal curve since it measures deviations from symmetry. Skewness takes a value of zero when the distribution is a complete, symmetric, bell-shaped curve. A positive value indicates that the cases are clustered more to the left of the mean with most of the extreme values to the right. A negative value indicates clustering to the right. Mathematically, skewness =

$$\frac{\sum_{i=1}^{N} [(X_i - \bar{X})/S]^3}{N} .$$

"Standard Error" helps to determine the potential degree of discrepancy between the sample mean and the usually unknown population mean. The standard error has properties very analogous to those of the standard deviations.

"Standard Deviation" measures the dispersion about the mean of an interval-level variable. It measures the square root of the variance:

$$\text{Standard Deviation} = \left[\frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^2\right]^{1/2}$$

The standard deviation is the positive square root of the variance and is another measure of variance. The standard deviation is, perhaps, the most important and most widely used measure of variability. A small volume of S denotes close clustering about the mean. A relatively large value represents wide scattering about the mean. The table shows relatively large values.

The range, maximum, and minimum present the usual extra data in the program. The statistical summary does not offer information adequately definitive to make conclusions required for the mission of this study. A Pearson's T test was performed with the *SPSS* package to establish any linear correlations among the various sets of data. A correlation of the various data sets was also performed with the IRM package in an attempt to further analyze the relationship between the various data sets.

The results of Pearson's correlation as performed with the *SPSS* are tabulated in Table 4. Output from this program included the correlation coefficient, the tests of significance, and the number of cases upon which the correlation coefficient was computed. Covariance, cross-product deviations for all combinations of pairs are shown in Table 5. In the graphical presentation of a typical set of data (Gravity, G, Figure 2), it appeared a priori unlikely to find a regression line, especially a straight one, which perfectly fits the data. Whether this is because the true relationship does not quite fit the curve being drawn or because of errors or imperfections in collecting the data, a measure of the goodness of fit of the regression line is desirable. The Pearson product moment correlation coefficient serves this purpose for linear regression. Where there is a fit (no error), it takes the value +1.0 or -1.0 where the sign is the same as the sign of the regression coefficient. A negative does not mean a bad fit; rather, it denotes an inverse relationship. When the linear-regression line is a poor fit to the data, it will be close to zero. The value of zero denotes the absence of a linear relationship.

If Pearson's coefficient is squared, we get another statistic which is a more easily interpreted measure of association when our concern is with the strength of relationship rather than with the direction of relationships. It varies from 0 to +10. (maximum). Its usefulness lies in the fact that the square of Pearson's coefficient is a measure of the proportion of variance in one variable explained by the other. A negative value for the correlation coefficient indicates a decrease in that property of the test with an increase in the compared property. Our test data in summary showed significant values of the correlation coefficients only for certain pairs (pp. 19 and 20).

Table 4. Results of Pearson's Correlation as Performed Using the SPSS

	G	FL	V_{210}	V_{100}	V_0EX	VI	TAN	CRAM	SAH	S	P	GA	PP
G	1.0000 (0) S= .001	.2194 (80) S= .051	-.3059 (80) S= .006	-.5897 (80) S= .001	-.3710 (80) S= .001	.5874 (80) S= .001	-.1197 (80) S= .290	-.2189 (80) S= .051	-.1664 (80) S= .140	-.1558 (80) S= .167	.1331 (80) S= .239	-.1234 (80) S= .275	-.0294 (80) S= .795
FL	.2194 (80) S= .051	1.0000 (0) S= .001	-.0111 (80) S= .922	-.1093 (80) S= .334	-.2136 (80) S= .057	.1551 (80) S= .169	-.2718 (80) S= .015	.0030 (80) S= .979	.1271 (80) S= .261	.0248 (80) S= .827	-.0641 (80) S= .573	.1306 (80) S= .248	.2201 (80) S= .050
V_{210}	-.3059 (80) S= .006	-.0111 (80) S= .922	1.0000 (0) S= .001	.5296 (80) S= .001	.1453 (80) S= .198	-.1900 (80) S= .091	.1820 (80) S= .106	.0264 (80) S= .816	.0603 (80) S= .595	-.0449 (80) S= .692	.0427 (80) S= .707	-.1111 (80) S= .326	-.0677 (80) S= .551
V_{100}	-.5897 (80) S= .001	-.1093 (80) S= .334	.5296 (80) S= .001	1.0000 (0) S= .001	.4414 (80) S= .001	-.7003 (80) S= .001	.0892 (80) S= .431	.0663 (80) S= .559	-.0193 (80) S= .865	-.1301 (80) S= .250	.0081 (80) S= .943	-.0360 (80) S= .751	-.0964 (80) S= .395
V_0EX	-.3710 (80) S= .001	-.2136 (80) S= .057	.1453 (80) S= .198	.4414 (80) S= .001	1.0000 (0) S= .001	-.5119 (80) S= .001	.0318 (80) S= .779	.0161 (80) S= .887	-.0676 (80) S= .551	-.0928 (80) S= .413	-.0474 (80) S= .676	-.0730 (80) S= .520	.0339 (80) S= .765
VI	.5874 (80) S= .001	.1551 (80) S= .169	-.1900 (80) S= .091	-.7003 (80) S= .001	-.5119 (80) S= .001	1.0000 (0) S= .001	-.0858 (80) S= .449	.0454 (80) S= .689	.1886 (80) S= .094	.1001 (80) S= .377	.0537 (80) S= .636	.2080 (80) S= .064	-.0691 (80) S= .543
TAN	-.1197 (80) S= .290	-.2718 (80) S= .015	.1820 (80) S= .106	.0892 (80) S= .431	.0318 (80) S= .779	-.0858 (80) S= .449	1.0000 (0) S= .001	.0022 (80) S= .984	-.0457 (80) S= .687	-.0627 (80) S= .580	-.0238 (80) S= .834	-.2105 (80) S= .061	.1346 (80) S= .234
CRAM	-.2189 (80) S= .051	.0030 (80) S= .979	.0264 (80) S= .816	.0663 (80) S= .559	.0161 (80) S= .887	.0454 (80) S= .689	.0022 (80) S= .984	1.0000 (0) S= .001	.7739 (80) S= .001	.0839 (80) S= .459	-.0268 (80) S= .814	.7167 (80) S= .001	-.1174 (80) S= .300
SAH	-.1664 (80) S= .140	.1271 (80) S= .261	.0603 (80) S= .595	-.0193 (80) S= .865	-.0676 (80) S= .551	.1886 (80) S= .094	-.0457 (80) S= .687	.7739 (80) S= .001	1.0000 (0) S= .001	.0339 (80) S= .765	-.0941 (80) S= .406	.3679 (80) S= .001	-.0266 (80) S= .815
S	-.1558 (80) S= .167	.0248 (80) S= .827	-.0449 (80) S= .692	-.1301 (80) S= .250	-.0928 (80) S= .413	.1001 (80) S= .377	-.0627 (80) S= .580	.0839 (80) S= .459	.0339 (80) S= .765	1.0000 (0) S= .001	-.0448 (80) S= .693	.1125 (80) S= .320	.0205 (80) S= .857
P	.1331 (80) S= .239	-.0641 (80) S= .573	.0427 (80) S= .707	.0081 (80) S= .943	-.0474 (80) S= .676	.0537 (80) S= .636	-.0238 (80) S= .834	-.0268 (80) S= .814	-.0941 (80) S= .406	-.0448 (80) S= .693	1.0000 (0) S= .001	-.0273 (80) S= .810	.0167 (80) S= .883
GA	-.1234 (80) S= .275	.1306 (80) S= .248	-.1111 (80) S= .326	-.0360 (80) S= .751	-.0730 (80) S= .520	.2080 (80) S= .064	-.2105 (80) S= .061	.7167 (80) S= .001	.3679 (80) S= .001	.1125 (80) S= .320	-.0273 (80) S= .810	1.0000 (0) S= .001	-.0536 (80) S= .644
PP	-.0294 (80) S= .795	.2201 (80) S= .050	-.0677 (80) S= .551	-.0964 (80) S= .395	.0339 (80) S= .765	-.0691 (80) S= .543	.1346 (80) S= .234	-.1174 (80) S= .300	-.0266 (80) S= .815	.0205 (80) S= .857	.0167 (80) S= .883	-.0536 (80) S= .644	1.0000 (0) S= .001

Table 5. Covariance, Cross-Product Diviations for all Combinations of Pairs

VARIABLES	CASES	MEAN	STD DEV
G	80	.0002	1.0003
FL	80	-.0000	1.0006
V210	80	-.0002	1.0002
V100	80	-.0000	.9999
VDEX	80	-.0008	1.0004
VI	80	.0002	1.0001
TAN	80	-.0425	.6244
CRAM	80	.0279	.9213
SAH	80	.0015	1.0012
S	80	.0007	.9996
P	80	-.0067	.6832
CA	80	.0293	.9894
PP	80	.0000	1.0001

VARIABLES		CASES	CROSS-PROD DEV	COVARIANCE	VARIABLES		CASES	CROSS-PROD DEV	COVARIANCE
G	FL	80	17.3491	.2196	G	V210	80	-24.1768	-.3060
G	V100	80	-46.5967	-.5898	G	VDEX	80	-29.3313	-.3713
G	VI	80	46.4248	.5877	G	TAN	80	-5.9046	-.0747
G	CRAM	80	-15.9353	-.2017	G	SAH	80	-13.1703	-.1667
G	S	80	-12.3108	-.1558	G	P	80	7.1873	.0910
G	CA	80	-9.6499	-.1222	G	PP	80	-2.3272	-.0295
FL	V210	80	-.8737	-.0111	FL	V100	80	-8.6460	-.1094
FL	VDEX	80	-16.8948	-.2139	FL	VI	80	12.2628	.1552
FL	TAN	80	-13.4120	-.1698	FL	CRAM	80	.2149	.0027
FL	SAH	80	10.0561	.1273	FL	S	80	1.9611	.0248
FL	P	80	3.4570	.0438	FL	CA	80	10.2170	.1293
FL	PP	80	17.4017	.2203	V210	V100	80	41.8448	.5297
V210	VDEX	80	11.4878	.1454	V210	VI	80	-15.0186	-.1901
V210	TAN	80	8.9767	.1136	V210	CRAM	80	1.9220	.0243
V210	SAH	80	4.7673	.0603	V210	S	80	-3.5498	-.0449
V210	P	80	2.3043	.0292	V210	CA	80	-8.6881	-.1100
V210	PP	80	-5.3512	-.0677	V100	VDEX	80	34.8787	.4415
V100	VI	80	-55.3272	-.7003	V100	TAN	80	4.3990	.0557
V100	CRAM	80	4.6279	.0611	V100	SAH	80	-1.5260	-.0193
V100	S	80	-10.2711	-.1300	V100	P	80	.4388	.0056
V100	CA	80	-2.8106	-.0356	V100	PP	80	-7.6123	-.0964
VDEX	VI	80	-40.4595	-.5121	VDEX	TAN	80	1.5700	.0199
VDEX	CRAM	80	1.1752	.0149	VDEX	SAH	80	-5.3521	-.0677
VDEX	S	80	-7.3300	-.0928	VDEX	P	80	-2.5590	-.0324
VDEX	CA	80	-5.7049	-.0722	VDEX	PP	80	2.6806	.0339
VI	TAN	80	-4.2323	-.0536	VI	CRAM	80	3.3058	.0418
VI	SAH	80	14.9209	.1889	VI	S	80	7.9017	.1000
VI	P	80	2.8970	.0367	VI	CA	80	16.2637	.2059
VI	PP	80	-5.4586	-.0691	TAN	CRAM	80	.1065	.0013
TAN	SAH	80	-2.2564	-.0286	TAN	S	80	-3.0921	-.0391
TAN	P	80	-.8016	-.0101	TAN	CA	80	-10.2736	-.1300
TAN	PP	80	6.6403	.0841	CRAM	SAH	80	56.1824	.7112
CRAM	S	80	6.1057	.0773	CRAM	P	80	-1.3315	-.0169
CRAM	CA	80	51.6129	.6533	CRAM	PP	80	-8.5478	-.1082

VARIABLES		CASES	CROSS-PROD DEV	COVARIANCE	VARIABLES		CASES	CROSS-PROD DEV	COVARIANCE
SAH	S	80	2.6830	.0340	SAH	P	80	-5.0867	-.0644
SAH	CA	80	67.9253	.8598	SAH	PP	80	-2.1043	-.0266
S	P	80	-2.4178	-.0306	S	CA	80	8.7920	.1113
S	PP	80	1.6153	.0204	P	CA	80	-1.4577	-.0185
P	PP	80	.9004	.0114	CA	PP	80	-4.1441	-.0525

Regression analyses were obtained to relate S & P, G & SAH, CRAM & N, CA & ZN, TBN & TAN, and V_{210} & V_{100} and to obtain equations for each pair. The correlation coefficient was obtained for each pair. These are shown in Table 4.

Pearson's correlation coefficient is used to measure the strength of relationship between two interval-level variables. In this case, the strength of relationship indicates both the goodness of fit of a linear regression line to the data and, when Pearson's coefficient is squared, the proportion of variance in one variable explained by the other.

Mathematically, Pearson's coefficient, r, is defined as the ratio of covariation to square root of the product of the variation in x and the variation in y where x and y symbolize the two variables. This corresponds to the formula:

$$r = \frac{\sum_{i=1}^{N} (X_i - \overline{X})(Y_i - \overline{Y})}{\left\{\left[\sum_{i=1}^{N} (X_i - \overline{X})^2\right]\left[\sum_{i=1}^{N} (Y_i - \overline{Y})^2\right]\right\}}$$

where:

X_i = ith observation of variable x
Y_i = ith observation of variable y
N = number of observations
$\overline{X} = \sum_{i=1}^{N} \frac{X_i}{N}$ = mean of variable x
$\overline{Y} = \sum_{i=1}^{N} \frac{X_i}{N}$ = mean of variable y

The formula employed by *SPSS* for computing Pearson's coefficient is as follows:

$$\text{Pearson's Correlation Coefficient} = \frac{\sum_{i=1}^{N} X_i Y_i - \left(\sum_{i=1}^{N} X_i\right)\left(\sum_{i=1}^{N} Y_i\right)/N}{\left\{\left[\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2/N\right]\left[\sum_{i=1}^{N} Y_i^2 - \left(\sum_{i=1}^{N} Y_i\right)^2/N\right]\right\}}$$

Significance tests are reported for each coefficient and are derived from the use of a student's T with N - 2 degrees of freedom for the computed quantity:

$$r\left[\frac{N-2}{1-r^2}\right]^{1/2}$$

where N = number of cases upon which the correlation coefficient was computed.

The tests of significance give information regarding the probability that the observed relationship could have happened by chance, i.e., probability that in a representative sample of a given size, the variables would exhibit a relationship as strong as the observed relationship. It has been accepted in other applications of *SPSS* to accept as statistically significant relationships which have a probability of occurrence by chance 5% of the time or less, i.e., in 5 out of 100 samples 0.05 or less. Applying this criteria, it can be noted that statistically significant results appear for the pairs listed below. A two-tailed list of significance as follows has been applied to the data:

Statistically significant	Pearson's correlation (absolute value)
G & FL	.2194
G & V_{210}	.3059
G & V_{100}	.5897
G & V_oEX	.3710
G & VI	.5874
G & TAN	.1197
G & CRAM	.2189
G & SAH	.1664
G & S	.1558
G & P	.1331
G & CA	.1284
FL & G	–
FL & V_{100}	.1093
FL & V_oEX	.2136
FL & VI	.1551
FL & TAN	.2718
FL & SAH	.1271
FL & CA	.1306
FL & pp	.2201
V_{210} & V_{100}	.5296
V_{210} & V_oEX	.1453
V_{210} & VI	.1900
V_{210} & TAN	.1820
V_{210} & SAH	.1271
V_{210} & CA	.1306
V_{210} & PP	.2201

Statistically Significant	Pearson's correlation (absolute value)
V_{100} & V_oEX	.4414
V_{100} & VI	.7703
V_{100} & TAN	.0802
V_{100} & CRAM	.0663
V_{100} & S	.1301
V_{100} & pp	.0964
V_oEX & VI	.5119
V_oEX & S	.0928
V_oEX & CA	.0730
VI & TAN	.0858
VI & SAH	.1886
VI & S	.1000
VI & P	.0537
VI & CA	.2080
VI & pp	.0691
TAN & CA	.2105
TAN & pp	.1346
TAN & S	.0627
CRAM & SAH	.7709
CRAM & S	.0839
CRAM & CA	.7167
CRAM & pp	.1174
SAH & P	.0941
SAH & CA	.8679
S & CA	.1125
CA & PP	.0530

Strong correlation relationships are shown between V_{210} and V_{100} (.5296), V_{100} and VI (.7703), V_oEX and VI (.5119), CRAM and SAH (.7709), CRAM and CA (.7167), and SAH and CA (.8679).

Because the correlation of a variable with itself is unity and the correlation of x with y is identical to the correlation of y with x, the redundant correlations are not included.

Factor Analysis

Factor analysis is a much more generalized procedure for evaluating and defining dimensional space among a relatively large number of variables. Because of the generality of factor analysis, it is difficult to present a capsule description of its functions and applications. The major use of factor analysis is to locate a small number of valid dimensions – clusters or factors contained in a larger set of independent items or variables. Factor analysis helps to determine the degree to which a given variable or several variables is part of a common, underlying phenomenon.

The single, most-distinctive characteristic of factor analysis is its data-reduction capability. From an array of correlation coefficients for a set of variables, factor-analysis techniques allow one to see whether some underlying pattern of relationships exists such that the data may be rearranged or reduced to a smaller set of factors or components that may be taken as source variables accounting for the observed interrelations in the data. In this study, one use of factor analysis was employed – exploratory – the exploration and detection of patterning of variables with a view toward the discovery of new concepts and a possible reduction of the data. Factor analysis is not a unitary concept. It subsumes a large number of procedures, the most general classification of which may be organized around major alternatives available at each of the customary steps. The steps are as follows: (1) the preparation of a correlation matrix, (2) the extraction of the initial factors – the exploration of possible data reduction, and (3) the notation to terminal solution – the search for simple and interpretable factors. The factor matrix for the oil data is shown in Table 6. This application applies the scheme of correlation of variables (association) which in *SPSS* analysis is called R-factor analysis.

In general, there are many tests available for characterizing mineral oil lubricants. When engine tests, rig tests, and functional tests such as oxidation are omitted, many tests still remain. Some inspection tests, elemental analysis, and carbon-type analysis of lubricants are considered here. The object of the work again is to examine interrelationships to draw some inference about the significance of the tests in themselves and when taken in conjunction with others. This study is cognizant of both physical/chemical tests and separate engine tests but limits itself to selected chemical and physical tests in order to maintain some degree of simplicity in the interrelations. A subsequent study of the relationships will include other tests performed on engines. Many of the tests have been known for a long time and are considered to be standard in the industry.

The first step in factor analysis requires the calculation of a measure of association for relevant variables. The correlation matrix shown in Table 7 serves as the measure of association for the variables. The complete applications of the *SFSS* scheme to the analysis of these lubricating oils includes, in addition to the correlation matrix, a

Table 6. Factor Matrix for the Oil Data

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10
G	.72724	-.39512	.22185	-.09165	.04617	-.15171	-.10337	.19750	.07638	.27003
FL	.34252	.05150	.40979	.67321	-.05209	.17880	-.26343	-.19117	.32561	.02073
V210	-.51210	.12174	.25120	.03045	.47135	.38239	-.35555	.36036	-.12724	.02741
V100	-.85154	.17342	.26002	.03915	.02330	.10003	-.11284	-.12846	-.02302	.01549
VOEX	-.63592	.06384	.03611	.02253	-.42897	-.26550	.07323	.47452	.31810	-.02147
VI	.83257	.00844	-.04902	-.15591	.26953	.09243	-.06393	.29976	.07242	-.15819
TAN	-.29452	.14059	-.60569	.00915	.60121	-.16879	-.04163	-.10771	.33433	.05257
CRAM	.08253	.86535	.05266	-.09291	-.02640	-.08005	.07005	-.00170	-.05622	.42537
SAH	.20534	.90483	.11755	.02623	.11951	-.10155	.03074	.05783	-.01621	-.13227
S	.12222	.15858	-.41534	.11690	-.23937	.76934	.30031	.10120	.07824	.07501
P	-.09151	-.09686	.53612	-.04575	.37842	.04491	.73005	-.01436	.11056	-.02463
CA	.30359	.87111	.02236	.02429	-.11374	-.08263	-.01060	-.00347	.00445	-.19159
PP	.00375	-.08476	-.24337	.82944	.13584	-.26348	.21519	.19551	-.25970	.02696

	FACTOR 11	FACTOR 12	FACTOR 13
G	.31019	-.08568	.01555
FL	-.09197	.00776	-.00464
V210	-.04572	-.09007	-.07999
V100	.27901	.21531	.10703
VOEX	-.04104	.00835	.00228
VI	-.07143	.26567	.06358
TAN	.05945	-.01868	-.01599
CRAM	-.15573	.10926	-.01842
SAH	.02013	-.18648	.20626
S	.10152	-.02884	.02745
P	-.00562	-.00124	-.04023
CA	.21126	.02525	-.20577
PP	.04537	.04870	.00677

VARIABLE	COMMONALITY
G	1.00000
FL	1.00000
V210	1.00000
V100	1.00000
VOEX	1.00000
VI	1.00000
TAN	1.00000
CRAM	1.00000
SAH	1.00000
S	1.00000
P	1.00000
CA	1.00000
PP	1.00000

Table 7. Correlation Matrix

	G	FL	V_{210}	V_{100}	V_0EX	VI	TAN	CRAM	SAH	S
G	1.00000	.21984	-.30594	-.58963	-.37076	.58733	-.30631	-.23634	-.16644	-.15437
FL	.21984	1.00000	-.01172	-.10981	-.21360	.15522	-.25916	.00622	.12702	.02540
V210	-.30594	-.01172	1.00000	.52957	.14472	-.18994	.16979	.02245	.06024	-.04558
V100	-.58963	-.10981	.52957	1.00000	.44090	-.70013	.13775	.05939	-.01928	-.13134
V0EX	-.37076	-.21360	.14472	.44090	1.00000	-.51108	.01169	.01967	-.06740	-.09272
VI	.58733	.15522	-.18994	-.70013	-.51108	1.00000	-.09324	.03570	.18855	.10079
TAN	-.30631	-.25916	.16979	.13775	.01169	-.09324	1.00000	.05597	.06465	-.02052
CRAM	-.23634	.00622	.02245	.05939	.01967	.03570	.05597	1.00000	.72808	.08808
SAH	-.16644	.12702	.06024	-.01928	-.06740	.18855	.06465	.72808	1.00000	.03309
S	-.15437	.02540	-.04558	-.13134	-.09272	.10079	-.02052	.08808	.03309	1.00000
P	.02733	-.01655	.08696	.12140	-.02184	-.03501	-.08479	-.03628	.01135	-.08736
CA	-.09984	.15033	-.13612	-.07600	-.07532	.23083	-.03079	.67886	.83188	.13069
PP	-.02945	.21997	-.06771	-.09631	.03399	-.06911	.15152	-.10596	-.02666	.02052

	P	CA	PP
G	.02733	-.09984	-.02945
FL	-.01655	.15033	.21997
V210	.08696	-.13612	-.06771
V100	.12140	-.07600	-.09631
V0EX	-.02184	-.07532	.03399
VI	-.03501	.23083	-.06911
TAN	-.08479	-.03079	.15152
CRAM	-.03628	.67886	-.10596
SAH	.01135	.83188	-.02666
S	-.08736	.13069	.02052
P	1.00000	-.14393	.00347
CA	-.14393	1.00000	-.04966
PP	.00347	-.04966	1.00000

principal-component analysis. In principal-component analysis, the given set of variables is transformed by a transformation matrix into new sets of composite variables or components that are uncorrelated to each other. The transformation matrix is shown in Table 8. No particular assumption about the underlying structure of the variables is required or assumed. What is sought is the best linear combination of variables – best in the sense that the particular combination of variables would account for more of the variance in the data as a whole than any other combination of variables. The first principal component is then viewed as the single best summary of linear relationships exhibited in the data. The second component is viewed as the second best linear combination of variables under the condition that the second component is orthogonal to the first. To be orthogonal to the first component, the second must account for the portion of variance not accounted for by the first. Thus, the second component may be defined as the linear combination of variables that accounts for the most residual variance after the effect of the first component is removed from the data. Subsequent components are defined similarly until all the variance in the data is exhausted. Unless at least one variable is perfectly determined by the remainder of the variables in the data, the principal-component solution requires as many components as there are variables. The principal-component model may be compactly expressed as

$$Z_j = a_{j1} F_1 + a_{j2} F_2 + a_{j3} F_3 \text{----} a_{jn} F_n$$

where each of the n-observed variables is described linearly in terms of n new uncorrelated components F_1, F_2, F_3 ----- F_n, each of which is, in turn, defined as a linear combination of the n original variables. Since each component is defined as the best linear summary of variance left in the data after the previous components are taken care of, the first m components – usually much smaller than the number of variables in the set – may explain most of the variance in the data. For factor-analytic purposes, the analyst normally retains only the first few components for further rotation. The *SPSS* subprogram employed for these studies is known as principal factoring with iteration. The immediate result of the initial factoring was the extraction of an unrotated factor matrix shown in Table 6. The factors are arranged in the order of their importance. The first factor is the most important, the second factor is the second most important, etc. The first factor tends to be a general factor; it has significant loading on every variable. Subsequent factors tend to be bipolar, that is, some factor loadings are positive and some are negative. The method includes defined factors. The interest here is to find whether some smaller number of components accounts for most of the variance. From the unrotated factor matrix, it is obvious that factor 2(FL) has primary influence on factor CA, SAH, and CRAM and has negative influence on G. Factor 3(V_{210}) has primary influence on TAN, FL, and P. Factor 4(V_{100}) has primary influence on pp and FL. Factor 5 (V_oEX) has primary influence on TAN, etc. Conversely, the influence of factor 2(FL) on VI and V_oEX is negligible.

Table 8. Transformation Matrix

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10
FACTOR 1	.21136	.57044	-.18450	-.35873	.07574	-.06197	-.39733	.01279	.23424	.43837
FACTOR 2	.91474	-.04896	.09524	.09507	.10459	-.15841	.14903	-.05792	.02110	-.21747
FACTOR 3	.09646	-.08278	-.54215	.25215	-.38333	.48844	.03897	-.22411	.36952	.16337
FACTOR 4	.00423	-.14475	.00779	.03265	.10642	-.04206	.12534	.76093	.61109	-.08157
FACTOR 5	.00164	.24755	.57925	.46265	-.23328	.37106	-.41534	.12878	-.04680	.04485
FACTOR 6	-.14153	.05706	-.16224	.39605	.77035	.04688	-.25901	-.26585	.17949	-.14475
FACTOR 7	.03753	-.05440	-.33344	-.38503	.31958	.76304	.07693	.22920	-.28706	-.03661
FACTOR 8	.04524	.42773	-.14719	.44634	.12748	-.01919	.59654	.25580	-.24763	.27177
FACTOR 9	-.03717	.12440	.50729	-.19854	.11461	.16249	.47931	-.38737	.43577	.12039
FACTOR 10	-.16335	-.27581	.10036	.04781	.11571	-.03104	-.03315	.03092	.02456	.47157
FACTOR 11	.18712	-.22004	.09086	-.04825	.16769	-.01459	-.06433	.06934	-.15152	.58959
FACTOR 12	-.15336	.49194	-.04173	-.15203	-.05375	-.00971	.03951	.07729	.01652	-.24821
FACTOR 13	-.00116	.10946	-.01742	-.11004	.03211	-.04743	.30455	.01598	-.01050	.00792

	FACTOR 11	FACTOR 12	FACTOR 13
FACTOR 1	-.30095	.00357	.01162
FACTOR 2	.07143	.26418	.00170
FACTOR 3	.15745	.02186	-.01979
FACTOR 4	.02241	-.06766	.00003
FACTOR 5	.00935	-.01740	-.06071
FACTOR 5	.07513	-.04191	.00436
FACTOR 7	-.08905	.05779	-.01315
FACTOR 8	-.14155	-.01228	-.02828
FACTOR 9	-.01915	-.07806	.01303
FACTOR 10	-.00995	.80173	-.05539
FACTOR 11	.54317	-.39904	.19321
FACTOR 12	.66251	.33296	.29577
FACTOR 13	.32317	-.02527	-.93099

Table 9 shows the terminal solution of the orthogonally rotated data. It is an orthogonal-factor matrix and stands for both a pattern and a structure matrix. The coefficients in the table represent both the regression weights and the correlation coefficients. The loadings, or numbers, in a given row represent regression coefficients to describe a given variable. In the principal-component matrix, the eigenvalue (Table 10) associated with each component represents the amount of the total variance accounted for by the individual factor in the factor matrix (Table 6).

The total variance of a variable accounted for by the combination of all common factors is referred to as the communality of the variable. This value indicates the amount of the variance of a variable that is shared by at least one other variable in the set. The proportion of total variance accounted for by a component is $\frac{\lambda_i}{n}$ where λ_i represents the eigenvalue of the ith component and n represents the number of variables in the set. From the data in Table 10, one sees that 85% of the total variance observed is accounted for by only 7 of the 13 pieces of analytical data about the lubricating oils. The number of significant components retained in the final rotation will be determined by the specification of the minimum eigenvalue criterion. The program retains and prints only components with eigenvalues greater than or equal to one. This criterion ensures that only components accounting for at least the amount of total variance of a single variable will be treated as significant.

Graphical representation of the notated data is shown in the illustrations that follow. *SPSS* in graphical representation allows the rotation of the factors one by one until every possible pair of factors has been plotted. Plotting the data in this fashion furnished information useful analytically in three ways: (1) the relative distance of the variable from the two axes, (2) the direction of the variable in relation to the axes (It may be either positive or negative loading.), and (3) the clustering of the variables and their relative position to each other. Information relative to the degree of correlation is furnished from these observations. In the examples included, rotation of the factors has been accomplished by varimax rotation.

IV. DISCUSSION OF VARIMAX FACTOR MATRIX

From Table 6, it can be seen that the primary influence of G is by factor 1 with strong contributions by factors 2, 3, 6, 7, 8, 10, and 11. Other factors are negligible. For F1, the primary influence is by factor 4 with strong contributions by factors 1, 3, 6, 7, 8, and 9. Other factors are negligible. For V_{210}, the primary influence is by factor 1 with strong contributions by factors 2, 3, 5, 6, 7, and 8. Other factors are negligible. The other variables such as V_{100}, V_oEX, VI, TAN, CRAM, SAH, S, P, CA, and pp are subject to the same analysis.

Table 9. Terminal Solution of the Orthogonally Rotated Data

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6	FACTOR 7	FACTOR 8	FACTOR 9	FACTOR 10
G	-.13526	.34829	-.16893	-.16213	-.11915	.01718	-.16737	-.02034	.11347	.85528
FL	.08392	.05006	-.13696	.01255	.00906	-.01156	-.09355	.12558	.96914	.07994
V210	-.01738	-.07764	.08135	.97852	-.01926	.04371	.05039	-.03847	.01297	-.10681
V100	-.00298	-.50945	.05788	.37269	-.08502	.08225	.21743	-.08470	-.01991	-.26765
VOEX	-.02891	-.23617	-.01887	.06063	-.04686	-.01961	.94994	.02409	-.10424	-.12637
VI	.14480	.90690	-.02411	-.06362	.05466	-.01196	-.24365	-.04561	.05571	.25346
TAN	.01775	-.02981	.97424	.08326	-.01514	-.04749	-.01675	.08892	-.13460	-.11516
CRAM	.71514	-.03993	.02904	.01573	.04380	-.00634	.01929	-.06581	-.02670	-.06096
SAM	.95498	.06737	.04507	.07327	-.01821	.04583	-.12899	.00057	.05411	-.05739
S	.04520	.04658	-.01403	-.02181	.99228	-.04258	-.04212	.01117	.03856	-.07356
P	-.03529	-.01732	-.04454	.04415	-.04211	.99566	-.01615	.00513	-.01046	.00930
CA	.94238	.09162	-.03299	-.10605	.07323	-.10117	-.01923	-.02323	.06718	-.02745
PP	-.03450	-.02472	.08560	-.04076	.01138	.00535	.02210	.98628	.11876	-.01275

	FACTOR 11	FACTOR 12	FACTOR 13
G	-.12958	-.03575	.00185
FL	-.00921	-.01101	.00083
V210	.10759	.00669	-.00744
V100	.57546	.00853	.00598
VOEX	.08311	.00791	.00024
VI	-.12924	-.01861	.00271
TAN	.02277	.01189	-.00304
CRAM	.00932	.68903	-.00335
SAM	-.03995	.06369	-.24431
S	-.02936	.01658	.00339
P	.02957	-.00232	-.00518
CA	.03551	.01030	.26194
PP	-.03016	-.02520	-.00097

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Table 10. Principal Component Matrix

VARIABLE	EST COMMONALITY	FACTOR	EIGENVALUE	PCT OF VAR	CUM PCT
G	1.00000	1	2.99759	23.0	23.0
FL	1.00000	2	2.59567	20.0	42.9
V210	1.00000	3	1.25517	9.7	52.6
V100	1.00000	4	1.20262	9.3	61.9
VOEX	1.00000	5	1.09254	8.4	71.3
VI	1.00000	6	1.00560	7.7	78.0
TAN	1.00000	7	.91102	7.0	85.0
CRAM	1.00000	8	.59471	4.6	89.6
SAM	1.00000	9	.43939	3.4	93.0
S	1.00000	10	.34584	2.7	95.6
P	1.00000	11	.27637	2.1	97.7
CA	1.00000	12	.18343	1.4	99.2
PP	1.00000	13	.11005	.8	100.0

The values given in Table 6 represent regression coefficients of the factors to describe a given variable. For example, for the variable, G:

$$G = 0.72694F_1 - 0.39589F_2 + 0.22149F_3$$

$$- 0.09300F_4 + 0.04745F_5 - 0.15405F_6$$

$$- 0.10191F_7 + 0.19609F_8 + 0.07906F_9$$

$$+ 0.26917F_{10} + 0.31050F_{11} - 0.08497F_{12}$$

$$+ 0.01619F_{13} \ .$$

where F_1 = factor 1, etc. The other variables such as FL, V_{210}, and V_{100} would be treated in a similar manner.

The general equation

$$Z_j = a_{j1}F_1 + a_{j2}F_2 + \text{-----} \ a_{jm}F_m + d_jU_j$$

expresses this relationship.

The variance accounted for by factor 1 is

$$(a_{11})^2 = (0.72694)^2 = 0.5284$$

The variance accounted for by factor 2 is

$$(a_{12})^2 = (0.39589)^2 = 0.1567,$$

etc.

$$\sum_{j=1}^{N} a^2_{j1} \quad j = 1, 2, 3, \text{---}, N$$

$$= (.72694)^2 + (.34124)^2 + \text{-----} + (.01003)^2 = 2.98602$$

$$= \text{respective eigenvalue.}$$

These values are shown in Table 6 along with cumulative percentages in Table 10.

Significantly, most of the variance in the data is accounted for by G and FL. The unrotated factors extracted through the factoring method may or may not give meaningful patterning of the variables. To supplant the data obtained from the unrotated factors, the factors are subsequently rotated to effect additional simplication.

Rotation in Subprogram

In this *SPSS* program, all the initial solutions extract orthogonal factors in order of their importance. The first factor so extracted tends to be a general factor; that is, it tends to load significantly on every variable. The second factor tends to be bipolar, that is, approximately half the variables have positive loadings and the other half have negative loading. The remaining factors also tend to be bipolar, and it is difficult to interpret such factors. Every variable tends to be decomposed into both positive and negative factors, and the complexity of each variable is usually greater than one.

The analytical method of rotation is designed to take a fixed number of factors and a fixed amount of variance accounted for by these factors and simplify the rows of the factor matrix and the column matrix to make as many values as possible in each row and column close to zero.

In the illustrations that follow, the *SPSS* depicts graphical presentation of rotated orthogonal factors employing a procedure termed varimax. Varimax centers on simplifying the columns of the factor matrix. A simple factor is defined in varimax as one with only 1s and 0s in the column. This simplification is equivalent to maximizing the variance of the squared loadings in each column. Since only two-dimensional space can be effectively plotted, every possible pair of factors is taken one by one. Significance of the graphs resides in three aspects: (1) the relative distance of a variable from the two axes, (2) the direction of a variable in relation to the axis (It may indicate either a positive or negative loading.), and (3) the clustering of variables and their relative position to each other. Conclusions relative to the degree of actual correlation between the factors are drawn from these observations.

In Figure 3, variables 2, 3, 5, 7, 10, and 13 load low on factor 2. Variables 8, 9, and 12 load high on factor 1, and variable 6 loads low on factor 1 and high on factor 2. In addition, variables 10, 2, 13, 7, 3, and 5 are close to the origin and have small loadings on both factor 1 and factor 2. As a whole, the graph separates cluster 10, 2, 13, 7, 3, and 5 from cluster 8, 9, and 12. The clustering of these groups indicates some degree of correlation between them. Variable 11 did not load significantly on either axis.

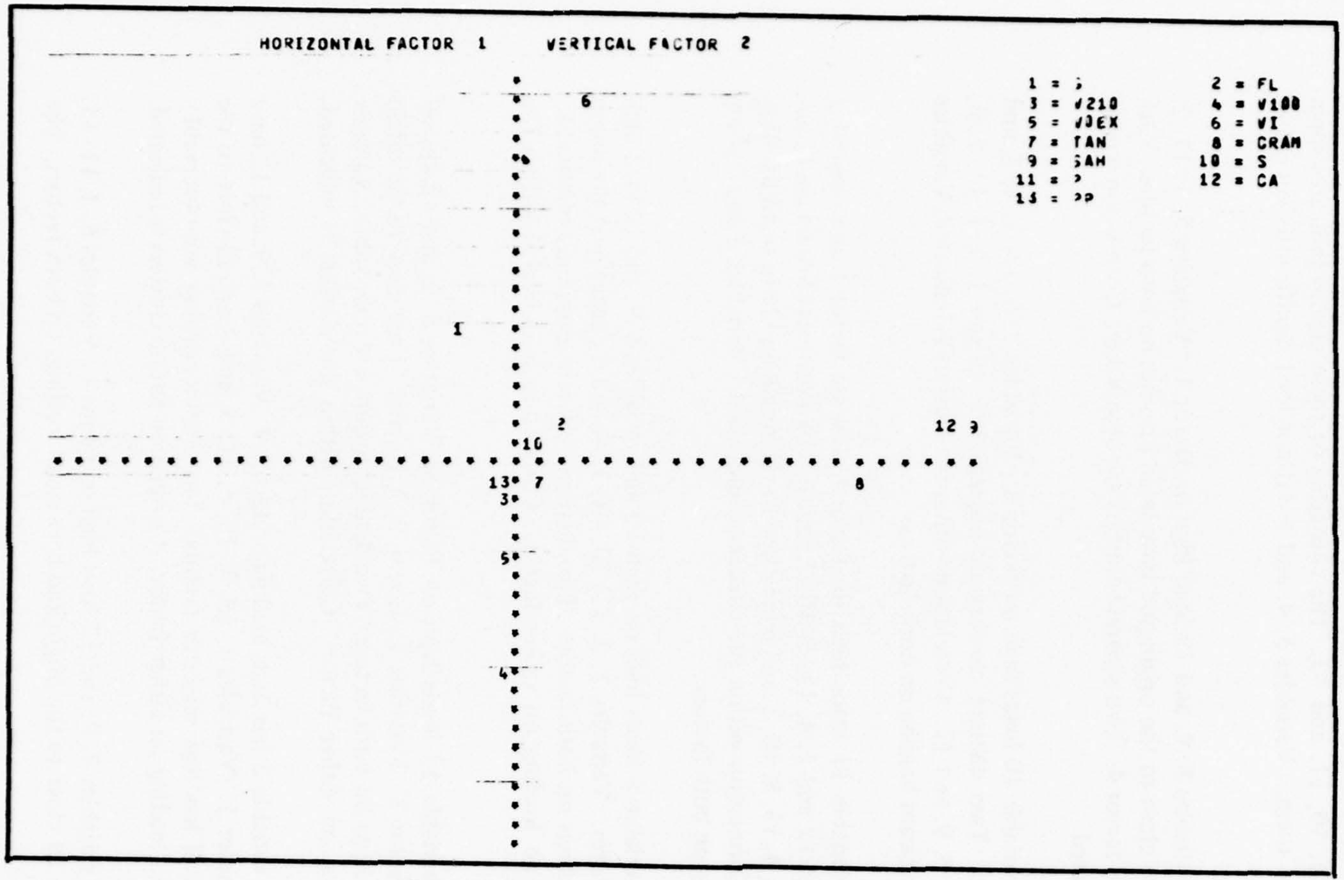


Figure 3. Horizontal Factor 1; Vertical Factor 2.

In Figure 4, variables 8, 9, and 12 load high on factor 1 and cluster. Variables 1, 2, 6, 10, 11, and 13 are all close to the origin and have small loadings on both factors Variable 7 loads high on factor 3. The graph separates cluster 8, 9, and 12 from cluster 1, 2, 6, 10, 11, and 13. The clustering of these groups indicates some correlation between them. Variables 3, 4, and 5 did not load significantly on either axis.

In Figure 5, variables 8, 9, and 12 load high on factor 1. Variables 5, 7, 11, 2, 13, 10, 6, and 1 are close to the origin and have small loadings on both factors. Variable 3 loads high on factor 4. Two separate clusters are observable. Correlation within the clusters is indicated.

In Figure 6, variable 10 loads high on factor 5. Variables 2, 6, 13, 1, 4, 7, and 11 show clustering. Two distinct clusters are observable – cluster 1, 4, 7, 11, 2, 6, and 13 and cluster 8, 9, and 12. Correlation within the clusters is indicated. Variables 3 and 5 show insignificant loading on both factors.

