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AN ANALYSIS OF ICE PROFILES OBTAINED BY SUBMARINE SONAR IN THE --ETC(U)
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SUBMARINE SONAR PROFILES FROM THE BEAUFORT SEA

Peter Wadhams
Raymond J. Horne

1978

Unclassified

MIL-STD-847A
31 January 1973

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (6)	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER (9)
4. TITLE (and Subtitle) AN ANALYSIS OF ICE PROFILES OBTAINED BY SUBMARINE SONAR IN THE AIDJEX AREA OF THE BEAUFORT SEA.		5. TYPE OF REPORT & PERIOD COVERED Data analysis and presentation 1976 rept. for period ending 1976
		6. PERFORMING ORG. REPORT NUMBER (14) SPRI-TR-78-1
7. AUTHOR(s) (10) Peter Wadhams and Raymond J. Horne		8. CONTRACT OR GRANT NUMBER(s) (15) N00014-78-G-0003
9. PERFORMING ORGANIZATION NAME AND ADDRESS Scott Polar Research Institute, University of Cambridge, England		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research, Arctic Programs, Arlington Va.		12. REPORT DATE (11) December 1978
		13. NUMBER OF PAGES 65
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Dept. of Oceanography, University of British Columbia, Vancouver B.C. Canada		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

(12) 69p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Beaufort Sea	Pressure ridges
Arctic Ocean	Ice thickness
Sea ice profiling	Submarine sonar

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A profile of the ice cover in the southern Beaufort Sea was obtained by the submarine USS GURNARD in April 1976, using a narrow-beam upward-looking sonar. The 1400 km profile consisted of three legs, of which the long south-north and east-west legs intersected near the Caribou camp of the AIDJEX experiment. A statistical analysis was carried out over contiguous 50 km → next page

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Figure 6. Report Documentation Page.

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sections to yield probability density functions of the drafts of ice and of level ice, distributions of keel spacings and drafts, and the frequencies and widths of leads. Two distinct types of ice cover were found. The first, nearest the coast in the south and west of the experimental area, consisted of heavily ridged ice with mean drafts of up to 5.1 m. The rest of the track (1200 km) consisted of a homogeneous ice cover with a mean draft of 3.7 m. The percentage of thin ice varied greatly between sections, with a range of 0.4 to 12.3% for ice of 0-1 m draft. Level ice, with a local gradient of less than 1 in 40, made up 56% of the homogeneous cover, with a preferred draft of 2.7 to 2.9 m. Keel spacings obeyed a negative exponential distribution, with a deficit at small spacings due to a keel shadowing effect and a surfeit at very large spacings due to the contribution of polynyas. The draft distribution of keels was also a negative exponential, differing from the distribution of Hibler *et al* (1972), probably because the narrow beam records a complex structure for every keel. The coastal ice had a greater frequency and mean draft of keels. Maximum keel draft was 31.12 m. The mean separation of leads was 212 m, with almost all leads being less than 50 m in width.

LEVEL II

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AN ANALYSIS OF ICE PROFILES OBTAINED BY SUBMARINE SONAR IN THE AIDJEX AREA OF THE BEAUFORT SEA

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TECHNICAL REPORT 78-1

CONTENTS

SUMMARY

A profile of the ice cover in the southern Beaufort Sea was obtained by the nuclear submarine USS GURNARD from 7 to 10 April 1976, using a narrow-beam upward-looking sonar. The 1400 km profile consisted of three legs, of which the long south-north and east-west legs intersected near the Caribou camp of the AIDJEX experiment. After initial processing at the Arctic Submarine Laboratory, San Diego, the data were passed via the AIDJEX Office to Scott Polar Research Institute for statistical analysis.

The analysis was carried out over contiguous 50 km sections of the profile, and yielded the following significant results:-

1. Probability density function of ice draft. The profile was found to consist of three distinct types of ice cover. Type 1, nearest the coast and extending from the 100 m isobath northwards to 71°30'N at 144°13'W, consisted of heavily ridged ice with a mean draft of 4.2-5.1 m. Type 2 comprised the major part of the profile (1200 km), and was shown statistically to be a homogeneous ice cover with the following distribution of ice drafts:- 0-0.5 m, 1%; 0.5-2 m, 10%; 2-5 m, 72%; over 5 m, 17%; mean draft 3.67 m. Type 3 was another short stretch of track at the far western end of the profile (151°W to 154°14'W at 72°42'N) which again consisted of more heavily ridged ice of mean draft 4.5-4.6 m. The proportion of thin ice was the most variable quantity in the distributions, having a range 0.1-3.5% (0-0.5 m draft) and 0.4-12.3% (0-1 m draft). The cumulative probability distributions for two 200 km sections crossing the AIDJEX camps were calculated for application to the AIDJEX model.

2. Level ice. Ice of less than 1 in 40 local gradient - i.e. mainly undeformed floes contaminated with a few keel bottoms - was found to comprise 56% of the ice in the type 2 cover. The most probable draft of level ice was 2.7-2.8 m, with secondary peaks at 2.0-2.1 m and 3.1-3.2 m. Thin ice had a peak at 0.8-0.9 m.

3. Keel spacings. The distribution of keel spacings was found to obey a negative exponential distribution, with departures at small spacings (a deficit due to a "keel shadowing" effect, where shallow keels are concealed by neighbouring deep keels) and at large spacings exceeding 300 m (a surfeit due to the presence of leads and polynyas).

4. Keel drafts. The distribution of drafts did not obey the theory of Hibler *et al* (1972) which predicts $P(h)\ \ \exp(-bh^2)$, but rather a simpler distribution of form

$$P(h)\,dh = B\exp(-bh)\,dh$$

with B, b as parameters. Hitherto only sails have been found to obey this relationship, and it is postulated that the cause is the very narrow beamwidth of the transducer, which enables it to probe into the fine structure of a keel and thus perceive multiple secondary "keels" where a wide-beam sounder sees only a single entity. Again types 1 and 3 have heavier ridging than type 2. For keels deeper than 9 m, type 2 ice has a mean of 1.39 keels per km with a mean draft of 11.79 m; types 1 and 3 have up to 5.1 keels per km with a mean draft of 12.1-12.7 m. Maximum keel draft was 31.12 m, with only one keel over 30 m in draft.

5. Leads and polynyas. A lead was defined as a continuous sequence of depth points where no point exceeds 1 m in draft. The average spacing of leads was 212 m over the whole profile, but almost all leads were less than 50 m in width. The mean spacing of leads over 50 m wide was 10.3 km, and of leads over 500 m wide was 237 km.

1. INTRODUCTION

During the period 7 to 10 April 1976 the nuclear submarine USS GURNARD (SSN-662) obtained a sonar profile of length 1400 km under the Beaufort Sea ice cover in the vicinity of the AIDJEX (Arctic Ice Dynamics Joint Experiment) main camp. Figure 1 shows the route followed by GURNARD, comprising south-north (OPQ) and east-west (RPS) legs intersecting near the Caribou camp (C), together with a connecting leg QR. GURNARD was equipped with a high-frequency, narrow-beam, upward-looking sonar installed by the Arctic Submarine Laboratory, Naval Undersea Center, San Diego. The sonar fed its output into a signal processing system that digitised the range to the ice underside, subtracted this from the transducer depth and thus generated a digital magnetic tape of ice drafts with a nominal resolution of 0.03 m. Mechanical limitations within the system reduced the absolute accuracy of ice draft to $\pm$ 0.3 m, with a demonstrated standard deviation for smooth ice of 0.09 m. Other details of the sonar system and of the submarine's depth and speed remain classified. The Arctic Submarine Laboratory did, however, supply information on the "surface beam diameter" (diameter of spread of the sonar beam at the surface, a function of beamwidth and cruising depth) and the "ping spacing" (horizontal distance between successive sound pulses, a function of ping frequency and submarine speed). Normally the ping spacing was between 1.3 and 1.5 m, and the surface beam diameter over almost the whole track was 3.17 m, implying a very narrow beam of less than 3° width.

Initial processing of the tapes was done at the Arctic Submarine Laboratory using a Univac 1108-1110 computer. Corrected depth data were merged with positional information to give a final tape in BCD format which was forwarded to the AIDJEX Project Office in Seattle for analysis. In turn the AIDJEX Office forwarded the tape to the Scott Polar Research Institute so that the data could be analysed using the same criteria and definitions as those employed in the analysis of data from HMS SOVEREIGN (Wadhams 1977a,b; Wadhams and Lowry, 1977). This report gives the results of such an analysis.

2. DATA PROCESSING

The initial processing at the Arctic Submarine Laboratory deleted spurious profile points caused by multiple echoes, fish, air bubbles etc., and set these to -10.0 ft. Isolated spurious points were then regenerated by linear interpolation. Appendix A describes this stage of processing in more detail, including the criteria for identifying spurious points. The data file created at San Diego consisted of a series of "blocks" each containing about 60 data points. The blocks were separated by single lines which usually contained all zeroes, but which at periodic intervals contained a position fix (i.e. latitude and longitude values). These position fixes thus split

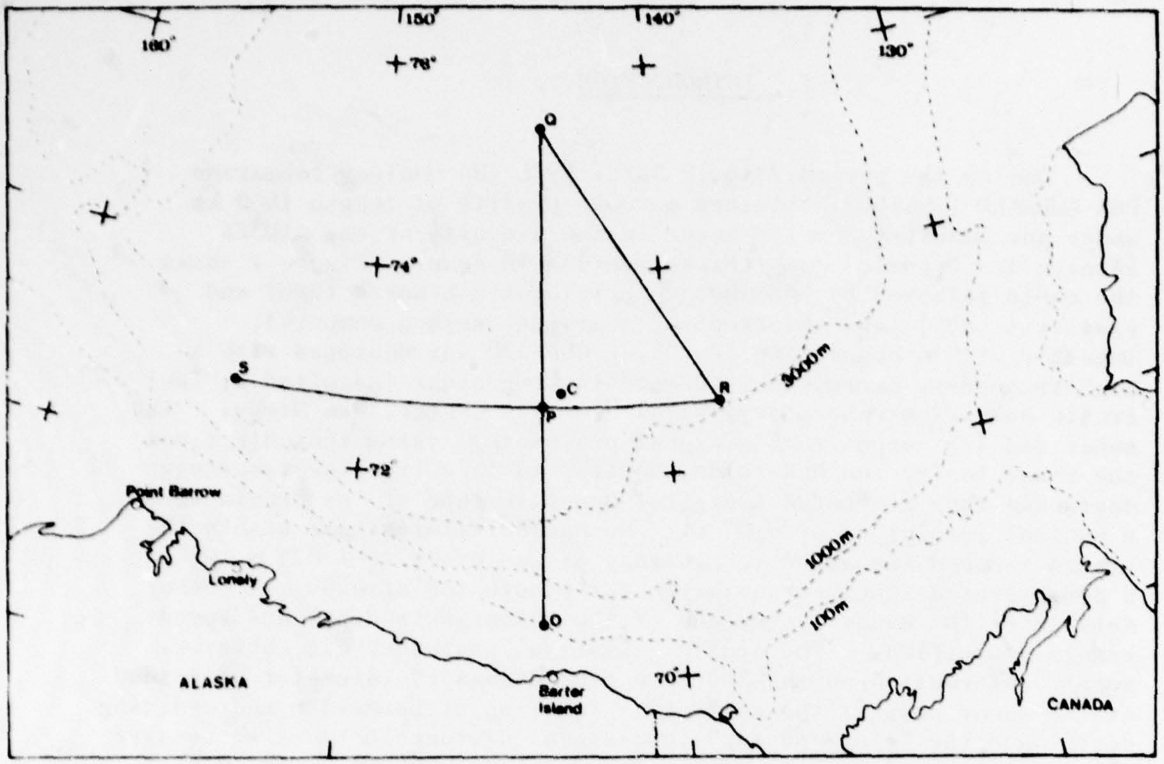


FIGURE 1 (above). Route of USS GURNARD, 7-10 April 1976.

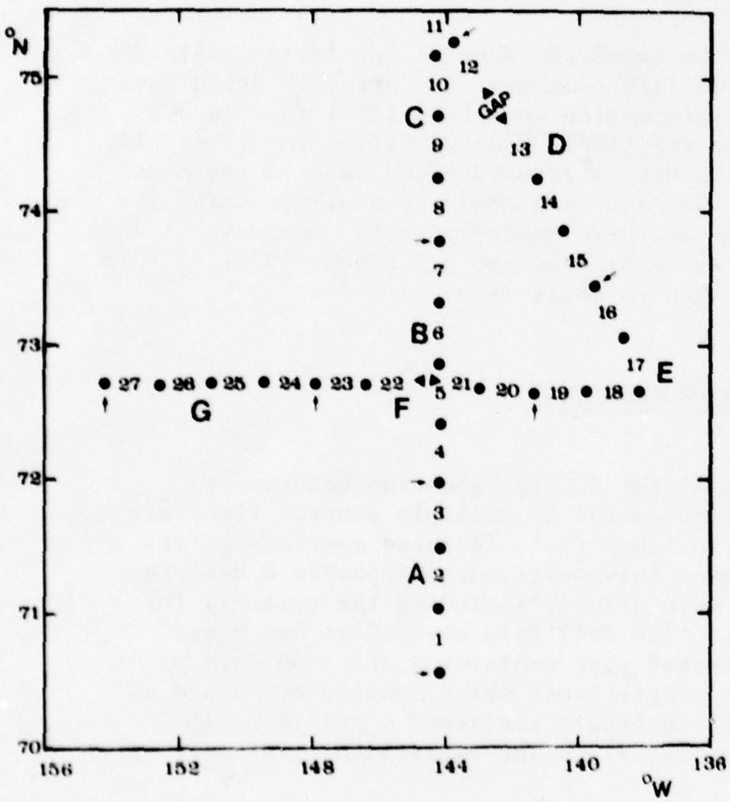


FIGURE 2 (left). Division of track into 50 km sections.

the data file into "intervals" for which the distance travelled is known.

The analysis at Scott Polar Research Institute was carried out on the University of Cambridge IBM 370/165 computer using an Algol 68C program which is listed and described in Appendix B. The program split the input file into "sections". Each section contained sufficient intervals to make up 50 km of data, and statistics were computed for each of the sections. Figure 2 shows the 27 sections involved (with an 18 km end-of-file gap between sections 12 and 13) and their precise positions and lengths are given in table 1.

Table 1. Positions of the 50 km sections.

Section		Latitude	Longitude	True length km
1	Starts	70° 35' 31.2"	144° 13' 14.4"	52.67
	Ends	71 03 49.2	144 13 18.0	
2	Ends	71 30 58.2	144 13 45.0	50.51
3	Ends	71 58 01.2	144 14 29.4	50.32
4	Ends	72 25 26.4	144 14 58.8	51.01
5	Ends	72 52 22.2	144 17 52.8	52.15
6	Ends	73 20 07.2	144 18 06.0	51.62
7	Ends	73 48 18.0	144 20 02.4	52.43
8	Ends	74 16 34.2	144 22 39.6	52.61
9	Ends	74 43 56.4	144 22 25.8	50.92
10	Ends	75 08 05.4	144 22 18.0	50.77
11	Ends	75 15 52.2	143 47 29.4	51.46
12	Ends	74 52 11.4	142 48 21.6	52.38
13	Starts	74 39 28.8	142 16 50.4	52.40
	Ends	74 15 43.2	141 20 22.8	
14	Ends	73 52 09.6	140 25 26.4	52.04
15	Ends	73 28 45.0	139 32 24.0	51.63
16	Ends	73 04 04.2	138 43 47.4	52.89
17	Ends	72 40 19.2	138 15 01.8	51.59
18	Ends	72 40 42.6	139 49 49.8	52.52
19	Ends	72 41 28.8	141 25 39.6	53.08
20	Ends	72 42 49.2	142 56 00.6	50.04
21	Ends	72 45 19.2	144 26 33.6	50.41
22	Starts	72 43 22.2	144 48 31.2	52.68
	Ends	72 43 14.4	146 23 51.6	
23	Ends	72 42 48.0	147 59 18.0	52.60
24	Ends	72 42 05.4	149 32 13.2	51.56
25	Ends	72 41 25.8	151 02 32.4	50.00
26	Ends	72 42 01.2	152 38 48.6	53.29
27	Ends	72 43 22.2	154 14 32.4	52.98

The actual depth data from the Arctic Submarine Laboratory came in the form of equally spaced depth points, the spacing being unspecified and varying from interval to interval. For every interval the Algol program therefore had to calculate an "interpolation length", the true spacing between depth points, by dividing the length of the interval (calculated assuming a Great Circle track between the position fixes for the beginning and end of the interval) by the number of points in the interval. The contribution made by each interval to the overall statistics for a 50 km section was then always weighted by the interpolation length, so that the resulting statistics are unbiased with respect to horizontal length.

The statistics generated are described and interpreted in the remainder of this report, and the full numerical values are given in Appendix C. The program produced two types of output file:-

(a) a line printer listing of the statistics, with details of the intervals making up the section (Appendix C);

(b) a file containing three frequency tables for each section (ice draft distribution, level ice and ridges) from which histograms were produced using a separate program. These appear in the body of the report.

Figure 3 shows a small part of the depth data plotted out. The two most obvious features are a high-frequency noise superimposed on the supposedly smooth ice bottom contour, and the occasional shallow depth point occurring within the structure of a pressure ridge. The noise is undoubtedly a feature of the recording system and, since it is random, it does not have a serious effect on probability densities of draft, although it will produce an anomalously high number of very small "pressure ridges" in the statistics. The shallow depth points are probably real features, caused by the very narrow sonar beam probing into fissures and crannies within the loose block structure of the ridge. The effect of these points may be quite large in causing a single pressure ridge to appear as multiple ridges in the statistics.

3. PROBABILITY DENSITY OF ICE DRAFT

3.1. Definition

The probability density function $P(h)$ of draft h is defined such that $P(h)\,dh$ is the probability that a random point on the ice underside has a draft between h and $(h + dh)$. $P(h)$ should really be expressed in the form $P(h, \underline{x}, t)$ since it is a function of time as well as of position. Further, although P can be stochastically defined at a point $\underline{x}$, an operational definition requires a profile to be taken over a finite length scale in order to arrive at an unbiased estimate of P. This length scale must be large enough to give a good estimator of P while small enough for the distribution not to change significantly within its compass. We have chosen 50 km, but some analyses have been done over shorter (17 km) and longer (200 km) length scales, where necessary.

$P(h)$ is related via the mean density of the ice to the thickness probability density function $g(h)$ of Thorndike *et al* (1975). $g(h)$ is important as an input parameter to various models of Arctic Ocean ice

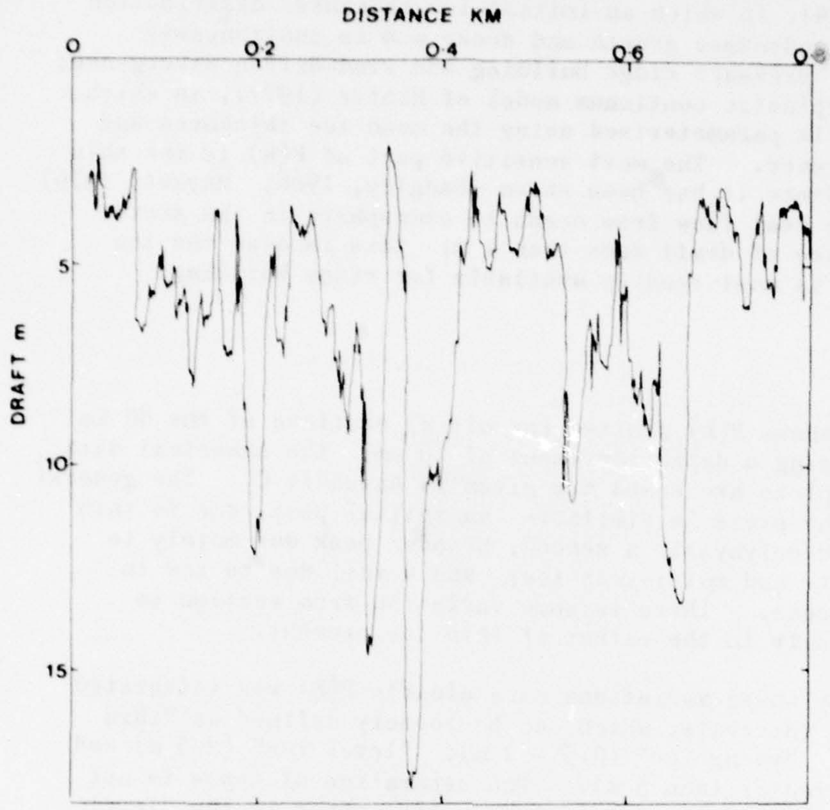


FIGURE 3. Plot of 800 m of ice depth data from section 1.

dynamics and thermodynamics. These include the AIDJEX model (Coon et al, 1974), in which an initial ice thickness distribution develops by thermodynamic growth and decay and is continuously redistributed by pressure ridge building and wind-driven divergence; and the viscous-plastic continuum model of Hibler (1977), in which a pressure term is parameterised using the mean ice thickness and percentage ice cover. The most sensitive part of P(h) is the thin ice component, since it has been shown (Badgley, 1966; Maykut, 1976) that most of the heat flow from ocean to atmosphere in the Arctic occurs through ice of draft less than 1 m; this is also the ice component which is most readily available for ridge building.

3.2. Results

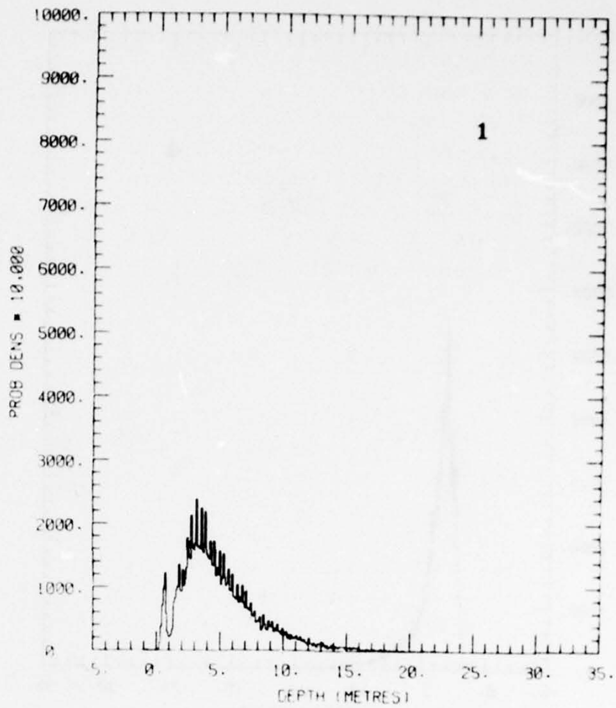
Figure 4 shows P(h) plotted for all 27 sections of the 50 km length scale, using a depth increment of 10 cm; the numerical data on which these plots are based are given in Appendix C. The general nature of all the plots is similar:- an initial peak, due to thin ice in leads and polynyas; a second, broader peak due mainly to undeformed first- and multi-year ice; and a tail due to ice in ridges and hummocks. There is some variation from section to section, especially in the extent of thin ice present.

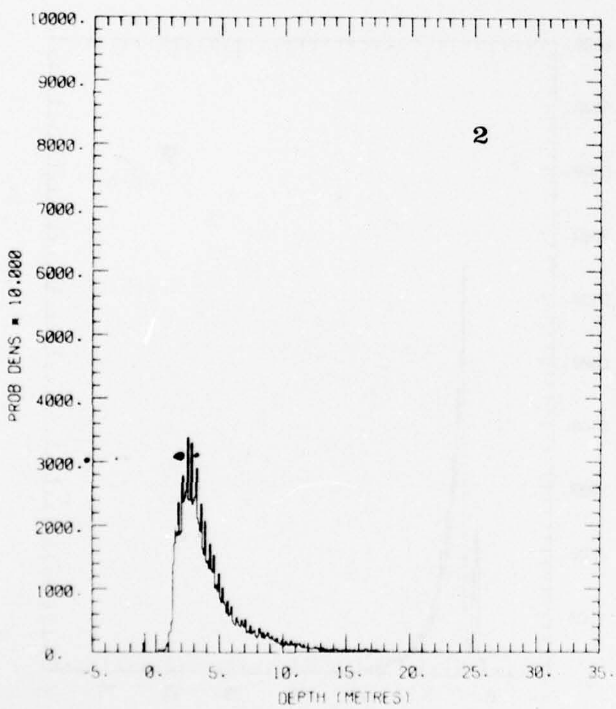
To display these variations more clearly P(h) was integrated over four depth intervals, which can be loosely defined as "thin ice" (0-0.5 m); "young ice" (0.5 - 2 m); "level ice" (2-5 m) and "ridged ice" (greater than 5 m). The separation of types is not perfect - parts of ridges, for instance, may appear in the "level ice" category - but the categories are indicative of changes in the nature of the ice cover. The results are given in *Table II*. The intervals were chosen so as to give a direct comparison with the data of Wadhams (1977b) from the heavily ridged offshore zone to the north of Greenland and Ellesmere Island. Wadhams found that the "thin ice peak" in his probability density functions usually occurred at less than 0.5 m draft, hence his choice of intervals, but the present results (fig. 4) usually show the peak at between 0.5 and 1 m, presumably because the profiles were done later in the winter (April compared to October for SOVEREIGN) so that the ice in polynyas is, on average, thicker. Thus we have also added a 0-1 m category in Table II to include all of the polynya ice.

