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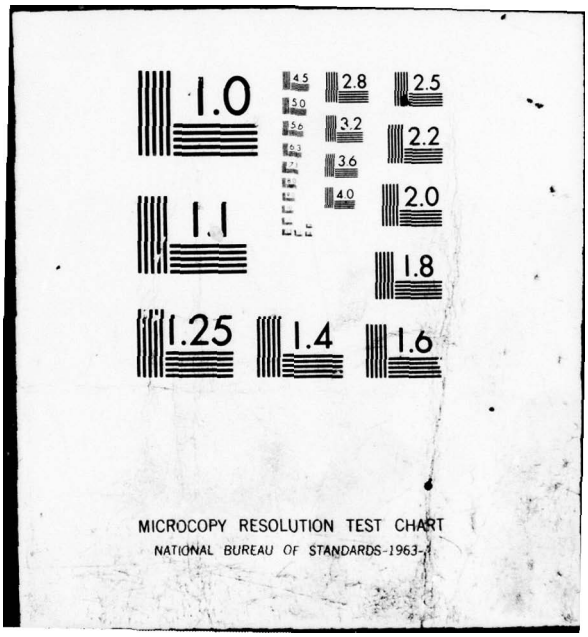
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NAVAL POSTGRADUATE SCHOOL

Monterey, California

ELEMENTS OF SOUND TRANSMISSION IN BEAMS

by

A.B. Coppens and O.B. Wilson, Jr.

14 April 1978 - 30 September 1978

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

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ELEMENTS OF SOUND TRANSMISSION IN BEAMS

by

A.B. Coppens and O.B. Wilson, Jr.

Abstract

The elements of longitudinal and shear waves in solids are reviewed and applied to the propagation of sound in an I beam. Comparison with experimental data reveals that principle modes of vibration in an I beam excited under a number of different source placements seem to be dominated by the first flexural mode. A variety of source transducers were constructed for the efficient excitation of motion in the beams.

Introduction

The propagation of mechanical waves in solids can often be described with the help of two potential functions. Longitudinal motion can be obtained from the scalar potential which satisfies the wave equation

$$\nabla^2 \phi = \frac{1}{c_L^2} \frac{\partial^2 \phi}{\partial t^2} \tag{1}$$

where c_L is the "longitudinal bulk" speed of sound.
Shear waves can similarly be described by the vector potential which obeys the wave equation

$$\nabla^2 \psi_i = \frac{1}{c_s^2} \frac{\partial^2 \psi_i}{\partial t^2} \qquad i = x, y, z \tag{2}$$

where c_s is the "shear" speed of sound.

The speeds of sound c_L and c_s associated with these two wave equations can be expressed in a variety of forms. If we define

Y = Young's Modulus

ρ = density

σ = Poisson's ratio

μ = Lame constant = Shear Modulus

λ = Lame constant

then these constants can be related by the following expressions[1]

$$Y = \mu \frac{3\lambda + 2\mu}{\lambda + \mu} \qquad \sigma = \frac{1}{2} \frac{\lambda}{\lambda + \mu} \tag{3}$$

$$\lambda = Y \frac{\sigma}{(1+\sigma)(1-2\sigma)} \qquad \mu = \frac{1}{2} Y \frac{1}{1+\sigma}$$

and the speeds of sound can be expressed as

$$c_L^2 = \frac{\lambda + 2\mu}{\rho} \tag{4}$$

$$c_s^2 = \frac{\mu}{\rho} \tag{5}$$

If we define the "longitudinal thin bar" speed of sound c_b

$$c_b^2 = \frac{Y}{\rho} \tag{6}$$

then the above speeds can be reexpressed as[2]

$$c_L^2 = c_b^2 \frac{1-\sigma}{(1+\sigma)(1-2\sigma)} \tag{7}$$

$$c_s^2 = c_b^2 \frac{1}{2} \frac{1}{1+\sigma} \tag{8}$$

From the relationships between the elastic constants, it is clear that σ must always satisfy $\sigma \leq 0.5$.

Obtaining the ratio

$$\left(\frac{c_s}{c_L}\right)^2 = 1 - \frac{1}{2} \frac{1}{1-\sigma} \tag{9}$$

reveals that the shear wave speed must always be smaller than the longitudinal wave speed,

$$0 < c_s < c_L/\sqrt{2} \tag{10}$$

Some acoustical data[1] for a few solids are presented the first five columns in Table 1. The values for Poisson's ratio were calculated from (8) and/or (9) and the tabulated speeds of sound c_b, and c_L, and c_s. If there was appreciable disagreement between the calculated values of σ, the average was listed. Values of c_b in parentheses were calculated from the listed value of σ and the other tabulated quantities.

