

AD-A066 341

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO  
FORM OF INSTABILITY OF STEADY CONVECTIVE MOVEMENT CAUSED BY INT--ETC(U)  
AUG 78 A A YAKIMOV

F/G 20/13

UNCLASSIFIED

FTD-ID(RS)T-1233-78

NL

| OF |  
AD  
AD 66 341



END

DATE  
FILMED

5--79

DDC

AD-A066341

1

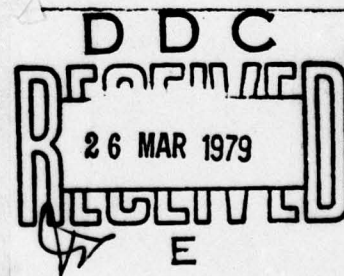
# FOREIGN TECHNOLOGY DIVISION



FORM OF INSTABILITY OF STEADY CONVECTIVE  
MOVEMENT CAUSED BY INTERNAL HEAT SOURCES

by

A. A. Yakimov



Approved for public release;  
distribution unlimited.

78 12 22 159

## EDITED TRANSLATION

FTD-ID(RS)T-1233-78

11 August 1978

MICROFICHE NR: *FD-78-C-001112*

FORM OF INSTABILITY OF STEADY CONVECTIVE MOVEMENT  
CAUSED BY INTERNAL HEAT SOURCES

By: A. A. Yakimov

English pages: 11

Source: Uchenyye Zapiski, Permskiy Gosudarstvennyy  
Universitet im. A. M. Gor'kogo, Nr. 293,  
Issue 4, 1972, Perm', PP. 37-42.

Country of origin: USSR

Translated by: Carol S. Nack

Requester: FTD/TQTA

Approved for public release; distribution unlimited

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
<i>A</i>	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-AFB, OHIO.

FTD -ID(RS)T-1233-78

Date 11 Aug 19 78



# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b>А а</b>	A, a	Р р	<b>Р р</b>	R, r
Б б	<b>Б б</b>	B, b	С с	<b>С с</b>	S, s
В в	<b>В в</b>	V, v	Т т	<b>Т т</b>	T, t
Г г	<b>Г г</b>	G, g	У у	<b>У у</b>	U, u
Д д	<b>Д д</b>	D, d	Ф ф	<b>Ф ф</b>	F, f
Е е	<b>Е е</b>	Ye, ye; E, e*	Х х	<b>Х х</b>	Kh, kh
Ж ж	<b>Ж ж</b>	Zh, zh	Ц ц	<b>Ц ц</b>	Ts, ts
З з	<b>З з</b>	Z, z	Ч ч	<b>Ч ч</b>	Ch, ch
И и	<b>И и</b>	I, i	Ш ш	<b>Ш ш</b>	Sh, sh
Й й	<b>Й й</b>	Y, y	Щ щ	<b>Щ щ</b>	Shch, shch
К к	<b>К к</b>	K, k	Ъ ъ	<b>Ъ ъ</b>	"
Л л	<b>Л л</b>	L, l	Ы ы	<b>Ы ы</b>	Y, y
М м	<b>М м</b>	M, m	Ь ь	<b>Ь ь</b>	'
Н н	<b>Н н</b>	N, n	Э э	<b>Э э</b>	E, e
О о	<b>О о</b>	O, o	Ю ю	<b>Ю ю</b>	Yu, yu
П п	<b>П п</b>	P, p	Я я	<b>Я я</b>	Ya, ya

\*ye initially, after vowels, and after Ъ, Ь; e elsewhere.  
When written as ë in Russian, transliterate as yë or ë.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

1233

# FORM OF INSTABILITY OF STEADY CONVECTIVE MOVEMENT CAUSED BY INTERNAL HEAT SOURCES

A. A. Yakinov

The stability of steady convective movement caused by internal heat sources was studied earlier in [1, 2]. The spectra of the decrements of the disturbances and the neutral curves were obtained for different values of the Prandtl number. This report considers the form of the disturbances of convective movement.

