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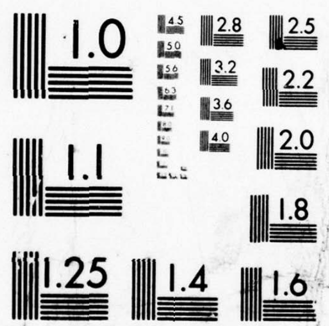


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# A HEURISTIC FOR MULTI-ECHELON MULTI-INDENTURED INVENTORY PROBLEMS

U.S. ARMY
INVENTORY
RESEARCH
OFFICE

December 1978



ROOM 800
U.S. CUSTOM HOUSE
2nd and Chestnut Streets
Philadelphia Pa. 19106

# A HEURISTIC FOR MULTI-ECHELON MULTI-INDENTURED INVENTORY PROBLEMS

TECHNICAL REPORT

BY

MEYER KOTKIN

DECEMBER 1978





US ARMY INVENTORY RESEARCH OFFICE
US ARMY LOGISTICS MANAGEMENT CENTER
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This report presents a heuristic for solving multi-echelon multi-indentured inventory problems. These problems generally have complex solution procedures that consume considerable amounts of computation time and the procedures are usually difficult to implement. The heuristic we present is easily implemented and provides near optimal solutions quickly. The heuristic has been used successfully on two echelon, two levels of indenture inventory problems. It is easily extendable to more levels of indenture and research is now being done to determine if the heuristic can also be extended to more echelons.

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Part of this research was performed while the author was at the US Army Tank-Automotive Materiel Readiness Command, Warren, MI.

## TABLE OF CONTENTS

## 1. INTRODUCTION

In 1973, Muckstadt [2] extended Sherbrooke's well known METRIC model [3] to explicitly account for multi-indentured items. By a reparable multi-indentured item we mean a reparable module that contains reparable components. The components themselves may contain reparable subcomponents, etc. Much of the Army's complex new equipment is designed with these indenture levels. The basic idea behind indentured items is that when a module fails, a failed component may be quickly removed and replaced with a serviceable component. Thus, the actual downtime of the module, that is, the time the module is not in a serviceable condition, may be less than if repair had to be done on the whole module. Muckstadt's MODMETRIC model leads to a complex solution technique that consumes a considerable amount of computer time. In this report we present a heuristic for reducing the time to solve MODMETRIC while yielding close to optimal solutions.

Section 2 presents a brief review of the MODMETRIC formulation while in Section 3 we discuss the solution procedure in more depth. Sections 4 and 5 discuss the heuristic we propose and other possible heuristics.

In this report we consider a two echelon system as in Figure 1.

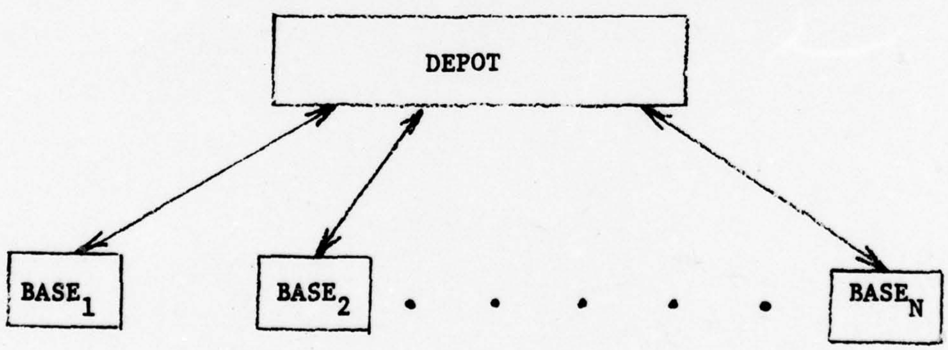
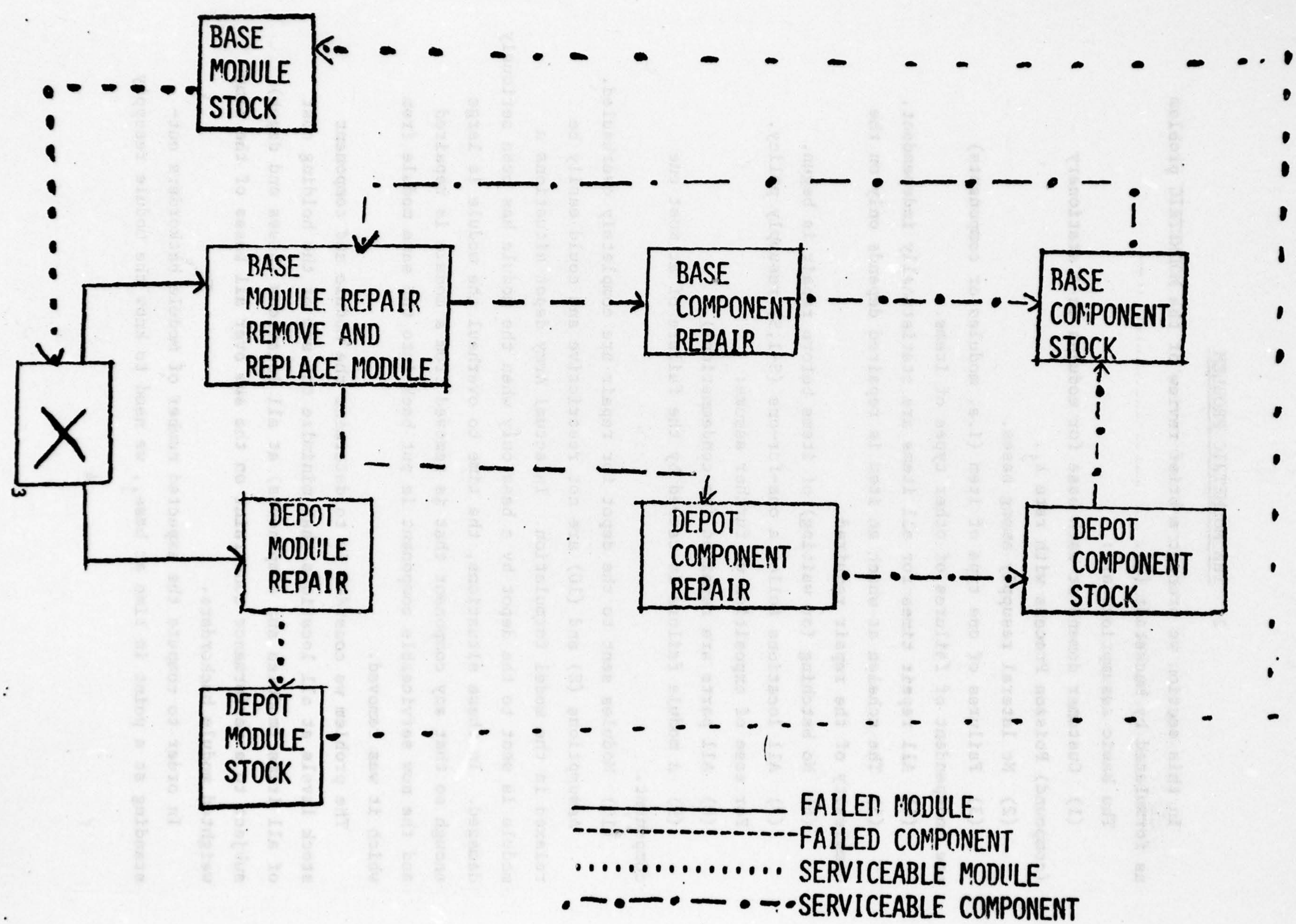



FIGURE 1

There are N bases and a depot. The dynamics of a multi-echelon multi-indentured system are depicted in Figure 2. We will consider a module with M components. When referring to the module and its components a 0 subscript will refer to the module; the components will be numbered 1, 2,...,M.


BASE MODULE STOCK
BASE MODULE REPAIR REMOVE AND REPLACE MODULE
BASE COMPONENT REPAIR
BASE COMPONENT STOCK
DEPOT MODULE REPAIR
DEPOT COMPONENT REPAIR
DEPOT COMPONENT STOCK
DEPOT MODULE STOCK
X
3
FAILED MODULE
FAILED COMPONENT
SERVICEABLE MODULE
SERVICEABLE COMPONENT

## 2. THE MODMETRIC PROBLEM

In this section we present a brief review of the MODMETRIC problem as formulated by Muckstadt [2].

The basic assumptions are:

(1) Customer demand at each base for modules is a stationary (compound) Poisson Process with rate $\lambda_j$.

(2) No lateral resupply among bases.

(3) Failures of one type of item (i.e. modules or components) are independent of failures of other types of items.

(4) All repair times for all items are statistically independent.

(5) The echelon at which an item is repaired depends only on the complexity of the repair required.

(6) No batching (or waiting) of items before repair is begun.

(7) All locations follow a one-for-one (S-1,S) resupply policy.

For ease of exposition we further assume:

(8) All parts are repaired (no condemnations).

(9) A module failure is caused by the failure of at most one component.

(10) Modules sent to the depot for repair are completely overhauled.

Assumptions (8) and (10) are not restrictive and could easily be relaxed in the model formulation. In actual Army depot situations a module is sent to the depot by a base only when the module has been seriously damaged. In these situations, the time to overhaul the module is large enough so that any component that is removed from a module is repaired and the now serviceable component is put back into the same module from which it was removed.

The problem we consider is to determine the module and component stock levels at all locations that minimize the sum of the holding cost of all items (modules and components) at all locations (bases and depot) subject to a performance constraint on the sum over all bases of the time weighted module backorders.

In order to compute the expected number of module backorders outstanding at a point in time at $\text{base}_j$, we need to know the module resupply time at $base_j$. It is through the expected module resupply time that MODMETRIC explicitly accounts for the effect of the components on module performance.

Let $T_j$ = expected module resupply time at $base_j$.

Then

(1) $$T_j = r_j(b_j + Z_j) + (1 - r_j)(O_j + D_j)$$

Where

$r_j$ = proportion of module failures repaired at $base_j$

$b_j$ = mean $base_j$ fault isolation and component remove and replace time

$Z_j$ = expected delay in module repair at $base_j$ due to unavailable serviceable components

$O_j$ = mean order and ship time between $base_j$ and depot

$D_j$ = expected delay at depot due to unavailable modules

Let $S_{kj}$ = spare stock level of item k at location j; k=0, 1,...,M (0 = module); j = 0,1,...,N (0 = depot)

$\lambda_j$ = expected daily module demand at $base_j$

$\lambda_o$ = expected daily module demand at depot

$$= \sum_{j=1}^{N} \lambda_j(1 - r_j)$$

$F_k$ = mean depot repair time for item k; k = 0, 1,...,M (0 = module)

The expected delay at the depot from the time $base_j$ places a module resupply request on the depot until a serviceable module is available for shipment to $base_j$ is given by

$$D_j = \frac{\text{expected number of module backorders outstanding at a point in time at depot}}{\text{expected daily demand for modules at depot}}$$

Given a depot module stock level of $S_{oo}$ the expected module backorders outstanding at the depot at a point in time is given by

(2) $$\sum_{x > S_{oo}} (x - S_{oo})\, p(x; \lambda_o F_o)$$

where $p(x;u_j)$ = probability that x units are in resupply at location j given a mean resupply time.