The particle velocity $\vec{u} = u_x\hat{x} + u_y\hat{y} + u_z\hat{z}$ of the mechanical wave in the solid can be obtained from the scalar and vector potentials,

$$\vec{u} = \nabla\phi + \nabla\times\vec{\psi} \tag{11}$$

and the stresses within the solid are given by

$$\sigma_{xx} = \lambda\nabla^2\phi + 2\mu\frac{\partial u_x}{\partial x} \qquad \sigma_{xy} = \mu\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \tag{12}$$

The other stress components are obtained by the usual permutation of subscripts. The subscript convention is that the first subscript designates the normal of the plane the stress is applied to and the second specifies the direction of the stress. Thus, <u>unlike</u> subscripts designate shearing stresses in the chosen coordinate system; these stresses are symmetrical, $s_{ij} = s_{ji}$. <u>Like</u> subscripts specify the principal stresses.

TABLE 1

Material	Speeds of Sound 10^5cm/sec c_b	c_L	c_s	Density g/cm^3 ρ	Characteristic Impedance 10^6g/cm^2-sec ρc_L	Possion's Ratio σ	$c_p(s_o)$
Aluminum 2S0	5.10	6.35	3.10	2.71	1.73	0.35	5.44
17ST	5.08	6.25	3.10	2.80	1.75	0.34	5.40
Beryllium	12.75	12.80	8.71	1.82	2.33	0.07	12.78
Brass 70-30	3.40	4.37	2.10	8.50	3.70	0.33	3.60
naval	3.49	4.43	2.12	8.10	3.61	0.35	3.73
Bronze (phosphor 5%)	3.43	3.53	2.23	8.86	3.12	0.17	3.48
Copper	3.71	4.68	2.26	8.90	4.18	0.35	3.96
Steel	5.17	5.85	3.23	7.80	4.56	0.28	5.39
mild	5.05	6.10	3.24	7.90	4.82	0.26	5.28
302	4.90	5.66	3.12	8.03	4.55	0.26	5.10
304L	(4.93)	5.64	3.07	7.90	4.45	0.29	5.15
410	5.03	7.39	2.99	7.67	5.67	0.41	5.48
416	(5.20)	6.02	3.23	7.70	4.64	0.30	5.46
Tin	2.73	3.32	1.67	7.30	2.42	0.33	2.89
Plastic acrylic resin	(1.88)	2.67	1.12	1.18	0.32	0.39	2.02
	(2.16)	2.68	1.32	1.20	0.32	0.34	2.30
Plexiglas	(1.88)	2.67	1.12	1.18	0.32	0.39	2.03
Polystyrene	(1.85)	2.35	1.12	1.06	0.23	0.35	1.96

Rayleigh Waves

Perhaps the simplest kind of a surface wave to consider is that which exists on the surface of a solid of infinite depth. Let the surface of the solid at z = 0 be completely free to move, and let the solid fill the half-space z > 0. If we assume a plane-wave disturbance of constant frequency traveling in the + x direction, then we can postulate the form of the potentials,

$$\underline{\phi} = Z_1(z)\, e^{i(\omega t - k_R x)}, \qquad \underline{\psi}_y = Z_2(z)\, e^{i(\omega t - k_R x)} \tag{13}$$

Substitution of these into their respective wave equations yields

$$\begin{aligned} \underline{\phi} &= A \exp\left[-\sqrt{k_R^2 - k_L^2}\, z + i(\omega t - k_R x)\right] \\ \underline{\psi} &= \underline{B} \exp\left[-\sqrt{k_R^2 - k_s^2}\, z + i(\omega t - k_R x)\right] \end{aligned} \tag{14}$$

for motion which will vanish in the limit of large z, where k_L and k_s are defined by

$$\begin{aligned} k_L &= \omega / c_L \\ k_s &= \omega / c_s \end{aligned} \tag{15}$$

There can be no stresses on the free surface of the solid, so that

$$\sigma_{zx} = \sigma_{zz} = 0 \quad \text{at} \quad z = 0 \tag{16}$$

the motion is independent of y so that all $\partial/\partial y$ must vanish. The boundary conditions then become

$$\sigma_{zx} = \mu\left(2\frac{\partial^2\phi}{\partial z\partial x} + \frac{\partial^2\psi_y}{\partial x^2} - \frac{\partial^2\psi_y}{\partial z^2}\right) = 0$$

$$\sigma_{zz} = \lambda\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) + 2\mu\left(\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\psi_y}{\partial z\partial x}\right) = 0$$