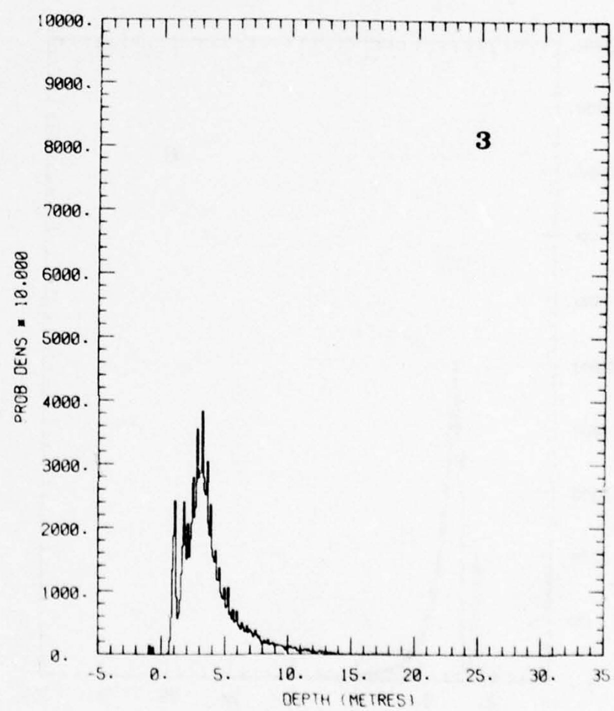
The results show a remarkable consistency of ice conditions over most of the experimental area. The exceptions are:-

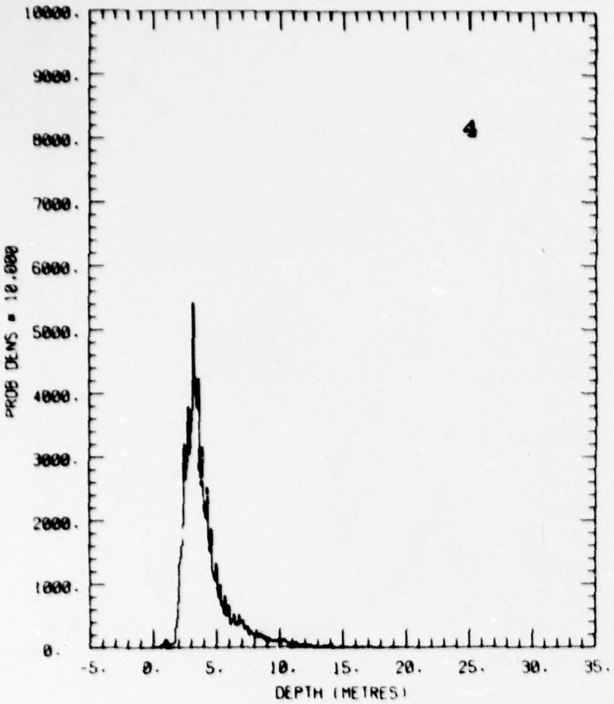
(i) the percentage of thin ice, which varies over a wide range (0.4 to 12.3% for the 0-1 m band) and with no apparent consistency of trend. The cause is partly statistical - thin ice is contained in a limited number of polynyas which are distributed non-uniformly along the submarine track - and partly real in the sense that thin ice has a transient existence and is constantly being destroyed by ridge-building so that changes in the wind field during the 3 days of the experiment may cause the thin ice to be radically redistributed.

FIGURE 4. Probability density function of ice draft for 50-km sections. Depth increment 10 cm.





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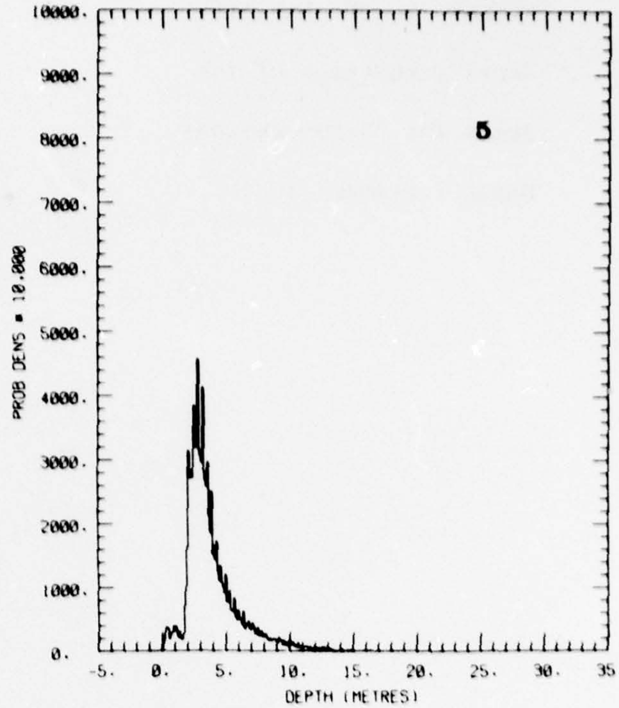
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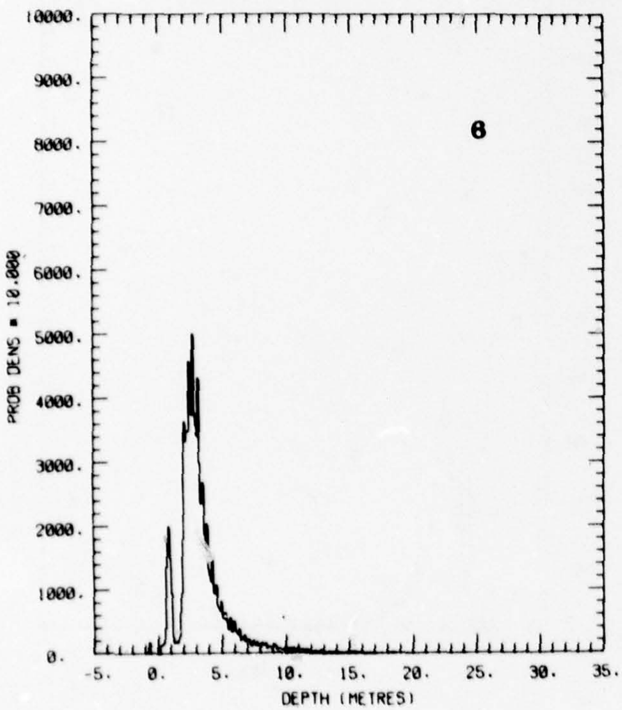
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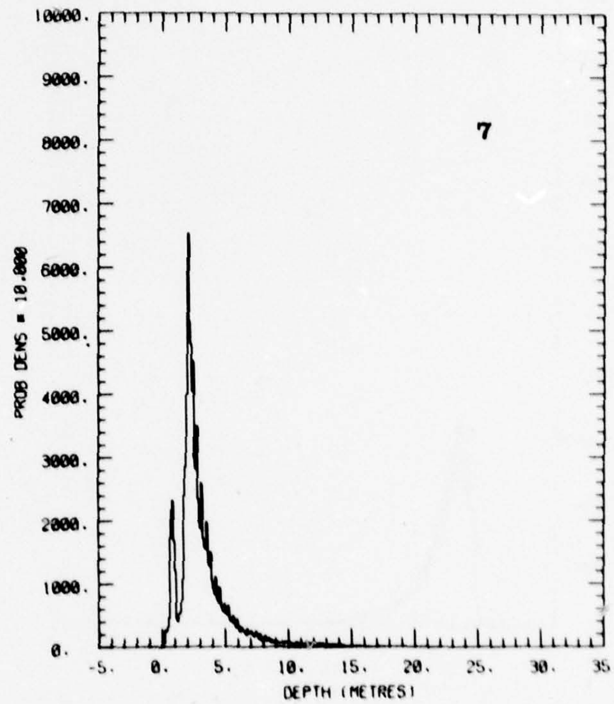
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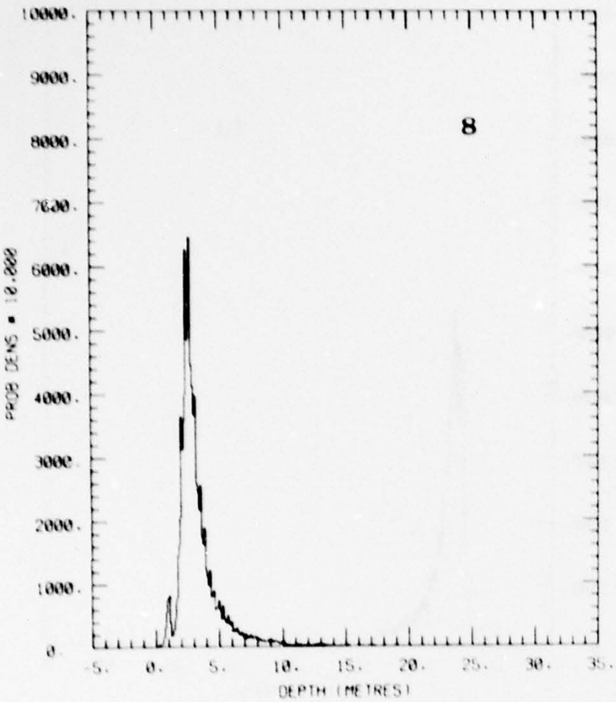
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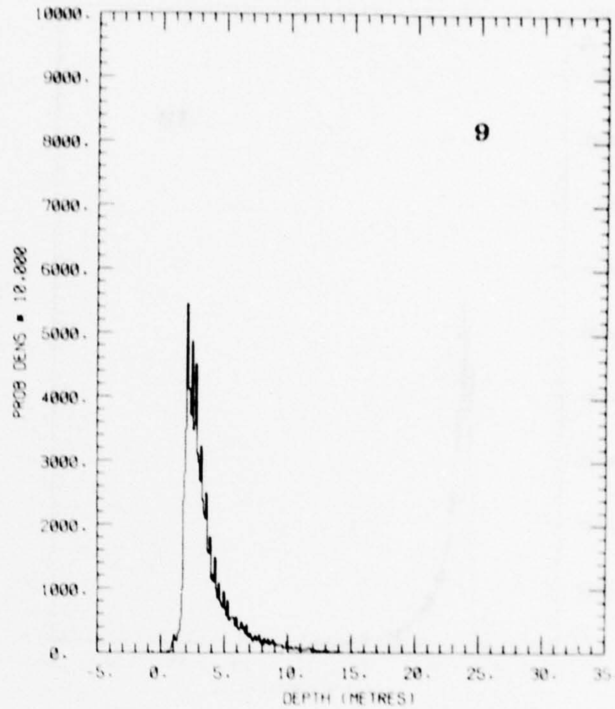
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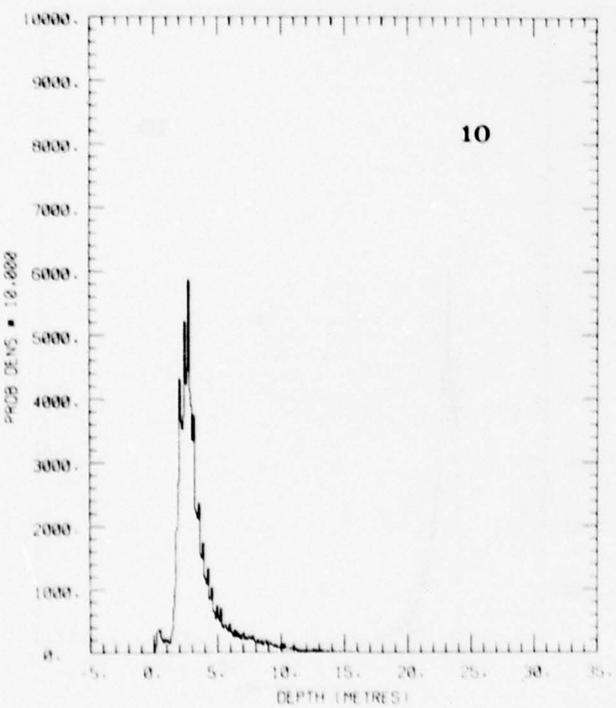
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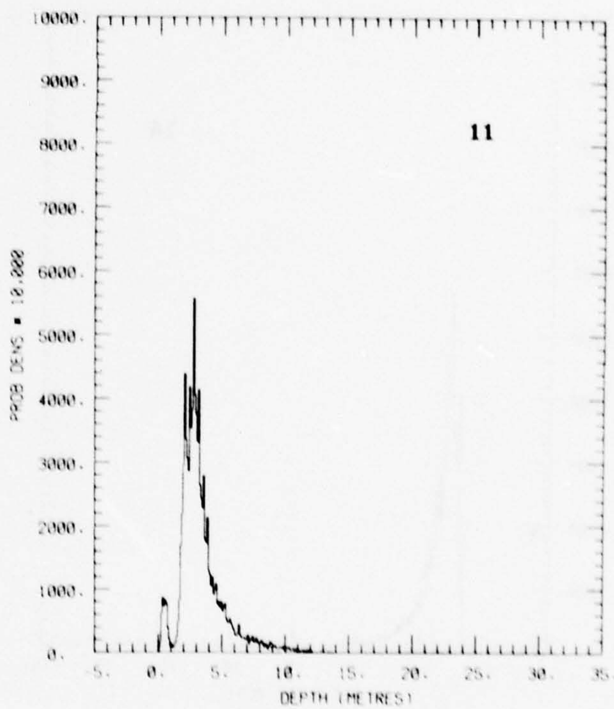
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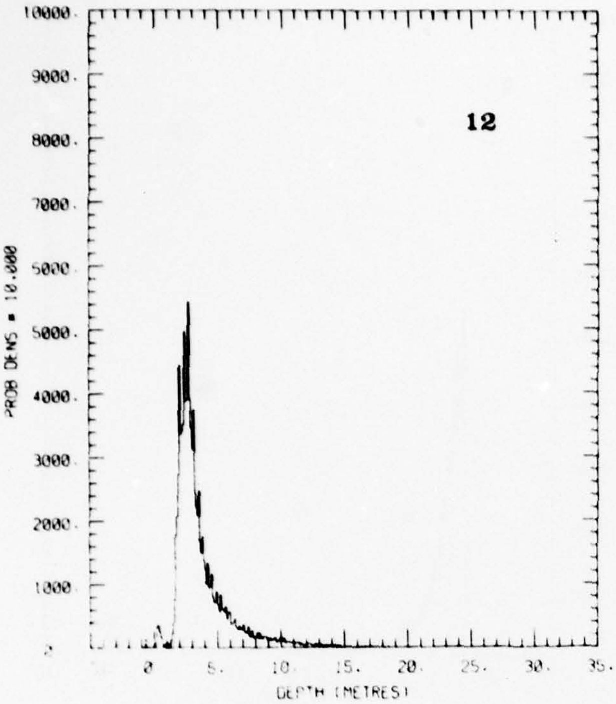
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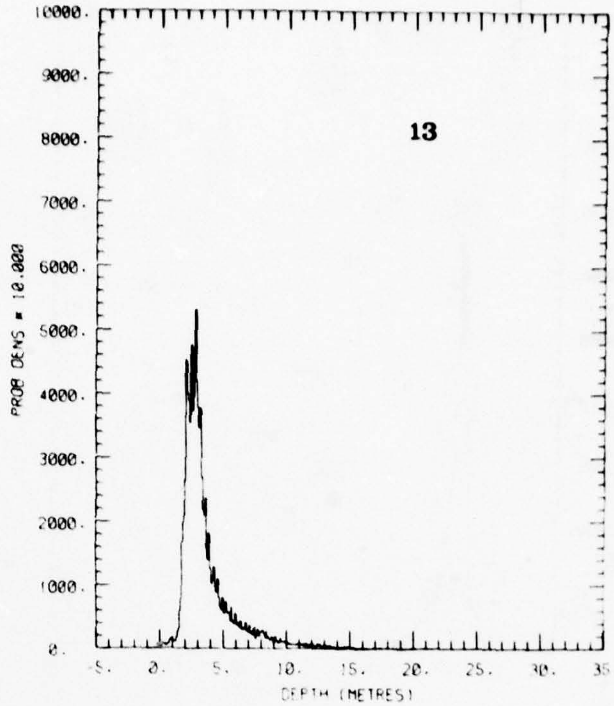
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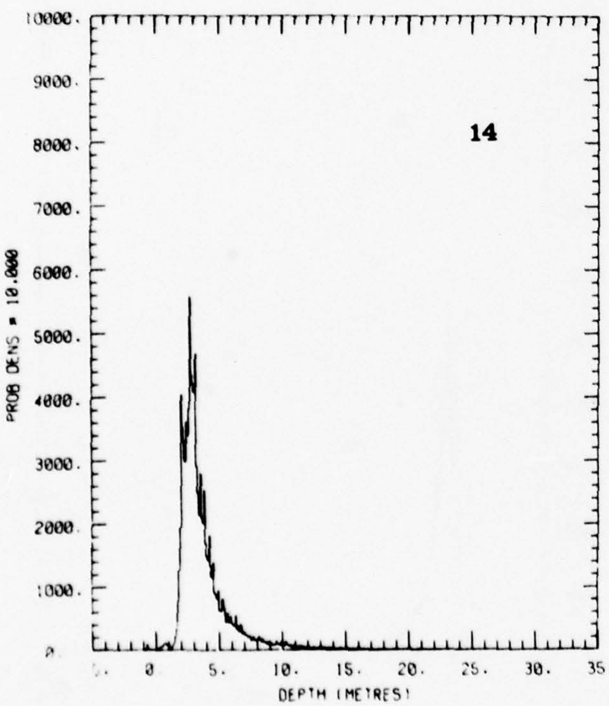
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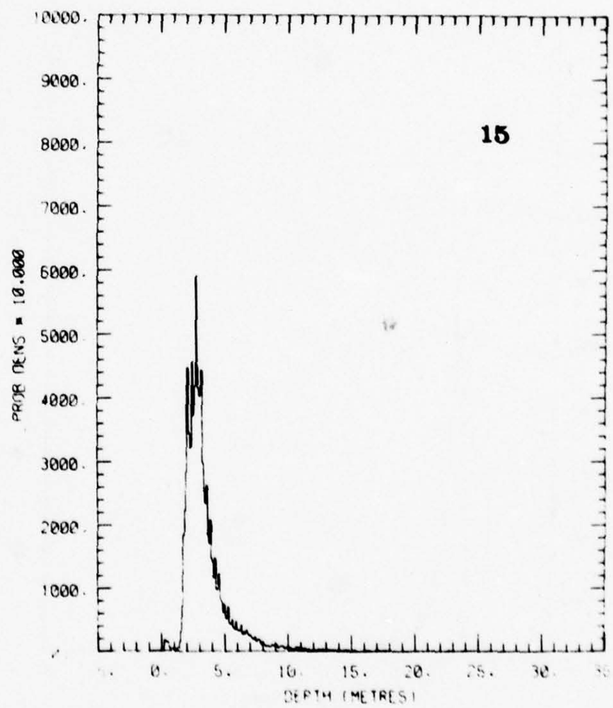
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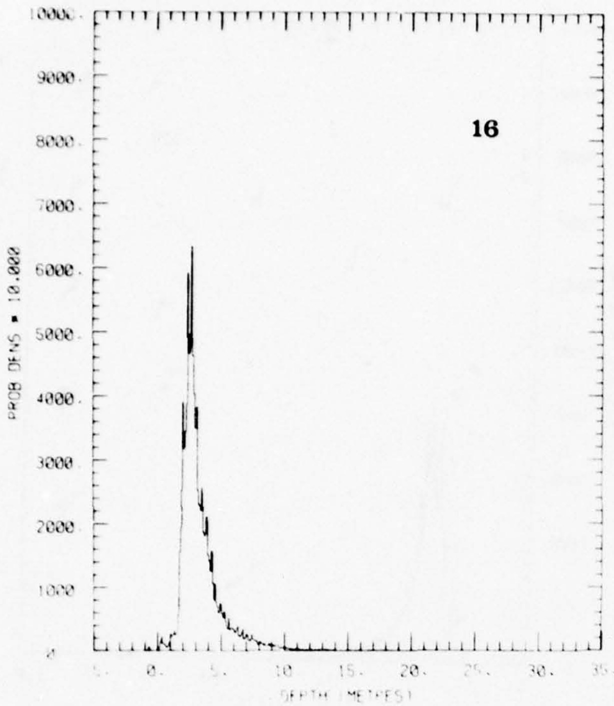
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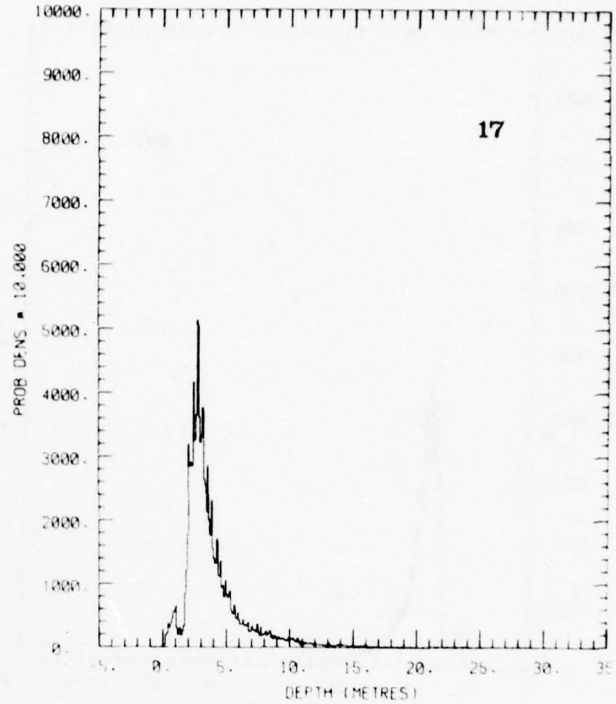
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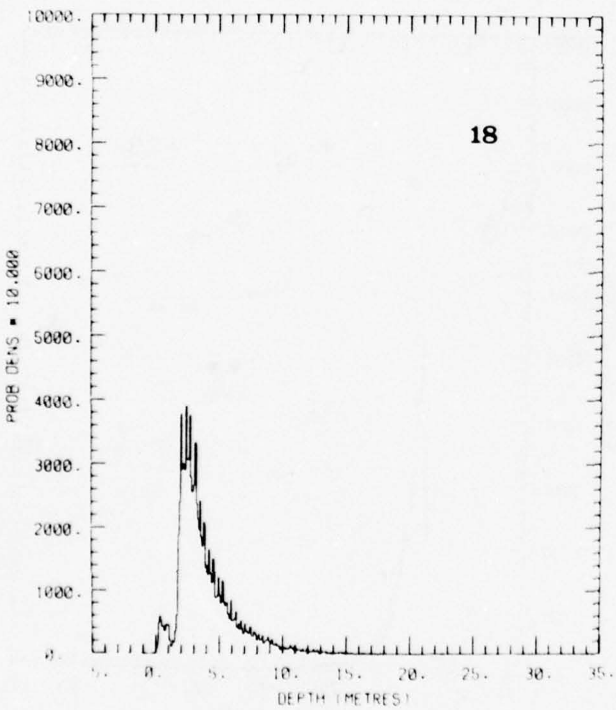
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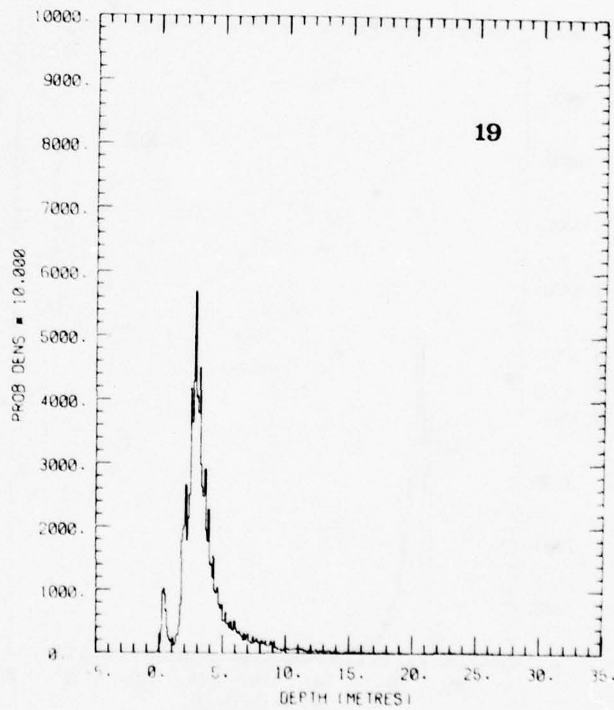
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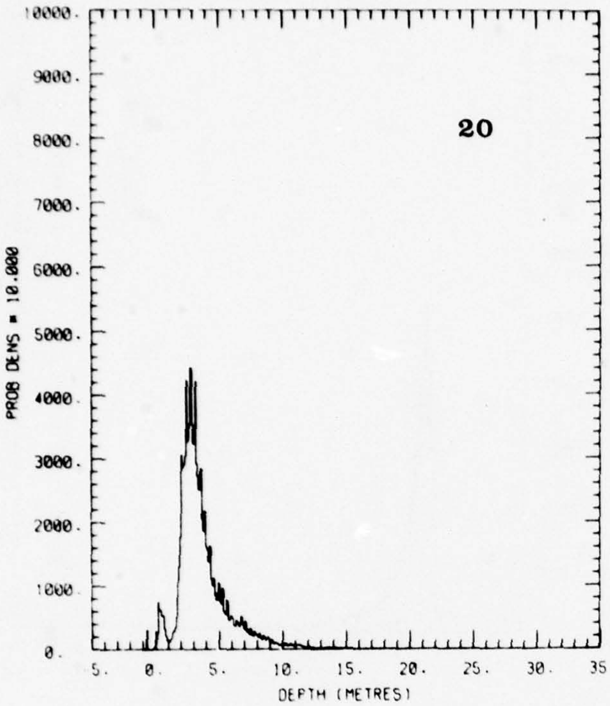
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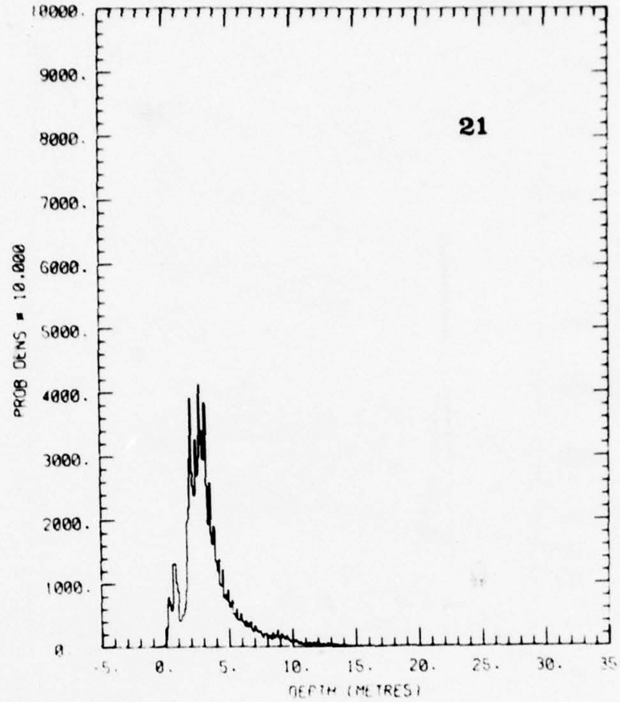
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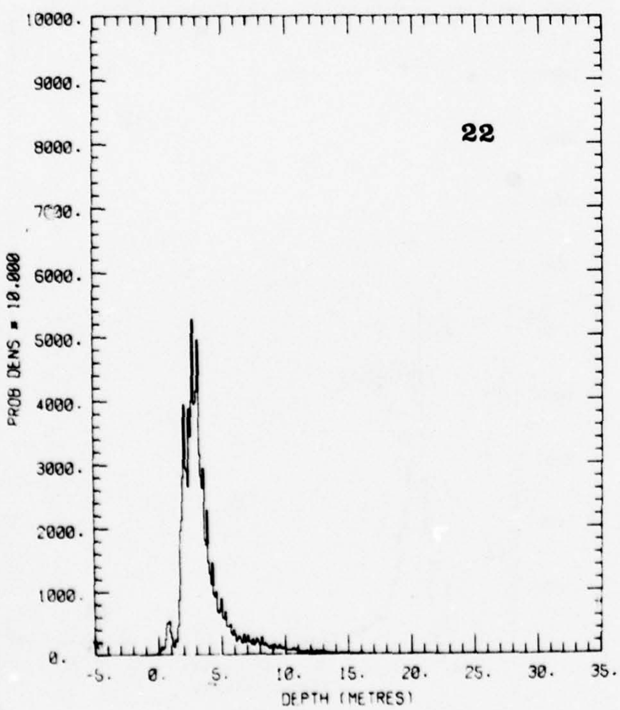
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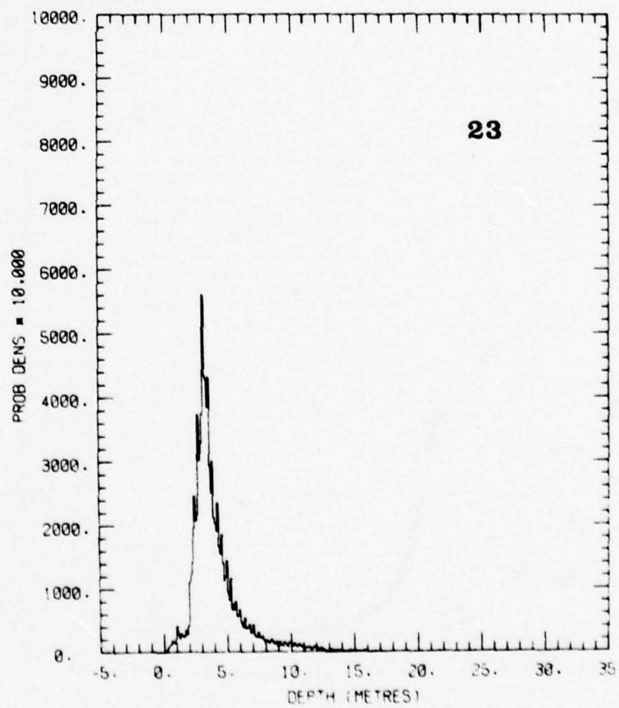
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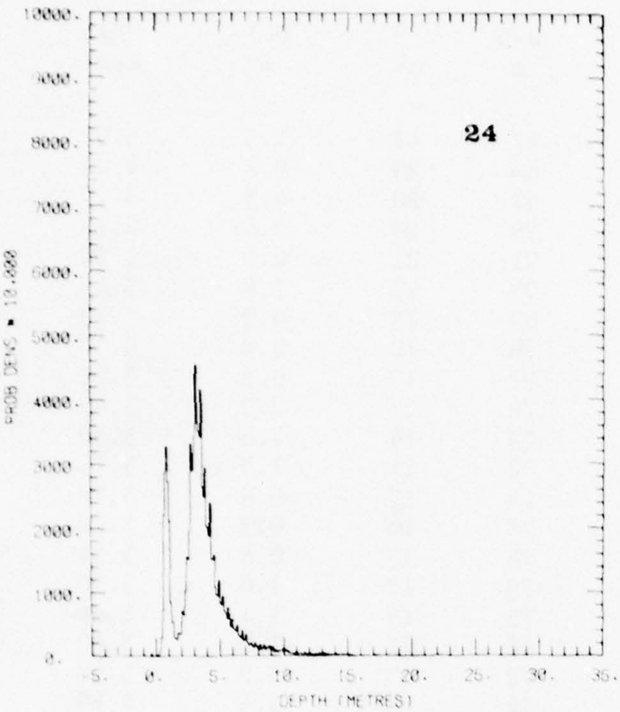
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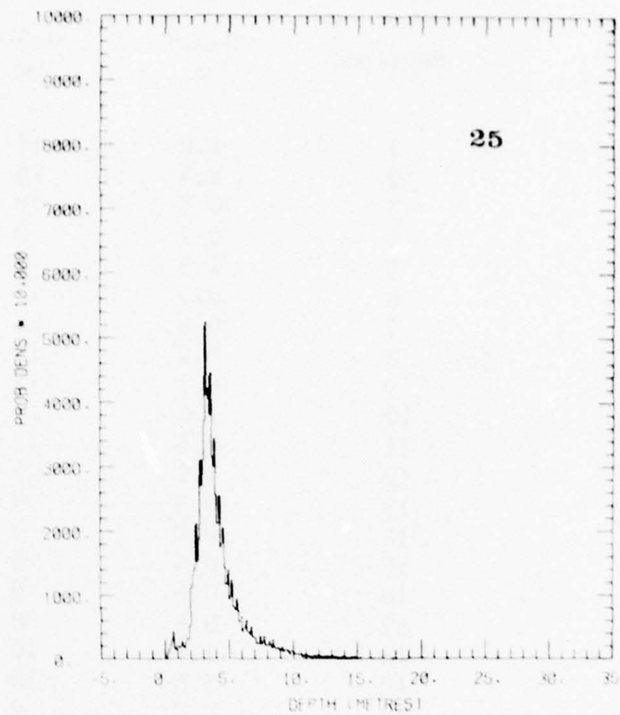
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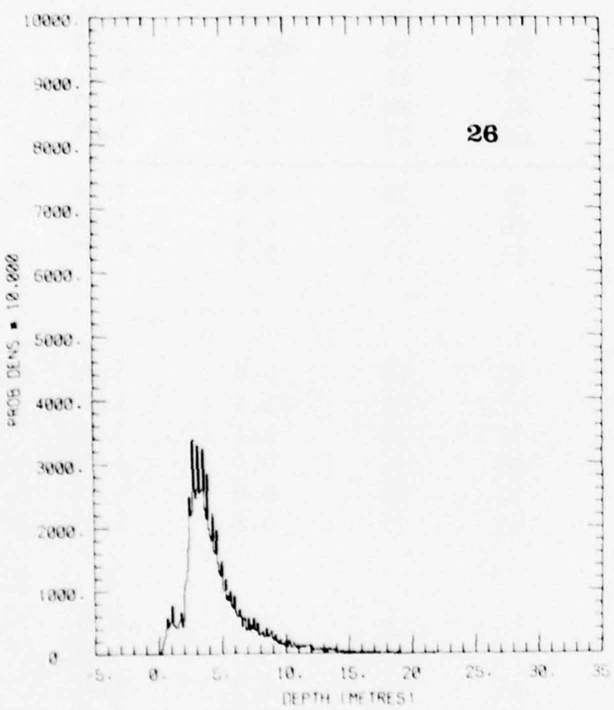
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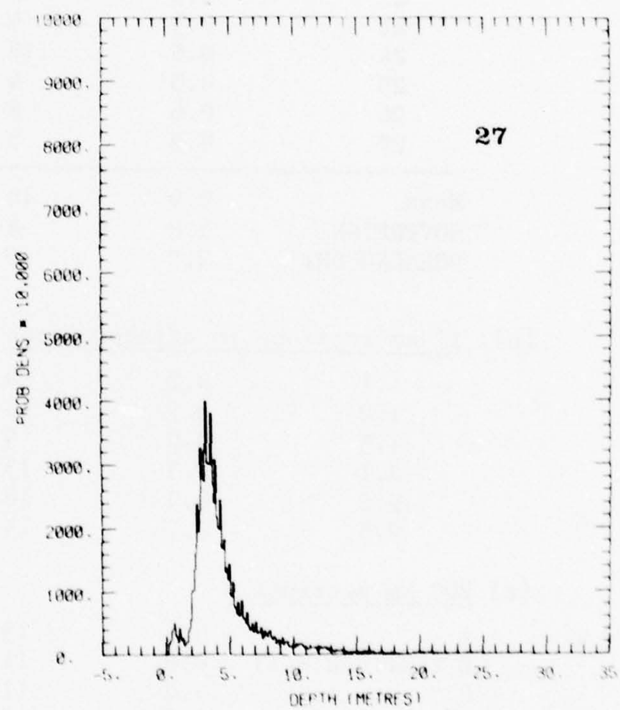
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PROB DENS * 10.000
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Table II. Percentages of ice cover in different ranges of draft

