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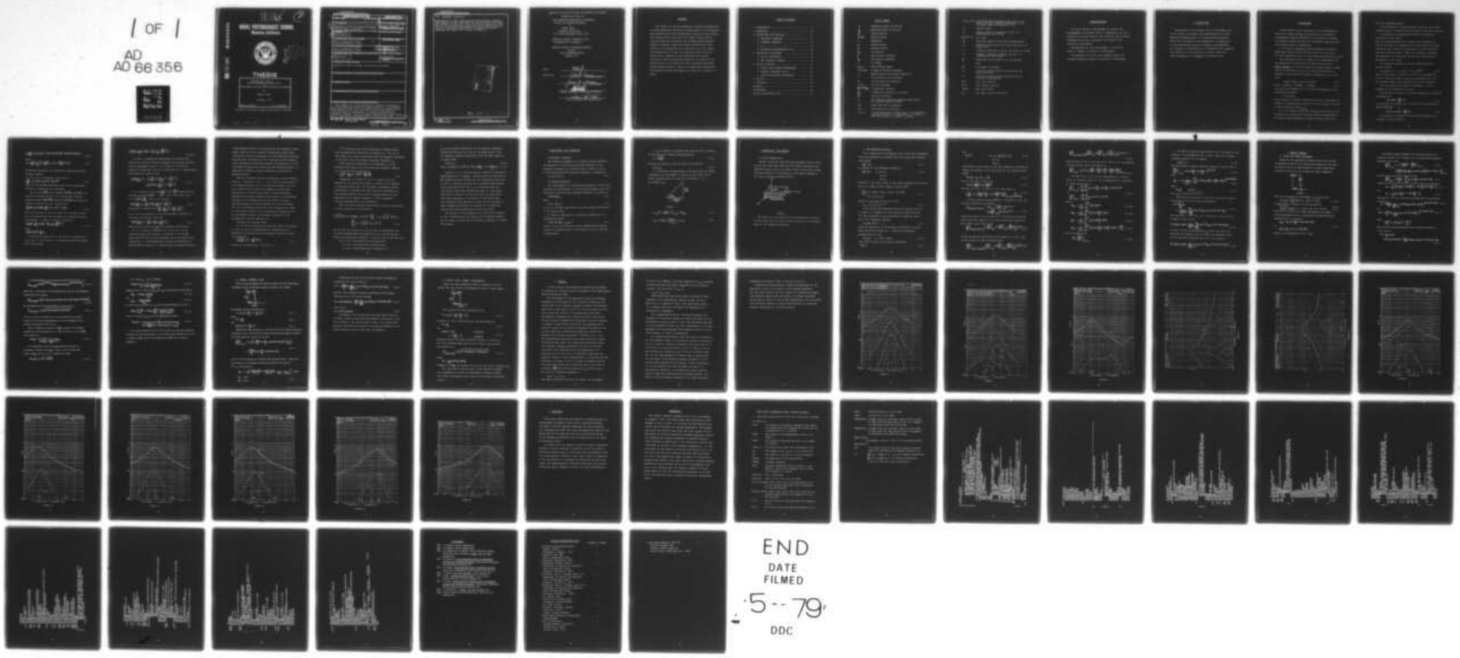

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

THEORETICAL STUDY OF
FINITE AMPLITUDE STANDING WAVES
IN
RECTANGULAR CAVITIES WITH PERTURBED BOUNDARIES

by

Mehmet Aydın

December 1978

Thesis Advisor: A. B. Coppens

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Theoretical Study of

Finite Amplitude Standing Waves in Rectangular Cavities with Perturbed Boundaries

by

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December 1978

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ABSTRACT

The effects of various geometrical boundary perturbations on finite-amplitude acoustical standing waves in a rectangular, rigid-walled cavity were investigated using non-linear theory. The standing waves that exist in an ideal cavity must be corrected when the boundaries are irregular.Three specific examples (stepped, linear and wedged perturbations) were worked out to demonstrate the corrections (in first order) near degeneracies for small perturbations.Those specific examples were compared to the experiments.The present theoretical model qualitatively predicts the effect of the perturbations on the behavior of the nonlinearly generated second harmonic.However, there are unexplained quantitative discrepancies between experiment and theory for a couple of cases.

TABLE OF CONTENTS

LIST OF SYMBOLS

ρ	Instantaneous density of the fluid
ρ_0	Equilibrium density of the fluid
Φ	Velocity potential
$\vec{u}=\nabla\Phi$	Particle velocity
s	Condensation
∇	Gradient operator
$\nabla\cdot$	Divergence operator
$\nabla\times$	Curl operator
η	Shear viscosity coefficient
η_B	Bulk viscosity coefficient
b	$(4/3)\eta/\eta_B$
$\gamma=C_p/C_v$	Ratio of specific heats
$C_0^2=(dP/d\rho)$	At $\rho=\rho_0$ for acoustical processes
C_0	Speed of sound in an unbounded volume of air
P	Instantaneous total pressure
P_0	Equilibrium total pressure
$p=P-P_0$	Acoustic pressure
$\Box^2=\nabla^2-\partial^2/\partial t^2$	D'Lambertian operator
$\Box_L^2$	D'Lambertian operator with losses
∇^2	Laplacian operator
c_p	The frequency dependent apparent phase speed for standing waves in cavity
RHS	Right hand side of equation
LHS	Left hand side of equation
(n,m,l)	A (time-independent) normal mode of a rectangular, rigid-walled cavity of dimensions L_x, L_y and L_z such that $k_x=n\pi/L_x$, $k_y=m\pi/L_y$, $k_z=l\pi/L_z$

$(n,m,l/w,\theta)$	A standing wave designation when the (n,m,l) mode is driven at angular frequency w; θ is the phase angle with respect to t=0.
Q	Quality factor
Q_n	Quality factor at resonance of the n th standing wave when driven
$\beta=(\gamma+1)/2$	For a gas
M_0	Peak Mach number of the driven standing wave
C_n	Effective phase speed associated with the n th normal mode
w	(Angular) frequency at which the cavity is driven
w_n	(Angular) resonance frequency of the n th standing wave when driven
Δ	Magnitude of perturbation on the boundary
t	Time
ϵ	Perturbation parameter
p_0	Classical linear solution for pressure for ideal boundaries
p'	First-order perturbation correction due to boundary irregularities
$\mathbf{1}(t)$	Unit step function
$\delta(t)$	Unit impulse function
$Re\{\ \}$	Real part of $\{\ \}$
a_0	0th order Fourier coefficient

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1. INTRODUCTION

The purpose of this research was to investigate some of the effects of boundary wall perturbations on finite amplitude standing waves in a rigid-walled rectangular cavity.The investigation was prompted by an examination of the experimental results of Coppens and Sanders[3], the research of DeVall[5] and of Kilmer[4],which suggested the existence of the excitation of modes other than those belonging to the family of the driven mode.

2.BACKGROUND

A plane elastic wave travelling in a non-dissipative fluid will change waveform as predicted by the relevant hydrodynamic equations [6],[7].If the problem is extended to absorptive media,only waves of relatively high amplitude will change waveforms appreciably.

At the Naval Postgraduate School,Coppens and Sanders [3],Kilmer [4],and DeVall [5] have dealt with the study of finite amplitude waves in rigid-walled rectangular cavities.

One interesting result of these cavity experiments was the appearence of excitations of modes which were not <u>family members</u> of the driven mode.For example,assume a rigid cavity of dimensions L_x, L_y, L_z is driven acoustically at frequency w,the resultant pressure standing wave is of the form

$$\cos k_x x \cos k_y y \cos k_z z \cos(wt+\theta) \qquad (2.1)$$

where $k_x = N\pi/L_x \ , \ k_y = M\pi/L_y \ , \ k_z = L\pi/L_z \qquad (2.2)$

and N,M,L are integers.Eq.(2.1) can be represented by the notational shorthand

$$(N,M,L/w,\theta)$$

If the cavity is driven to excite the (0,M,0) mode,then the family of standing waves consist of all of those of the form $(0,nM,0/nw,\theta_n)$ when $nw \doteq nw_{0,M,0}$.

As it is stated in [3],"The standing waves which can be excited in any real cavity deviate from the predictions of the linear wave equation with ideal boundary conditions "for the following reasons:

(a).The presence of boundary-layer losses at the cavity surfaces yields a dispersive contribution to the wave equation.

(b).Geometrical irregularities alter the effective dimensions of the cavity.

Both of these mechanism can be treated as equivalent as long as the shift in frequency are so small that the actual resonances are close to the theoretical values resulting from the classical model." These are treated by assuming the dimensions are exact,and the apparent phase speed is determined on that basis.

The resonance frequency for each standing wave is defined as [3]

$$w_n = C_n\left[(n_x\ k_x)^2 + (n_y\ k_y)^2 + (n_z\ k_z)^2\right]^{1/2} \tag{2.3}$$

where k's are given by Eq.(2.2) and n is a shorthand for the set (n_x, n_y, n_z) where n_x, n_y, n_z are integers,and C_n is the apparent phase speed appropriate for that frequency.

The non-linear wave equation applicable to this problem can be obtained as follows:

The continuity equation for wave propagation in Eulerian coordinates is

$$\nabla.(\rho\vec{u}) + \frac{\partial\rho}{\partial t} = 0 \tag{2.4}$$

this equation can be written in terms of the condensation,

$s \equiv (\rho - \rho_0)/\rho_0$ as

$$\nabla.[(1+s)\vec{u}] + \frac{\partial s}{\partial t} = 0 \tag{2.5}$$

The equation of motion in Eulerian coordinates for a contained viscous fluid is

$$\rho\left[\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}\right]=-\nabla P+b\eta\nabla(\nabla.\vec{u})-\eta\nabla\times\nabla\times\vec{u}+ODAT \tag{2.6}$$

where

$$P=\frac{\rho_0 C_0^2}{\gamma}\left(\frac{\rho}{\rho_0}\right)^{\gamma}\doteq\frac{\rho_0 C_0^2}{\gamma}\left[1+\gamma s+\frac{\gamma(\gamma-1)}{2}s^2\right] \tag{2.7}$$

ODAT=Other dispersive and absorptive terms arising from boundary effects.

Eq.(2.6) can be rearrange in the form of

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}+\frac{1}{\rho}\nabla P=\frac{1}{\rho}\mathcal{L}\vec{u} \tag{2.8}$$

where the operator $\mathcal{L}$ describes those physical processes leading to absorption and dispersion.

One can write $\nabla\times\vec{u}=0$ and therefore $\vec{u}=\nabla\Phi$,where Φ is the velocity potential,based on the irrotational velocity assumption.Hence, $\mathcal{L}\vec{u}=\mathcal{L}\nabla\Phi$ .Replacing $\vec{u}=\nabla\Phi$ and using the condensation,Eq.(2.8) can be written as

$$\frac{\partial}{\partial t}\nabla\Phi+\frac{1}{2}\nabla(\nabla\Phi)^2+\frac{C_0^2}{\gamma-1}\nabla(1+s)^{\gamma-1}=\nabla\mathcal{L}\Phi \tag{2.9}$$

Now,with the help of Eq.(2.5) and(2.9), and after a good deal of manipulation the non-linear wave equation may be approximated in terms of velocity potential

$$c_p^2\Box_L^2\Phi\doteq\frac{\partial}{\partial t}\left[(\nabla\Phi)^2+\frac{\gamma-1}{2}\frac{1}{C_0^2}\left(\frac{\partial\Phi}{\partial t}\right)^2\right] \tag{2.10}$$

where

$$c_p^2\Box_L^2\equiv C_0^2\Box^2+\frac{\partial}{\partial t}\mathcal{L}$$

It should be noted that if the fluid is lossless and $c_p=C_0$ then (2.10) reduces to a previously known non-linear wave equation [9]

$$C_0^2 \Box^2 \Phi \doteq \frac{\partial}{\partial t}\left[(\nabla\Phi)^2 + \frac{\gamma-1}{2}\frac{1}{C_0^2}\left(\frac{\partial\Phi}{\partial t}\right)^2\right]$$

(2.10a)

In order to express the approximate non-linear wave equation in terms of acoustic pressure and particle velocity, one can rearrange the Eq.(2.9) in terms of p and $\vec{u}$ and combine that equation with (2.10).The result is a quadratically non-linear wave equation [2]

$$C_p^2 \Box_L^2\left(\frac{p}{\rho_0 C_0^2}\right) \doteq -\frac{1}{2}\frac{\partial^2}{\partial t^2}\left[\gamma\left(\frac{p}{\rho_0 C_0^2}\right)^2 + \left(\frac{\vec{u}}{C_0}\right)^2\right] + \frac{1}{2}C_0^2\nabla^2\left[\left(\frac{p}{\rho_0 C_0^2}\right)^2 - \left(\frac{\vec{u}}{C_0^2}\right)^2\right] \qquad (2.11)$$

If it chances to be that $\left(\frac{p}{\rho_0 C_0^2}\right)^2$ and $\left(\frac{\vec{u}}{C_0}\right)^2$ nearly satisfy the wave equation, $C_0^2\Box^2(\quad) = 0$,then on the RHS of Eq. (2.11) $C_0^2\nabla^2 \doteq \frac{\partial^2}{\partial t^2}$ and (2.11) becomes

$$C_p^2 \Box_L^2\left(\frac{p}{\rho_0 C_0^2}\right) \doteq -\frac{\partial^2}{\partial t^2}\left[\frac{\gamma-1}{2}\left(\frac{p}{\rho_0 C_0^2}\right)^2 + \left(\frac{\vec{u}}{C_0}\right)^2\right] \qquad (2.12)$$

Further,if it happens that $\frac{\partial^2}{\partial t^2}\left(\frac{p}{\rho_0 C_0^2}\right)^2 \doteq \frac{\partial^2}{\partial t^2}\left(\frac{\vec{u}}{C_0}\right)^2$,

as is true for solutions to the wave equation separated in cartesian coordinates, then [2]

$$C_p^2 \Box_L^2\left(\frac{p}{\rho_0 C_0^2}\right) \doteq -\frac{\partial^2}{\partial t^2}\left[\frac{\gamma+1}{2}\left(\frac{p}{\rho_0 C_0^2}\right)^2\right] \qquad (2.13)$$

(Note that this is true only for cartesian coordinates.)

As it is stated in [3]"The LHS of Eq.(2.13) is the classical,linear wave equation pertinent to the system under study.The RHS can be interpreted as a forcing function consisting of a three-dimensional spatial distribution of phase-cohorent sources.In a second-order perturbation theory, "this forcing function is obtained from the classical (first-order) solution of the acoustic problem.The second-order perturbation solution describes the nonlinearities resulting from the self interaction of the classical solution.Higher-order perturbation solutions consider the interaction of the non-linear solution with itself,and the forcing function is composed of products of both classical and nonlinearly generated terms.