Application of these boundary conditions to our solutions yields the amplitude of ψ in terms of A,

$$|\underline{B}| = \frac{2k_R\sqrt{k_R^2 - k_L^2}}{2k_R^2 - k_s^2} A$$

and a characteristic equation for the wave number k_R

$$4k_R^2\sqrt{k_R^2 - k_L^2}\sqrt{k_R^2 - k_s^2} - (2k_R^2 - k_s^2)^2 = 0 \qquad (17)$$

Under the condition $k_R > k_s$ (recall that we must have $k_s > 2k_L$ from (10)), which is required for a surface wave which goes to zero for large z, it can be shown that the characteristic equation has only one real root, which corresponds to propagation in the +x direction (an imaginary or complex root would correspond to exponential damping in the +x direction). This root k_R is given approximately by[3]

$$\frac{k_s}{k_R} = \frac{c_R}{c_s} \doteq \frac{0.87 + 1.12\sigma}{1 + \sigma} \qquad (18)$$

Thus, for the allowed values of Poisson's ratio ($0 < \sigma < 0.5$), the values of the phase speed of a Rayleigh wave must satisfy the inequalities

$$0.87 < \frac{c_R}{c_s} < 0.95 \tag{19}$$

The "skin depth" z_R of the Rayleigh wave is determined by the exponential decays of the two potentials. Roughly,

$$z_R \sim \frac{1}{\sqrt{k_R^2 - k_s^2}} \lesssim \frac{3.2}{k_R} \sim \frac{\lambda_R}{2} \tag{20}$$

Lamb Waves

When the solid medium is not infinitely thick, there must be a set of boundary conditions to be applied to both surfaces of the material, and this can change the properties of the propagation appreciably.

Let the solid be of thickness z and loaded on both surfaces with air. The boundary conditions are then the same as before, vanishing stress at the interface. For the plane wave disturbance of constant frequency traveling in the +x direction, the form of the potentials is as before,

$$\underline{\phi} = Z_\phi(z)\, e^{i(\omega t - kx)}, \quad \underline{\psi}_y = Z_\psi(z)\, e^{i(\omega t - kx)} \tag{21}$$

Now, however, instead of restricting attention to solutions to the wave equation representing waves attenuating with large z, we must allow for the possibility of standing waves.

Suitable trial solutions are then

$$\underline{\Phi} = \left[A_1 \cos\sqrt{k_L^2 - k^2}\, z + \underline{A}_2 \sin\sqrt{k_L^2 - k^2}\, z \right] e^{i(\omega t - kx)}$$

$$\underline{\Psi} = \left[\underline{B}_1 \sin\sqrt{k_s^2 - k^2}\, z + \underline{B}_2 \cos\sqrt{k_s^2 - k^2}\, z \right] e^{i(\omega t - kx)} \tag{22}$$

and the boundary conditions are

$$\sigma_{zx} = \sigma_{zz} = 0 \quad \text{at} \quad z = 0, Z \tag{23}$$

Application of the boundary conditions results in two separate characteristics

$$\frac{\tan\left(\sqrt{k_s^2 - k^2}\, Z/2\right)}{\tan\left(\sqrt{k_L^2 - k^2}\, Z/2\right)} = -\frac{4k^2\sqrt{k_s^2 - k^2}\sqrt{k_L^2 - k^2}}{\left(2k^2 - k_s^2\right)^2} \tag{24}$$

$$\frac{\tan\left(\sqrt{k_s^2 - k^2}\, Z/2\right)}{\tan\left(\sqrt{k_L^2 - k^2}\, Z/2\right)} = -\frac{\left(2k^2 - k_s^2\right)^2}{4k^2\sqrt{k_s^2 - k^2}\sqrt{k_L^2 - k^2}} \tag{25}$$

whose independent solutions together yield the allowed values of k. These equations reveal that the propagation of Lamb waves in usually highly dispersive, as to be expected with a waveguide-like system. The cutoff frequencies for the various allowed modes of vibration can be obtained from the requirement $k \to 0$, except for the two lowest modes of vibration which can exist in the limit of frequency approaching zero.

The motions of the Lamb waves can be classified as either symmetric or antisymmetric. In the former case, governed by (24), there is a plane of symmetry at $z = Z/2$ so that the displacements of the surfaces at $z = 0, Z$ are mirror images of each other. For the case of antisymmetric motion, governed by (25), the displacements of the two surfaces are identical and the distortion of the plate resembles that of a transversely vibrating membrane.