In Figure 7, variable 11 loads high on factor 6, low on factor 1 and clustering occurs for 8, 9, and 12 and 1, 4, 13, 5, 10, 2, and 6. Two distinct clusters are observable. Variables 1, 4, 13, 5, 10, 2, and 6 are close to the origin and have small loadings on both factors. Correlation within the clusters is indicated. Variables 3 and 7 show insignificant loading on both factors.

In Figure 8, variable 5 loads high on factor 7 and variables 8, 9, and 12 load high on factor 1 and cluster. Variables 1, 2, 11, 10, 13, 3, 4, and 6 cluster near the origin and have small loadings on both factors. Two distinct, separate groups are observable. Variable 7 shows no loading on either factor. Correlation is indicated within the clusters.

In Figure 9, variable 13 loads high on factor 8. Variables 8, 9, and 12 cluster and load high on factor 1. Variables 1, 3, 4, 6, 2, 7, 10, and 11 are close to the origin and have small loadings on both factors. Two distinct groups are observable. Variable 5 shows no loading on either factor. Correlation within the clusters is indicated.

In Figure 10, variable 2 has high loading on factor 9. Variables 8, 9, and 12 have high loadings on factor 1. Variables 1, 13, 3, 10, 6, 11, 5, and 7 are all close to the origin and have small loadings on both factors. Two distinct groups are observable. Variable 4 shows no loading on either factor. Correlation within clusters is indicated.

In Figure 11, variables 8, 9, and 12 load high on factor 1. Variables 6, 2, 11, 13, 10, 5, 7, and 4 are all close to the origin and have small loadings on both factors. The

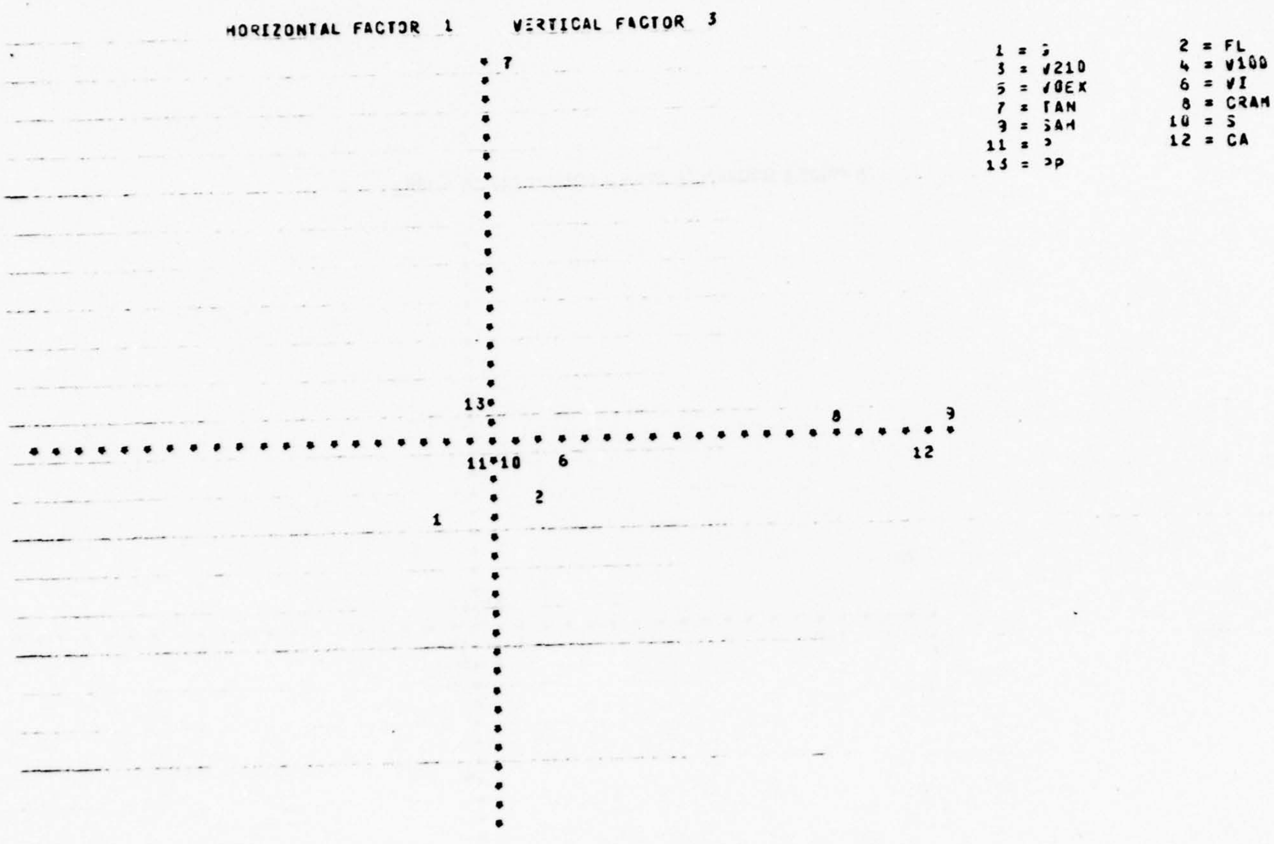


Figure 4. Horizontal Factor 1; Vertical Factor 3.

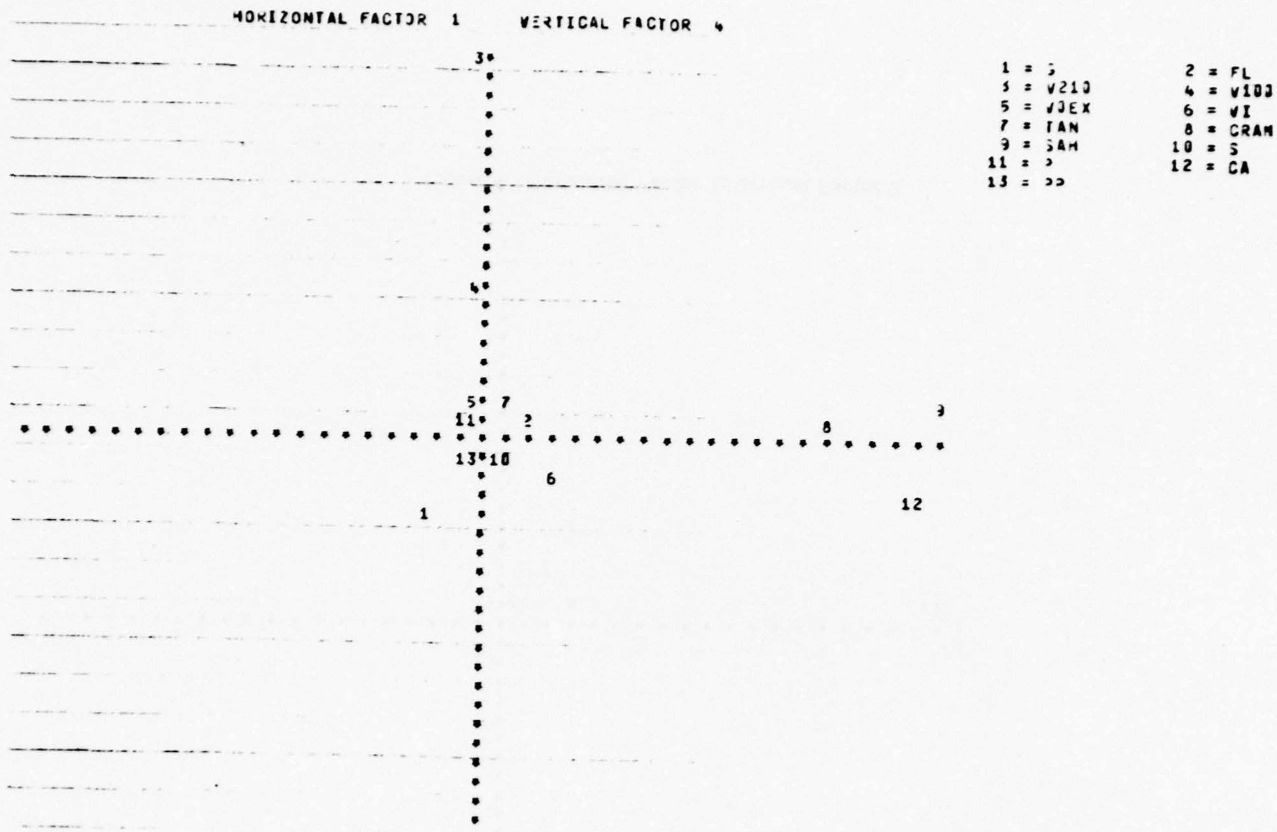


Figure 5. Horizontal Factor 1; Vertical Factor 4.

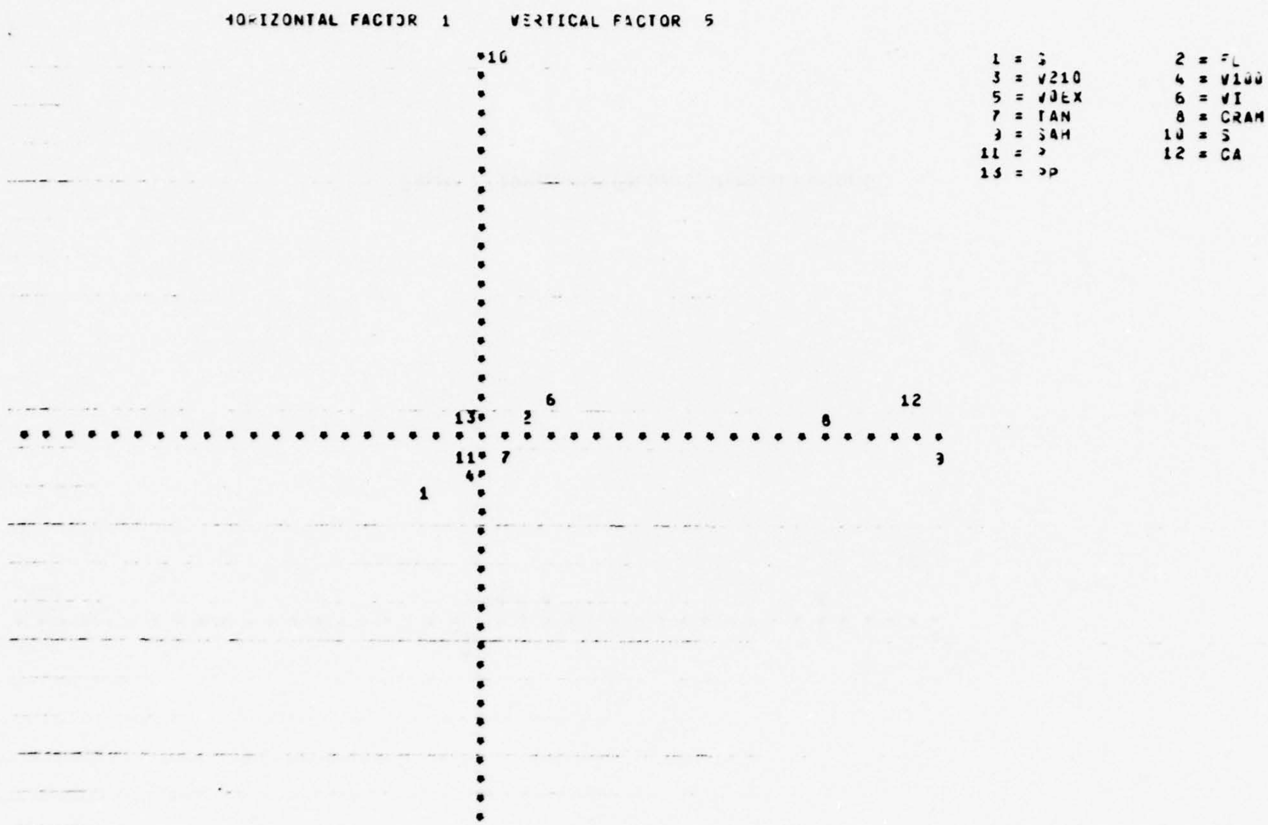


Figure 6. Horizontal Factor 1; Vertical Factor 5.

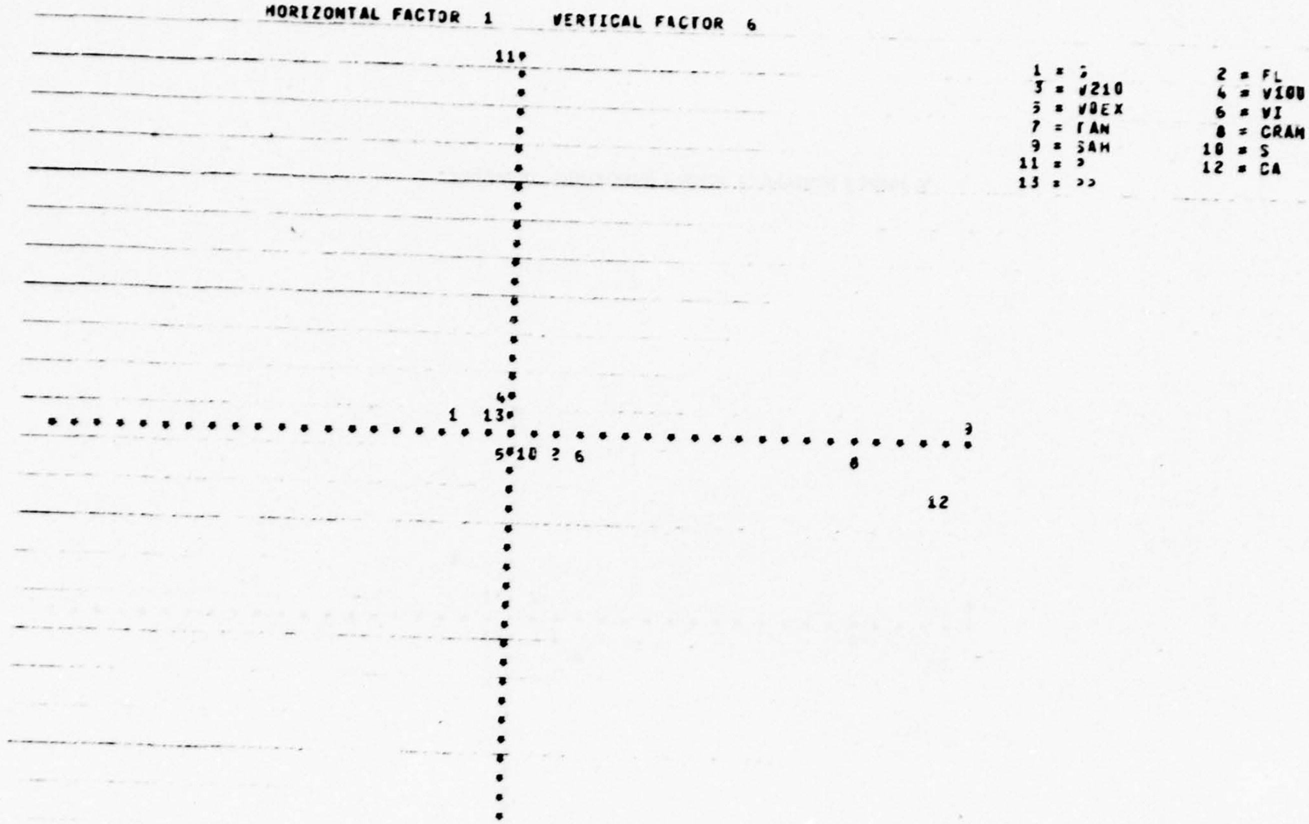


Figure 7. Horizontal Factor 1; Vertical Factor 6.

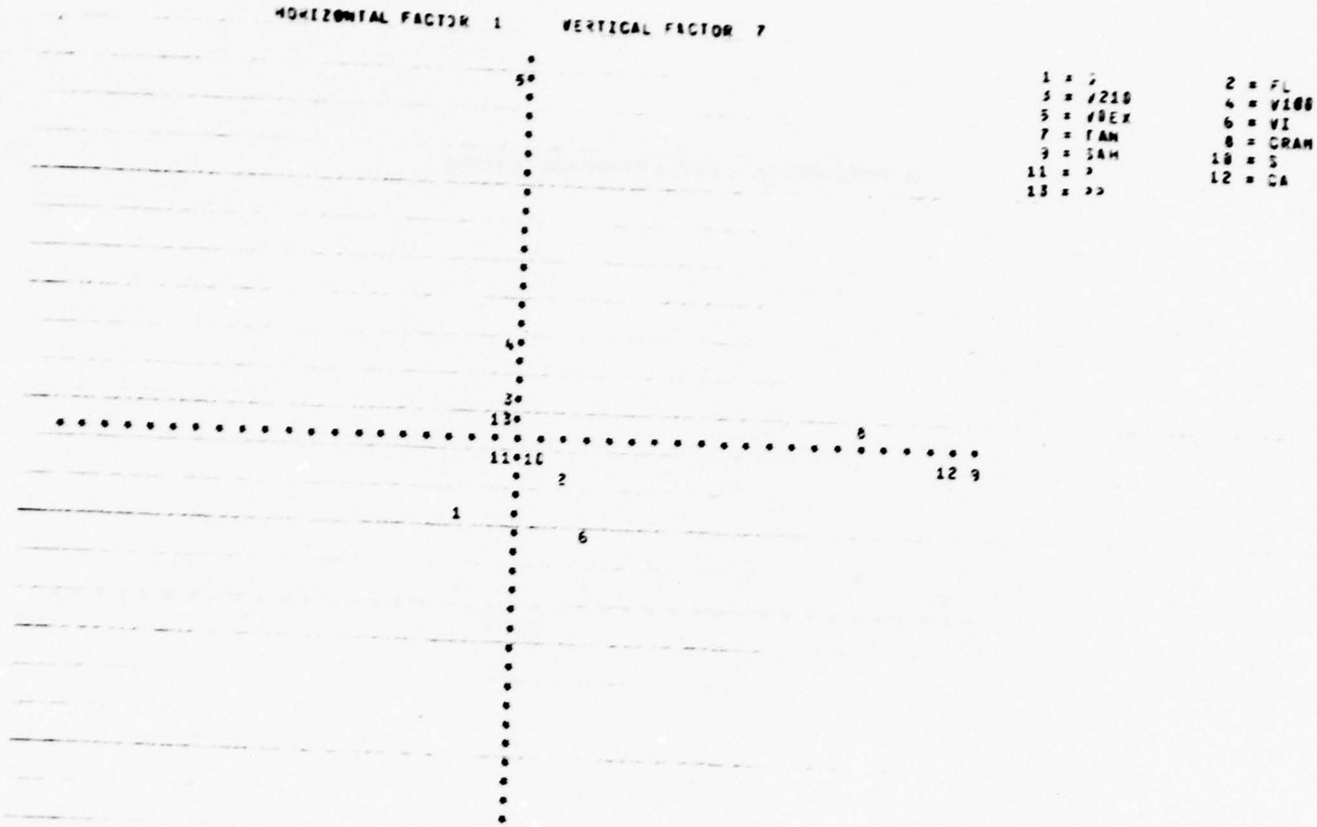


Figure 8. Horizontal Factor 1; Vertical Factor 7.

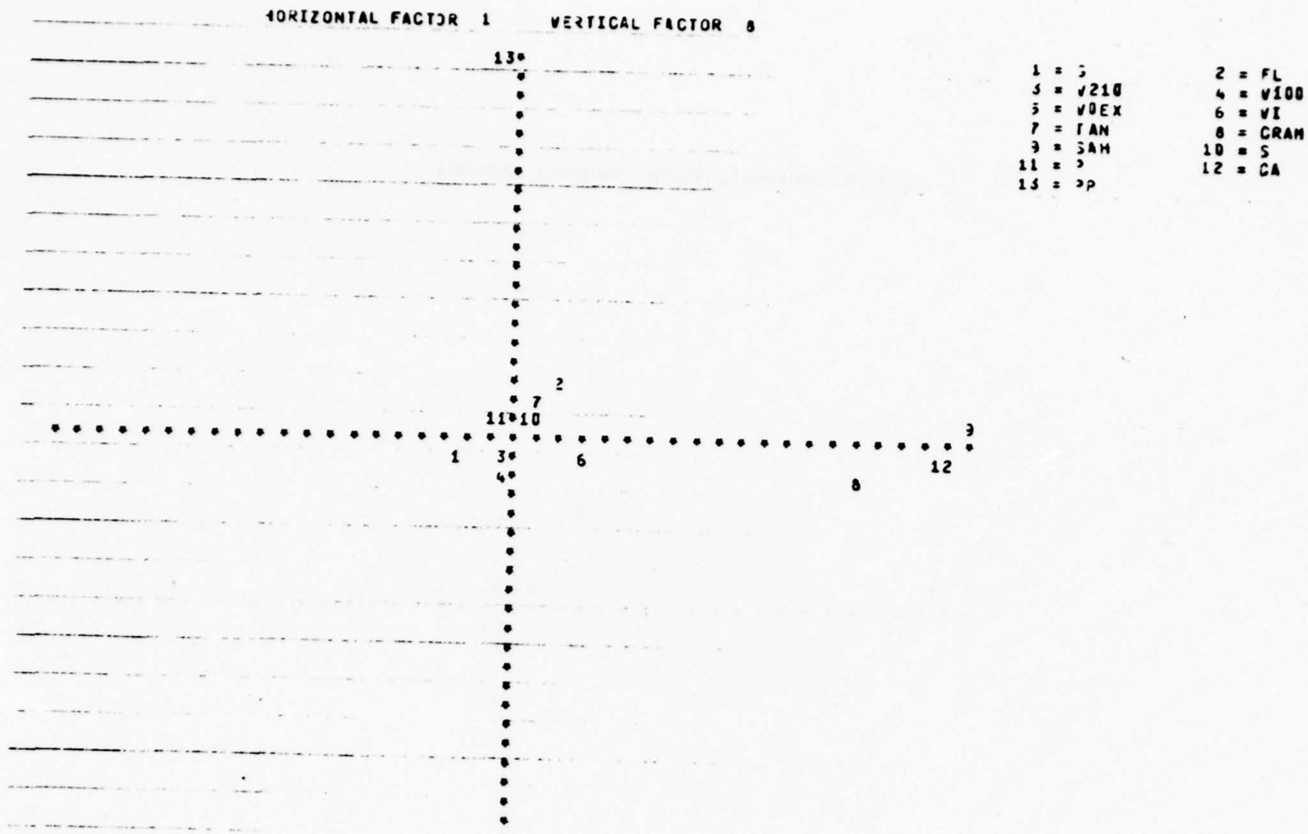


Figure 9. Horizontal Factor 1; Vertical Factor 8.

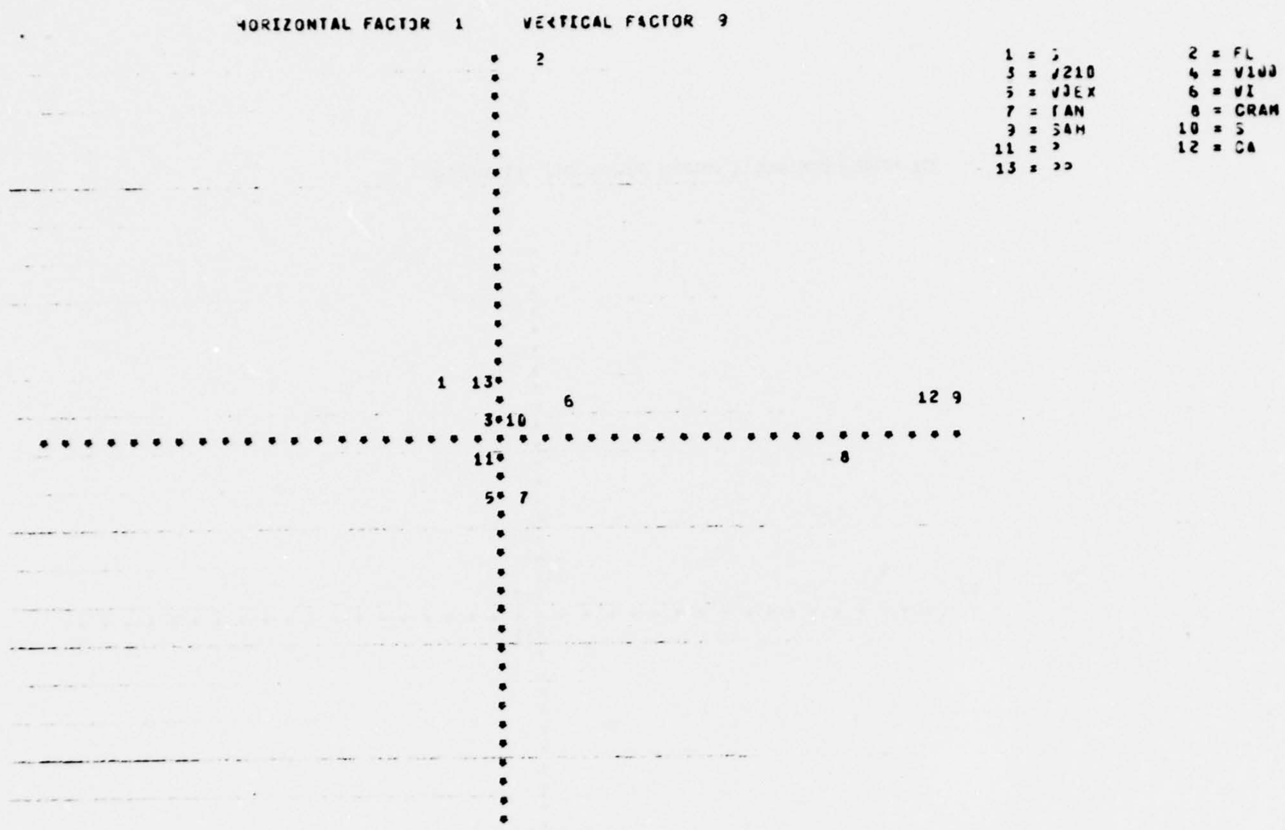


Figure 10. Horizontal Factor 1; Vertical Factor 9.

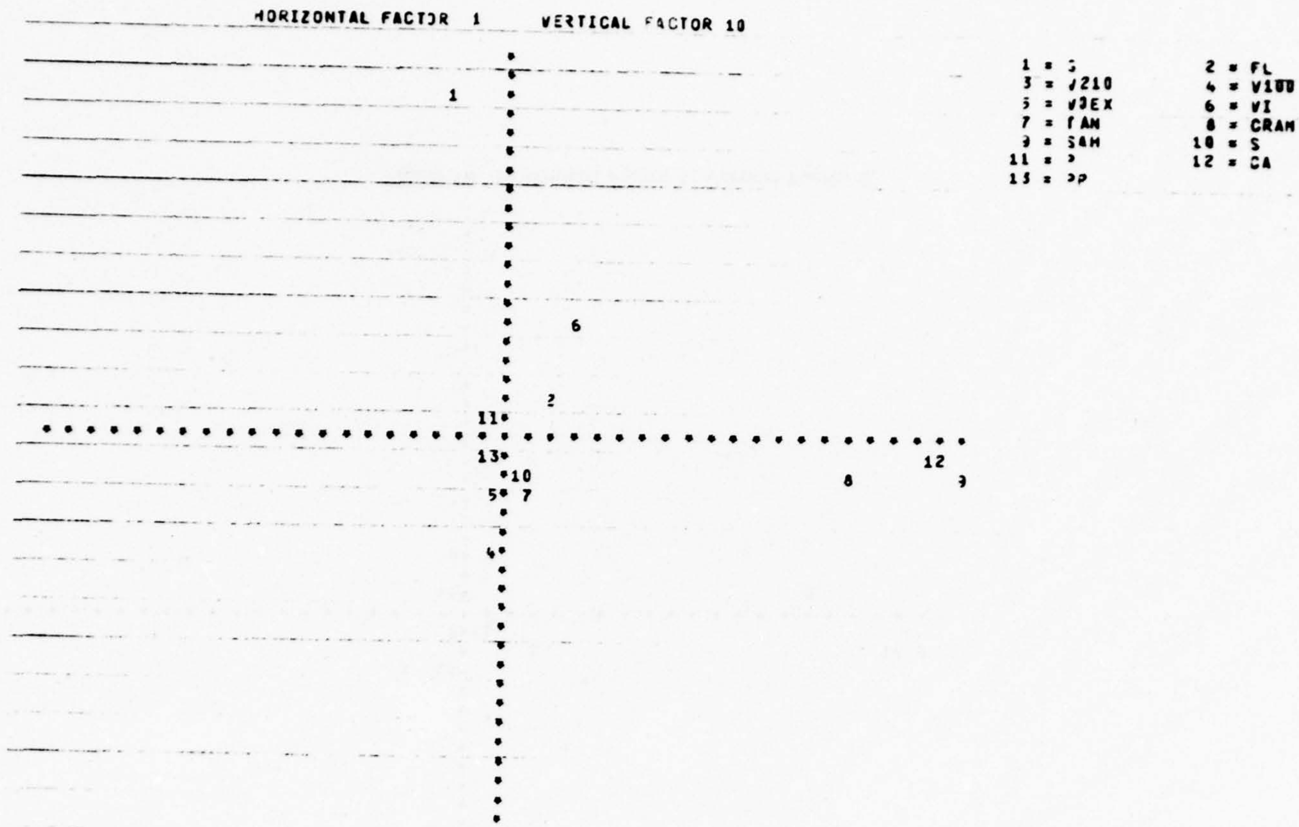


Figure 11. Horizontal Factor 1; Vertical Factor 10.

clusters of variables 8, 9, and 12 and 6, 2, 11, 13, 10, 5, and 7 separate the variables into two groups. Correlation within the two groups is indicated. Variable 3 shows no significant loading on either factor.

In Figure 12, variable 4 loads moderately high on factor 11. Variables 1, 6, 13, 10, 2, 11, 7, 5, and 3 are all close to the origin and have small loadings on both factors. Variables 8, 9, and 12 load high on factor 1 and have small loadings on factor 11. Two separate clusters are indicated with correlation within the clusters.

In Figure 13, variables, 1, 13, 2, 6, 5, and 10 all are close to the origin and have small loadings on both factors. Variables 9 and 12 have high loadings on factor 1. Variable 8 has significant loading on both factor 12 and factor 1. Variables 3, 4, 7, and 11 have no significant loading on either factor. Correlation within the clusters is indicated.

In Figure 14, variables 1, 5, 10, 2, 6, 13, and 7 all cluster near the origin and have small loadings on both factors. Variables 8, 9, and 12 load high on factor 1 and cluster less close; variables 9 and 12 show significant loading on factor 1 and are negative. Two groups are observable. Variables 3, 4, and 11 show insignificant loading on either factor. Correlation between the two groups is indicated.

In Figure 15, variable 7 loads high on factor 3. Variable 6 loads high on factor 2. Variables 5, 3, 13, 8, 9, 10, 11, 12, and 2 are all close to the origin and have small loadings on both factors. Clustering is indicative of correlation among the variables. Variable 4 loads moderately high on factor 2 and is negatively correlated.

In Figure 16, variable 3 loads high on factor 4. Variable 6 loads high on factor 2. Variables 5, 7, 9, 11, 2, 13, 10, and 12 all are close to the origin and have small loadings on both factors. Variable 4 loads moderately high on both factors and negatively on factor 4. Correlation is indicated within variables 5, 7, 9, 11, 2, 13, 10, and 12. Variable 8 does not load significantly on either factor.

In Figure 17, variable 10 loads high on factor 5 and insignificantly on factor 2. Variable 6 loads high on factor 2 and insignificantly on factor 5. Variables 13, 12, 2, 5, 3, 11, and 9 all are close to the origin and have small loadings on either factor. Variable 4 loads moderately heavy on factor 2 and negatively. Variables 12, 13, 2, 5, 3, 11, and 9 indicate correlation within. Variables 7 and 8 have insignificant loading on both factors.

In Figure 18, variable 11 loads high on factor 6 and insignificantly on factor 2. Variable 6 loads high on factor 2. Varible 4 loads moderately high and negative on factor 2. Variables 3, 2, 13, 9, 5, 8, 10, and 12 cluster and are all close to the origin.

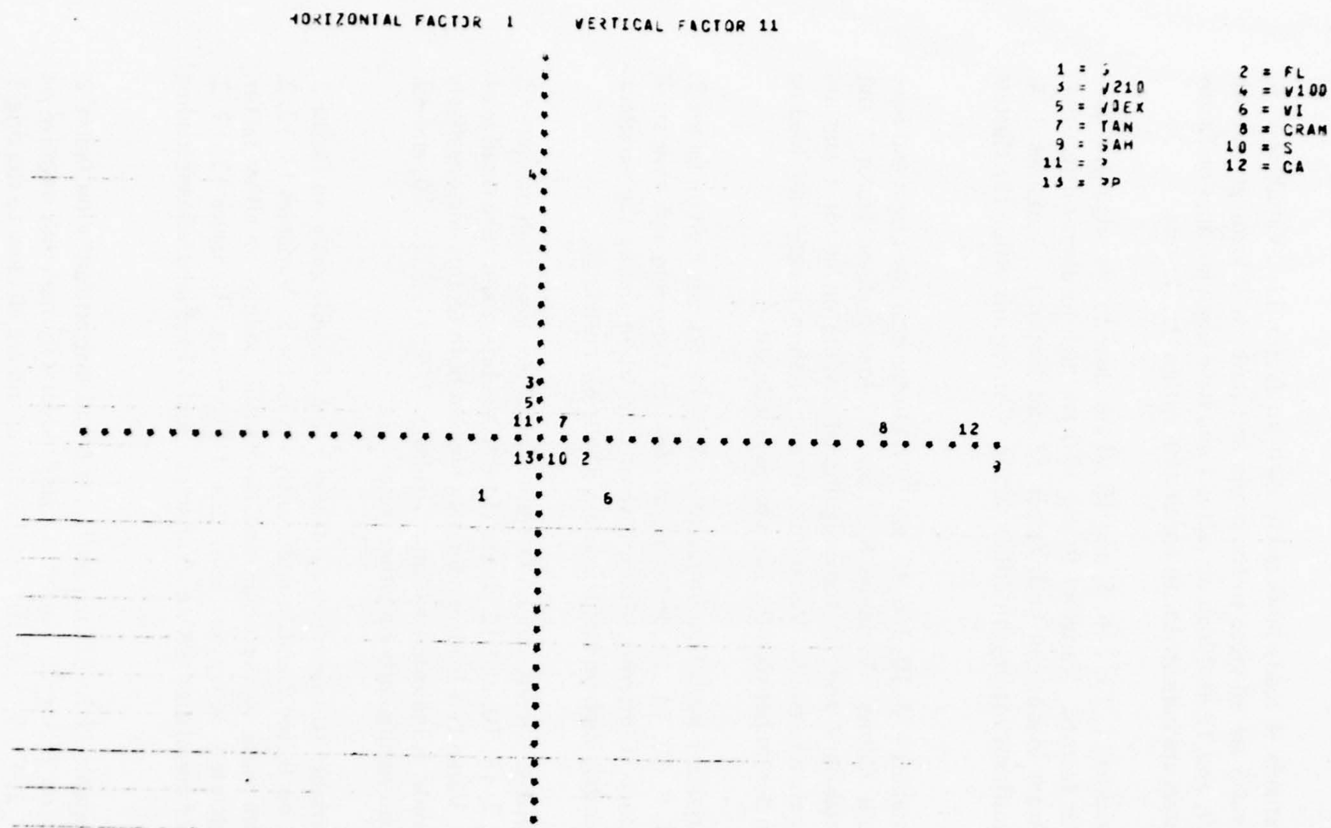


Figure 12. Horizontal Factor 1; Vertical Factor 11.

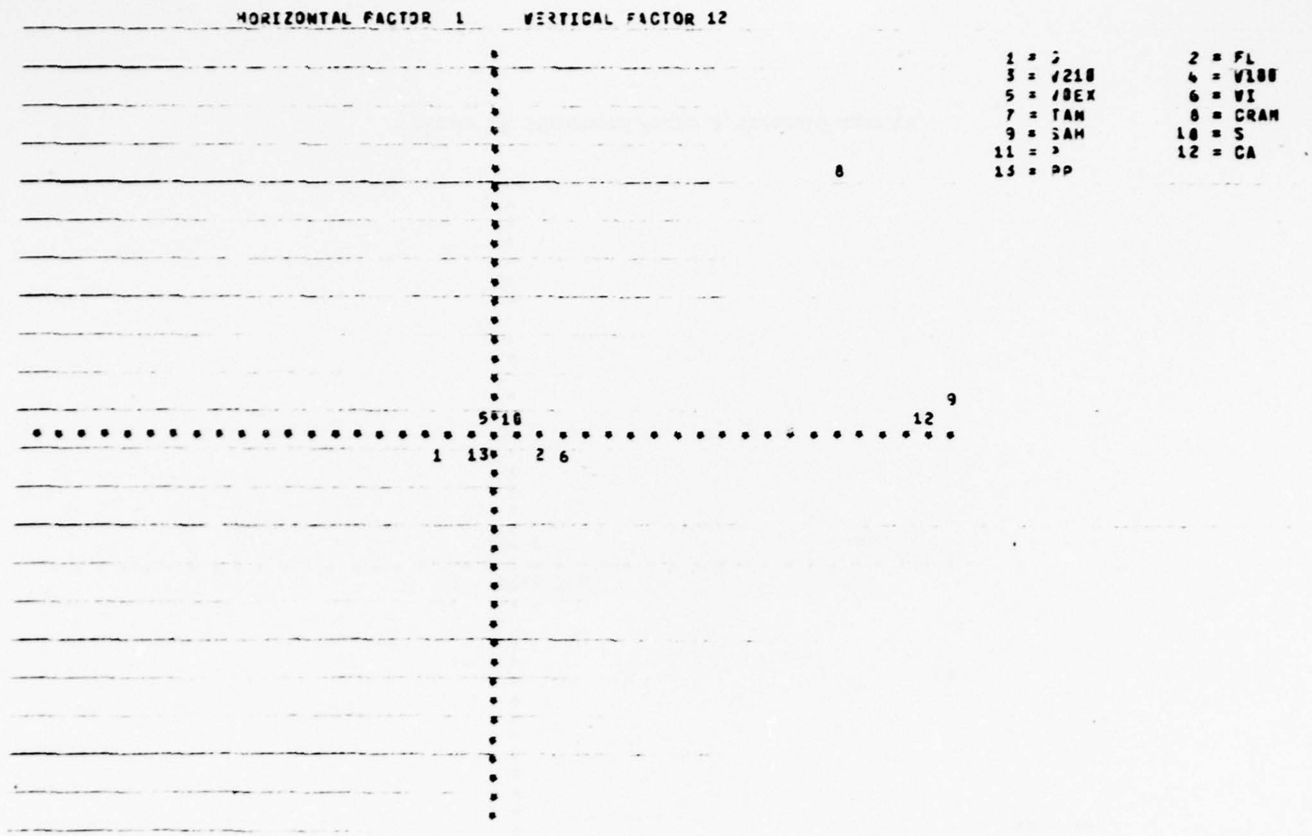


Figure 13. Horizontal Factor 1; Vertical Factor 12.

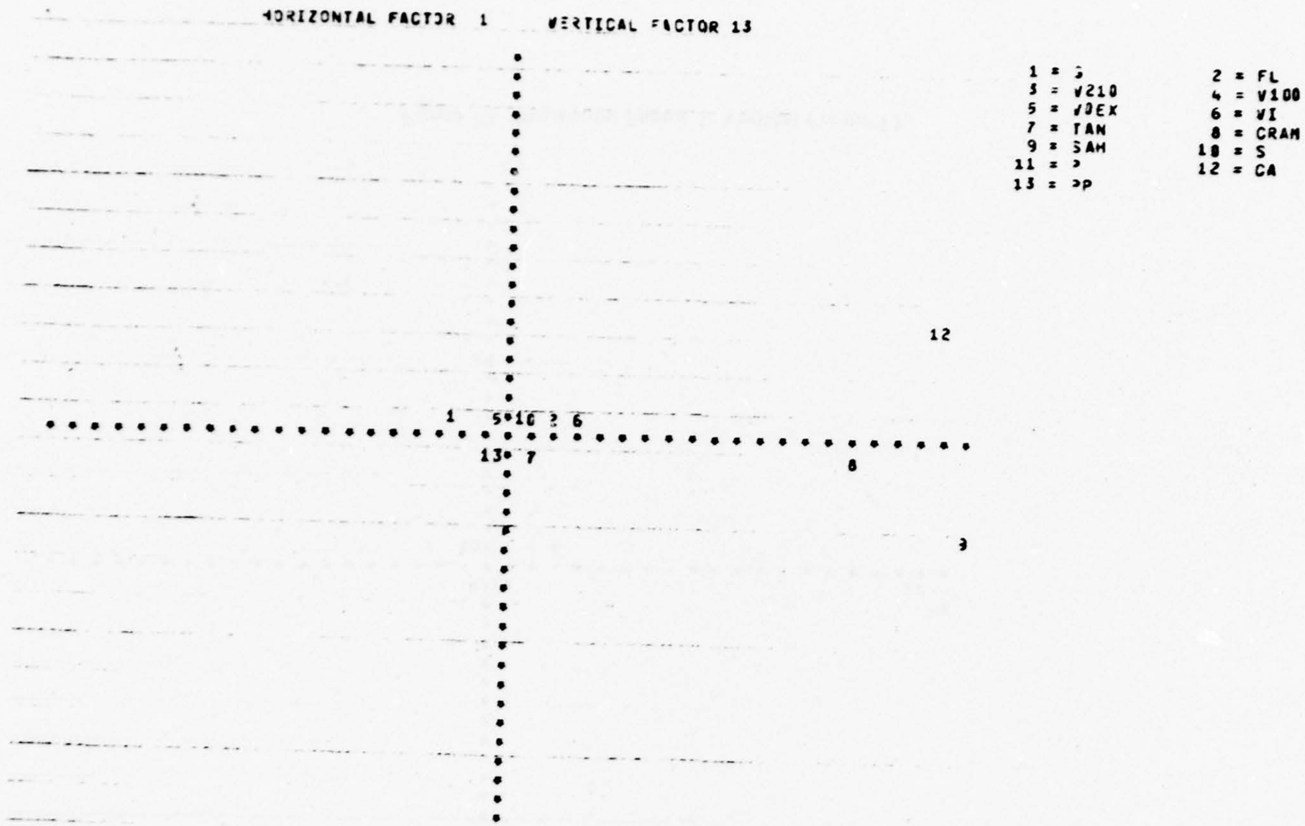


Figure 14. Horizontal Factor 1; Vertical Factor 13.

