





FINAL

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#### THE APPLICATION OF QUANTITY

#### DISCOUNTS IN ARMY PROCUREMENTS

BY

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#### EXECUTIVE SUMMARY

A. <u>BACKGROUND</u>. The Department of Defense (DOD) has directed the use of basic Economic Order Quantity (EOQ) principles in the acquisition of secondary items. One of the assumptions that the EOQ model makes is that there is no control over acquisition price; yet it has been established that the unit cost of an item is not always independent of the quantity procured. Often, discounts are offered for the purchase of larger quantities than that directed by the EOQ. Presently, material is bought in lots established by an EOQ; however, if offerors were asked to submit offers on an EOQ and larger alternate quantities, then savings could be gained by acquisition of more material at a reduced unit cost. This savings would be realized by reducing the annual purchase costs.

B. <u>OBJECTIVE</u>. The objective of this study is to explore the feasibility of introducing the concept of quantity discounts (QD) into the Army acquisition procedures.

C. <u>METHODOLOGY</u>. The study and research methods employed consisted of a review and analysis of the QD program as implemented by the Air Force; interviews with selected individuals at Air Force procurement activities; and the development of a total variable cost equation to evaluate offers.

D. <u>CONCLUSIONS AND RECOMMENDATIONS</u>. The Air Force QD program has been successful notwithstanding self imposed conservative parameters. The US Army Materiel Development and Readiness Command (DARCOM) should benefit by implementation of a QD program similar to the Air Force QD program but with less conservative parameters. It is recommended that a test QD program be conducted at one or more appropriate Materiel Readiness Commands.

E. <u>IMPLEMENTATION</u>. Headquarters DARCOM has approved a test of the QD program, and testing has been initiated at US Army Missile Materiel Readiness Command, Redstone Arsenal, AL.

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#### CHAPTER I

#### INTRODUCTION

#### A. BACKGROUND.

Inventory theory was one of the earliest operations research techniques to be applied in business, industry and the public sector. The Department of Defense (DOD) has directed the use of basic Economic Order Quantity (EOQ) principles in the acquisition of secondary items. DOD Instruction (DODI) 4140.39, Procurement Cycles and Safety Levels of Supply for Secondary Items, dated 17 July 1970, establishes policies for determining procurement cycles and safety levels of supply at Inventory Control Points (ICP's) for secondary items, and illustrates the basic mathematical functions and their application in an inventory model. One of the assumptions that the EOQ model makes is that there is no control over acquisition price; yet it has been established that the unit price of an item is not always independent of the quantity procured. Often, discounts are offered for the purchase of larger quantities than that dictated by the EOQ. Presently, material is bought in lots established by an EOQ; however, if offerors were asked to submit offers on an EOQ and larger alternate quantities, then savings could be gained by acquiring more material at a reduced unit cost. This savings would be realized by reducing the annual purchase costs.

#### B. STUDY OBJECTIVE.

The objective of this study is to explore the feasibility of introducing the concept of quantity discounts (QD) into the Army acquisition procedures.

#### C. SCOPE.

This study will focus on the development of procedures for determining when it is economically advantageous to attempt to obtain a QD in the acquisition of secondary items.

#### D. METHODOLOGY.

The approach planned to accomplish the study objective is to (i) review the QD program as implemented by the Air Force; (ii) interview selected individuals in Air Force buying activities; (iii) develop a total variable cost (TVC) equation to evaluate offers, allowing the selection of that quantity offered for which TVC is a minimum.

#### E. REPORT ORGANIZATION.

Chapter II briefly discusses the concept of EOQ and the QD principle. Chapter III reviews and provides an analysis of the Air Force QD program. Chapter IV proposes a QD program tailored for the U.S. Army Materiel Development and Readiness Command (DARCOM) activities. Chapter V summarizes the conclusions, recommendation, and implementation.

#### F. SPECIAL NOTE.

Throughout this report, the convention 2 X EOQ, 3 X EOQ, etc., will be used to denote multiples of a base EOQ amount.