The lowest modes are the zeroth order symmetric (s_0) and antisymmetric (a_0) modes. These can be found by allowing the arguments of the trignometric functions to vanish. In this limit, we have

$$k_{s0} = \frac{1}{2} \frac{k_s^2}{\sqrt{k_s^2 - k_L^2}} \tag{26}$$

from (24) for the lowest symmetrical mode (s_0). The phase speed for this mode can be seen to be

$$c_p(s_0) = \frac{c_b}{\sqrt{1-\sigma^2}} = c_s \sqrt{\frac{2}{1-\sigma}} \tag{27}$$

which is independent of frequency, at least for small frequencies. Because of this independence it follows that the group speed has the same value. In this low frequency limit, it is seen that propagation in this mode is dispersionless. Values of $c_p(s_0)$ are presented in Table 1 as calculated from the given values of c_b, c_s, and σ. If the calculations led to slightly different answers, the results were averaged. For the lowest antisymmetric mode, (a_0), on the other hand, complicated analysis[3] reveals that in the low frequency limit the group speed is

$$c_g(a_0) \sim 2\sqrt{\omega z/2}\ \sqrt[4]{c_b^2/3(1-\sigma^2)} = 1.35\sqrt{c_p(s_0)\, f z} \tag{28}$$

where f is the frequency. and the phase speed is

$$c_p(a_0) \sim c_g(a_0)/2 \tag{29}$$

so that there is considerable dispersion for propagation in this mode.

The cutoff frequencies for the higher modes can be determined by finding the frequencies for which $k \to 0$. These frequencies occur when

$$\frac{\tan\left(\sqrt{k_s^2 - k_L^2}\, Z/2\right)}{\tan\left(\sqrt{k_L^2 - k^2}\, Z/2\right)} \to 0 \tag{30}$$

for symmetric modes and

$$\frac{\tan\left(\sqrt{k_s^2 - k_L^2}\, Z/2\right)}{\tan\left(\sqrt{k_L^2 - k^2}\, Z/2\right)} \to \infty \tag{31}$$

for antisymmetric modes. The results are

$$\begin{aligned} k_L Z/2 &= \pi/2,\ 3\pi/2,\ 5\pi/2,\ \ldots \\ k_s Z/2 &= \pi,\ 2\pi,\ 3\pi,\ \ldots \end{aligned} \tag{32}$$

for symmetric modes and

$$\begin{aligned} k_L Z/2 &= \pi,\ 2\pi,\ 3\pi,\ \ldots \\ k_s Z/2 &= \pi/2,\ 3\pi/2,\ 5\pi/2,\ \ldots \end{aligned} \tag{33}$$

for antisymmetric modes.

At very high frequencies, the s_0 and a_0 modes resemble Rayleigh waves propagating on both of the surfaces of the solid, and the group and phase speeds for both modes monotonically approach the Rayleigh wave limit c_R. For all higher symmetric and antisymmetric modes, the high frequency limit of the group and phase speeds approaches the shear speed c_s.

Results for Steel Plates

Study of Table 1 reveals that representative values for the various phase speeds and Poisson's ratio in steels are

$$c_L \sim 5.8 \times 10^5 \text{ cm/sec}$$

$$c_p(a_o) \sim 5.3 \times 10^5 \text{ cm/sec}$$

$$c_b \sim 5.1 \times 10^5 \text{ cm/sec}$$

$$c_s \sim 3.2 \times 10^5 \text{ cm/sec}$$

$$c_R \sim 3.0 \times 10^5 \text{ cm/sec}$$

$$\sigma \sim 0.28$$

On the basis of these values, the cutoff frequencies for some of the lowest modes in ½ in. and ¼ in. steel plate have been calculated and presented in Table 2, and estimated value of $c_p(a_o)$ presented in Table 3 for a number of frequencies.

The Rayleigh speed $c_R = 3.0 \times 10^5$ cm/sec is the asymptotic limit of $c_p(a_o)$, so values of $c_p(a_o)$ calculated from the approximate expression (29) which exceed c_R have been replaced by that quantity. Calculated values approaching c_R should be considered as only estimates. The group speed $c_g(a_o)$ can be taken as nearly c_R for all cases listed in the Table 2.

While little can be said in general about the absorption of Lamb waves in plates, the fact that the propagation is in modes requires that absorption be greatest where phase speed dispersion is greatest. Thus, greatest losses at a given frequency should occur for those modes of propagation having the highest cutoff frequencies. Additionally, the attenuation of Lamb waves should be determined primarily by shearing losses for those modes whose motion is predominately shear (such as a_o) and by longitudinal losses for those modes whose motion is dominantly longitudinal (such as s_o)

TABLE 2

CUTOFF FREQUENCIES FOR THE LOWER
LAMB WAVE MODES IN STEEL PLATE

Mode	Condition	Cutoff Frequency in kHz 1/4 in. Steel Plate	1/2 in. Steel Plate
s_0		0	0
a_0		0	0
a_1	$z = \lambda_s/2$	252	126
s_1	$z = \lambda_L/2$	457	228
s_2	$z = \lambda_s$	504	251
a_2	$z = 3\lambda_s/2$	756	378