(a) 50 km sections

Section	0-0.5 m	0.5-2 m	2-5 m	5 m+	0-1 m	Mean draft
1	1.2	11	47	41	4.5	5.09
2	0.4	15	60	25	0.8	4.22
3	0.3	18	62	20	4.3	3.77
4	0.1	2	78	20	0.4	4.14
5	1.3	7	71	21	2.8	3.92
6	0.7	13	73	13	7.8	3.45
7	0.9	23	63	13	9.3	3.19
8	0.2	10	78	12	2.0	3.57
9	0.1	12	71	17	0.5	3.61
10	1.2	11	74	14	2.2	3.47
11	2.5	11	73	14	4.8	3.40
12	1.2	9	72	18	1.5	3.74
13	0.3	9	74	17	0.8	3.70
14	0.2	5	78	16	0.7	3.78
15	0.5	9	78	13	0.8	3.51
16	0.6	9	79	12	1.0	3.38
17	0.8	8	72	19	3.1	3.89
18	1.8	10	66	22	3.9	3.87
19	3.5	10	72	15	5.2	3.62
20	2.7	8	72	18	4.4	3.69
21	2.7	17	64	16	8.2	3.47
22	0.5	11	75	14	2.6	3.53
23	0.3	4	76	20	1.1	4.14
24	0.5	18	65	16	12.3	3.63
25	0.5	4	75	21	1.7	4.17
26	0.6	8	63	28	3.0	4.50
27	0.8	5	68	27	2.5	4.61
Mean	0.9	10	71	19	3.4	3.81
SOVEREIGN	4.8	4	28	63	6.5	7.22
DREADNOUGHT	2.7	9	55	33	3.3	4.29

(b) 17 km sections in offshore zone

1.1	0.2	9	42	48	0.6	5.58
1.2	3.7	14	47	35	13.3	4.45
1.3	0.0	9	53	38	0.1	5.18
2.1	0.3	13	56	31	0.9	4.51
2.2	0.3	18	57	24	0.6	4.14
2.3	0.7	13	66	21	0.8	4.03

(c) 200 km sections

A	0.6	15	56	29	3.2	4.36
B (Caribou n-s)	0.8	11	71	17	5.1	3.68
C	1.0	11	74	14	2.4	3.46
D	0.6	8	76	16	1.0	3.68
E	1.7	9	72	17	3.3	3.69
F (Caribou e-w)	1.6	10	72	17	4.1	3.71
G	0.6	9	68	23	4.9	4.23

(ii) Ridged ice at the southernmost (and, to a lesser extent, the westernmost) end of the profile is significantly greater in quantity. The percentage of ridged ice is very high in section 1 and diminishes to a fairly steady "equilibrium value" by section 3. Clearly the first 2 sections represent the "offshore province" of Weeks et al (1971), a heavily ridged coastal province where the mean onshore tendency of ice drift leads to net convergence and ridge building. To a lesser extent sections 26-27 mark the outer edges of this province further to the west. Figure 1 shows that sections 1 and 2 occur over the continental slope (the profile itself commences at the 100 m depth contour) and that section 27 ends just as this slope is again approached off Point Barrow. The remainder of the sections are samples of what seems a very homogeneous ice cover.

The mean values over all 27 sections are compared in table II with the mean results from SOVEREIGN (a 1000 km profile from 81°N 0°W to 84°50'N 70°W) and DREADNOUGHT (a 560 km profile from 85° to 90°N at 6°E in the ice of the Trans Polar Drift Stream; Williams et al, 1975). The best agreement is with the DREADNOUGHT data, although the GURNARD data show a somewhat lower mean draft which can be ascribed to the beamwidth of the DREADNOUGHT's echo sounder. Clearly the ice encountered by SOVEREIGN was far more heavily ridged and thicker than even the heaviest section of GURNARD.

The 50 km gauge is short enough to resolve most real variations, but to investigate the rapidly changing ice conditions at the beginning of the profile (sections 1 and 2) a 17 km gauge was used, i.e. each section was split into three. The results, in table II(b), show a steady decrease in the percentage of ridged ice as the submarine travels north away from the Alaskan coast. Note the isolated value of 13.3% for 0-1 m ice in section 1.2.

Finally, to obtain very reliable statistics for large tracts of the ice cover, the 50 km sections were combined into a 200-km length gauge as shown in figure 2, lettered A to G (A is 150 km only). B and F are now the appropriate sections for the crossings under Caribou camp, and have the length scale recommended in the AIDJEX model and by Thorndike et al (1975). The results for B and F agree very closely. Again, the 200 km statistics show that the character of the ice is essentially constant over most of the track (B to F), with an increase in mean draft and percentage of ridged ice at the western end (G) and, particularly, the southern end (A).

3.3 Statistical reliability

It is clear from table II(b) that there are progressive changes in the nature of the ice cover over sections 1 and 2, and that these sections (and probably 26 and 27) differ in nature from sections 3-25. The question that remains is whether the variations between sections in 3-25 are statistical artefacts, i.e. due to finite sampling length, or whether they are due to real variations, albeit minor, in the nature of the ice cover. Our null hypothesis is that the ice cover over a substantial part of the southern Beaufort Sea (the area sampled by sections 3-25) is a homogeneous cover in which any given statistical parameter tends to the same value everywhere if sampled over a sufficient length of track.

The question of "sufficient length" is a crucial one. For a given sampling length, some parameters are estimated more accurately than others. For instance, 50 km of track usually contains enough ice types to make it a good estimator of mean draft, and most probably of percentage ridged ice. It may not be a good estimator, however, of thin ice percentage since this ice is contained in a small number of polynyas which may not happen to fall uniformly within the length gauge. A longer gauge may be required for a good estimate of this parameter, and also of such parameters as mean polynya spacing or the frequency of very deep pressure ridges.

We can test for the homogeneity of the data using a non-parametric run test (Bendat and Piersol, 1971). The data is divided into n sections for which a given statistic S takes values S_j (j=1,n). The mean value of S is calculated, and each section is classified as (+) or (-) according as $S_j > \overline{S}$ or $S_j < \overline{S}$. The number of runs of consecutive (+) or (-) classes in the n sections is found and tested for significance. An exceptionally small number of runs signifies a trend or a clustering in S; an exceptionally large number signifies a factor tending to cause alternation of high and low S values. The 23 50-km sections (3-25) were tested in this way, and an additional test was afforded by 30 17-km sections corresponding to sections 3-12 (sections 1-12 were analysed at 17-km gauge for use in table II). The mean values of the statistics of table II over 3-25 and the results of the test are as follows (* signifies non-significance):-

	Mean and standard error	Test - 50 km n=23	Test - 17 km n=30
0-0.5 m percentage cover	1.0 ± 0.2	2.5%	*
0.5-2 m " "	10.3 ± 0.9	*	*
2-5 m " "	72 ± 1	*	*
over 5 m " "	16.6 ± 0.7	*	*
0- m " "	3.5 ± 0.7	*	*
mean draft " "	3.67 ± 0.06	*	*

Only one statistic - the 0-0.5 m percentage cover - was rejected by this test, at the 25% significance level and for the 50 km sections. This implies that the thin ice percentage does not come from a homogeneous population, i.e. that there are significant trends or clusterings in this statistic (which can be seen by inspection of table II) suggestive of a process acting with a wavelength much greater than 50 km and tending to cause large amounts of thin ice over a number of sections followed by small amounts. This process must be the wind stress field which causes divergence in one zone of the ice cover and convergence in another, on a length scale of hundreds of km.

Otherwise we can accept the hypothesis that the ice draft distributions over 1150 km of track in the southern Beaufort Sea (3-25) come from a homogeneous ice cover with constant statistical properties. The best values to take for the mean draft and percentages of ice in various depth ranges are given above, together with the standard error. It can be seen that this standard error is virtually constant for each class (except 0-0.5 m), implying a greater fractional error in the estimates of uncommon ice types than in those of common types - a result to be expected.

3.4 Cumulative probability

The cumulative probability G(h) is defined by

$$G(h) = \int_0^h P(h)\, dh$$

It is used as a major parameter in the AIDJEX model (Coon et al, 1974) where it is known as the "ice thickness distribution". We have computed G(h) for the two 200 km sections B and F which cross the Caribou camp in the south-north and east-west directions. The results are shown in figure 5.

The two distributions differ slightly but are similar in general shape. The median depth (G(h) = 0.5) is reached at 3.2 m and the graph is plotted as far as G(h) = 0.99, which is reached at 12.2 m. G(h) = 0.999 is reached at 16.4 m.

4. LEVEL ICE

4.1 Definition

Williams et al (1975), in their analysis of the DREADNOUGHT data, sought a way of determining the preferred thickness or thicknesses of undeformed floes, and the percentage of the ice cover occupied by floes of this type. By trial and error they decided that the best working definition of "level ice" is that the draft point concerned should have draft points 4 m to each side of it differing in depth by less than 20 cm, i.e. a local gradient of less than 1 in 40 measured on an 8 m gauge length across the point. On account of the high-frequency structure in the GURNARD profile (fig. 3), we have relaxed this definition slightly and we define a level ice point as one whose draft differs from a point 10 m away to either side by less than 25 cm, i.e. a 1 in 40 gradient in one direction moving away from the point.

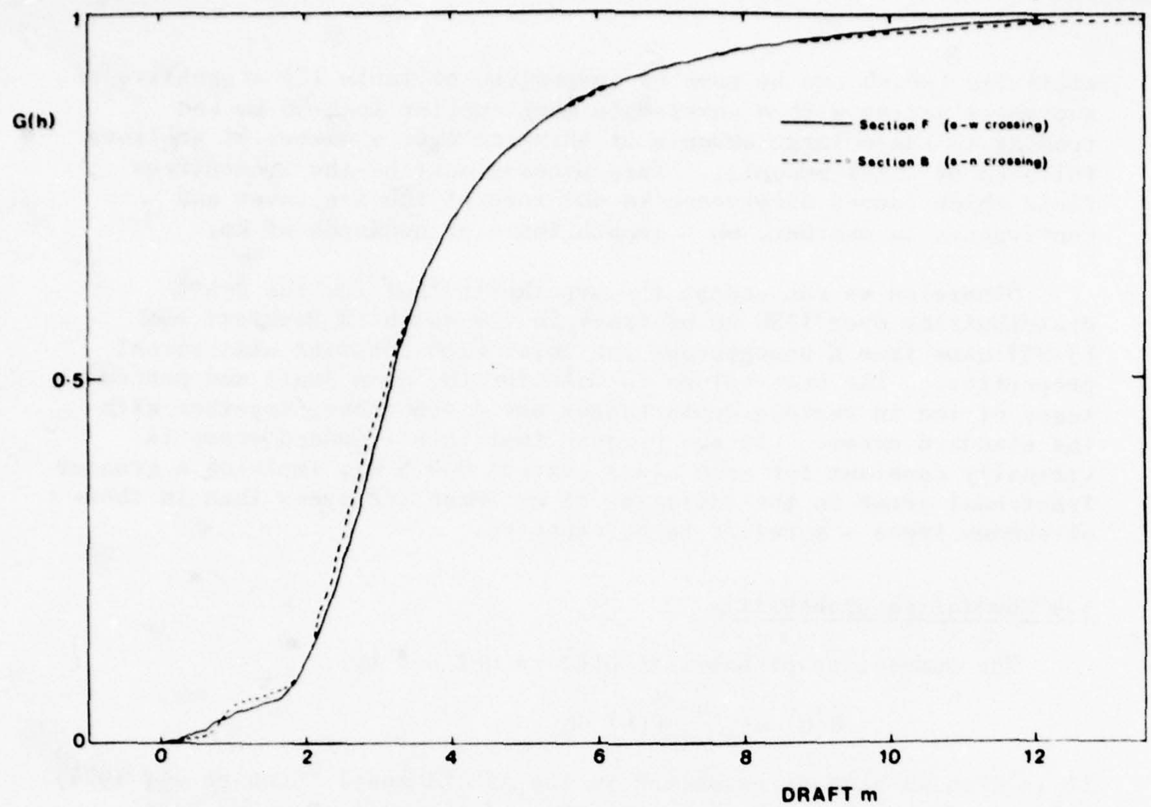


FIGURE 5 (above). Cumulative probability distribution G(h) of ice draft for the two 200 km sections which straddle the Caribou camp.

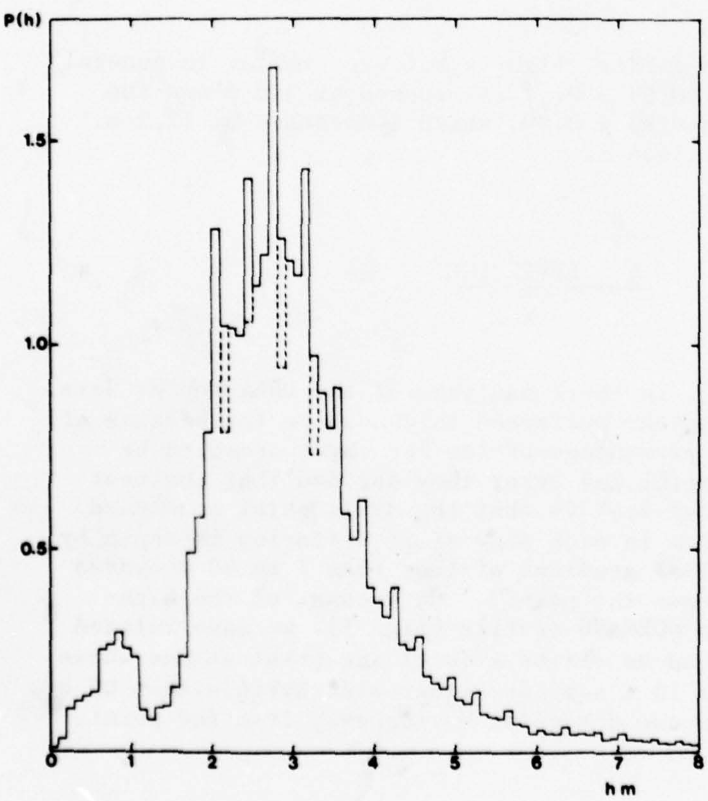


FIGURE 7 (left). Probability density function P(h) of level ice draft for whole submarine track. Bin size 0.1 m. Dotted lines show bins between 2 and 4 m draft after normalisation (see text).

4.2 Results

Figure 6 shows probability density functions of level ice draft for all 27 sections at 50 km gauge, again using a 10 cm depth increment. Any "level ice" occurring beyond about 5 m draft must be on the bottoms of ridges and hummocks. The inferences to be drawn from these plots concern:-

(i) positions of peak or peaks, indicating preferred drafts of level ice of varying ages (young, first-year, second-year, etc.);

(ii) mean draft of level ice;

(iii) percentage of total ice cover occupied by "level ice".