#### CHAPTER II

#### BACKGROUND DISCUSSION OF EOQ AND QD

#### A. EOQ THEORY.

DODI 4140.39 states that the objective of DOD policy is: "To minimize the total of variable order and holding costs subject to a constraint on time weighted, essentiality-weighted requisitions short " (4, p.2). The total variable cost consists of cost to order, cost to hold and the implied shortage cost. Procurement cycles (i.e., EOQ) and safety levels are determined through minimization of these costs for any given group of items in an inventory. The total variable cost, variable cost to order, and variable cost to hold formulas are discussed in the various enclosures to DODI 4140.39. The implied shortage cost is a function of other management decisions which are made outside the scope of DODI 4140.39.

B. QD THEORY.

Of the assumptions made in the EOQ model (fixed unit price, stationary demand, deterministic lead time), the assumption that there is no control over unit price is the basis of effort accomplished to date on the QD principle. This assumption is invalid because experience in industry and the military shows that discounts are offered when large quantities are acquired. This means that the optimum EOQ computed by the total variable cost EOQ model may not be optimum when discounts are offered, and the acquisition of a larger quantity may be more economical than the EOQ. This consideration is recognized in DODI 4140.39 which permits acquisitions larger than the EOQ when price breaks are sufficiently large (i.e., cost effective).

The principle of a QD is to acquire more of an item when it is to the Government's advantage to do so. It is to the Government's advantage when the total annual cost is lower for a larger quantity acquired. In essence, the EOQ is solicited along with several larger quantities, and the quantity ultimately acquired is the one which results in the lowest total annual cost.

The annual costs include the variable costs of acquisition, ordering and holding.

 Acquisition Cost. The acquisition cost is the element which is not presently included in the DODI 4140.39 EOQ model, and the element which will provide different results for the optimum buy quantity.

2. <u>Cost of Ordering</u>. The cost of ordering is the variable administrative cost associated with processing a purchase request. It varies among the Materiel Readiness Commands (MRCs) and also depends on the complexity of the purchase, e.g., it is lower for small purchases. DODI 4140.39 states: "Costs to be considered in determining cost to order will be those variable direct labor and support costs which begin with the output of the requirement notice, through the mailing of the contract or order and will include processing the physical asset into the proper warehouse location after receipt from the contractor. Average contract administration cost will also be a part of the cost to order an item of inventory " (4 p.6). The DODI specifies in functional detail how the order costs are to be compiled.

3. <u>Holding Cost</u>. The elements of holding cost are investment, storage, deterioration and loss, and obsolescence. Before a decision is made to buy a larger quantity, it must be recognized that about 10% of the purchase price

must be given up for the loss of investment opportunity, i.e., we will have funds invested in inventory which substantially decreases its liquidity. It will also cost about 1% of the purchase price for storage, and about 2% for deterioration, losses, etc. The values for investment, storage and deterioration are standard within DOD at the wholesale level and are not dependent on a particular commodity. Finally, there is always a risk of obsolescence as a result of diminished demand rates that result in excess inventory. Therefore, before purchasing the larger quantity, the possibility of disposal of a portion of the assets must be properly evaluated. The chance that this will happen depends on the commodity and therefore the rate for the MRCs range between 8 and 15%. Therefore, the total holding cost could range from 21 to 28% of the purchase price.

#### CHAPTER III

#### AIR FORCE QUANTITY DISCOUNT PROGRAM

#### A. INTRODUCTION.

This chapter will discuss the Air Force (AF) QD program in terms of authority and operation, evaluate the results of the program; and discuss observations about the program. The observation must be taken into account in the consideration of the feasibility of adoption of a QD program in the DARCOM acquisition procedures.

1. <u>Authority</u>. Research effort sponsored by Air Force Logistics Command (AFLC) indicated that AFLC could obtain substantial dollar savings by implementation of a QD program (3). The AF QD program received DOD approval and was documented in AFLC Regulation 70-23, dated 30 June 1976 (1). The program was officially implemented at all of the Air Logistics Centers (ALCs) in October 1976, and has been successful (2 p.11).

2. <u>Purpose</u>. The prime motivation for the AF program is to obtain lower prices and thus reduce acquisition costs. Reduced cost of ordering also result from a decrease in the number of item acquisitions. Reduction in the purchase request (PR) volume was not considered to be a beneficial by-product and not a major factor for supporting a QD program.