TABLE 3

$c_p(a_o)$ IN REPRESENTATIVE STEEL PLATES

Frequency in kHz	Approximate Value of $c_p(a_o)$ in 10^5 cm/sec	
	1/4 in. Steel Plate	1/2 in. Steel Plate
50	1.8	2.5
100	2.5	3.0
150	3.0	3.0
200	3.0	3.0

Experimental Study

A short series of experiments was performed at Naval Weapons Center with the participation of Mr. Myron Iverson. The purpose was to study the propagation of sound from source to receiver in an I beam as a function of frequency and of transducer placement and separation. The I beam measured approximately 21 ft. long, has 3 in. flanges, a 5 in. web, and the thickness of all portions was nominally 0.25 in.

The transmitter was affixed at several positions very close to one end of the beam, and the receiver was positioned in the middle of the web (between the flanges) at distances of approximately 4, 8, 12, 16 and 21 ft. from the transmitter. The transmitter was excited with a 3 V peak-to-peak, four cycle, tone burst of either 170 or 360 kHz signal. The received signal was amplified 40 dB and displayed on a CRO which was synched to the excitation of the transmitter. Photographs were taken of the displayed waveforms and are presented in Table 4.

The transmitter was a longitudinal vibrator fabricated at the Naval Postgraduate School. Its active element was a 1 in. diameter 0.25 in. thick ceramic disc. It was attached to the I beam with a 3/8 - 24 threaded stud. The mating surface of the I beam was ground down to remove rust and sclae and well-greased to provide good acoustical contact with the surface of the transmitter.

The receiver was an accelerometer furnished by Naval Weapons Center. It was hand-held or gravity coupled to the I beam with a thin film of oil between it and the ground surfaces of the I beam.

Analysis of the photographs reveals the following significant points:

(1) In all photographs, the first signal reaches the receiver with a speed of about 4900 m/sec. This can be identified with the low-frequency

group and phase speed $C_p(s_o)$ for the S_o mode. This signal is then a symmetric compressional wave traveling through the flanges and web from source to receiver. It is interesting to note that it is always relatively weak compared to the subsequent signal composed of propagation in other modes and reverberation.

(2) In all of the photographs for which the frequency was 170 kHz, there is a very clear, unambiguous arrival with a speed of about 3200 m/sec. This signal is particularly prominent in Table 4a. This would appear to be identifiable as a wave propagating in the a_o mode since the calculated group speed for this mode at this frequency is c_R and this value agrees very well with the values in Table 3 for freqeuncies between 150 and 200 kHz. Further supporting evidence is that the geometry of Table 4a is one which should emphasize propagation in the flexural wave modes and all higher s and a modes should be cut off.

(3) The photographs for which the frequency is 360 kHz reveal evidence of considerable multipathing and dispersion. This is plausible because at this frequency the a_1 mode can also be excited.

It must be understood that the identification of the "longitudinal thin bar speed of sound" and the flexural wave modes a_o and s_o is really only an approximation because the geometry of an I beam obviously deviates from that of an infinite thin plate or bar. However, the propagation in the I beam can be characterized as very similar to these modes, with the flexural modes far outweighing the longitudinal modes.

Source Position

Frequency = 170 kHz

Source to receiver (ft) = 4

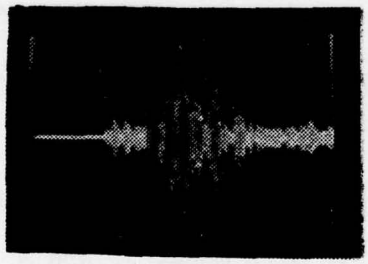
Horizontal scale (ms/div) = 0.2

Vertical scale (V/div) = 0.5

8

0.2

0.5



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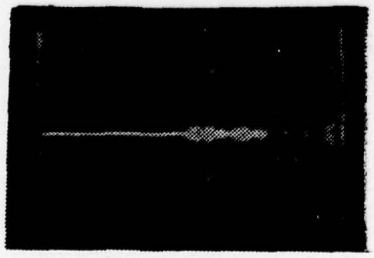

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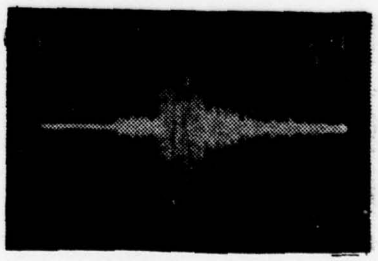

