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APPROXIMATE CALCULATION OF A FLOW IN AN INCOMPRESSIBLE WAKE CON--ETC(U)  
AUG 78 M P DANILOV, N G ZEMLYANOV, L I LESOV  
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APPROXIMATE CALCULATION OF A FLOW IN AN INCOMPRESSIBLE  
WAKE CONTAINING A SMOOTH THIN PLATE  
IN ITS PLANE OF SYMMETRY

By

M. P. Danilov, N. G. Zemlyanoy, and L. I. Lesov



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# U. S. BOARD ON GEOGRAPHIC NAMES transliteration SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b><i>А а</i></b>	A, a	Р р	<b><i>Р р</i></b>	R, r
Б б	<b><i>Б б</i></b>	B, b	С с	<b><i>С с</i></b>	S, s
В в	<b><i>В в</i></b>	V, v	Т т	<b><i>Т т</i></b>	T, t
Г г	<b><i>Г г</i></b>	G, g	У у	<b><i>У у</i></b>	U, u
Д д	<b><i>Д д</i></b>	D, d	Ф ф	<b><i>Ф ф</i></b>	F, f
Е е	<b><i>Е е</i></b>	Ye, ye; E, e*	Х х	<b><i>Х х</i></b>	Kh, kh
Ж ж	<b><i>Ж ж</i></b>	Zh, zh	Ц ц	<b><i>Ц ц</i></b>	Ts, ts
З э	<b><i>З э</i></b>	Z, z	Ч ч	<b><i>Ч ч</i></b>	Ch, ch
И и	<b><i>И и</i></b>	I, i	Ш ш	<b><i>Ш ш</i></b>	Sh, sh
Й й	<b><i>Й й</i></b>	Y, y	Щ щ	<b><i>Щ щ</i></b>	Shch, shch
К к	<b><i>К к</i></b>	K, k	Ъ ъ	<b><i>Ъ ъ</i></b>	"
Л л	<b><i>Л л</i></b>	L, l	Ы ы	<b><i>Ы ы</i></b>	Y, y
М м	<b><i>М м</i></b>	M, m	Ь ь	<b><i>Ь ь</i></b>	'
Н н	<b><i>Н н</i></b>	N, n	Э э	<b><i>Э э</i></b>	E, e
О о	<b><i>О о</i></b>	O, o	Ю ю	<b><i>Ю ю</i></b>	Yu, yu
П п	<b><i>П п</i></b>	P, p	Я я	<b><i>Я я</i></b>	Ya, ya

\*ye initially, after vowels, and after ъ, ь; e elsewhere.  
When written as ё in Russian, transliterate as yě or ě.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian	English
rot	curl
lg	log

APPROXIMATE CALCULATION OF A FLOW IN  
AN INCOMPRESSIBLE WAKE CONTAINING A  
SMOOTH THIN PLATE IN ITS PLANE OF SYMMETRY

M. P. DANILOV, N. G. ZEMLYANOY, and  
L. I. LESOV

The effect of viscosity on the general nature and macrostructure of a flow can be disregarded for a free turbulence at very large Reynolds numbers. However, in the case of a turbulence near the wall, at any Reynolds number, there is always a zone the nature of flow in which is determined by the viscosity of the fluid and, consequently, by the Reynolds number if the wall is smooth.

Under the conditions of a turbulent flow along hard walls, the turbulence is under the direct influence of the wall in the zone close to it. In the case of a smooth wall, this effect is expressed through the interaction of viscous stresses.

At a certain distance from the wall the direct influence of the viscosity of the fluid on the macrostructure of turbulence can weaken considerably and become negligibly small; thus, in this case, one observes a similarity with a free turbulent flow.