Thus,if a system is driven at frequency w,the nonlinear term in..."equation (2.13) "... will force the existence of all integer multiples nw of the driving frequency and the full solution must contain all harmonics of the input frequency.In a closed cavity,each of those nonlinearly generated waves whose frequency lies close to the resonance frequency of a standing wave of the cavity and whose associated spatial function matches that of the standing wave can be strongly excited.Just how strongly will depend on the quality factor Q for the particular resonance and the difference between the resonance frequency of the standing wave and the harmonic nw.....

"Consider two limiting cases.

"(1) If the forcing function does not have its frequency nw close to w_n,this standing wave is being forced at a frequency far removed from its resonance.Since this yields the inequality

$$\left| c_0^2 \Box^2 p \right| \gg \left| \frac{\partial}{\partial t} \mathcal{L} p \right| \tag{2.14}$$

losses can be ignored in..."Eq.(2.13).

"(2) If $nw \sim w_n$, then the standing wave is being forced near resonance, and losses must be retained in..."Eq.(2.13). "The value of C_n can be determined from the apparent dimensions of the cavity and the measured resonance frequency w_n.

"The losses are described by the measured Q_n of the resonance. This means that the linear-wave equation operator for the system can be written as

$$C_0^2 \Box^2 + \frac{\partial}{\partial t} \mathcal{L} \doteq C_n^2 \Box^2 - \frac{nw}{Q_n} \frac{\partial}{\partial t} \tag{2.15}$$

"Comparison of cases (1) and (2) reveals that the responce of the cavity when $nw \not\sim w_n$ is order of 1/Q compared to that when $nw \sim w_n$. Thus for the high-Q resonances usually encountered in cavities with rigid walls, the components in the forcing function which excite standing waves far from resonance can be ignored compared to those components exciting standing waves near resonance....

"The non-linear, coupled, transcendental equation appliciable to this problem can be expressed as

$$R_n \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} (\phi_n - \theta_n) = N_0 M_0 \beta Q_n \cos\theta_n \Bigg[1/2 \sum_{j=1}^{n-1} R_j R_{n-j} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} (\phi_j + \phi_{n-j}) - \sum_{j=1}^{\infty} R_{n+j} R_j \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} (\phi_{n+j} - \phi_j) \Bigg] \tag{2.16}$$

for all $n>1$. The values of Q_n and w_n must be determined from the infinitesimal-amplitude behavior of the cavity. The Mach number M_0 and driving frequency w are known and N_0 has the value

N_0=1/2 for a one-dimensional standing wave

1/4 for a two-dimensional standing wave

1/8 for a three-dimensional standing wave."

R_n is the Fourier coefficient of nth harmonic component, normalized such that $R_1=1$. ϕ_n is the phase angle of the nth harmonic component, where $\phi_1=0.$, and the phase angle θ_n is given by [3]

$$\tan\theta_n = -F_n \qquad (2.17)$$

where $F_n = Q_n\left[(nw)^2 - w_n{}^2\right]/(nw)^2 \doteq 2Q_n\left(\frac{nw-w_n}{w_n}\right)$ for $\frac{nw-w_n}{w_n} \ll 1$ (2.18)

Equation (2.16) can be solved by a method of successive approximations on a digital computer. This has been done by [3] and [5] and both decided that the theoretical model can be used to identify the modes of a non-ideal, rigid-walled cavity provided quantities e_n to be defined later are sufficiently small. The theoretical model in its present form fails to account for the excitation of modes other than those belonging to the family of the driven mode. This excitation was observed to occur only in the case of nearly degenerate modes. It is believed to be caused by some linear coupling mechanism within the cavity.

The purpose of this research is to see if the presence of wall irregularities can explain how non-family members may be strongly excited, and to present an example to support this theory.

3. DEFINITIONS AND NOTATIONS

A. FREQUENCY PARAMETER

The frequency parameter is a quantity which indicates the position of the driving frequency relative to the resonance frequency, f_1, of driven mode in terms of the Q_1 of the driven mode. The frequency parameter is defined by

$$F_1=2Q_1(f-f_1)/f_1 \tag{3.1}$$

B. STRENGTH PARAMETER

The investigation of the pressure waveform in the cavity required the calculation of the strength parameter from the observable quantities. The strength parameter is defined as

$$STRPM=M_0\beta Q_1 \tag{3.2}$$

where

$$M_0=P_1/(\rho_0 c_0^{\,2}) \tag{3.3}$$

and P_1 is the peak amplitude of p_1, the pressure distribution of the driven mode.

In terms of observable or calculable quantities it is reformulated as [5]

$$STRPM=\sqrt{2}\, V\beta Q_1/(S_m\rho c_0)^2 \tag{3.4}$$

where V and S_m are the rms voltage reading and microphone sensitivity respectively of the receiver used to sense the standing wave.

C. e_n is defined to indicate the position of w_n relative to the classical harmonic frequencies, nw_1.

$$e_n = \frac{w_n - nw_1}{nw_1} \tag{3.5}$$

and one can relate e_n with F_n from (2.18) such as

$$F_n \doteq 2Q_n e_n \tag{3.6}$$

D. A pictorial representation of F_n which will be useful throughout the development is given in Fig.1. From now on three subscripts will be used for convenience. i.e. F_n becomes F_{nml}.

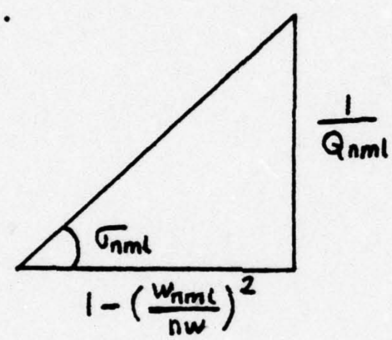


FIGURE 1

$$F_{nml} = \left[1-\left(\frac{w_{nml}}{nw}\right)^2\right] Q_{nml} = \cot\sigma_{nml} \tag{3.7}$$

$$Q_{nml} \sin\sigma_{nml} = \frac{Q_{nml}}{(1+F^2_{nml})^{1/2}} = S_{nml} \tag{3.8}$$

4. THEORETICAL DEVELOPMENT

A. CAVITY DESCRIPTION

Assume a perfectly rigid-walled rectangular cavity which has one wall perturbed such that the cavity dimensions are $L_x[1+\epsilon f(y,z)]$, L_y and L_z as shown in Fig.2 below. Also assume the perturbation on the boundary is very small compared to the cavity dimensions, $|\epsilon f(y,z)| \ll 1$.

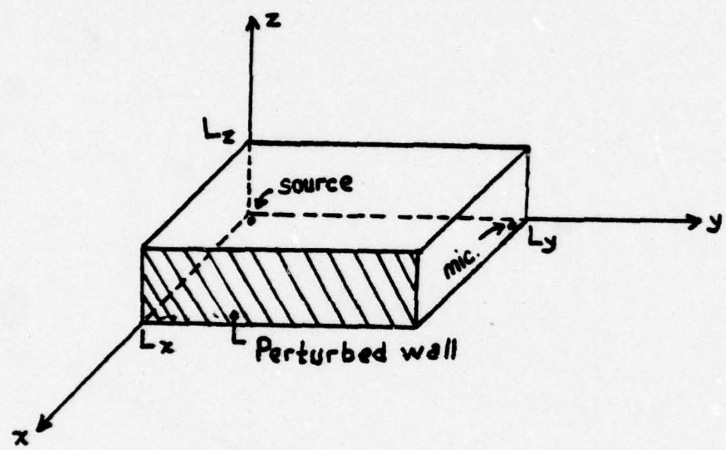


FIGURE 2

The cavity is to be excited by a source near the origin in such a way that the (N,M,L) mode is driven at a frequency close to its resonance frequency.

B. THE PERTURBED BOUNDARY

For a rigid-walled rectangular cavity with ideal boundaries (ϵ=0),the pressure p_0 obtained from the linear wave equation with losses

$$\square_L^2 \, p_0=0 \qquad (4.1)$$

is subject to the following conditions,

$$\nabla p_0 \cdot \vec{n}=0 \quad \text{at } x=0,L_x \quad y=0,L_y \quad z=0,L_z \qquad (4.2)$$

where $\vec{n}$ is the local normal to the ideal boundary.The solution for p_0 in terms of Mach number is given by[4]

$$\frac{p_0}{\rho_0 c_0{}^2}=M_0 \cos k_x x \cos k_y y \cos k_z z \cos(wt+\theta) \qquad (4.3)$$
$$=M_0 \, (N,M,L/w,\theta)$$

where k's are given in Eq.(2.2), and

$$w=c_p \, (k_x{}^2+k_y{}^2+k_z{}^2)^{1/2} \qquad (4.4)$$

If the cavity has perturbed walls,the solution will be in terms of a summation of the classical linear solution for ideal boundaries plus perturbation correction terms due to the irregular boundary:

$$p=p_0+\epsilon p'+\epsilon^2 p''+\ldots.. \qquad (4.5)$$

Since the magnitude of the boundary perturbation is kept small,second and higher terms in ϵ can be considered insignificant,so that

$$p=p_0+\epsilon p' \quad \text{(to first order)} \qquad (4.6)$$

and p must satisfy the following conditions,

$$\square^2 p=0 \qquad (4.7)$$

and

$$\nabla p \cdot \vec{n} = 0 \qquad \text{at } x=0, L_x[1+\epsilon f(y,z)] \qquad (4.8)$$
$$y=0, L_y$$
$$z=0, L_z$$

where $\vec{n}$, the local normal to the real surface, is obtained by taking the gradient of the equation for the boundary, given by [2]

$$\vec{n} = \nabla\left\{x - L_x[1+\epsilon f(y,z)]\right\} \qquad (4.10)$$

Thus, to the first order in ϵ,

$$\vec{n} = \hat{x} - \epsilon L_x \frac{\partial f(y,z)}{\partial y}\hat{y} - \epsilon L_x \frac{\partial f(y,z)}{\partial z}\hat{z} \qquad (4.11)$$

and when Eq.(4.11) is used in (4.8) the result is

$$\left[\frac{\partial p}{\partial x} - \epsilon L_x \frac{\partial f(y,z)}{\partial y}\frac{\partial p}{\partial y} - \epsilon L_x \frac{\partial f(y,z)}{\partial z}\frac{\partial p}{\partial z}\right]_{x=L_x[1+\epsilon f(y,z)]} = 0 \qquad (4.12)$$

A Taylor series expansion [4] for p evaluated at the real boundary $L_x[1+\epsilon f(y,z)]$ produces

$$p\Big|_{x=L_x[1+\epsilon f(y,z)]} = p\Big|_{x=L_x} + \frac{\partial p}{\partial x}\Big|_{x=L_x}\epsilon L_x f(y,z)$$
$$+ \frac{1}{2}\frac{\partial^2 p}{\partial x^2}\Big|_{x=L_x}[\epsilon L_x f(y,z)]^2 + \cdots \qquad (4.13)$$

Substituting Eq.(4.6) into RHS of (4.13), taking the partial derivative with respect to x on both sides and keeping the first-order terms in ϵ, yields

$$\frac{\partial p}{\partial x}\Big|_{x=L_x[1+\epsilon f(y,z)]} = \frac{\partial p_0}{\partial x}\Big|_{x=L_x} + \epsilon\frac{\partial p'}{\partial x}\Big|_{x=L_x} + \frac{\partial^2 p_0}{\partial x^2}\Big|_{x=L_x}\epsilon L_x f(y,z) + \cdots \qquad (4.14)$$

Taking the partial derivatives with respect to y and z and using exactly the same procedure gives

$$\frac{\partial p}{\partial y}\Big|_{x=L_x[1+\epsilon f(y,z)]} = \frac{\partial p_0}{\partial y}\Big|_{x=L_x} + \epsilon\frac{\partial p'}{\partial y}\Big|_{x=L_x} + \frac{\partial^2 p_0}{\partial y\partial x}\Big|_{x=L_x}\epsilon L_x f(y,z) + \cdots \qquad (4.15)$$

$$\left.\frac{\partial p}{\partial z}\right|_{x=L_x[1+\epsilon f(y,z)]} = \left.\frac{\partial p_0}{\partial z}\right|_{x=L_x} + \epsilon \left.\frac{\partial p'}{\partial z}\right|_{x=L_x} + \left.\frac{\partial^2 p_0}{\partial z \partial x}\right|_{x=L_x} \epsilon L_x f(y,z) + \cdots\cdots \tag{4.16}$$

Substituting (4.14),(4.15) and (4.16) into (4.12) and keeping the first-order terms in ϵ results in

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = L_x\left[-f(y,z)\frac{\partial^2 p_0}{\partial x^2} + \frac{\partial f(y,z)}{\partial y}\frac{\partial p_0}{\partial y} + \frac{\partial f(y,z)}{\partial z}\frac{\partial p_0}{\partial z}\right]_{x=L_x} \tag{4.17}$$

The RHS of Eq.(4.17) can be represented as a Fourier series in cosines,so that p' can be expressed as a summation of normal modes.Hence,

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} a_{ml} \cos\frac{m\pi}{L_y}y \cos\frac{l\pi}{L_z}z \cos(wt+\theta)$$

or (4.18)

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} a_{ml}\,(0,m,l/w,\theta)$$

where

$$a_{00} = \frac{1}{L_y L_z}\int_0^{L_y}\int_0^{L_z}[G]\,dy\,dz \tag{4.19a}$$

$$a_{m0} = \frac{2}{L_y L_z}\int_0^{L_y}\int_0^{L_z}[G]\cos\frac{m\pi}{L_y}y\,dy\,dz \tag{4.19b}$$

$$a_{0l} = \frac{2}{L_y L_z}\int_0^{L_y}\int_0^{L_z}[G]\cos\frac{l\pi}{L_z}\,dy\,dz \tag{4.19c}$$

$$a_{ml} = \frac{4}{L_y L_z}\int_0^{L_y}\int_0^{L_z}[G]\cos\frac{m\pi}{L_y}y\cos\frac{l\pi}{L_z}z\,dy\,dz\ ,\ \begin{matrix}m\neq 0\\ l\neq 0\end{matrix} \tag{4.19d}$$

and G is defined as

$$G \equiv \left.\frac{\partial p'}{\partial x}\right|_{x=L_x} \tag{4.19e}$$

In order to find the contribution to p' from each of the terms, it is stated[2] that for a cavity forced by a dynamic boundary condition at a boundary

$$c_p^2 \Box_L^2 p' = 0 \qquad (4.20)$$

and the dynamic boundary condition from the (m,l)th term is

$$\left.\frac{\partial p'_{ml}}{\partial x}\right|_{x=L_x} = A \cos\frac{m\pi}{L_y}y \cos\frac{l\pi}{L_z}z \; e^{i(wt+\theta)} \qquad (4.21)$$

then

$$p'_{ml} = -A \frac{1}{\left(\frac{w}{C_o}\right)^2 L_x} \sum_{n=0}^{\infty} \Delta_n (-1)^n S_{nml} \cos\frac{n\pi}{L_x}x \cos\frac{m\pi}{L_y}y \cos\frac{l\pi}{L_z}z \; e^{i(wt+\theta+\sigma_{nml})} \qquad (4.22)$$

where

$$\Delta_n = \begin{cases} 1 & \text{if } n=0 \\ 2 & \text{if } n=1,2,3,\ldots. \end{cases} \qquad (4.23)$$

and S_{nml} is given by Eq.(3.8).