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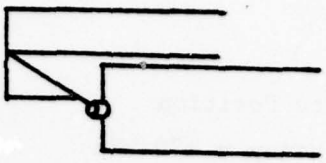
Source Position

Frequency = 170 kHz



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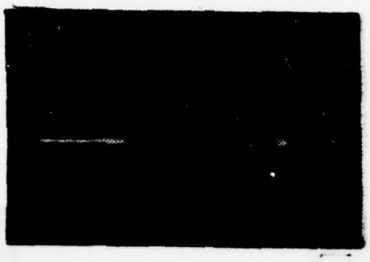

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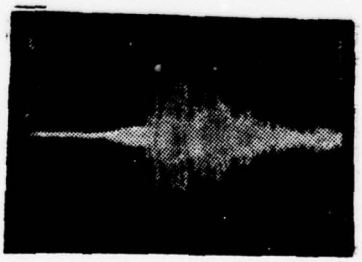
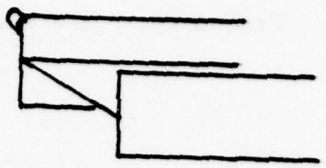


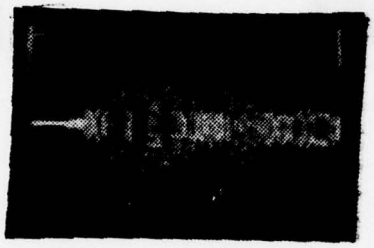
4b

Source Position

Frequency = 170 kHz



4
0.2
0.2



8
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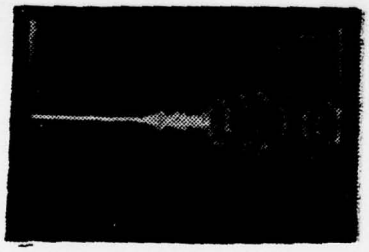
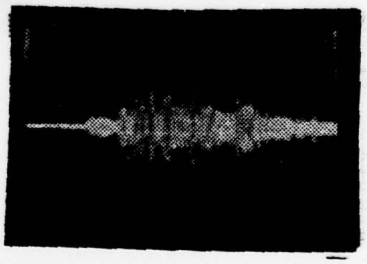
0.2

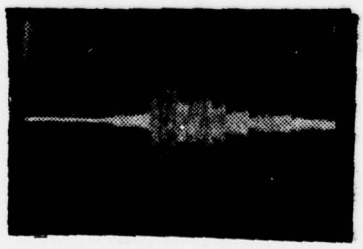
12
0.2
0.2



16
0.5
0.2



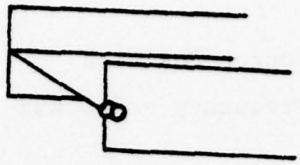
21
0.5
0.2



4c

Source Position

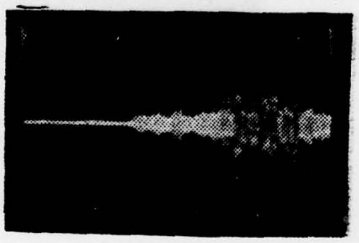
Frequency = 170 kHz



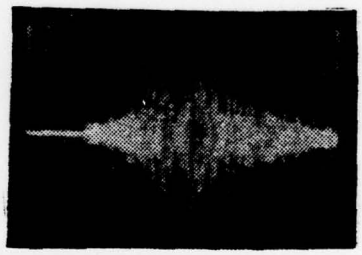
4
0.2
0.2

8
0.2
0.2

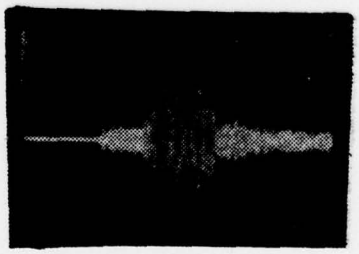
12
0.2
0.2



16
0.5
0.1



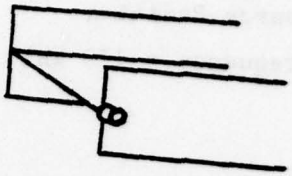
21
0.5
0.1



4d

Source Position

Frequency = 360 kHz



4

0.1

0.1

8

0.2

0.1

12

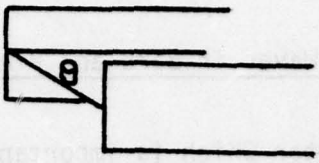
0.5

20

4e

Source Position

Frequency = 360 kHz



4

0.1

0.1

8

0.2

50

12

0.2

50

16

0.5

20

4f

Excitation of Waves in Bars and Plates

One parameter which is important in the transfer of power from a source of vibratory motion to the elastic waves in a plate or bar is the mechanical impedance at the driving point. Also important is the internal impedance of the source device.

$$\text{Mechanical impedance } Z_m = \frac{\text{Force}}{\text{Velocity}} = \frac{F}{U}$$

For simple harmonic motions, the power delivered by a Force $F = F_o e^{i\omega t}$ is $Wi = \frac{F_o \cdot U_o}{\sqrt{2}} \cos\theta$ where θ is the phase angle between F and U.