1. We arrange the system of coordinates as shown in Fig. 1.

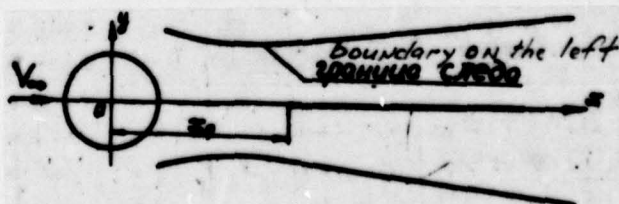


Fig. 1.



Figure 2 taken from [1] shows a hypothetical velocity profile in a turbulent boundary layer

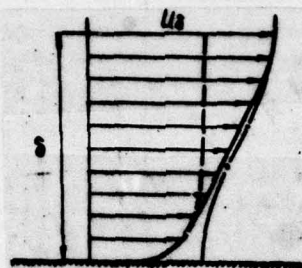


Fig. 2.

- total profile of medium velocity;
- - - velocity distribution according to the law of the wall;
- . - velocity distribution in the wake.

2. We will present the velocity distribution across the wake without a plate in the form of the power series

$$\frac{u}{u_\delta} = \sum_{n=0}^{\infty} a_n \left(\frac{y}{\delta}\right)^n, \quad (1)$$

where  $u$  - longitudinal velocity;  $u_\delta$  - longitudinal velocity at the border of the wake;  $y$  - running ordinate;  $\delta$  - halfwidth of the wake.

We determine the coefficients of the polynomial using the conditions on the axis of the wake ( $y=0$ ) and on its boundary ( $y=\delta$ )

$$\begin{aligned} y=0 \quad u=u_m; \quad \frac{\partial u}{\partial y} &= 0; \quad (u_m - \text{velocity at the wake's axis}) \\ y=\delta \quad u=u_\delta; \quad \frac{\partial u}{\partial y} &= 0; \end{aligned}$$

$$a_0 = \frac{u_m}{u_\delta}; \quad a_1 = 0; \quad a_2 = 3\left(1 - \frac{u_m}{u_\delta}\right); \quad a_3 = -2\left(1 - \frac{u_m}{u_\delta}\right).$$

$$\frac{u}{u_\delta} = \frac{u_m}{u_\delta} + 3\left(1 - \frac{u_m}{u_\delta}\right)\frac{y^2}{\delta^2} - 2\left(1 - \frac{u_m}{u_\delta}\right)\frac{y^3}{\delta^3}.$$

The connection between the halfwidth of the wake  $\delta$  and the velocity at the wake's axis  $u_m$  we establish from the condition of preservation of an excessive pulse (the flow is assumed to be isobaric)

$$\int_0^\delta \frac{u}{u_\delta} \left(1 - \frac{u}{u_\delta}\right) dy = \frac{1}{4} C_x h; \quad (3)$$

$$\delta = \frac{35 u_s^2 C_x h}{2(9 u_s^2 + 17 u_m u_s - 26 u_m^2)} \quad (4)$$

3. In the case of a wake with a plate, we will divide the flow into two regions. In the inner region adjacent to the plate we have a characteristic near-the-wall dependence. The external region represents a turbulent wake. We will carry out the calculation by joining the solutions for the external and internal regions.

The thickness of the region near the wall we will define with the assumption that the plate is streamlined by a homogeneous flow at the velocity  $u_{m1}$  (velocity at the wake's axis when  $x=x_n$ ) and the boundary layer, beginning at the spout, is turbulent

$$\delta_1 = 0.37(x-x_n) \left[ \frac{u_{m1}(x-x_n)}{\nu} \right]^{-1/4}, \quad (5)$$

where  $\nu$  - kinematic coefficient of viscosity. The boundary of the external region  $\delta_2 = \delta + \delta_1^*$ , where  $\delta$  - halfwidth of the wake without the plate;  $\delta_1^*$  - displacement thickness. When  $y=\delta_1$ , using expressions (2) and (4), we obtain the boundary conditions for calculating the internal region