3. <u>Applicability</u>. The AF program is applicable only to items with low reorder periods (approximately 6 months or less) and for this reason includes only those items with an average annual demand dollar value from \$500 to \$50,000. Dollar value in this context refers to annual sales at the wholesale

level of management. The program is limited to stock funded items only, and for obvious reasons includes only stable items. Small purchases are excluded from the program in order to retain the administrative simplicity of small purchase procedures. Negotiated procurements are limited to an EOQ value of less than \$60 thousand to preclude the need for cost and pricing data. (The Defense Acquisition Regulation (DAR) requires submission and certification of cost and pricing data in non-competitive acquisitions exceeding \$100 thousand). AF experience has shown that discounts are usually sufficient to result in offers below \$100 thousand when solicitations are up to \$120 thousand (i.e., 2 X EOQ).

4. <u>Procedures</u>. Solicitations for QD candidate acquisitions are based on the following three quantities: the EOQ; 1.5 X EOQ; and 2.0 X EOQ. In addition to appropriate evaluation and delivery provisions, the solicitation includes a clause stating that the AF will acquire the larger quantity if it yields lower annual costs to the Government, provided funds are available. This clause precludes possible legal difficulties in the event of fund shortages.

The AF program is an off-line program which requires much manual processing; however, the establishment of records, an audit trail, and provision for status of the PR at any point prior to closeout is automated.

The buyer evaluates the prices and determines which offer represents the lowest overall unit price for each of the three quantities solicited. A computer program is then utilized to determine if discount offers are cost effective and which offer is the most advantageous to the AF. The buyer will

review the computer product for input accuracy and then forward to the Item Manager (IM) for a decision on the quantity to acquire. Reporting on the experience of each quantity discount PR is required by AFLC Regulation 70-23.

#### B. AF RESULTS.

DOD has supported the AF QD program to the extent of providing \$8.8 million (1% of the stock fund) for Fiscal Year (FY) 1978, and the AF requested \$9.6 million for FY 1979.

1. <u>Savings</u>. The gross savings realized from this program were \$1.7 million in FY 1977 and \$1.4 million through April of FY 1978. Savings per unit are calculated by comparing price per unit for the EOQ with the price per unit for the larger quantity actually acquired. Gross savings are calculated as the savings per unit times the number of units acquired. Net savings, which the AF does not measure, would result from the addition of savings due to processing fewer PRs and the subtraction of the additional holding cost.

2. <u>Discounts</u>. The discounts are negligible in some cases, and large in others, 20 to 35%. The average discount is 6.5%. The average discount does not include 53% of the QD solicitations which do not receive any offered discount. The buyers queried suppliers to determine why no discounts were offered and learned that the two prime reasons are: (1) that the largest solicitation quantity of 2 X EOQ is too small to offer a discount; and (2) that some suppliers offer no discount under any conditions.

3. <u>Volume of Purchase Requests (PRs)</u>. The AF does not keep statistics on the impact of the QD program on PR volume.

#### C. OBSERVATIONS.

Although there is no data on the impact of the AF program on the PR volume,

it is obvious that the impact is small because: (1) discounts are offered in only 47% of the QD solicitations; (2) the solicitation quantities are small, i.e., limited to 2 X EOQ; and (3) the program does not apply to small purchases which constitute the bulk of procurements. The AF program also has little impact on the volume of changes to PR's. Changes to procurement actions appreciably add to the procurement workload. The AF buyers indicated they had hoped the QD program would assist in this area; however, they stated that there had been no discernible improvement.

The last observation is that the average discounts offered justify acquisitions larger than 2 X EOQ. Table I represents a break even analysis which indicates the discounts which must be received to acquire multiples of the EOQ for various EOQ months. (See Appendix for derivation of the formula used to compute the table entries). The table actually represents a trade-off between additional holding costs and reduced ordering costs and lower unit prices. As an example, the chart indicates that for an item which has an EOQ of six months, it would pay to acquire 18 months worth of stock (3 X EOQ) provided that the discount is 7.02% or greater.