Hence,

(3) $$D_j = \frac{\sum_{x > S_{oo}} (x - S_{oo})\, p(x; \lambda_o F_o)}{\lambda_o}$$

$D_j$ reflects the interaction of the two echelon supply system on module performance at $base_j$. Similarly $Z_j$ reflects the effect of the components on module performance at $base_j$. We now show how $Z_j$ may be calculated.

Let $P_k$ = Probability that a module failure was caused by a failure of component k k=1,...,M.

$g_{kj}$ = Expected delay in $base_j$ module repair given that the module failed due to a failure of component k.

$\lambda_{kj}$ = Daily demand for component k at location j k=1,...,M j=0,...,N

Then, clearly

(4) $$Z_j = \sum_{k=1}^{M} g_{kj} P_k$$

Since module failures that require $base_j$ repair follow a Poisson Process with rate $r_j \lambda_j$ then component k failures follow a Poisson Process with rate $\lambda_{kj} = P_k \lambda_j r_j$

Hence,

(5) $$P_k = \frac{\lambda_{kj}}{r_j \lambda_j}$$

To calculate $g_{kj}$ we need to know the $base_j$ resupply time for component k. If $T_{kj}$ = $base_j$ resupply time for component k, then similar to (1).

(6) $$T_{kj} = r_{kj} b_{kj} + (1 - r_{kj}) (O_{kj} + D_{kj})$$

where

$r_{kj}$ = Proportion of component k failures repaired at $base_j$.

$b_{kj}$ = Mean component k repair time at $base_j$.

$O_{kj}$ = Component k order and ship time between depot and $base_j$.

$D_{kj}$ = Delay at depot due to unavailable serviceable component k stock.

Given a depot spare stock level of $S_{ko}$ of component k, we have, as before, that

(7) $$D_{kj} = \frac{\sum_{x > S_{ko}} (x - S_{ko})\, p(x; \lambda_{ko} F_k)}{\lambda_{ko}}$$

Hence, all the terms in (6) have been calculated and similar to (3) we see

(8) $$g_{kj} = \frac{\sum_{x > S_{kj}} (x - S_{kj})\, p(x; \lambda_{kj} T_{kj})}{\lambda_{kj}}$$

Substituting (5) and (8) into (4) we have finally that

(9) $$Z_j = \sum_{k=1}^{M} \frac{\sum_{x > S_{kj}} (x - S_{kj})\, p(x; \lambda_{kj} T_{kj})}{r_j \lambda_j}$$

With this, all the terms in the expected $base_j$ module resupply time equation (1) have been calculated.

In summary then, given the system parameters and module and component stock levels at all locations, to calculate the $\text{base}_j$ module resupply time, we:

(a) calculate the delay at depot due to unavailable components of type k; $k = 1,\ldots,M$.

(b) calculate the component resupply time at $\text{base}_j$ for each component.

(c) calculate expected component backorders at $\text{base}_j$ and then the delay in $\text{base}_j$ module repair due to unavailable components.

(d) determine expected delay at depot due to unavailable modules.

(e) calculate base module resupply time.

It should be clear from the above discussion that to explicitly model the component - module relationship leads to some rather complex expressions and a difficult optimization problem. In the next section we explore solution procedures for the optimization problem in more depth.

## 3. OPTIMIZATION PROBLEM

The MODMETRIC Optimization Problem can now be formulated as

$$\text{(10)} \quad \text{Min} \sum_{j=0}^{N} C_H^0 S_{0j} + \sum_{k=1}^{M} C_H^k S_{kj}$$

$$\text{S.T.} \sum_{j=1}^{N} \sum_{x > S_{0j}} (x - S_{0j}) \, p(x; \lambda_j \, T_j) \leq \gamma_M$$

Where $\underline{S} = (S_{00}, S_{01}, \ldots, S_{0N}; S_{10}, \ldots, S_{1N}, S_{20}, \ldots, S_{MN})$ is a vector of non-negative integers, $\gamma_M$ is a specified module performance level and $C_H^k$ is the unit price for item k, k=0,...,M.

Problem (10) is not convex and optimization is difficult due to the complex interactions between echelons and between modules and components as expressed in the resupply time equations. Kotkin [1] has developed bounds on the optimal module and component stock levels (and hence bounds on the optimal module and component budget expenditures). Shay and O'Malley [4] have developed a solution procedure that is not as complex as the procedure suggested by Muckstadt. Even so, no relatively easy solution to (10) is known.

Muckstadt [2] suggests partitioning (10) into 2 subproblems:

$$\text{(11a)} \quad \text{Min} \sum_{j=0}^{N} \sum_{k=1}^{M} C_H^k S_{kj}$$

$$\text{S.T.} \sum_{j=1}^{N} \sum_{k=1}^{M} \sum_{x > S_{kj}} (x - S_{kj}) \, p(x; \lambda_{kj} T_{kj}) \leq \gamma_C$$

Then, after obtaining the solution to (11a) we solve:

$$\text{(11b)} \quad \text{Min} \sum_{j=0}^{N} C_H^0 S_{0j}$$

$$\text{S.T.} \sum_{j=1}^{N} \sum_{x > S_{0j}} (x - S_{0j}) \, p(x; \lambda_j \overline{T}_j) \leq \gamma_M$$

where $\bar{T}$ is the module resupply time at base$_j$ given the component stock levels determined in (11a). Note the variables in (11a) are the component stock levels while in (11b) the variables are the module stock levels. Note that (11a) and (11b) are problems of the METRIC [3] type. Muckstadt solved the dual problems to (11a) and (11b) (the dual problems being to minimize the sum over all bases of the expected item backorders outstanding at a point in time subject to a budget constraint) and suggested the following procedure for solving problem (10). Set lower and upper bounds on component investment and a component budget increment. Then for a fixed total budget solve the dual of (11a) with the component budget set at its lower bound and then solve the dual of (11b) with the module budget = total budget - component budget. Next, increment the component budget and repeat the procedure. The approximate solution to (10) is then the best mix of component and module budgets determined by this procedure.

The procedure Muckstadt suggests for solving (10) can involve solving many subproblems of the type (11a) and (11b). This procedure may consume a considerable amount of computer time and leads to long, complex computer programs.

The purpose of this report is to present a heuristic procedure for solving (10) that involves solving subproblems (11a) and (11b) only once each. This savings in solution complexity and running time is especially desirable since in most practical situations a tradeoff curve of budget versus performance is desired rather than a solution to (10) for a particular performance level $\gamma_M$. To construct this trade off curve involves solving (10) with various values for $\gamma_M$ and thus the need for a good heuristic for solving (10) is apparent.

By introducing a generalized Lagrange multiplier $B_C$ in (11a) and multiplier $B_M$ in (11b) these problems can be rewritten as:

$$\text{(12a)} \qquad \text{Min} \sum_{j=1}^{N} \sum_{k=1}^{M} [C_H^k S_{kj} + B_C \sum_{x > S_{kj}} (x - S_{kj})\, p(x;\lambda_{kj} T_{kj})] + \sum_{k=1}^{M} C_H^k S_{ko}$$

and

(12b) $$\text{Min} \sum_{j=1}^{N} [C_H^O S_{0j} + B_M \sum_{x>S_{oj}} (x - S_{0j})\, p(x;\lambda_j \bar{T}_j)] + C_H^O S_{oo}$$

Note (12a) is separable by component. Furthermore, the multiplier $B_M$ in (12b) is the same as the multiplier we might use in (10) if we were to try and solve (10) directly. By specifying $B_M$ in (10) (and hence in (12b)) we are implicitly specifying a target module performance value $\gamma_M$. We then ask if given the multiplier $B_M$ can we a priori determine the "optimal" multiplier value $B_C^*$ in (12a)? If this can be done then solving problems (12a) and (12b) once each will yield a satisfactory solution to (10). The heuristic we present will be an attempt to find the "optimal" $B_C^*$ given $B_M$.

The entire approximation procedure that we employ will be:

a. Set Module backorder penalty, $B_M$, in (10).

b. Set Component backorder penalty, $B_C$.

c. Use $B_C$ in (12a) to determine component stock levels.

d. Adjust module resupply time to reflect component delays.

e. Use $B_M$ in (12b) with adjusted module resupply time to determine optimal module stock levels.

f. Return to step 1 and adjust $B_M$ to achieve the desired target module performance.

This paper focuses on a heuristic for determining $B_C$ in step b. above.

## 4. HEURISTICS

In the next section we present a heuristic for determining $B_C^*$. Before proceeding with this heuristic we will examine two other possible heuristics for $B_C^*$.

An apparent heuristic would be to set $B_C = B_M$ i.e. charge the system as much for a component backorder as we do for a module backorder. This would imply that a component backorder is considered as bad as a module backorder. The constraint in (10) is a constraint on module backorders. But not every component backorder is causing a module backorder. If a component is backordered this means a module is waiting to be repaired. It does not necessarily mean that the waiting module is backordered. So a component backorder may only mean repair on a module is being delayed and in this case we would not want to charge the same penalty as we do on a module backorder. Hence, it would seem that $B_M$ is an upper bound on the component backorder cost. By charging too high a component backorder penalty in (12a) we would be investing more money in components than we would normally do in an optimal solution to (10), thereby causing an under investment in modules.

Note that since $B_C = B_M$ in this heuristic (called the $B_M$ heuristic), as $B_M$ increases the heuristic will increase $B_C$ and thus increase the component budget as well as the module budget to meet higher module performance targets.

Another possible heuristic would be to set $B_C$ = module unit price (MUP = $C_H^O$). Since a component backorder means a module is waiting to be repaired and hence is not in a serviceable condition, it may be reasonable to charge the system the unit price of a module that is no longer available to it. This approach however, also does not differentiate between the possible effects of component backorders. Contrary to the $B_M$ heuristic, the module unit price heuristic (MUP heuristic) says component backorders should only be charged for delaying repair of modules and not for causing module backorders. It seems reasonable (and we shall show this later) that the MUP should be a lower bound on the true value of the component backorder penalty cost. Underestimating $B_C^*$ results in an underinvestment in components thereby causing an overinvestment in modules.

Note that in the MUP heuristic, since the module unit price does not change, component stock levels do not change for different values of $B_M$. Although this heuristic is extremely easy to implement (since it involves solving (12a) only once and these component levels are the stock levels for all values of $B_M$) it seems reasonable to expect that the component levels should vary for different values of the module backorder penalty cost.

Figure 1 through Figure 4 illustrates how these heuristics compare with Muckstadt's solution procedure. In these graphs, we have plotted expected backorders versus total budget.