If $Z_m = R + i\,X = |Z_m| e^{i\theta}$

$$\text{Tan } \theta = \frac{X}{R} \qquad \text{Cos } \theta = \frac{R}{|Z_m|}$$

$$\text{So that } Wi = \frac{F_o^2 R}{\sqrt{2}\,|Z_m|^2} = \frac{U_o^2}{\sqrt{2}} R$$

Generally it is found that both R and X may vary with frequency and it is convenient to calculate expressions for power delivered by a constant force generator or a constant velocity generator, in analogy with electrical circuit practice.

The driving point impedance of a complex mechanical structure may vary rapidly with frequency in the neighborhood of resonances. It is possible to calculate resonance frequencies and the driving point impedances for only the most simple structures. It is useful to examine the results of calculations for the non resonant cases of an infinite beam or plate. The implication of infinite length is that not only is the length large compared to the wavelength but that is large enough to insure that waves reflected from the far end have a negligible effect on the driving point impedance.

For the case of flexural waves in an infinite beam, driven at a free end, there is no moment at this end and the result differs from the case for the driver located at the center of the infinite beam.

Ross (Ref 4 p. 137) gives the driving point impedance for flexural wave excitation at the forced end of an infinite bar,

$$Z_m = \frac{YI}{\omega} k_f \gamma \cdot \frac{k_f + i\gamma}{\frac{k_f}{\gamma} + \frac{\gamma}{k_f}}$$

Where ω = Angular frequency

Y = Young's modulus

I = Moment of inertia of beam cross section

k_F = Wave number for Flexural wave in beam

$\gamma = k_{F_o} \frac{v_F}{v_{F_\ell}} = k_{F_o} \sqrt{\sqrt{1+\delta^2} - \delta} = k_F \left(\sqrt{1+\delta^2} - \delta \right)$

k_{F_o} = Low Frequecy value of wave number

V_F = Phase speed of Flexural wave

V_{F_ℓ} = Low frequency value of phase speed

$\delta = \frac{\Gamma(1+\epsilon)}{2} \left(\frac{\omega}{\Omega} \right)$

Γ = Shear parameter = $\frac{Y}{K G}$

G = Shear Modulus

K = A constant which takes account of warping of cross section during bending, which depends on shape and Poisson's ratio, Normally $K < 1$

ϵ = Ratio of mass of entrained fluid to that of structure

$\Omega = c_\ell^2 \sqrt{\frac{\mu}{YI}} = \frac{c_\ell}{K} \sqrt{1+\epsilon}$ is a reference angular frequency

$C_e = \sqrt{Y/\rho}$ = long bar compressive wave speed

μ = Total mass per unit length of beam which is involved in the motion.

ρ = Volume density

At high frequencies, $\delta > 3$, and the reactive part of Z becomes very small and $V_F = V_{F_h} \simeq V_{F_\ell} / \sqrt{\Gamma \frac{\omega}{\Omega}}$

For a constant force source,

$$W_i \simeq \overline{F^2} / \mu v_f$$

where $\overline{F^2}$ is the mean square of the driving Force.

For a constant velocity source,

$$W_i = R \overline{U^2}$$
$$\simeq \frac{\mu v_f \overline{U^2}}{1 + (\gamma / k_f)^2}$$

Where $\overline{U^2}$ is the mean square velocity at the driven point.

For a source located at the midpoint of an infinite beam:

For a constant Force:

$$W_i = \frac{1 - \frac{\delta}{\sqrt{1+\delta^2}}}{4 \mu v_f} \overline{F^2}$$

for a constant velocity:

$$W_i = 2 \mu v_f \overline{U^2}$$

It appears that if the driving point is away from a free end, the impedance increases and the beam becomes increasingly resistant to absorption of power from a constant applied force as the frequency increases.

Impedance For A Thin Plate

The computation of an expression for the input impedance for a point force exciting flexural waves in a thin plate involves several types of Bessel's Functions and considerable manipulation, yet it results in a surprisingly simple formula. (Similar considerations for a thick plate may not be so simple).