# TABLE I

# BREAK-EVEN ANALYSIS

REQUIRED DISCOUNTS TO BREAK EVEN WHEN LARGER QUANTITIES ARE BOUGHT ARE SHOWN BELOW.

BUY QUANTI 3.81 7.02 9.76 12.12	18 6.82 11.49	12 5.00 8.5	9 3.95 6.8	6 2.78 4.8	<u>20</u> <u>2.5</u> 3 1.47% 2.6	EOQ MONTHS
ПЛ	16 00	8.57 12.12	6.84 9.76	4.86 7.02	<u>2.5</u> 30 2.613.81	BUY QUAN
	16.00 20.22	12.12 15.53	9.76 12.61	7.02	<u>30</u> 3.81	BUY QUANTITY
	24.11	18.75	15.34	11.25	4 <u>0</u> 6.25	

#### CHAPTER IV

#### PROPOSED DARCOM QD PROGRAM (TEST)

#### A. INTRODUCTION.

The success the AF has enjoyed from their QD program, leads to the conclusion that a DARCOM sponsored test of the QD application at one or more MRCs appears appropriate at this time.

1. <u>Purpose</u>. The number of acquisitions over the past years has been steadily increasing and has presently reached such a volume that it can not be handled efficiently by the authorized procurement resources at the MRCs. Analysis of the AF QD program indicates that simple duplication of AF procedures would not provide sufficient assistance in solving the workload problem. A program is needed that will: reduce the acquisition cost, reduce the workload to a level that can be efficiently handled by present personnel resources, and maintain the reduced workload. The workload can be reduced by lowering the volume of Procurement Work Directives (PWDs) and by reducing the volume of changes to PWDs. The purpose of a test, therefore, is to determine if an expanded QD program can substantially reduce both acquisition cost and the procurement workload while providing data necessary to make a decision on implementation of a full QD program.

2. <u>Applicability</u>. Such an expanded QD program would apply to all stable Army Stock Fund (ASF) items that are stocked at the wholesale level. The program would apply to both small and large purchases for the candidate items; however, there is no intention of losing the advantages of the simplified small

purchase procedures, (i.e., the solicitation of any acquisition which is processed initially under small purchase procedures will not be allowed to result in an offer estimated to exceed \$10,000. For example, an EOQ of 100 units at a unit price of \$50 would preclude multiples in excess of 2 X EOQ. An EOQ of 100 units at a unit price of \$90 would not be processed under QD procedures). Non-competitive acquisitions will be limited to less than \$100,000 to preclude problems with cost and pricing data.

#### B. PROCEDURES.

A formal DARCOM QD program is envisioned to be a normal Commodity Command Standard System (CCSS) operation with minimum manual intervention. Technical procedures will have to be coordinated with the test Materiel Readiness Command (MRC).

1. <u>Automation</u>. The identification of QD candidate items, printing of applicable statements on PWDs, evaluation of offers, and collection of data for evaluation will be automated to the maximum extent possible. This will preclude work disruption or the need for additional personnel resources.

2. <u>Solicitation increments.</u> Table 2 is a schedule of the eight increments to be solicited, provided the funds and method of procurement do not preclude solicitation up to 4.5 X EOQ. The advantages of eight range quantities as compared to the three specific quantities the AF uses are: (1) flexibility up to the time of award to acquire exactly what is needed, thereby precluding the need for changes to PWDs in process; (2) providing a "range" of eight increments which may more closely approximate the offerors economic production quantity.

3. Delivery schedule. Small producers may not be capable of producing

NUMBER 00 SOLICITATION INCREMENTS TABLE 2 3.0 X EOQ TO 3.5 X EOQ - 1 2.0 X EOQ TO 2.5 X EOQ - 1 0.5 X EOQ TO EOQ - 1 4.0 X EOQ TO 4.5 X EOQ - 1 3.5 X EOQ TO 4.0 X EOQ - 1 2.5 X EOQ TO 3.0 X EOQ - 1 1.5 X EOQ TO 2.0 X EOQ - 1 EOQ TO 1.5 X EOQ - 1 INCREMENT 13

the larger quantities in the same time frame as larger producers. Therefore, in order to keep the smaller producers in the competition, delivery schedules will be constructed to provide, within limits, the flexibility to produce according to capacity. To maintain the integrity of evaluation of delivery schedules, required delivery of 1 X EOQ will be structured under normal procedures; quantities in excess of 1 X EOQ will be required to be delivered at a minimum rate of 1 X EOQ every procurement cycle. The solicitation will be structured so as to state the required delivery schedule in terms of quantities per time period; however, the offeror will be allowed to offer alternate delivery terms, provided they do not exceed the required schedules established in the solicitation.