The four curves compare the $B_M$ and MUP solution heuristics with Muckstadt's MODMETRIC. From these figures we can make the following observations:

a. For low ready reates (ready rate is the percentage of time no module backorders are outstanding) both the $B_M$ and MUP heuristics work well. (The numbers next to the MODMETRIC points indicate the ready rate).

b. As the ready rate increases the $B_M$ heuristic begins to do better than the MUP heuristic. (Figures 1 and 2).

c. As the ready rate increases even further, the MUP heuristic begins to do considerably better than the $B_M$ heuristic. However, both are generally inferior to the MODMETRIC solution obtained by Muckstadt's procedure (Figures 3 and 4).

Since in most practical cases we are interested in ready rates in the 90 - 100% range, Figures 3 and 4 imply that for higher ready rates it is better to overinvest in modules than to underinvest in modules. This corresponds to an empirical observation made by Muckstadt [2] in his original MODMETRIC paper.

To gain some insights into the observations we shall explore (10), (12a) and (12b) in more depth.

<u>Claim</u>: If the module stock is constrained to be zero at all bases, then $B_C^* = B_M$ for all $B_M$.

<u>Proof</u>: By introducing the multiplier $B_M$ in (10) we get the new problem.

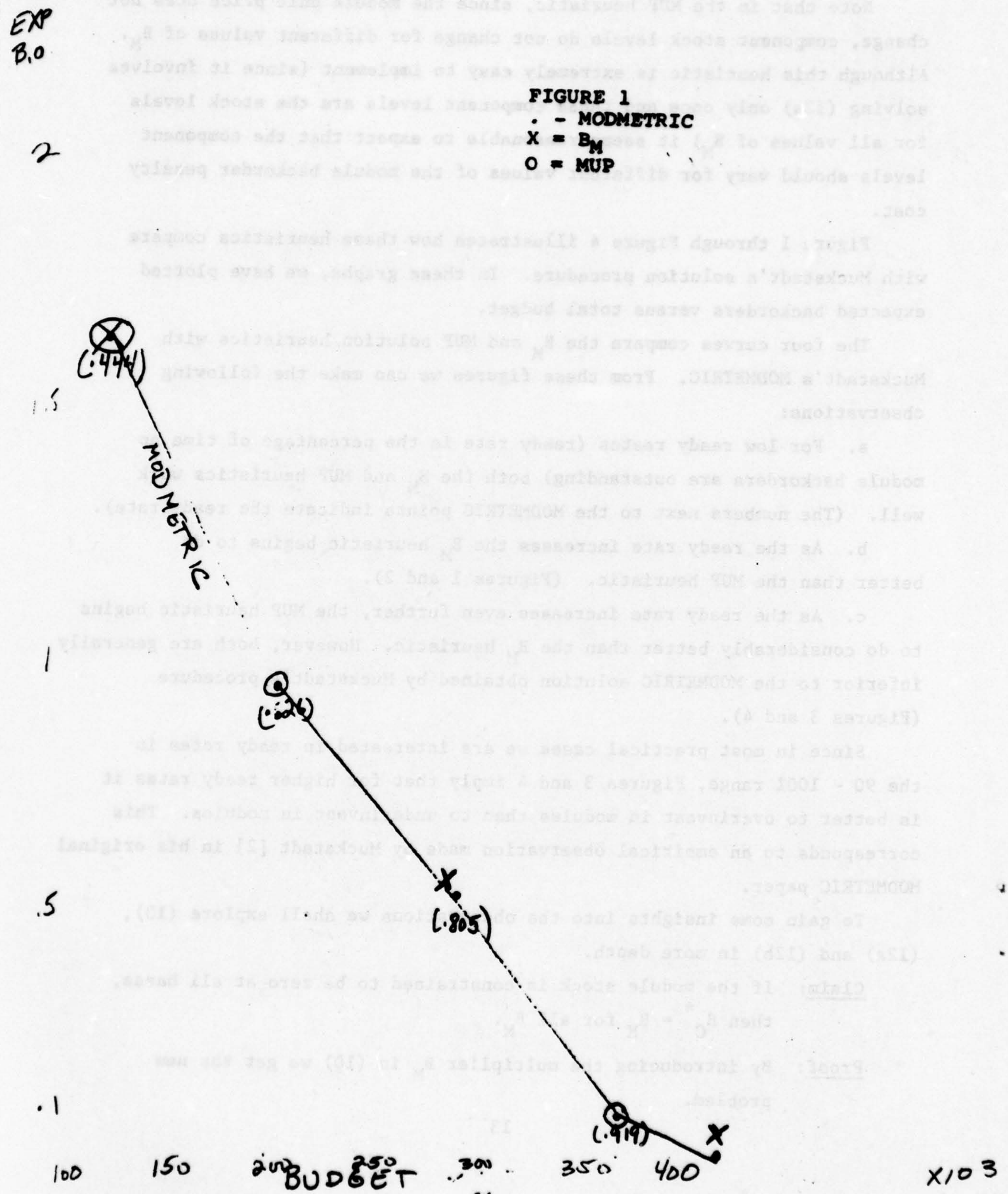


FIGURE 1
. - MODMETRIC
X = $B_M$
O = MUP
EXP B.O
2
1.5
1
.5
.1
(.444)
(.626)
(.805)
(.919)
MODMETRIC
100
150
200
250
300
350
400
X10 3
BUDGET

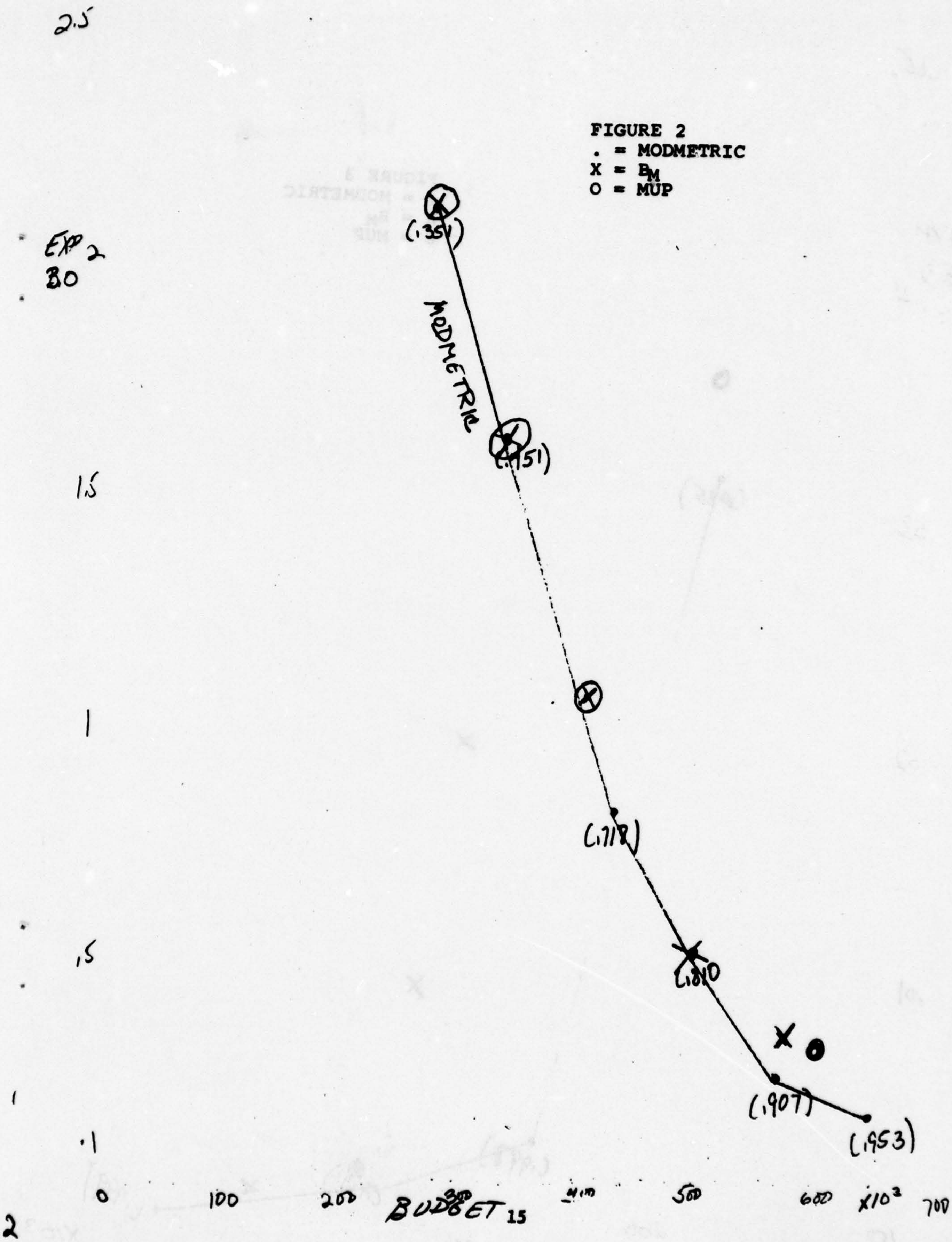


FIGURE 2
. = MODMETRIC
X = $B_M$
O = MUP
EXP 2
BO
MODMETRIC
(.351)
(.751)
(.718)
(.810)
(.907)
(.953)
2.5
1.5
1
.5
.1
2
0
100
200
300
400
500
600
700
X10³
BUDGET 15