Applying this solution to Eq.(4.18) gives the complete solution for the first-order perturbation

$$p' = \sum_{\substack{m=0 \\ l=0}}^{\infty} p'_{ml}$$

$$= -\frac{1}{\left(\frac{w}{C_o}\right)^2 L_x} \sum_{\substack{n=0 \\ m=0 \\ l=0}}^{\infty} a_{ml} \Delta_n (-1)^n S_{nml} (n,m,l / w, \theta + \sigma_{nml}) \qquad (4.24)$$

and combining (4.24) with (4.6) yields the total acoustic pressure in the cavity.

$$p = (N,M,L/W,\theta) - \frac{\epsilon}{\left(\frac{w}{C_o}\right)^2 L_x} \sum_{\substack{n=0 \\ m=0 \\ l=0}}^{\infty} a_{ml} \Delta_n (-1)^n S_{nml} (n,m,l / w, \theta + \sigma_{nml}) \qquad (4.25)$$

If this is near a resonance, $w \sim w_{nml}$, then this term will dominate the summation and all other non-degenerate terms can be omitted. Consequently, the Eq.(4.25) becomes

$$p = (N,M,L/W,\theta) - \frac{\epsilon}{\left(\frac{w}{C_o}\right)^2 L_x} a_{ml} \Delta_n (-1)^n S_{nml} (n,m,l/w, \theta + \sigma_{nml}) \qquad (4.26)$$

5. SPECIFIC EXAMPLES

A. CAVITY WITH STEPPED PERTURBATION

Assume that the rigid-walled rectangular cavity given in Fig.2 is perturbed as shown in Fig.3 below, and also assume that the cavity is driven in the (0,1,0) mode resulting (0,1,0/w,θ) standing wave and that the (0,2,0) and (1,0,0) modes are (nearly) degenerate.

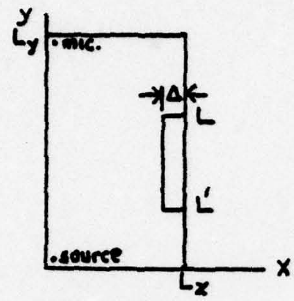


FIGURE 3

From Fig.3 the equation for the boundary at L_x can be found,

$$x = L_x\left\{1 - \frac{\Delta}{L_x}\left[\mathbb{1}(y-L') - \mathbb{1}(y-L)\right]\right\} \tag{5A.1}$$

By means of Eq.(4.8) ϵ and f(y,z) can be written as

$$\epsilon = \Delta/L_x \tag{5A.2}$$

$$f(y,z) = -\left[\mathbb{1}(y-L') - \mathbb{1}(y-L)\right] \quad \text{at} \quad L' < y < L \tag{5A.3}$$

Since the (0,2,0) and (1,0,0) modes are degenerate the emphasis of this development will be on these particular modes. The pressure distribution of (0,2,0) mode is

$$p_{020} = P_2 \cos\frac{2\pi}{L_y} y \cos(2wt + \theta_2)$$

or (5A.4)

$$p_{020} = P_2\,(0,2,0/2w,\theta_2)$$

where P_2 is the amplitude of (0,2,0) mode.

Utilizing the theory developed in the preceeding sections and using the equations (4.17) through (4.26), the first-order perturbation correction can be found

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = L_x\left[\frac{\partial f(y,z)}{\partial y}\quad\frac{\partial p_{020}}{\partial y}\right]_{x=L_x} \tag{5A.5}$$

$$= \frac{2\pi P_2 L_x}{L_y}\left\{\left[\sin\frac{2\pi}{L_y}y\cos(2wt+\theta_2)\right]\left[\delta(y-L')-\delta(y-L)\right]\right\}_{x=L_x}$$

Eq.(5A.5) can be written as a Fourier series

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = \frac{2\pi P_2 L_x}{L_y}\sum_{m=0}^{\infty} a_m \cos\frac{m\pi}{L_y}y\cos(2wt+\theta_2) \tag{5A.6}$$

Inversion of the Eq.(5A.5) and (5A.6) yields the Fourier coefficients

$$a_m = \frac{2}{L_y}\left[\sin\left(\frac{2\pi}{L_y}L'\right)\cos\left(\frac{m\pi}{L_y}L'\right) - \sin\left(\frac{2\pi}{L_y}L\right)\cos\left(\frac{m\pi}{L_y}L\right)\right] \tag{5A.7}$$

and

$$a_0 = \frac{1}{L_y}\left[\sin\left(\frac{2\pi}{L_y}L'\right) - \sin\left(\frac{2\pi}{L_y}L\right)\right] \tag{5A.8}$$

Recalling Eq.(4,23) and (4.24), first order perturbation correction, p', is found as

$$p' = -\frac{2\pi P_2 L_x}{L_y}\frac{a_0}{\left(\frac{2w}{C_0}\right)^2 L_x}(2)(-1)^1 Q_{100}\sin\sigma_{100}\cos\frac{\pi x}{L_x}\,e^{i(2wt+\theta_2+\sigma_{100})} \tag{5A.9}$$

and

$$\epsilon p' = \frac{4\pi P_2}{L_y}\epsilon a_0\frac{1}{\left(\frac{2w}{C_0}\right)^2}Q_{100}\sin\sigma_{100}\cos\frac{\pi x}{L_x}\,e^{i(2wt+\theta_2+\sigma_{100})} \tag{5A.10}$$

where

$$\left(\frac{2w}{C_0}\right)^2 = (2k_{100})^2 = \left(\frac{4\pi}{L_y}\right)^2$$

Hence, the total pressure, associated with the angular frequency 2w, in the cavity is

$$p = p_{020} + \epsilon p'$$

$$= P_2(0,2,0/2w,\theta_2) + \frac{P_2 L_y}{4\pi}\epsilon a_0 Q_{100}\sin\sigma_{100}\,(1,0,0/2w,\theta_2+\sigma_{100}) \tag{5A.11}$$

The total pressure at the microphone position, x=0 and $y=L_y$, is

$$p\Big|_{mic.\,position} = P_2\, \mathrm{Re}\left\{ e^{i(2wt+\theta_2)} + \left[\frac{L_y}{4\pi}\epsilon\partial_0 Q_{100}\sin\sigma_{100}\right] e^{i(2wt+\theta_2+\sigma_{100})} \right\} \quad (5A.12)$$

Define B=[ ] and after a little manipulation and use of trigonometric identities, (5A.12) becomes

$$p\Big|_{mic.\,position} = P_2\left\{ (1+B\cos\sigma_{100})\cos(2wt+\theta_2) - (B\sin\sigma_{100})\sin(2wt+\theta_2) \right\} \quad (5A.13)$$

and the amplitude of the total pressure in the cavity is

$$P\Big|_{mic.\,position} = P_2\sqrt{(1+B\cos\sigma_{100})^2 + (B\sin\sigma_{100})^2} \quad (5A.14)$$

Eq.(5A.14) is the corrected value of the amplitude of the second harmonic of the driving mode, obtained by Eq.(2.16), because of the boundary irregularity given in Fig.3.

Now, it is desired to express $\sin\sigma_{100}$ in terms of the frequency parameter of the driving mode, (0,1,0). With the help of Fig.1, $\sin\sigma_{100}$ can be written as

$$\sin\sigma_{100} = \frac{1}{\left\{1+Q_{100}^2\left[1-\left(\frac{f_{100}}{2f}\right)^2\right]^2\right\}^{1/2}} \quad (5A.15)$$

If f approaches to zero then σ_{100} approaches to π, and if f approaches to infinity then σ_{100} is close to zero. In these same limits $\cos\sigma_{100}$ goes to -1 and +1 respectively. Hence

$$\cos\sigma_{100} = \pm\sqrt{1-\sin^2\sigma_{100}} \quad (5A.16)$$

For $2f \sim f_{100}$,(5A.15) becomes

$$\sin\sigma_{100} \doteq \frac{1}{\left\{1+\left(2Q_{100}\frac{2f-f_{100}}{f_{100}}\right)^2\right\}^{1/2}} \tag{5A.17}$$

Recalling Eq.(3.1) and (3.5),F_{100} and e_{100} can be written in the form of

$$F_{010} = 2Q_{010}\frac{f-f_{010}}{f_{010}} \tag{5A.18}$$

and

$$e_{100} = \frac{f_{100}-2f_{010}}{2f_{010}} \tag{5A.19}$$

Eq.(5A.19) can be solved for f_{100} and this substituted into (5A.17)

$$2Q_{100}\frac{2f-f_{100}}{f_{100}} = 2Q_{010}\frac{Q_{100}}{Q_{010}}\frac{f-f_{010}(1+e_{100})}{f_{010}(1+e_{100})} \tag{5A.20}$$

Use of $1/(1+e_{100}) \doteq 1-e_{100}$ and little manipulation reveals

$$\sin\sigma_{100} \doteq \frac{1}{\sqrt{1+\left\{\frac{Q_{100}}{Q_{010}}\left[F_{010}-2Q_{010}e_{100}\right](1-e_{100})\right\}^2}} \tag{5A.21}$$

As a result,equations (5A.14),(5A.16),(5A.21) are the final amplitude correction of the second harmonic of the driving mode obtained by Eq.(2.16) . A computer program for this was developed by author and is given in appendix A.

B. LINEARLY PERTURBED CAVITY

Using the same assumptions in section A, assume that the rigid-walled rectangular cavity is perturbed linearly as shown in Fig.4 below.

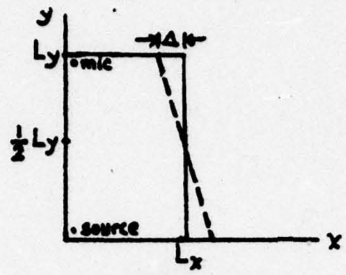


FIGURE 4

The equation for this perturbation is

$$x = L_x\left[1+\frac{\Delta}{L_x}\left(1-\frac{2}{L_y}y\right)\right] \tag{5B.1}$$

Hence,

$$\epsilon = \frac{\Delta}{L_x} \tag{5B.2}$$

and

$$f(y,z) = \left(1-\frac{2}{L_y}y\right) \tag{5B.3}$$

Applying the same procedure as in section A, the first-order perturbation correction and the total acoustic pressure associated with angular frequency 2w in that particular cavity can be found

$$\left.\frac{\partial p'}{\partial x}\right|_{x=L_x} = L_x\left\{\left[-\frac{2\pi P_2}{L_y}\sin\frac{2\pi}{L_y}y\cos(2wt+\theta_2)\right]\left(-\frac{2}{L_y}\right)\right\}$$

$$= \frac{4\pi P_2 L_x}{L_y^2}\sin\frac{2\pi}{L_y}y\cos(2wt+\theta_2) \tag{5B.4}$$

Eq.(5B.4) can be written as a Fourier series and the Fourier coefficient, a_m, is found by an integration procedure evaluated in the interval 0 to L_y. The result is

$$a_m = -\frac{1}{\pi}\left\{\frac{\cos(2-m)\pi}{(2-m)}+\frac{\cos(2+m)\pi}{(2+m)}-\frac{1}{(2-m)}-\frac{1}{(2+m)}\right\}, m\neq 2$$

$$a_0 = 0.0$$

$$a_2 = 0.0 \tag{5B.5}$$

Recalling the Eq. (4.24), the first-order perturbation correction for (0,2,0) mode is

$$p' = -\frac{4\pi\beta_2 L_x}{L_y^2}\frac{a_0}{\left(\frac{2w}{C_0}\right)^2 L_x}(2)(-1)^1 S_{100}(1,0,0/2w,\theta_2+\sigma_{100}) \quad (5B.6)$$

and the total acoustic pressure associated with angular frequency 2w in the cavity becomes

$$p = (0,2,0/2w,\theta_2) - \frac{4\pi\beta_2}{L_y^2}\frac{Ga_0}{\left(\frac{2w}{C_0}\right)^2}(2)(-1)^1 S_{100}(1,0,0/2w,\theta_2+\sigma_{100}) \quad (5B.7)$$

since $a_0=0.0$

$$p = (0,2,0/2w,\theta_2) \quad (5B.8)$$

According to the calculation developed above there is no need to make a first-order perturbation correction to the (0,2,0) mode in the cavity shown in Fig.4. As a result, the pressure distribution is equal to the second harmonic of the driven mode, since $a_0=0.0$ and this yields $p'=0.0$

C. CAVITY WITH WEDGED PERTURBATION

Under the same assumptions made in sections A and B, assume that the cavity is perturbed as shown in Fig.5 below.

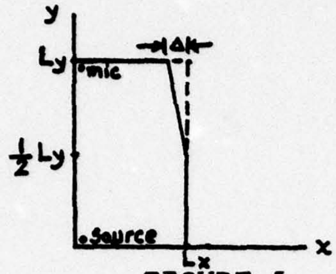


FIGURE 5

The equation for this perturbation is

$$x = L_x \left[1 - \frac{\Delta}{L_x} \left(\frac{2y}{L_y} - 1 \right) \right] \tag{5C.1}$$

By means of Eq.(4.8) ϵ and f(y,z) can be written as

$$\epsilon = \frac{\Delta}{L_x} \tag{5C.2}$$

and

$$\begin{aligned} f(y,z) &= 0.0 \qquad\qquad , \ y \leq L_y/2 \\ &= -\left(\frac{2y}{L_y} - 1\right) \qquad , \ y > L_y/2 \end{aligned} \tag{5C.3}$$

Applying exactly the same procedure followed in section A, the total pressure amplitude in the cavity(in first-order perturbation) is

$$P\Big|_{mic.\ position} = P_2\sqrt{(1 + B\cos\sigma_{100})^2 + (B\sin\sigma_{100})^2} \tag{5C.4}$$

where

$$B = \frac{1}{2\pi} \partial_0 \epsilon Q_{100} \sin\sigma_{100}$$

$\sin\sigma_{100}$ and $\cos\sigma_{100}$ are given by Eq.(5A.21) and (5A.16) respectively.