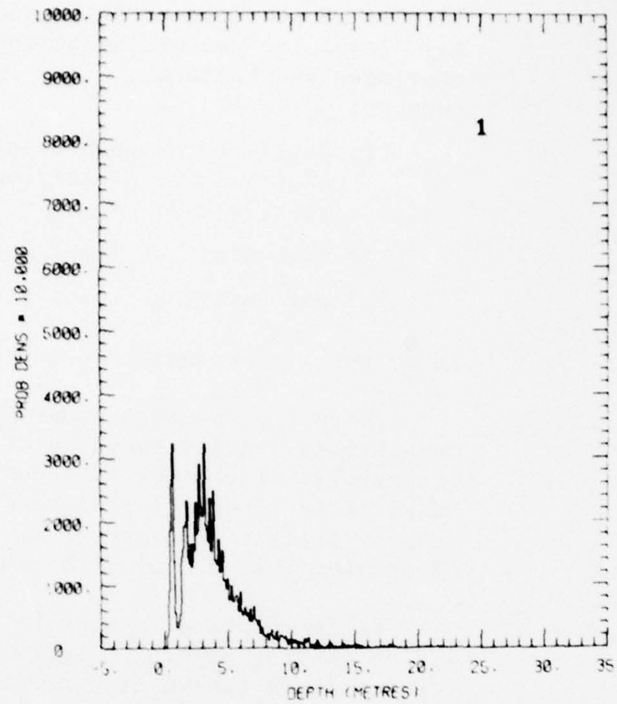
These results are gathered in table III.

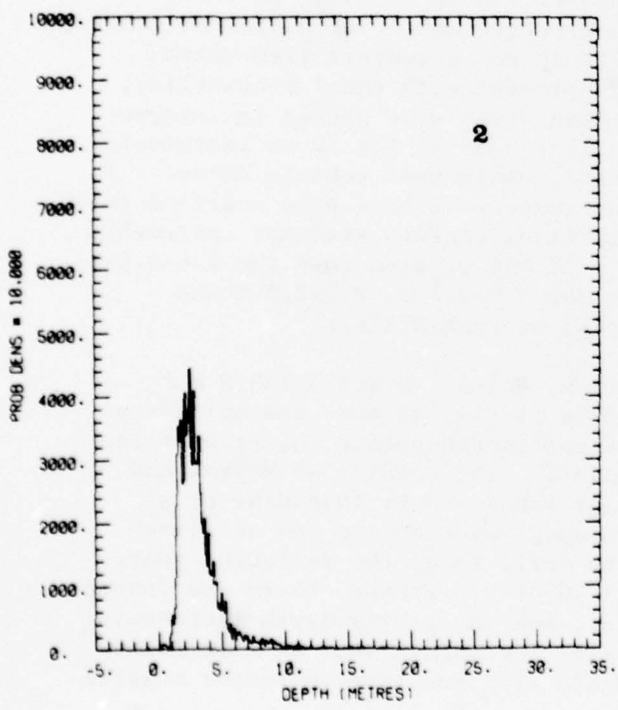
There are two methods of estimating (i). One is to plot the probability density function of level ice over the whole track length (figure 7); the other is to use a ranking technique on the individual 50 km plots to detect preferred drafts. In table III this has been done by listing in order of size every bin with a probability density of greater than 0.5 plus the thin ice peak if it exceeds 0.1.

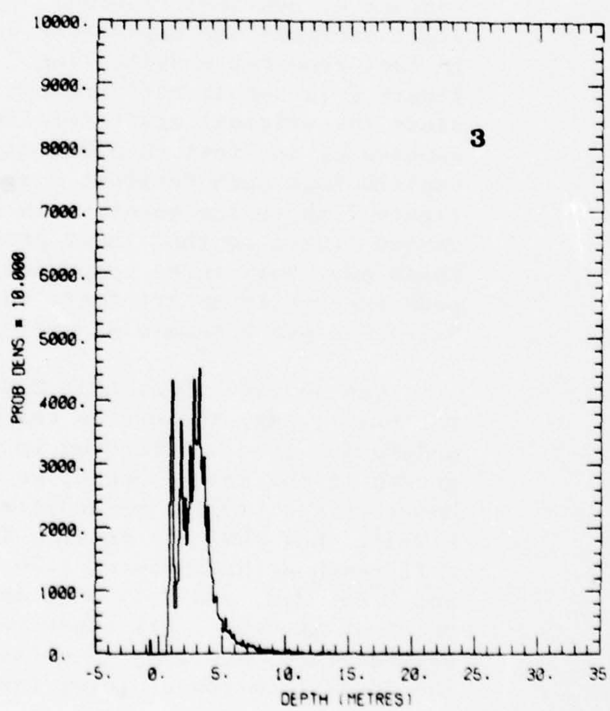
It is tempting to ascribe the preferred depths of table III to ice of varying ages. Clearly the 2.7-2.8 m category is the most popular depth (shown also by figure 7) with 2.3-2.5 m and 2.0-2.1 m as lesser peaks. Thick level ice of 3.1 depth and above is confined to regions in the far west and far south of the profile, i.e. the regions of heaviest ridging. However, figure 7, which at first sight confirms the hypothesis of a discrete number of preferred drafts, in fact required modification. The 10 cm increments from which figure 7 is constructed are not all present with equal probability, since the original draft data from San Diego were quoted in integral numbers of decifeet (0.03048 m). Thus some of the 10 cm increments contain four such decifeet components, while most contain three. In figure 7 those increments with four components have been modified (with dashed lines) so that their probabilities conform with the increments which have only three components. It can be seen that the 2.4-2.5 m peak was really an artefact, while the 2.0-2.1 m, 2.7-2.8 m and 3.1-3.2 m peaks remain as valid peaks of probability.

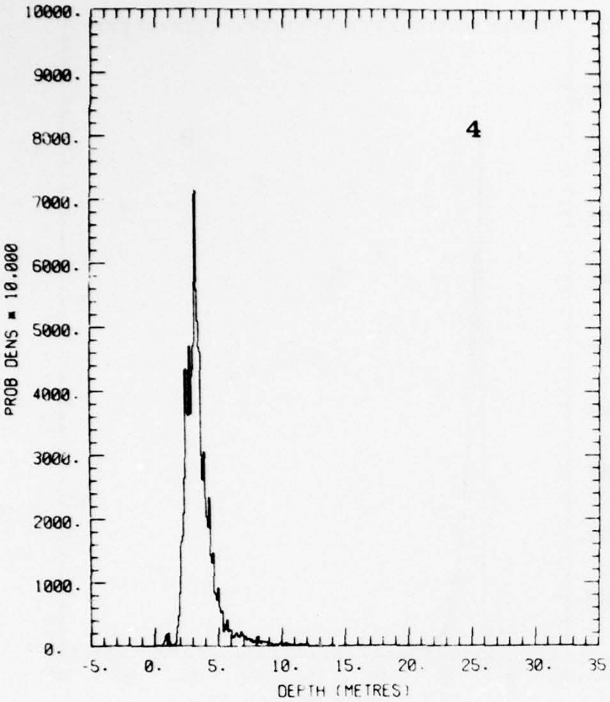
Can we say, then, that 2.0-2.1 m, 2.7-2.8 m and 3.1-3.2 m represent, say, the drafts reached by first-, second- and multi-year undeformed ice? According to present thermodynamic theories of ice growth in the Arctic Ocean, we cannot. The results of Maykut and Untersteiner (1971), as modified and tabulated by Thorndike et al (1975), show that ice growing from open water at the end of summer will reach a thickness of 1.76 m by April 10 of the following year, and 2.04, 2.21 and 2.35 m by April 10 of its second, third and fourth years of growth. All these values, and the yearly depth increments between them, are less than the level ice values that we have found. The identification of level ice draft with ice type therefore remains an open question.

FIGURE 6. Probability density function of level ice draft for 50-km sections. Depth increment 10 cm.





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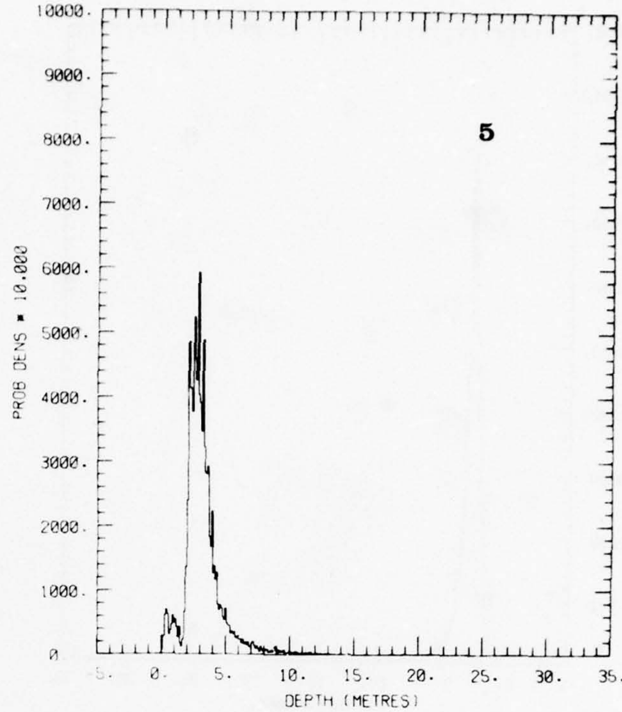
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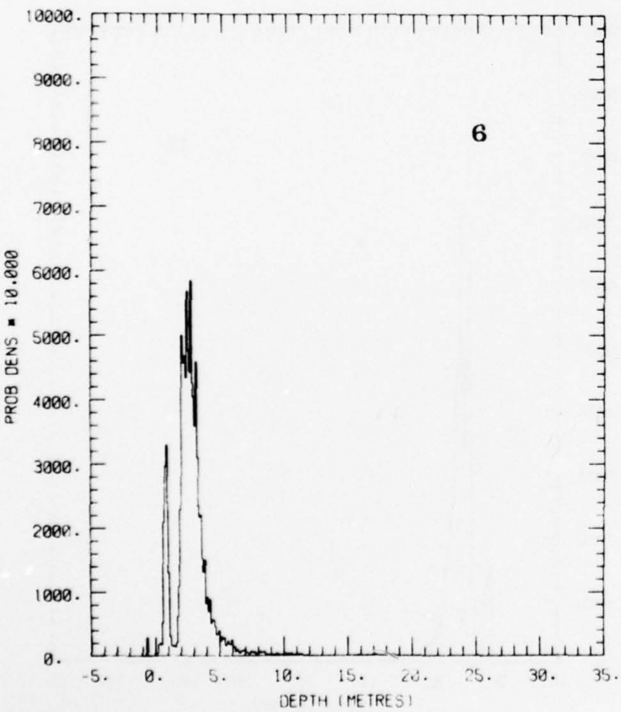
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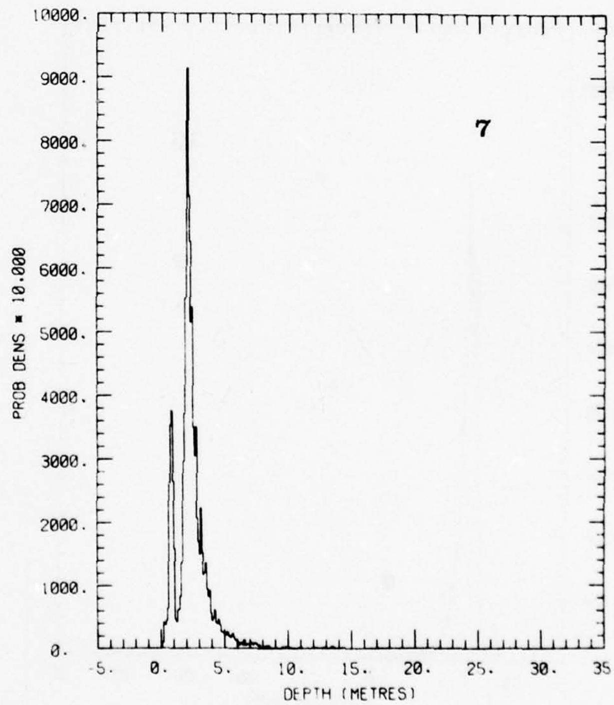
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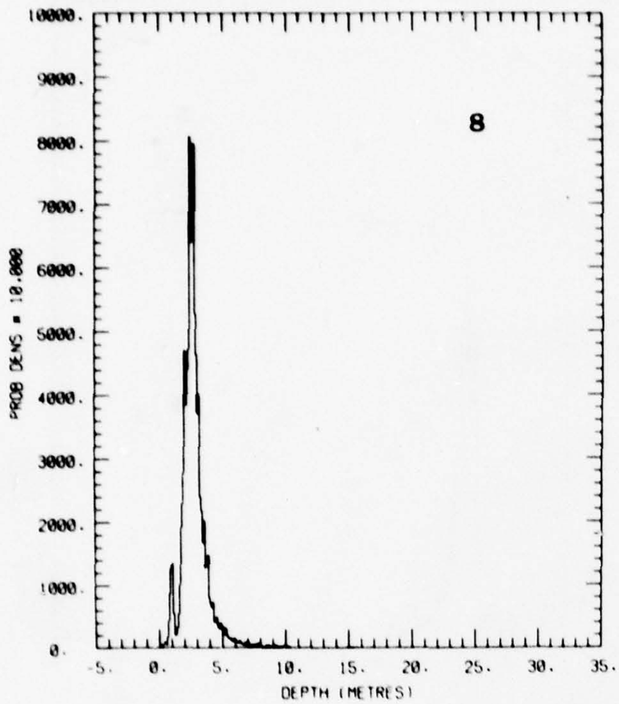
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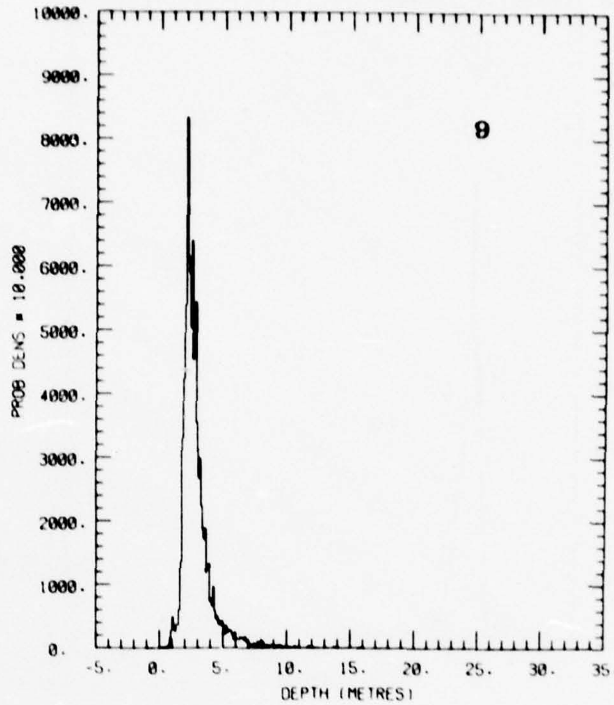
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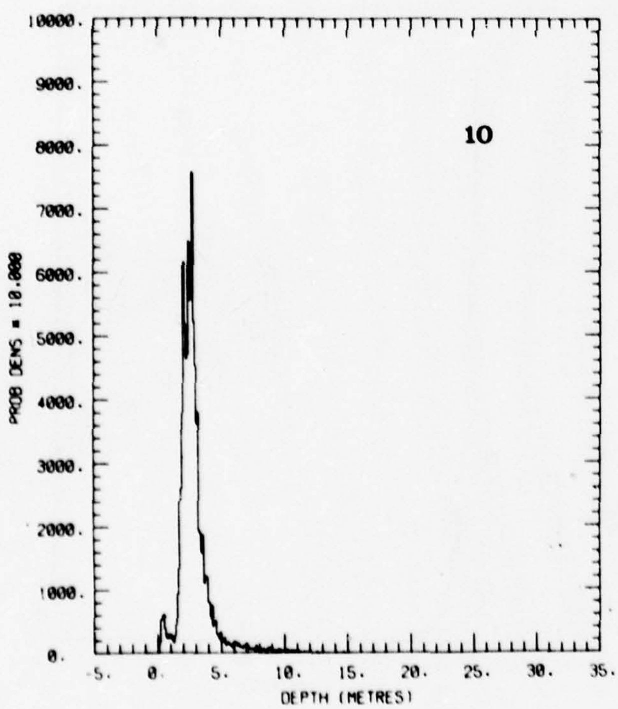
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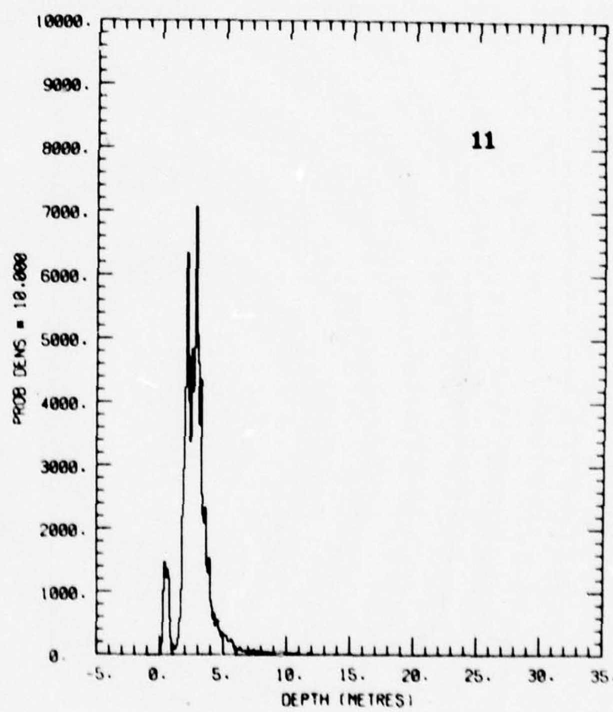
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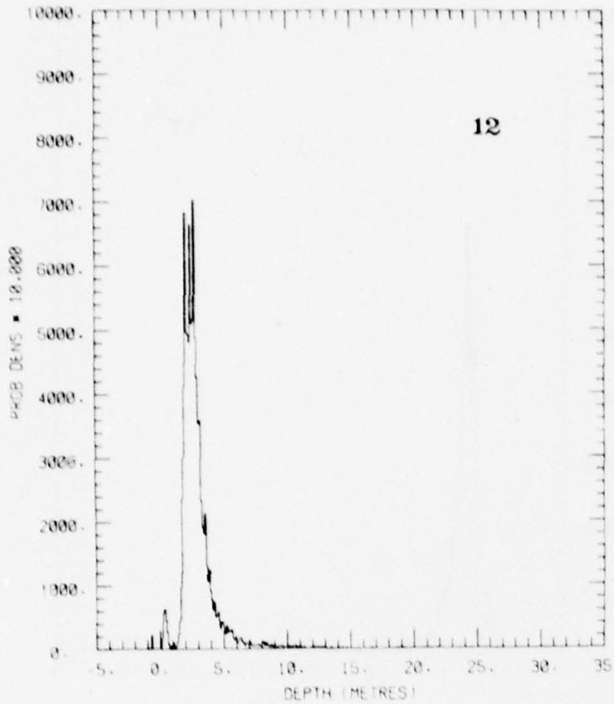
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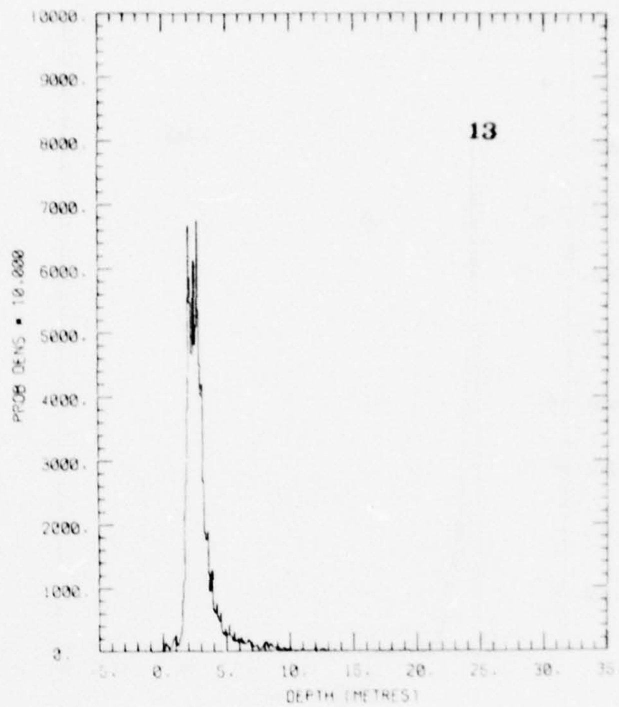
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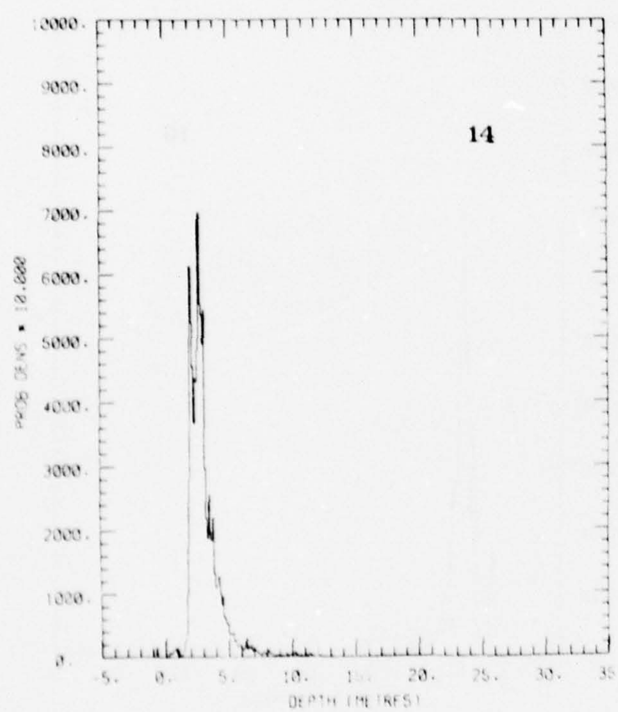
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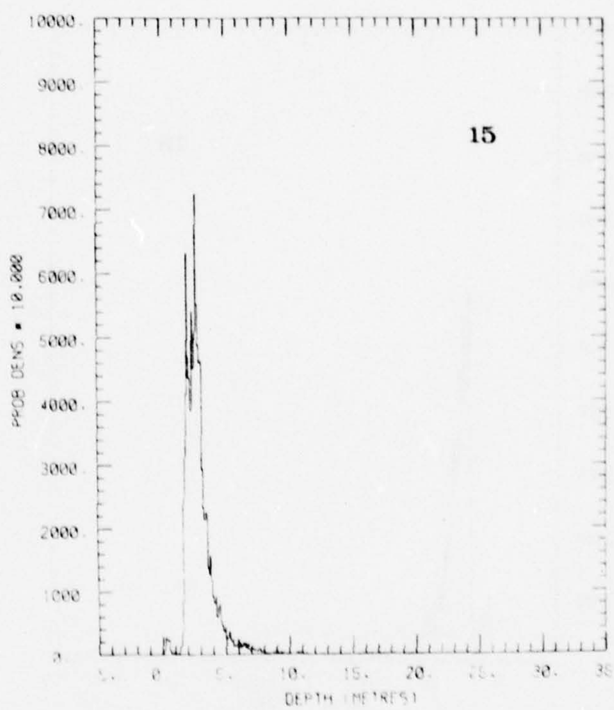
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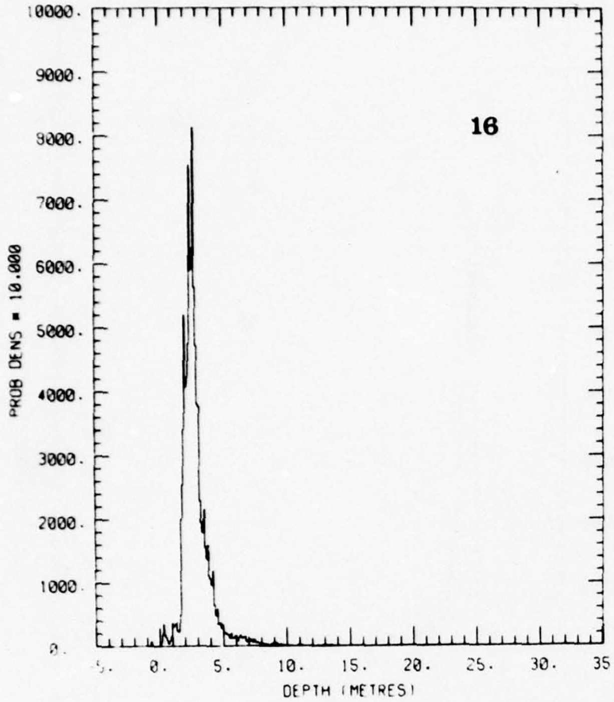
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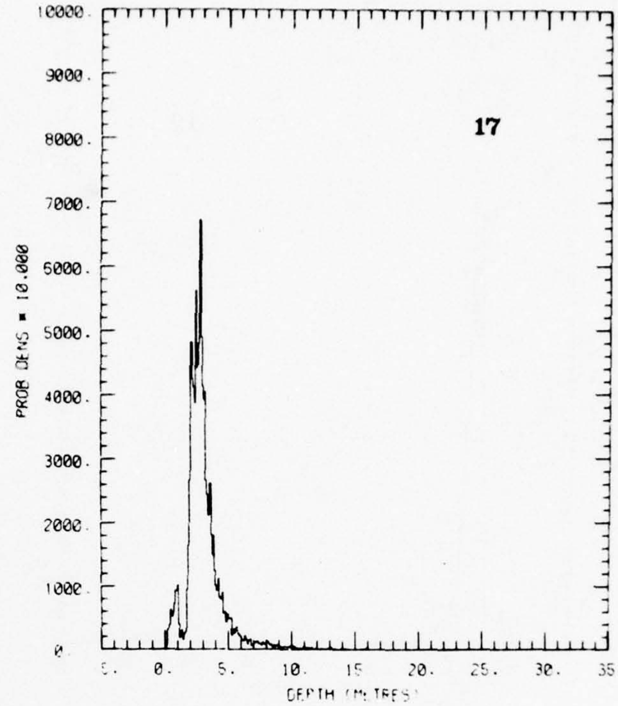
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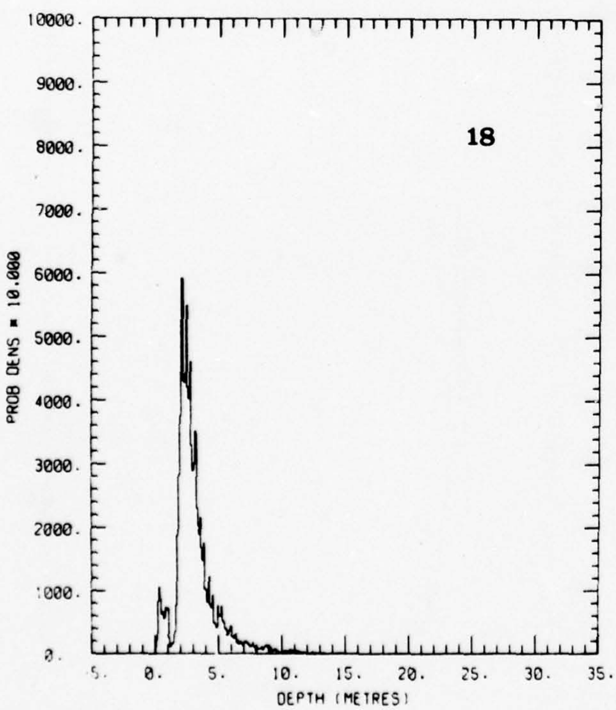
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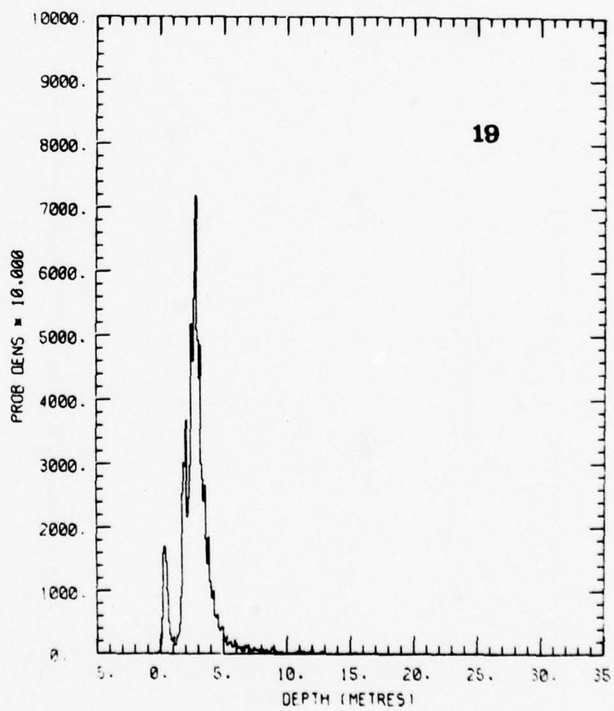
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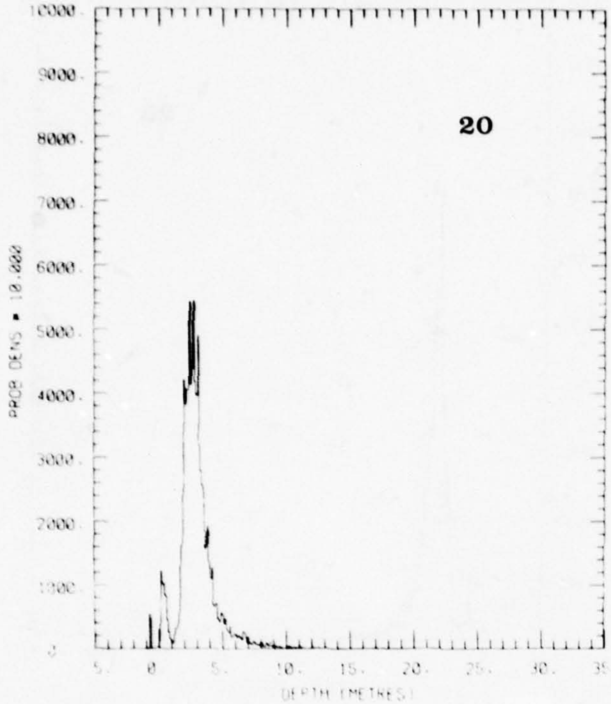
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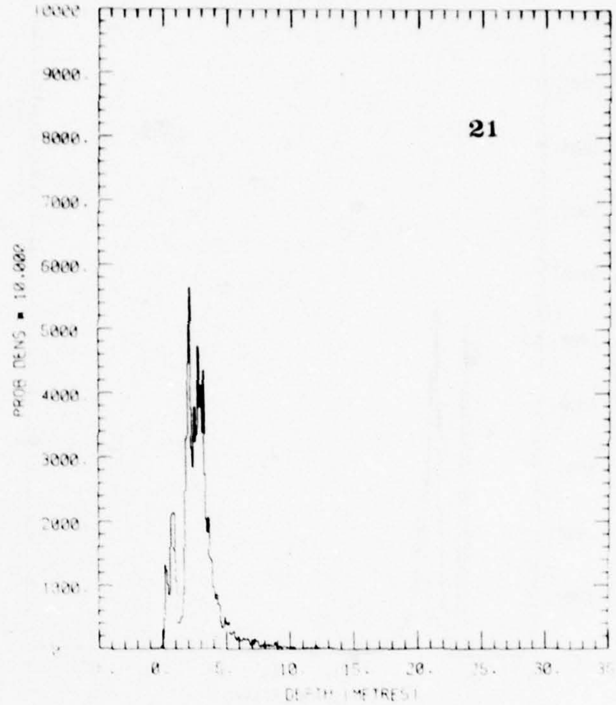
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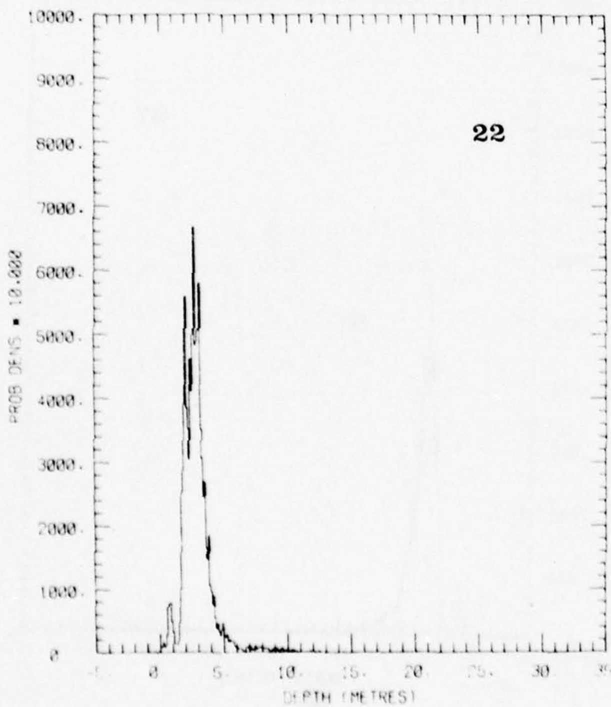
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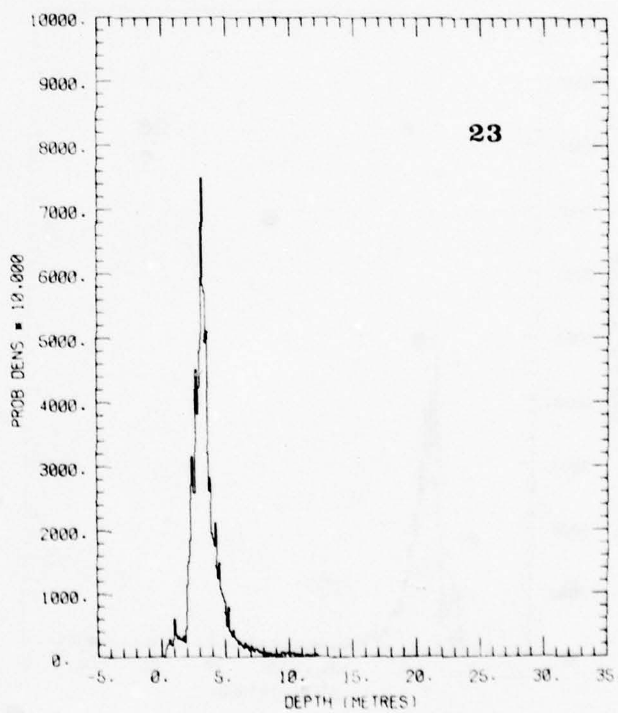
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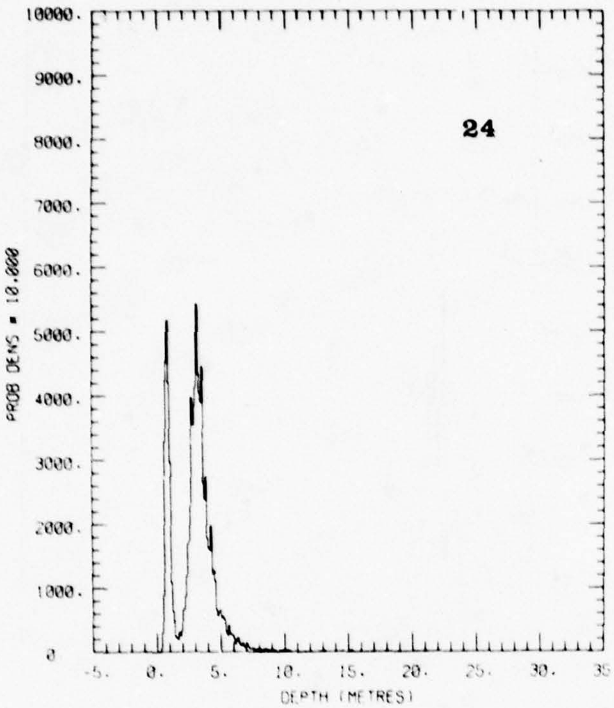
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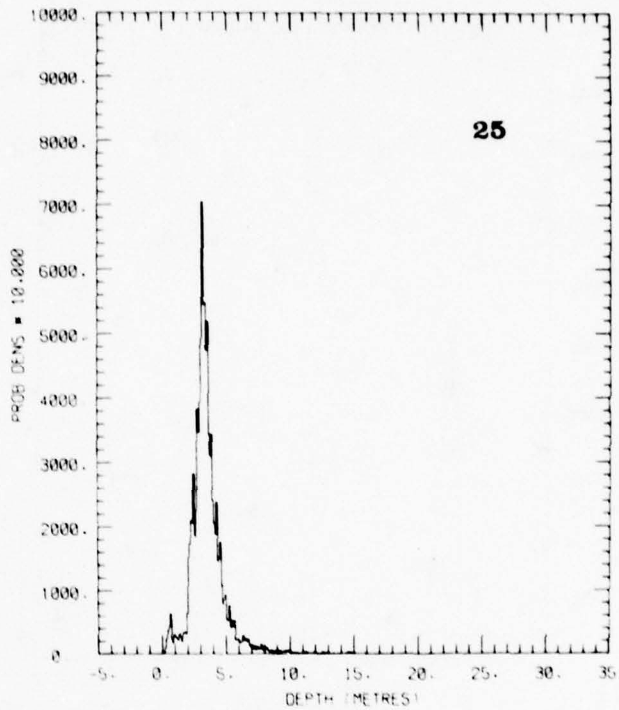
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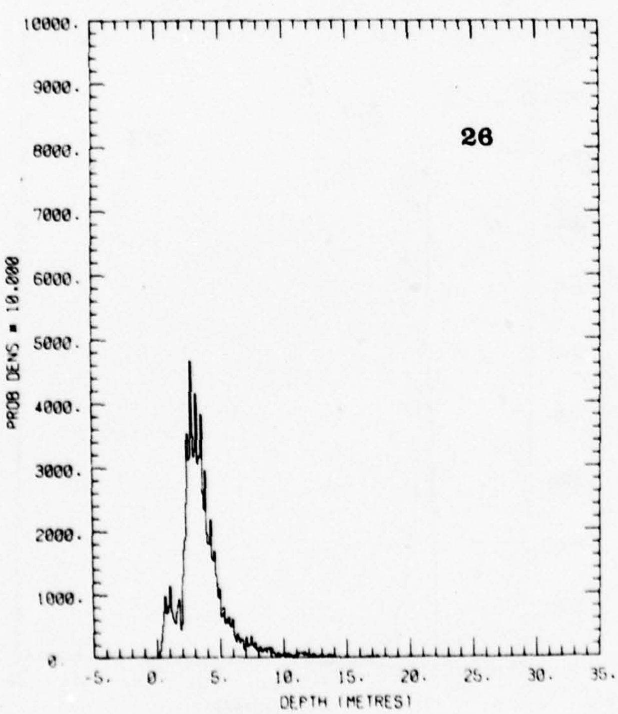
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DEPTH (METRES)