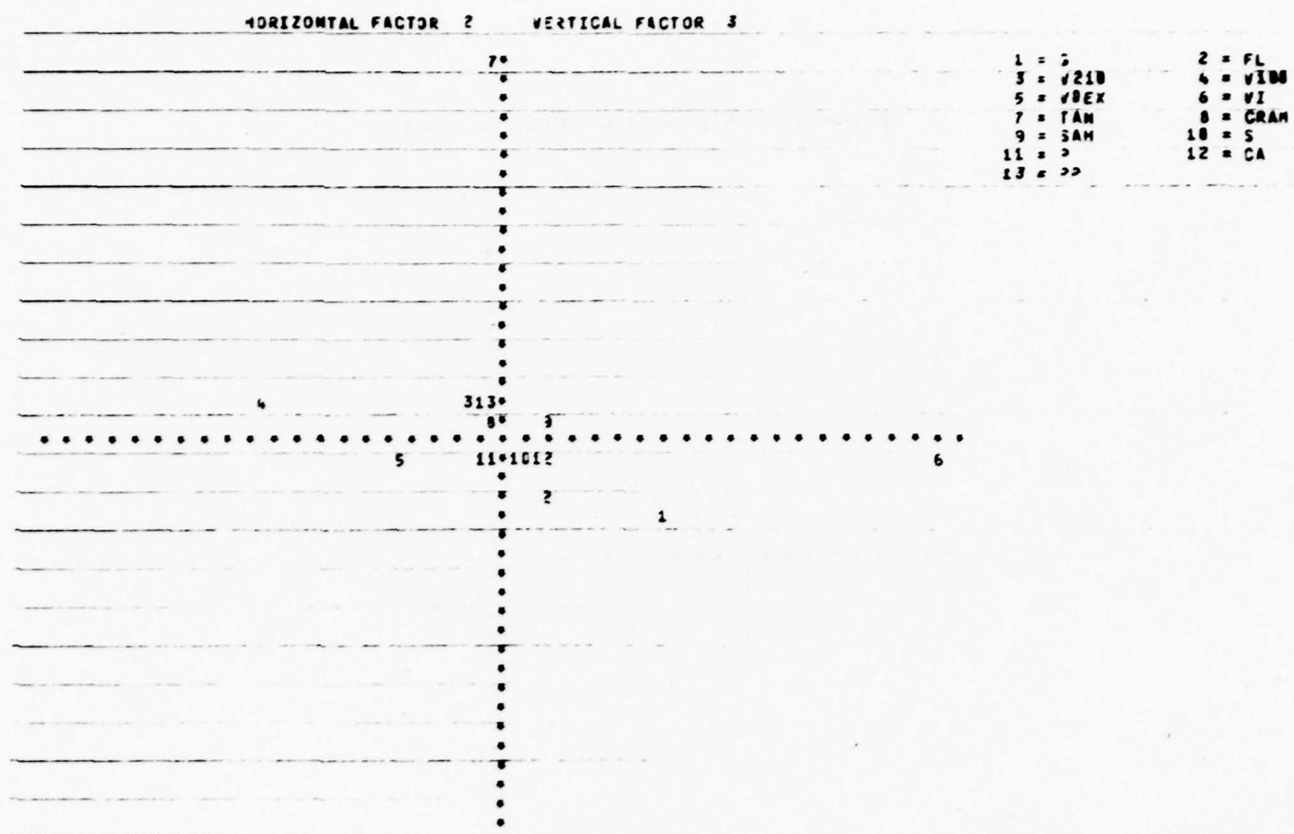


Figure 15. Horizontal Factor 2; Vertical Factor 3.

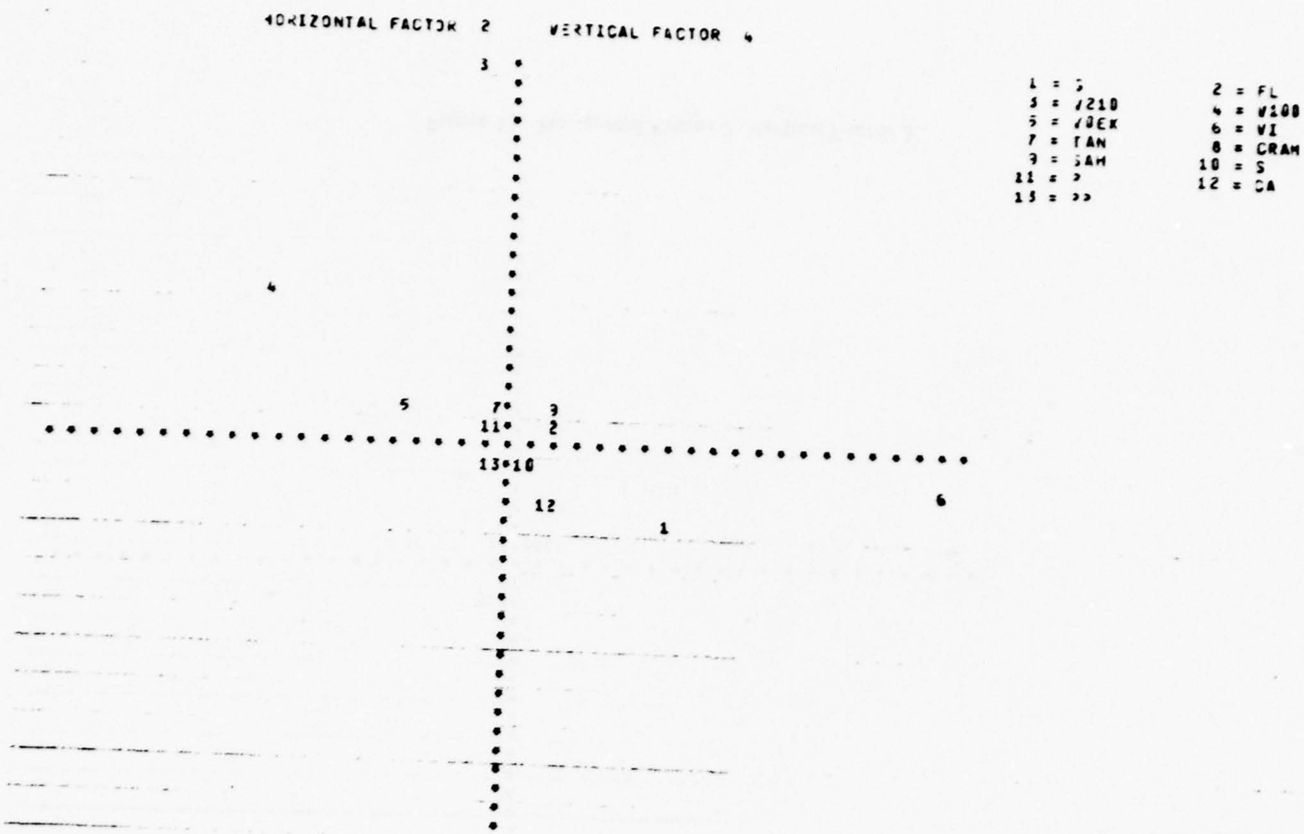


Figure 16. Horizontal Factor 2; Vertical Factor 4.

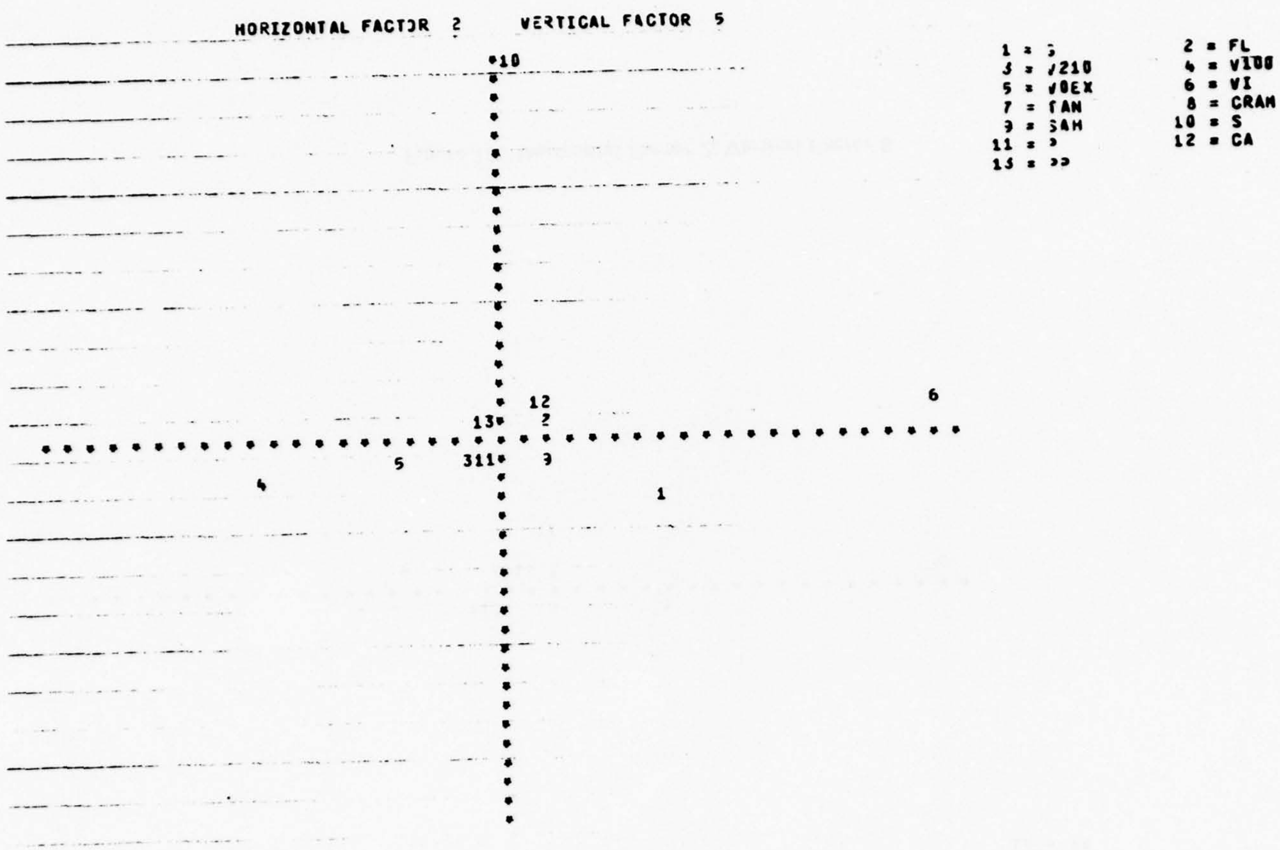


Figure 17. Horizontal Factor 2; Vertical Factor 5.

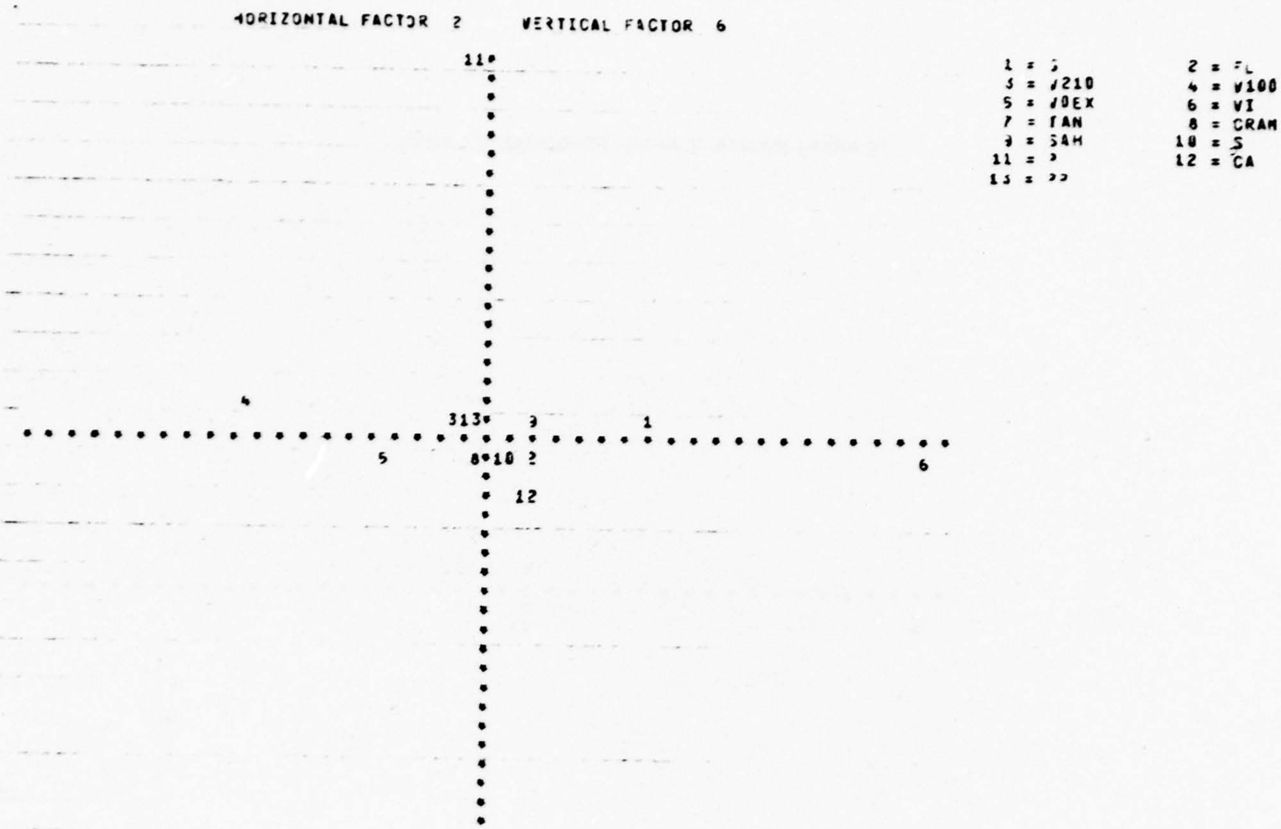


Figure 18. Horizontal Factor 2; Vertical Factor 6.

They have small loadings on both factors and indicate some internal correlation. Variable 7 loads insignificantly on both factors.

In Figure 19, variables 5 and 6 load high (negatively) on factors 7 and 2 respectively. Variables 4 and 1 load negatively and moderately on factors 2 and 7 respectively. Variables 3, 13, 10, 11, 12, and 2 cluster and are close to the origin with small loadings on each factor. Variables 3, 13, 10, 11, 12, and 2 correlate.

In Figure 20, variable 13 loads high on factor 8. Variable 6 loads high on factor 2. Variables 2, 7, 5, 9, 10, 11, 3, 8, 12, and 1 cluster and are close to the origin. Also, they all have small loadings on both factors. Variable 4 loads negatively and moderately high on factor 2.

In Figure 21, variable 6 loads high on factor 2. Variable 2 loads high on factor 9. Variables 3, 5, 7, 10, 11, 12, and 13 cluster and are close to the origin. Small loadings on both factors and correlation are indicated. Variables 8 and 9 load insignificantly on both factors.

In Figure 22, variables 2, 3, 5, 7, 8, 9, 10, 11, 12, and 13 are close to the origin and have small loadings on both factors. Correlation is indicated. Variable 1 loads high on factor 10. Variable 6 loads high on factor 2. Both variables 1 and 6 load on factors 10 and 2 respectively. Variable 4 loads negatively and moderately high on factor 2.

In Figure 23, variables 5, 3, 11, 12, 10, 13, and 9 cluster, are near the origin, and have small loadings on both factors. A degree of correlation is indicated. Variable 6 loads high on factor 2. Variable 4 loads moderately high on factors 2 and 11. Variable 2, 7, and 8 show insignificant loadings on both factors.

In Figure 24, variable 6 loads high on factor 2 and insignificantly on factor 12. Variable 8 loads high on factor 12. Variables 5, 3, 7, 9, 10, 12, 13, and 2 cluster, are close to the origin, and have small loadings on both factors. Correlation is indicated. Variable 11 exhibits insignificant loading on both factors. Variables 4, 5, and 1 load moderately high on factor 2 with variables 4 and 5 having a negative correlation.

In Figure 25, variables 3, 13, 2, 10, and 5 cluster and are close to the origin. There is no significant loading on either factor. The degree of cluster formation lessened. Correlation is indicated for variables 3, 2, 13, 10, and 5. Variable 6 loads high on factor 2. Variables 4, 3, 5, 10, 2, 1, and 6 have insignificant loading on factor 13. Variables 4 and 5 have a negative correlation. Variables 2, 7, and 8 show insignificant loading on either factor. Correlation as it relates to factor 2 and factor 13 is more limited. Variables 9 and 12 load on factor 13 with variable 9 having a negative correlation.

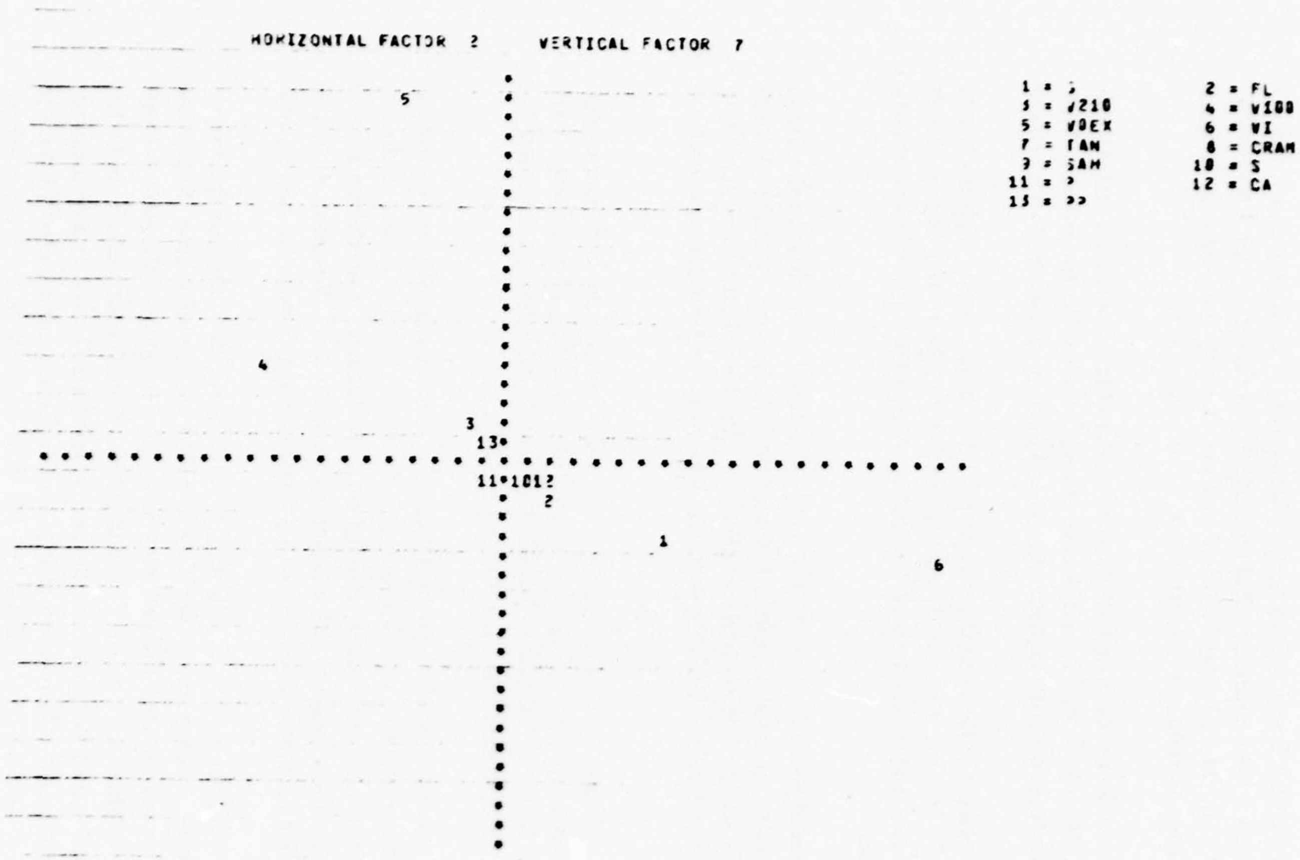


Figure 19. Horizontal Factor 2; Vertical Factor 7.

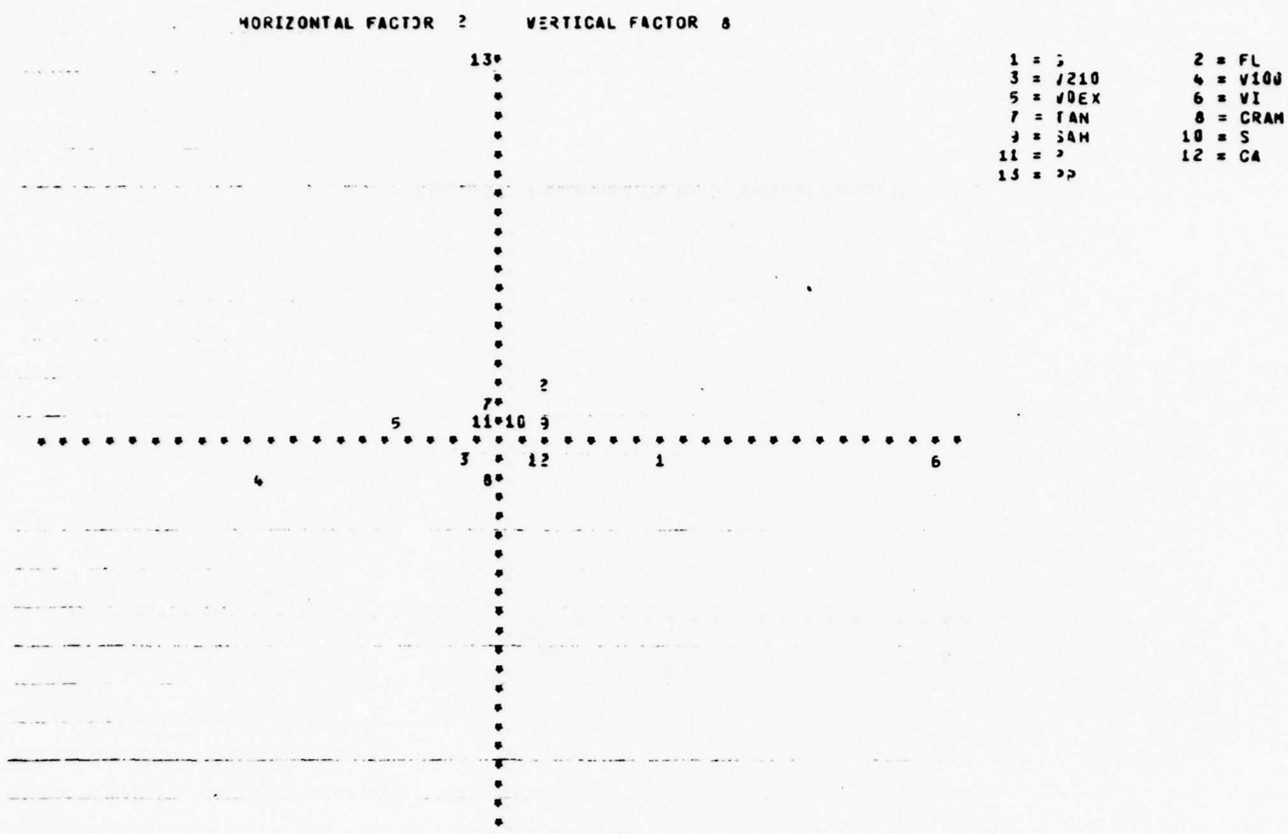


Figure 20. Horizontal Factor 2; Vertical Factor 8.

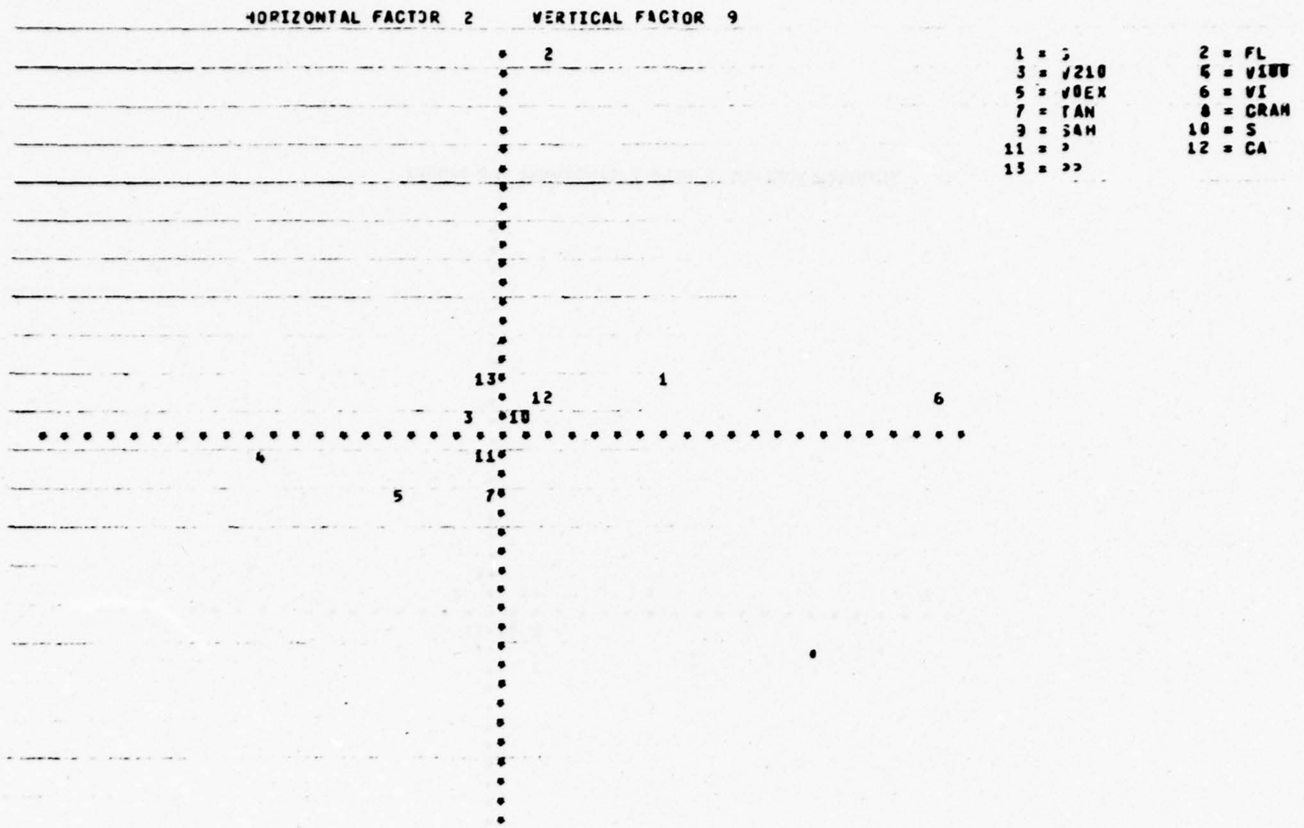


Figure 21. Horizontal Factor 2; Vertical Factor 9.

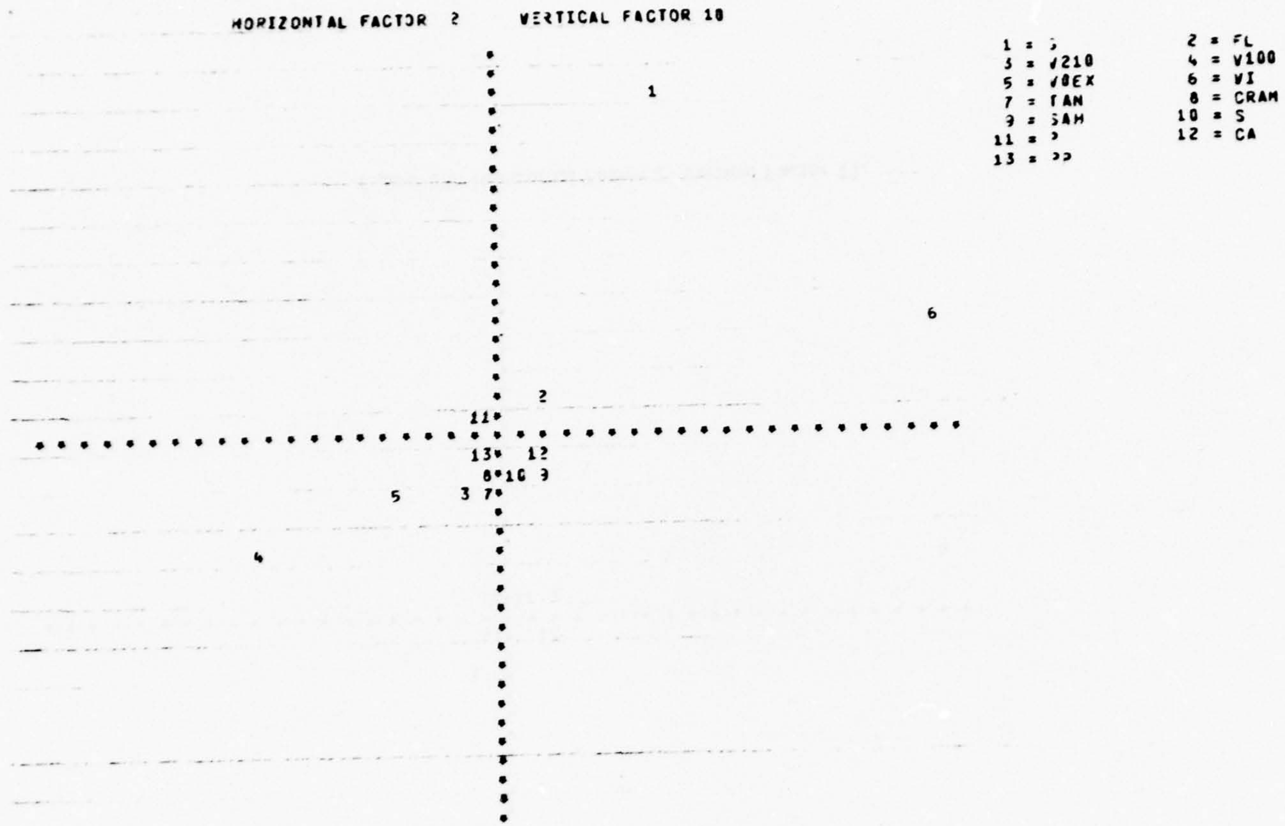


Figure 22. Horizontal Factor 2; Vertical Factor 10.

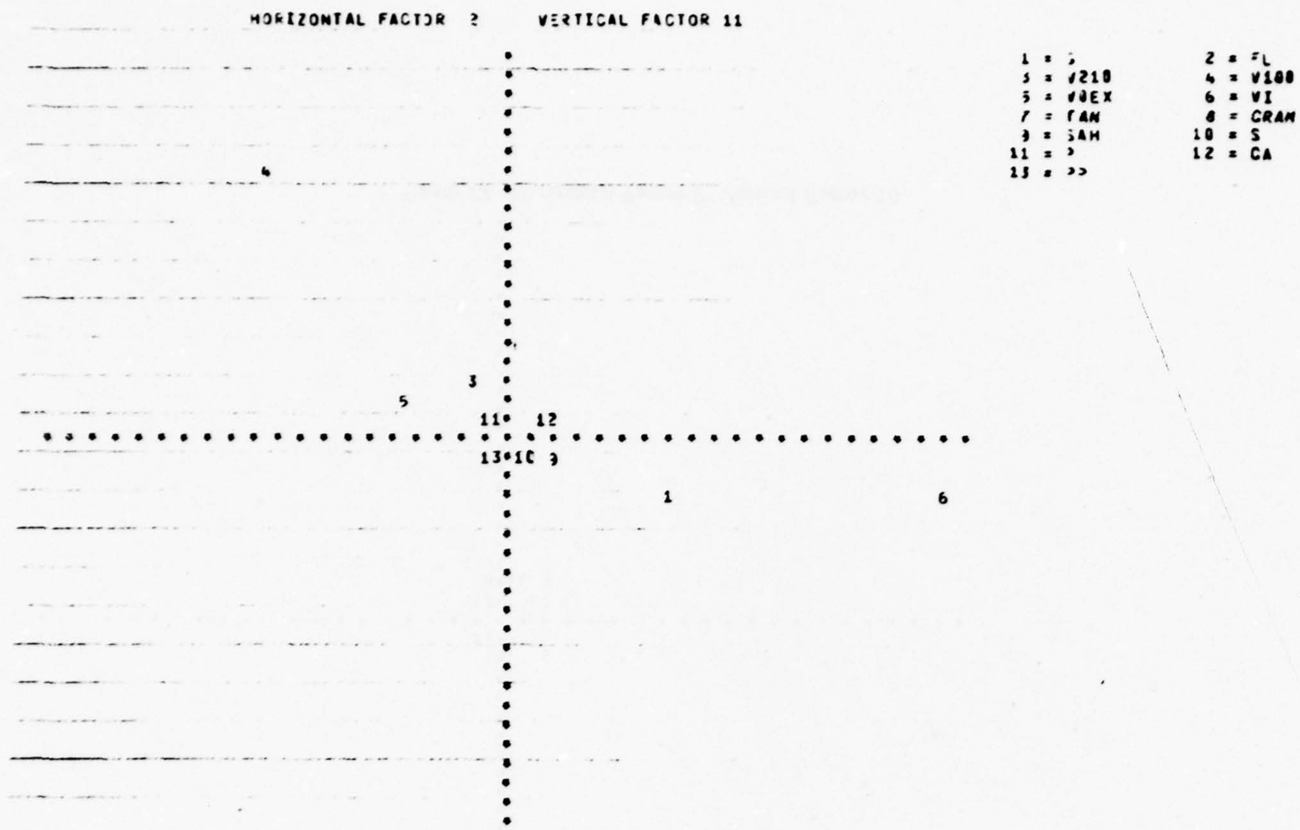


Figure 23. Horizontal Factor 2; Vertical Factor 11.

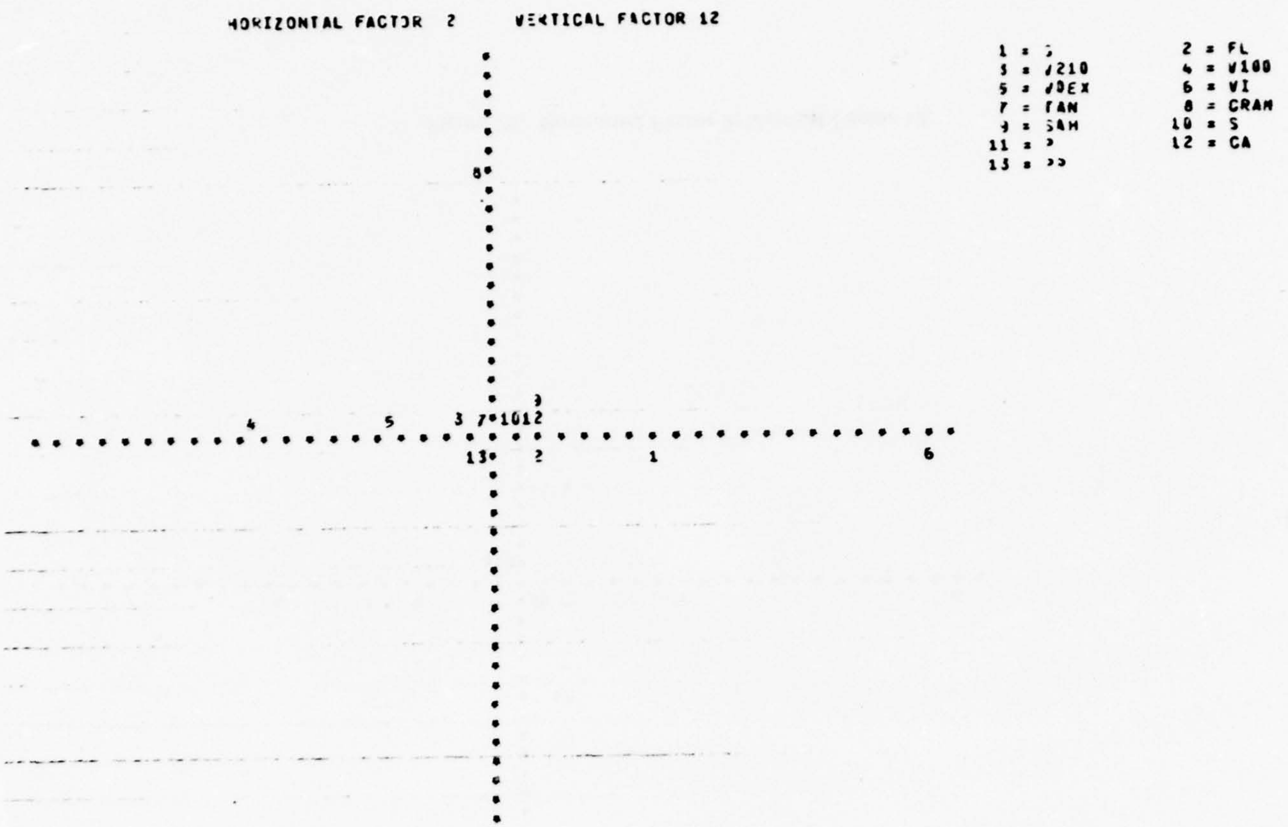


Figure 24. Horizontal Factor 2; Vertical Factor 12.

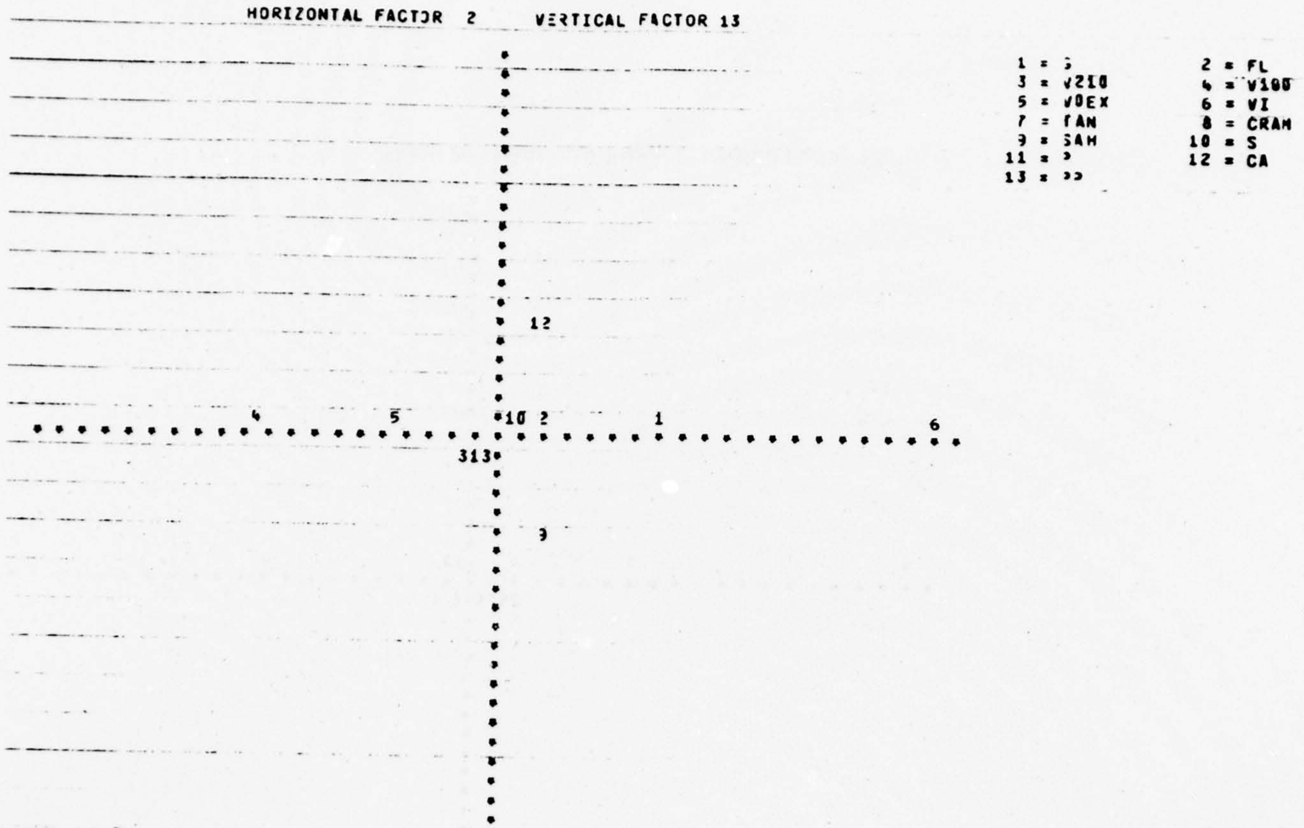


Figure 25. Horizontal Factor 2; Vertical Factor 13.

In Figure 26, variables 2, 5, 9, 11, 8, 10, 13, 6, and 12 cluster and are close to the origin. Group correlation is indicated. Variable 7 loads high on factor 3. Variable 3 loads high and variable 4 loads moderately on factor 4.

In Figure 27, variables 1, 2, 3, 4, 8, 9, 11, 12, and 13 cluster and are close to the origin. Small loadings on either factor and correlation are indicated. Variable 7 loads high on factor 3. Variable 10 loads high on factor 5. Only one grouping of variables is indicated. Variables 5 and 6 have insignificant loading on both factors.