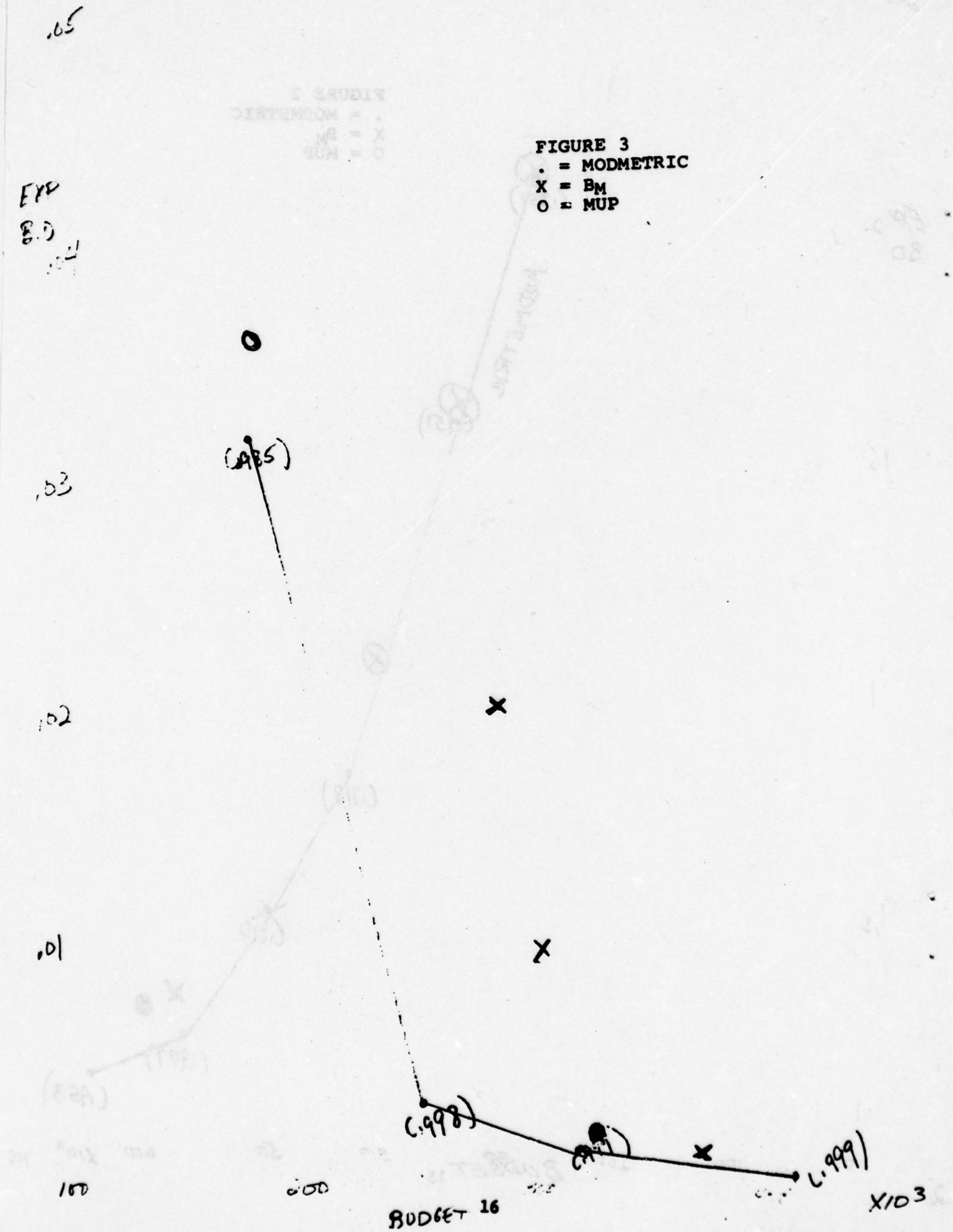


FIGURE 3
. = MODMETRIC
X = B_M
O = MUP
.05
EXP
.04
.03
.02
.01
(.985)
(.998)
(.999)
100
200
BUDGET 16
X10^3

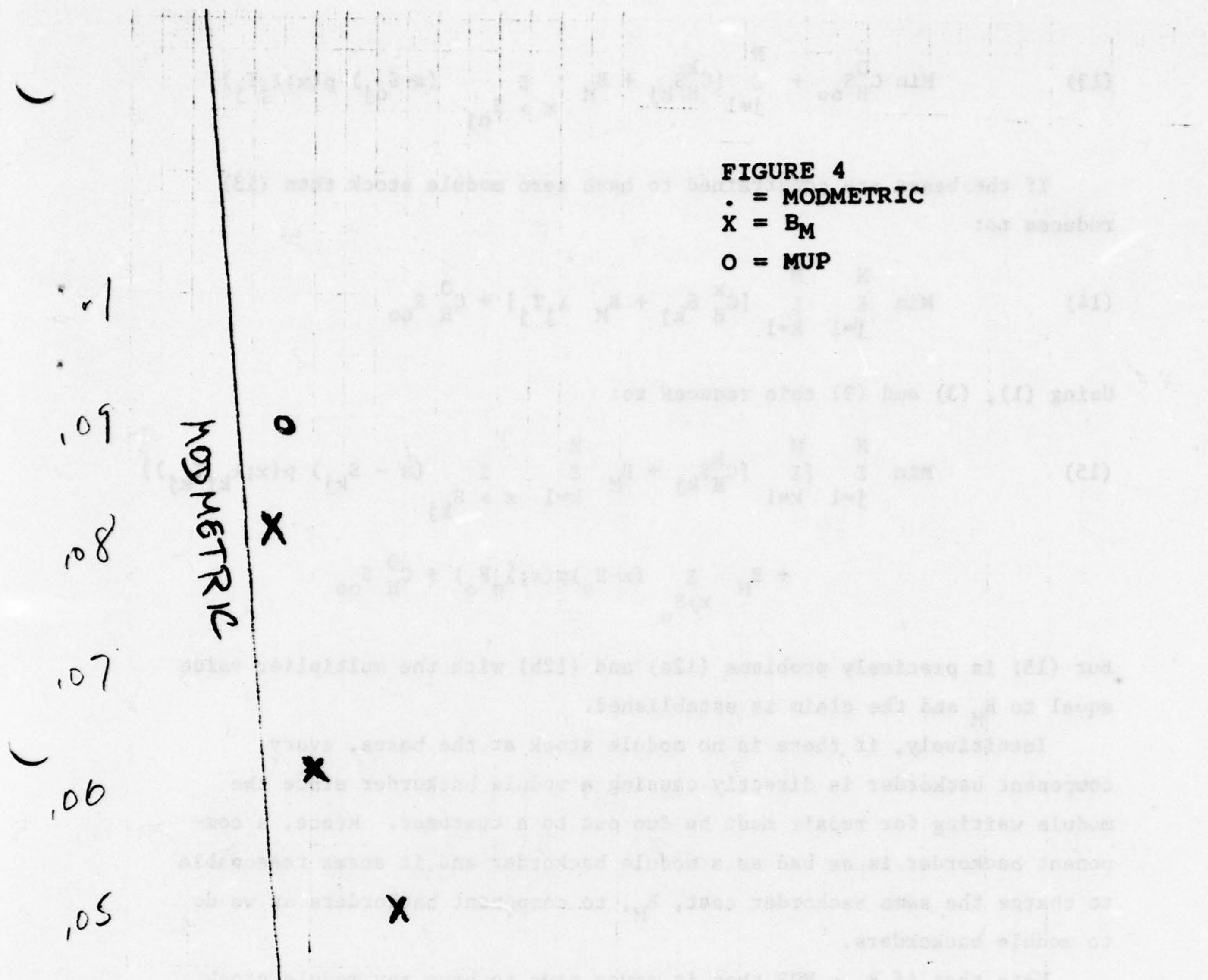


FIGURE 4
. = MODMETRIC
X = $B_M$
O = MUP
MODMETRIC
.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
(.981)
.955
(.983)
(.995)
.998
600
700
800
900
1000
1100
1200
1300
x10³
BUDGET





4

$$\text{(13)} \qquad \text{Min } C_H^O S_{oo} + \sum_{j=1}^{N} [C_H^k S_{kj} + B_M \sum_{x > S_{oj}} (x-S_{oj})\, p(x;\lambda_j T_j)]$$

If the bases are constrained to have zero module stock then (13) reduces to:

$$\text{(14)} \qquad \text{Min } \sum_{j=1}^{N} \sum_{k=1}^{M} [C_H^k S_{kj} + B_M \lambda_j T_j] + C_H^O S_{oo}$$

Using (1), (3) and (9) this reduces to:

$$\text{(15)} \qquad \text{Min } \sum_{j=1}^{N} [\sum_{k=1}^{M} [C_H^k S_{kj} + B_M \sum_{k=1}^{M} \sum_{x > S_{kj}} (x - S_{kj})\, p(x;\lambda_{kj} T_{kj})]$$

$$+ B_M \sum_{x>S_o} (x-S_o) p(x;\lambda_o F_o) + C_H^O S_{oo}$$

But (15) is precisely problems (12a) and (12b) with the multiplier value equal to $B_M$ and the claim is established.

Intuitively, if there is no module stock at the bases, every component backorder is directly causing a module backorder since the module waiting for repair must be due out to a customer. Hence, a component backorder is as bad as a module backorder and it seems reasonable to charge the same backorder cost, $B_M$, to component backorders as we do to module backorders.

Note that if $B_M \leq$ MUP then it never pays to have any module stock since it costs less to have one unit year of module backorders than to hold the module for one year. Hence, to force base module stock to be zero, set $B_M$ = MUP. Unless the module has a low demand rate, zero module stock levels at the bases will result in a relatively low ready rate. Hence, for low ready rates we would expect to see that the two heuristics agree with each other and with MODMETRIC.

The above observations help to illustrate why MUP is a practical lower bound on $B_C^*$. In most problems of interest $B_M \geq$ MUP since we desire the system to have spare modules, and since we would expect the component backorder cost to increase as the module backorder cost increases, MUP should be a lower bound on $B_C^*$.

As $B_M$ is increased above MUP and we start to put module stock at the bases, the two heuristics begin to give different answers. Initially the overinvestment in components by the $B_M$ heuristic is less damaging than the underinvestment in components by the MUP heuristic. This is probably due to the fact that for low ready rates, achieving a module performance target can be done by investing in either modules or components. However, for higher and higher ready rate targets we would expect modules to play a more significant factor in achieving the ready rate target. Hence, the MUP heuristic which buys more modules than the $B_M$ heuristic will tend to do better for higher ready rates.

A procedure for solving (10) similar to the Muckstadt solution procedure would be to specify a multiplier increment $d_c$. Then, for a particular value of $B_M$, solve problems (12a) and (12b) with $B_C$ = MUP, $B_C$ = MUP + $d_c$, $B_C$ = MUP + $2d_c$,........$B_C = B_M$. The solution would then be the best of these trial solutions. Here again, however, many solutions of (12a) and (12b) are required for a particular value of $\gamma_M$. The next heuristic we present will attempt to take advantage of the knowledge gained from the MUP and $B_M$ heuristics.

## 5. THE FINAL HEURISTIC

Previously, we mentioned that specifying $B_M$ in (10) and (12b) implicitly specifies a module performance target. Particularly, regardless of the component levels, specifying $B_M$ establishes a lower bound

$$(16) \qquad \underline{A} = 1 - \frac{MUP}{B_M}$$

on the module ready rate at a base. To see this, note that if the component levels are fixed at all locations and the depot module stock is fixed, then (10) (or 12b) reduces to

$$(17) \qquad \text{Min} = \sum_{j=1}^{N} [C_H^O S_{oj} + B_M \sum_{x > S_{oj}} (x - S_{oj}) \, p(x;\lambda_j \bar{T}_j)]$$

where $\bar{T}$ reflects the fixed component and depot module levels. (17) is separable by base and for each base (17) is convex in $S_{oj}$ (This is well known and a proof can be found in Sherbrooke [3]). The problem for base j is

$$(18) \qquad \text{Min } Q_j(S_{oj}) = C_H^O S_{oj} + B_M \sum_{x > S_{oj}} (x - S_{oj}) \, p(x;\lambda_j \bar{T}_j)$$

The optimal base j module stock, $S_{oj}*$, must satisfy the optimality conditions:

(a) $Q_j (S_{oj}*) - Q_j (S_{oj}* - 1) \leq 0$

(19) (b) $Q(S_{oj}* + 1) - Q_j (S_{oj}*) \geq 0$

(c) or $S_{oj}* = 0$

Using these conditions we see that $S_{oj}*$ must satisfy

$$(20) \qquad \underline{P} (S_{oj}* + 1;\lambda_j \bar{T}_j) \leq \frac{C_H^O}{B_M} = \frac{MUP}{B_M} \leq \underline{P} (S_{oj}*;\lambda_j \bar{T}_j)$$

where

$$\underline{P}\,(S_{oj}^{*};\mu) = \sum_{k=S_{oj}^{*}}^{\infty} p(k;\mu)$$

$\underline{P}\,(S_{oj}^{*} + 1)$ is the probability that $(S_{oj}^{*} + 1)$ or more modules are in resupply to base j. But this is just the probability that there is at least one module backorder at base j and hence the ready rate = 1 - probability of one or more module backorders, is bounded below by (16).