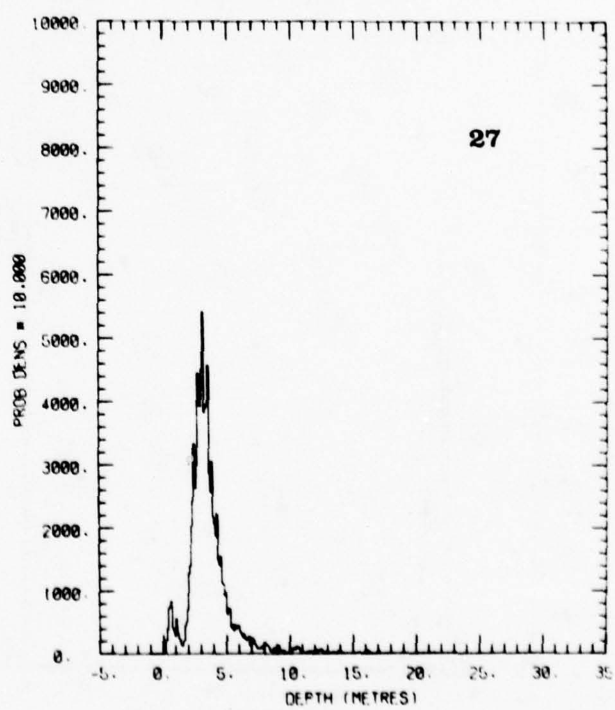

27
PROB DENS * 10.000
DEPTH (METRES)

Table III. Level ice summary

Section	Mean draft	% level ice	Thin ice peak	Level ice drafts with probability density >0.5
1	3.81	35.0	0.6	
2	3.17	43.5		
3	2.96	52.5	1.0	
4	3.54	54.2		3.1
*5	3.15	53.2		2.7 2.4
6	2.78	58.8	0.8	2.7 2.4
7	2.43	59.8	0.8	2.0 2.1 2.2 1.9 2.4 2.3
8	2.81	58.5	1.0	2.4 2.7 2.6 2.5 2.8
9	2.88	52.9		2.0 2.4 2.1 2.2 2.7 1.9 2.3
10	2.82	57.0		2.7 2.4 2.0 2.6 2.5 2.8 2.1
11	2.77	56.9	0.3	2.7 2.0 2.8
12	2.94	54.1		2.7 2.0 2.4 2.6 2.5
13	2.97	55.3		2.7 2.0 2.4 2.1 2.2 2.8 2.6
14	3.12	58.1		2.7 2.0 2.8 3.1 2.9 2.1
15	2.96	59.1		2.7 2.0 2.8 2.4 2.6
16	2.91	59.3		2.7 2.4 2.6 2.5 2.8 2.0
17	3.06	53.3	1.0	2.7 2.4
18	3.09	51.0	0.3	2.0 2.4
19	2.00	58.0	0.3	2.7 2.6 2.4 2.8
20	3.03	55.1	0.2	2.7 2.4
*21	2.73	54.5	0.8	2.0
22	2.93	58.5		2.7 3.1 2.0 3.0
23	3.52	54.4		3.1 3.2 3.3 3.5
24	2.97	61.1	0.8	3.1
25	3.51	52.6		3.1 3.3 3.2 3.5
26	3.61	47.9	1.0	
27	3.68	49.4		3.1
Mean	3.07	54.3		
Mean (sections 3-25)	2.98	56.1±0.7		

* Caribou crossings

The values of mean level ice draft in table III are not very meaningful, because some ice from keel bottoms appears in the histograms of figs. 6 and 7 and biases the mean draft upwards. However, the figure for percentage of level ice in table III is strongly indicative of the real percentage of undeformed ice present, although necessarily an overestimate because of keel bottoms. Again we see a low figure for sections 1 and 2 with their heavy ridging; a fairly steady equilibrium value for sections 3 to 25 (which passes a run test for homogeneity); and a final decrease of level ice occurrence in sections 26 and 27. The thin ice peak, as shown in fig. 7 and table III, is concentrated either around 0.2 to 0.4 m or around 0.8 to 1.1 m. Fig. 7 shows 0.8-0.9 m as the most probable category, and the distribution of probabilities in the depth range 0-1.5 m in fig. 7 is a measure of the relative frequencies of polynyas of varying ages.

5. DISTRIBUTION OF KEEL SPACINGS

5.1 Independent keels

The extent of an independent keel is defined using the criterion that the troughs on either side of the keel crest (point of maximum draft) must descend at least half way towards the local level ice surface, in this case defined arbitrarily as a draft of 2.5 m. This is analogous to the Rayleigh criterion for resolving spectral lines in optics and is identical to that used by Williams et al (1975), Wadhams (1976, 1977b) and Weeks et al (1977) for the analysis of submarine and aircraft profiles (for airborne laser profiles the identification of the "local level ice surface" is much easier). It differs from the criterion of Hibler et al (1974), where the troughs must descend a fixed distance (61 cm for surface ridges) from the peak; Hibler (1975) has discussed the effect of this difference in definition on the resulting distribution.

5.2 Theory of spacings

Hibler et al (1972) showed that if ridges occur at random along a track the distribution of spacings between ridges is given by

$$P_r(x)\ dx = \mu \exp(-\mu x)\ dx \qquad (2)$$

where μ is the mean number of ridges per unit length of track and $P_r(x)\,dx$ is the probability that a given spacing lies between x and (x + dx) in length. Mock et al (1972) tested this relationship for surface ridges using aerial photographs, and found good agreement except for an excess of ridges at small spacings. On a purely random theory, however, we expect a deficit of ridges or keels at small spacings, on account of the so-called "ridge shadowing" effect (Wadhams, 1977b). This occurs because keels have a finite slope angle so that their crests cannot lie closer than a certain minimum distance x_{crit}. Within this distance the shallower ridge is not detected and the ridge-picking criterion selects only the deeper ridge. Figure 8 illustrates this effect for two keels of relief h, h' (h'>h) relative to the level ice bottom, each ridge being of triangular cross-section with slope α. Under these circumstances

$$x_{crit} = h' \cot\alpha \tag{3}$$

A theoretical treatment of the modification of (2) by (3) is complex, but an approximate solution is given in a paper awaiting publication (Lowry and Wadhams, unpubl.). The net effect is that close spacings and shallow ridges tend to be lost preferentially from the distributions of spacing and draft.

5.3 Results

The complete numerical results for keel spacings are given in Appendix C. The results are presented for each of the 50 km sections, with a spacing increment of 20 m. Distributions are presented for all keels deeper than 5 m and for all keels deeper than 9 m. This is because Wadhams (1977b) found that a number of deep floe bottoms appeared in the draft range 5 - 9 m and that the theoretical keel draft distribution function was valid only beyond 9 m. It was felt, therefore, that by taking 9 m as a cutoff a more valid distribution of spacings of "real" keels could be obtained. The results from Appendix C were added together to yield an overall keel spacing distribution for the whole profile; this is shown in figure 9.

Both distributions (> 5 m and > 9 m) show general agreement with (2), with the expected deficit at small spacings. This deficit shows itself only in the spacing range 0 - 40 m, as opposed to the results of Wadhams (1977b, fig. 4), where the deficit extends its influence to 120 m. This is probably because of the transducer beamwidth in the SOVEREIGN profile, which makes ridges seem broader and less steep than they really are (Wadhams, 1977c). At large spacings fig. 9 shows a positive deviation from (2), which must be due to an additional effect upsetting the purely random distribution. I suggest that this effect is simply the presence of leads and polynyas, which interpose occasional smooth stretches of ice into the otherwise random icefield and thus generate an anomalous number of large keel spacings.

It should be noted that the lines of best fit to the rectilinear parts of fig. 9 do not have a gradient of $-\mu$. For the 5 m cutoff the

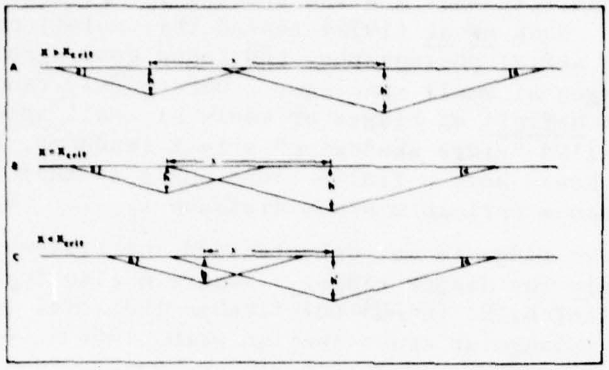


FIGURE 8. Illustration of the keel shadowing effect for two keels of separation x and relief h,h' relative to local level ice draft.

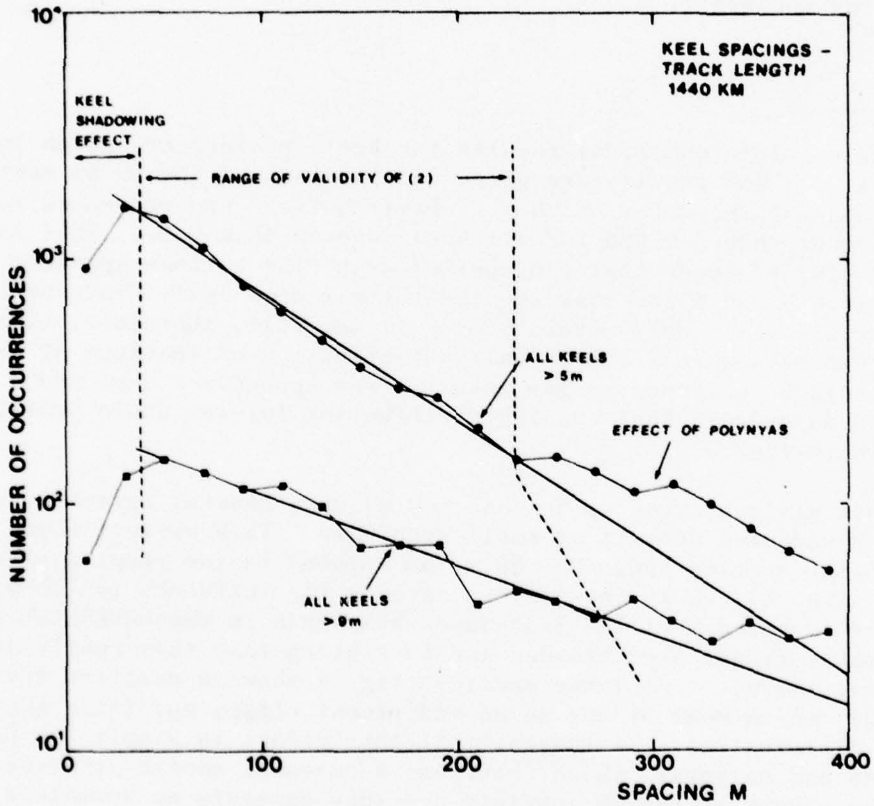


FIGURE 9. Distribution of keel spacings over whole submarine track. Bin size 20 m. Results are plotted for keels deeper than 5 m and 9 m, and a straight line is fitted to the central portion of each curve.

gradient is 5.2 (= 7.3) while for 9 m it is 2.9 (μ= 1.7). One expects a gradient of magnitude greater than μ because keel shadowing implies that the original "population" has been reduced before entering the statistics. This is so for 9 m but not for 5 m, again implying that 9 m is a better cutoff to use so as to obtain a population of "pure" keels.

Appendix C also presents a tabulation of spacings classified according to the depth of the deeper keel of the pair which defines the spacing, i.e. a tabulation of x against (h' + 2.5) in figure 8. This is an attempt to find the best α for use in (3). In Appendix C the tabulation is given for all 27 sections and for 5 m and 9 m cutoff. We have taken the results for 9 m cutoff and plotted them in bins of 2 m depth increment and 20 m spacing increment. The curves (figure 10) show a peak at a spacing which progressively increases with depth. In fig. 10 the position of the highest peak in each curve has been plotted against the relevant depth (the heavy black dots), and it can be seen that the increase with depth is roughly linear. A line of best fit has been drawn through these points which, when applied to (3), gives a value of 13.3^o for α. Of course, these curves show that there is no one value of α, otherwise there would be a sharp spacing cutoff within which no keel pairs are to be found. Instead, there is a range of α, a range which is spread out still further by the fact that the keels are not being profiled orthogonally but at various angles of encounter. Wadhams (1977c) dealt with this statistical averaging problem. Our value of 13.3^o is, in any case, an underestimate because it refers to the peak of each spacing distribution rather than to the spacing at which keel pairs begin to be found. However it is indicative of the validity of the "keel shadowing" concept. Again, Wadhams (1977c) found by actually measuring the slopes of keels on sonar profiles that the slope angle distribution had a peak in the range $16\text{-}20^o$ but, when adjusted to take account of angle of encounter, the mean value of α came to 32^o. Thus our 13.3^o figure does not appear unreasonably low.