1. Heat sources with volumetric density  $Q$  are uniformly distributed in a plane vertical layer of viscous fluid with a width of  $2h$ . The layer is assumed to be closed from above and below, and the vertical walls which bound it are held at the same temperatures. Stationary plane-parallel convective movement with velocity and



temperature profiles which are even relative to the axis of the channel, found from the ordinary equations of convection with consideration of internal heat sources [1], originate in this channel due to internal heating. If we take  $h$ ,  $h^2/\nu$ ,  $g\beta qh^4/2\nu$ ,  $qh^2/2$ ,  $\rho g\beta h^2/2$  ( $q = Q/\rho c_p \chi$ ) as the units of distance, time, velocity, temperature, and pressure, respectively, the dimensionless velocity and temperature profiles will be

$$v_0 = 1/60(1 - 6x^2 + 5x^4), \quad (1)$$

$$T_0 = 1 - x^2. \quad (2)$$

#### Small plane normal disturbances

$$\psi(x, z, t) = \varphi(x) \exp(-\lambda t + ikz), \quad (3)$$

$$T(x, z, t) = \theta(x) \exp(-\lambda t + ikz), \quad (4)$$

are imposed on the main flow, where  $\varphi$  and  $\theta$  are the amplitudes of the oscillations,  $\lambda$  is the decrement, and  $k$  is the wave number.

Substituting (3) and (4) in the convection equation and considering the smallness of the disturbances, we will obtain a system of linear homogeneous differential equations for determining intensities  $\varphi(x)$  and  $\theta(x)$  [1]:

$$\Delta^2 \varphi - ikGH\varphi + \theta' = -\lambda \Delta \varphi, \quad (5)$$

$$P^{-1} \Delta \theta + ikG(T_0' \varphi - v_0 \theta) = -\lambda \theta \quad (6)$$

$$(\Delta \varphi = \varphi'' - k^2 \varphi, H\varphi = v_0 \Delta \varphi - v_0' \varphi, G = g\beta qh^4/2\nu^2, P = \nu/\chi)$$

with the homogeneous boundary conditions

$$\varphi = \varphi' = 0, \theta = 0 \quad \text{at } x = \pm 1. \quad (7)$$

2. We will use the Galerkin method to solve boundary problem (5)-(7). We will find  $\varphi$  and  $\theta$  in the form of the superimposition of the basic functions

$$\varphi = \sum_{i=1}^N a_i \varphi_i, \quad \theta = \sum_{k=1}^M b_k \theta_k. \quad (8)$$

We will take the intensity of the disturbances in a quiescent fluid, determined from the boundary problem

$$\Delta^2 \varphi_i = -\mu_i \Delta \varphi_i, \quad \varphi_i = \varphi'_i = 0, \quad \text{at } x = \pm 1, \quad (9)$$

$$(i = 1, 2, \dots, N),$$

as the basic functions  $\varphi_i$ , and the intensity of the temperature perturbations, determined by the problem

$$P^{-1} \Delta \theta_k + \nu_k \theta_k = 0, \quad \theta_k = 0 \quad \text{at } x = \pm 1 \quad (10)$$

$$(k = 1, 2, \dots, M)$$

as the basic functions  $\theta_k$  (the explicit form of the basic functions is given in [3], for example).

The requirement of the orthogonality of the discrepancies in the basic functions leads to a system of homogeneous linear algebraic equations for coefficients  $a_i$  and  $b_k$ .



The condition of the existence of a nonzero solution for this system determines the spectrum of the characteristic decrements of disturbances  $\lambda$  depending on the Grashof number  $N$ , the Prandtl number  $F$  and the wave number  $k$ . The characteristic decrements  $\lambda$  are defined as the intrinsic values of the system matrix.

The expansion coefficients  <sup>$a_i$  and  $b_k$</sup>  are the components of the latent vector which corresponds to characteristic number  $\lambda$ . This vector was found as follows. The value of  $\lambda$  was substituted in the characteristic equation

$$|A - \lambda E| = 0, \quad (11)$$

where  $A$  is the system matrix and  $E$  is the unit matrix. One equation was deleted from the system obtained. For the best conditionality of the matrix obtained, the equation with the minimum modulus of the coefficient in the diagonal term was selected as this equation. One of the unknowns ( $a_k$  or  $b_k$ ) was assigned a random value (e.g., "-1"), and the system thus obtained (with complex elements) was solved by the method of primary elements.