The theoretical predictions of these specific examples were examined with series of experiments developed by [8]. The further discussions about these will be given in the next section.

6. RESULTS

In this section the theoretical predictions performed in sections 5A, 5B and 5C will be compared to the experimental results obtained by [8].

The information on the empirical losses and resonance frequencies is contained in the Q's and e's.These are the values used in the computer program to predict the harmonic distortion on the basis of Eq.(2.16) and is plotted as thin solid curves.The results of including the first-order perturbation correction are plotted as thick solid curves for each specific example.The theoretical curves in figures 6 through 16 were plotted along with the experimentally-measured values for the cavity configurations shown on top of each figure.The theoretically-predicted values were generated for frequency-parameter intervals of 0.2, and the experimentally-measured values were plotted as square-blocks.Data were taken, and theoretical predictions made, for different strength parameters for the (0,n,0) mode associated with different cavity configurations.It is important to note that the n=2 distortion peaks when the system is driven at this frequency.That is, when the driving frequency w is equal to (1/2) w_2,there is maximum content of P_2.The point where this occurs for each P_n/P_1 curve is indicated by an arrow with the label of F_{020}.At that point the value of frequency parameter is

$$F_{020}=2\ Q_{010}\ e_{020}$$

(The same thing could be done, of course, for any member of the (0,n,0) family). The arrow labelled as F_{100} indicates the position where the nearly degenarate (1,0,0) mode is resonant, and the value of F_{100} is

$$F_{100}=2\ Q_{010}\ e_{100}$$

The theoretical predictions made in section 5B were compared to the experimental results as seen in Fig.7. When Fig.7 is compared to Fig.6, the unperturbed cavity, it is clearly seen that the theory and experiment are excellently in agreement.

For a wedged perturbation, the theory predicts the frequency of the second harmonic at which the effect of the perturbation occurs as seen in Fig.8.The predicted magnitude of the perturbation effect for this configuration is in good agreement with the experiment.The anomolous behavior of the third harmonic in Fig.8 is unexplained.

For stepped perturbation, when the cavity is perturbed, but the geometry leads to no predicted correction as seen in Fig.9 or leads to predicted correction less than about 0.02 as in Fig.12 or less than about 0.05 as in Fig.14, then it was observed that there was very little or no effect from the (1,0,0) mode.Agreement for these cases is good except for the region lying between frequency parameter 4 and 9 in Fig.9.What happened in that region is also unexplained, but it was observed one time only.When the amount of perturbation correction is increased the theory predicts effects larger than experimentally observed.However, the effect of the perturbation appears at the right frequency parameter as is seen in Fig.'s 10,11,13,15 and 16.

Choosing the shim position, length and magnitude is very important as well as is choice of the strength parameter. For the shims, Δ=0.04 and 0.25 inches for stepped and wedged perturbations respectively.The effect of strength parameter can be seen in Fig.'s 15 and 16.The experimental data associated with the third harmonic in Fig.15 were believed to come from harmonic distortion in the piston motion.

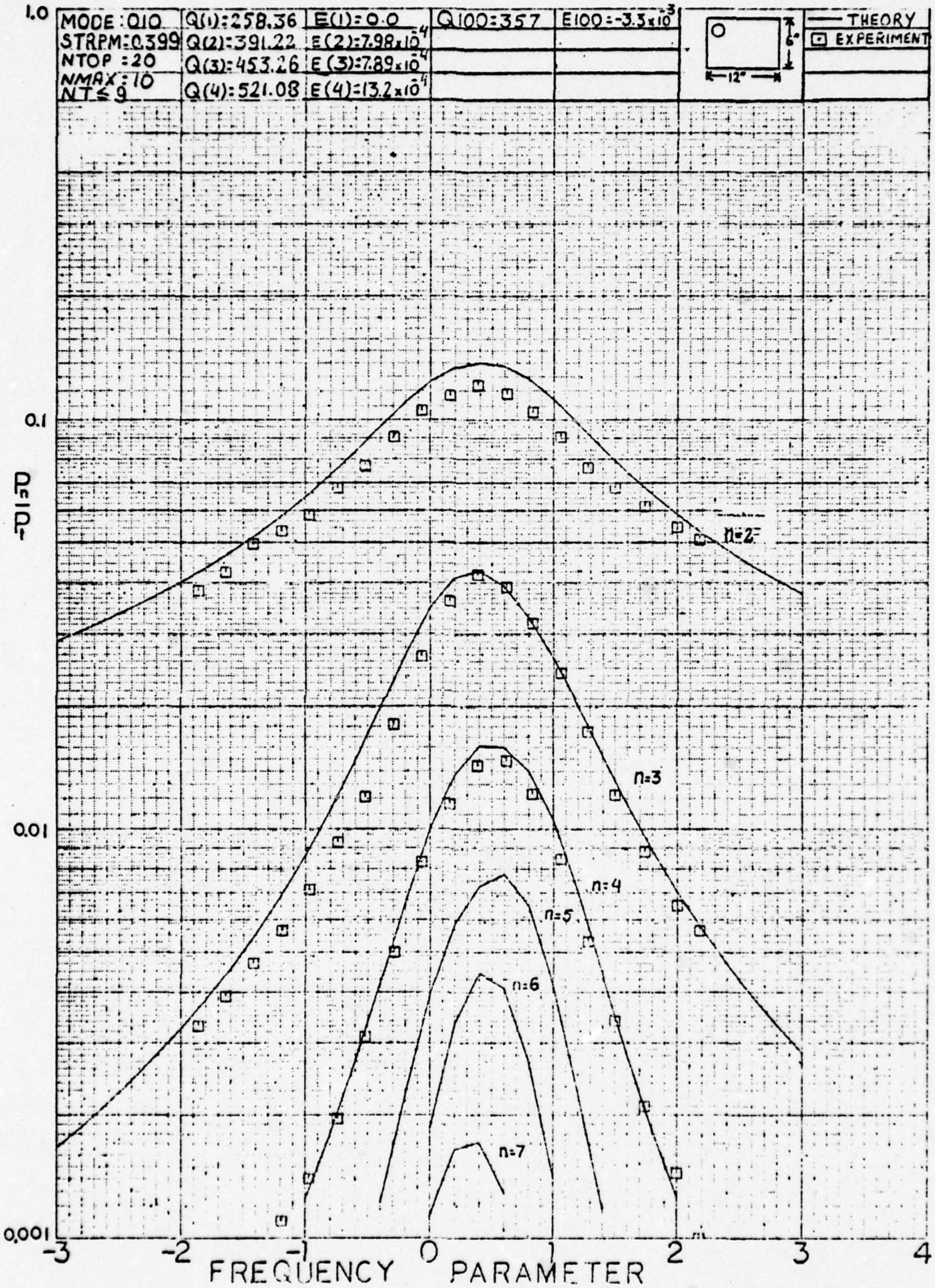
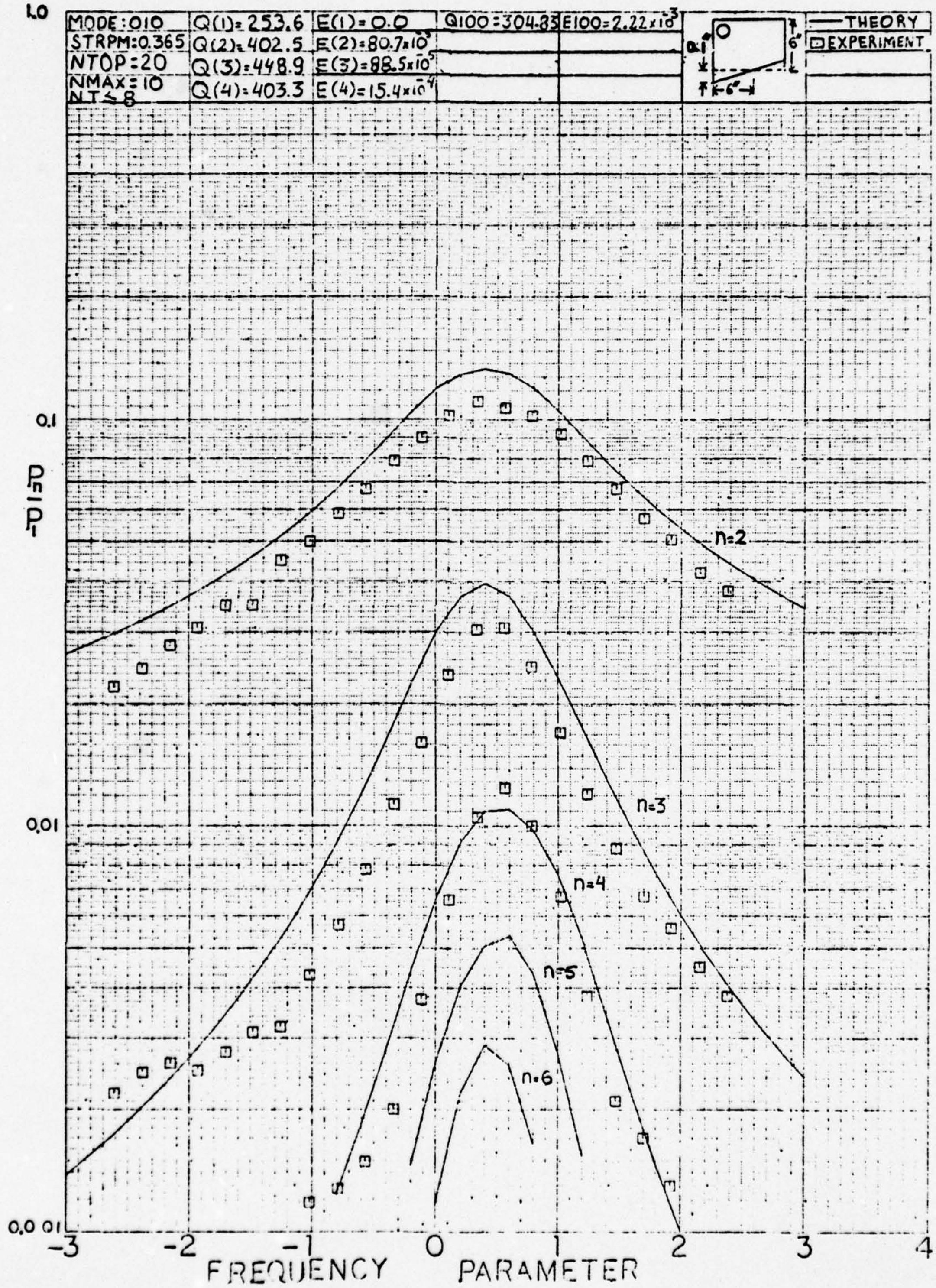

MODE: 010
STRPM: 0.399
NTOP: 20
NMAX: 10
NT ≤ 9
Q(1): 258.36
Q(2): 391.22
Q(3): 453.26
Q(4): 521.08
E(1) = 0.0
E(2) = 7.98 x 10^4
E(3) = 7.89 x 10^4
E(4) = 13.2 x 10^4
Q100 = 357
E100 = -3.3 x 10^-3
12"
6"
THEORY
EXPERIMENT
1.0
0.1
0.01
0.001
n=2
n=3
n=4
n=5
n=6
n=7
-3
-2
-1
0
1
2
3
4
FREQUENCY PARAMETER

FIGURE 6



1.0
MODE:010
STRPM:0.365
NTOP:20
NMAX:10
NT≤8
Q(1)=253.6
Q(2)=402.5
Q(3)=448.9
Q(4)=403.3
E(1)=0.0
E(2)=80.7x10⁻⁵
E(3)=88.5x10⁻⁵
E(4)=15.4x10⁻⁴
Q100=304.83
E100=2.22x10⁻³
6"
6"
THEORY
EXPERIMENT
0.1
0.01
0.0 01
n=2
n=3
n=4
n=5
n=6
−3
−2
−1
0
1
2
3
4
FREQUENCY PARAMETER

FIGURE 7

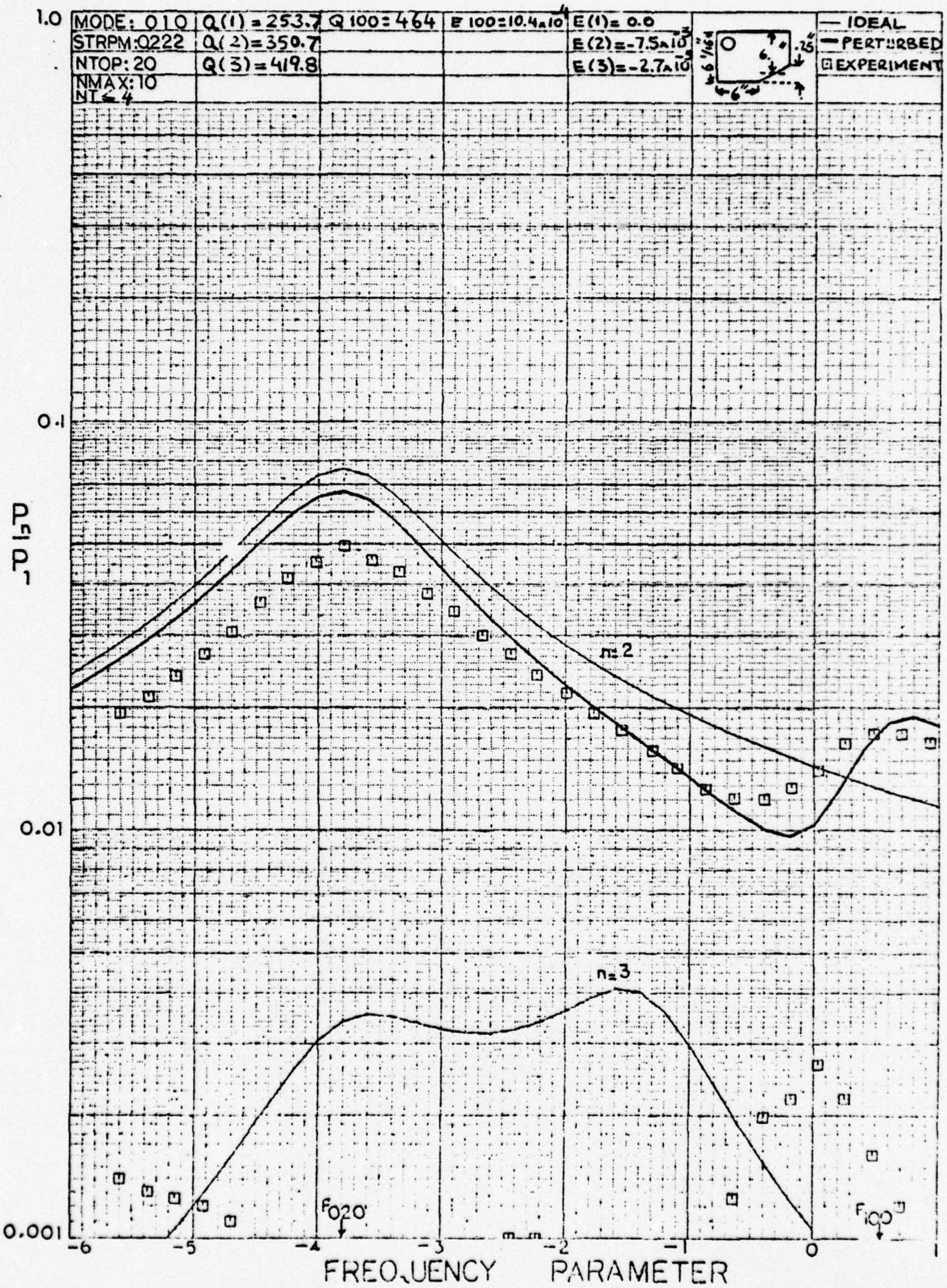

MODE: 010
STRPM:0222
NTOP: 20
NMAX: 10
NT≤4
Q(1)=253.7
Q(2)=350.7
Q(3)=419.8
Q 100=464
E(1)=0.0
— IDEAL
— PERTURBED
EXPERIMENT
n=2
n=3
FREQUENCY PARAMETER