Given that a component is backordered, it is possible that a module backorder exists as well. If this is the case it seems reasonable to charge $B_M$ for the component backorder. If there is no module backordered then we charge MUP for the component backorder. However, $\underline{A}$ in (16) gives an approximation to the percentage of time that there will be no module backorders and hence $1 - \underline{A}$ is the proportion of time there are module backorders. Combining the above observations, the heuristic we propose is to set

$$\text{(21)} \qquad B_C^* = \underline{A}\,(MUP) + (1 - \underline{A})\,(B_M)$$

$$= \left(1 - \frac{MUP}{B_M}\right) MUP + \left(\frac{MUP}{B_M}\right) B_M$$

In summary, the heuristic says that if on a day on which there is a component backorder there are also modules backordered, charge $B_M$ per component backorder day. Otherwise charge MUP per component backorder day. $\underline{A}$ underestimates the true ready rate since $\underline{A}$ is only a target and it is usually exceeded (see equation (20)). However, the fact that there is a component backordered affects (lowers) the probability that there are no modules backordered at the same time. Hence, we use $\underline{A}$ as an approximation to the probability of no module backorders.

The heuristic (21) has several desirable properties. From (21) we note that:

(a) MUP $\leq B_C^* < B_M$

(b) $B_C^*$ increases as $B_M$ increases

(c) As $B_M$ goes to infinity, $B_C^*$ goes to 2 MUP

Properties (a) and (b) are desirable in light of the discussion of the previous section. There seems to be no intuitive significance to the value 2 MUP. As $B_M$ increases, $B_C^*$ approaches this value of 2 MUP. Hence, for large values of $B_M$, $B_C^*$ at worst underestimates the true optimal component backorder cost. As mentioned previously, for higher ready rates it is better to underinvest in components than to overinvest.

Figures 5, 6 and 7 compare the heuristic and MODMETRIC results. These are again plots of expected backorders versus total budget. Tables 1, 2 and 3 give the corresponding numerical values for the points plotted in Figures 5, 6 and 7 respectively. From the graphs and tables we note that:

(a) the heuristic generates some but not all of the MODMETRIC points.

(b) when the heuristic generates points that are not MODMETRIC points, the heuristic points lie on or near the extended MODMETRIC curve. This extended curve is simply a straight line completion of the MODMETRIC points.

(c) most of the later points generated by the heuristic are obtained by increasing the module stock levels and not the component levels. This is because all $B_C^*$ for these later points are nearly equal to 2 MUP.

(d) for very high ready rates, the heuristic underinvests in components and hence overinvests in modules.

When the heuristic and MODMETRIC points agree, both the component and module stock levels at all locations agree. In principle, Muckstadt's MODMETRIC solution procedure will generate a solution for any budget value. Many of the heuristics points could be generated by the MODMETRIC algorithm if in the search algorithm the component budget increment is made small enough. In general, however, the heuristic will generate fewer points than

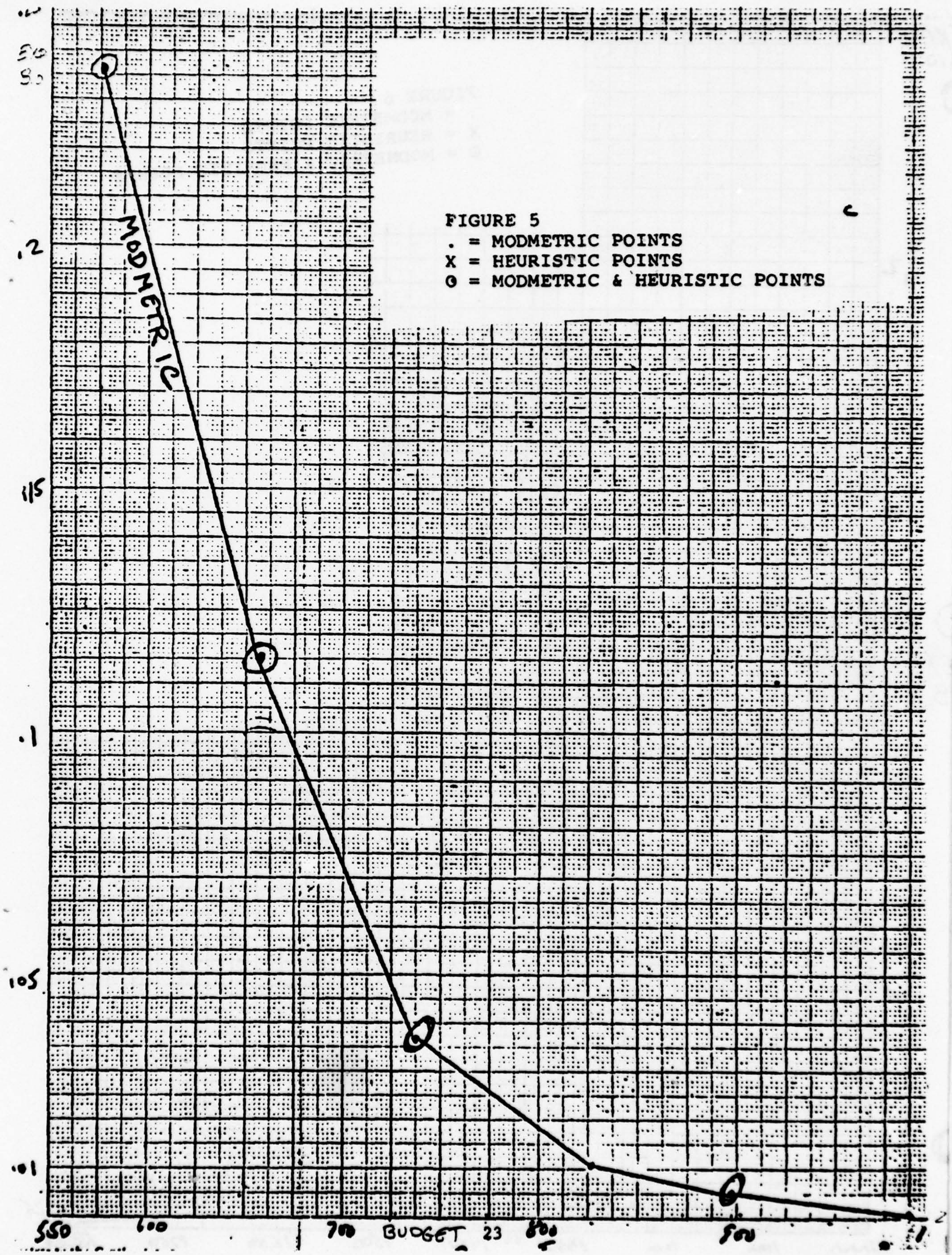


FIGURE 5
. = MODMETRIC POINTS
X = HEURISTIC POINTS
⊙ = MODMETRIC & HEURISTIC POINTS
MODMETRIC
.2
.15
.1
.05
.01
550
600
700
BUDGET 23
800
900

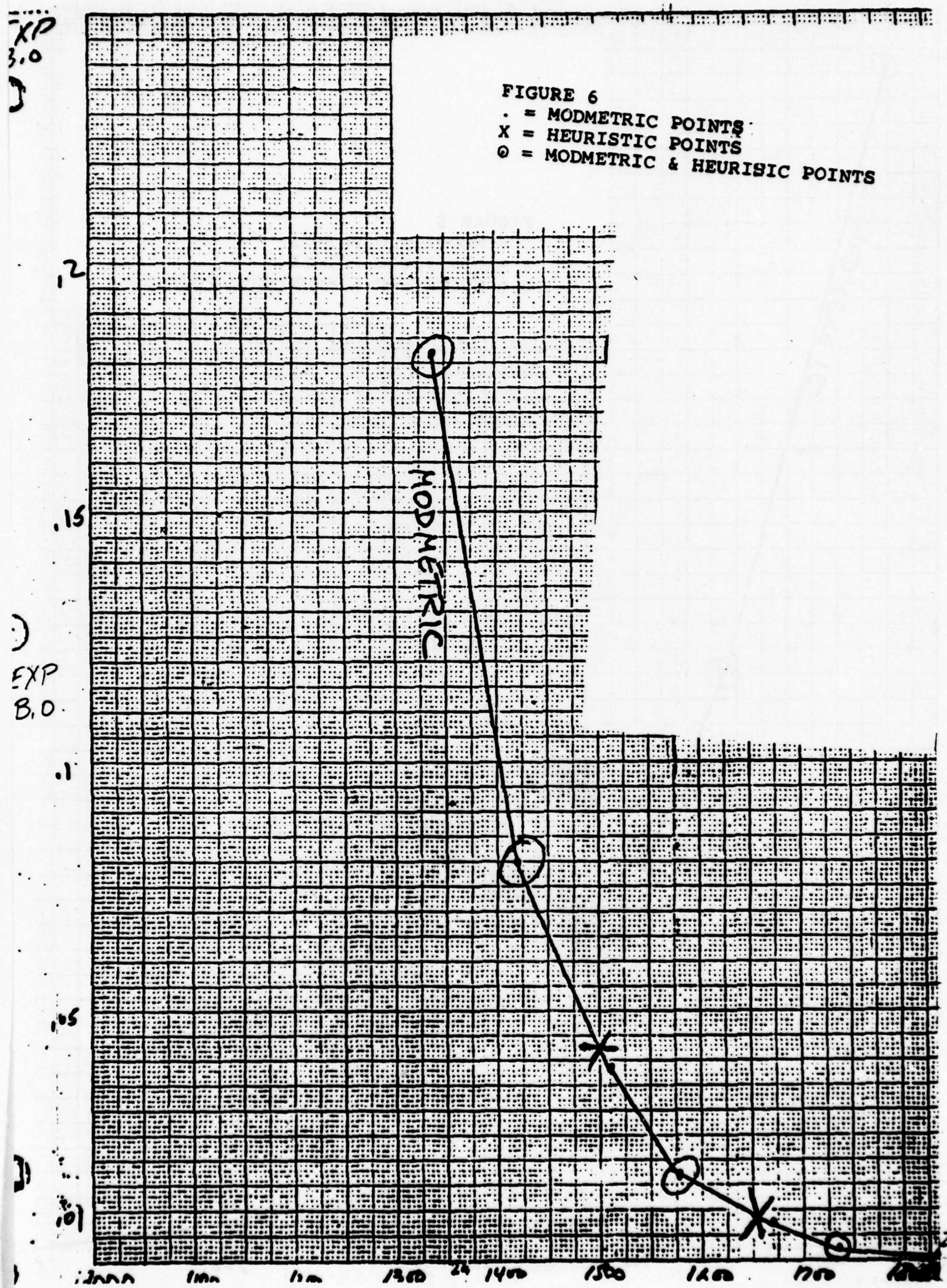


FIGURE 6
. = MODMETRIC POINTS
X = HEURISTIC POINTS
⊙ = MODMETRIC & HEURISIC POINTS
MODMETRIC
EXP
B.0
.2
.15
.1
.05
1300
1400
1500
1600
1700