#### 4. Evaluation of prices.

A computer program will be furnished to the MRC for use in evaluation of prices. The actual formula is TOTAL ANNUAL COST=ACQUISITION COST+ORDER COST+HOLD COST=

AYD X EP + C X 
$$\frac{AYD}{Q}$$
 +  $\frac{Q}{2}$  X EP X H

Where:

EP = the computation of effective unit price considering the offered unit price and, if applicable, transportation costs, first article costs, and prompt payment discounts. These costs are prorated over the purchase quantity.

AYD = annual yearly demand.

C = administrative variable order cost per order.

Q = buy quantity (i.e., lower bound in each of the eight solicitation increments in table 2.

H = holding cost factor as a percent of effective unit price.
Translating the formula, AYD X EP is the annual acquisition cost.
C X AYD/Q is the annual order cost; Q/2 X EP X H is the annual holding cost.
5. Use of Graphs in Evaluation of Prices.

Figure 1 is a graph that may be used to evaluate prices if demand and lead time are deterministic. The figure may also be used if the Wilson EOQ given by the square root formula below is a good approximation of the optimum EOQ procured by the operational system.

Wilson EOQ = 
$$2 \times AYD \times C$$
  
H X EP

An example of use of the figure is: if the EOQ is 12 months of supply, and the discount offered relative to the EOQ price is 10% for buying 3 X EOQ, then buying 3 X EOQ would not be economical. The graph shows that in order for 3 X EOQ to be an economical buy when the EOQ is 12 months of supply, a minimum discount of 12.1% is needed.

#### C. TEST DETAILS.

1. <u>Applicability</u>. Because of the uniqueness of this concept, as well as on going programs which could affect the quantity of test candidate items, testing at any MRC should be on a voluntary basis. Any MRC which volunteers to participate in a test should be aware that participation in the test will have positive benefits such as improved item availability and lower PWD volume in later years. However, additional personnel resources will not be made available for purposes of running the test. Additional obligation authority for acquiring the larger inventory should be provided by DARCOM when the test MRC needs it.



2. <u>Procedures</u>. Prior to implementation of the actual test, procedures must be developed at the MRC. Such procedures would include as a minimum: (1) a method of identifying candidate items; (2) development of the required information for the Procurement Work Directive (PWD); (3) structure of the appropriate schedules in the solicitation packages; (4) development of an offer abstract and evaluation procedure; (5) test evaluation data; and (6) assigned responsibility for the formation of a written procedure or guide to be used by the participating directorates.

#### D. DATA DESIRED FROM TEST.

The purpose of a test of the QD program is to measure the ability of the program to meet the objectives of reduced acquisition cost and workload reduction while providing data necessary to make a decision on implementation aspects should the test show positive results. Data obtained during the test process should provide answers to the following questions:

(1) How much additional obligation authority will be needed to support a QD program?

(2) What is the extra cost to process a QD acquisition, and should such cost be considered in the offer evaluation model?

(3) What is the effect on Procurement Administrative Leadtime (PALT)?

(4) Which classes of items will result in the most cost effective use of the QD procedure?

(5) What will be the acquisition savings, and the net savings?

(6) Will QD work at the MRCs?

(7) What are the most appropriate implementation procedures?

#### CHAPTER V

#### CONCLUSIONS, RECOMMENDATION, IMPLEMENTATION

A. <u>CONCLUSIONS</u>. Experience indicates that the assumption of no control over acquisition price in the EOQ model is invalid. Discounts are offered when large quantities are acquired. The Air Force QD program has successfully obtained discounts ranging from very small up to 35% notwithstanding the constraints they have placed on the program.