FIGURE 8

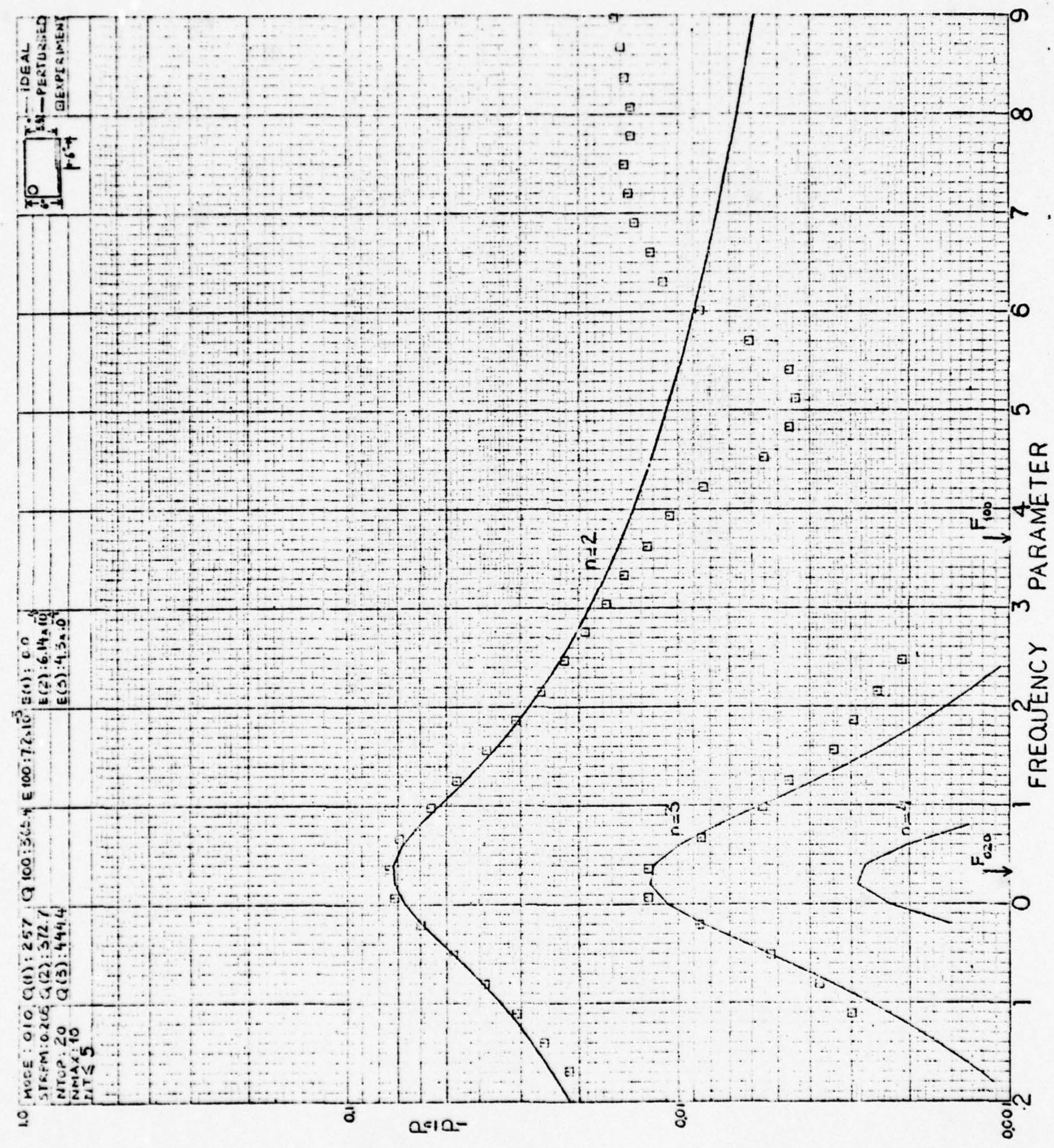
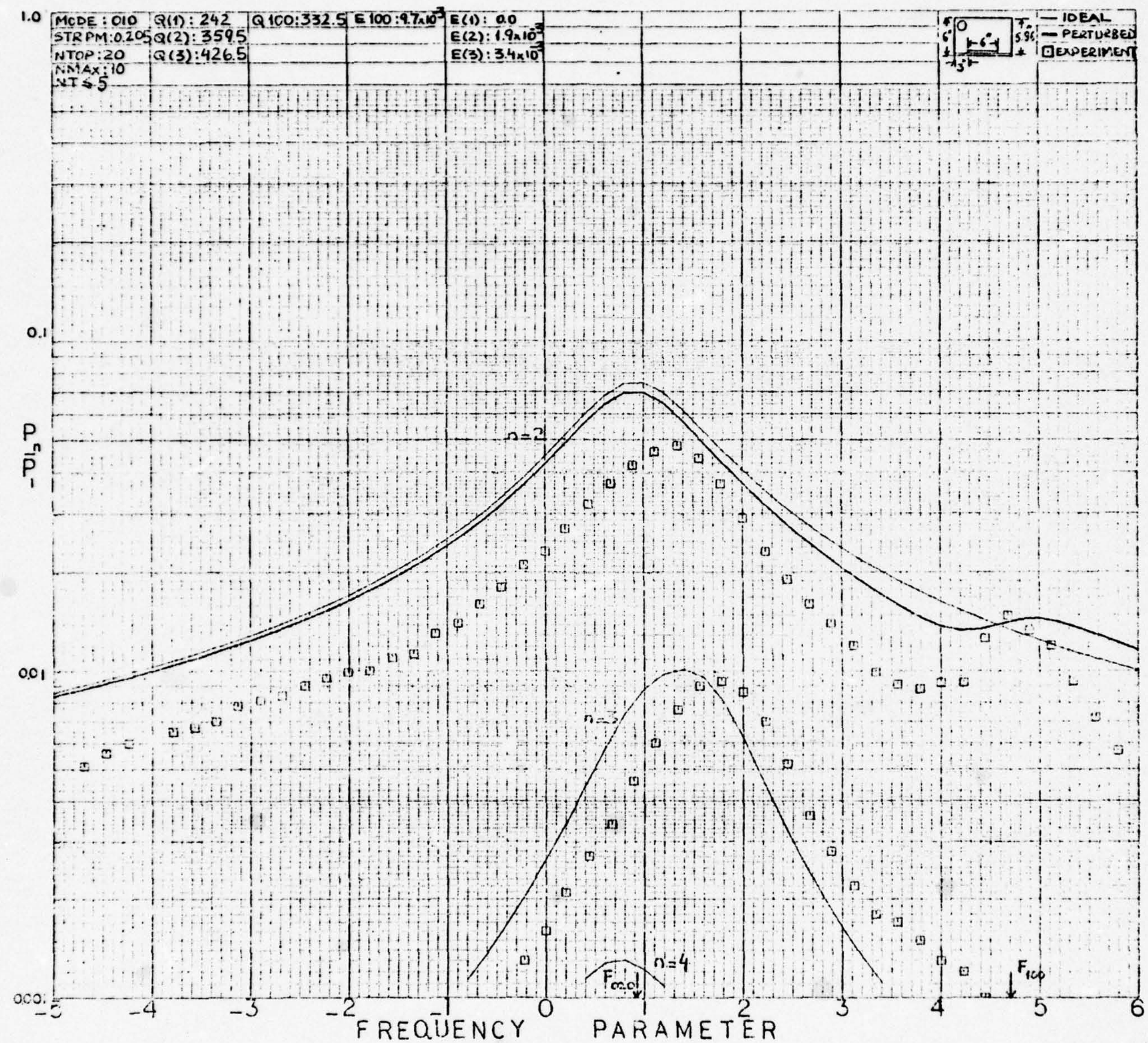
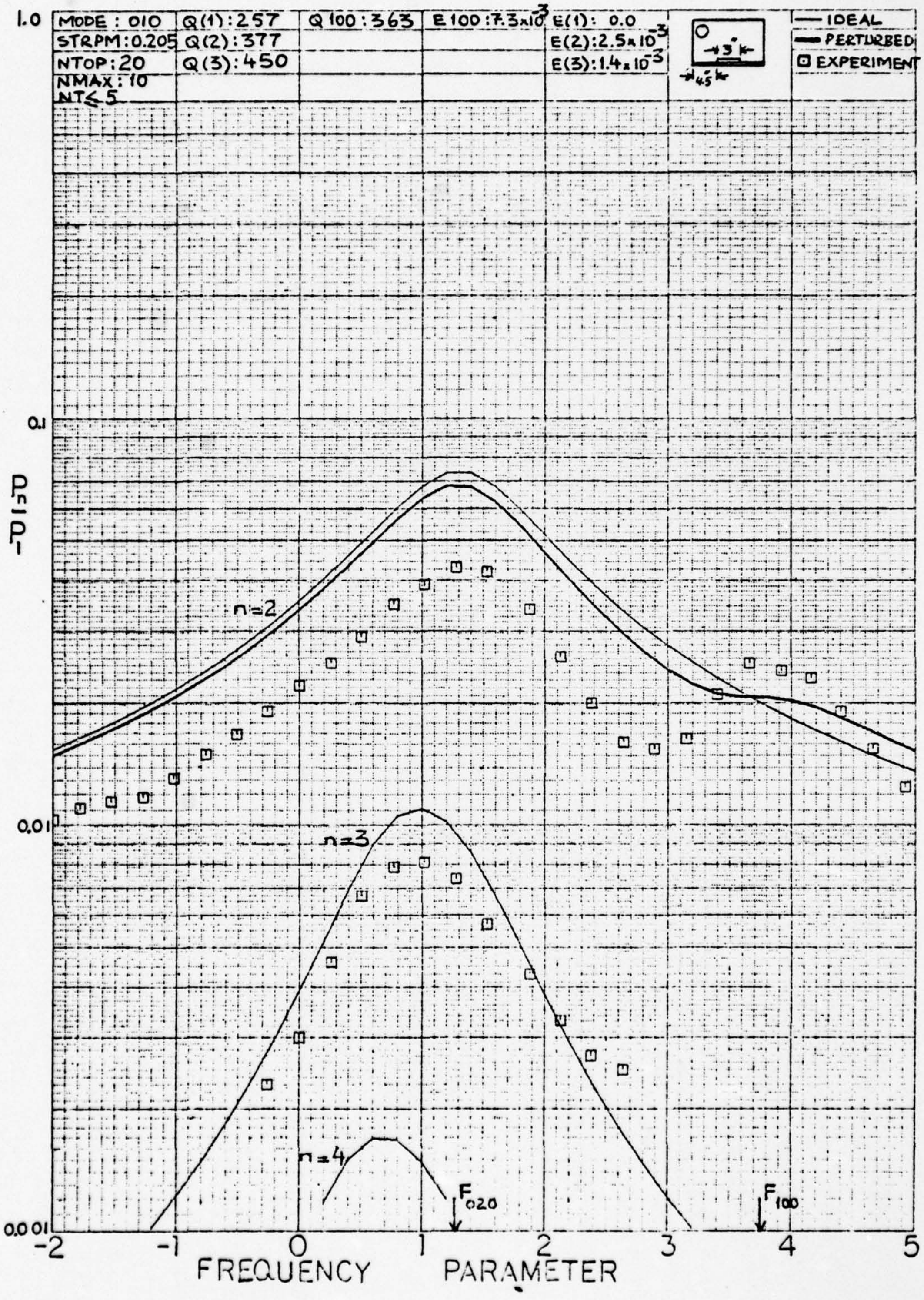