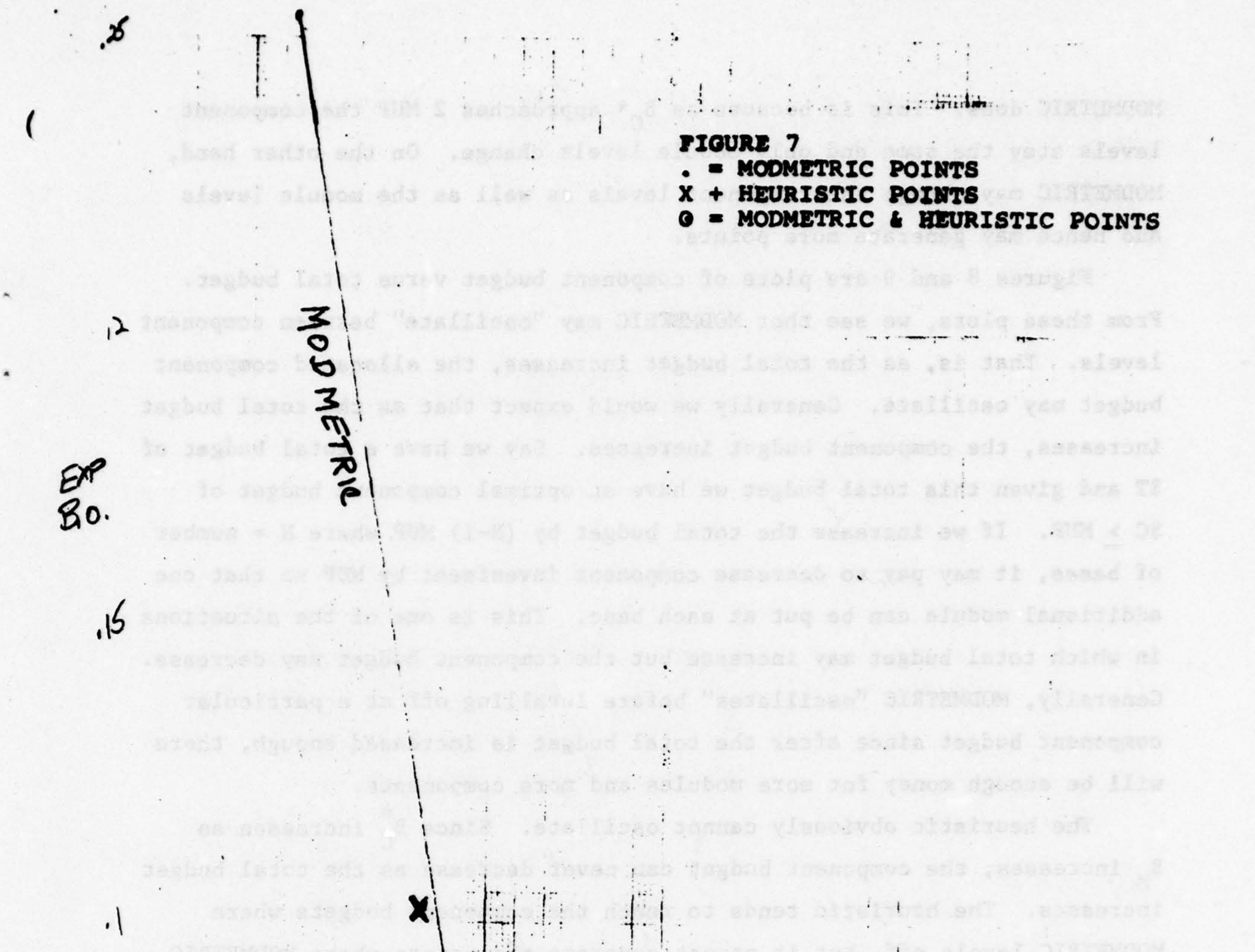


FIGURE 7
. = MODMETRIC POINTS
X + HEURISTIC POINTS
0 = MODMETRIC & HEURISTIC POINTS
MODMETRIC
EXP
BO.
.25
.2
.15
.1
.05
.01
700
800
900
1000
1100
1200
BUDGET
X10³

MODMETRIC does. This is because as $B_C$* approaches 2 MUP the component levels stay the same and only module levels change. On the other hand, MODMETRIC may change the component levels as well as the module levels and hence may generate more points.

Figures 8 and 9 are plots of component budget verus total budget. From these plots, we see that MODMETRIC may "oscillate" between component levels. That is, as the total budget increases, the allocated component budget may oscillate. Generally we would expect that as the total budget increases, the component budget increases. Say we have a total budget of \$T and given this total budget we have an optimal component budget of \$C $\geq$ MUP. If we increase the total budget by (N-1) MUP where N = number of bases, it may pay to decrease component investment by MUP so that one additional module can be put at each base. This is one of the situations in which total budget may increase but the component budget may decrease. Generally, MODMETRIC "oscillates" before levelling off at a particular component budget since after the total budget is increased enough, there will be enough money for more modules and more components.

The heuristic obviously cannot oscillate. Since $B_C^*$ increases as $B_M$ increases, the component budget can never decrease as the total budget increases. The heuristic tends to reach the component budgets where MODMETRIC levels off, but it cannot generate the points where MODMETRIC oscillates. The heuristic may generate the total budgets $T_o$ and ($T_o$ + N MUP) while MODMETRIC may generate points $T_o$, ($T_o$ + (N-1) MUP) and ($T_o$ + N MUP). This helps to explain why MODMETRIC will usually generate more points than the heuristic.

The figures 5, 6, 7, 8 and 9 are typical of the results we had. Experience has indicated that the heuristic yields points satisfactorily close to the MODMETRIC points to be a viable alternative. Note that the heuristic (21) adjusts itself for different priced modules and for varying system parameters.

A good decision making policy might be as follows. Use the heuristic to generate the performance versus budget tradeoff curve. Then a manager may choose the point he wishes to be at. Once this point is determined, the more precise solution procedure (or the MODMETRIC solution procedure) may be used to obtain a more exact answer for the chosen point. However, even for the chosen point, experience indicates the heuristic solution will be satisfactory without more complex and time consuming procedures.

## TABLE 1

| MODMETRIC COST | MODMETRIC EXP B.O. | MODMETRIC READY RATE | HEURISTIC COST | HEURISTIC EXP B.O. | HEURISTIC READY RATE |
|---|---|---|---|---|---|
| 275,500 | 2.096 | .3507 | 275,500 | 2.096 | .3507 |
| 336,500 | 1.592 | .4512 | 461,500 | .6436 | .7632 |
| 435,500 | .7971 | .7182 | 496,500 | .4940 | .8103 |
| 496,500 | .4940 | .8130 | 576,500 | .2352 | .9013 |
| 576,500 | .2352 | .9013 | 656,500 | .1146 | .9532 |
| 656,500 | .1146 | .9532 | 736,500 | .0378 | .9834 |
| 736,500 | .0378 | .9834 | 896,500 | .0047 | .9979 |
| 826,500 | .0106 | .9951 | 1,056,500 | .0004 | .9998 |
| 896,500 | .0047 | .9979 | | | |
| 986,500 | .00085 | .9996 | | | |
| 1,072,600 | .0003 | .9999 | | | |



## TABLE 2

| MODMETRIC COST | MODMETRIC EXP B.O. | MODMETRIC READY RATE | HEURISTIC COST | HEURISTIC EXP B.O. | HEURISTIC READY RATE |
|---|---|---|---|---|---|
| 893,900 | 2.830 | .2430 | 629,300 | 5,4014 | .0671 |
| 960,800 | 1.834 | .4804 | 880,800 | 2.0515 | .3565 |
| 1,040,800 | 1.303 | .5897 | 960,800 | 1.8340 | .4804 |
| 1,120,800 | .7946 | .7462 | 1,095,800 | .9710 | .6708 |
| 1,177,900 | .5668 | .8075 | 1,175,800 | .5733 | .8056 |
| 1,257,900 | .3078 | .8859 | 1,257,900 | .3078 | .8859 |
| 1,337,900 | .1817 | .9337 | 1,337,900 | .1817 | .9337 |
| 1,417,900 | .0796 | .9688 | 1,417,900 | .0796 | .9688 |
| 1,512,900 | .0382 | .9843 | 1,577,900 | .0171 | .9930 |
| 1,577,900 | .0174 | .9930 | 1,657,900 | .0075 | .9968 |
| 1,672,900 | .0067 | .9971 | 1,737,900 | .0031 | .9981 |
| 1,737,900 | .0031 | .9987 | 1,817,900 | .0011 | .9995 |
| 1,832,900 | .0010 | .9996 | 1,897,900 | .0005 | .9998 |
| 1,897,900 | .0005 | .9998 | | | |

## TABLE 3

| MODMETRIC | | | HEURISTIC | | |
|---|---|---|---|---|---|
| COST | EXP B.O. | READY RATE | COST | EXP B.O. | READY RATE |
| 393,600 | 2.857 | .2397 | 331,000 | 3.5423 | .1701 |
| 490,000 | 1.894 | .4695 | 393,600 | 2.857 | .2397 |
| 553,600 | 1.336 | .5821 | 553,600 | 1.336 | .5820 |
| 633,600 | .8393 | .7063 | 618,600 | .9198 | .6843 |
| 724,600 | .4043 | .8404 | 698,600 | .4897 | .8117 |
| 793,600 | .2520 | .9044 | 778,600 | .2885 | .8992 |
| 884,600 | .0850 | .9645 | 859,600 | .1119 | .9542 |
| 976,100 | .0415 | .9819 | 939,600 | .0591 | .9747 |
| 1,044,600 | .0140 | .9939 | 1,019,600 | .0203 | .9914 |
| 1,136,100 | .0054 | .9976 | 1,099,600 | .0086 | .9962 |
| 1,204,600 | .0019 | .9992 | 1,179,600 | .0030 | .9987 |
| 1,296,100 | .0006 | .9997 | 1,259,600 | .0010 | .9995 |
| 1,364,600 | .0002 | .9999 | | | |