6. DISTRIBUTION OF KEEL DRAFTS

6.1 Theory

The theory of keel drafts which has been most extensively tested against observation is that of Hibler <u>et al</u> (1972). They used a variational calculation which gives the most likely distribution of geometrically congruent ridges that will yield a given volume of deformed ice. The result is

$$P_r(h)\ dh = 2\lambda\bar{h}\ \exp\ (\lambda h_o^2)\ \exp(-\lambda h^2)\ dh \qquad (4)$$

where $P_r(h)$ dh is the probability that the draft lies between

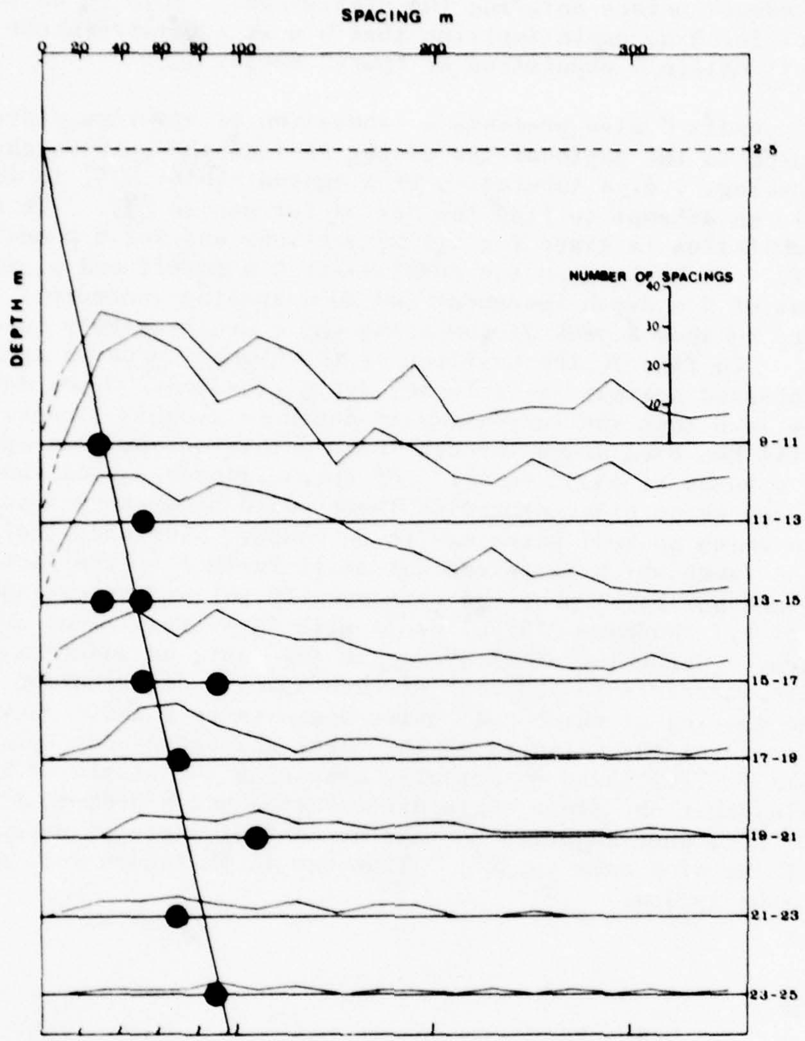


FIGURE 10. Spacing distributions for keels of draft greater than 9 m, in 2 m draft increments. The spacing corresponding to the maximum of each distribution has been plotted as a heavy black dot. This spacing increases linearly with keel relief.

h and (h + dh);

$\bar{h}$ is the mean draft;

h_o is a low value cutoff below which keels are not included in the statistics;

λ is a parameter which must be derived by iteration from

$$\exp(-\lambda h_o^2) = \bar{h}\ (\lambda\pi)^{\frac{1}{2}}\ \mathrm{erfc}(\lambda^{\frac{1}{2}}\ h_o) \tag{5}$$

This has been shown to give a better fit to submarine sonar observations than an alternative distribution proposed on empirical grounds by Diachok (1975):

$$P_r(h)\ dh = \frac{2h}{a^2}\ \exp\ (-h^2/a^2)\ dh \tag{6}$$

with $a = 2\bar{h}/\pi^{\frac{1}{2}}$.

Again, (4) is modified by the ridge shadowing effect (Lowry and Wadhams, unpubl.), but the modification has only a small effect at the low-draft end of the distribution where it causes a slight deficit of keels; this is a much less drastic modification than that applied to the spacing distribution.

Recently it has been found that surface ridge sails, to which this theory was also thought to apply, actually obey a simpler negative exponential distribution of form

$$P_r(h)\ dh = B\ \exp\ (-bh)\ dh \tag{7}$$

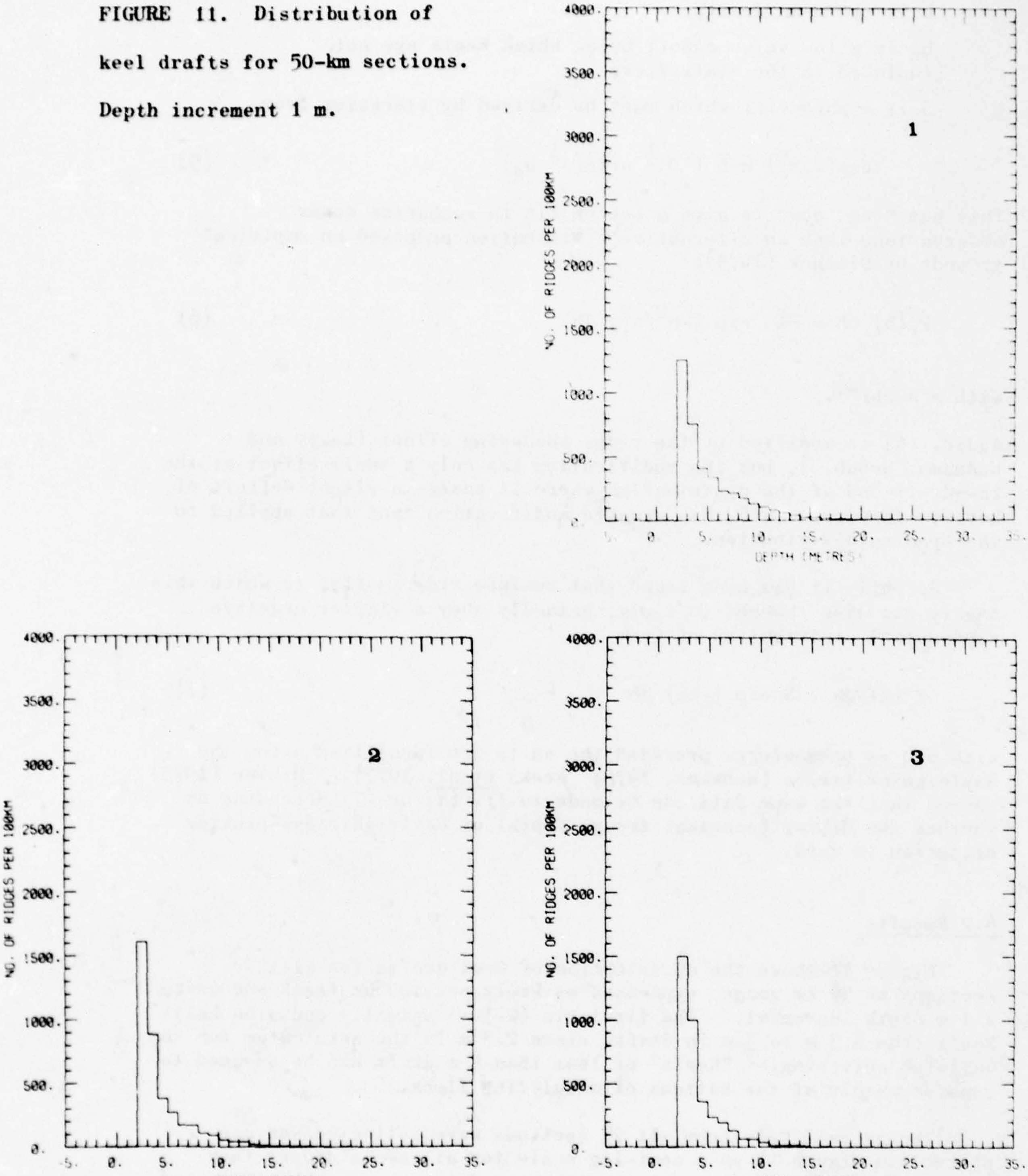
with B,b as parameters, provided the sails are identified using the Rayleigh criterion (Wadhams, 1976; Weeks et al, 1977). Hibler (1975) showed that the same data can be made to fit (4) or (7) depending on whether the Hibler (constant trough depth) or Rayleigh ridge-picking criterion is used.

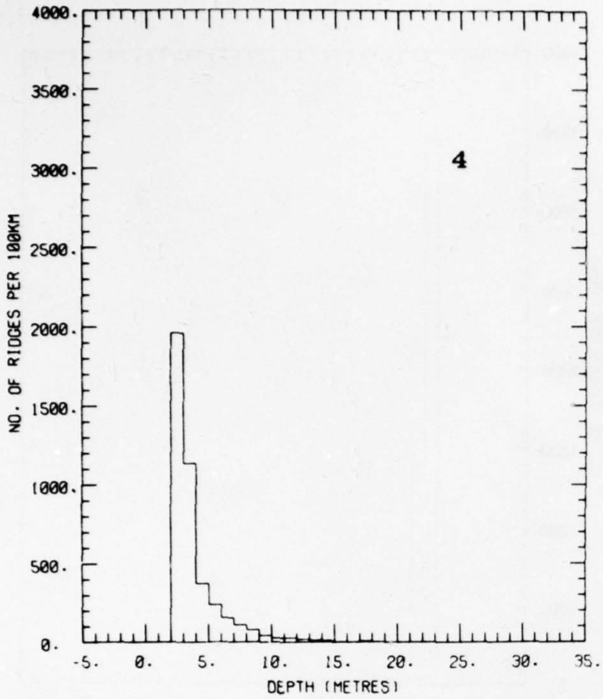
6.2 Results

Figure 11 shows the distribution of keel drafts for all 27 sections at 50 km gauge, expressed as keels per 100 km track and using a 1 m depth increment. The first bin (2-3 m) actually contains only keels from 2.5 m to 3 m in draft, since 2.5 m is the zero datum for the Rayleigh criterion. "Keels" of less than 5 m draft can be assumed to consist mainly of the bottoms of undulating floes.

The overall data from all 27 sections were collected and are plotted in figure 12 on a semi-log scale for all keels deeper than 5 m. The result is an exceptionally good fit to a straight line, not

FIGURE 11. Distribution of keel drafts for 50-km sections. Depth increment 1 m.

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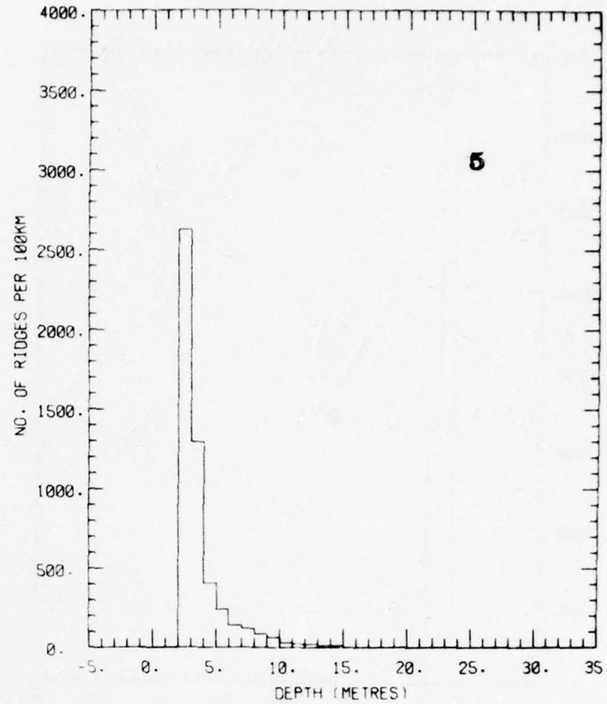
-5. 0. 5. 10. 15. 20. 25. 30. 35.
DEPTH (METRES)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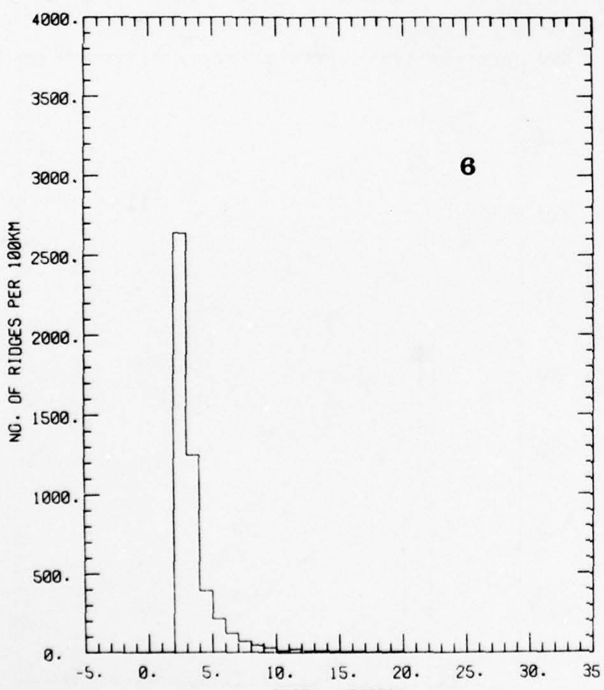
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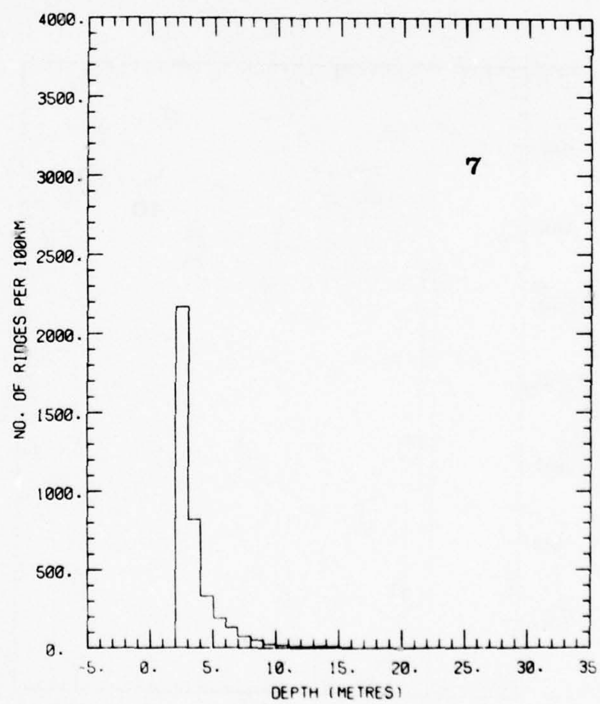
-5. 0. 5. 10. 15. 20. 25. 30. 35.
DEPTH (METRES)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NO. OF RIDGES PER 100KM
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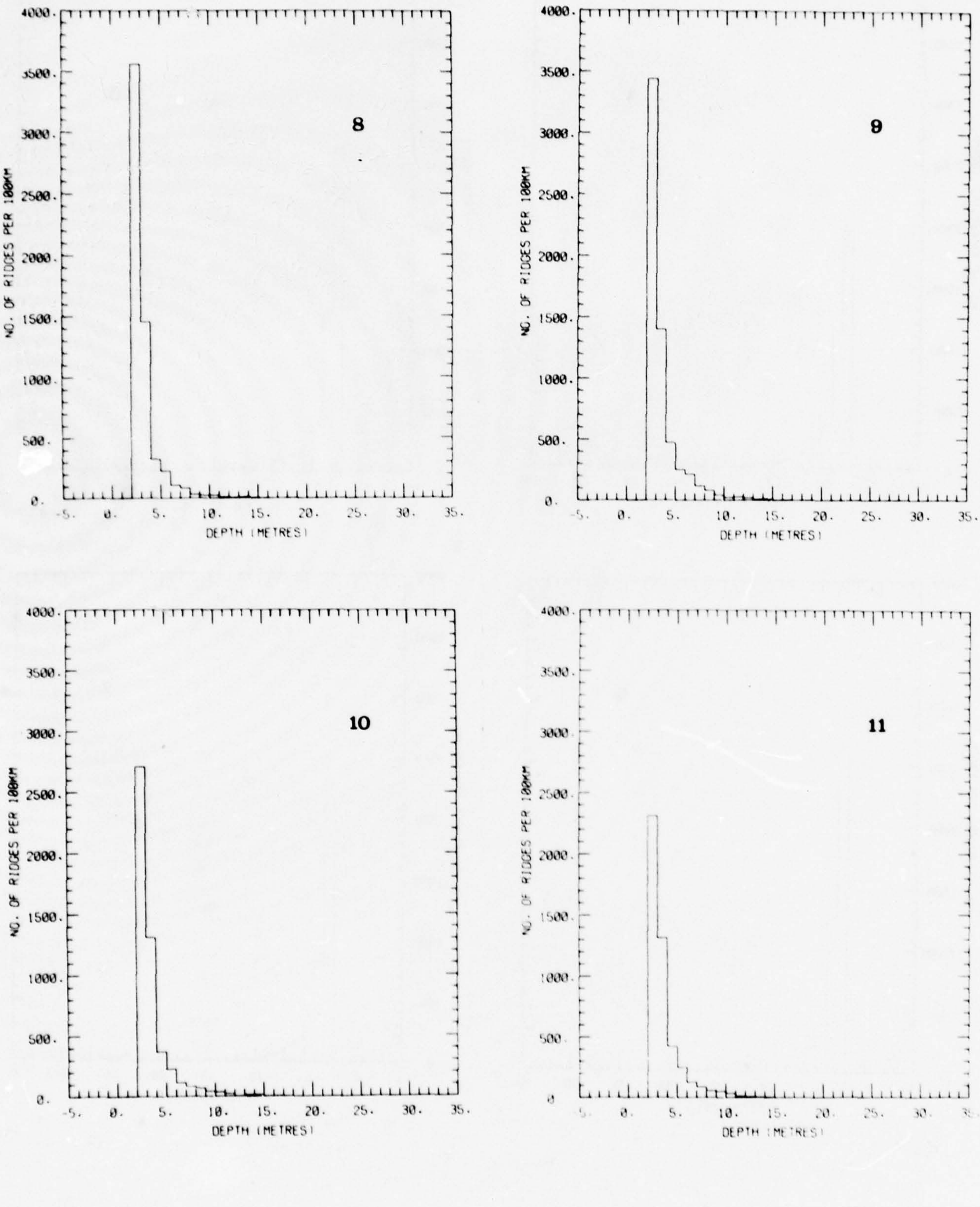
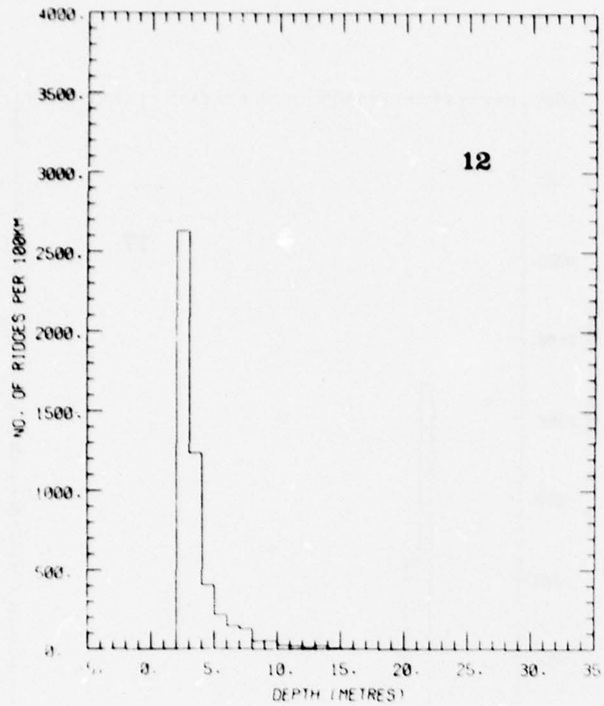
DEPTH (METRES)


8
9
10
11
NO. OF RIDGES PER 100KM
DEPTH (METRES)
0.
500.
1000.
1500.
2000.
2500.
3000.
3500.
4000.
-5.
5.
10.
15.
20.
25.
30.
35.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DEPTH (METRES)
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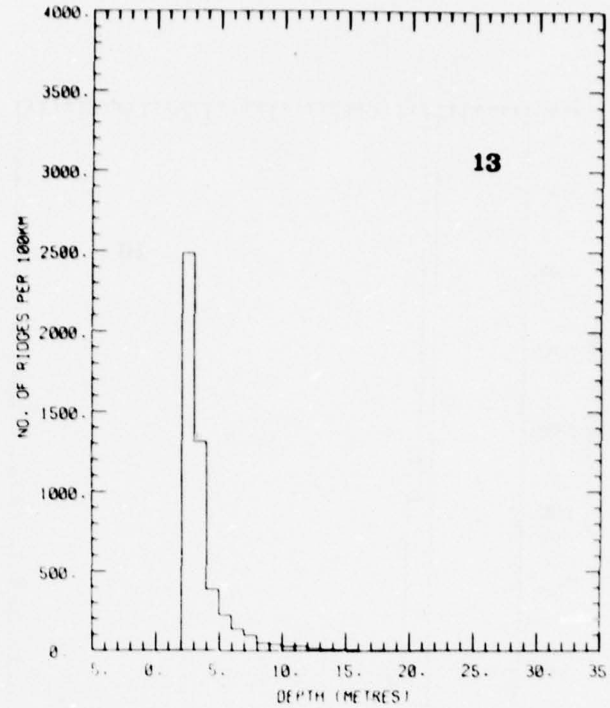
-5. 0. 5. 10. 15. 20. 25. 30. 35.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DEPTH (METRES)
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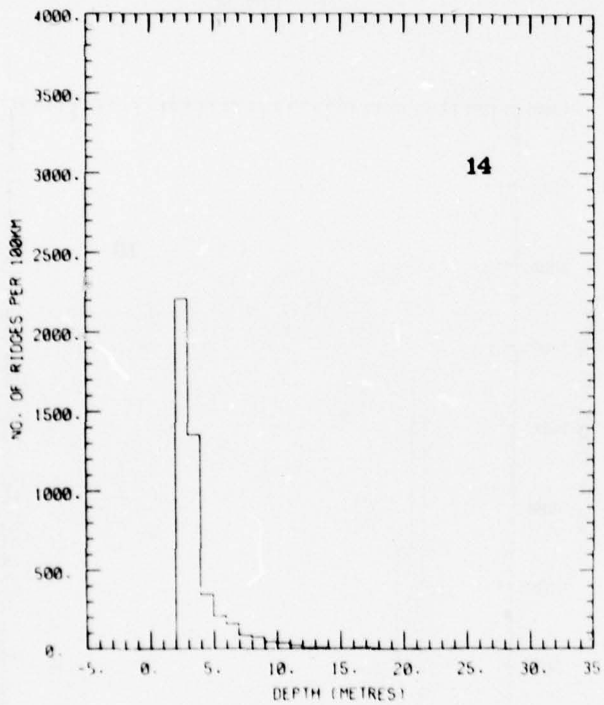
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NO. OF RIDGES PER 100KM
DEPTH (METRES)
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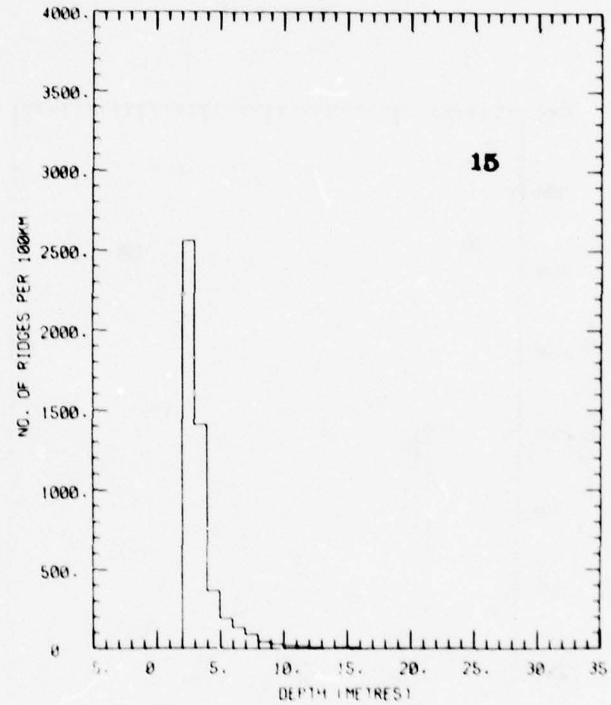
-5. 0. 5. 10. 15. 20. 25. 30. 35.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DEPTH (METRES)
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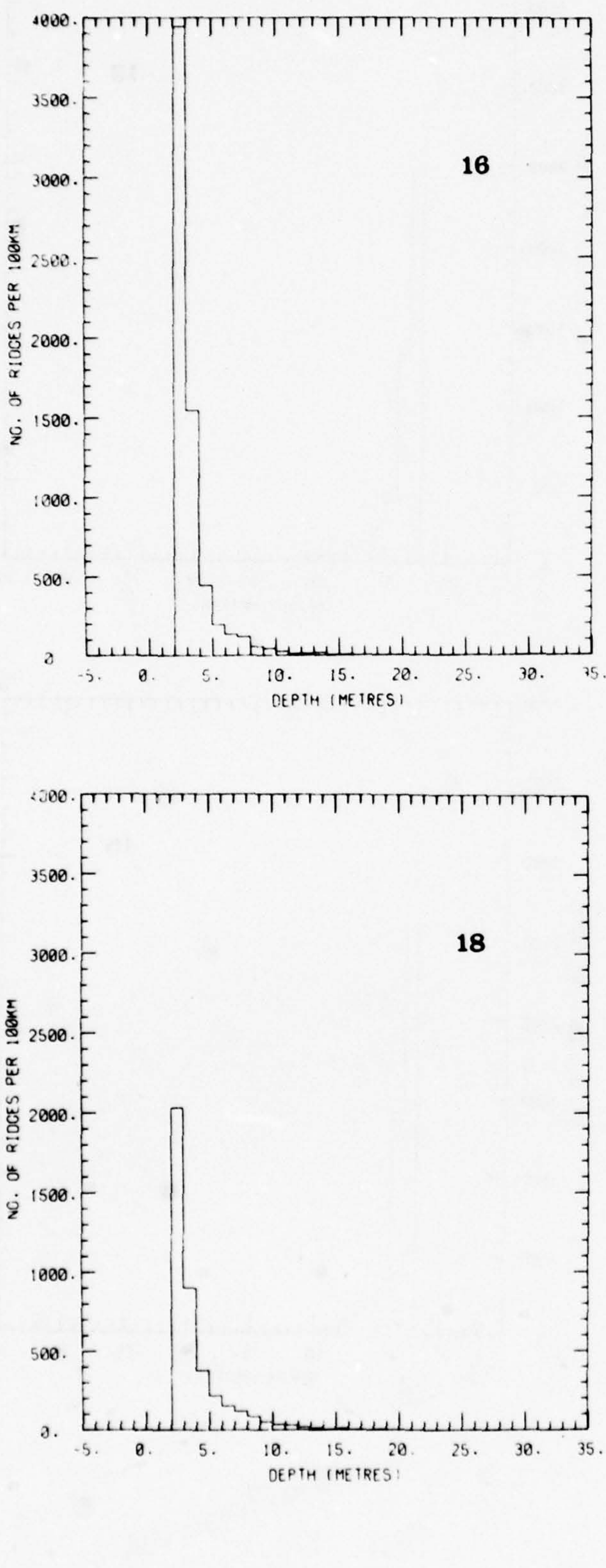
5. 0 5. 10. 15. 20. 25. 30. 35.