$$a) y=\delta_1 \quad \frac{u_{\delta_1}}{u_s} = \frac{u_m}{u_s} + 3 \left[ \frac{17 - \sqrt{1225 - 1820 C_x h / \delta}}{52} \left( \frac{\delta_1}{\delta} \right)^2 - 2 \left( \frac{35 + \sqrt{1225 - 1820 C_x h / \delta}}{52} \right) \left( \frac{\delta_1}{\delta} \right)^3 \right]; \quad (6)$$

$$\frac{\partial u}{\partial y} \Big|_{y=\delta_1} = 6 u_s \left( \frac{35 + \sqrt{1225 - 1820 C_x h / \delta}}{52} \right) \left( \frac{\delta_1}{\delta} - \frac{\delta_1^2}{\delta^2} \right) = \lambda. \quad (7)$$

b)  $y=0, u=0$ , we find  $\frac{\partial u}{\partial y} \Big|_{y=0}$ .

According to the Newton's hypothesis the shearing stress  $\tau = \mu \frac{\partial u}{\partial y}$ ;

where  $\mu$  - dynamic viscosity coefficient.

On the wall  $\tau_{cr} = \frac{c_f \rho u_s^2}{2}$ , from this  $\frac{\partial u}{\partial y} \Big|_{y=0} = \frac{\tau_{cr}}{\mu} = \frac{c_f u_s^2}{2\nu}$ , where  $c_f$  - friction coefficient. According to the Prandtl formula

$$c_f = 0.0128 \frac{(u_s \delta_1^{**})^{-1/2} u_s^2}{\nu^{1/4}}, \quad \frac{\partial u}{\partial y} \Big|_{y=0} = 0.0064 \frac{(u_s \delta_1^{**})^{-1/2} u_s^2}{\nu^{1/4}} B. \quad (8)$$

To calculate  $\delta_1^{**}$ , in the first approximation we will use a power law for the distribution of the velocity



$$u = u_s \left( \frac{y}{\delta_1} \right)^{1/2}; \quad (9)$$

$$\delta_1^{**} = \int_0^{\delta_1} \frac{u}{u_s} \left( 1 - \frac{u}{u_s} \right) dy = \frac{7}{12} \delta_1. \quad (10)$$

4. Approximating the velocity distribution, when  $0 < y < \delta_1$ , by a power series, we obtain

$$\frac{u}{u_s} = \sum_{n=0}^{\infty} C_n \left( \frac{y}{\delta_1} \right)^n. \quad (11)$$

We determine the  $C_n$  coefficient using the boundary conditions when  $y=0$  and  $y=\delta_1$ . We have

$$\frac{u}{u_s} = B_1 \frac{y}{\delta_1} + (3-2B_1-A_1) \frac{y^2}{\delta_1^2} + (A_1+B_1-2) \frac{y^3}{\delta_1^3}, \quad (12)$$

where  $B_1 = \frac{u}{u_s}$ ;  $A_1 = \frac{1}{u_s} \frac{du}{dy}$ .

In the second approximation, when calculating  $\delta_1^{**}$ , we will use expression (12)

$$\begin{aligned} \delta_1^{**} = \int_0^{\delta_1} \frac{u}{u_s} \left( 1 - \frac{u}{u_s} \right) dy = \int_0^{\delta_1} \left[ B_1 \frac{y}{\delta_1} + (3-2B_1-A_1) \frac{y^2}{\delta_1^2} + (A_1+B_1-2) \frac{y^3}{\delta_1^3} \right] \times \\ \times \left[ 1 - B_1 \frac{y}{\delta_1} + (3-2B_1-A_1) \frac{y^2}{\delta_1^2} + (A_1+B_1-2) \frac{y^3}{\delta_1^3} \right] dy. \end{aligned} \quad (13)$$

The approximations can continue until  $\delta_{1n}^{**} - \delta_{1n+1}^{**} = \Delta$ ; in this case,  $\Delta$  is a relatively small assigned value.

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