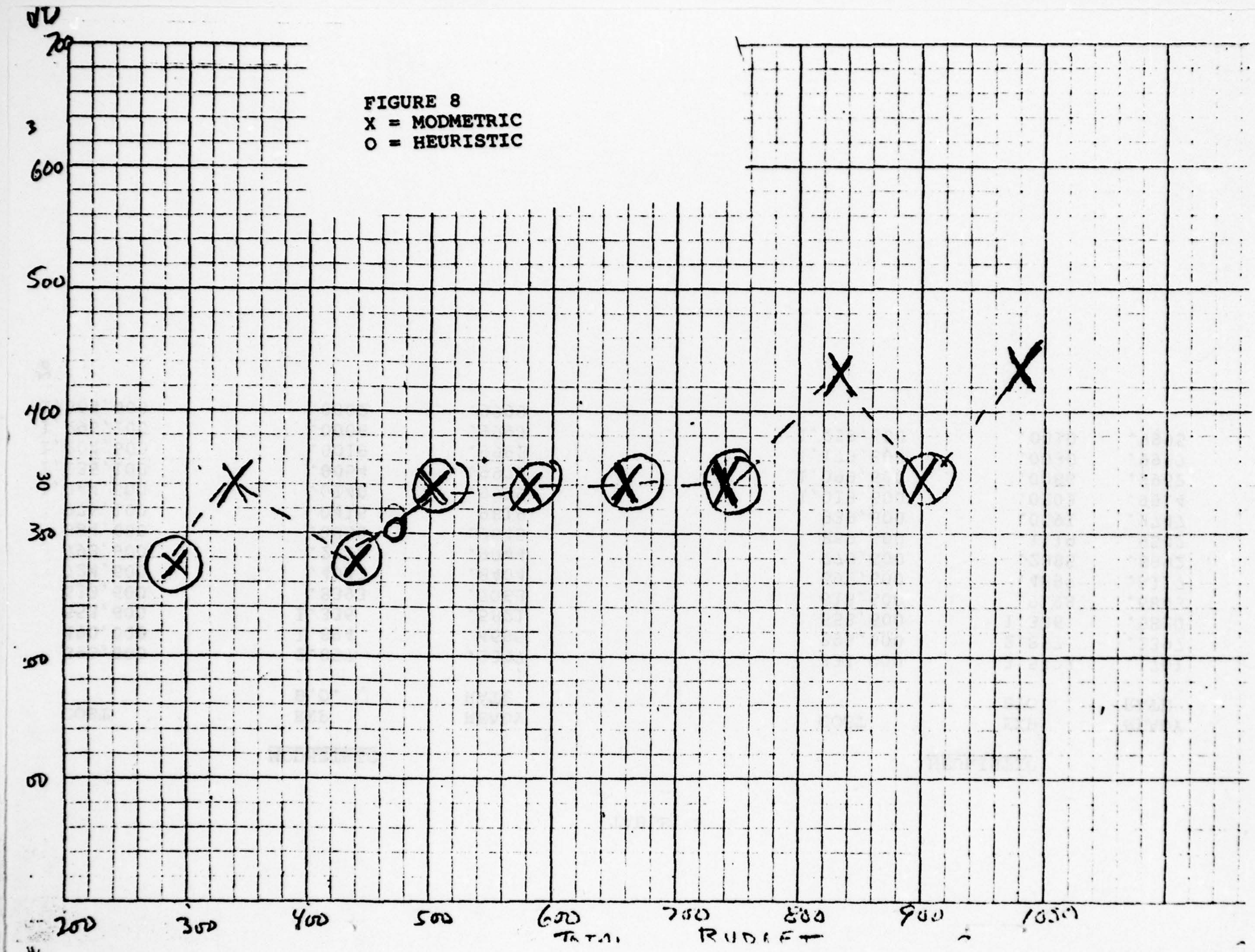


FIGURE 8
X = MODMETRIC
O = HEURISTIC
700
600
500
400
30
350
200
300
400
500
600
700
800
900
1050
TOTAL BUDGET

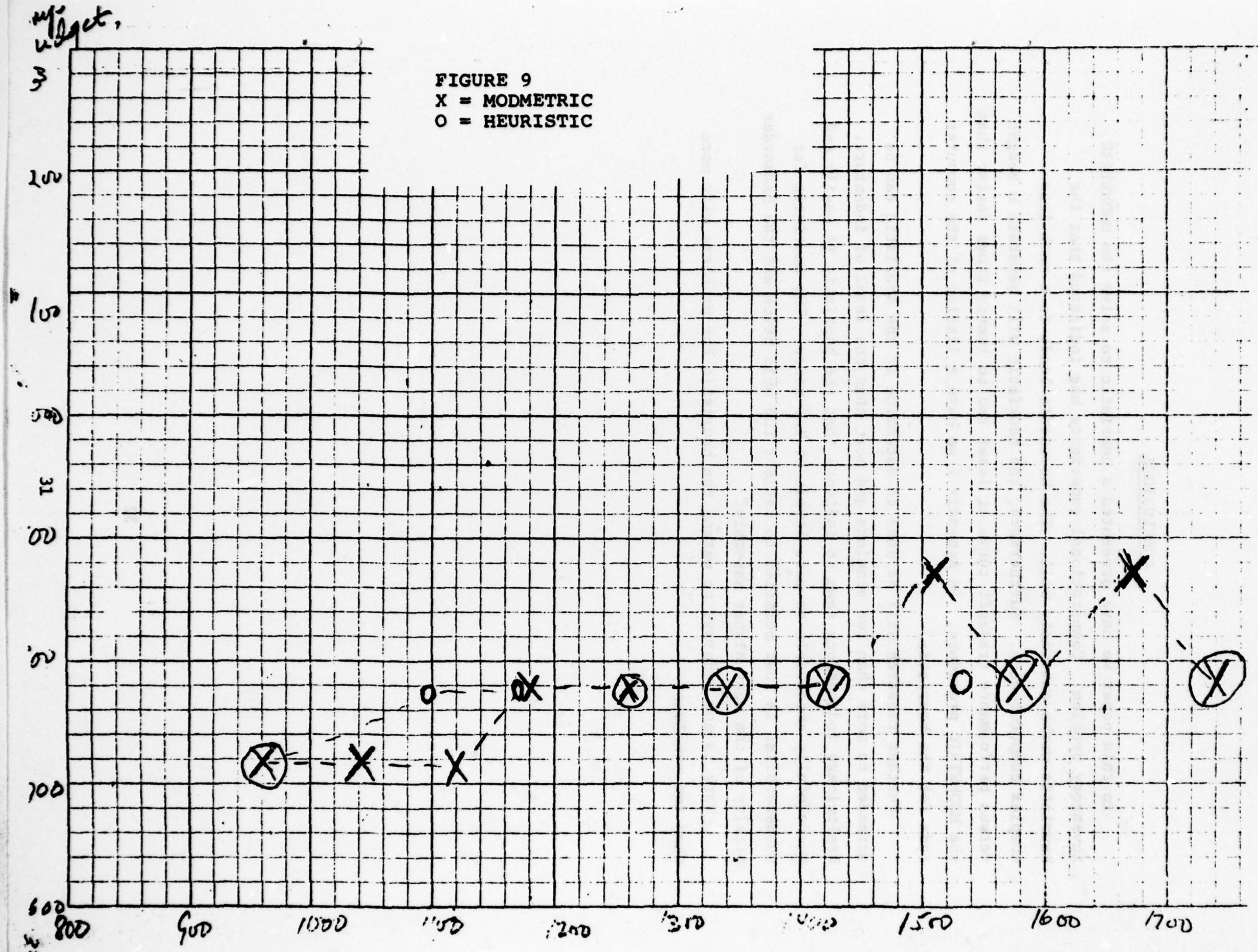


FIGURE 9
X = MODMETRIC
O = HEURISTIC

## CONCLUSIONS

In this paper we have presented a heuristic to solve the MODMETRIC inventory problem. Computational experience has indicated that the heuristic works extremely well. The heuristic leads to a much less complex computer code. Furthermore, the heuristic will generate a budget versus performance tradeoff curve at least ten to twenty times faster than the MODMETRIC procedure. In Appendix I we have a listing of the computer code for the heuristic.

Further research will be done to determine if the heuristic can be extended to more than two echelons and more than one level of indenture. Preliminary indications seem to indicate that the heuristic is easily and successfully extendable to more levels of indenture. The heuristic for subassemblies is just modified to reflect the unit price and the backorder penalty of the next higher assembly.

Work is now underway to extend the heuristic for problems with more than two echelons.

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1 Kotkin, Meyer, "Some Results for Multi-Echelon Multi-Indentured Inventory Problems," US Army Tank Automotive Materiel Readiness Command, To Be Published.

2 Muckstadt, John A., "A Model for a Multi-Item Multi-Echelon, Multi-Indenture Inventory System," Management Science, Vol 20, No. 4, Dec 1973, pp 472-481.

3 Sherbrooke, Craig C., "METRIC-Multi-Echelon Technique for Recoverable Item Control," Operations Research, Vol 16, 1968, pp 122-141.

4 Shay, F. M. and T. J. O'Malley, "An Efficient Optimization Procedure for a levels of Indenture Inventory Model," AF-605 (draft), Logistics Management Institute, Washington, DC, February 1978.