FIGURE 9



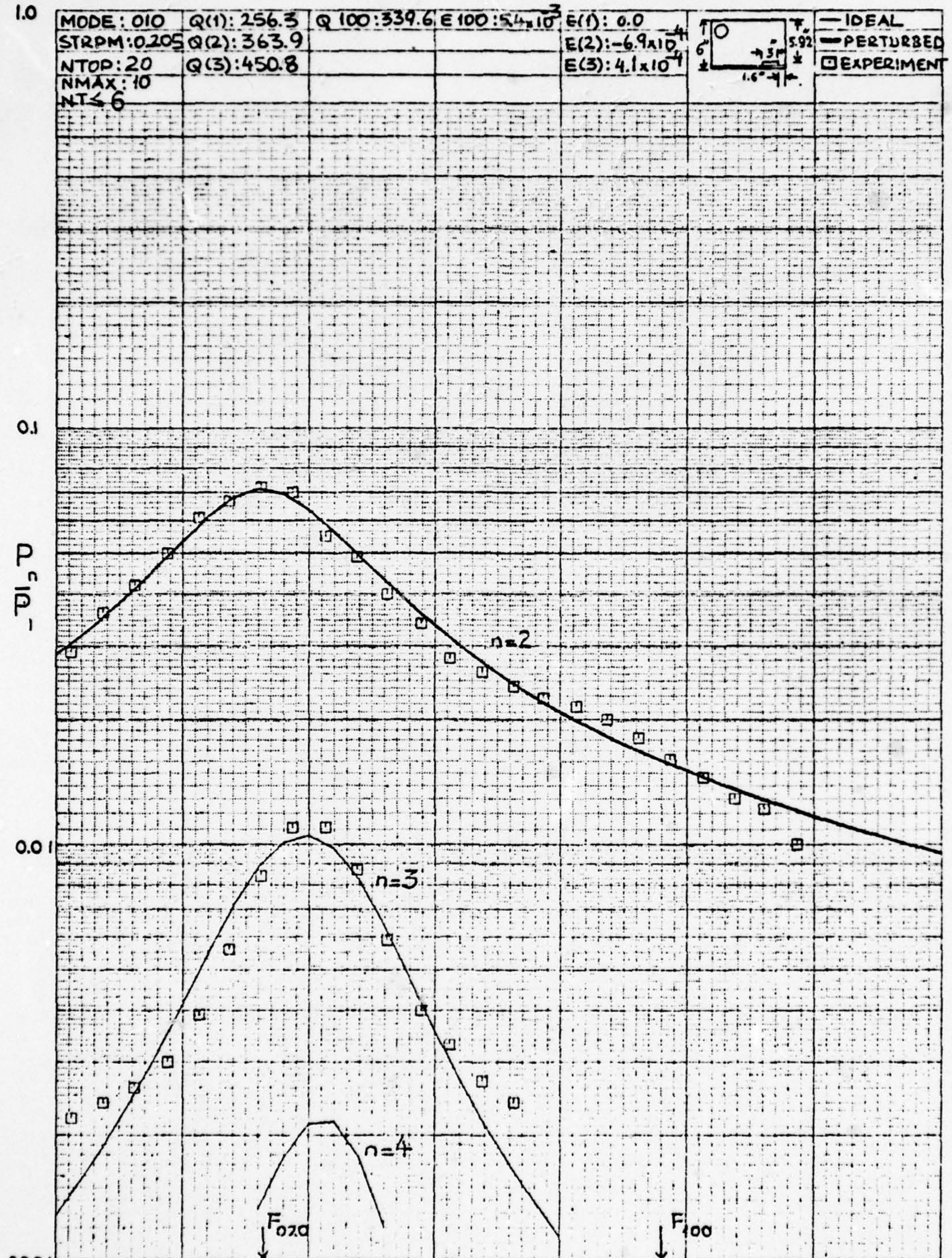
MODE: 010
STRPM: 0.205
NTOP: 20
NMAX: 10
NT ≤ 5
Q(1): 242
Q(2): 3595
Q(3): 426.5
Q 100: 332.5
E 100: 9.7x10-3
E(1): 0.0
E(2): 1.9x10-3
E(3): 3.4x10-3
— IDEAL
— PERTURBED
EXPERIMENT
1.0
0.1
0.01
0.001
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
n=2
n=3
n=4
FREQUENCY PARAMETER

FIGURE 10



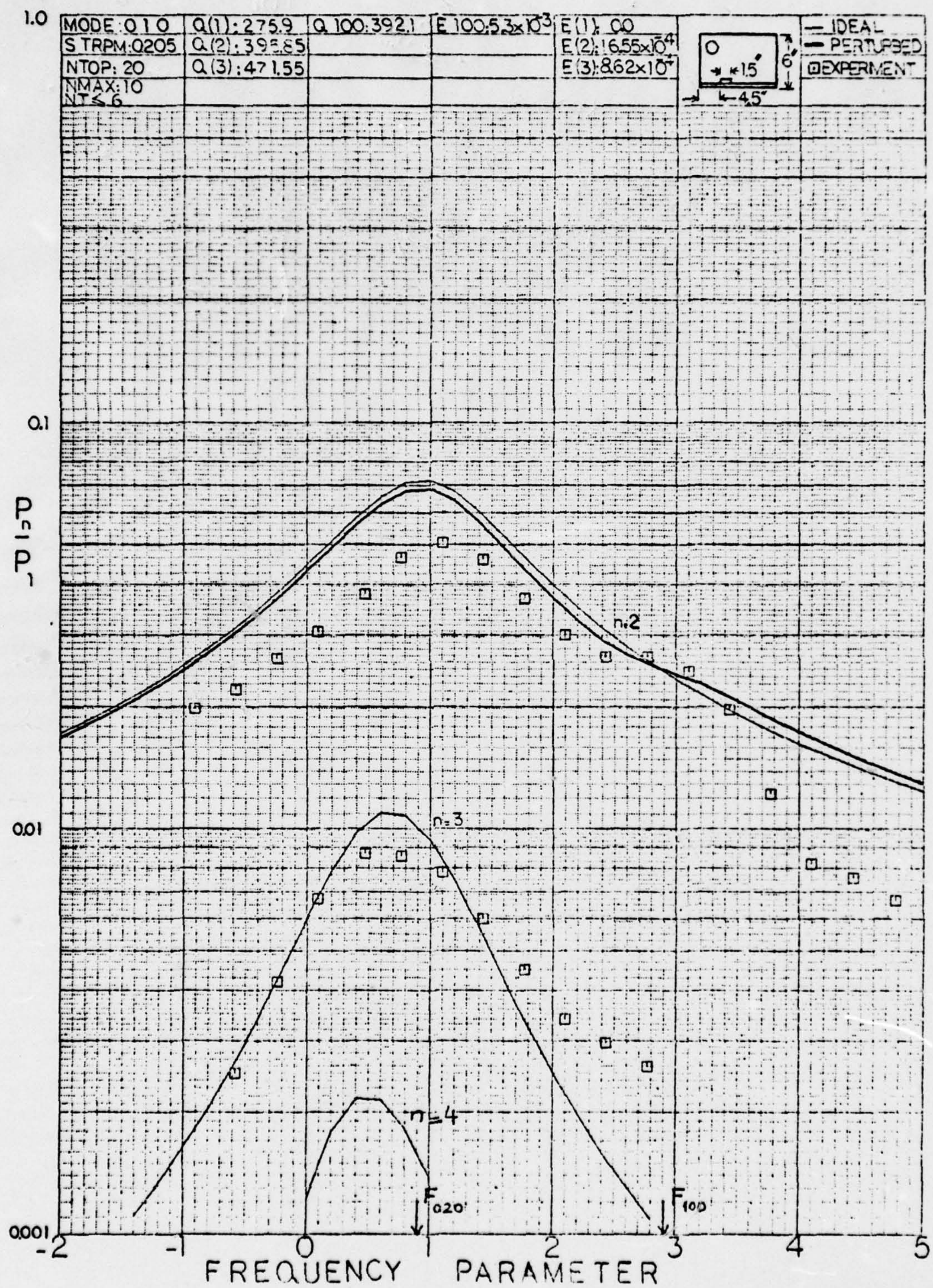
MODE : 010
STRPM : 0.205
NTOP : 20
NMAX : 10
NT ≤ 5
Q(1) : 257
Q(2) : 377
Q(3) : 450
Q 100 : 363
E 100 : 7.3 x 10^-3
E(1) : 0.0
E(2) : 2.5 x 10^-3
E(3) : 1.4 x 10^-3
3"
4.5"
IDEAL
PERTURBED
EXPERIMENT
1.0
0.1
0.01
0.001
P_n / P_1
n=2
n=3
n=4
F_020
F_100
-2
-1
0
1
2
3
4
5
FREQUENCY PARAMETER

FIGURE 11



MODE: 010
STRPM: 0.205
NTOP: 20
NMAX: 10
NT≤6
Q(1): 256.3
Q(2): 363.9
Q(3): 450.8
Q 100: 339.6
E 100: 5.4 x 10⁻³
E(1): 0.0
E(2): -6.9 x 10⁻⁴
E(3): 4.1 x 10⁻⁴
IDEAL
PERTURBED
EXPERIMENT
1.0
0.1
0.01
0.001
P_n / P_1
n=2
n=3
n=4
F_020
F_100
-2
-1
0
1
2
3
4
5
FREQUENCY PARAMETER

FIGURE 12



MODE: 0 1 0
S TRPM: 0205
NTOP: 20
NMAX: 10
NT ≤ 6
Q(1): 275.9
Q(2): 395.85
Q(3): 471.55
Q 100: 392.1
E 100: 5.3×10⁻³
E(1): 0.0
E(2): 16.55×10⁻⁴
E(3): 8.62×10⁻⁴
6"
1.5"
4.5"
IDEAL
PERTURBED
EXPERMENT
1.0
0.1
0.01
0.001
Pn/P1
n=2
n=3
n=4
F020'
F100'
-2
-1
0
1
2
3
4
5
FREQUENCY PARAMETER

FIGURE 13

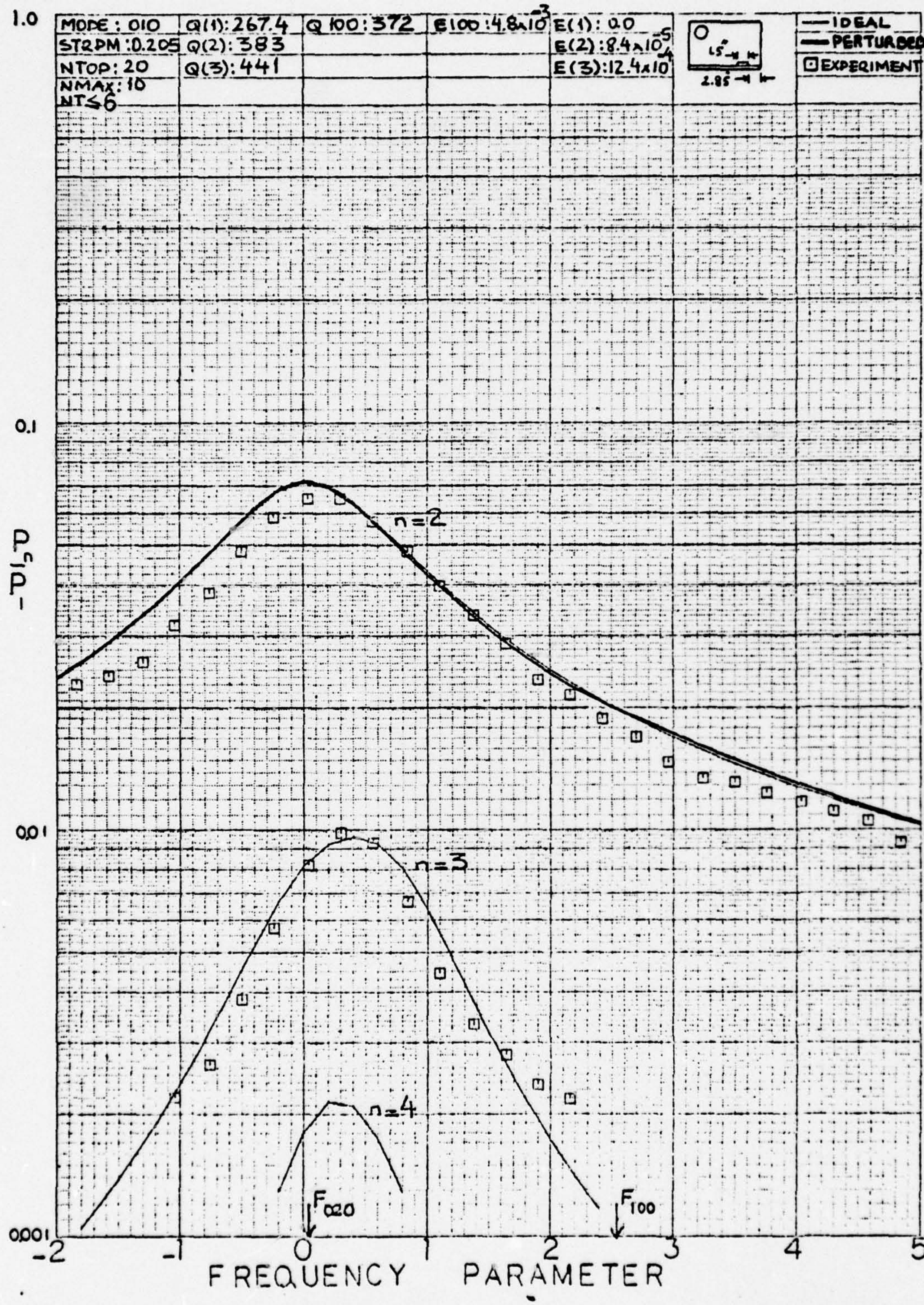

MODE: 010
STRPM: 0.205
NTOP: 20
NMAX: 10
NT≤6
Q(1): 267.4
Q(2): 383
Q(3): 441
Q 100: 372
E100: 4.8x10^-3
E(1): 0.0
E(2): 8.4x10^-5
E(3): 12.4x10^-4
IDEAL
PERTURBED
EXPERIMENT
n=2
n=3
n=4
F020
F100
1.0
0.1
0.01
0.001
-2
-1
0
1
2
3
4
5
FREQUENCY PARAMETER

FIGURE 14

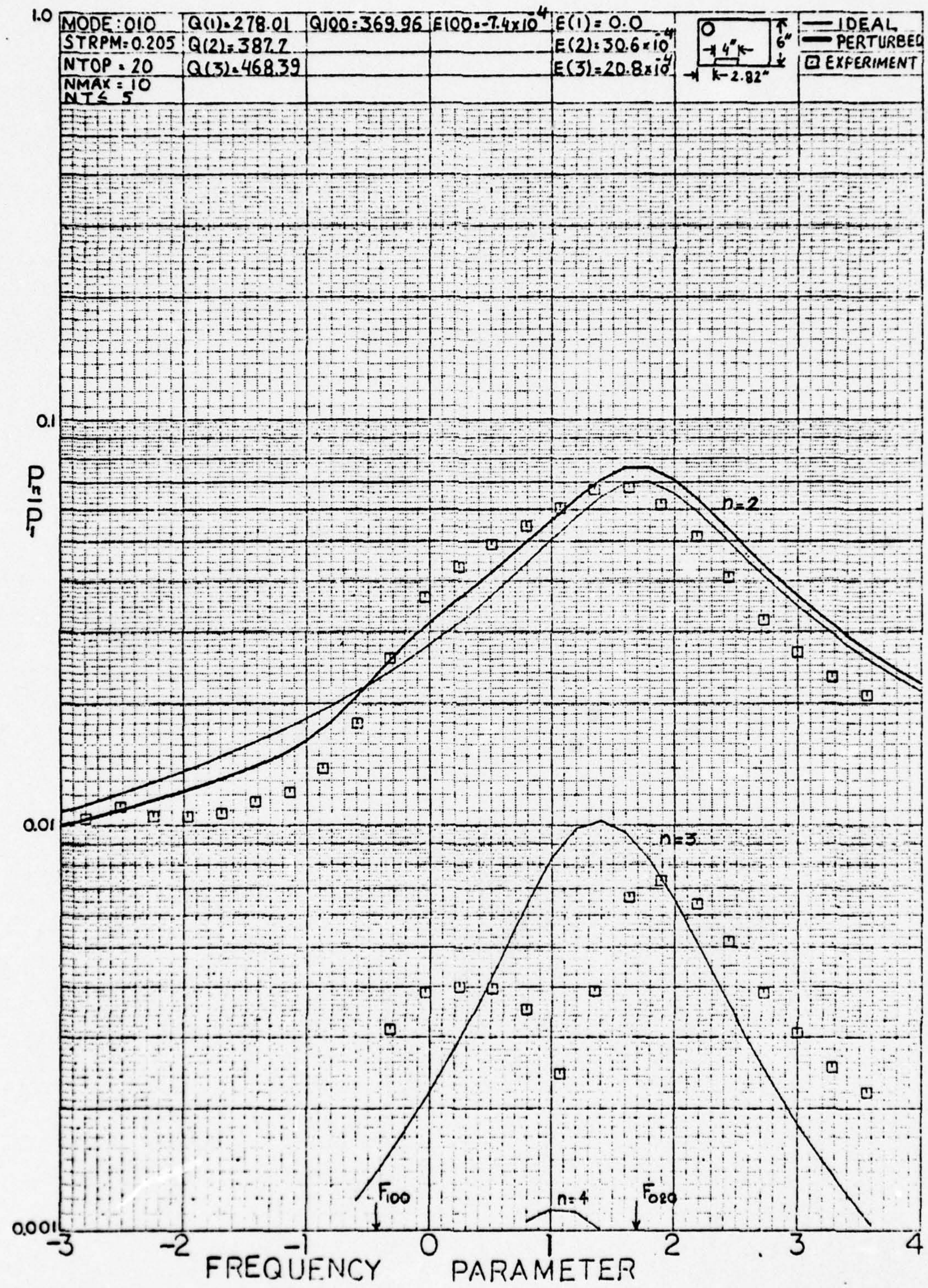

MODE: 010
STRPM: 0.205
NTOP = 20
NMAX = 10
NT ≤ 5
Q(1) = 278.01
Q(2) = 387.7
Q(3) = 468.39
Q100 = 369.96
E100 = -7.4x10^-4
E(1) = 0.0
E(2) = 30.6x10^-4
E(3) = 20.8x10^-4
4"
6"
2.82"
IDEAL
PERTURBED
EXPERIMENT
1.0
0.1
0.01
0.001
P_n/P_1
n=2
n=3
n=4
F100
F020
-3
-2
-1
0
1
2
3
4
FREQUENCY PARAMETER