16
17
18
19
NO. OF RIDGES PER 100KM
DEPTH (METRES)
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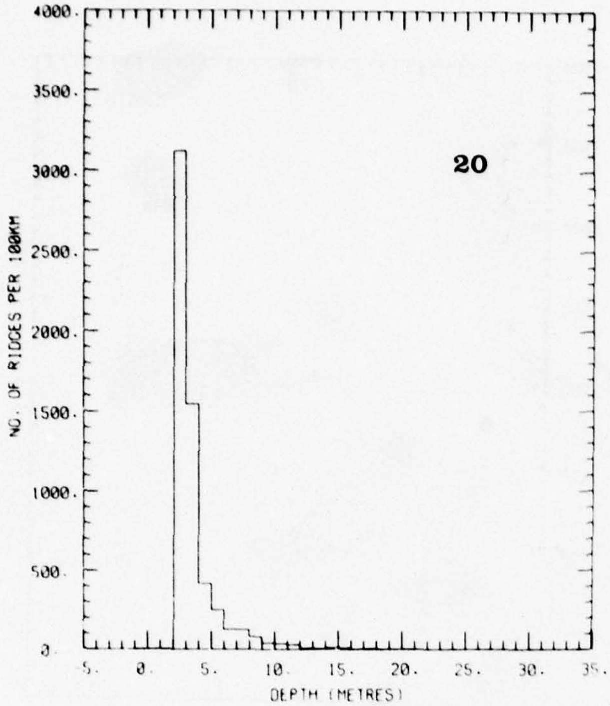
500.
1000.
1500.
2000.
2500.
3000.
3500.
4000.
-5.
0.
5.
10.
15.
20.
25.
30.
35.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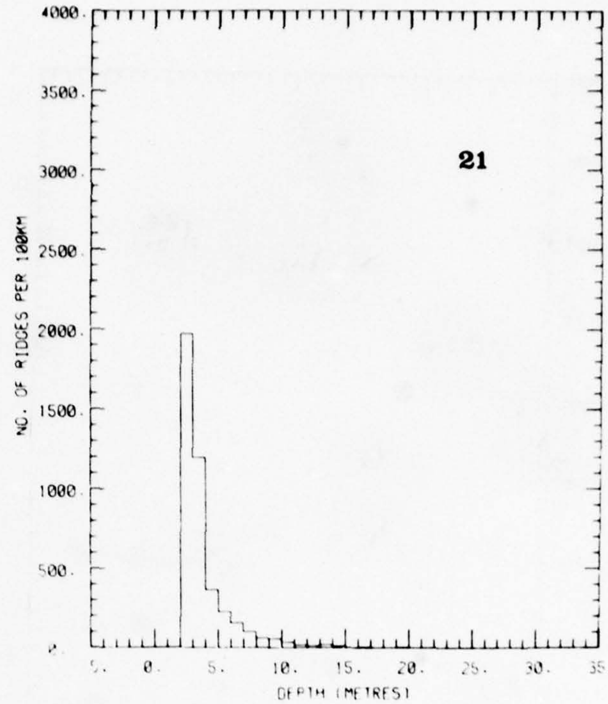
DEPTH (METRES)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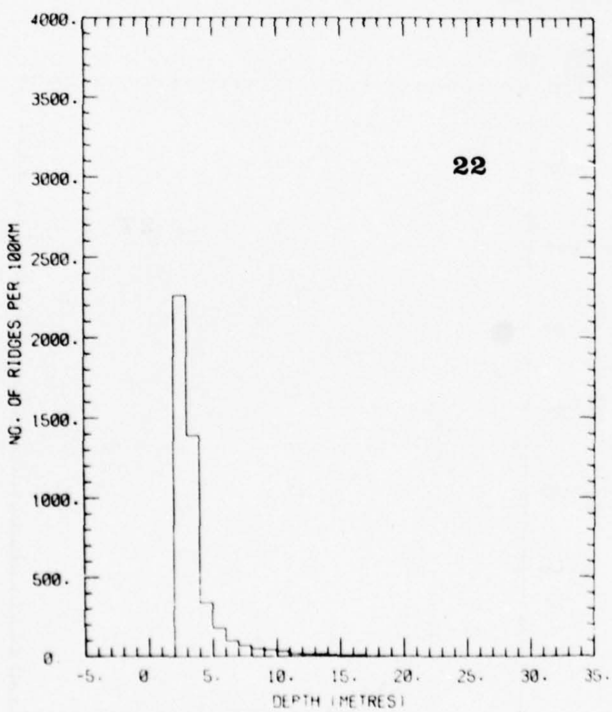
DEPTH (METRES)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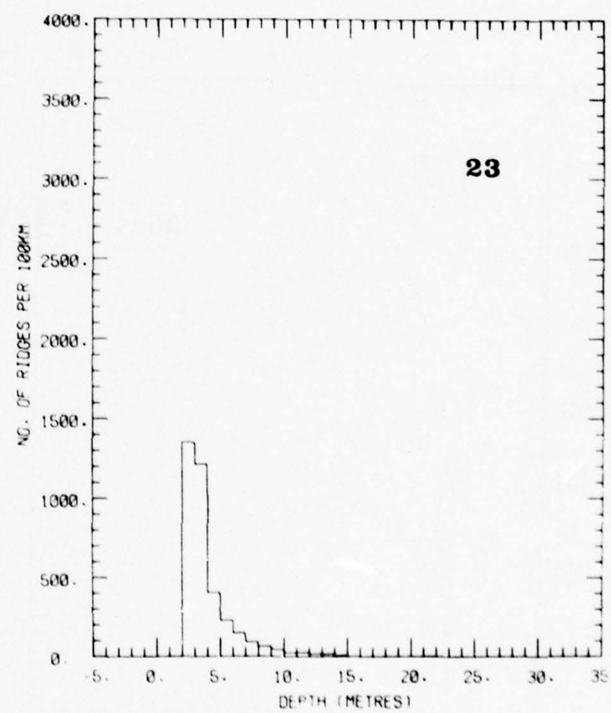
DEPTH (METRES)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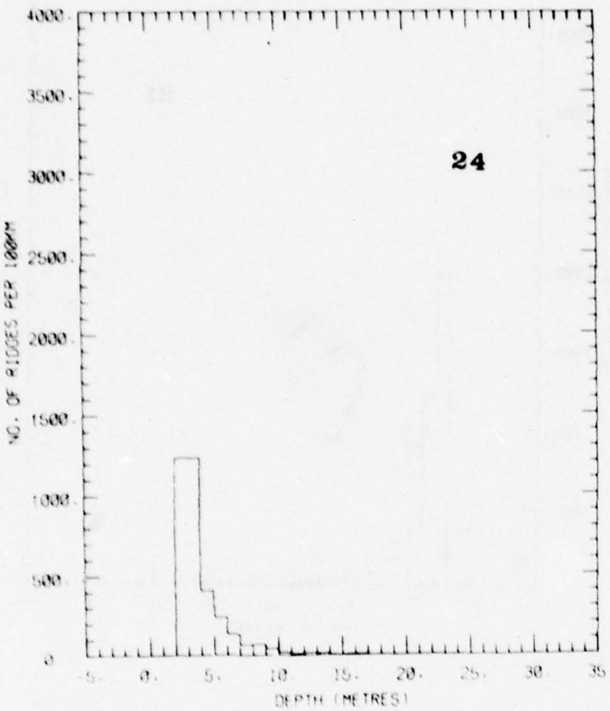
NO. OF RIDGES PER 100KM
DEPTH (METRES)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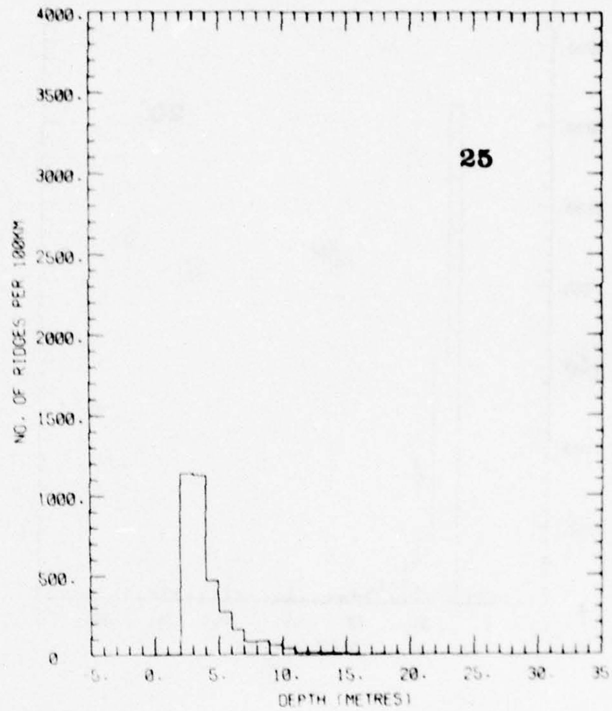
DEPTH (METRES)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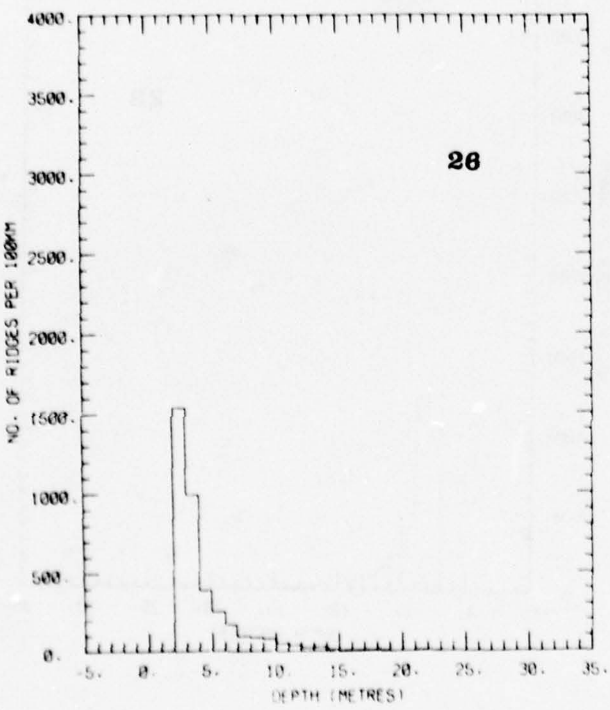
NO. OF RIDGES PER 100KM
DEPTH (METRES)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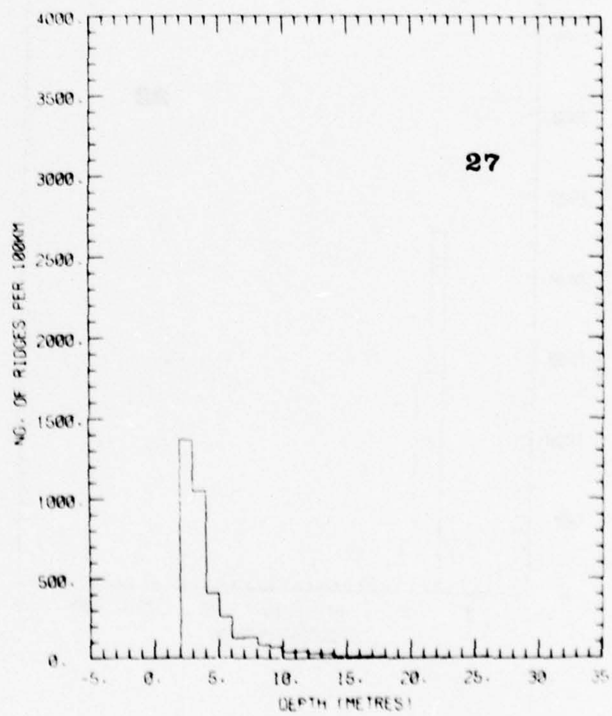
DEPTH (METRES)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DEPTH (METRES)

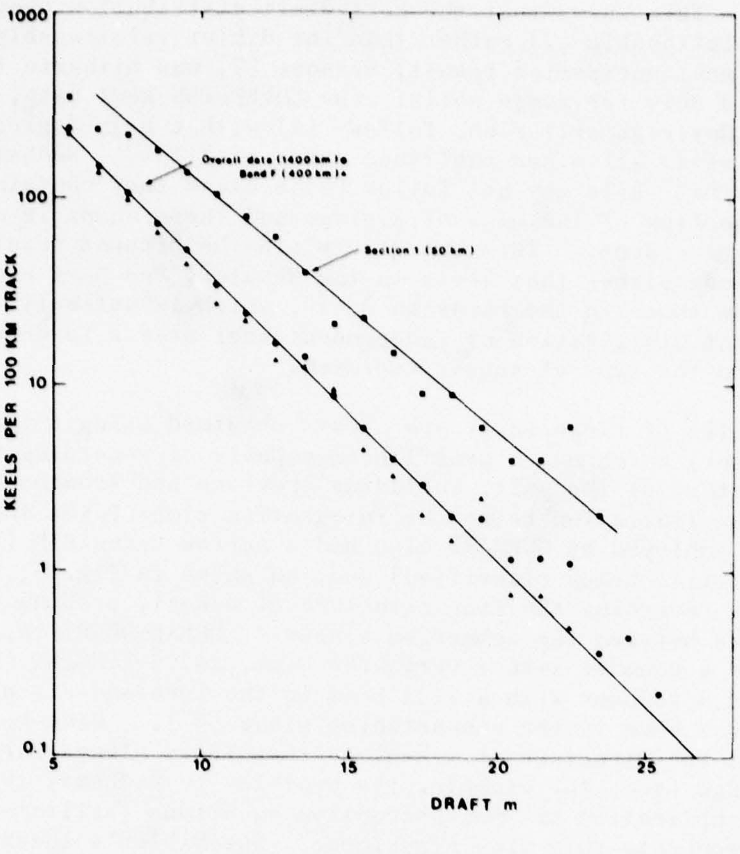


FIGURE 12. Distribution of keel drafts plotted on a semi-log scale for data at 1400 km, 400 km and 50 km length gauges.

only for the overall data but also for data at 200 km gauge (e.g. section F, also shown in fig. 12) and even 50 km gauge (section 1, fig. 12). This shows that the keel draft distribution obeys the simpler relationship (7) rather than the Hibler relationship (4). This is a most unexpected result, because (7) was hitherto thought to be valid only for ridge sails; the SOVEREIGN keel data, analysed using the Rayleigh criterion, follow (4) with a high degree of exactness as do all other published sonar profiles. Wadhams (1977b) suggested that sails may not follow (4) because they contain only a small proportion of the mass of a ridge and their shape is determined largely by accident. This cannot explain the present result. We must conclude either that keels in the Beaufort Sea have a different nature from those in the Eurasian Basin, which is unlikely, or that the apparent distribution of independent keel drafts is dependent in some way on the type of sensor employed.

Profiles of ridge sails are always obtained using a laser profilometer, which has a pencil beam capable of recording much of the fine structure of the sail, including crevices and troughs between the blocks (the limitation being the integration time of the laser electronics). The sonar employed by GURNARD also had a narrow beamwidth (its precise characteristics being classified) and, as shown in fig. 3, it also appears capable of recording the fine structure of a keel, probing into clefts and hollows between the submerged blocks. DREADNOUGHT (Williams et al, 1975) used a sounder with a very wide beam, and SOVEREIGN (Wadhams, 1977b) had a sounder with a wide beam in the fore-and-aft plane (17^o) and a narrow beam in the athwartships plane (5^o). Wide-beam sounders smooth out the structure of a keel so that it is always perceived as a single wedge (see, for example, the profiles in Wadhams, 1977c), and even the application of reconstruction equations (Williams et al, 1975) cannot regenerate this fine structure. Now Hibler's theory depends on the concept of geometrically congruent ridges, each an entity of the same shape possessing mass and potential energy which depend only on its depth. A wide-beam sounder forces keels to approximate to this concept by smoothing out any incidental structure that they may possess and leaving them as discrete entities. Thus narrow-beam echo sounders and laser profilometers produce one type of ice profile with ridge height characteristics obeying (7), while wide-beam sounders produce another type, obeying (4). A narrow-beam sounder, by splitting many ridges into multiple "ridges", sees a greater ridge frequency than a wide-beam sounder (e.g. in Wadhams, 1977b, the sail frequency is a multiple of the keel frequency for the same ice cover).

This hypothesis now covers all results except laser profilometer data reported by Hibler et al (1974), which still obeyed (4) but which were not analysed on the Rayleign criterion. A crucial test of the hypothesis would be to smooth the GURNARD profile artificially by convolving it with the beam pattern of a wide-beam echo sounder, and to observe the effect on the resulting statistics. We hope to report on this computer simulation in a later publication.

It was shown in Wadhams (1977b) that the parameters B,b in (7) can be expressed as simple functions of $\bar{h}$ and μ. If (7) is rewritten in the form

$$n(h)\ dh = B \exp(-b\ h)\ dh \qquad (8)$$

where n(h) is the number of keels per km track per metre draft increment, then

$$\mu = \int_{h_o}^{\infty} n(h)\ dh \qquad (9)$$

and

$$\bar{h} = \int_{h_o}^{\infty} h\ n(h)\ dh \qquad (10)$$

so that

$$b = (\bar{h} - h_o)^{-1} \qquad (11)$$

and

$$B = \mu b \exp(b\ h_o) \qquad (12)$$

Thus μ and $\bar{h}$ are the two parameters of the keel draft distribution from which the whole shape of the distribution can be deduced using (8) to (12). Table IV shows these parameters tabulated for all the 50 km sections, using h_o = 5 m and 9 m so as to be consistent with the statistics of Wadhams (1977b). The results are also plotted in figure 13. The following conclusions can be drawn:-

(a) The major part of the profile (sections 3-25) has a ridging distribution which is very homogeneous and which falls within narrow limits of variation. These limits are extremely narrow for h_o = 5 m (5.4-8.2 for μ; 7.2-7.8 m for $\bar{h}$) and somewhat wider for h_o = 9 m, probably because of the smaller number of keels involved. Mean and standard deviation for these four parameters are given; run tests show that we can accept the hypothesis of a homogeneous ice cover with respect to ridging intensity.

(b) Sections 1-2 and 26-27 fall clearly outside the range of variation of the other sections (fig. 13), indicating much heavier ridging (greater μ) in these parts of the track. This result agrees with what we have found from the probability density functions; at these two extremities of the track there are more pressure ridges per unit length, a greater mean keel draft, a greater mean ice draft and a greater proportion of deformed ice.

(c) There is no clear positive correlation between μ and $\bar{h}$, although the four heavily ridged sections have both a high μ and a high $\bar{h}$.

Table IV. Pressure ridge frequencies and mean drafts for 50 km sections

Section	Draft > 5 m No. per km	Draft > 5 m Mean draft	Draft > 9 m No. per km	Draft > 9 m Mean draft	Maximum draft m
1	14.77	8.57	5.11	12.15	23.13
2	10.28	8.30	3.07	12.20	28.83
3	8.11	7.69	1.81	11.79	21.64
4	7.41	7.62	1.55	11.75	19.60
5	7.92	7.78	1.90	11.61	22.65
6	5.68	7.57	1.10	12.77	22.65
7	5.49	7.43	0.99	12.06	20.54
8	5.47	7.20	0.91	11.84	20.73
9	8.17	7.39	1.43	11.51	20.09
10	6.40	7.51	1.40	11.34	16.86
11	6.59	7.26	1.24	11.25	22.59
12	7.08	7.65	1.58	11.75	23.65
(Short	6.17	7.78	1.06	13.10	31.12)
13	6.60	7.65	1.49	11.89	20.97
14	6.69	7.68	1.38	12.13	24.93
15	5.46	7.32	0.89	12.11	23.38
16	6.24	7.39	1.13	11.23	18.96
17	7.15	7.79	1.65	12.31	24.84
18	7.46	7.64	1.64	11.40	18.35
19	5.50	7.53	1.11	11.93	26.73
20	7.25	7.50	1.38	11.71	20.36
21	6.98	7.52	1.53	11.38	19.99
22	5.43	7.71	1.37	11.73	19.66
23	7.01	7.65	1.56	12.04	24.48
24	6.36	7.49	1.22	12.01	22.01
25	7.82	7.66	1.78	11.72	22.07
26	8.82	8.25	2.65	12.32	29.23
27	8.72	8.34	2.60	12.69	29.14
Mean of main group (3-25)	6.71 ±0.20	7.55 ±0.03	1.39 ±0.06	11.79 ±0.07	
Mean of Caribou crossings (4-7, 20-23)	6.65	7.60	1.42	11.88	
SOVEREIGN	5.68	11.74	3.74	14.19	43±1
DREADNOUGHT	4.20	9.57	2.00	12.57	30±1

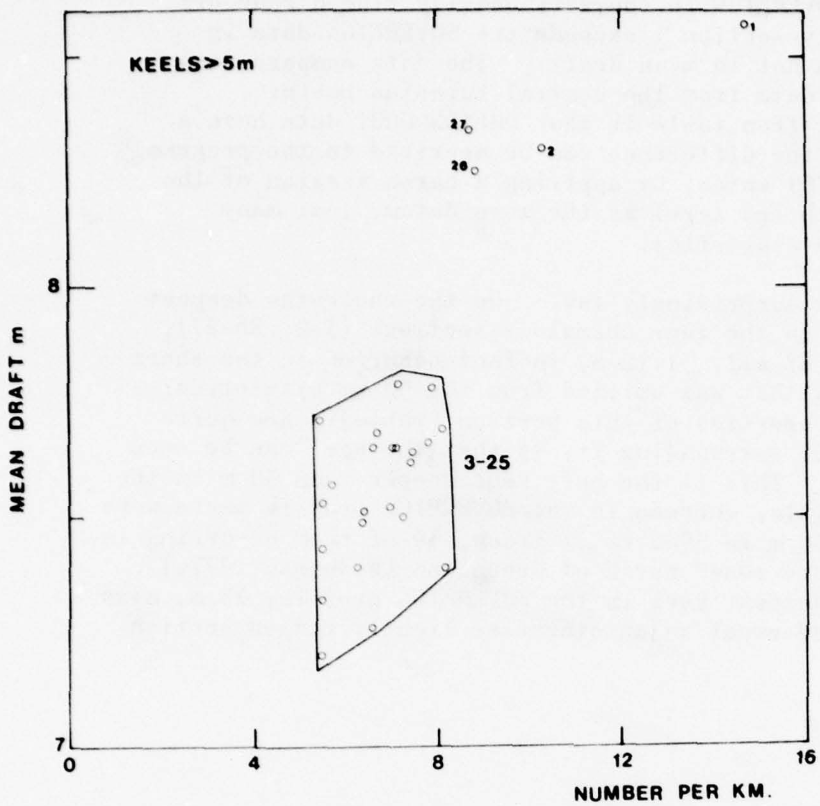
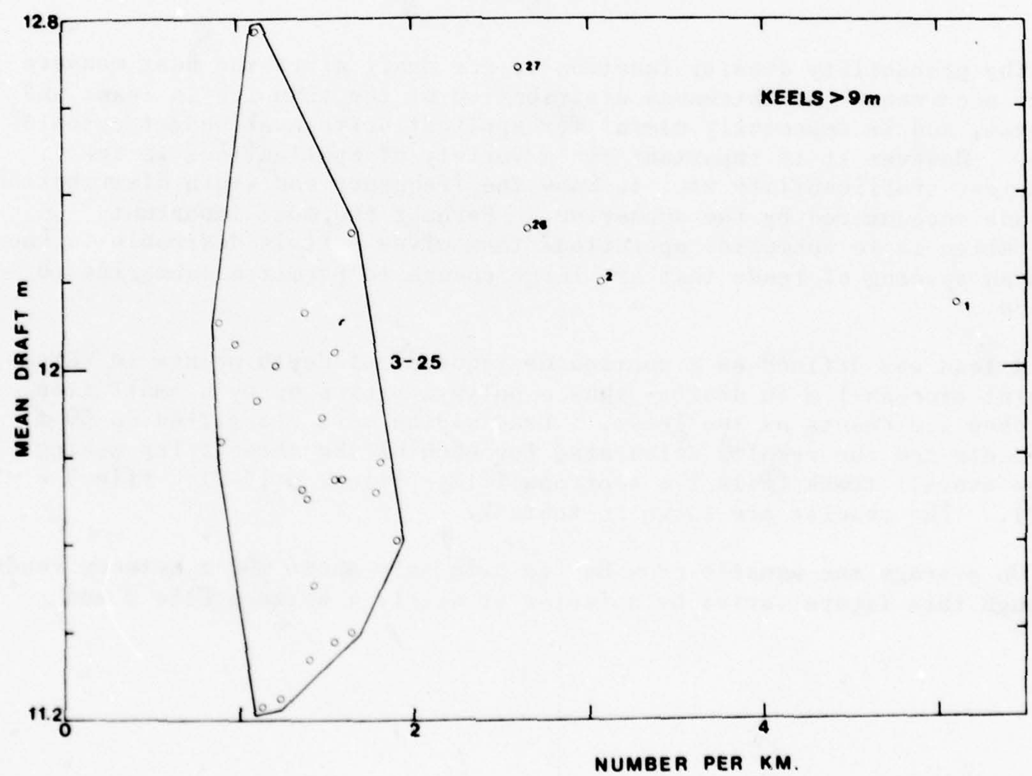


FIGURE 13. Mean keel draft plotted against mean number of keels per km for keels deeper than 5 m and 9 m. Each point represents a 50 km section.