## APPENDIX I

## COMPUTER LISTING

```
      PROGRAM TESTER(INPUT,OUTPUT,TAPE5)
      DIMENSION OPSTOCK(2),ORGSPER(4),TAT(4),OST(4),XLB(4),P(4)
     X,PSUM(4)
      DIMENSION TOTH(10),TOTB(10)
      REAL MINCOST
    2 PRINT*,"INPUT BOCNHA AND INDEX"
      READ*,BOCNHA,INDEX
      IF(INDEX .EQ. 0)STOP
      DO 100 I=1,10
      TOTH(I)=0.
      TOTB(I)=0.
  100 CONTINUE
      REWIND 5
      READ(5,*)ORGSPER,OST,FH,YNRTS,YMTBD,UPNHA,DRTLRU,DENS
      YRATE=1./YMTBD
      NSRUS=0
    1 READ(5,*)XMTBD,UP,P,TAT
      IF(EOF(5).NE.0)GO TO 30
      FRATE=1./XMTBD
      NSRUS=NSRUS+1
      DEM=FH*(1.-YNRTS)*FRATE/30.
      OLDEM=DEM*ORGSPER(1)*(1.-P(1))*TAT(2)
      DO 10 I=1,INDEX
      IF(I.EQ.1)BOCOST=UPNHA*(2-(UPNHA/BOCNHA))
      GO TO 15
   20 SYS=1.-UPNHA/BOCNHA
      XINDIV=SYS**(1./DENS)
      BOCOST=DENS*(1.-XINDIV)*BOCNHA
      GO TO 15
   15 CRATIO=UP/HOCOST
      ECHSTK=1000.
      CALL TWOECH(OLDEM,TAT,OST,P,PSUM,DEM,ORGSPER,CRATIO,ECHSTK,
     XXLB,OPSTOCK,MINCOST,AVAIL)
      HOCOST=(OPSTOCK(1)*ORGSPER(1)+OPSTOCK(2))*UP
      BACK=MINCOST-(HOCOST/BOCOST)
      TOTH(I)=TOTH(I)+HOCOST
      TOTB(I)=TOTB(I)+BACK
   10 CONTINUE
      GO TO 1
   30 DO 25 I=1,INDEX
      DEM=FH*YRATE/30.
      OLDEM=DEM*ORGSPER(1)*YNRTS*DRTLRU
      CRATIO=UPNHA/BOCNHA
      TAT(1)=TOTB(I)/((1.-YNRTS)*DEM*ORGSPER(1))
      TAT(2)=DRTLRU
      P(1)=1.-YNRTS
      CALL TWOECH(OLDEM,TAT,OST,P,PSUM,DEM,ORGSPER,CRATIO,ECHSTK
     X,XLB,OPSTOCK,MINCOST,AVAIL)
      PRINT*," ********ASSEMBLY VALUES ******************************"
      PRINT*,OPSTOCK
      PRINT*," "
      HA=(OPSTOCK(1)*ORGSPER(1)+OPSTOCK(2))*UPNHA
      21
```

```
      PRINT*," ASS. HOLDING COST= ",HA
      PRINT *," TOTAL ASS. AND SUB. HOLDING COST = ",HA+TOTH(1)
      PRINT*,"SUB. DELAY = ",TAT(1)
      PRINT*,"EXPECTED ASS, B.O. ",MINCOST-HA/HOCHHA
      PRINT*," ASS. AREADY RATE ",AVAIL
   25 CONTINUE
      PRINT*,"*************************************************"
      GO TO 2
      STOP
      END
      FUNCTION BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      REAL LBTLEC
      TEMP=(UBTC-LBTLEC)/CRATIO
      BOUND=AMIN1(TEMP,ECHSTK)
      RETURN
      END
      SUBROUTINE TWOECH(DLDEM,TAT,OST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,STOCK,MINCOST,AVAIL)
      REAL NRAR,MOPT,LBTLEC,MINCOST
      DIMENSION STOCK(2),TAT(4),P(4),PSUM(4),OST(4),ORGSPER(4)
      DIMENSION XLB(4)
      RLTD(S)=BLTD+DEM1*BACKO(DLDEM,S)/DEM2
      DEM2=REMOVES*ORGSPER(1)*(1.-P(1))
      DEM1=DEM2/ORGSPER(1)
      BLTD=REMOVES*(P(1)*TAT(1)+(1.-P(1))*OST(1))
      NRAR=AINT(BLTD*2+5.)
      NRAR=ONEECH(CRATIO,RLTD(XLB(2)),NRAR,XLB(1))
      UBTC=ORGSPER(1)*(NRAR*CRATIO+BACKO(RLTD(XLB(2)),NRAR))+XLB(2)
     X*CRATIO
      STOCK(1)=NRAR
      MOPT=XLB(2)
      MINCOST=UBTC
      NRAR=ONEECH(CRATIO,BLTD,NRAR,XLB(1))
      LBTLEC=ORGSPER(1)*(NRAR*CRATIO+BACKO(BLTD,NRAR))
      NRAR=STOCK(1)
      J=XLB(2)
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
  200 J=J+1
      IF(UPBOUND .LT. J)GO TO 20
      STOCK(2) = J
      NRAR=ONEECH(CRATIO,RLTD(STOCK(2)),NRAR,XLB(1))
      COST=J*CRATIO+ORGSPER(1)*(NRAR*CRATIO+BACKO(RLTD(STOCK(2)),NRAR))
      IF (COST.GT.MINCOST) GO TO 200
      UBTC=COST
      MINCOST = COST
      STOCK(1) = NRAR
      MOPT = STOCK(2)
   15 GO TO 200
   20 STOCK(2) = MOPT
      AVAIL=POISLT(RLTD(MOPT),STOCK(1))
      RETURN
      END
      FUNCTION ONEECH(CRATIO,PIPE,INITST,XLOBND)
      REAL NRAR, INITST
C
      NRAR = INITST
      STK = NRAR
      COSTAL = BACKO(PIPE,NRAR) + CRATIO*NRAR
      DO 10 I = 1, 100
      NRAR = NRAR - 1.
      IF (NRAR.LT.XLOBND) GO TO 20
      COSTAR = BACKO(PIPE,NRAR) + CRATIO*NRAR
```

```
C     ASSUME CONVEXITY
      IF (COSTAR.[illegible].COSTAL) GO TO 20
      STK = NRAR
      COSTAL = COSTAR
      PRINT [illegible]
   20 ONEECH = STK
      RETURN
      END
```

# APPENDIX II

The test module used to verify the heuristic had the following characteristics:

Module unit price: $80,000

Mean hours between module demands: 260

Depot module repair time: 60 days

All components required depot repair. The component data was:

| Component # | Mean Hours Between Demands | Unit Price | Depot Repair Time |
|---|---|---|---|
| 1 | 1000 | 25000 | 45 |
| 2 | 800 | 1000 | 45 |
| 3 | 3800 | 35000 | 45 |
| 4 | 4000 | 1100 | 45 |
| 5 | 2400 | 30000 | 45 |
| 6 | 2200 | 1500 | 45 |

The order and ship time between bases and depot was 15 days for all items. There were two identical bases. We assumed 4, 8 and 12 module failures per base per month in Figures 5, 6 and 7 respectively.

Other cases with different unit prices, repair times, failure rates, etc., were run and the results were comparable to those presented in the report.

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| 1 | George Washington University, Inst of Management Science & Engr., 707 22nd St., N.W., Wash., DC 20006 |
| 1 | Naval Postgraduate School, ATTN: Dept of Opns Anal, Monterey, CA 93940 |
| 1 | Air Force Institute of Technology, ATTN: SLGQ, Head Quantitative Studies Dept., Dayton, OH 43433 |
| 1 | US Army Military Academy, West Point, NY 10996 |
| 1 | Librarian, Logistics Mgt Inst., 4701 Sangamore Rd., Wash.,DC 20016 |
| 1 | University of Florida, ATTN: Dept of Industrial Systems Engr., Gainesville, FL 32601 |
| 1 | RAND Corp., ATTN: S. M. Drezner, 1700 Main St., Santa Monica, CA 90406 |
| 1 | US Army Materiel Systems Analysis Activity, ATTN: DRXSY-CL, Aberdeen Proving Ground, MD 21005 |
| 1 | Commander, US Army Logistics Center, ATTN: Concepts & Doctrine Directorate, Ft. Lee, VA 23801 |
| 1 | ALOG Magazine, ATTN: Tom Johnson, USALMC, Ft. Lee, VA 23801 |
| 1 | Commander, USDRC Automated Logistics Mgt Systems Activity, P.O. Box 1578, St. Louis, MO 63188 |
| 1 | Director, DARCOM Logistics Systems Support Agency, Letterkenny Army Depot, Chambersburg, PA 17201 |
| 1 | Commander, Materiel Readiness Supply Activity, Lexington, KY 40507 |
| 1 | Director, Army Management Engineering Training Agency, Rock Island Arsenal, Rock Island, IL 61202 |
| 1 | Defense Logistics Agency, Cameron Sta, Alexandria, VA 22314 |
| 1 | Dep Chf of Staff (I&L), HQ USMC-LMP-2, ATTN: MAJ Sonneborn, Jr., Wash. DC 20380 |
| 1 | Commander, US Army Depot Systems Command, Letterkenny Army Depot, ATTN: DRSDS-LL, Chambersburg, PA 17201 |
| 1 | US Army Logistics Evaluation Agency, New Cumberland Army Depot, New Cumberland, PA 17070 |
| 1 | HQ, Dept of the Army, (DASG-HCL-P), Wash., DC 20314 |
| 1 | Operations Research Center, 3115 Etcheverry Hall, University of California, Berkeley, CA 94720 |

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| 1 | Mr. Ellwood Hurford, Scientific Advisor, ATCL-SCA, Army Logistics Center, Ft. Lee, VA 23801 |
| 1 | Commandant, USA Armor School, ATTN: MAJ Harold E. Burch, Leadership Dept, Ft. Knox, KY 40121 |
| 1 | Prof Robert M. Stark, Dept of Stat & Computer Sciences, University of Delaware, Newark, DE 19711 |
| 1 | Prof E. Gerald Hurst, Jr., Dept of Decision Science, The Wharton School, University of Penna., Phila., PA 19174 |
| 1 | Logistics Studies Office, DRXMC-LSO, ALMC, Ft. Lee, VA 23801 |
| 1 | Procurement Research Office, DRXMC-PRO, ALMC, Ft. Lee, VA 23801 |
| 1 | Dept of Industrial Engr. & Engr. Management, Stanford University, Stanford, CA 94305 |
| 1 | Commander, US Army Communications Command, ATTN: Dr. Forrey, CC-LOG-LEO, Ft. Huachuca, AZ 85613 |
| 1 | Commander, US Army Test & Evaluation Cmd, ATTN: DRSTE-SY, Aberdeen Proving Ground, MD 21005 |
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| 1 | Dr. John Voelker, EES Bldg. 11, Argonne National Laboratory, 9700 S. Cass Ave., Argonne, IL 60439 |
| 1 | DARCOM Intern Training Center, ATTN: Jon T. Miller, Bldg. 468, Red River Army Depot, Texarkana, TX 75501 |
| 1 | Prof Leroy B. Schwarz, Dept of Management, Purdue University, Krannert Bldg, West Lafayette, Indiana 47907 |
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| 1 | US General Accounting Office, ATTN: Mr. J. Morris, Rm 5840, 441 G. St., N.W., Wash., DC 20548 |
| 1 | Operations & Inventory Analysis Office, NAVSUP (Code 04A) Dept of Navy, Wash., DC 20376 |
| 1 | US Army Research Office, ATTN: Robert Launer, Math. Div., P.O. Box 12211, Research Triangle Park, NC 27709 |
| 1 | Prof William P. Pierskalla, Dept of Ind. Engr. & Mgt. Sciences, Northwestern University, Evanston, IL 60201 |
| 1 | US Army Materiel Systems Analysis Activity, ATTN: DRXSY-MP, Aberdeen Proving Ground, MD 21005 |
| 1 | Air Force Logistics Management Center, ATT: AFLMC/LGY, Gunter Air Force Station, AL 36114 |
| 1 | Engineer Studies Center, 6500 Brooks Lane, Wash., DC 20315 |