3. The normal disturbances  $\psi(x, z, t)$  and  $T(x, z, t)$  with a certain amplitude  $a$ , which remains arbitrary when staying within the bounds of the linear theory of stability, are added to the main flow and distort it. We will plot the current lines and isotherms of the disturbed total movement.



At an arbitrary fixed point in time  $t_0$ , the equation of the current line of disturbed movement is

$$\psi_0(x) + a\psi(x, z, t_0) = C_1,$$

where  $\psi_0(x)$  is the current function of the primary flow, and  $C_1$  is a certain constant.

If disturbances  $\psi(x, z, t)$  are determined from formula (3), the current line equation assumes the following form:

$$\psi_0(x) + a_1\varphi(x)e^{ikz} = C_1. \quad (12)$$

Factor  $e^{ikt_0}$  only affects the intensity of the disturbances; therefore, we can set  $t_0 = 0$  without affecting continuity.

Considering the complex form of  $\varphi(x)$  and  $C_1$ , equation (12) can be rewritten as:

$$\psi_0(x) + a_1[\varphi_r(x)\cos kz - \varphi_i(x)\sin kz] = A_1, \quad (13)$$

where

$$\varphi_r(x) = \operatorname{Re} \varphi(x), \quad \varphi_i(x) = \operatorname{Im} \varphi(x),$$

and  $a_1$  and  $A_1$  are real constants.

The isolines are plotted as follows. The corresponding values of  $x$  are found from equation (13) for a given value of  $A_1$  and a fixed value of  $z$ . Breaking down the change interval  $x$  into a sufficiently

large number of parts, we will obtain the current line for the selected value of  $\Delta_1$ . Varying  $\Delta_1$  with a certain spacing, we will find the family of equidistant current lines. The equation for the isotherms

$$T_0(x) + a_1 [\theta_r(x) \cos kz - \theta_i(x) \sin kz] = B_1, \quad (14)$$

is found analogously, where

$$\theta_r(x) = \operatorname{Re} \theta(x), \quad \theta_i(x) = \operatorname{Im} \theta(x),$$

and the family of isotherms of the disturbed movement is plotted.

4. The results of the numerical calculations are given below. The intrinsic values of the system matrix were found using the QR algorithm realized on Aragats and M-220M computers of the Computer Center of the Perm State University [4]. Six to fifteen functions of each type were taken in expansions (8). The latent vector, isolines and isotherms were found on an M-220M computer.

Figure 1 shows the neutral curve for the Prandtl number  $P = 10$  plotted from the materials in [2]. As this study establishes, at sufficiently large values of the Prandtl number, the neutral curve consists of two branches. The short-wave branch corresponds to hydrodynamic disturbances drifting slowly along the channel. The long-wave branch corresponds to heat wave disturbances, the phase velocity of which is close to the maximum flow velocity<sup>1</sup>.



Footnote: For comparison, the broken line in Fig. 1 shows the neutral curve for  $P = 0$  taken from [1], which characterizes the development of hydrodynamic disturbances alone. End footnote

It is interesting to trace the form of the disturbances corresponding to both branches.



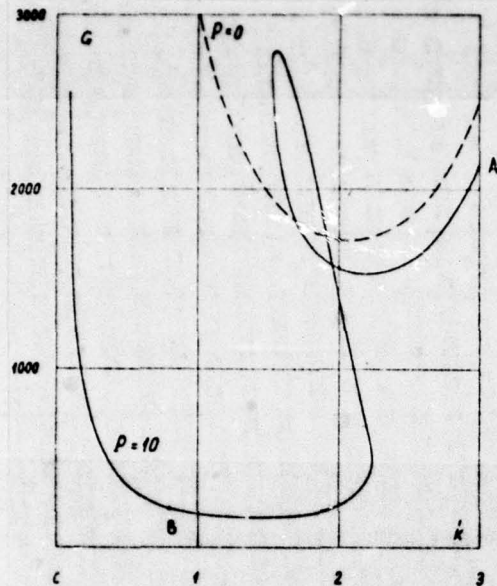


Fig. 1.

Figure 2 shows the current lines and isotherms of the total convective movement plotted for point A (see Fig. 1), which lies on the hydrodynamic branch of the neutral curve ( $k = 3$ ;  $G = 2180$ ). Figure 3 corresponds to point B, which is located on the thermal branch ( $k = 0.8$ ;  $G = 230$ ). For convenience of illustration, the vertical scale in this figure is one-fourth of the full scale.