In Figure 28, variables 1, 4, 9, 13, 2, 10, 8, and 12 cluster and are close to the origin. Small loading exists on either factor. Correlation is indicated. Variable 11 loads high on factor 6. Variable 7 loads high on factor 3. Variables 5, 3, and 6 show no loading.

In Figure 29, variables 4, 3, 8, 13, 12, 9, 2, 1, and 6 cluster and are close to the origin. Small loading exists on either factor. Correlation is indicated. Variable 5 loads high on factor 7. Variable 7 loads high on factor 3. Variables 10 and 11 show no significant loading on factor 3 or factor 7 respectively.

In Figure 30, variables 2, 9, 11, 1, 12, 3, 8, and 4 cluster and are close to the origin. Small loading exists for either factor. Correlation is indicated. One main grouping exists. Variable 7 loads high on factor 3. Variable 13 loads high on factor 8 – one group of the data indicated. Variables 5, 6, and 10 show no loading on either factor.

In Figure 31, variables 4, 8, 11, 5, 3, 10, 12, 9, 1, and 13 are clustered. All are near the origin and have small loadings on either factor. Internal correlation is indicated. Variable 2 loads high on factor 9. Variable 7 loads high on factor 3. Variable 6 shows no loading on either factor.

In Figure 32, variables 2, 6, 11, 12, 13, 10, 9, 5, and 3 cluster and all are close to the origin. Internal correlation is indicated. Variable 7 loads high on factor 3. Variable 1 loads high on factor 10. Variable 8 shows no loading of significance.

In Figure 33, variables 3, 5, 12, 8, 2, 10, 9, 13, 1, and 6 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 7 loads high on factor 3. Variable 4 loads high on factor 11. One group of variables is indicated. Variable 11 shows no loading on either factor.

In Figure 34, variables 4, 9, 12, 1, 2, 11, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 7 loads high on factor 3. Variables 3, 5, 6, and 10 have no significant loading on either factor. Variable 8 loads high on factor 12.

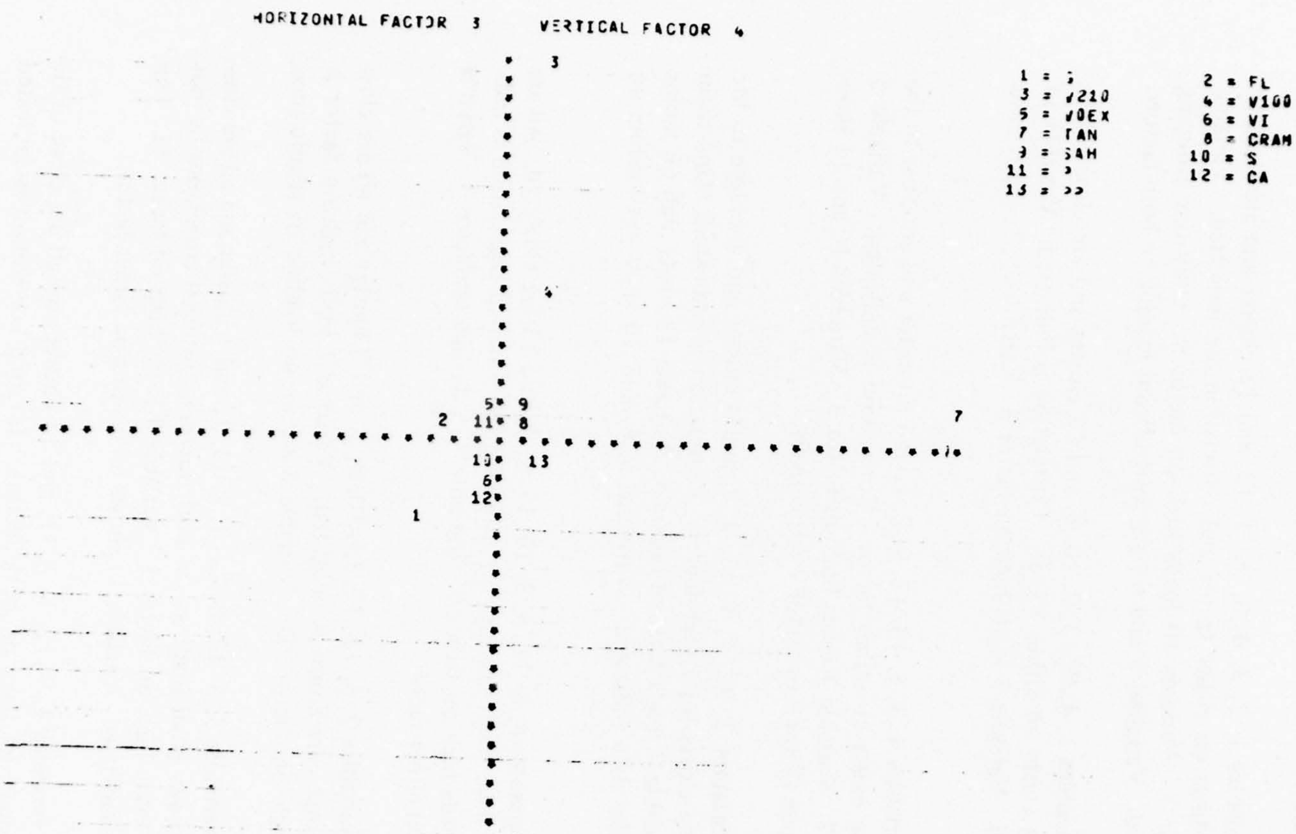


Figure 26. Horizontal Factor 3; Vertical Factor 4.

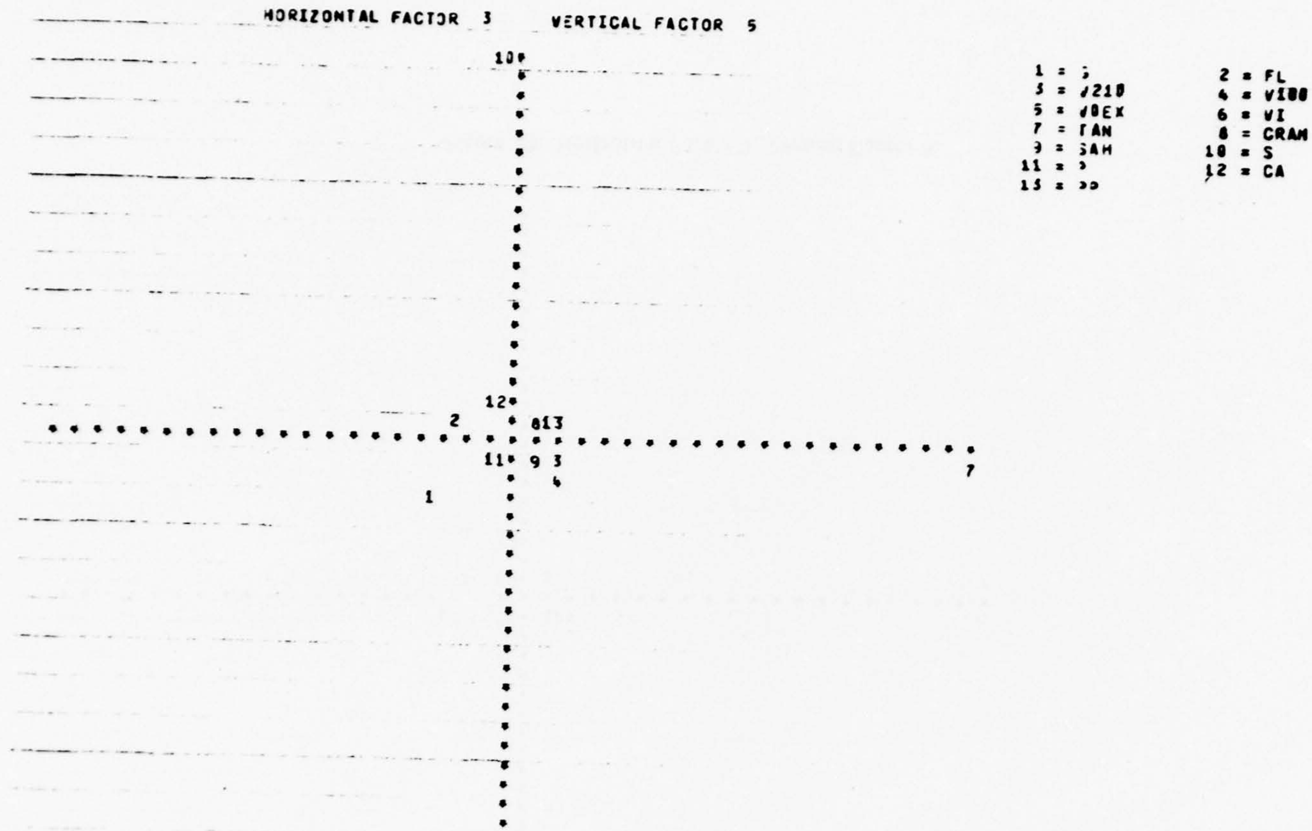


Figure 27. Horizontal Factor 3; Vertical Factor 5.

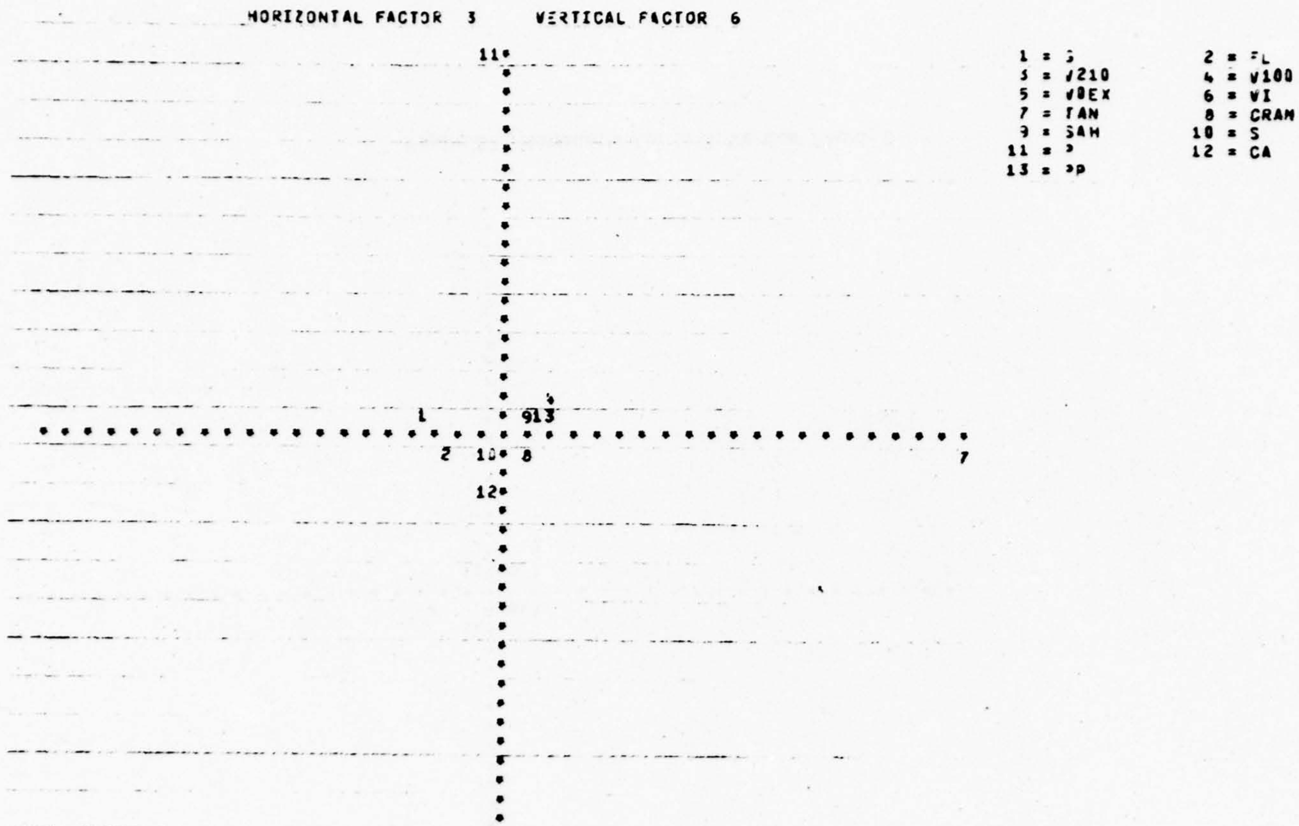


Figure 28. Horizontal Factor 3; Vertical Factor 6.

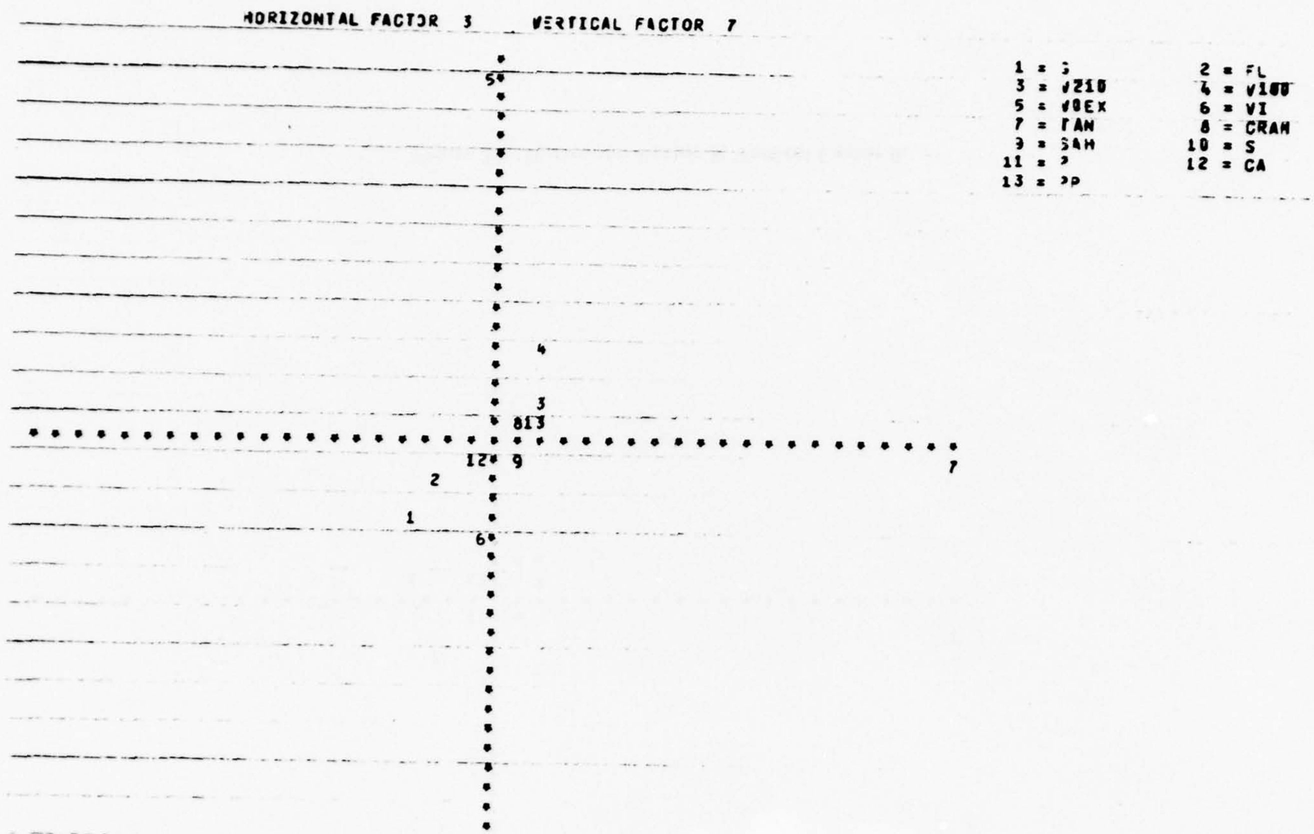


Figure 29. Horizontal Factor 3; Vertical Factor 7.

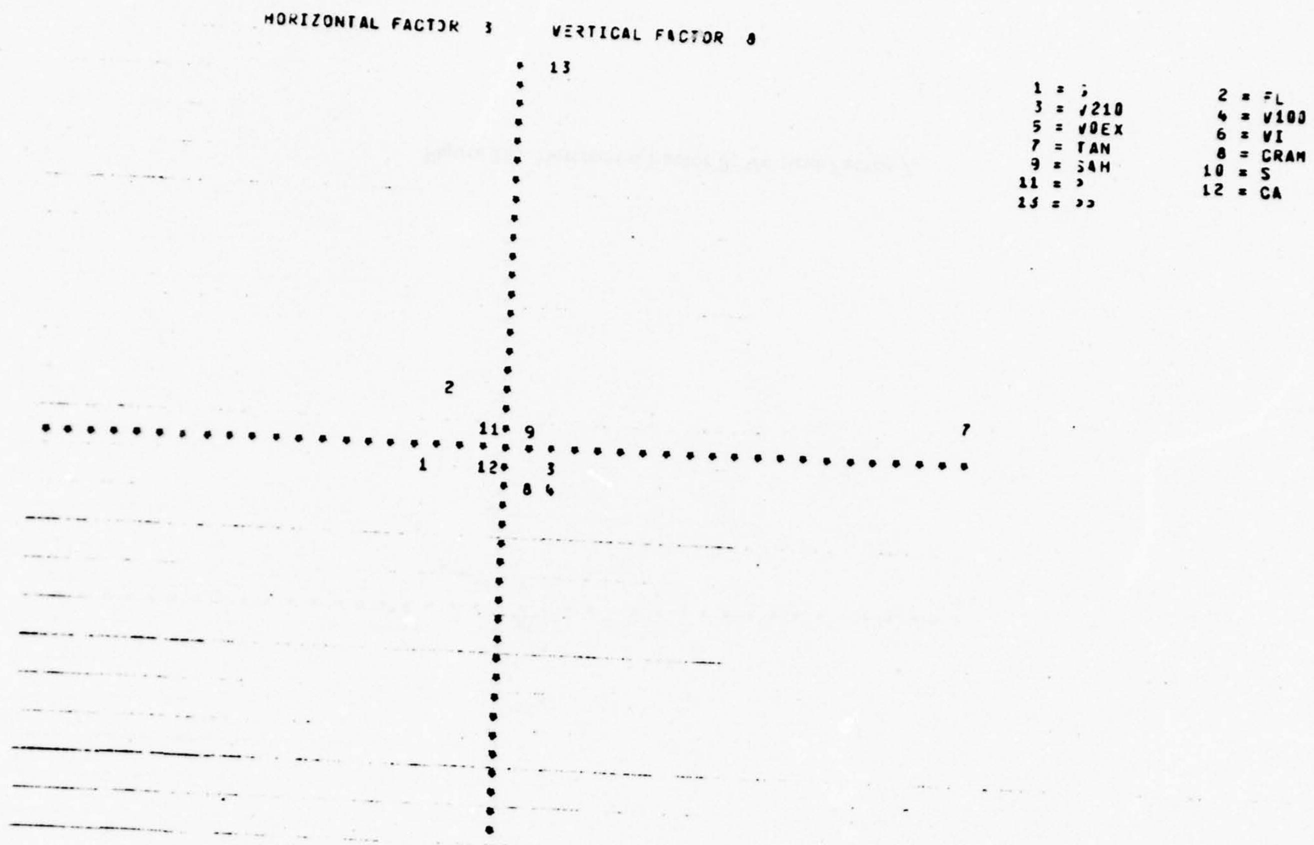


Figure 30. Horizontal Factor 3; Vertical Factor 8.

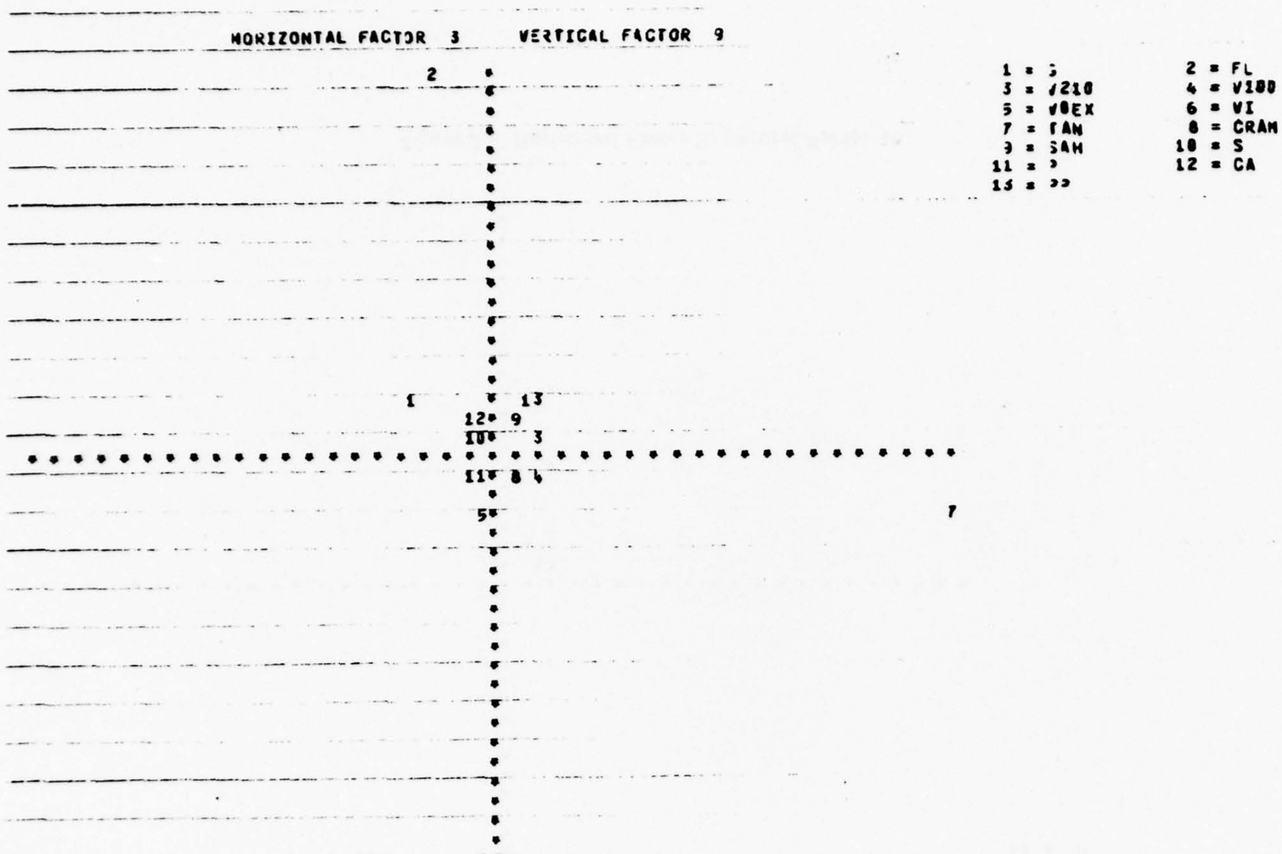


Figure 31. Horizontal Factor 3; Vertical Factor 9.

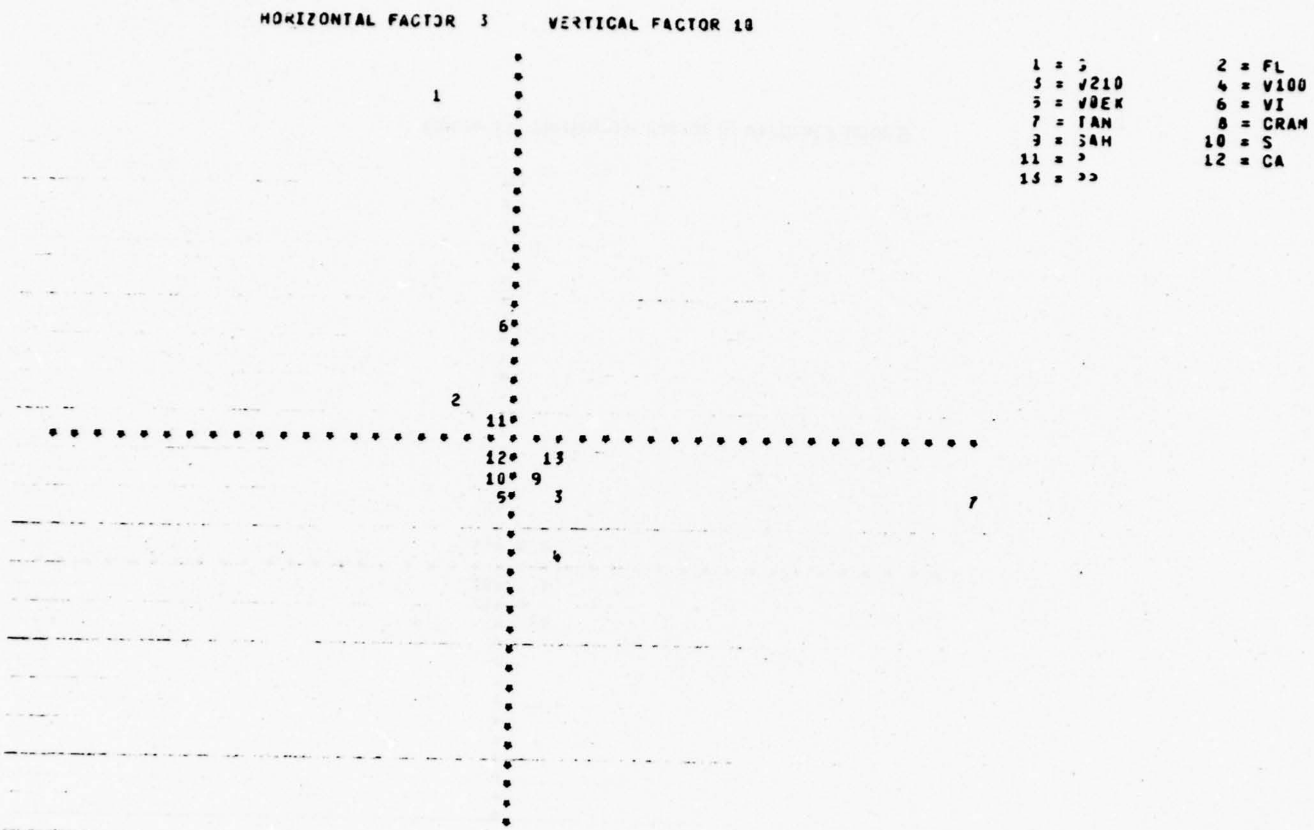


Figure 32. Horizontal Factor 3; Vertical Factor 10.

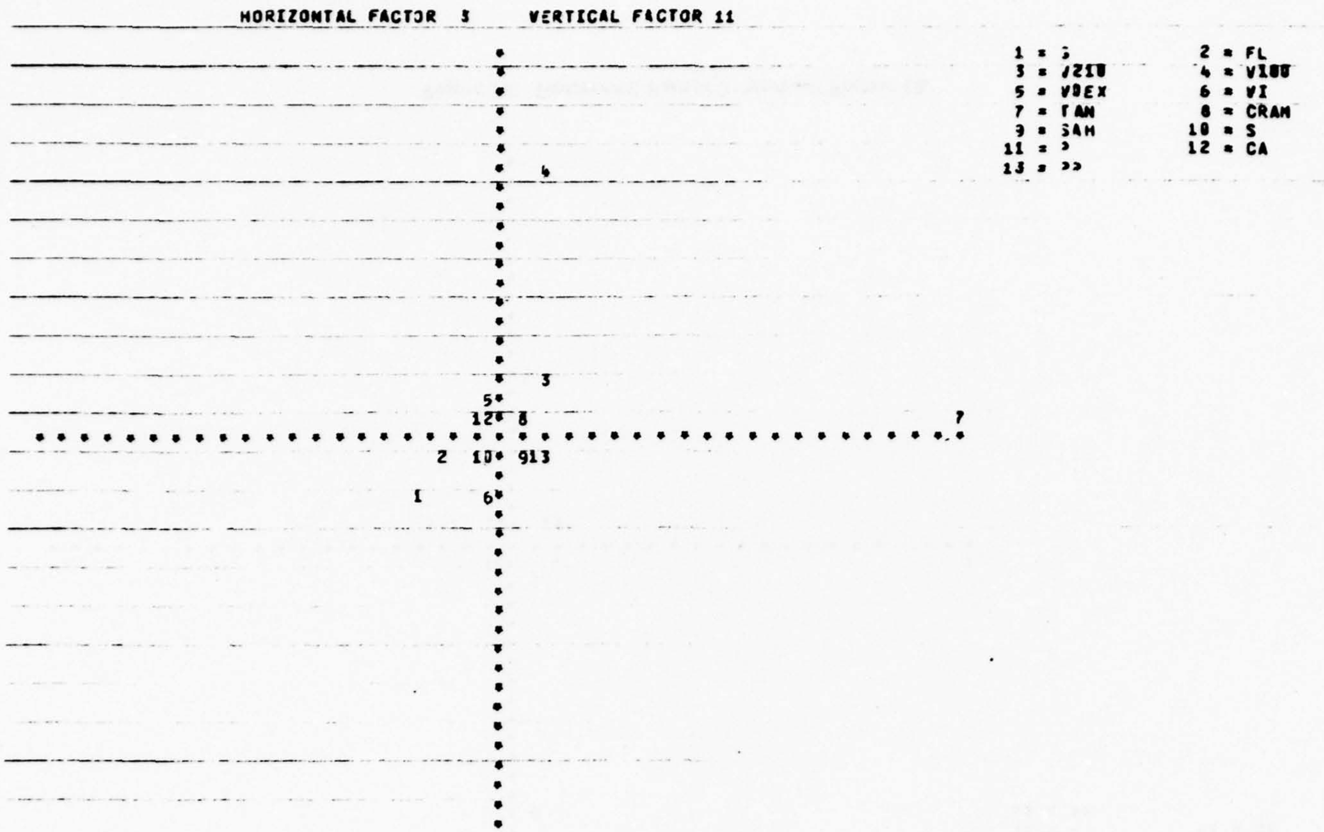


Figure 33. Horizontal Factor 3; Vertical Factor 11.

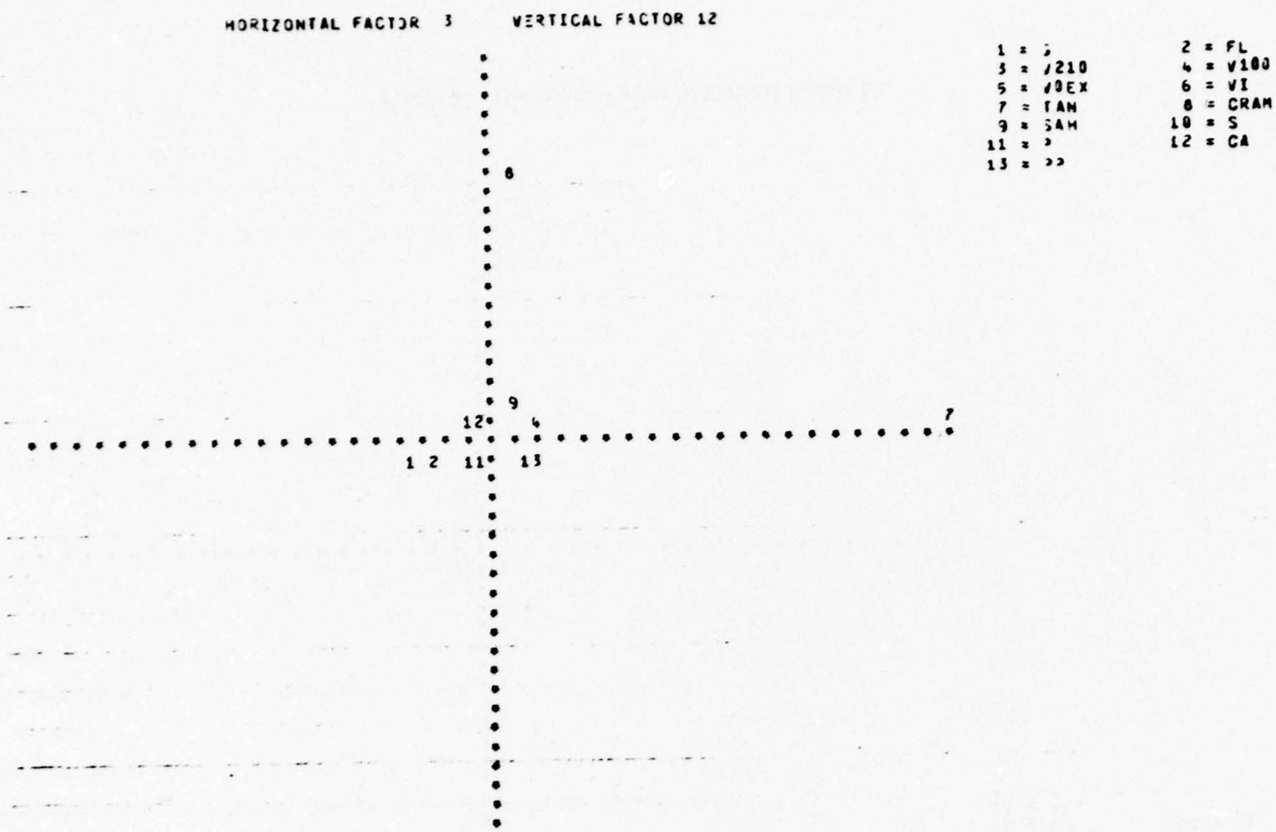


Figure 34. Horizontal Factor 3; Vertical Factor 12.

In Figure 35, variables 1, 2, 10, 4, 12, 11, 8, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 7 loads high on factor 3. Variables 3, 5, and 6 show insignificant loadings on both factors.

In Figure 36, variables 12, 6, 13, 8, 11, and 9 cluster and are all close to the origin All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4; variable 10 loads high on factor 5. Variables 2, 5, and 7 do not load significantly on either factor.

In Figure 37, variables 1, 13, 9, 6, 10, 8, 7, and 12 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4; variable 6 loads high on factor 4. Variables 2 and 5 show no significant loading on either factor.

In Figure 38, variables 13, 8, 12, 10, 11, 9, and 2 cluster and are all close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4; variable 5 loads high on factor 7. Variable 7 shows insignificant loading on both factors.

In Figure 39, variables 2, 7, 10, 11, 9, 1, 12, 6, and 8 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4. Variable 13 loads high on factor 8. Variable 5 shows insignificant loading on both factors.

In Figure 40, variables 1, 13, 12, 6, 10, 9, and 11 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4; variable 2 loads high on factor 9. Variables 5 and 8 show insignificant loading on either factor.

In Figure 41, variables 2, 11, 12, 13, 10, 8, 9, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4. Variable 1 loads high on factor 10. Variable 5 shows insignificant loading on either factor.

In Figure 42, variables 5, 7, 11, 12, 2, 9, 13, 1, and 6 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4. Variable 4 loads moderately high on both factors. Variable 10 shows insignificant loading on either factor.

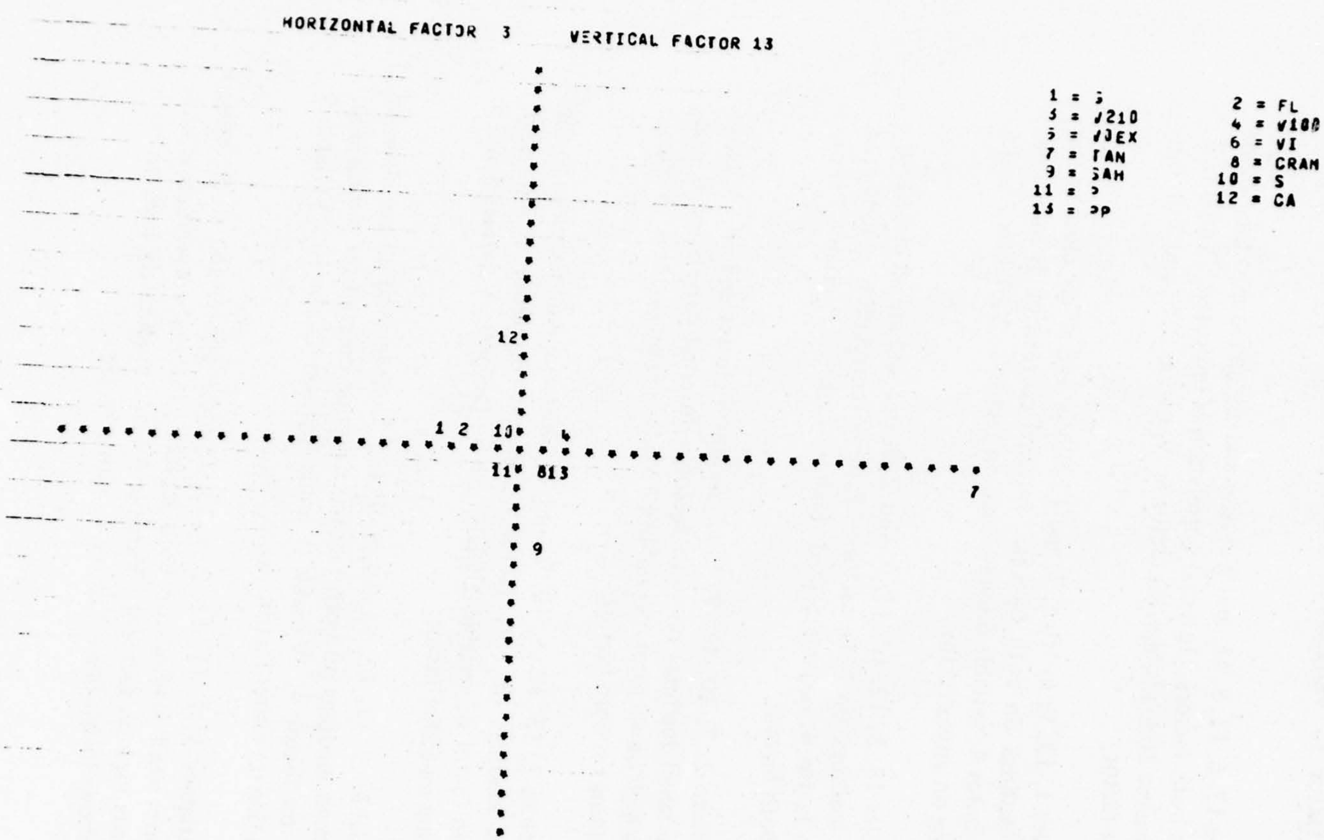


Figure 35. Horizontal Factor 3; Vertical Factor 13.

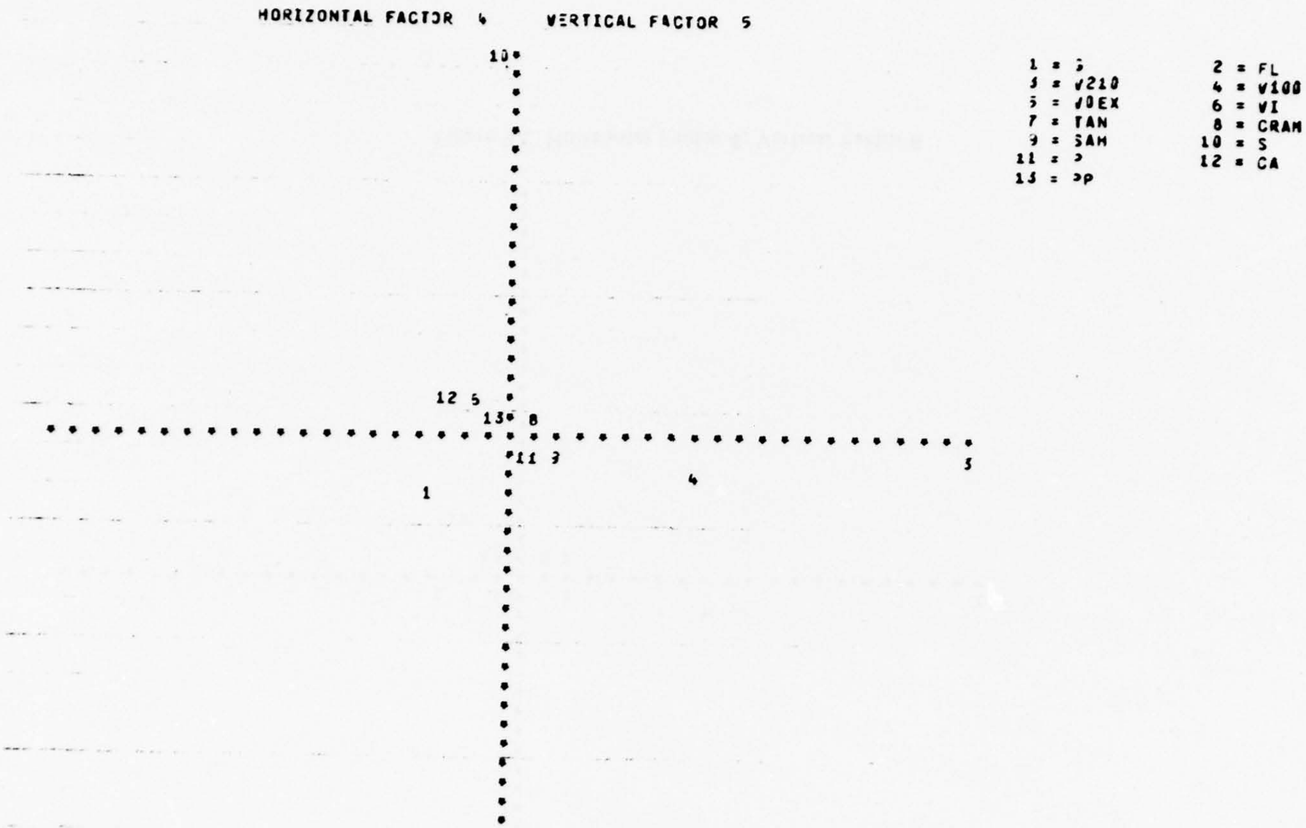


Figure 36. Horizontal Factor 4; Vertical Factor 5.