Ross gives the result (P162)

$$Z \simeq 8\sqrt{\mu' B_p} = \frac{4}{\sqrt{3}}\rho_s C_p h^2 = \frac{4}{\sqrt{3}}\mu' C_p h$$

Where μ' = mass per unit area

$= \rho_s h$

ρ_s = volume density

h = plate thickness

C_p = longitudinal wave speed in the plate

$= \sqrt{Y/\rho_s(1-\sigma^2)}$

$B_p = \dfrac{Y k^2 S}{\sqrt{1-\sigma^2}}$ = Bending rigidity of plate per unit area

σ = Poisson's Ratio

S = Cross sectional area per unit width

k = radius of gyration of cross section

When resonances exist, there are maxima and minima in Z coinciding with resonances and anti-resonances. Ross quotes results from others indicating that geometric mean values in such cases agree well with this result for the characteristic impedance.

An example for a plate of thickness, $h = 1$ cm, $\rho_s = 7800\ kg/m^3$, $C_p = 5250\ m/sec$, $\mu' = 78\ kg/m^2$, gives $Z \simeq 9500\ N\cdot sec/m$

As a comparison, the order of magnitude of the characteristic impedance of a typical piezoelectic ceramic of 1 cm diameter.

C_ℓ ~ 5000 m/sec

ρ ~ 7000 kg/m^3

area $A = \frac{\pi\ 10^{-4}}{4}$ m^2

$A = \rho C_\ell A \simeq 2,700$ N·sec/m

It is seen that the order of magnitude of the characteristic driving point impedance for a thin steel plate is comparable to that of a piezoelectric ceramic vibrator having a diameter equal to the plate thickness. This suggests that the problem of obtaining some reasonable matching of impedances is not severe.

Transducer Fabrication

As part of the effort in this program, several electroacoustic transducers were designed, tested and delivered to the sponsor. Two each of four different models were furnished. The designs were intended to be most effective as sound sources. Different models had differenct resonance frequencies. The intent was to provide sources which radiated relatively efficiently at a number of frequencies in the range from about thirty kHz to two or three hundred kHz. These could facilitate measurements of transmission loss through structures and particularly help in assessing an optimum frequency. The following paragraphs provide a brief description of their construction and free resonance frequencies. All transducers used piezoelectric ceramic elements and the dominant mode of vibration was that of a longitudinal vibrator. The base of each transducer was designed to accept a threaded stud for clamping to the structure.

Although only a few resonance frequencies are listed, there were a number of higher frequencies at which resonances also occurred.

Transducer Mod 2

Ceramic: Gulton Mfg. Co. HS 21 Ceramic
Radially polarized cylinder
1/2" OD x 1/4" ID x 1 1/2" Long

Base: Brass Hexagonal Rod 5/8" x 5/8" Long

Joint: Epoxy Resin, Unreinforced

Blocked Capacitance: C_o = 3500 p F

Resonance Frequencies Noted: 26 kHz, 56 kHz

Transducer Mod 3

Ceramic: Channel Industries Channelite 5800

Axially polarized Cylinder

3/4" OD x 1/4" ID x 3/4" Long

Base: Brass, Round 3/4" OD x 3/4" Long

Joint: Epoxy Resin, Unreinforced.

Capacitance: C_o = 130 p F

Resonance Frequencies noted: 43, 75, 83, 95 kHz

Transducer Mod 4

Ceramic: Channelite 5800

Axially polarized Discs 1/2" OD x 1/8" ID x 1/4" Long

Two discs mounted end to end with reversed polarity -

Driven electrically in parallel

Base: Brass, Hexagonal 5/8" x 5/8" Long

Joint: Epoxy resin reinforced by compression screw

Capacitance: C_o = 400 p F

Resonance Frequencies Noted: 50, 75, 85 kHz

Transducer Mod 5

Ceramic: Channelite 5800 Disc Axially polarized 1" OD x 1/4" thick

Base: Brass, Round 1" OD x 1/2" Long

Joint: Epoxy Resin, Unreinforced

Capacitance: C_o = 800 p F

Resonance Frequencies Noted: 69.5, 74, 84.5 kHz

References

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3. Viktorov, I. A., Rayleigh And Lamb Waves, Plenum Press (1967).

4. Ross, D., Mechanics of Underwater Noise, Pergamon Press (1976).

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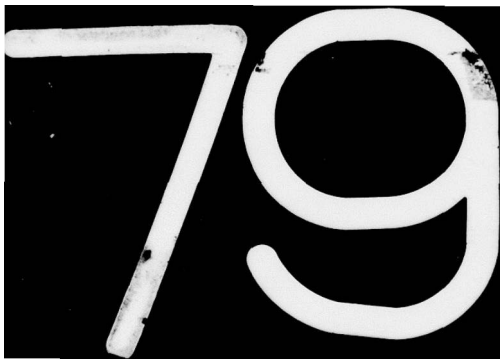
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