FIGURE 15

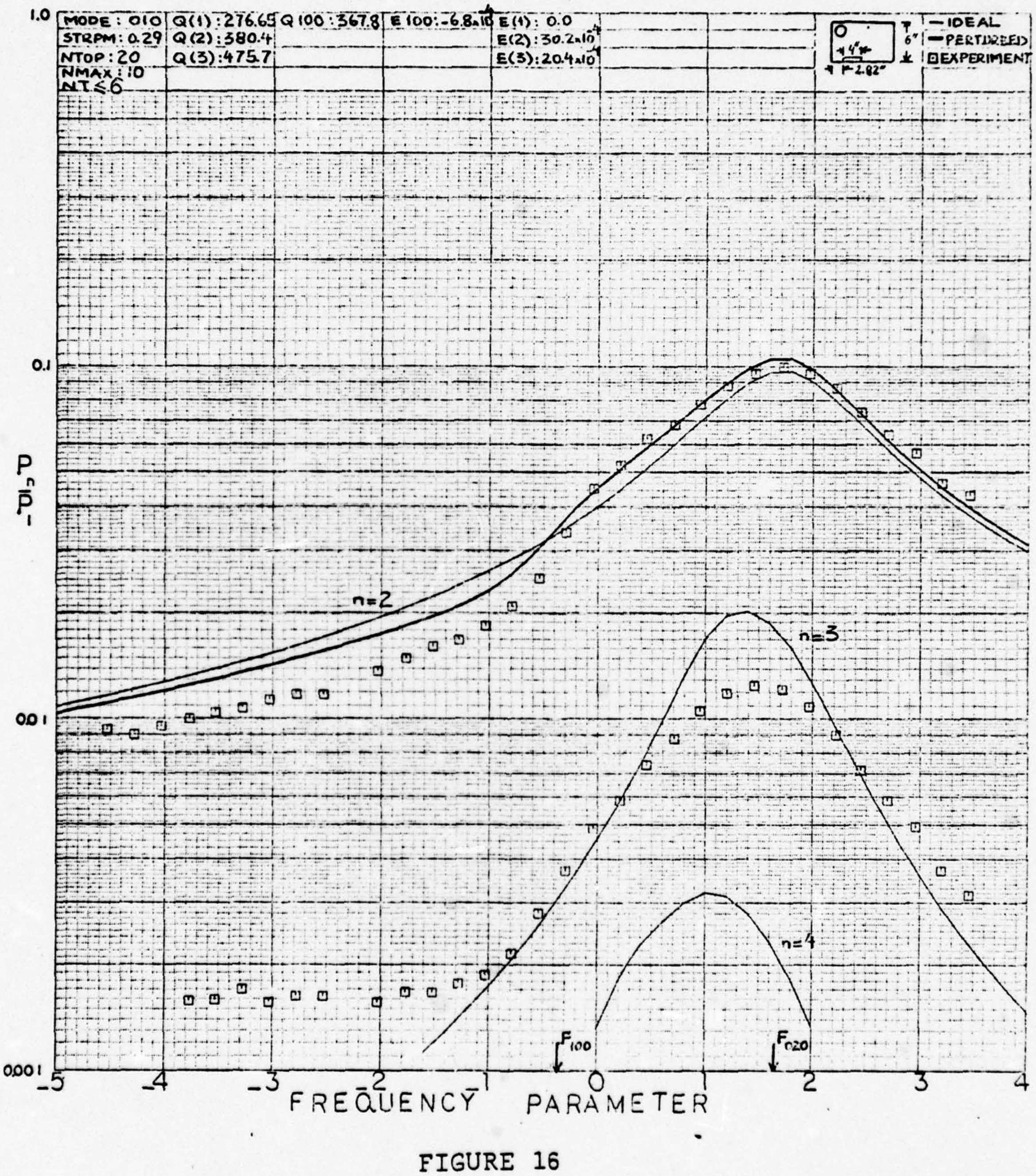

MODE : 010
STRPM : 0.29
NTOP : 20
NMAX : 10
NT ≤ 6
Q(1) : 276.65
Q(2) : 380.4
Q(3) : 475.7
Q 100 : 367.8
E 100 : -6.8x10⁻⁴
E(1) : 0.0
E(2) : 30.2x10⁻⁴
E(3) : 20.4x10⁻⁴
4"
6"
2.82"
— IDEAL
— PERTURBED
□ EXPERIMENT
1.0
0.1
0.01
0.001
P_n / P_1
n=2
n=3
n=4
F_100
F_020
-5
-4
-3
-2
-1
0
1
2
3
4
FREQUENCY PARAMETER

FIGURE 16

7. CONCLUSION

Non-linear theory has been applied to standing waves in a rigid-walled rectangular cavity with a perturbed boundary in order to find one possible mechanism for the excitation of a standing wave other than those belonging the family of the driven mode.It was observed that such an excitation exists if the boundary perturbation and the dimensions of the cavity are favorably chosen.

It appears that the present theoretical model succesfully predicts the major features of harmonic content for finite-amplitude standing waves in the cavity when the geometry leads to no perturbation correction (Fig.7 and 9).When the magnitude of the perturbation is increased the predicted features were larger than experimentally observed.Second-order perturbation corrections may be needed to account for these discrepancies.

APPENDIX A

The original computer program for Eq.(2.16) was prepared by Coppens in 1973, and author made some extentions to that program so that it would (1) calculate the perturbation correction and (2) present the results graphically. The program calculates the relative amplitudes and phase angles of standing waves in cavities keeping the strength parameter constant and changing the frequency parameter to generate response curves showing the amplitudes of the nonlinearly excited standing waves as function of the frequency parameter F_1. It also calculates the perturbation correction according to Eq.(4.24) and then finds the total relative pressure amplitude using Eq.(4.26). The program also draws the graph of the relative pressure amplitudes of the ideal cavity,total relative pressure amplitude of the perturbed cavity and the experimental data on a 3 cycle semilog paper.The Versatec Graphics Plotting Manual [10] was used for the graphical processes on the IBM 360 of the Randolph Church Computer Center,Naval Postgraduate School.

SOME USEFUL INFORMATION ABOUT COMPUTER PROGRAM

* Quantities marked with (*) must be controlled or changed for each run.

*KON	The number of iterations throughout the region of interest.For this program the iterations are performed with 0.2 intervals.
*NCUR	The number of the experimental curves to be drawn + 1
*NDAT	The number of experimental data in the region of interest
BUR(I,J)	The array that stores the experimental data
*XL	The length of the cavity in the x-direction
*YL	The length of the cavity in the y-direction
*DELTA	The magnitude of the perturbation
*STRPM	Strength parameter
*FREQ	Frequency parameter stored in ATA(I,1) and ZER(I,1).Input as the maximum value of FREQ in the region of interest
XDAT(JET)	The x coordinate
YDAT(JET)	Value of the curve f(x) for XDAT
ATA(I,N),N≠1	The array that stores the logarithmic value of the pressure amplitudes of the harmonics of the driving mode
ZER(I,N),N≠1	The array that stores the linear value of the pressure amplitudes of the harmonics of the driving mode
*Q(I)	Quality factors of driving mode and harmonics of it
*E(I)	e's value of driving mode and harmonics of it