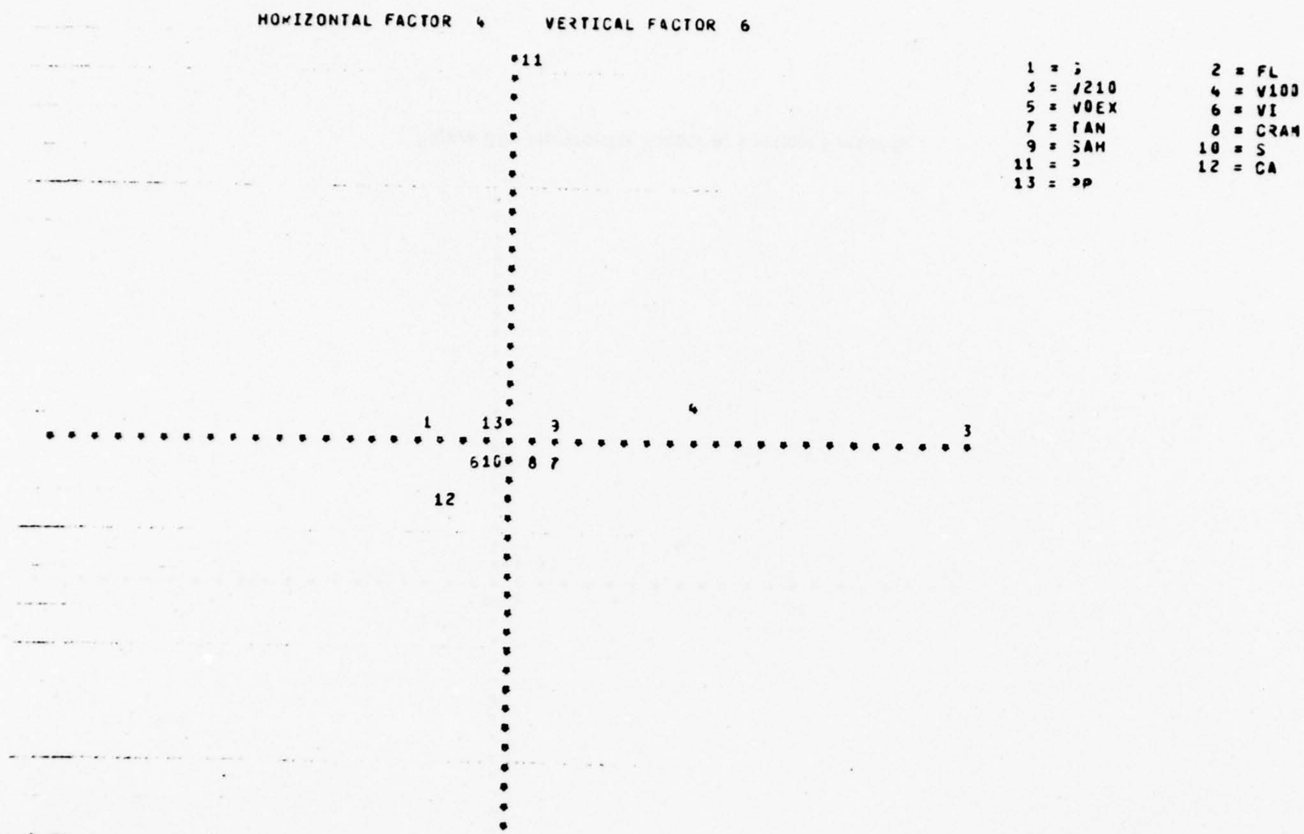


Figure 37. Horizontal Factor 4; Vertical Factor 6.

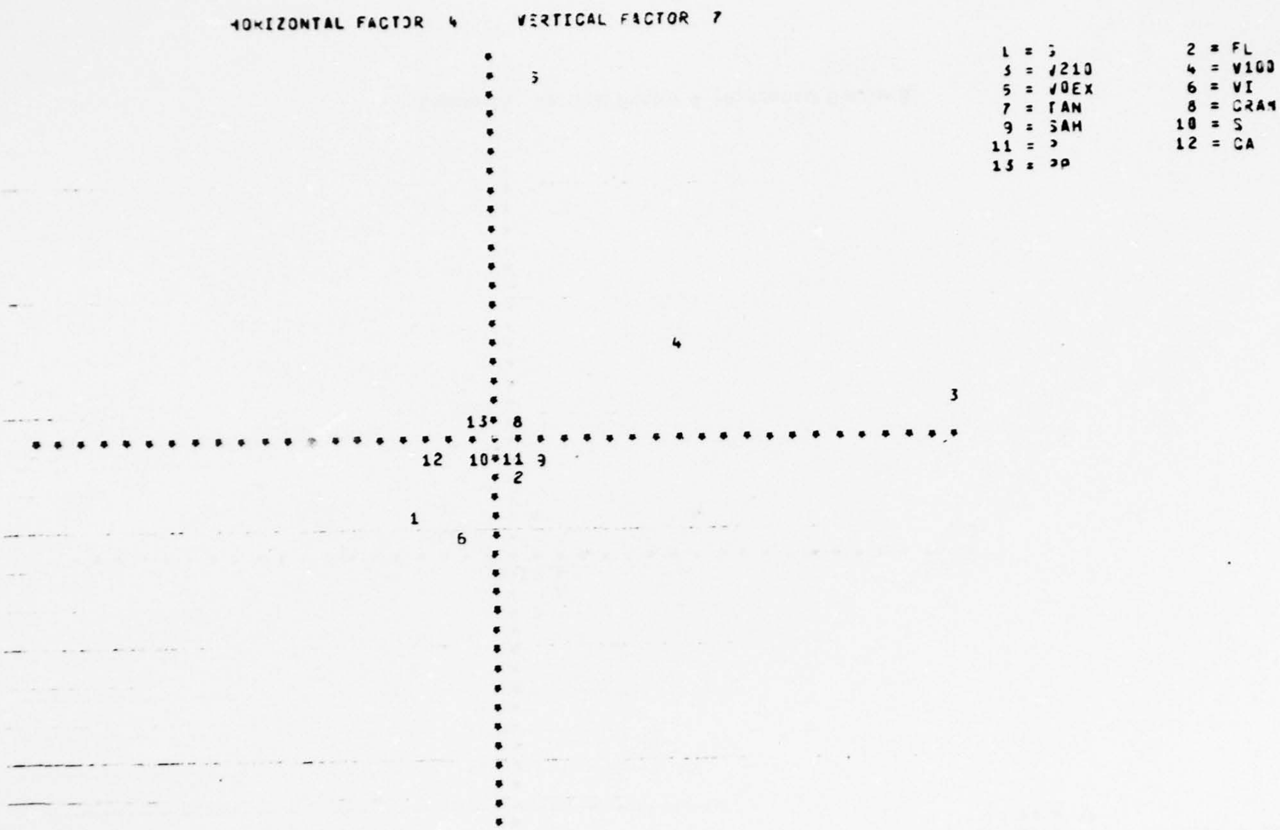
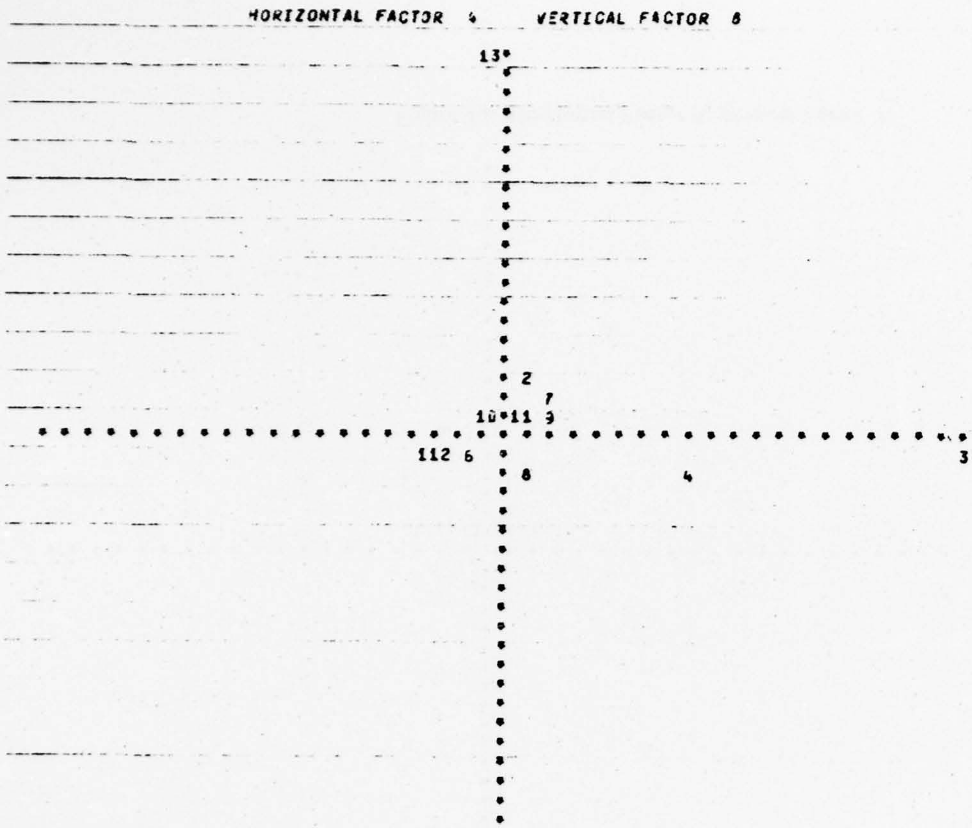


Figure 38. Horizontal Factor 4; Vertical Factor 7.



1 = ;	2 = FL
3 = V210	4 = V100
5 = V0EX	6 = VI
7 = TAN	8 = CRAM
9 = SAM	10 = S
11 = >	12 = CA
13 = >>	

Figure 39. Vertical Factor 4; Horizontal Factor 8.

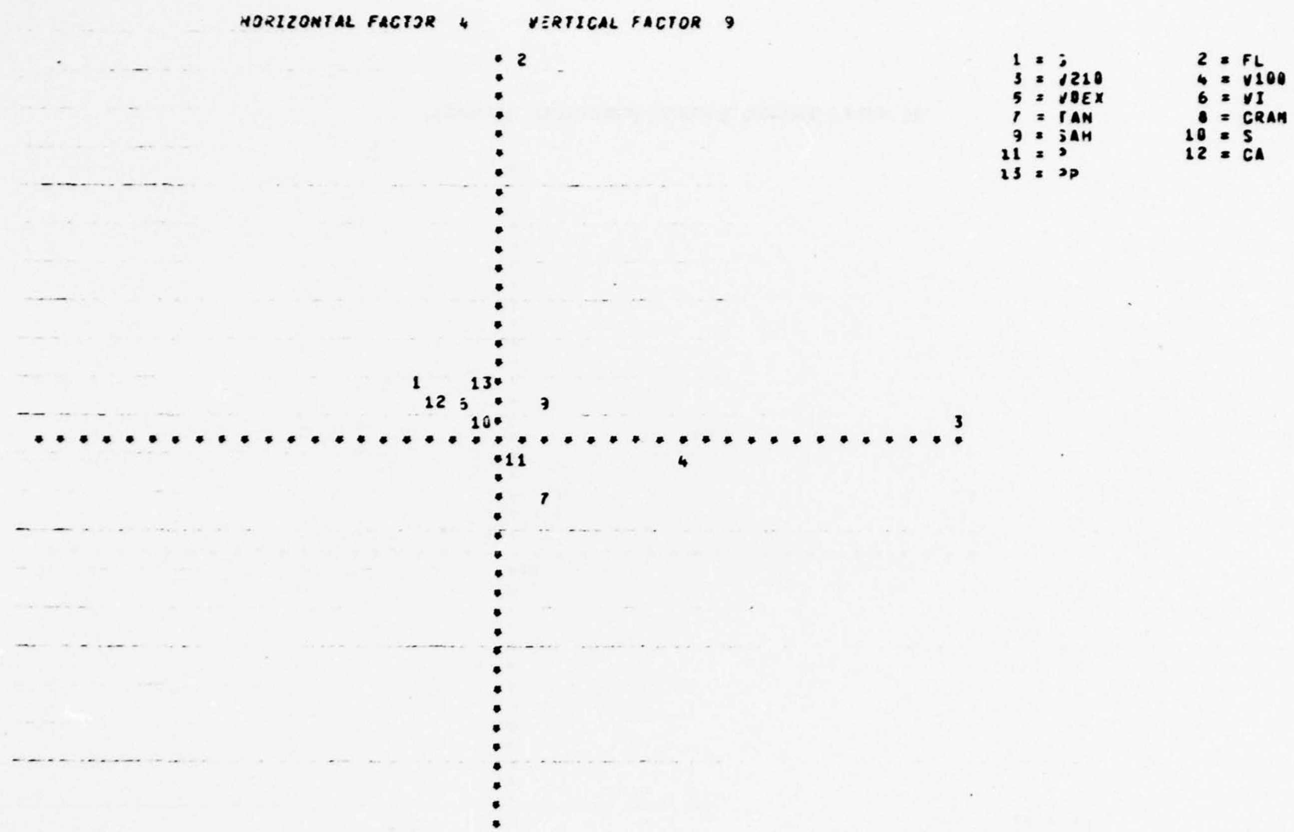


Figure 40. Horizontal Factor 4; Vertical Factor 9.

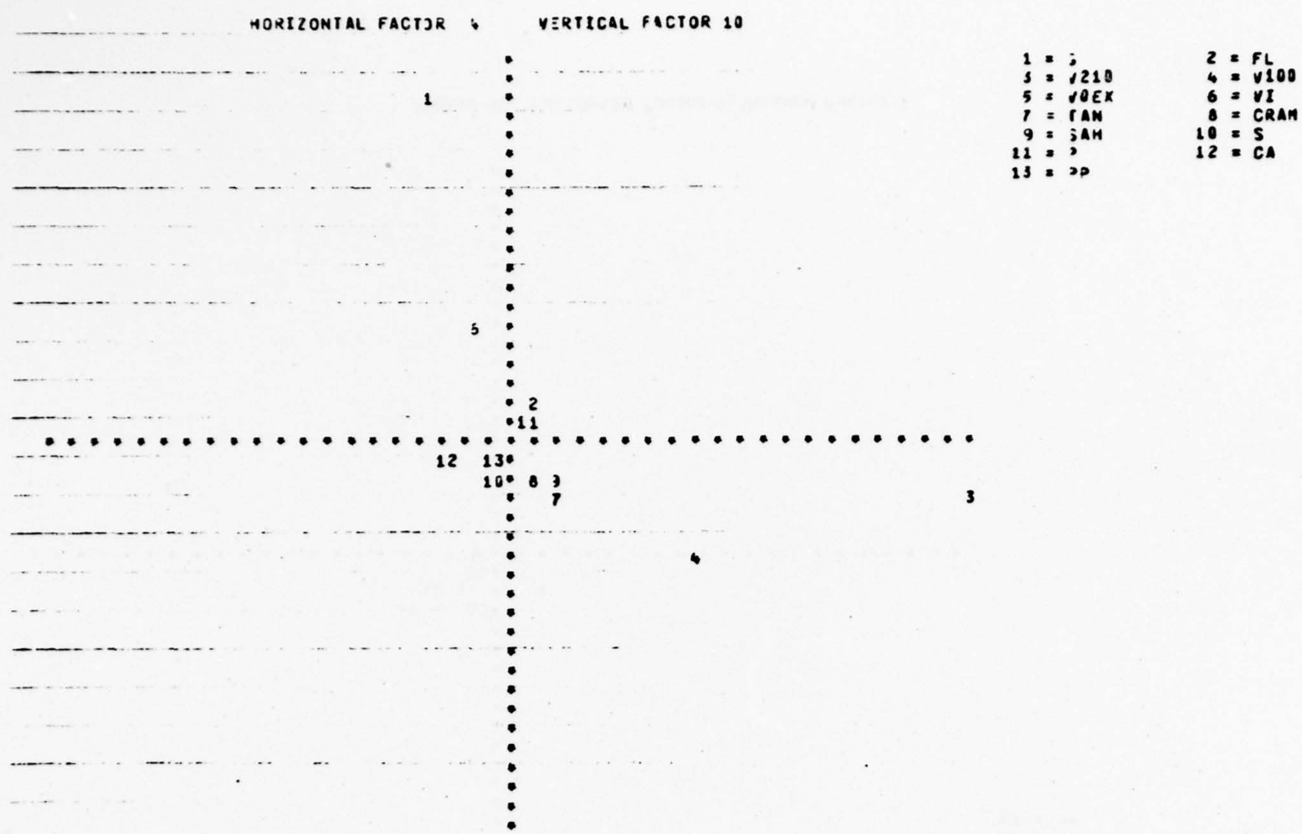


Figure 41. Horizontal Factor 4; Vertical Factor 10.

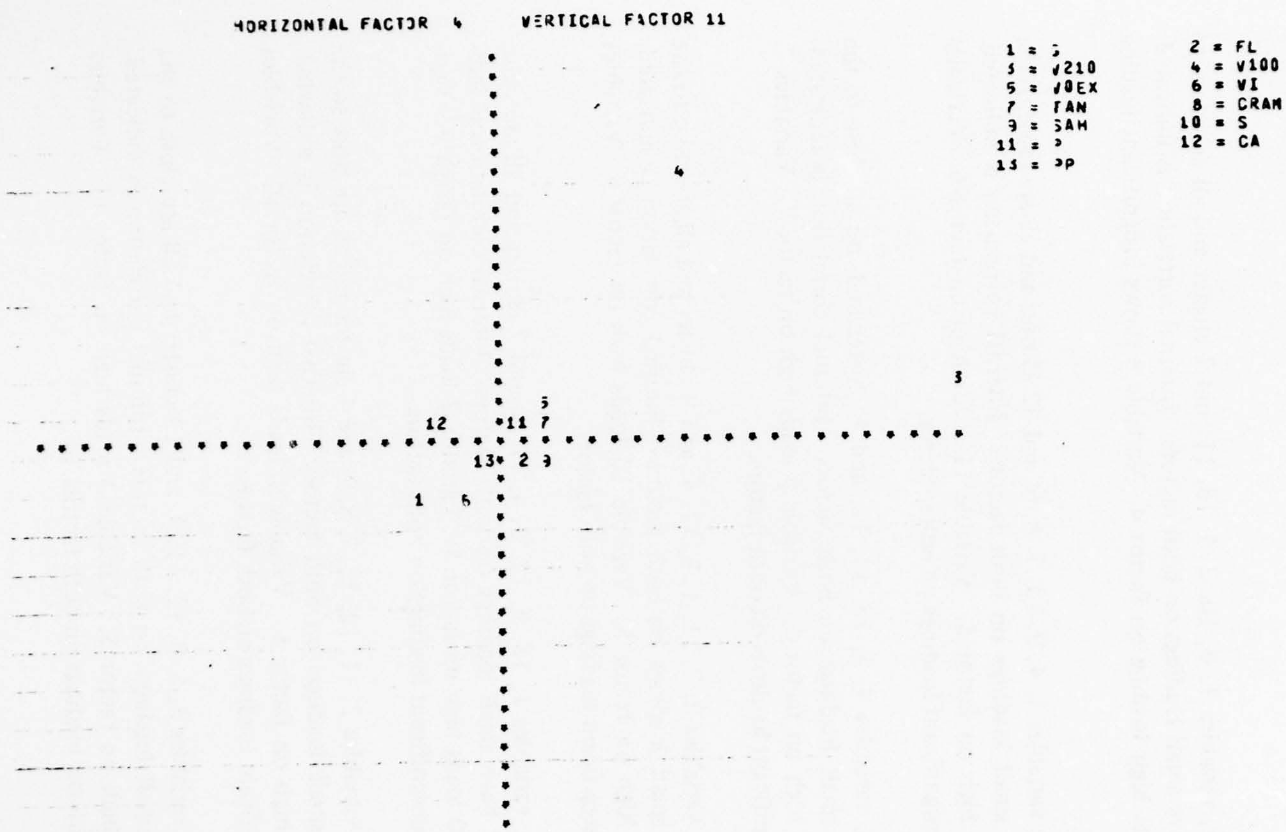


Figure 42. Horizontal Factor 4; Vertical Factor 11.

In Figure 43, variables 9, 10, 11, 12, 13, 1, 6, and 7 cluster and are all close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 loads high on factor 4. Variable 8 loads high on factor 12. Variable 4 loads moderately high on factor 4. Variables 2 and 5 show insignificant loading on both factors.

In Figure 44, variables 1, 6, 10, 2, 5, 13, 11, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 3 exhibits high loading on factor 4. Variable 8 shows insignificant loading on either factor.

In Figure 45, variables 1, 4, 9, 13, 7, 8, 6, and 12 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 11 loads high on factor 6. Variables 2, 3, and 5 show insignificant loadings on either factor.

In Figure 46, variables 4, 3, 13, 11, 12, and 1 cluster and are all close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 5 loads high on factor 7. Variables 7, 8, and 9 have insignificant loadings on both factors.

In Figure 47, variables 2, 7, 11, 1, 3, 12, 4, and 8 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 13 loads high on factor 8. Variables 5, 6, and 7 have insignificant loadings on both factors.

In Figure 48, variables 1, 13, 9, 12, 3, 4, 11, 8, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 2 loads high on factor 9. Variables 5 and 6 have insignificant loadings on both factors.

In Figure 49, variables 2, 11, 12, 13, 7, 8, 9, and 4 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 1 loads high on factor 10. Variables 5 and 3 have insignificant loadings on both factors.

In Figure 50, variables 3, 5, 8, 12, 11, 9, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 4 loads high on factor 11. Variables 2 and 7 have insignificant loadings on both factors.

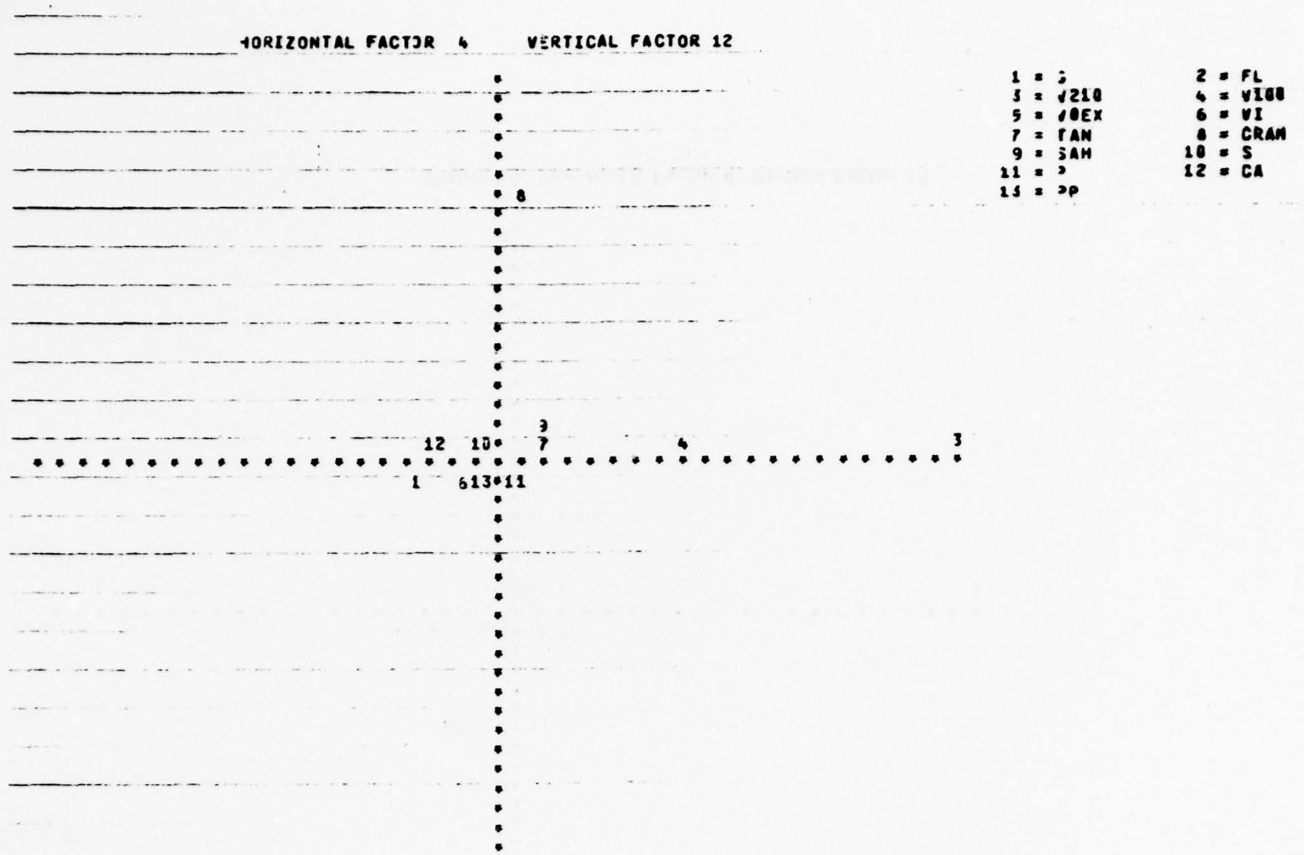


Figure 43. Horizontal Factor 4; Vertical Factor 12.

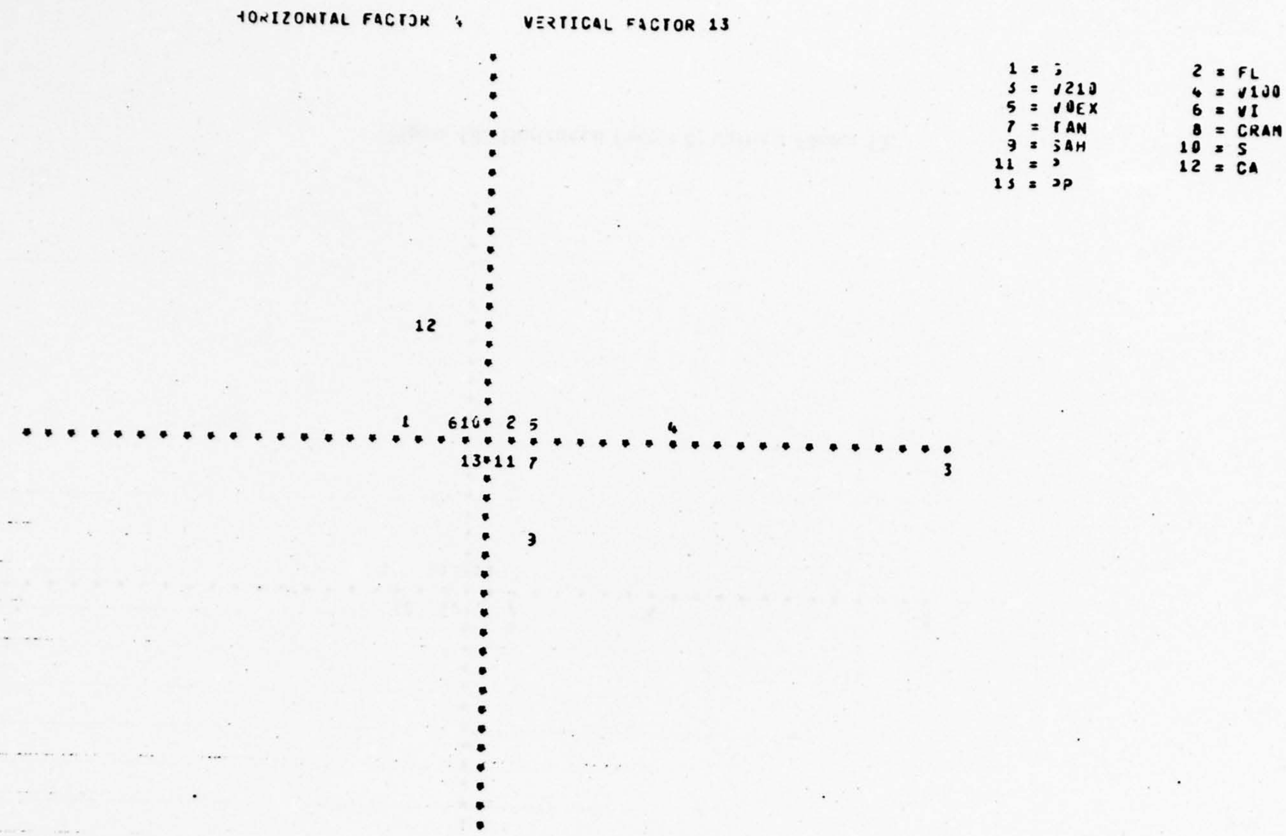


Figure 44. Horizontal Factor 4; Vertical Factor 13.

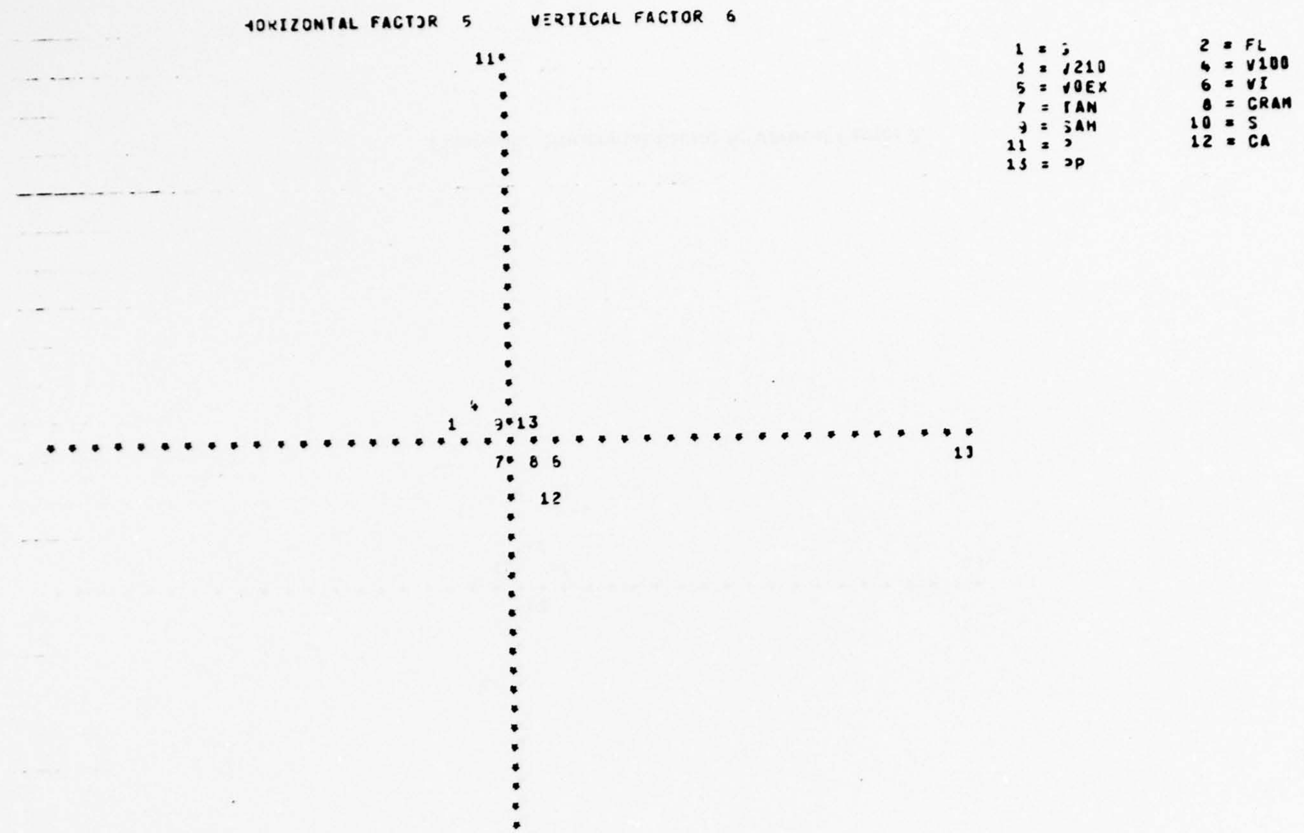


Figure 45. Horizontal Factor 5; Vertical Factor 6.

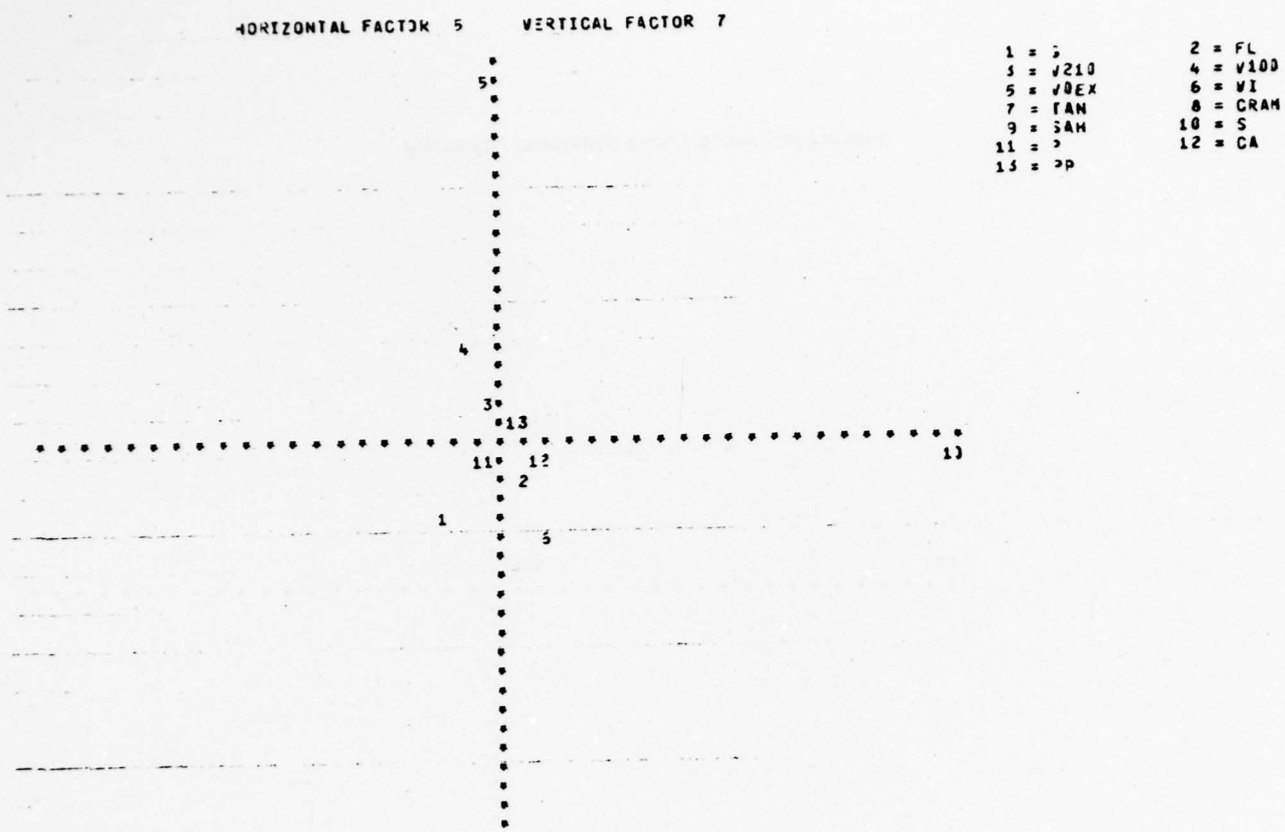


Figure 46. Horizontal Factor 5; Vertical Factor 7.

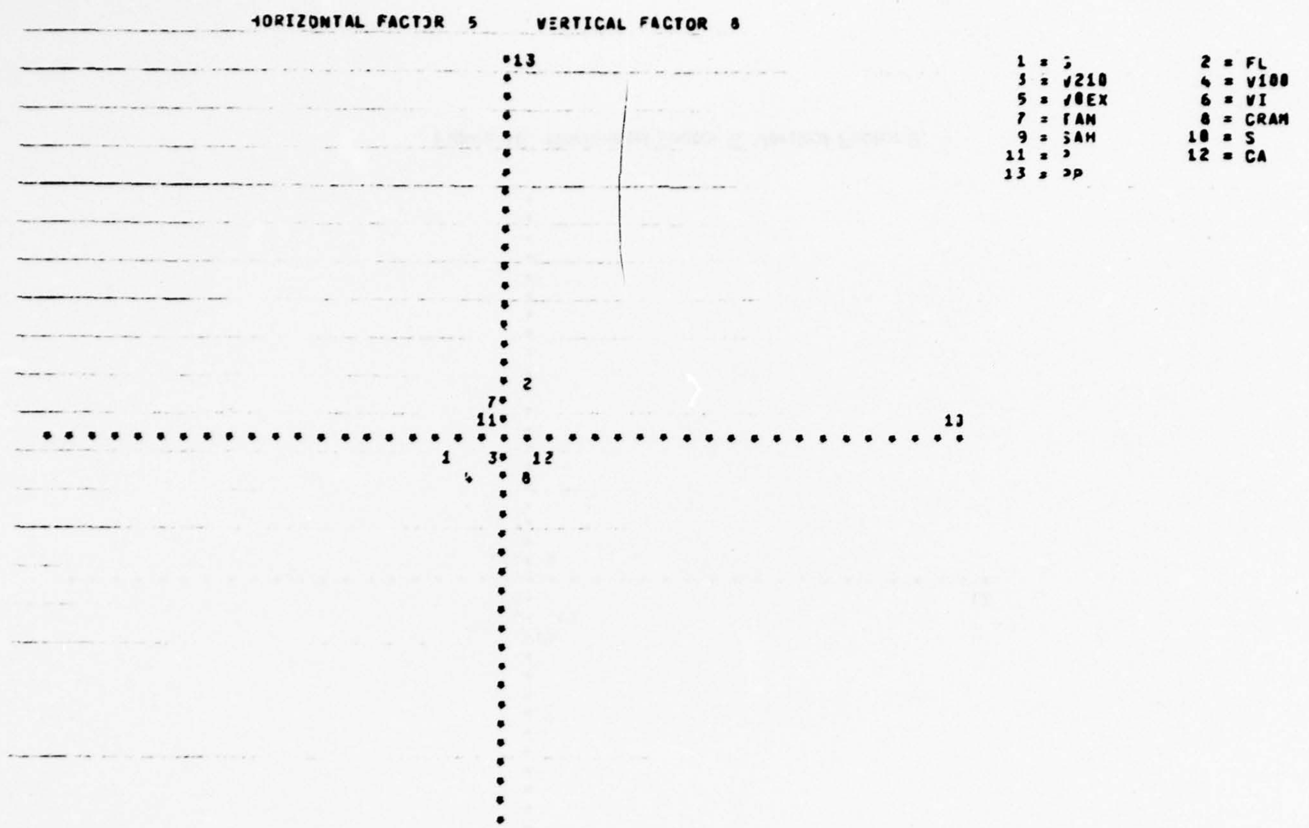
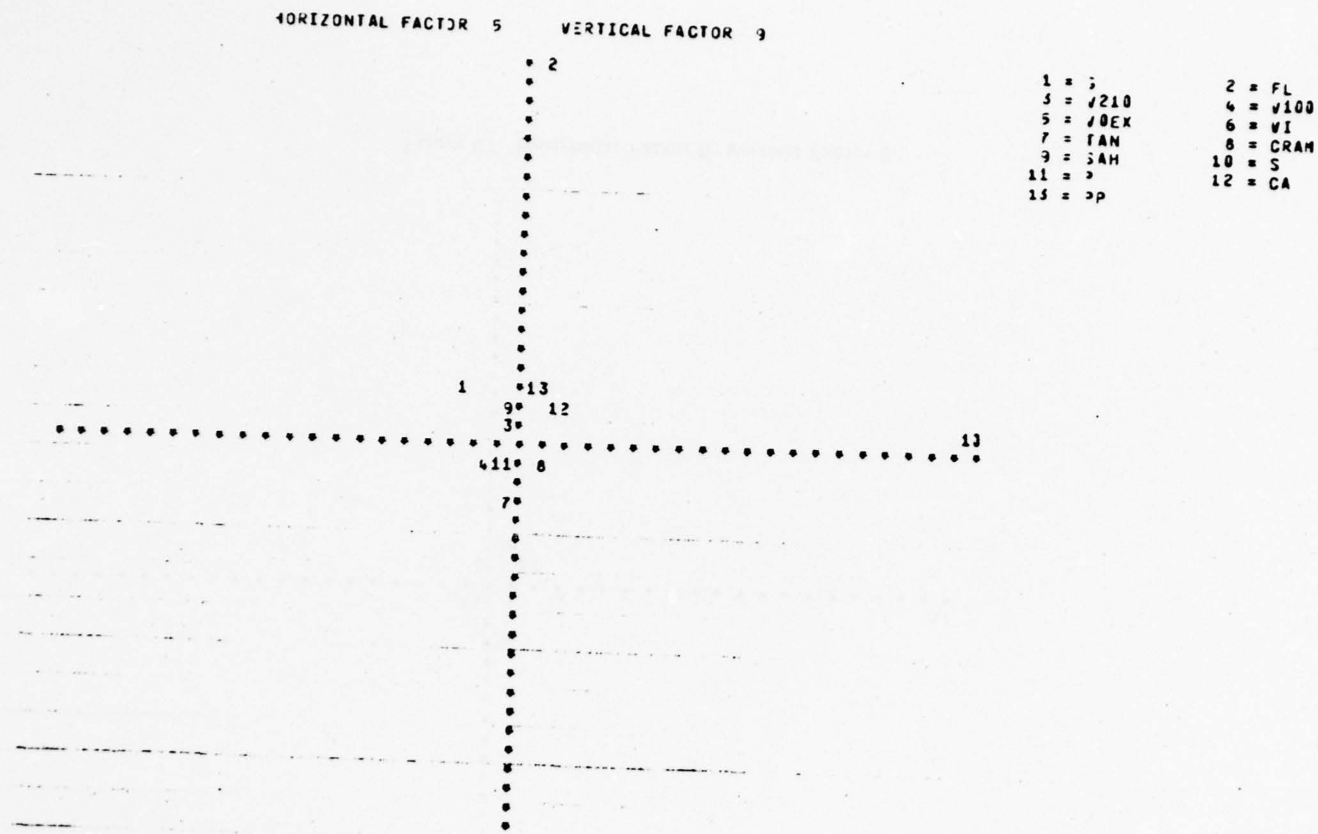


Figure 47. Horizontal Factor 5; Vertical Factor 8.