Expansion of the Air Force QD program by soliciting a series of range quantities from the EOQ up to 4 X EOQ, and applying the program to small purchases, is expected to result in: (1) more discounts offered; (2) larger discounts offered; and (3) a long run reduction in administrative workload. B. <u>RECOMMENDATION</u>. A proposed DARCOM QD program has been developed as discussed in Chapter IV. Since there are many unanswered questions, it is recommended that the DARCOM QD program be tested at one or more MRCs to determine the degree of success a full QD program can be expected to accomplish and also provide data necessary to make a decision on implementation of a full QD program.

C. <u>IMPLEMENTATION</u>. The test QD program has been initiated at the US Army Missile Materiel Readiness Command based upon Headquarters DARCOM prior approval.

#### APPENDIX

#### BREAK EVEN ANALYSIS

Let:

Q	=	EOQ, the economic order quantity when discounts are not offered.
Qm	=	Q expressed in months of supply.
Qd	=	Quantity Discount buy expressed in months of supply.
d	=	Discount for buying a quantity larger than Q. (Example: $d =$
		0.15 is a 15% discount).
k	=	Factor for quantity procured expressed as a multiple of Q.
		(Example: if buy $3 \times Q$ , then $k = 3$ ).
QD	=	Quantity Discount buy expressed in units ( i.e., $QD = k \cdot Q$ ).
h	=	Factor for variable cost to hold. (Example: if holding cost is
		25% of the item unit price, h = 0.25).
р	=	Variable cost to order in dollars.
AYD	=	Average yearly demand (i.e., average consumption of stock in one
		year).
UP	=	Unit price of item when buying Q.
	F	ware of anolysis second demand is deterministic. Then, if puise

For purposes of analysis assume demand is deterministic. Then, if price does not depend on quantity bought, the acquisition cost is fixed. Minimization includes only the hold and order cost. The optimum Q is given by (1) below:

(1)  $Q = \sqrt{2 \cdot AYD \cdot p/h \cdot UP}$ 

If price is dependent on the quantity bought then acquisition cost is variable and the equation to be minimized becomes:

(2) Cost = acquisition + order + hold.

The alternatives we have are to buy Q or QD. For Q, (2) becomes: AYD Q

(3) Cost = UP·AYD + Q p + 
$$2 \cdot h \cdot UP$$

For QD, (2) becomes:

(4) Cost = (1-d) UP·AYD +  $\frac{AYD}{k.Q} + \frac{k.Q}{2 \cdot h} \cdot (1-d)$  UP

The difference between (3) and (4) is:

(5) Difference =  $d \cdot UP \cdot AYD + \frac{AYD}{Q \cdot P} (1-k) - 2 \cdot h \cdot UP (k-kd-1)$ 

Break-even occurs when the difference is not less than zero. Setting the difference to zero and solving for  $\underline{d}$  gives the break-even value of discount:

(6) 
$$d = \frac{Q}{2 \cdot h \cdot UP} \frac{(k-1)}{(UP \cdot AYD} - \frac{AYD}{Q \cdot p} \frac{k-1}{(k)}$$
 From (1)  $p = \frac{Q^2 \cdot h \cdot UP}{2 \cdot AYD}$ 

This value of  $\underline{p}$  is the same whether we buy Q or QD. Thus, substitution for  $\underline{p}$  in (6) and simplifying, gives.

(7) 
$$d = \frac{q^2 \cdot h \cdot (k-1)^2}{k (AYD + k \cdot 2 \cdot h)}$$

The demand parameter can be eliminated from (7) by writing Q in terms of its equivalent months of supply, Qm

 $Qm = \frac{Q}{AYD}$  12 Substituting in (7) gives

(8) 
$$d = \frac{h \cdot Qm}{(24 + k \cdot h \cdot Qm)} \cdot \frac{(k-1)^2}{k}$$

A typical value for the holding cost factor at the DARCOM MRC's is 25%. Substituting 0.25 for <u>h</u> and Qd/Qm for <u>k</u> gives the simplified, parameter free, result below used in developing table 1 of the main report.

(9) 
$$d = \frac{(Qd - Qm)^2}{Qd (96 + Qd)}$$

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#### STUDY TEAM COMPOSITION

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Economic Order Quantity						
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