*Q100	Quality factor of (1,0,0) mode
*E100	e-value of (1,0,0) mode
*XDAT(JET+1)	Integer value of left-hand corner on the x-axis. It must have the same value as the 7th argument of subroutine CALL AXIS for x-axis.
*YDAT(JET+1)	Integer value of left-hand corner on the y-axis. It must have the same value as the 7th argument of subroutine CALL AXIS for y-axis.
XDAT(JET+2), YDAT(JET+2)	Increment value of x and y for scalling purposes
HUM	The linear value of the total acoustic pressure amplitude associated with angular frequency 2w.
*B	$\frac{\epsilon}{\pi} Q_{100} \sin\sigma_{100}(1/4)\, L_y\, a_0$ for stepped perturbation $\frac{\epsilon}{\pi} Q_{100} \sin\sigma_{100}(a_0/2)$ for wedged perturbation and a_0 is the 0th Fourier coefficient.

```
//  EXEC  FORTCLGW
//FORT.SYSIN DD *
C          -----------------------------------------
C                                    FREQ CHANGE
C          STANDING WAVES IN CAVITIES-------------
C          NO DATA READIN-------TO START OFF ON SKIRTS
C          AND GENERATE A STARTING DECK
C         THIS PROGRAM CALCULATES RAMP(N) AND PHI(N) FOR
C          STANDING WAVES IN THREE DIMENSIONAL CAVITIES
C          THE PROGRAM KEEPS STRPM THE SAME BUT
C          CHANGES FREQ TO GENERATE A RESPONSE CURVE
C          -----------------------------------------
      DIMENSION RAMP(50),PHI(50),THETA(50),FAC(50),S(50),C(50),
     2    RATIO(50),DPHI(50),FIX(50),E(50),Q(50),CCRF(50),CCRR(50),
     3 ZER(62,10),BUR(62,10),Y(200),V(100),XDAT(62),YDAT(62),ATA(62,10)
      DATA LMASK1/-30584/,LMASK2/-21846/
      DATA ZER,BUR,ATA/1860*0./
      DATA Y,V/300*0./
      DATA XDAT,YDAT/124*0./
      R=10./3.
      L=0
      K=0
      KON=36
    1 FORMAT(//33X,'STRENGTH PARAMETER  =',F5.3,
     2     /33X,'FREQUENCY PARAMETER =',F6.3,
     3     /33X,'NTOP                =',I5,
     4     /33X,'NMAX                =',I5,
     5     /33X,'NT                  =',I5//)
    2 FORMAT(17X,I5,4F15.4)
    3 FORMAT(//16X,'I',8X,'N',6X,'AMPLITUDE',8X,'PHI',
     2     11X,'RATIO',9X,'DPHI',/)
    4 FORMAT(22X,I5,1E15.4,1F14.5)
    5 FORMAT(12X,I5)
    6 FORMAT(22X,I5,1E15.4,3F14.5)
    7 FORMAT(10X,2F10.3,I10,1F10.3)
    8 FORMAT(8E10.3)
    9 FORMAT(//20X,'NT',9X,'RINF',11X,'RSLP',
     2     11X,'DPINF',10X,'DSLP',/)
   10 FORMAT(1H1)
   11 FORMAT(27X,I5,1E15.4,1F14.5)
C     -----------------------------------------------------------
C       DO 475 READS THE EXPERIMENTAL DATAS WITH FORMAT 1001
C       NCUR=NUMBER OF CURVES TO BE DRAWN+1
C     -----------------------------------------------------------
      NCUR=3
      NDAT=24
      DO 475 I=1,NDAT
  475 READ(5,1001) (BUR(I,J),J=1,NCUR)
```

```
 1001 FORMAT(4F8.6)
      DO 50 N=1,50
      FIX(N)=0.0
      RAMP(N)=0.0
      PHI(N)=0.0
      RATIO(N)=0.0
      DPHI(N)=0.0
      THETA(N)=0.0
      FAC(N)=0.0
      CORP(N)=0.0
      CORR(N)=0.0
      S(N)=0.0
      C(N)=0.0
      E(N)=0.0
   50 Q(N)=0.0
      NI=1
      PI=3.14159
      RAMP(1)=1.0
      PHI(1)=0.0
      S(1)=0.0
      C(1)=1.0
C     ------------------------------------------------------------------------
C     INPUT VALUES FOR PERTURBATION CORRECTION
      Q100=339.6
      E100=5.43E-3
      XL=5.96
      YL=12.
      DELTA=0.04
C         ---------------------------------------
C                                 INPUT PARAMETERS
C         RECTANGULAR CAVITY
C         FAMILY OF MODES GIVEN BY VALUE OF RELABS
C         KEEP NTOP, NMAX LE 50
      RELABS=010.
      STRPM=0.205
      FREQ=5.2
      NMAX=10
      NTOP=20
      FXMX=0.75
      RMIN=1.0 E-4
C        HERE THRU 90 CALCULATES INFINITESIMAL-AMPLITUDE
C        RESONANCE PARAMETERS FOR NON-R-K CASE
      Q(1)=256.25
      Q(2)=363.85
      Q(3)=450.8
      E(1)=0.0
      E(2)=-6.85E-4
      E(3)=4.085E-4
```

```
      DC 89 N=4,NMAX
      XN=FLOAT(N)
      Q(N)=Q(1)*SQRT(XN)
      E(N)=0.001*COS(XN-5.0)
   89 CONTINUE
         WRITE(6,11)(N,Q(N),E(N),N=1,NMAX)
      DC 4000 LS=1,KON
      FRUP=FREQ-0.2
      DC 90 N=1,NMAX
      TEMP=2.0*E(N)*Q(1)
      XNUM=-FRUP+TEMP
      XNBF=-FREQ+TEMP
      XDEN=Q(1)/Q(N)
      XD2=XDEN*XDEN
      TEMP=SQRT(XNUM**2+XD2)
      TMBF=SQRT(XNBF**2+XD2)
      THETA(N)=ATAN2(XNUM,XDEN)
      ARGN=XNUM-XNBF
      ARGD=XDEN+(XNUM*XNBF/XDEN)
      CORP(N)=ATAN2(ARGN,ARGD)
      CORR(N)=TMBF/TEMP
   90 FAC(N)=0.5*STRPM/TEMP
      FREQ=FRUP
C        -----------------------------------------
C        HERE THROUGH 383 INITIALIZES RATIO(N) AND DPHI(N)
C        DO 380 CALCULATES INITIAL VALUES FOR RATIO(N) AND DPHI(N)
C        AND OBTAINS THE VALUE OF N=NT AFTER WHICH RATIC =0.0 AND
C        RAMP LE RMIN
      NT=NMAX
      T1=1.0
      T2=0.0
      DC 380 N=2,NMAX
      IF(RAMP(N)-RMIN)369,369,370
  369 NT=N-1
      DC 382 J=N,NMAX
      RATIO(J)=0.0
  382 DPHI(J)=0.0
      GO TO 383
  370 RATIO(N)=(RAMP(N)/RAMP(N-1))*0.5*(1.0+CORR(N))
      DIFF=PHI(N)-PHI(N-1)
      IF(ABS(DIFF)-PI)374,371,371
  371 IF(DIFF-PI)373,372,372
  372 DIFF=DIFF-2.0*PI
      GO TO 374
  373 DIFF=DIFF+2.0*PI
  374 DPHI(N)=DIFF+CORP(N)
      T1=T1*RATIO(N)
      T2=T2+DPHI(N)
```

```
      IF(ABS(T2)-PI)474,471,471
  471 IF(T2-PI)473,472,472
  472 T2=T2-2.0*PI
      GO TO 474
  473 T2=T2+2.0*PI
  474 CONTINUE
         WRITE(6,6)N,T1,T2,RATIO(N),DPHI(N)
  380 CONTINUE
  383 CONTINUE
      DO 1000 LN=1,7
         WRITE(6,5)LN
C        HERE THRU 81 DETERMINES EXTRAPOLATION PARAMETERS
C        OVER N=2,NT AND THEN CALCULATES RATIO(N), PHI(N),
C        FOR N=NT,NTOP.  IF (NT LE 2), PROGRAM SKIPS TO 81
      IF(NT-2)81,81,189
  189 TR=0.0
      TD=0.0
      TN=0.0
      TRN=0.0
      TDN=0.0
      TNN=0.0
      XD=0.0
      DO 190 N=2,NT
      XN=FLOAT(N)
      X=1.0/XN
      TR=TR+RATIO(N)
      TD=TD+DPHI(N)
      TN=TN+X
      TRN=TRN+RATIO(N)*X
      TDN=TDN+DPHI(N)*X
      TNN=TNN+X*X
      XD=XD+1.0
  190 CONTINUE
      RDEN=1.0/(TNN*XD-TN*TN)
      XUMR=TNN*TR-TN*TRN
      XUMD=TNN*TD-TN*TDN
      RINF=XUMR*RDEN
      DPINF=XUMD*RDEN
      RSLP=(RINF*XD-TR)/TN
      DSLP=(DPINF*XD-TD)/TN
C         IF RINF G 1.0, RECALCULATION BASED UPON
C         RINF=1.0 AND RATIO(2)
      IF(RINF-1.0)192,192,191
  191 RINF=1.0
      RSLP=2.0*(1.0-RATIO(2))
  192 CONTINUE
         WRITE(6,2)NT,RINF,RSLP,DPINF,DSLP
      DO 80 N=NT,NTOP
```

```
      XN=FLOAT(N)
      TEMP=1.0/XN
      RATIO(N)=RINF-RSLP*TEMP
      DPHI(N)=DPINF-DSLP*TEMP
   80 CONTINUE
   81 CONTINUE
      DO 300 LM=2,50
C           HERE THRU 278 CALCULATES RAMP(N) AND PHI(N) FROM
C           N=(LP LE NMAX),(NT-1) AND EXTRAPOLATES THEM FROM
C           N=NT,NTOP.  ALL RAMP AND PHI BEYOND RAMP(N)=RMIN
C           ARE SET TO 0.0.  THE ASSOCIATED S(N) AND C(N) ARE FOUND.
      IF(LM-NMAX)260,260,261
  260 LP=LM
      GO TO 262
  261 LP=NMAX
  262 DO 276 N=LP,NTOP
      IF(RAMP(N-1)-RMIN)270,270,271
  270 DO 277 J=N,NTOP
      RAMP(J)=0.0
      PHI(J)=0.0
      S(J)=0.0
  277 C(J)=0.0
      GO TO 278
  271 RAMP(N)=RAMP(N-1)*RATIO(N)
      PHI(N)=PHI(N-1)+DPHI(N)
      IF(ABS(PHI(N))-PI)275,272,272
  272 IF(PHI(N)-PI)274,273,273
  273 PHI(N)=PHI(N)-2.0*PI
      GO TO 275
  274 PHI(N)=PHI(N)+2.0*PI
  275 S(N)=RAMP(N)*SIN(PHI(N))
      C(N)=RAMP(N)*COS(PHI(N))
  276 CONTINUE
  278 CONTINUE
C           HERE THRU 300 CALCULATES NEW VALUES
C           OF FIX(N) FOR N=1,LP
      XLM=FLOAT(LM)
      DO 300 N=1,LP
      XN=FLOAT(N)
      FIX(N)=FXMX/(XLM-XN+1.0)
  300 CONTINUE
C           HERE THRU 131 CALCULATES NEW VALUES FOR RAMP(N),
C           PHI(N), S(N), AND C(N) FOR N=2,LP.  WHEN
C           (RAMP(IN) L RMIN), THEN FOR (IN LE N LE LP) ALL
C           ARE SET TO ZERO
      DO 100 N=2,LP
      SUMC1=0.0
      SUMC2=0.0
```

```
      SLMS1=0.0
      SUMS2=0.0
      M=N-1
      IF(M)105,105,102
  102 DO 104  J=1,M
      K=N-J
      SLMS1=SLMS1+S(J)*C(K)+C(J)*S(K)
  104 SUMC1=SUMC1+C(J)*C(K)-S(J)*S(K)
  105 M=NTOP-N
      IF(M)109,109,106
  106 DO 108  J=1,M
      K=N+J
      SLMS2=SUMS2+S(K)*C(J)-C(K)*S(J)
  108 SUMC2=SUMC2+C(K)*C(J)+S(K)*S(J)
  109 F=0.5*SUMS1-SUMS2
      G=0.5*SUMC1-SUMC2
      TEST=F**2+G**2
      RAMP(N)=RAMP(N)+FIX(N)*(FAC(N)*SCRT(TEST)-RAMP(N))
      IF(RAMP(N)-RMIN)110,111,111
  110 DC 131  J=N,LP
      RAMP(J)=0.0
      PHI(J)=0.0
      S(J)=0.0
  131 C(J)=0.0
      GC TO 58
  111 PEST=ATAN2(F,G)
      TEST=PEST+THETA(N)-PHI(N)
      IF(ABS(TEST)-PI)113,113,122
  122 IF(TEST-PI)124,123,123
  123 TEST=TEST-2.0*PI
      GC TO 113
  124 TEST=TEST+2.0*PI
  113 PHI(N)=PHI(N)+FIX(N)*TEST
      IF(ABS(PHI(N))-PI)129,129,27
   27 IF(PHI(N)-PI)29,28,28
   28 PHI(N)=PHI(N)-2.0*PI
      GO TO 129
   29 PHI(N)=PHI(N)+2.0*PI
  129 S(N)=RAMP(N)*SIN(PHI(N))
  100 C(N)=RAMP(N)*COS(PHI(N))
   58 CONTINUE
 3000 CONTINUE
C           HERE THRU 183 CALCULATES NEh RATIO(N) AND DPHI(N)
C           FOR ALL NONZERO RAMP(N).  RAMP(NT) IS HIGHEST
C           NONZERO RAMP, BUT NT IS NEVER GREATER THAN NMAX.
C           RATIO(N) AND CPHI(N) ARE SET TC ZERO FOR
C           (NT L N LE NMAX)
      NT=NMAX
```

```
      DO 180 N=2,NMAX
      IF(RAMP(N))169,169,170
  169 NT=N-1
      DO 182 J=N,NMAX
      RATIO(J)=0.0
  182 DPHI(J)=0.0
      GO TO 183
  170 RATIO(N)=RAMP(N)/RAMP(N-1)
      DIFF=PHI(N)-PHI(N-1)
      IF(ABS(DIFF)-PI)174,171,171
  171 IF(DIFF-PI)173,172,172
  172 DIFF=DIFF-2.0*PI
      GO TO 174
  173 DIFF=DIFF+2.0*PI
  174 DPHI(N)=DIFF
  180 CONTINUE
  183 CONTINUE
         WRITE(6,11)(N,RAMP(N),PHI(N),N=1,NT)
 1000 CONTINUE
          WRITE(6,10)
         WRITE(6,1)STRPM,FREQ,NTOP,NMAX,NT
          WRITE(6,3)
          N=1
          WRITE(6,4)N,RAMP(N),PHI(N)
          WRITE(6,6)(N,RAMP(N),PHI(N),RATIO(N),DPHI(N),N=2,NMAX)
      ZER(LS,1)=FREQ
      ATA(LS,1)=FREQ
      DO 1789 MEH=2,10
      IF(RAMP(MEH).LE.0.001) GO TO 1788
      ATA(LS,MEH)=RAMP(MEH)
 1789 ZER(LS,MEH)=R*ALOG10(RAMP(MEH)*1000.0)
 1788    M=NMAX+1
         WRITE(6,4)(N,RAMP(N),PHI(N),N=M,NTOP)
         WRITE(6,9)
         WRITE(6,2)NT,RINF,RSLP,DPINF,DSLP
         WRITE(6,10)
 4000 CONTINUE
C      STARTING TO DRAW A SEMI-LOG(3*70) GRAPH AS A BACKGROUND
      CALL PLOTS(0,0,0)
      DO 1111 I=1,3
      Z=1.0
      DO 2222 J=1,60
      K=K+1
      IF(Z.GE.4.0) GO TO 3333
      Y(K)=R*(ALOG10(Z+.1)-ALOG10(Z))
      Z=Z+.1
      GO TO 2222
 3333 Y(K)=R*(ALOG10(Z+.2)-ALOG10(Z))
```

```
      Z=Z+.2
 2222 CCNTINUE
 1111 CONTINUE
      DC 4444 I=1,3
      Z=1.0
      DC 5555 J=1,9
      L=L+1
      V(L)=R*(ALOG10(Z+1.)-ALOG10(Z))
 5555 Z=Z+1.
 4444 CCNTINUE
      CALL GRID(0.,0.,70,0.1,1180,Y,LMASK1)
      CALL GRID(0.,0.,7,1.0,1027,V,LMASK2)
C       STARTING TO SCALE THE AXIS
C       LABELING THE AXIS
      CALL AXIS(0.,0.,'FREQUENCY  PARAMETER',-20,7.,0.,-2.,1.)
      CALL AXIS(0.,0.,'P(N)/P(1)',+9,10.,90.,0.,1.)
C       STARTING TC DRAW THEORETICAL CURVES
      DO 1787 I=2,10
      JET=0
      KAT=0
      DC 1786 J=1,KON
      IF(ZER(J,I).NE.0.) GO TO 1785
      KAT=KAT+1
      GO TO 1786
 1785 JET=JET+1
      XCAT(JET)=ZER(J,1)
      YCAT(JET)=ZER(J,I)
 1786 CCNTINUE
      IF(KAT.EQ.KCN) GO TO 1002
      XDAT(JET+1)=-2
      XCAT(JET+2)=1.
      YDAT(JET+1)=0.0
      YCAT(JET+2)=1.0
      CALL LINE(XCAT,YDAT,JET,1,0,0)
 1787 CONTINUE
C       STARTING TO CRAW EXPERIMENTAL CURVES
 1002 DO 1784 I=2,NCUR
      JET=0
      DC 1783 J=1,NDAT
      IF(BUR(J,I).EQ.0.) GO TO 1783
      JET=JET+1
      XCAT(JET)=BUR(J,1)
      YDAT(JET)=R*ALOG10(BUR(J,I)*1000.)
 1783 CCNTINUE
      XCAT(JET+1)=-2
      XCAT(JET+2)=1.
      YCAT(JET+1)=0.0
      YDAT(JET+2)=1.0
```

```
      CALL LINE(XCAT,YDAT,JET,1,-1,0)
 1764 CONTINUE
C     STARTING TO DRAW PERTURBATION CORRECTION
      ZIR=2.*Q(1)*E100
      ZOR=1.-E100
      ZAR=Q130/Q(1)
      EPSILO=DELTA/XL
      JET=0
      DC 2953 I=1,KCN
      IF(ATA(I,2).EQ.0.) GO TO 2950
      JET=JET+1
      SINSIG=1./SQRT(1.+(ZAR*(ATA(I,1)-ZIR)*ZOR)**2)
      CCSSIG=SQRT(1.-SINSIG**2)
      IF(ATA(I,1).LE.ZIR) COSSIG=-COSSIG
      B=(0.037*EPSILO*Q100*SINSIG)/PI
      HUM=ATA(I,2)*SQRT((1.+B*COSSIG)**2+(B*SINSIG)**2)
      YDAT(I)=R*ALOG10(HUM*1000.)
      XCAT(I)=ATA(I,1)
 2950 CONTINUE
      XDAT(JET+1)=-2
      XDAT(JET+2)=1.
      YDAT(JET+1)=0.0
      YCAT(JET+2)=1.0
      CALL NEWPEN(3)
      CALL LINE(XDAT,YDAT,JET,1,0,0)
      CALL PLOT(0.,0.,+999)
      WRITE(6,2951)
 2951 FORMAT('1',20X,'FREQ.PARAM.     P(2)/P(1)         P(3)/P(1)         P(4)/P(
     11)        P(5)/P(1)        P(6)/P(1)',/,20X,6(11('-'),3X),/)
      DC 2952 I=1,KON
      WRITE(6,2953) (ATA(I,J),J=1,6)
 2953 FORMAT(22X,F5.2,9X,5(F8.3,6X),/)
 2952 CONTINUE
      STOP
      END
```

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