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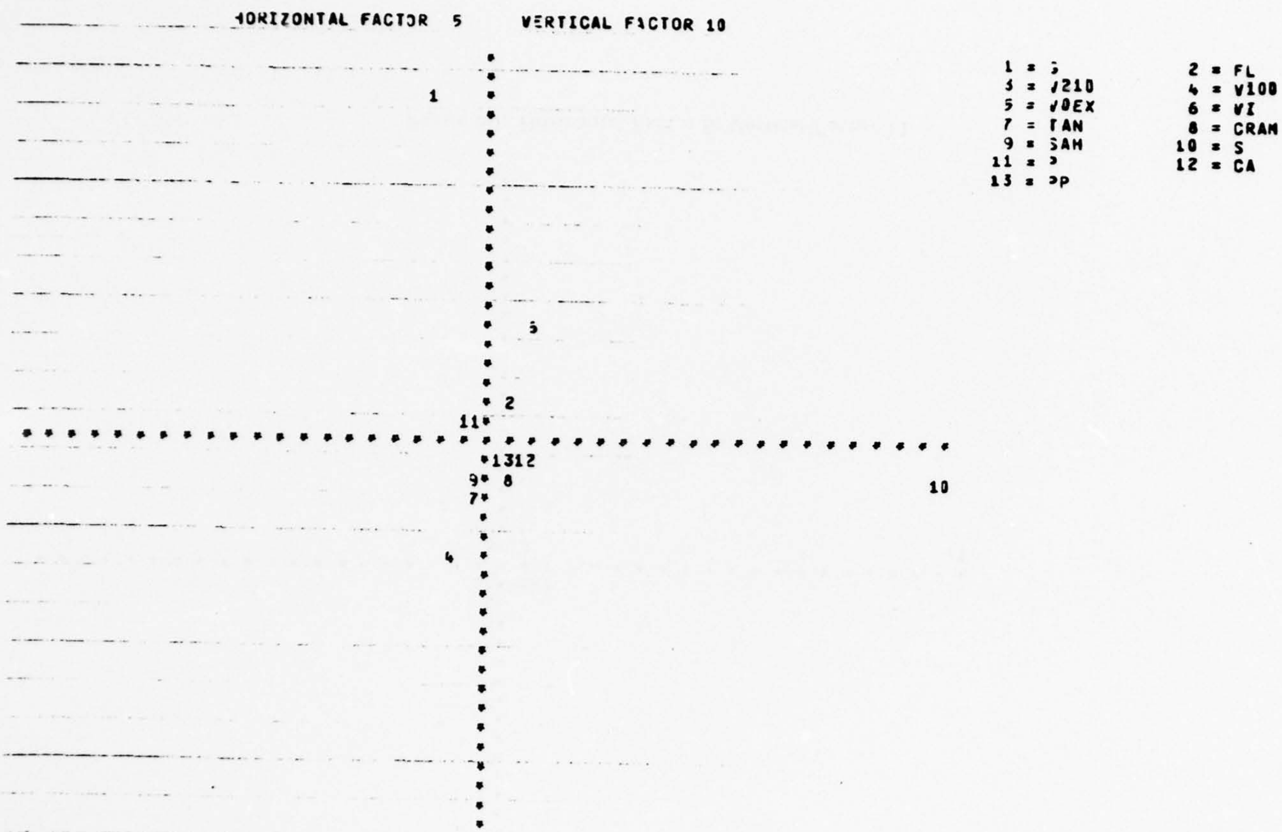

Figure 49. Horizontal Factor 5; Vertical Factor 10.

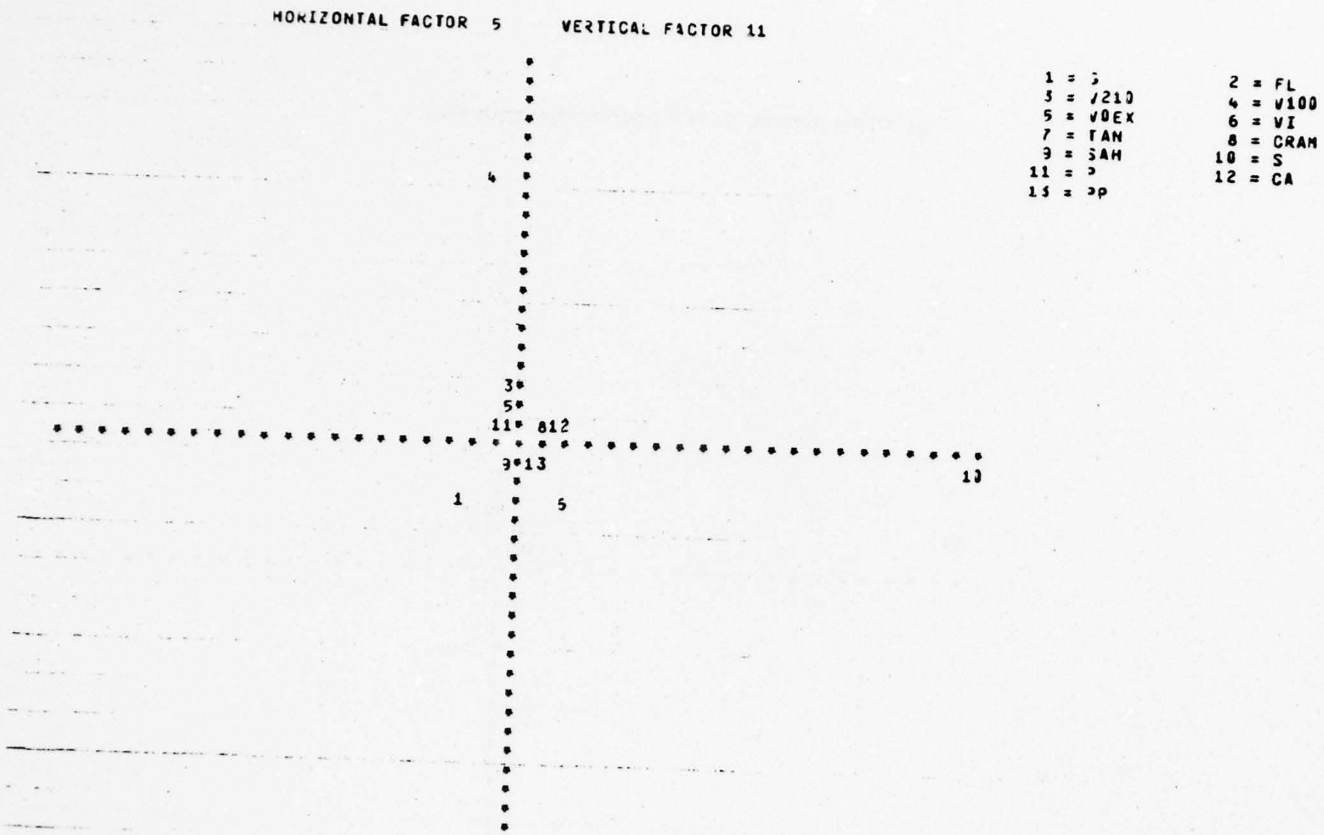

Figure 50. Horizontal Factor 5; Vertical Factor 11.

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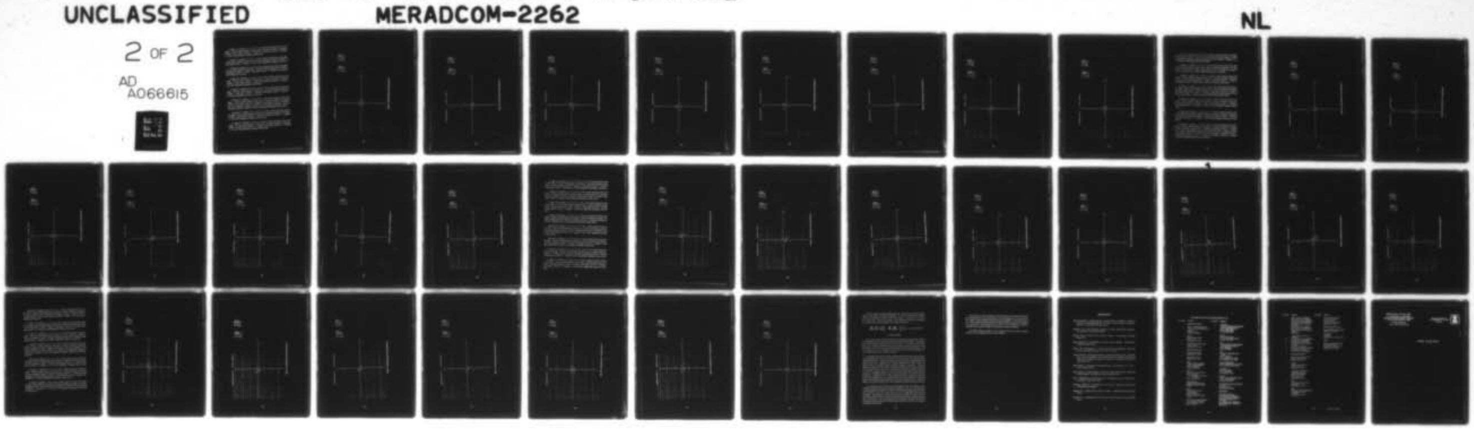

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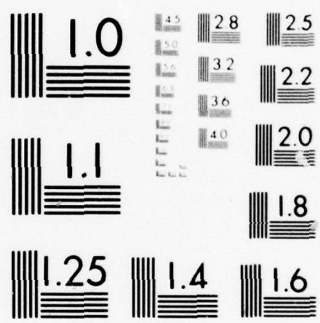

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In Figure 51, variables 1, 9, 4, 7, 12, 11, 13, and 6 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variable 8 loads high on factor 12. Variables 2, 3, and 5 have insignificant loadings on both factors.

In Figure 52, variables 1, 2, 4, 5, 6, 11, and 13 cluster and are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 10 loads high on factor 5. Variables 3, 7, and 8 show insignificant loading on both factors.

In Figure 53, variables 3, 8, 13, 12, 10, 9, and 2 cluster and are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. Variable 5 loads high on factor 7. Variable 7 shows insignificant loading on both factors.

In Figure 54, variables 2, 7, 9, 10, 12, 6, 3, 8, and 4 cluster and are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. Variable 13 loads high on factor 8. Variables 1 and 5 show insignificant loadings on both factors.

In Figure 55, variables 13, 12, 6, 9, 10, 3, 8, 4, and 7 cluster and are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. Variable 2 loads high on factor 9. Variables 1 and 5 show insignificant loadings on both factors.

In Figure 56, variables 2, 12, 13, 10, 9, 7, and 3 cluster and are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 1 loads high on factor 10. Variable 11 loads high on factor 6. Variables 6 and 4 have significant loading on factor 10 and are negatively correlated. Only one grouping is indicated. Variables 5 and 8 show insignificant loading on either factor.

In Figure 57, variables 3, 5, 8, 12, 10, 13, 6, and 1 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. Variable 4 loads high on factor 11. Variables 2, 7, and 9 show insignificant loading on both factors.

In Figure 58, variables 12, 10, 9, 3, 4, 6, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. Variable 8 loads high on factor 12. Variables 1, 2, 5, and 7 show insignificant loading on either factor.

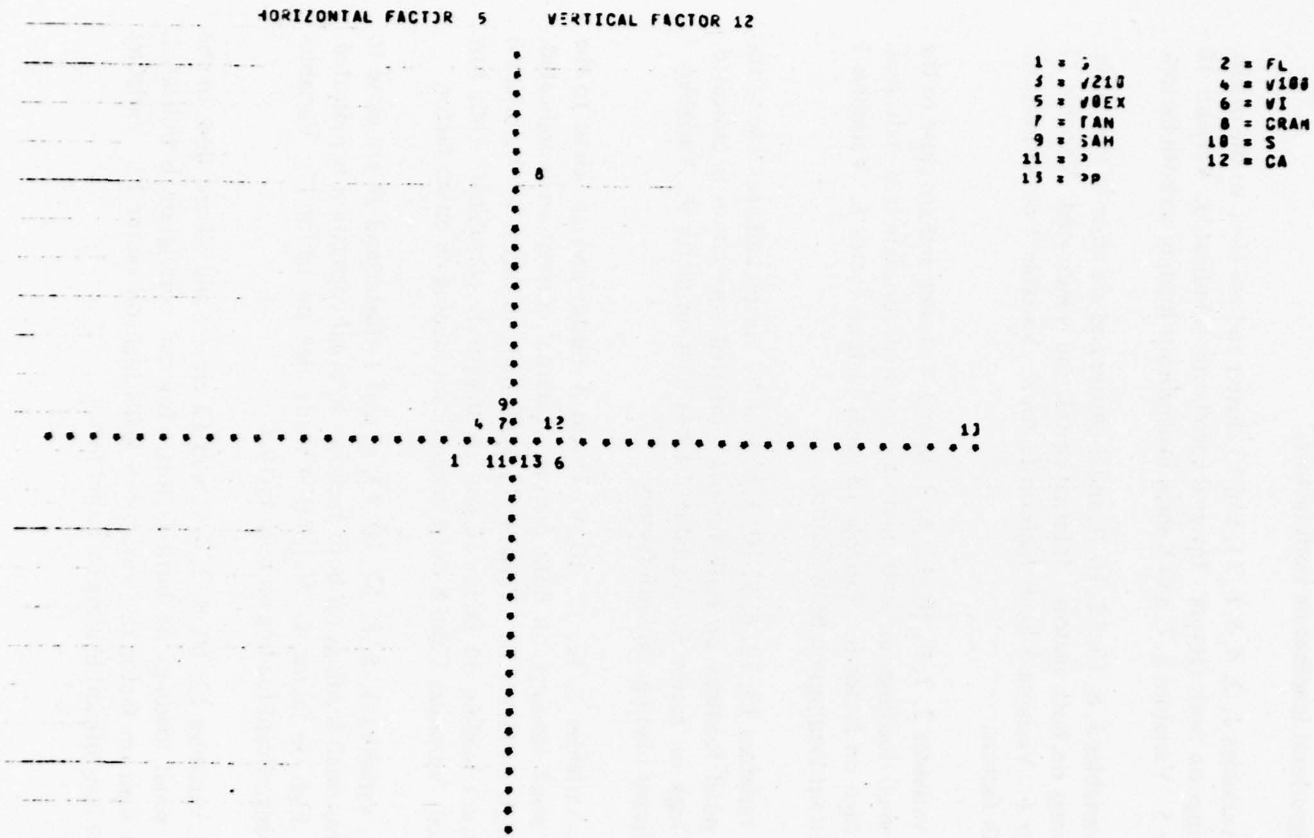

Figure 51. Horizontal Factor 5; Vertical Factor 12.

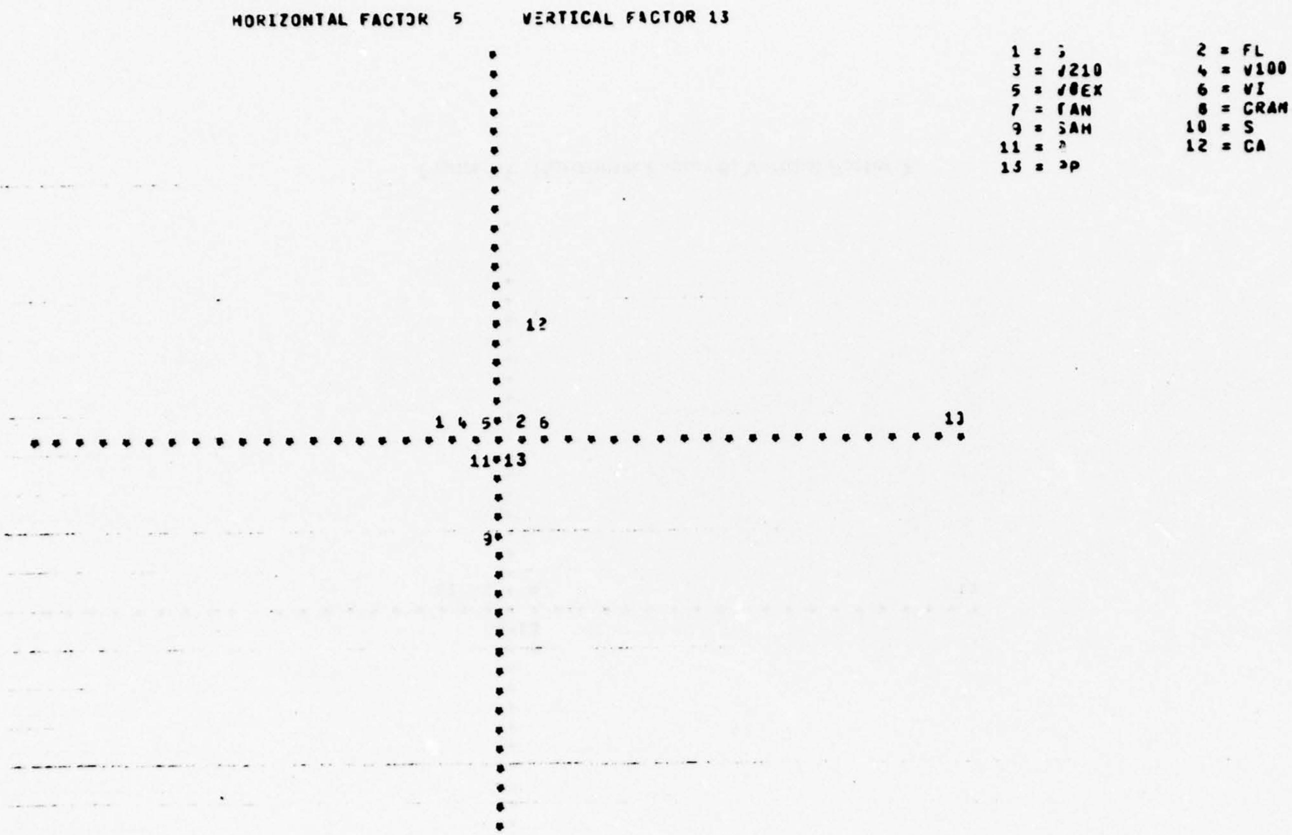

Figure 52. Horizontal Factor 5; Vertical Factor 13.

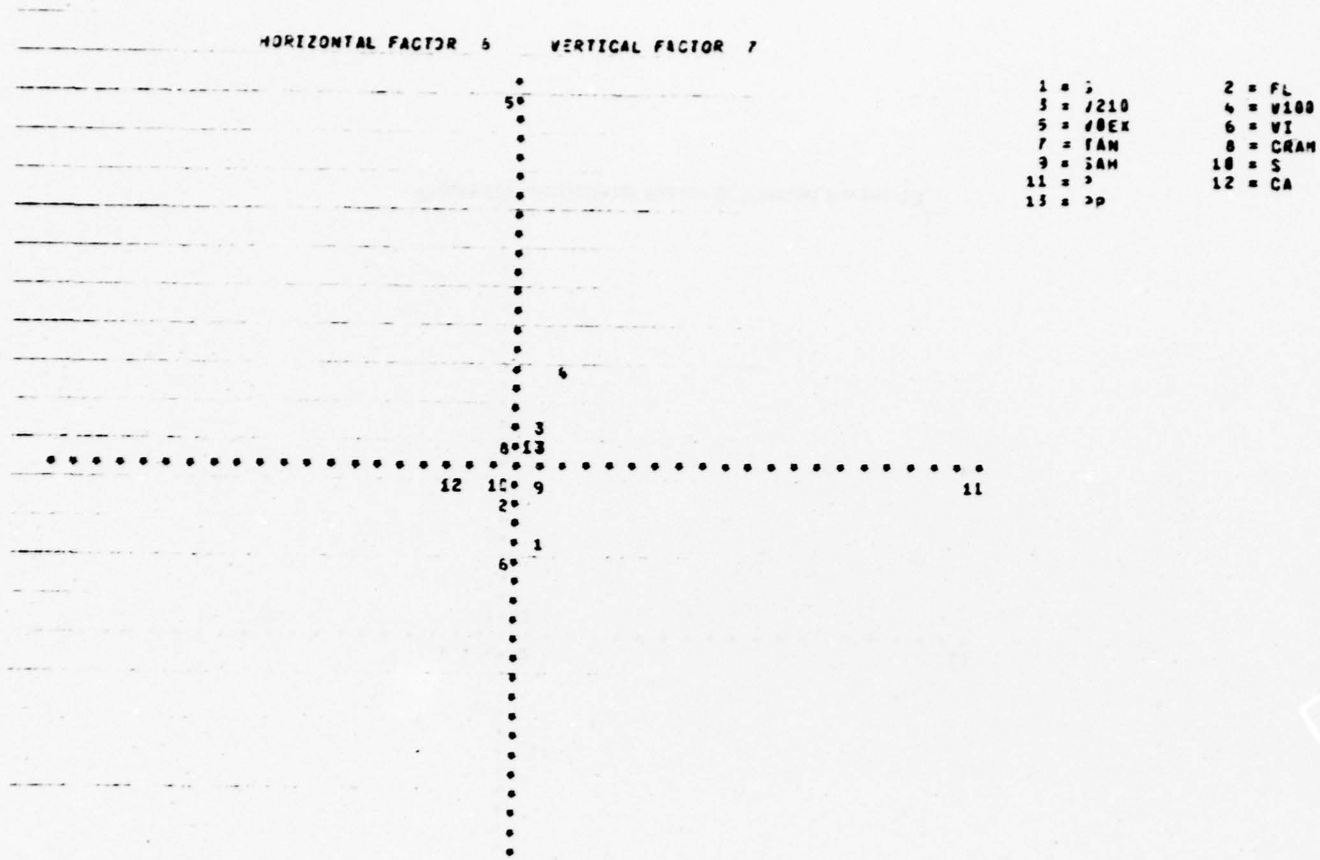

Figure 53. Horizontal Factor 6; Vertical Factor 7.

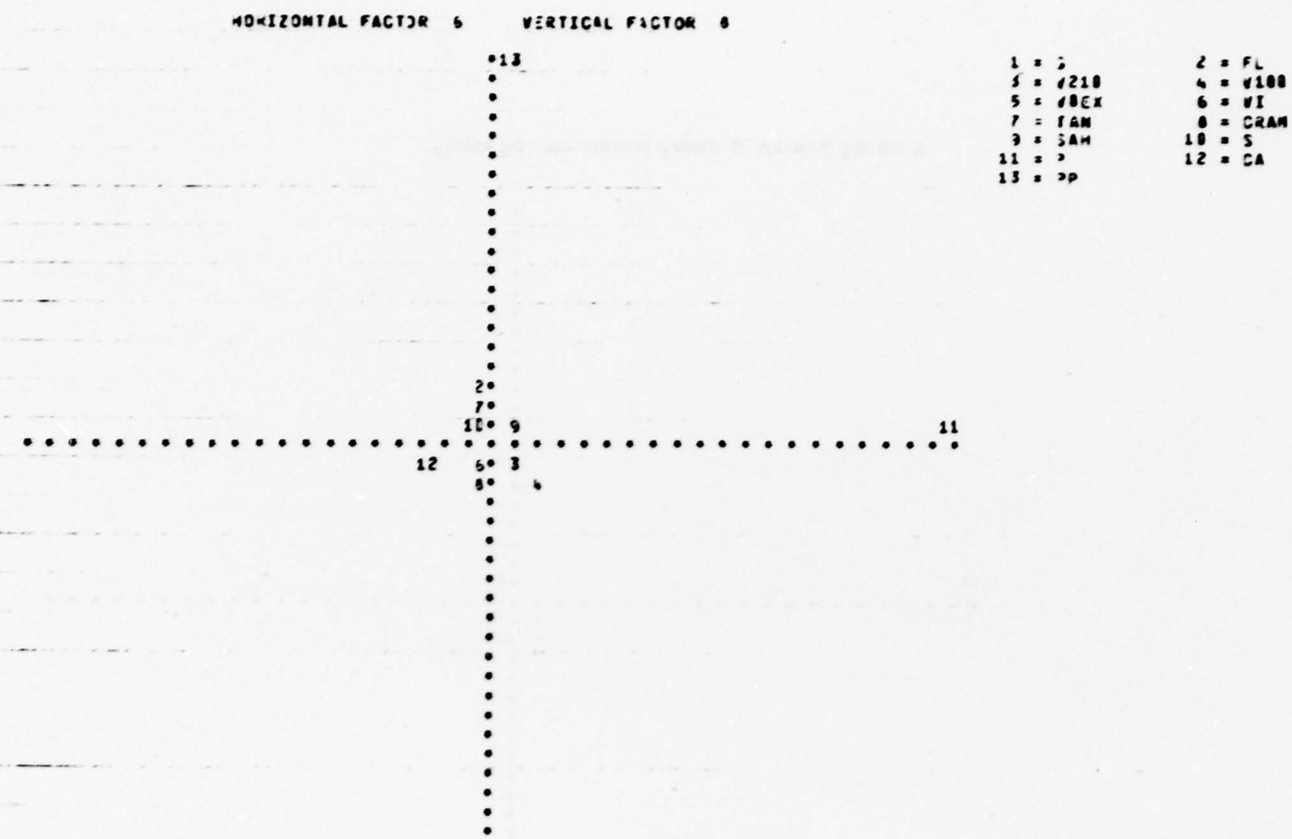

Figure 54. Horizontal Factor 6; Vertical Factor 8.

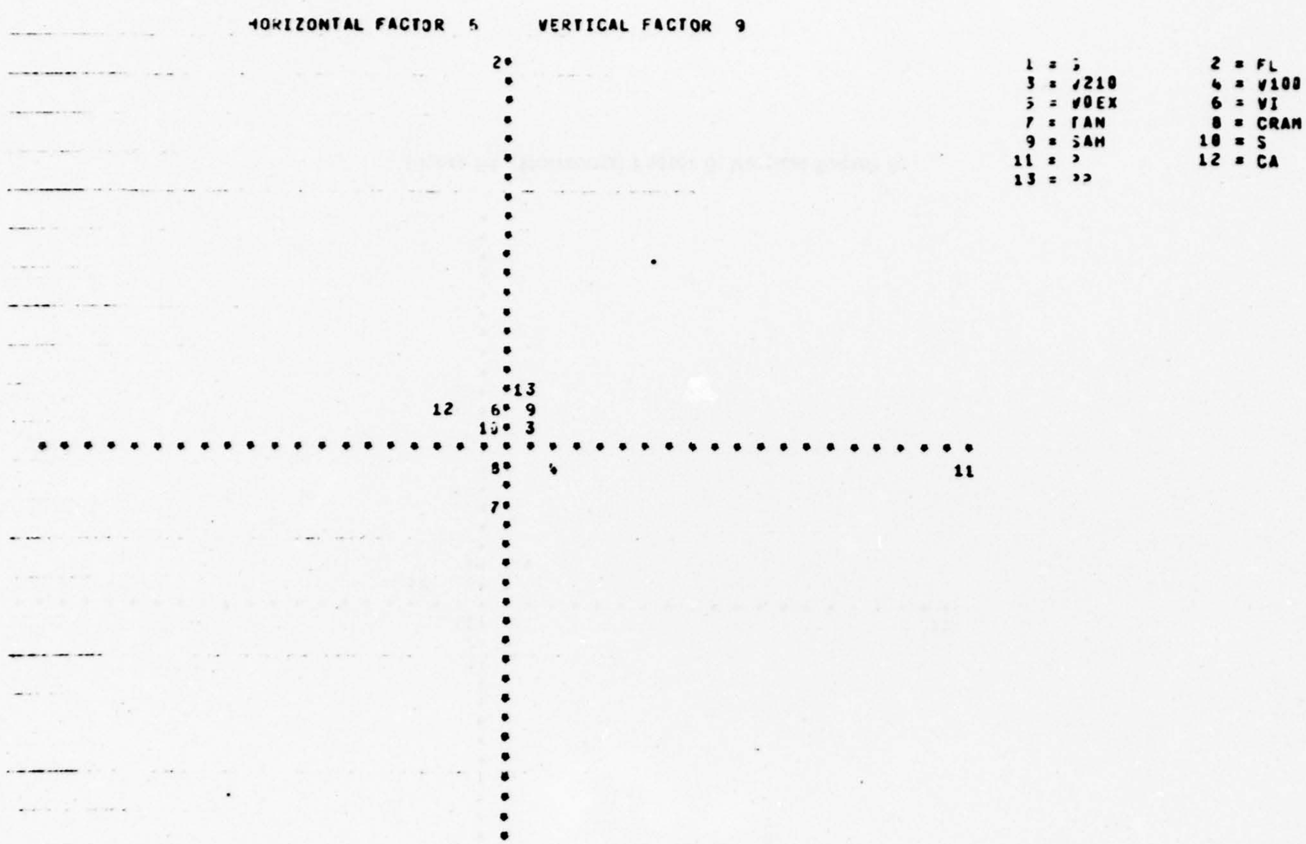

Figure 55. Horizontal Factor 6; Vertical Factor 9.

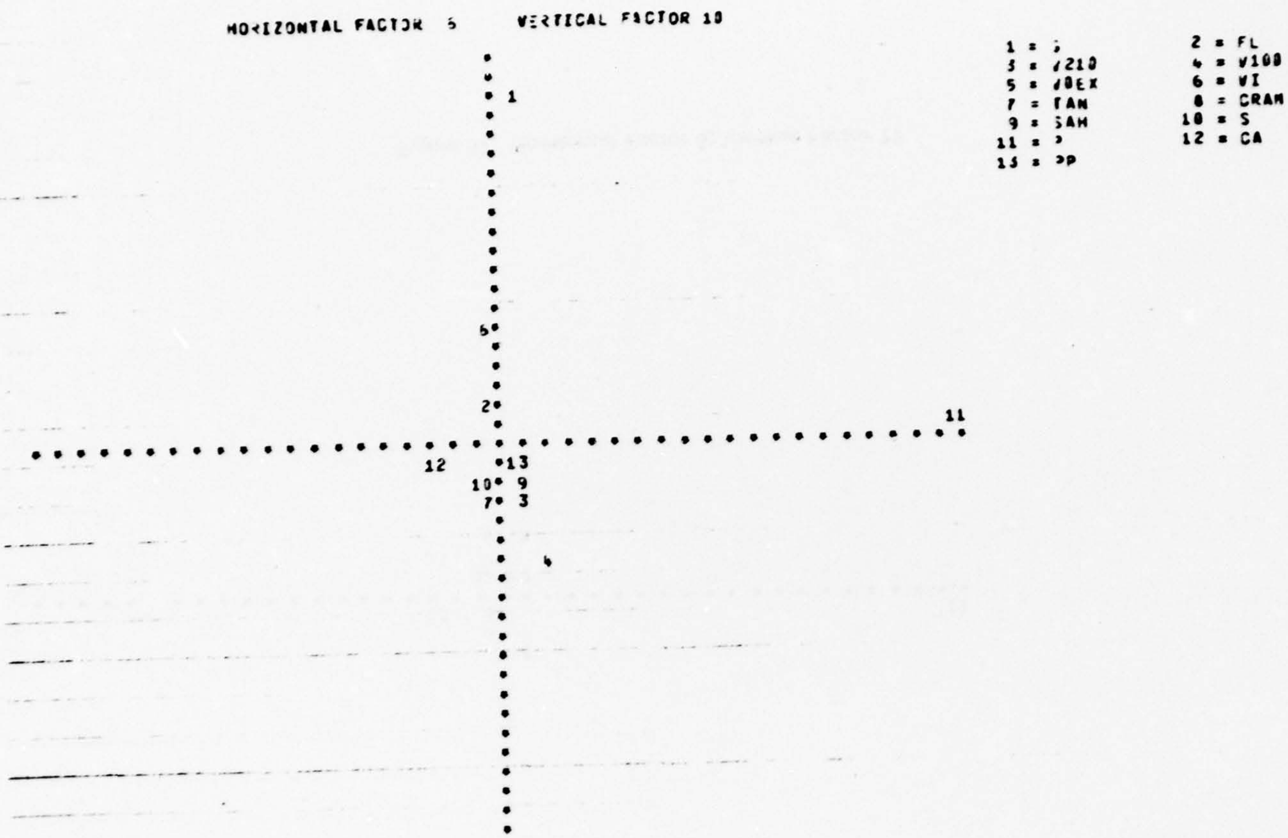

Figure 56. Horizontal Factor 6; Vertical Factor 10.

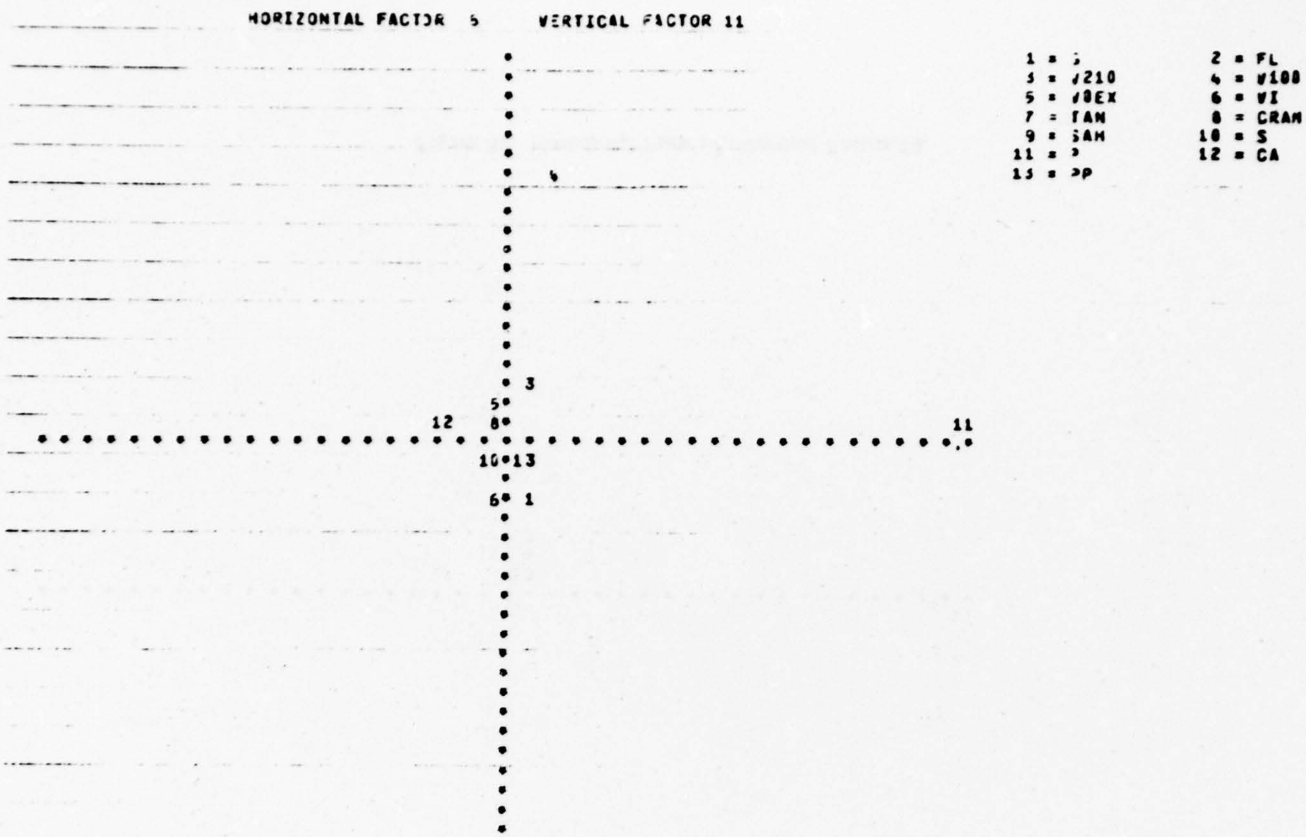

Figure 57. Horizontal Factor 6; Vertical Factor 11.

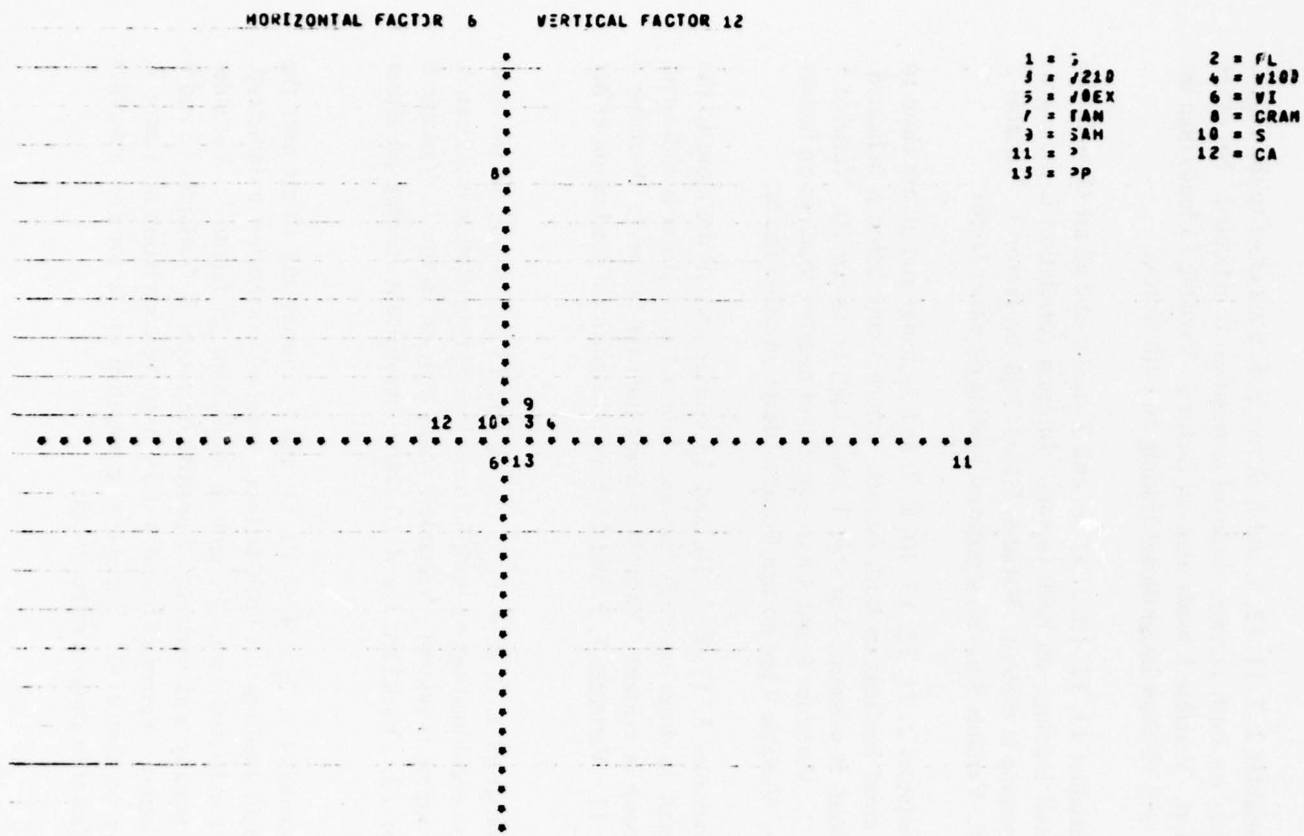

Figure 58. Horizontal Factor 6; Vertical Factor 12.

In Figure 59, variables 10, 1, 4, 8, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. Variable 11 loads high on factor 6. There is no other grouping of variables. Variables 2, 3, 5, and 7 have no significant loading on either factor.

In Figure 60, variables 2, 7, 11, 12, 3, and 8 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other discrete set is indicated. Variable 5 loads high on factor 7. Variable 13 loads high on factor 8. Variables 9 and 10 show insignificant loading on both factors.

In Figure 61, variables 13, 12, 10, 3, 11, 8, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other discrete grouping is evident. Variable 5 loads high on factor 7. Variable 2 loads high on factor 9. Variable 9 has no significant loading on either factor.

In Figure 62, variables 2, 11, 12, 13, 10, 8, 7, and 3 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other discrete group is evident. Variable 1 loads high on factor 10. Variable 5 loads high on factor 7. Variables 6 and 4 have significant negative loadings on factors 10 and 7 respectively. Variable 9 has no significant loadings on either factor.

In Figure 63, variables 3, 12, 8, 2, 10, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other discrete group is evident. Variable 5 loads high on factor 7. Variable 4 loads high on factor 11. Variables 7, 9, and 11 have no significant loading on either factor.

In Figure 64, variables 1, 2, 3, 4, 6, 9, 11, 12, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other discrete grouping is evident. Variable 5 loads high on factor 7. Variable 8 loads high on factor 12. Variables 7 and 10 show insignificant loading on either factor.

In Figure 65, variables 1, 2, 3, 4, 6, 10, 11, and 13 cluster and all are near the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is indicated here. Variable 5 loads high on factor 7. Variables 4, 12, 6, and 9 load equally with variable 12 positive on factor 7. Variables 12 and 9 show negative correlation. Variables 6 and 4 show negative correlation on factor 7. Variable 4 is positive on factor 13. Variable 6 is negative on factor 13. Variables 7 and 8 show insignificant loading on either factor.