Fig. 2. KBY: (1) Current lines. (2) Isotherms.

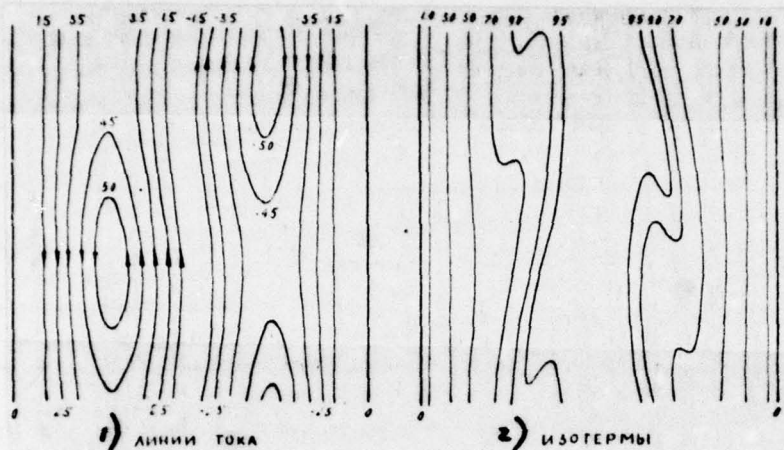
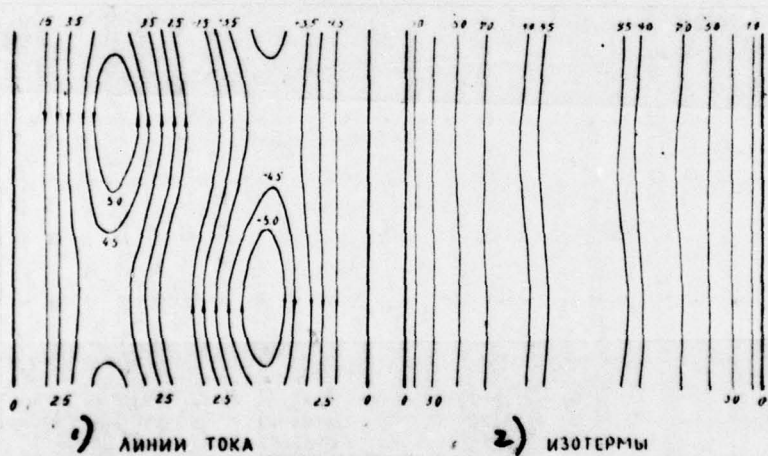


Fig. 3. KBY: (1) Current lines. (2) Isotherms.



The values of the current function indicated in the figures are increased  $10^3$  times, and the temperature values - ten times. When comparing the figures one should remember that constant  $a_1$  used in formulae (13) and (14) was 2.5 times larger for point A than for point B when plotting the isolines.

These figures show that in both cases, instability develops in the form of two vortex chains which alternate on the interfaces of the convective flows. Thus, although hydrodynamic disturbances and rising heat flux disturbances are related to two different modes of the instability spectrum, there is no essential difference in their form. However, the difference is that heat waves have a relatively high phase velocity compared to hydrodynamic disturbances.

#### Bibliography

1. G. Z. Gershuni, Ye. M. Zhukhovitskiy, A. A. Yakimov. PNM, 1970, 34, No. 4.
2. G. Z. Gershuni, Ye. M. Zhukhovitskiy, A. A. Yakimov. PNM, in print.



3. B. I. Rudakov. PNH, 1966, 30, No. 2.

4. A. A. Yakinov. Sci. Notes of Perm' University, 1971, No. 259, coll. "Algorithms and Programs for the Araks Computer."

End-1233

# DISTRIBUTION LIST

## DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>	<u>ORGANIZATION</u>	<u>MICROFICHE</u>
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
E344 DIA/RDS-3C	9	E403 AFSC/INA	1
C043 USAMIIA	1	E404 AFDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D	1	E410 ADTC	1
LAB/FIO		E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NICD	3
C019 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NICD	2
H300 USAICE (USAFEUR)	1		
P005 ERDA	1		
P005 CIA/CRS/ADB/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/ID	1		