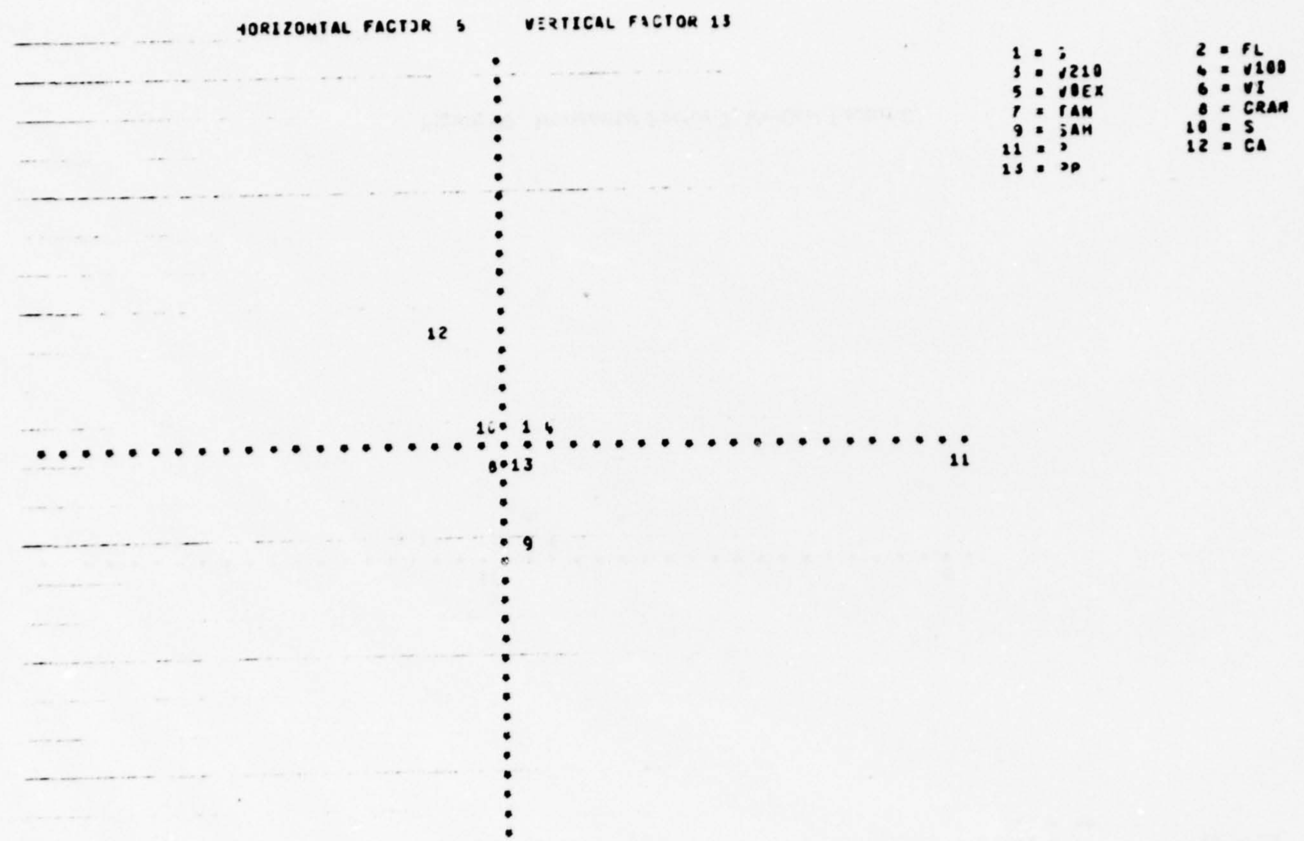

Figure 59. Horizontal Factor 6; Vertical Factor 13.

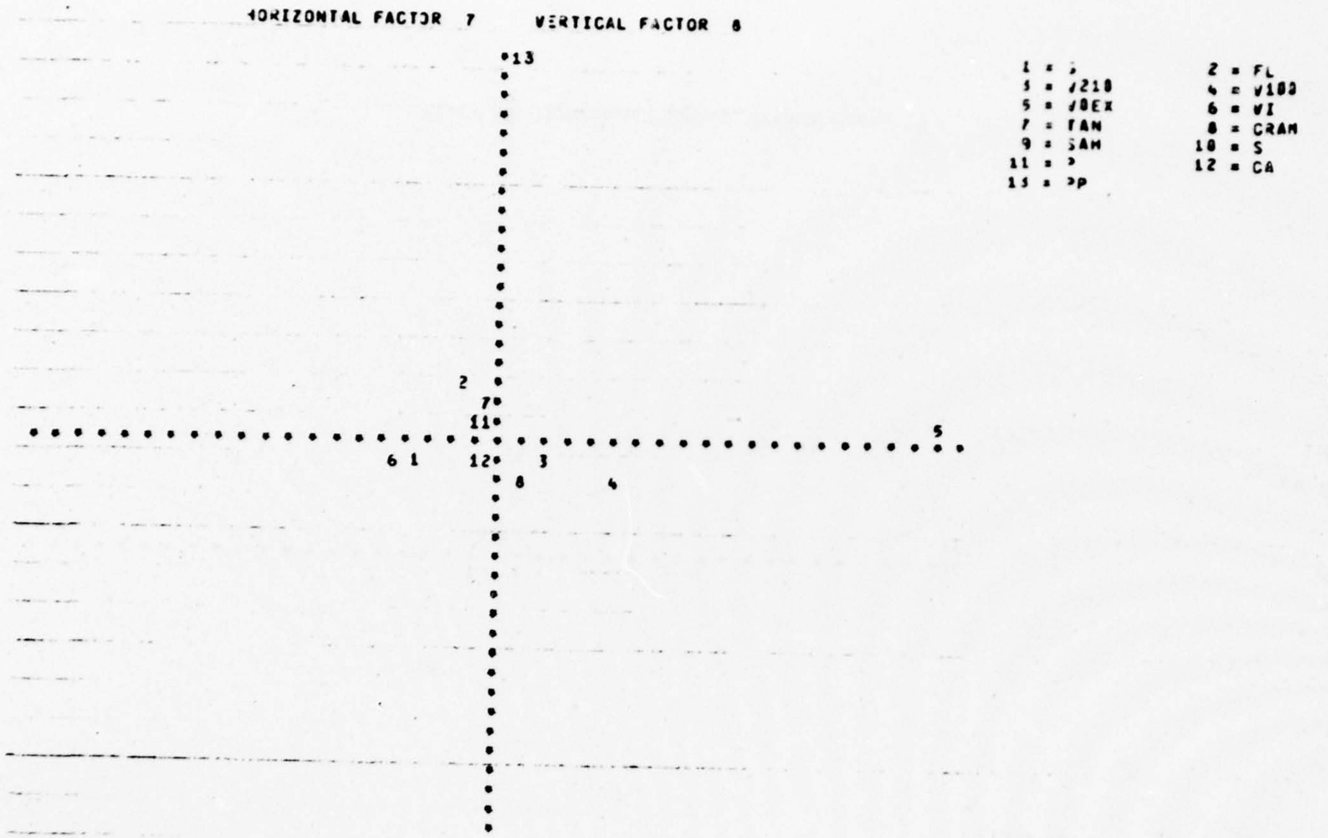

Figure 60. Horizontal Factor 7; Vertical Factor 8.

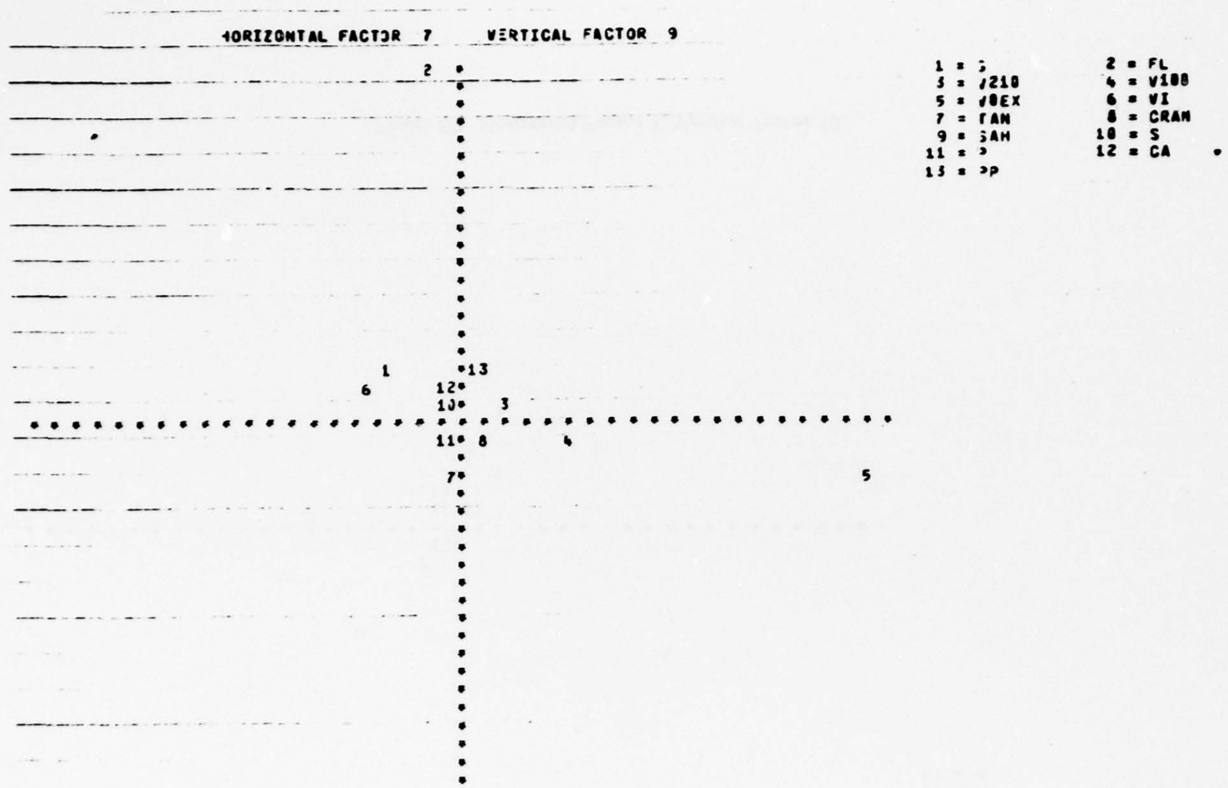

Figure 61. Horizontal Factor 7; Vertical Factor 9.

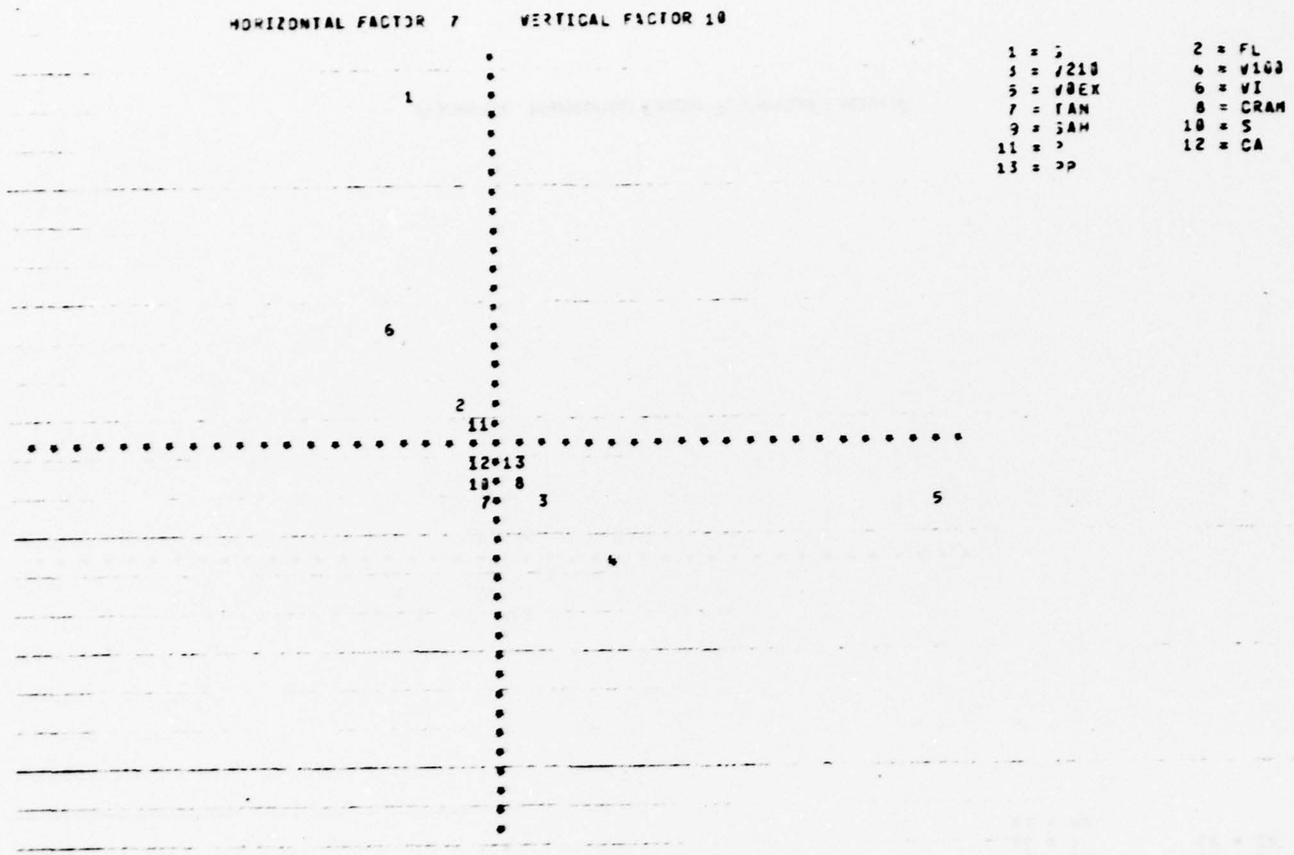

Figure 62. Horizontal Factor 7; Vertical Factor 10.

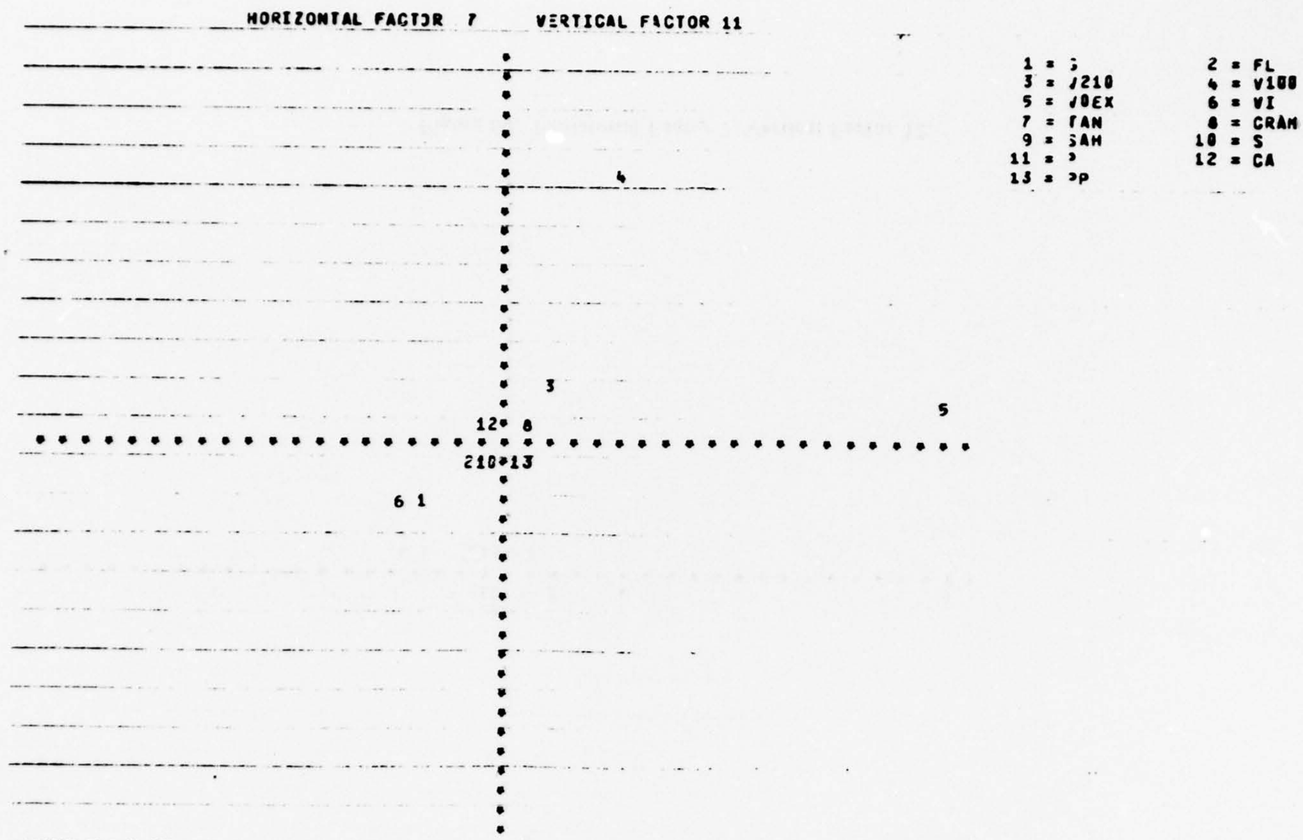

Figure 63. Horizontal Factor 7; Vertical Factor 11.

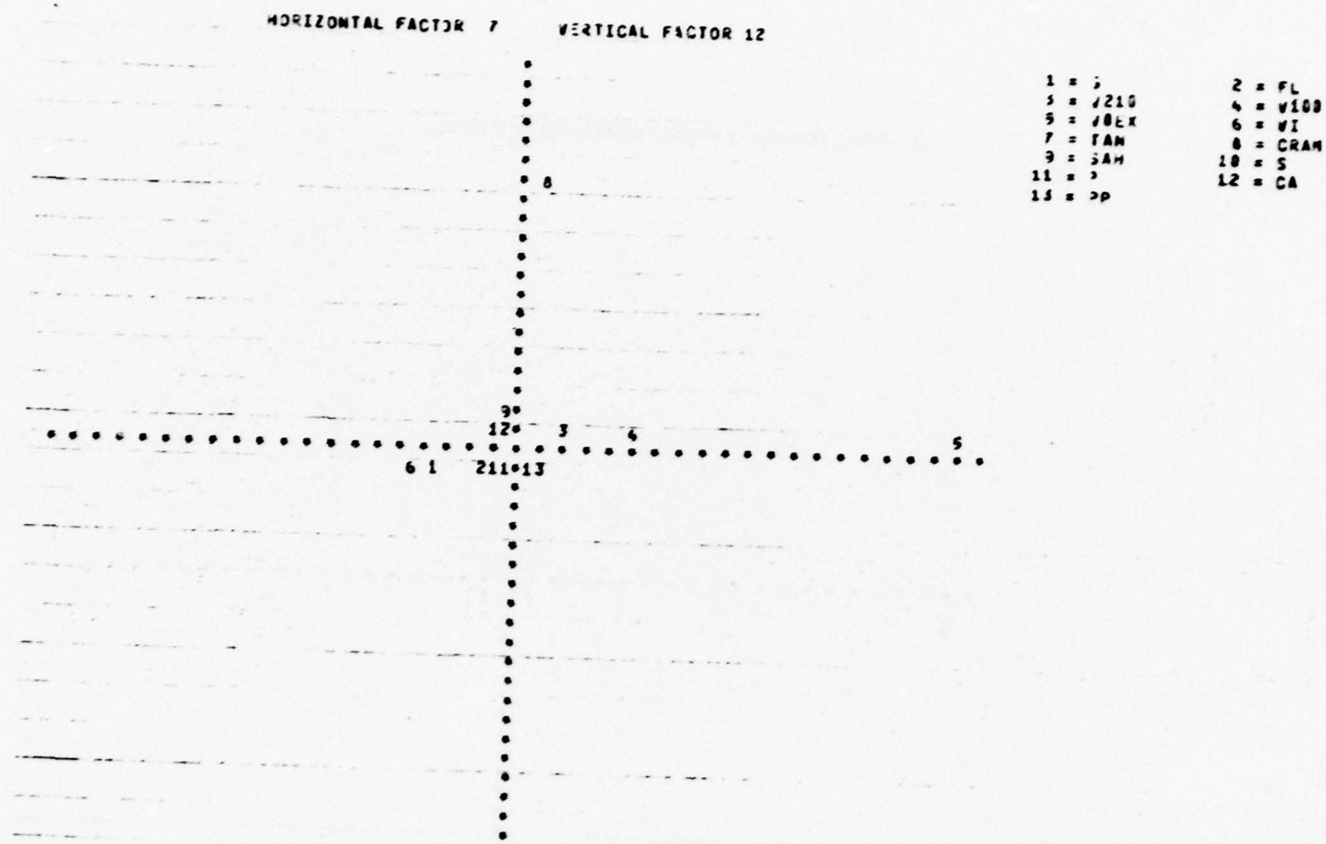

Figure 64. Horizontal Factor 7; Vertical Factor 12.

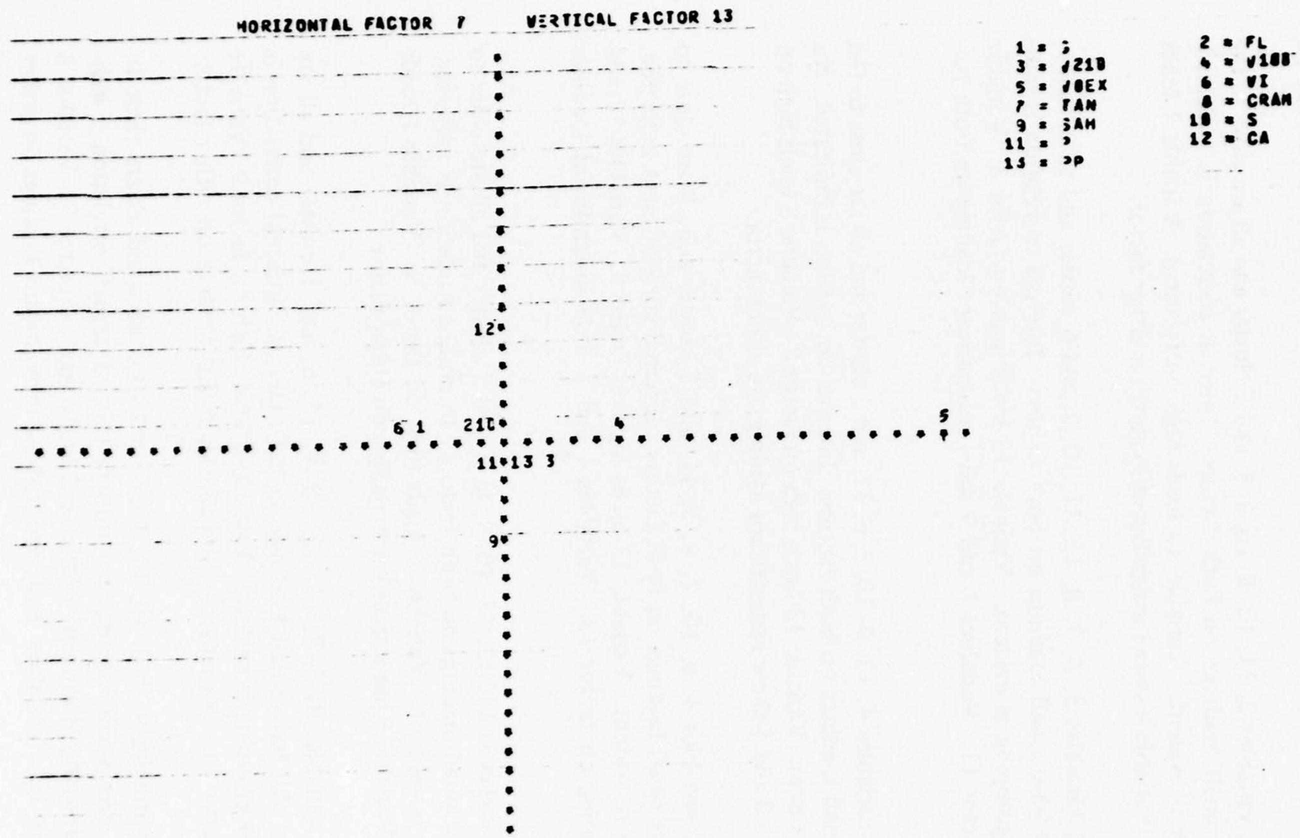

Figure 65. Horizontal Factor 7; Vertical Factor 13.

In Figure 66, variables 1, 12, 9, 3, 10, 8, 11, 5, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 13 loads high on factor 8. Variable 2 loads high on factor 9. Variables 4 and 6 show insignificant loadings on either factor.

In Figure 67, variables 2, 11, 12, 8, 10, 3, 5, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 13 loads high on factor 8. Variable 1 loads high on factor 10. Variable 9 shows insignificant loading on either factor.

In Figure 68, variables 3, 5, 7, 8, 12, 11, 10, 2, and 6 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 13 loads high on factor 8. Variable 4 loads high on factor 11. Variables 1 and 9 show insignificant loading on both factors.

In Figure 69, variables 4, 12, 9, 10, 7, 6, 11, and 2 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 13 loads high on factor 8. Variable 8 loads high on factor 12. Variables 3 and 5 show insignificant loadings on either factor.

In Figure 70, variables 4, 6, 10, 2, 8, 3, 11, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 13 loads high on factor 8. Variables 12 and 9 have modest loading on factor 13. Variables 1 and 5 have insignificant loadings on either factor.

In Figure 71, variables 11, 12, 13, 10, 9, 8, 7, and 3 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 2 loads high on factor 9. Variable 1 loads high on factor 10. Variable 5 has insignificant loading on either factor.

In Figure 72, variables 3, 5, 7, 11, 12, 10, 9, 13, 6, and 1 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 2 loads high on factor 9. Variable 4 loads high on factor 11. Variable 8 shows insignificant loading on either factor.

In Figure 73, variables 9, 7, 4, 10, 12, 11, 6, and 13 cluster and all are close to to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 2 loads high on factor 9. Variable 8 loads high on factor 12. Variables 1, 3, and 5 show insignificant loadings on either factor.

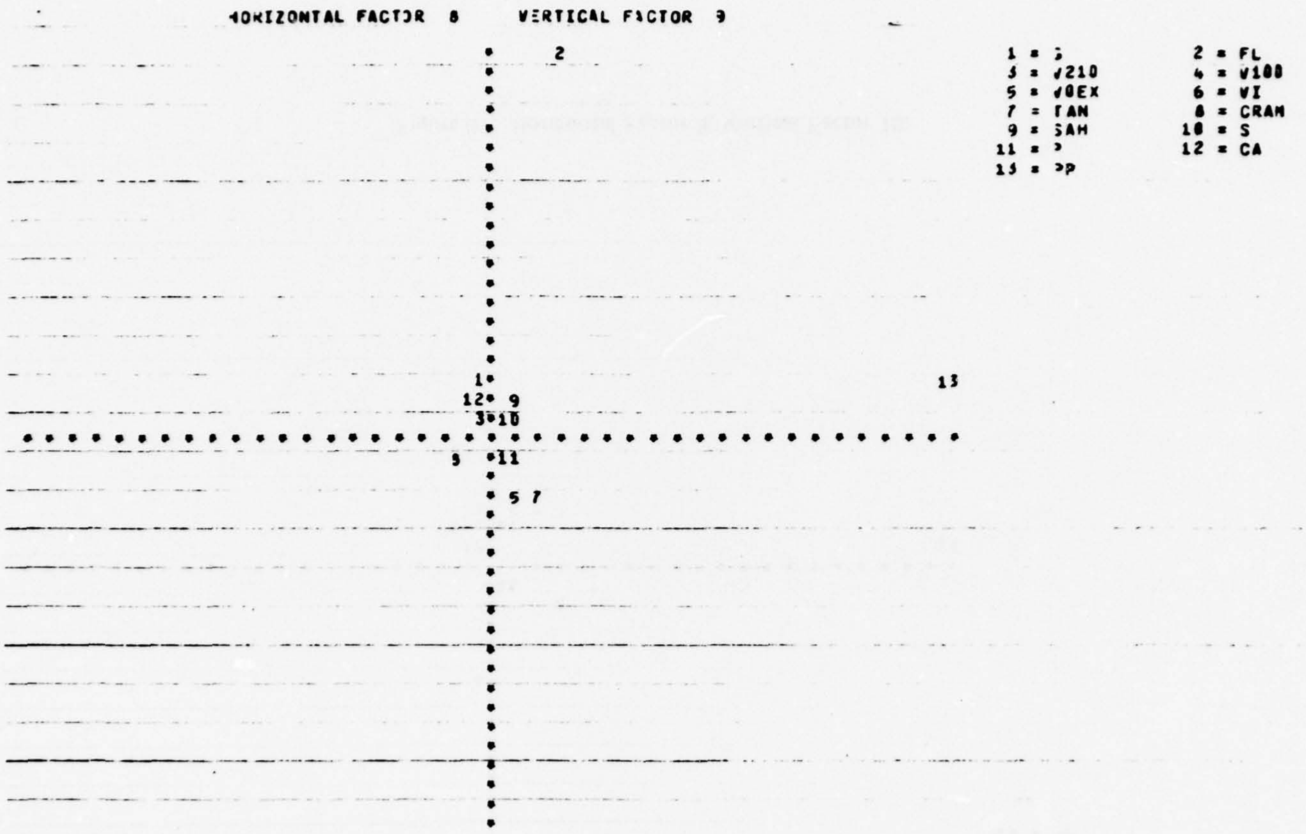

Figure 66. Horizontal Factor 8; Vertical Factor 9.

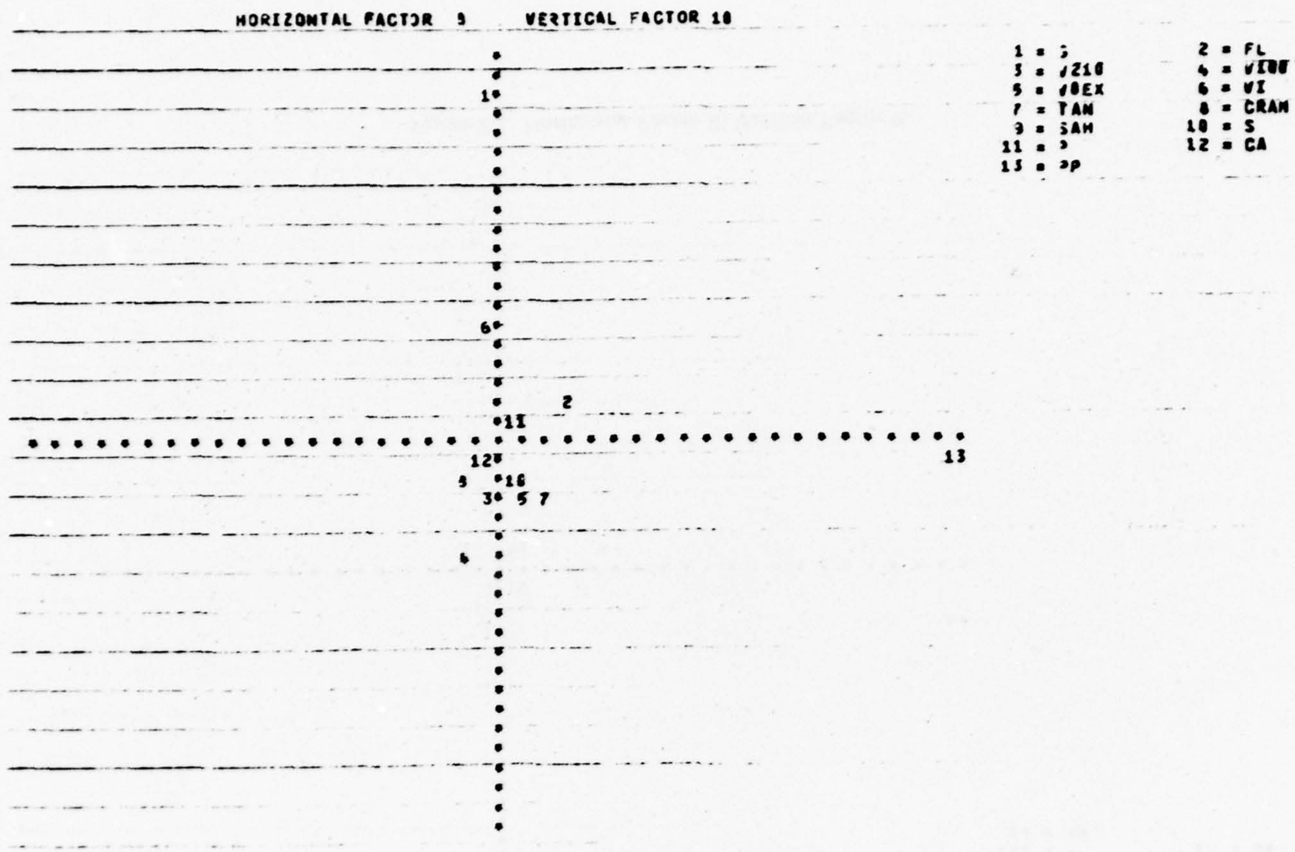

Figure 67. Horizontal Factor 8, Vertical Factor 10.

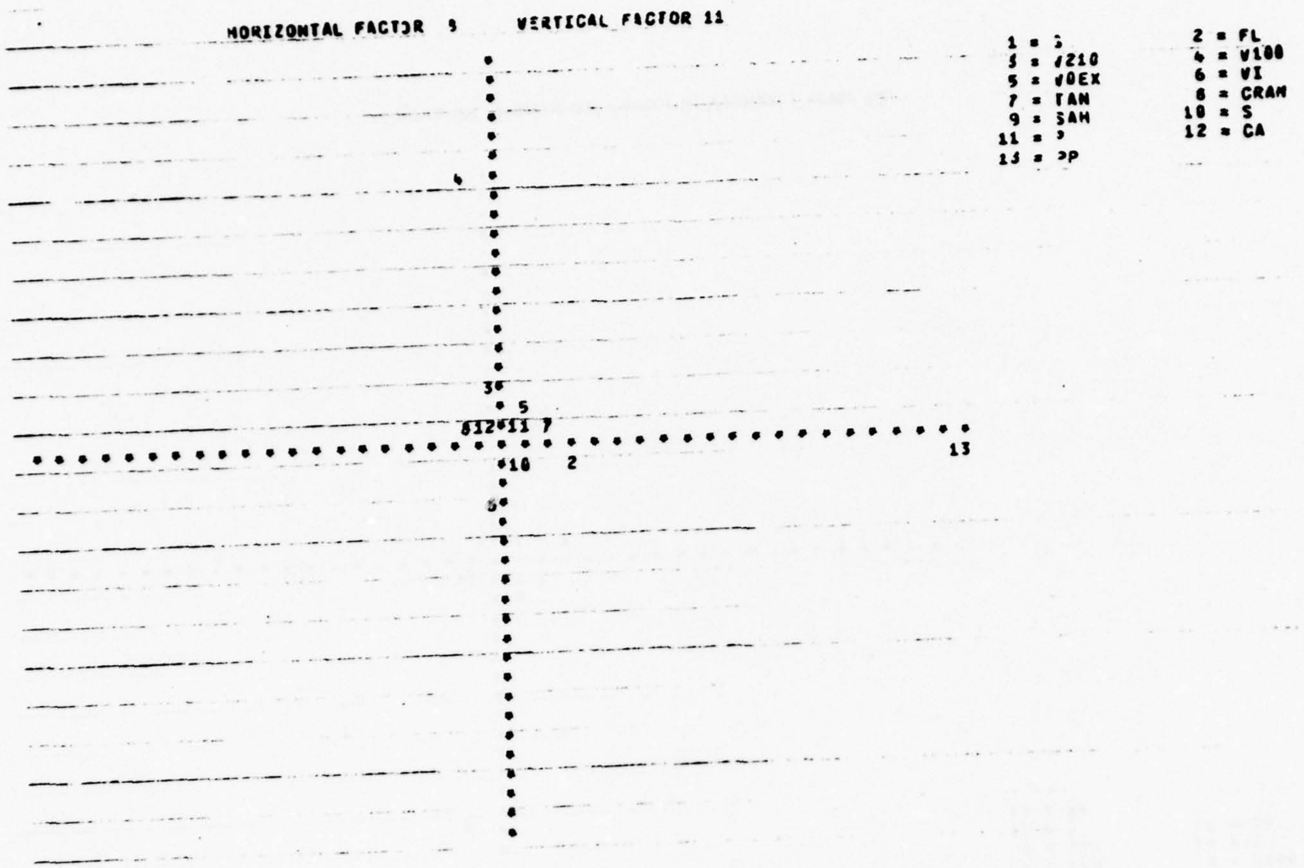

Figure 68. Horizontal Factor 8; Vertical Factor 11.

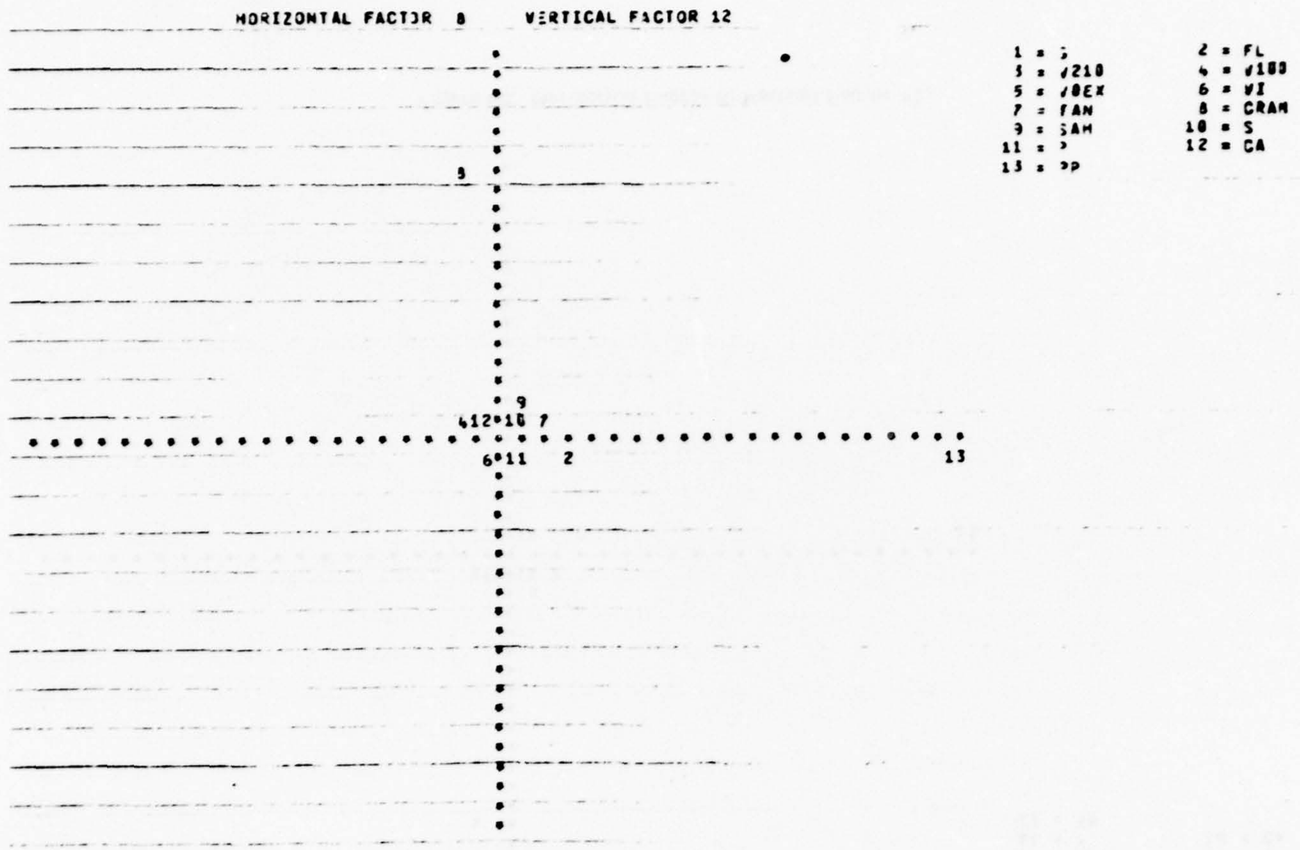

Figure 69. Horizontal Factor 8; Vertical Factor 12.

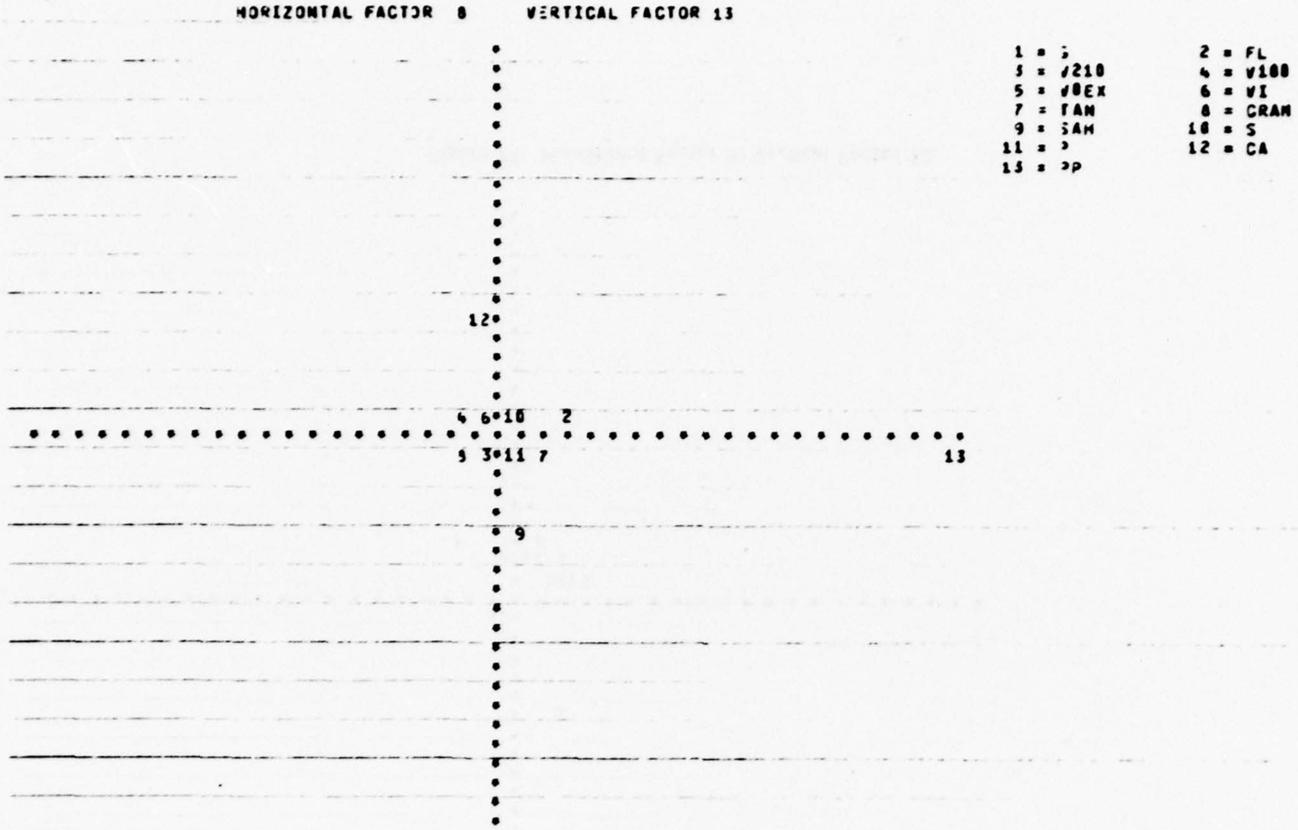

Figure 70. Horizontal Factor 8; Vertical Factor 13.

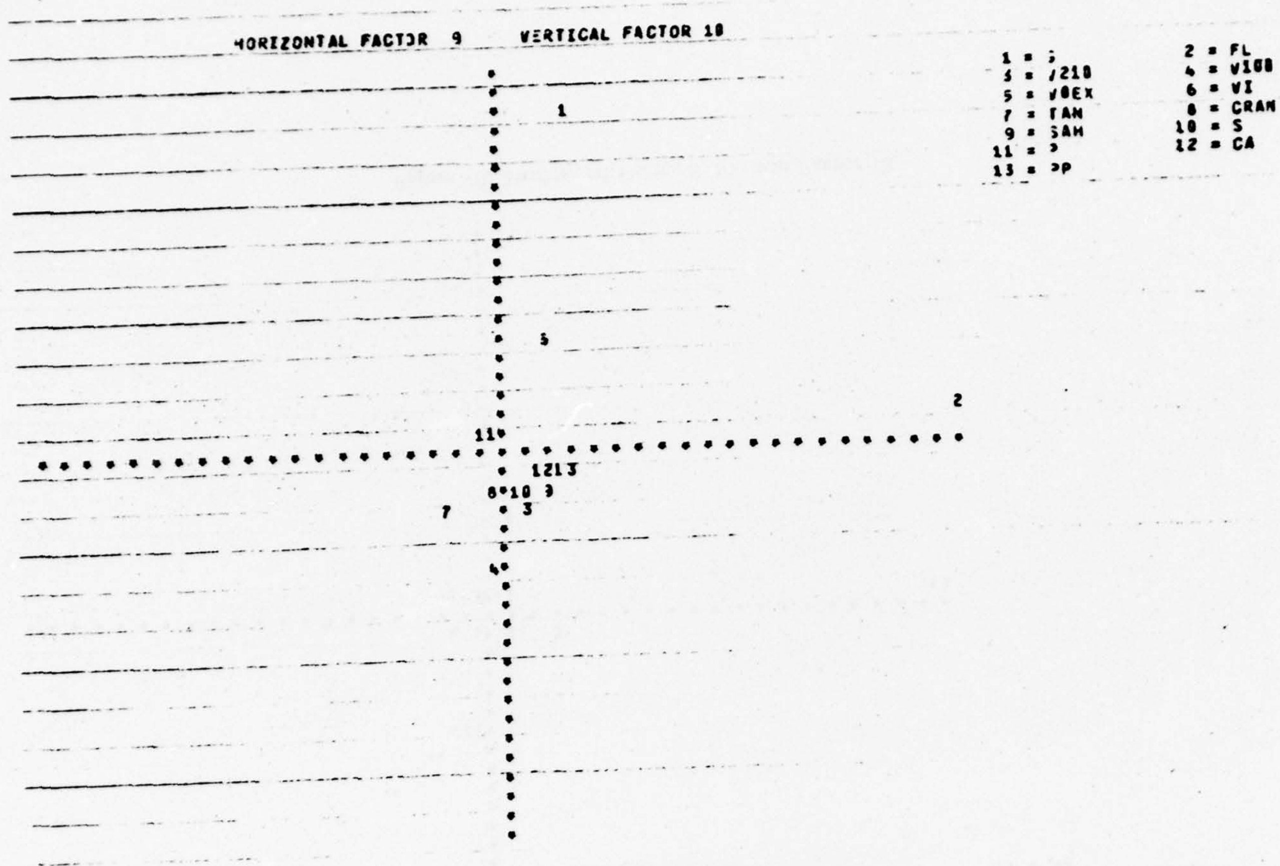

Figure 71. Horizontal Factor 9; Vertical Factor 10.

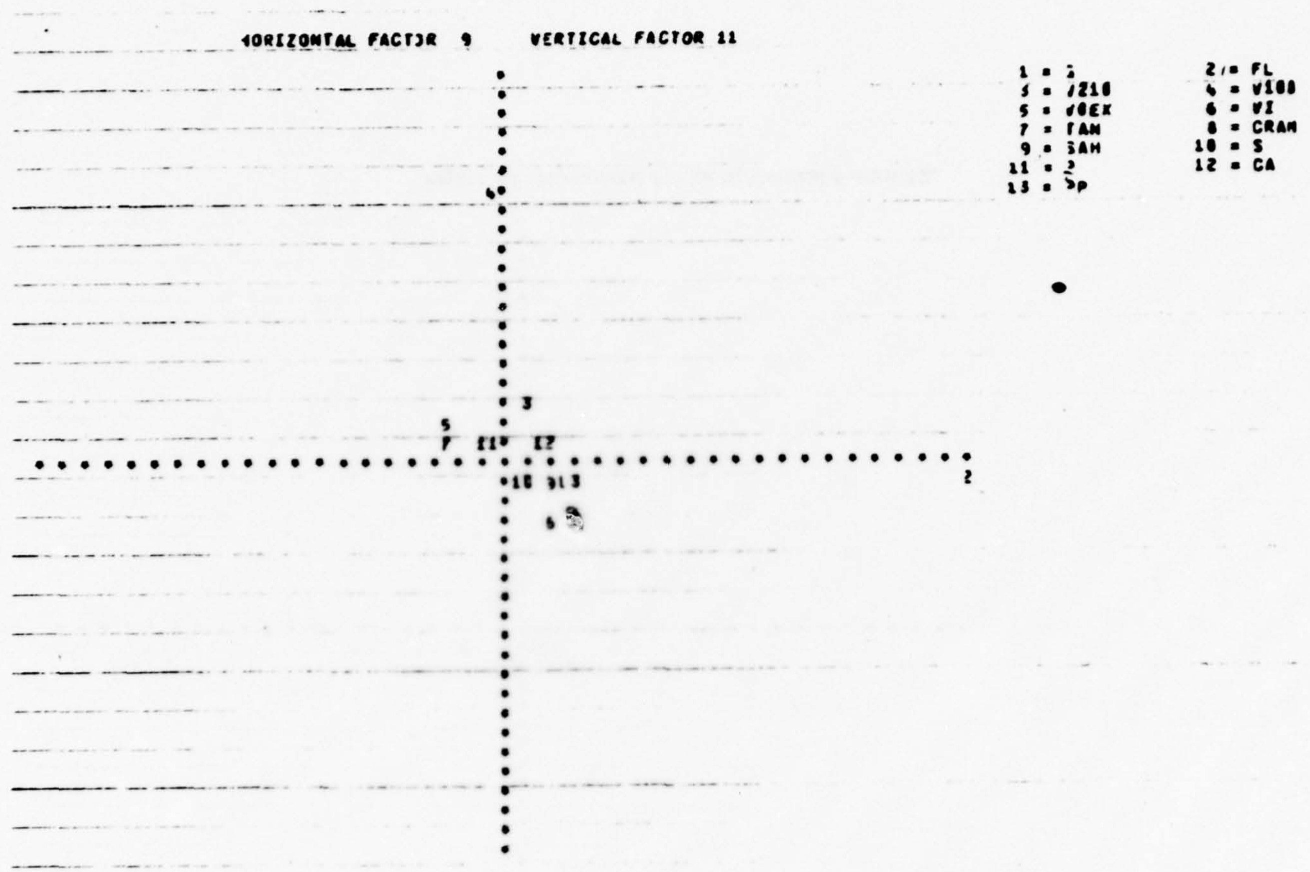

Figure 72. Horizontal Factor 9; Vertical Factor 11.

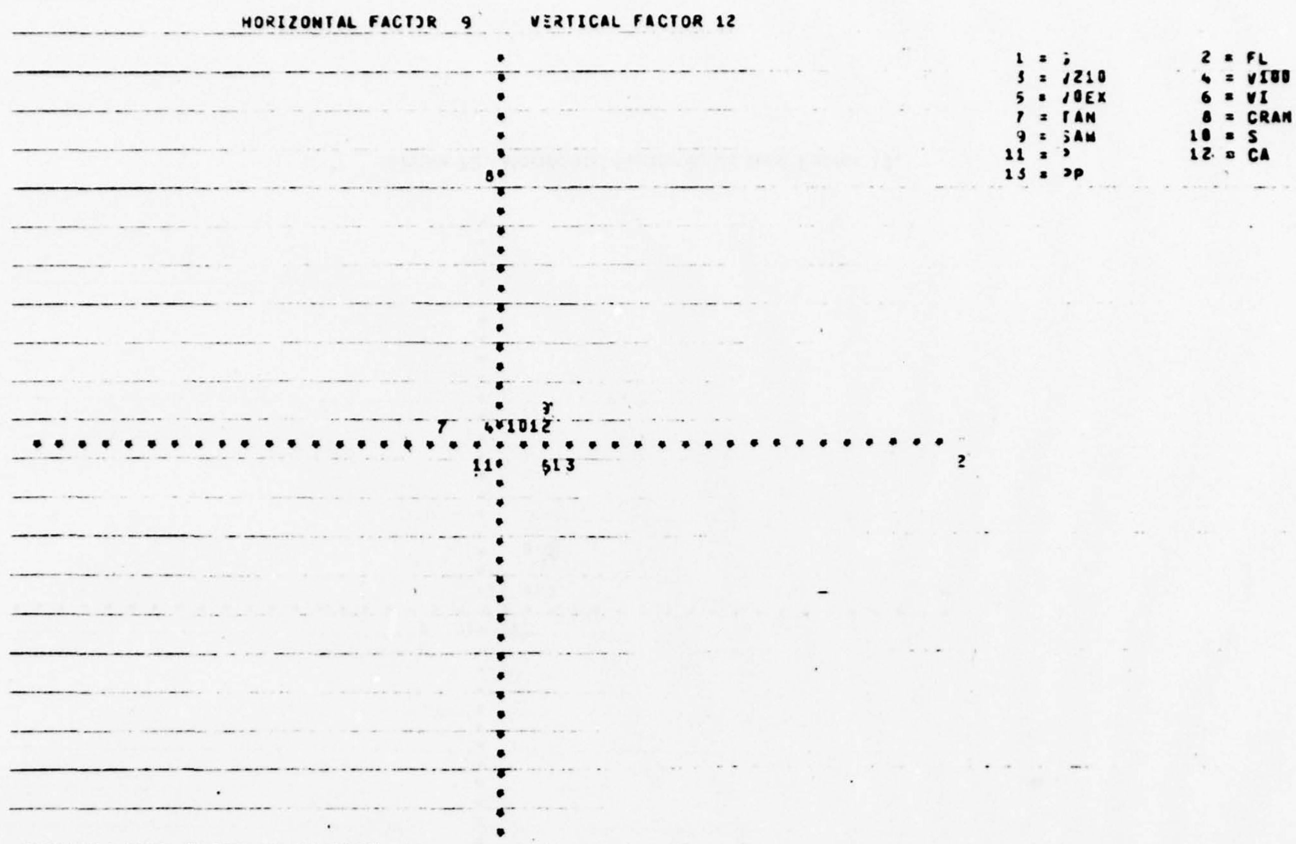

Figure 73. Horizontal Factor 9; Vertical Factor 12.

In Figure 74, variables 5, 4, 10, 6, 1, 7, 11, 3, and 13 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 2 loads high on factor 9. Variables 12 and 9 (negative) load on factor 13 with a negative correlation. Variable 8 shows insignificant loading on either factor.

In Figure 75, variables 3, 5, 7, 8, 12, 11, 10, 13, and 2 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 1 loads high on factor 10. Variable 4 loads high on factor 11. Variable 9 shows insignificant loading on either factor.

In Figure 76, variables 9, 7, 10, 12, 13, 11, and 2 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 1 loads high on factor 10. Variable 8 loads high on factor 12. Variables 3 and 5 show insignificant loading on either factor.

In Figure 77, variables 5, 10, 2, 7, 8, 13, and 11 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 1 loads high on factor 10. Variables 6, 2, and 4 have loadings at the origin of factor 10. Variable 3 has insignificant loadings on either factor. Variables 12 and 9 have significant loadings on factor 13 and correlate negatively.

In Figure 78, variables 6, 10, 5, 13, 11, 9, 3, and 12 cluster and are all close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 4 loads high on factor 11. Variable 8 loads high on factor 12. Variables 1, 2, and 7 have insignificant loadings on either factor.

In Figure 79, variables 6, 10, 5, 13, 11, and 3 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 4 loads high on factor 11. Variables 12 and 9 (negative) have small loadings on factor 13 with variable 9 having a negative value. Variables 1, 2, 7, and 8 have insignificant loadings on either factor.

In Figure 80, variables 6, 10, 13, and 7 cluster and all are close to the origin. All have small loadings on both factors. Internal correlation is indicated. No other grouping is evident. Variable 8 loads high on factor 12. Variable 12 has small loading on factor 13. Variables 1, 2, 3, 4, 5, and 11 have insignificant loadings on both factor 13 and factor 12.

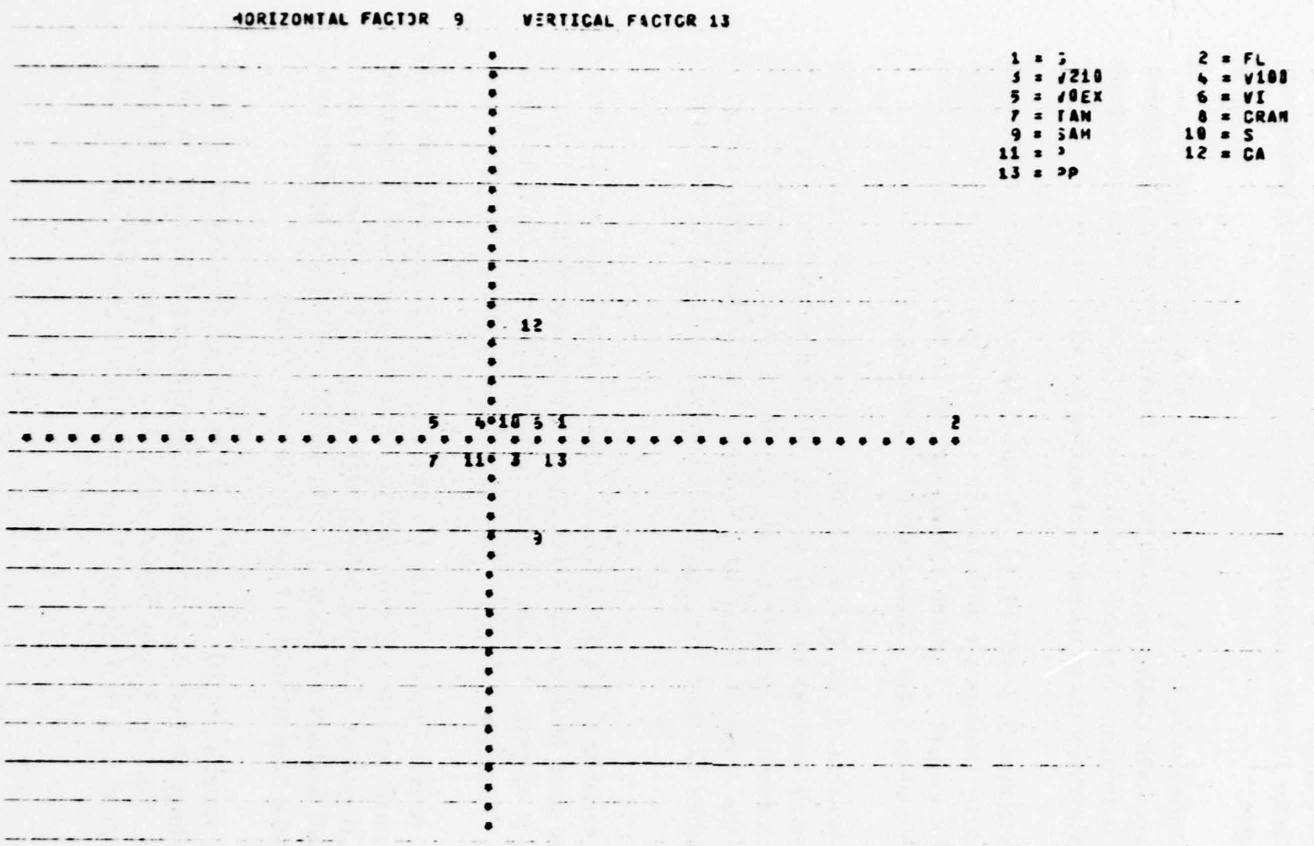

Figure 74. Horizontal Factor 9; Vertical Factor 13.

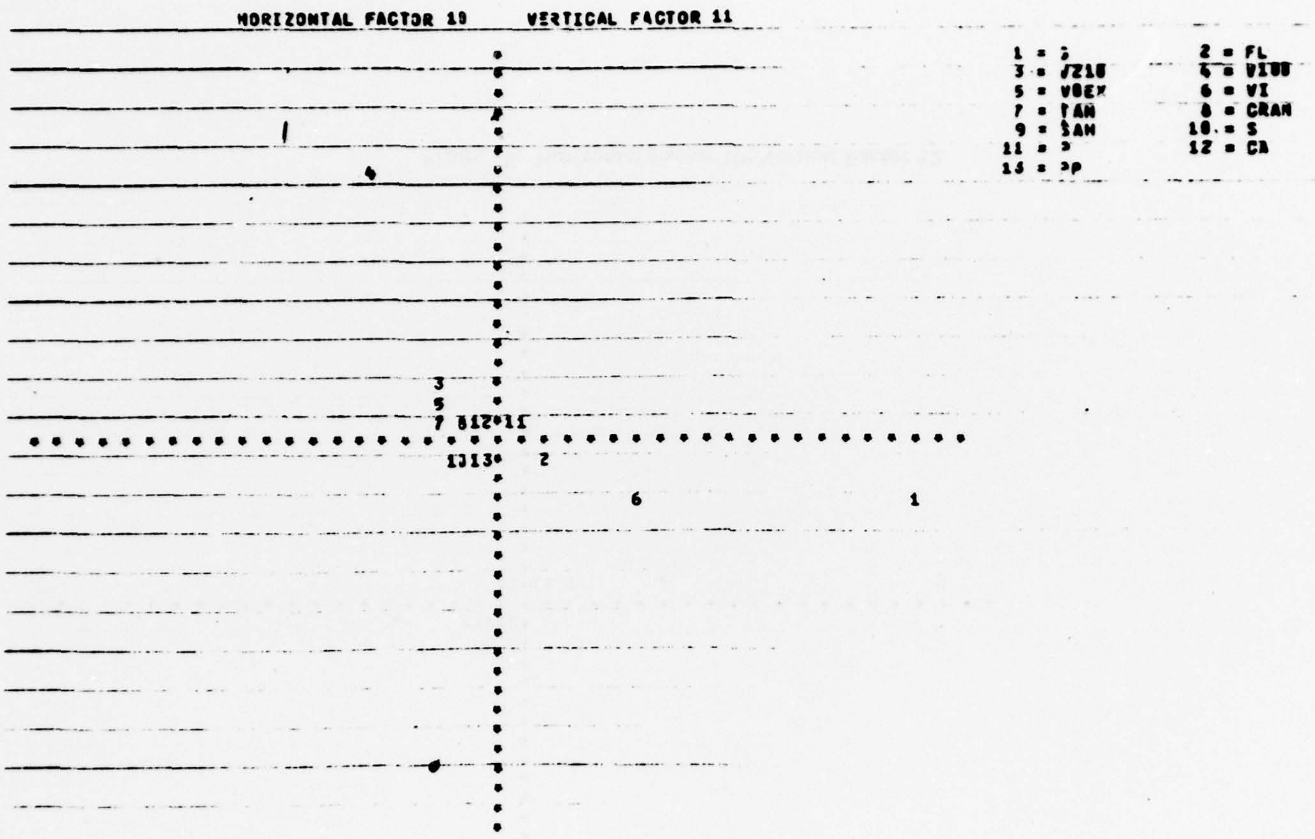

Figure 75. Horizontal Factor 10; Vertical Factor 11.

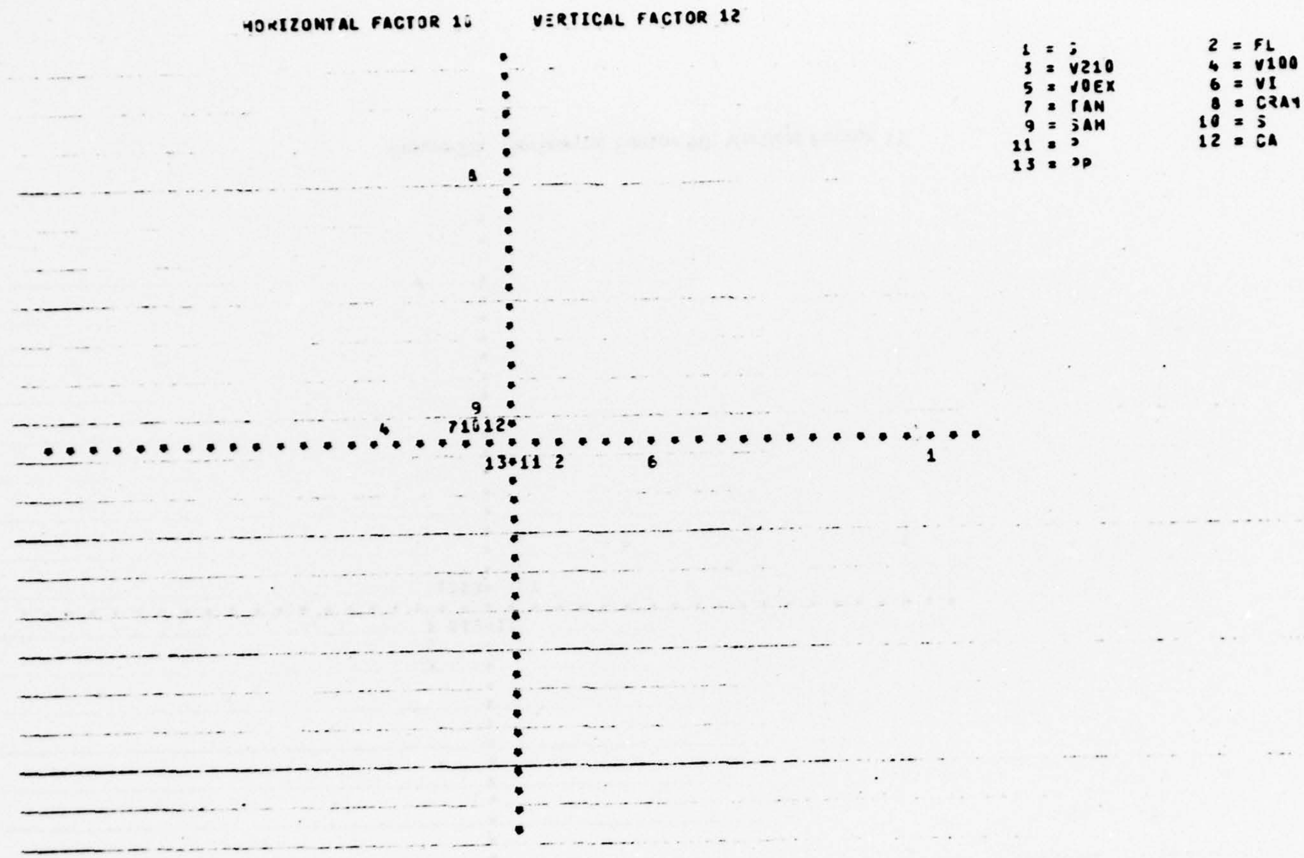

Figure 76. Horizontal Factor 10; Vertical Factor 12.

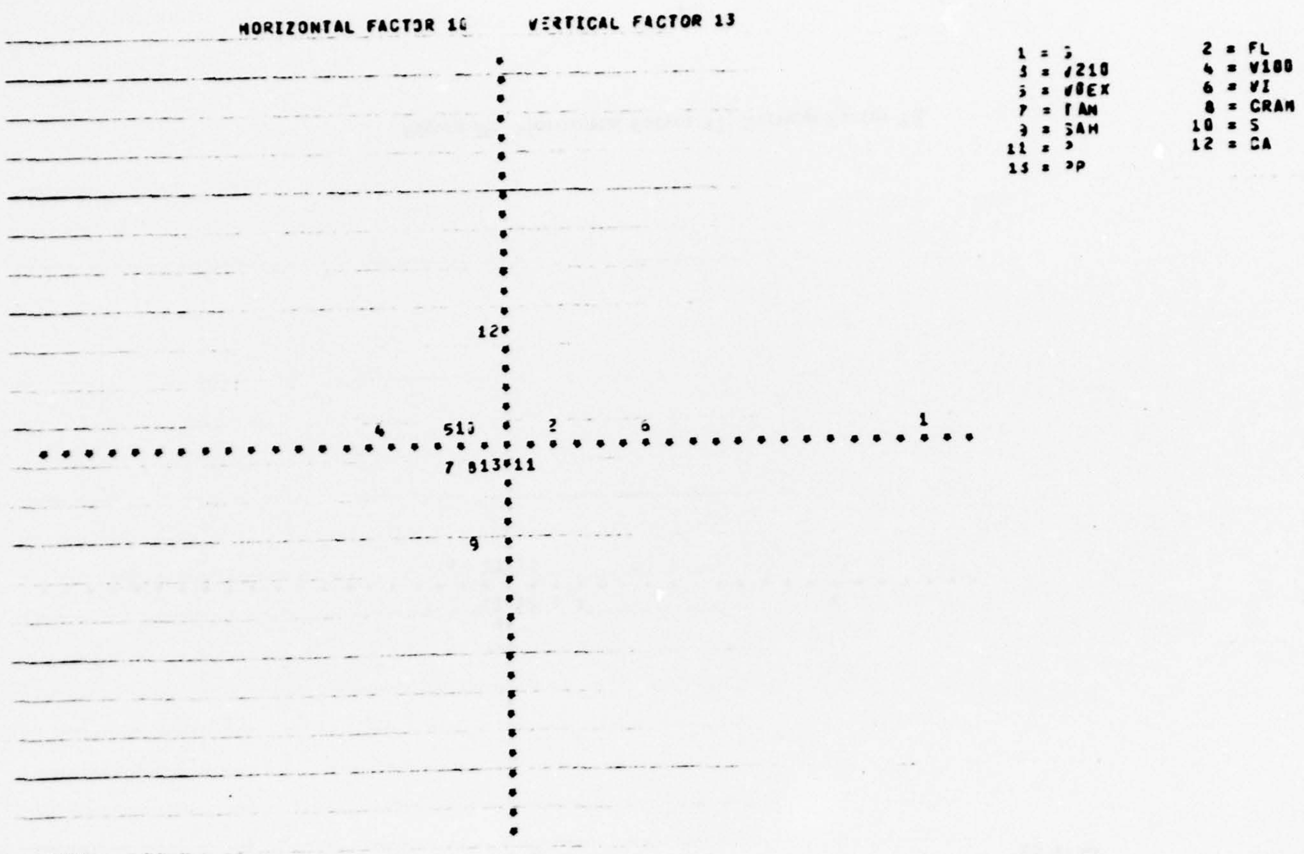

Figure 77. Horizontal Factor 10; Vertical Factor 13.

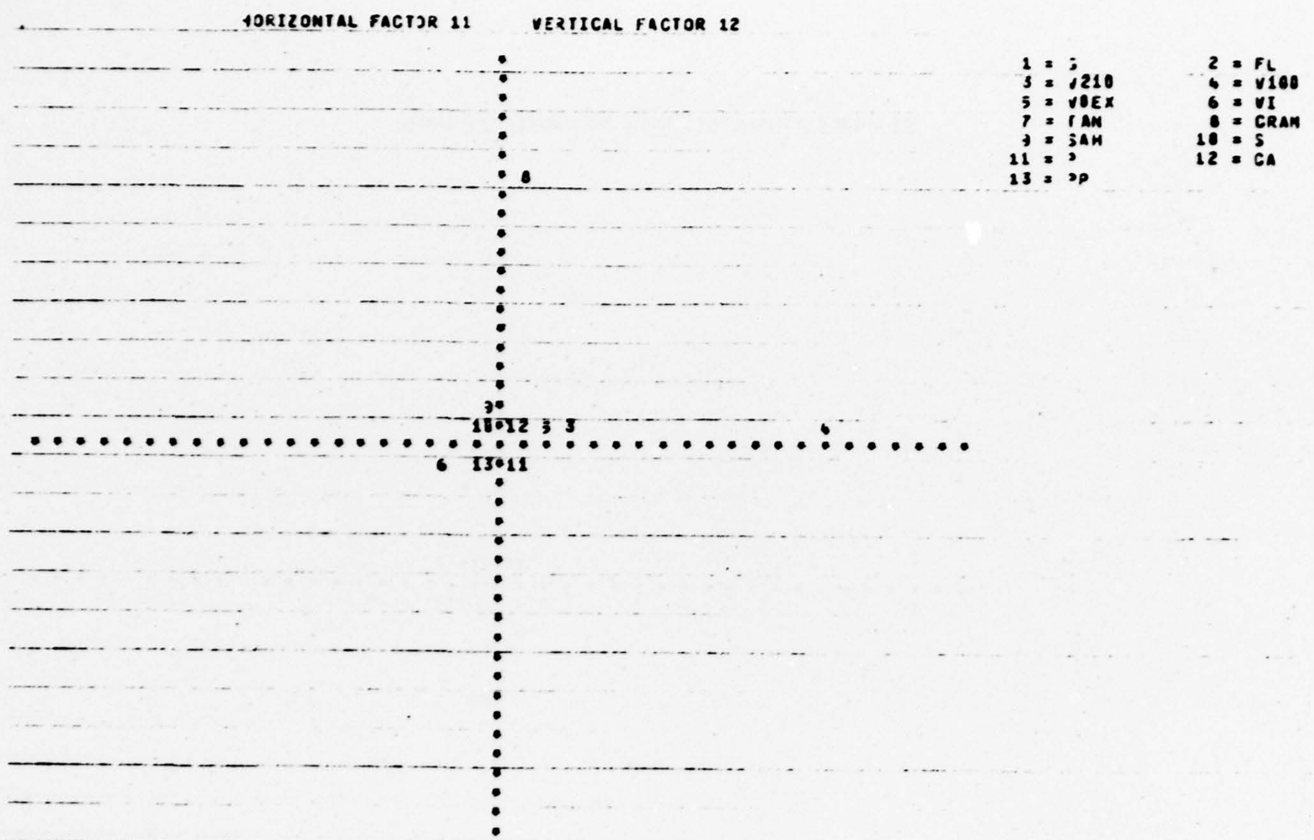

Figure 78. Horizontal Factor 11; Vertical Factor 12.

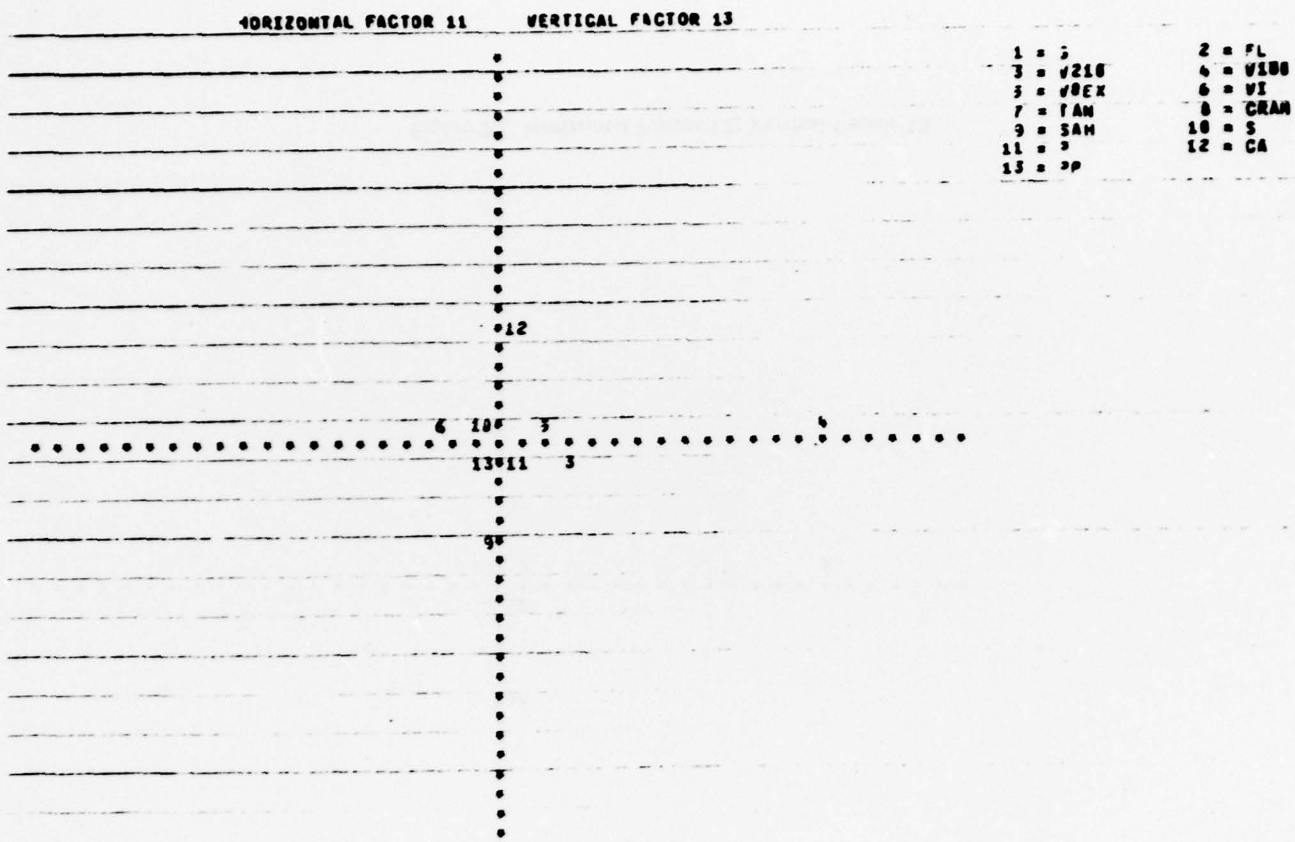

Figure 79. Horizontal Factor 11; Vertical Factor 13.

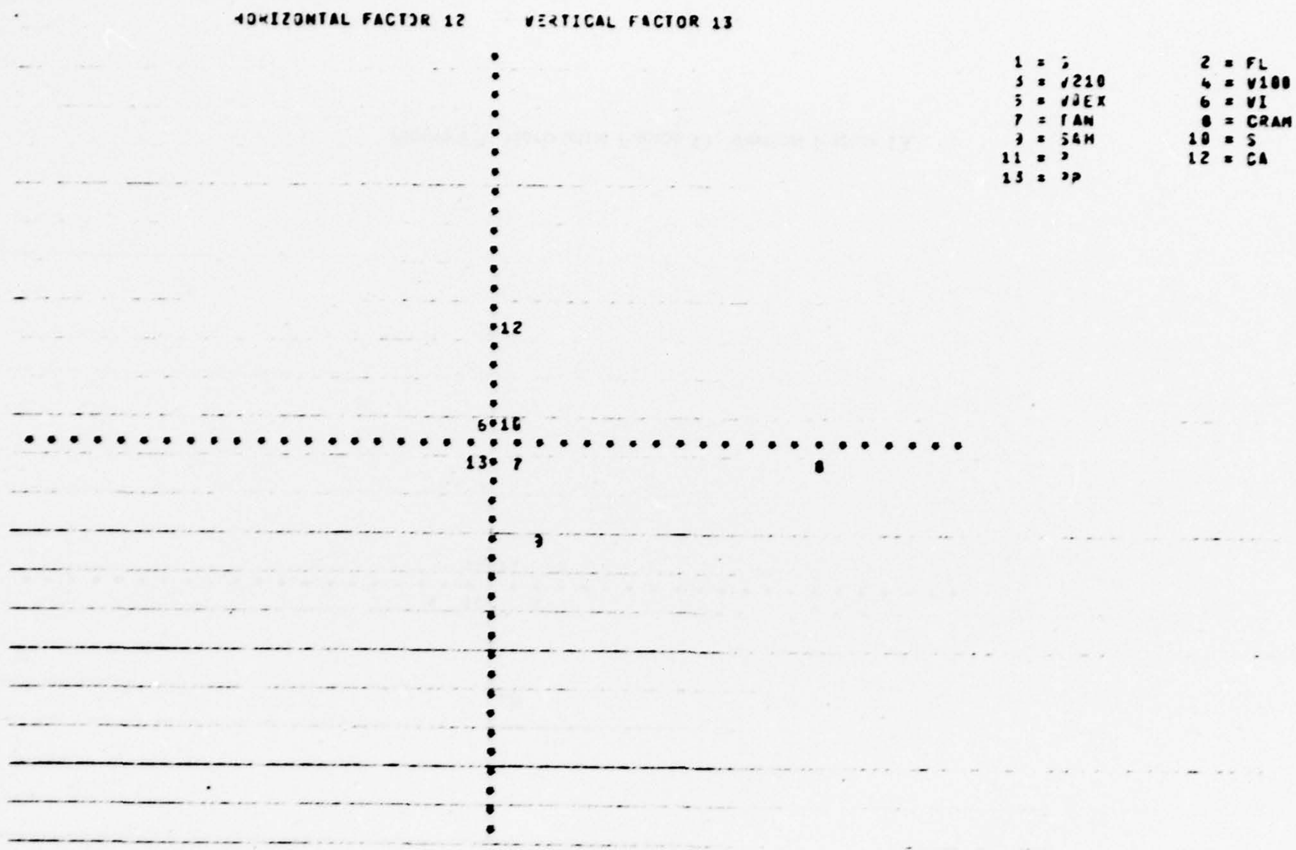

Figure 80. Horizontal Factor 12; Vertical Factor 13.

Varimax rotation as discussed analytically above represents one factoring solution applied to the data by the *SPSS* subprogram. In varimax application, the criterion centers on simplying the columns of the factor matrix. A simple factor is defined as one with only 1s and 0s in the column. This simplication amounts to maximizing the variance of the squared loadings in each column according to

$$n\sum_{p=1}^{n}\sum_{j=1}^{n}\left(\frac{ajp}{hj}\right)^4 - \sum_{p=1}^{n}\left(\sum_{j=1}^{n}\frac{a^2jp}{h^2j}\right)^2 \longrightarrow \text{maximum.}$$

V. CONCLUSIONS

An overall look has been made at laboratory chemical and physical data obtained on 80 lubricating oils meeting the requirements of Specification MIL-L-2104 employing an *SPSS* statistical package and a CDC computer. The purpose of this effort was to seek internal correlations among the data and dynamometer results and field performance and to seek ways to relate laboratory data and field performance. Out of a possible 26 pieces of analytical data collected for each of the 80 samples, 13 pieces of information were selected for scrutiny.

It was found that, overall, the data exhibited substantial variance, departed from a normal distribution in all cases, and showed, in general, a lack of internal correlation among the data items. There was exhibited, however, significant correlations between selected pairs of data. Sulphated ash vs calcium, sulphated ash vs carbon residue, and carbon residue vs calcium showed favorable Pearson's correlation. From the unmatched, unrotated, factor matrix, flash point was influenced by calcium, sulphated ash, and carbon residue. Flash point was negatively influenced by gravity. Viscosity at 210°F was strongly influenced, or vice versa, by total acid number, flash point, and phosphorus. Negligible influence was noted for flash point or viscosity at 0 (extrapolated) and viscosity index, etc. Other examples of paired correlation were indicated by the data. Significant in the observations was the fact that about 80% of the total variance of the observations was attributed to only three analytical items – flash point, gravity, and viscosity measurements. The importance of this deserves further scrutiny.

The statistical package, SPSS, that was employed allows the exhibition of the orthogonal rotation of the data in graphical presentation. When this is accomplished, certain groupings of the data become evident. These groupings suggest internal correlation of groups of data and could point the way to considerable data-reductions possibilities. Dynamometer and field results were not available in this study, but the possibility exists that if these results were included along with the graphical presentations, information on the relationships between laboratory, chemical, and physical observations and dynamometer and fuel observations could be obtained. The graphical presentations are included.

The application of these statistics to selected chemical and physical data obtained from lubricating oils has been performed in a very preliminary way. The results so far only suggest certain correlations and data-reduction possibilities. To be more meaningful, a more complete inclusion of the data on a greater number of samples is warranted. The uniqueness of the distributions, the failure of the distributions to approach normality, and the apparent negative correlations need further explanation.

The SPSS statistical package or some variation does seem to offer a possible method for the accomplishment of the task undertaken.

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