(d) For h_0 = 9 m both the frequency and the mean draft are much lower than those found by SOVEREIGN in the very heavily ridged zone off north Greenland. Only section 1 exceeds the SOVEREIGN data in keel frequency, though not in mean draft. The data compare very well with DREADNOUGHT data from the central Eurasian Basin: although it may appear from table IV that DREADNOUGHT data have a lower μ and higher $\bar{h}$, the difference can be ascribed to the program of Williams et al (1975) which, by applying a harsh version of the Rayleigh criterion with sea level as the zero datum, lost many shallow keels from the statistics.

(e) The maximum drafts are surprisingly low. On the whole the deepest keel drafts are found in the four anomalous sections (1-2, 26-27), but the deepest draft of all, 31.12 m, in fact occurred in the short 18 km portion of track that was omitted from the 50 km statistics. The general ridging properties of this portion (table IV) are quite typical of the sections surrounding it, so that the keel can be seen as an isolated event. This is the only keel deeper than 30 m in the entire 1400 km of profile, whereas in the SOVEREIGN profile there were 45 keels deeper than 30 m in 3900 km of track, 39 of them occurring in the 1050 km of "offshore zone" north of Greenland (Wadhams, 1977c). By a coincidence the deepest keel in the SOVEREIGN profile, 43 m, also occurred as an isolated event in an otherwise lightly ridged section of ice cover.

7. LEADS AND POLYNYAS

The probability density function of ice draft gives the best measure of the occurrence and thickness distribution of the thin ice in leads and polynyas, and is especially useful for application to heat budget calculations. However it is important for a variety of applications in ice mechanics, trafficability etc. to know the frequency and width distribution of leads encountered by the submarine. Perhaps the most important application is to submarine operations themselves - it is desirable to know the mean spacing of leads that are large enough to permit a submarine to surface.

A lead was defined as a continuous sequence of depth points in which no point exceeds 1 m in draft - thus a polynya broken up by a small floe of broken ice counts as two leads. Lead widths were classified in 50 m increments and the results calculated for each of the three files making up the overall track (File 1 = sections 1-12; File 2 = 13-21; File 3 = 22-27). The results are shown in table V.

On average the aquatic crow has to swim only about 200 m between leads - although this figure varies by a factor of nearly 4 between File 2 and

Table V (a) Distribution of lead widths : number of leads encountered per 100 km of track.

Lead width m	File 1	File 2	File 3	Overall
0-50	389	257	915	463
50-100	5.6	6.2	4.4	5.6
100-150	0.9	1.5	2.8	1.5
150-200	0.5	2.1	0.9	1.1
200-250	0.3	0.2	0.3	0.3
250-300	0.6	0.6	0.6	0.6
300-350	0	0	0	0
350-400	0.2	0	0	0.1
400-450	0	0	0	0
450-500	0	0.2	0	0.1
500	0.6	0.4	0	0.4

(b) Mean distance travelled between leads

Width m	Distance between encounters
0	212 m
50	10.3 km
100	24.1 km
150	38.4 km
200	67.6 km
500	237 km

File 3. However, the lead which he reaches is likely to be very narrow, and few leads exceeded 50 m in width. The "exceedence table" in V(b) shows that a submarine which requires a 200 m lead for a safe surfacing will have to travel 68 km to find one. In fact a submarine trying to surface usually investigates every lead wider than about 50 m in case the cross-track dimension is sufficient to permit surfacing; 50 m leads occurred every 10 km in the southern Beaufort Sea during the period of this experiment.

APPENDIX A. ARCTIC SUBMARINE LABORATORY PROCESSING TECHNIQUE

The digitized magnetic tapes were unpacked and processed by the Arctic Submarine Laboratory on a UNIVAC 1108 - 1110 computer system at the Naval Undersea Center. Time marks were removed, and the corrected data was then merged with position information and converted to BCD format for transmittal to Project AIDJEX.

After the initial unpacking, spurious profile points resulting from multiple echoes, fish, air bubbles, etc., were eliminated by two correction programs. Any points that were deleted were set to -10.0. Multiple adjacent deleted points were left at this value, but single points were filled by linear interpolation. These were flagged by adding 200 to the interpolated value.

The first correction program deleted any points that were greater than or equal to 150 feet. At the beginning of a tape, or following one or more zeros recorded by the system, the program searched for the first valid data point by computing the absolute difference between adjacent points ($|X_i - X_{i+1}|$), and requiring that the difference be less than 5.0 feet. For example, if the difference between the first two points satisfied this criterion, the program assumed that X_1 was correct and used it as its current X_i value. If, however, the difference was greater than 5.0, it deleted X_1, made X_2 the current X_i value, and computed a difference ($|X_2 - X_3|$). It continued in this manner until it found a difference that was less than 5.0, at which time it accepted the current X_i value as the first valid data point.

After the program chose a valid X_i value, it computed the absolute difference between adjacent points and compared the difference with a threshold number that was set to 20.0 feet. If the D_1 difference ($|X_i - X_{i+1}|$) was less than the threshold, the program accepted X_{i+1} as a valid point and used it as its next X_i value. However, if the difference was greater than 20, it computed a D_2 equal to $|X_i - X_{i+2}|$, and, if necessary, a D_3 equal to $|X_i - X_{i+3}|$. The program accepted X_{i+1} as a valid point only if both D_2 and D_3 were also greater than the threshold value. If D_2 was less than 20, X_{i+1} was deleted, and X_{i+2} was selected as the next X_i value. If D_2 was greater than 20 but D_3 was less, both X_{i+1} and X_{i+2} were deleted, and X_{i+3} was chosen as the next X_i value.

Since the system recorded zeros during periods when it didn't receive any data, zero words were not used to compute the differences. The program required that if D_1 ($|X_i - X_{i+1}|$) was greater than the threshold, <u>both</u> D_2 and D_3 must be greater than 20 or the data were rejected. In the case that D_1 was greater than 20 and X_{i+2} was zero, the program deleted X_{i+1}. If D_1 and D_2 were greater than 20, but X_{i+3} was zero, both X_{i+1} and X_{i+2} were deleted. In either case, or if X_{i+1} was zero, the program searched for the next valid X_i value by using the technique described above (third paragraph).

In order to maintain continuity between data records, the program included the first three data points from the succeeding record with the current record when it computed the differences.

The second program subtracted a surface offset correction from each of the profile points. This correction was determined from an analysis of both analog and digital data, and is used to shift the <u>mean</u> data points in areas of open water to read zero. A few points that were already very near zero became negative. The offset was usually one to two feet.

This program also eliminated spurious points that might have been missed by the first program by deleting any points during a variable time period that exceeded the maximum possible ice depth which was determined by examining the analog records. Points deleted by this program were not interpolated and appear as -10.0.

APPENDIX B. SPRI COMPUTER PROGRAM FOR DATA PROCESSING

The program is written in Algol 68C. It consists of three parts:-

(a) the main program, which controls the whole computation, reading in data, accumulating statistics and outputting results;

(b) a series of statistics procedures for initialising, updating and outputting tables, identifying level ice and ridges etc. Note that two data-structures are defined, MODE TABLE and MODE TWOWAYTABLE. Each of these structures contains details about the classification defining the table, as well as space for storing the accumulated frequencies etc.;

(c) various miscellaneous "utility" procedures, which may be considered as just an extension to the Algol 68C standard prelude.

```
#...AIDJEX STATISTICS...#
'BEGIN
 FILE datin,datout;  FILE in=standin, out=standout;
 (open(datin,"datin",standin channel)¬=0 |  GOTO err open);
 (open(datout,"datout",standout channel)¬=0 |  GOTO err open);
 standin:=datin;

 STRING er="******error - ", wn="******warning - ";
 INT prob dens fac =1000000;
 INT dim prof=5000, dim ridge=1000, dim level=5000;
 REAL level criterion dist =1000,   #(cm)#
      level criterion grad =1/40;

 MODE  TABLE = STRUCT( INT min,nf,div, REF [ ] REAL freq,
                       INT below, above, npts,  REAL npts weighted, sum);
 MODE  TWOWAYTABLE = STRUCT ( INT minx,miny,nfx,nfy,divx,divy,
                              REF [,] INT freq,  REF [ ] INT freqx,freqy,
                              INT belowx,belowy,abovex,abovey,
                                  sumx,sumy,npts);

 REAL section distance limit =16; #km#
 PROC distance =( REAL lat1,long1,lat2,long2) REAL :
   BEGIN  #..computes great circle distance in km..#
   REAL a1=lat1*pi/180, a2=lat2*pi/180, b=(long2-long1)*pi/180;
   1.86019 * 60 * (180/pi) * arccos( sin(a1)*sin(a2) + cos(a1)*cos(a2)*cos(b))
   END ;
 [1:45] CHAR line;
 PROC set posn =( REF  REAL lat,long) VOID :
   BEGIN lat:=valu(line[1:5]) + valu(line[6:10])/60 + valu(line[11:15])/360000;
   long:=valu(line[16:20]) + valu(line[21:25])/60 + valu(line[26:30])/360000;
   print("   posn. - ");  PRINT line [1:30]
   END ;
 REAL total dist:=0, last lat, last long;
 print(newline,"AIDJEX STATISTICS",newline,newline);
 print("level criterion dist=",level criterion dist,
      " (cm),    level criterion grad=",level criterion grad,
      newline,newline);
 BOOL started:= FALSE ;
 line:=rdrc(45);
 IF  NOT (line[1:5]= RC "00000")
 THEN started:= TRUE ;
     set posn (last lat, last long)
 FI ;
 INT nsection:=0;
 WHILE  NOT (line[1:4]= RC "FINI")
 DO #...process section...#
    REAL section distance:=0; nsection PLUS 1;
    standout:=out;
    print(newline,newline,"SECTION",nsection);
   #..initialise statistics..#
    INT nintvl:=0;
    TABLE tab prof, tab level, tab ridges;
    freq OF tab prof:= LOC [1:460] REAL ; initialise table(tab prof,-99,460,10);
    freq OF tab level:= LOC [1:460] REAL ;
    initialise table(tab level,-99,460,10);
    freq OF tab ridges:= LOC [1:46] REAL ;
    initialise table(tab ridges,-99,46,100);
```

```
TWOWAYTABLE tab spac5, tab spac9;
freqx OF tab spac5:= LOC [1:20] INT ;
freqy OF tab spac5:= LOC [1:20] INT ;
freq  OF tab spac5:= LOC [1:20,1:20] INT ;
initialise twowaytable (tab spac5,0,500,20,20,20,100);
freqx OF tab spac9:= LOC [1:20] INT ;
freqy OF tab spac9:= LOC [1:20] INT ;
freq OF tab spac9:= LOC [1:20,1:20] INT ;
initialise twowaytable (tab spac9,0,900,20,16,20,100);

[] INT special pts =(1,16,31,61,461);

[1:4] INT ridge summary1, ridge summary2;
ridge summary1:= ridge summary2:= (1000000,-1000000,0,0);

WHILE  NOT (line[1:4]= RC "FINI"  OR
            section distance > section distance limit)
DO #..process interval..#
   [1:dim prof] INT profile;  INT nprofile:=0, nblocks:=0;
   nintvl PLUS 1;
   print(newline,"    INTERVAL",nintvl);
   WHILE #.read block.#
       TO 4
       DO read(newline);
          TO 15
          DO  INT p=valu(rdrc(5));
              IF p¬=-9990  AND p¬=-100
              THEN nprofile PLUS 1;
                 (nprofile<=dim prof
                   | profile[nprofile]:= ROUND ((p<2000|p|p-2000)*3.048))
              FI
          OD
       OD ;
       nblocks PLUS 1;
       read(newline); line:=rdrc(45);
       IF line[1:5]= RC "     "
       THEN print(newline,".....churk boundary");
          read(newline); line:=rdrc(45)
       FI ;
       line[1:5]= RC "00000"
   DO  SKIP  OD ;
  #..compute statistics on profile, and update stats tables..#
   print("    nprofile,nblocks",nprofile,nblocks);
   (nprofile>dim prof | print(newline,"********error",
                            "dim prof exceeded");
                            nprofile:=dim prof);
   IF line[1:4]= RC "FINI"
   THEN print(newline,"+++++this interval excluded from statistics because"
                      " no posn. fix at end")
   ELIF  NOT started
   THEN print(newline,"+++++this interval excluded from statistics because"
                      " no posn. fix at start",newline,newline);
       set posn (last lat, last long);
       started:= TRUE
   ELSE  REAL lat,long; set posn(lat,long);
       REAL interval distance =distance(last lat,last long,lat,long);
       REAL interp int =interval distance*100000/nprofile; #(cm)#
       section distance PLUS interval distance;
```

```
            last lat:=lat; last long:=long;
            [1:dim ridge] INT xridge,yridge;   INT nridges;
            identify ridges(profile[1:nprofile],xridge,yridge,nridges);
            [1:nridges] INT xspace5, yspace5, xspace9, yspace9;
            INT nspace5, nspace9;
            setup spacings (xridge[1:nridges], yridge[1:nridges], 500,
                            xspace5, yspace5, nspace5, interp int);
            setup spacings (xridge[1:nridges], yridge[1:nridges], 900,
                            xspace9, yspace9, nspace9, interp int);
            [1:dim level] INT level;  INT nlevel;
            INT level criterion npts:=
                  ROUND (level criterion dist/interp int);
            (level criterion npts=0 | level criterion npts:=1);
            INT level criterion ydiff =
                  ROUND (level criterion grad*level criterion npts*interp int);
            identify level points(profile[1:nprofile],level,nlevel,
                                  level criterion npts, level criterion ydiff);

            print(newline,"interval dist, dist per pt., nridges, nlevel",
                    ROUND (interval distance*1000(),"  dm",
                    ROUND interp int," cm", nridges, nlevel);

            update table(profile[1:nprofile],tab prof,interp int);
            update table(yridge[1:nridges],tab ridges,1);
            update table(level[1:nlevel],tab level,interp int);
            update twowaytable (xspace5[1:nspace5], yspace5[1:nspace5], tab spac5);
            update twowaytable (xspace9[1:nspace9], yspace9[1:nspace9], tab spac9);

            update summary(yridge[1:nridges],500,4500,ridge summary1);
            update summary(yridge[1:nridges],900,4500,ridge summary2)

         FI
       #..end process interval..#
     OD ;
     total dist PLUS section distance;
    #..output statistics for section..#
     print(newline,newline,"STATISTICS FOR SECTION",nsection);
     IF section distance > 0
     THEN
        print(newline,"section distance=",
                 ROUND (section distance*1000(),"  dm      ",
               "no. intervals=",nintvl);
        print(newline,"ICE PROFILE -");
        print table details(tab prof,special pts);
        [ ] INT dummy=(0);
        print(newline,"LEVEL ICE -");
        print table details(tab level,dummy);
        print(newline,"INDEPENDENT RIDGES -");
        print table details(tab ridges,dummy);
        print summary(ridge summary1,500,4500);
        print summary(ridge summary2,900,4500);
        print(newline,"RIDGE SPACINGS BY DEEPER KEEL DEPTH -");
        print(newline,"  x values (spacings) in metres, y values"
                 " (depths) in cm");
        print(newline,"   RIDGES DEEPER THAN 5 METRES",newline);
        print twowaytable (tab spac5);
        print(newline,newline,"RIDGES DEEPER THAN 9 METRES",newline);
        print twowaytable (tab spac9);
```

```
          scale table freqs(tab prof,prob dens fac);
          scale table freqs(tab level,prob dens fac);
         #.normalise ridge freqs to no. per 100km of track.#
          FOR I  TO nf OF tab ridges
          DO (freq OF tab ridges)[I]:= ROUND ((freq OF tab ridges)[I]*100/
                   section distance)
          OD ;
         #.write output file.#
          standout:=datout;
          output table(tab prof);
          output table(tab level);
          output table(tab ridges)
      ELSE print(newline,"+++++zero length section - ignored")
      FI
     #...end process section...#
    OD ;
    standout:=out; print(newline,"END OF RUN **************************",newline,
                     " total distance=",total dist)  EXIT
     dim level exc: msg(er,"dim level exceeded")  EXIT
     dim ridge exc: msg(er,"dim ridge exceeded") EXIT
     err open: msg(er,"opening cataset")
    END
```

HASP-II***HASP-II.......END JOB 602....12.29.18 PM 8 JUN 78....POST RJH6...J0602RJH6....

```
#..statistics procedures...#
 PROC identify ridges =([ ] INT profile, REF [ ] INT xridge,yridge,
                                REF   INT nridges) VOID :
 #..identifies independent ridges (by Rayleigh Criterion) from profile,
    and sets xridge,yridge and nridges..#
   BEGIN
   BOOL ridge:= FALSE , max:= TRUE ;
   INT kridge;  REAL ht:=100000; nridges:=0;
   REAL yy:=profile[1]-250, yynext; (yy<0 | yy:=0);
   FOR i  TO  UPB profile - 1
   DO yynext:=profile[i+1]-250; (yynext<0 | yynext:=0);
     IF max
     THEN  IF yynext>yy
           THEN #..min found..# max:= FALSE ;
              IF ridge
              THEN  IF yy < ht/2
                    THEN #..ridge confirmed..#
                      nridges PLUS 1;
                      (nridges> UPB xridge |  GOTO dim ridge exc);
                      xridge[nridges ]:=kridge; yridge[nridges ]:= ROUND (ht+250);
                      ridge:= FALSE ; ht:=yy
                    FI
              ELSE (yy<ht | ht:=yy)
              FI
           FI
     ELSE  IF yynext < yy
           THEN #..max found..# max:= TRUE ;
              IF ridge
              THEN (yy>ht | kridge:=i; ht:=yy)
              ELSE (yy>2*ht | ridge:= TRUE ; ht:=yy; kridge:=i)
              FI
           FI
     FI ;
     yy:=yynext
   OD
   END ;

PROC identify level points= ([ ] INT y, REF [ ] INT level, REF   INT n,
       INT level criterion npts, level criterion ydiff) VOID :
   BEGIN  INT np=level criterion npts, yd=level criterion ydiff;
   n:=0;
   FOR i  TO  UPB y
   DO  BOOL level left =
           (i-np>=1 |  ABS (y[i]-y[i-np]) <= yd |  FALSE ),
         level right =
           (i+np<= UPB y |  ABS (y[i]-y[i+np]) <= yd |  FALSE );
      IF level left  OR level right
      THEN n PLUS 1;
         (n> UPB level |  GOTO dim level exc);
         level[n]:=y[i]
      FI
   OD
   END ;

PROC update summary= ([ ] INT y, INT lower, upper,  REF [ ] INT summary) VOID :
   BEGIN  REF  INT ymin=summary[1], ymax=summary[2], sum=summary[3],
                   ny=summary[4];
   FOR i  TO  UPB y
```

```
    DO  INT yy=y[i];
        IF yy>=lower  AND yy<=upper
        THEN ny PLUS 1; sum PLUS yy;
            (yy>ymax | ymax:=yy); (yy<ymin | ymin:=yy)
        FI
    OD
    END ;

PROC print summary= ([ ] INT s, INT lower, upper) VOID :
    BEGIN
    print(newline,"      of the values in range",lower,"     to",upper,"     -",
        newline,"            no. values =",s[4],"     mean=",
        ROUND ratio(s[3],s[4]),"     minimum =",s[1],"     maximum =",s[2])
    END ;

PROC setup spacings= ([ ] INT xridge, yridge,  REAL min depth,
                     REF [ ] INT xspace, yspace,  REF  INT nspacings,
                     REAL interp int) VOID :
  #..sets xspace to spacings between ridges (m),
     and yspace to deeper keel depths (cm)..#
  #..considers only ridges of depth > min depth..#
    BEGIN
    INT i:=0, j:=0, n:=0;
    DO  WHILE j PLUS 1;
            (j> UPB xridge |  GOTO fin);
            yridge[j] <= min depth
        DO  SKIP  OD ;
        IF i>0
        THEN n PLUS 1;
            xspace[n] :=  ROUND ((xridge[j]-xridge[i])*interp int/100);
            yspace[n] := max(yridge[i], yridge[j])
        FI ;
        i:=j
    OD ;
     fin: nspacings:=n
    END ;

PROC print table details= ( TABLE t,[ ] INT special pts) VOID :
  #..(assumes freq values have not been scaled)..#
    BEGIN  INT min=min OF t, nf=nf OF t, div=div OF t;
    INT max=min+div*nf-1;
    print(newline,"      values are classified from",min,"     to",max,"     in",
          nf,"   divisions of",div);
    print(newline,"      no. of pts. included =",npts OF t,"     mean value =",
            ROUND ratio(sum OF t,npts weighted OF t),
            "     weighted no. of pts included =", ROUND npts weighted OF t);
    print(newline,"      no. of values excluded because out of range: below-",
          below OF t,"     above-",above OF t);
    FOR i  TO  UPB special pts - 1
    DO  REAL s:=0;  INT n1=special pts[i], n2=special pts[i+1] - 1;
        FOR j  FROM n1  TO n2
        DO s PLUS (freq OF t)[j]  OD ;
        print(newline,"      percentage of pts in range", min+div*n1-div, "     to",
              min+div*n2-1, "     is",  ROUND ratio(s*100,npts weighted OF t))
    OD
    END ;

PROC print twowaytable= ( TWOWAYTABLE t) VOID :
```

```
    BEGIN print(newline,"excluded:  x too low, y too low, x too high,"
                "y too high : ",belowx OF t,belowy OF t,
               abovex OF t,abovey OF t,
               newline,"no. pts. included",npts OF t,
               "     mean x", ROUND ratio(sumx OF t,npts OF t),
               "     mean y", ROUND ratio(sumy OF t,npts OF t));
    print(newline,newline,"x values:               ");
    FOR ix  TO nfx OF t
    DO prnt(minx OF t + divx OF t*(ix-1), 4)  CD ;
    print(newline,newline,"x frequencies           ");
    FOR ix  TO nfx OF t
    DO prnt((freqx OF t)[ix], 4)  CD ;
    print(newline,"  y values y frequencies");
    FOR iy  TO nfy OF t
    DO print(newline);
       prnt(miny CF t + divy OF t*(iy-1), 10);
       prnt((freqy OF t)[iy], 10);
       FOR ix  TO nfx OF t
       DO prnt((freq CF t)[ix,iy], 4)  OD
    OD ;
    print(newline,"                ")
    END ;

PROC initialise table= ( REF  TABLE t, INT min,nf,div) VOID :
    BEGIN min OF t:=min; nf OF t:=nf; div OF t:=div;
    below CF t:=above OF t:=npts OF t:=0;
    sum OF t:=npts weighted OF t:=0;
    CLEAR freq OF t
    END ;

PROC initialise twowaytable= ( REF  TWOWAYTABLE t, INT minx,miny,nfx,nfy,
                              divx,divy) VOID:
    BEGIN minx OF t:=minx; miny OF t:=miny; nfx OF t:=nfx; nfy CF t:=nfy;
    divx OF t:=divx; divy CF t:=divy;
    belowx OF t:= belowy OF t:=abovex CF t:=abovey OF t:=sumx CF t:=
       sumy OF t:=npts OF t:=0;
    CLEAR freq OF t;  CLEAR freqx CF t;  CLEAR freqy OF t
    END ;

PROC update table =([ ] INT y, REF  TABLE t, REAL weight) VOID
 #..updates table using values in y, weighting contributions by "weight"..#
    BEGIN  REF  INT below=below OF t, above=above OF t, npts=npts OF t;
           REF  REAL sum=sum OF t, npts weighted=npts weighted CF t;
    INT min=min OF t, nf=nf OF t, div=div OF t;
    INT max=min+div*nf-1;
    FOR i  TO  UPB y
    DO  INT yy=y[i];
       IF yy<min
       THEN below PLUS 1
       ELIF yy>max
       THEN above PLUS 1
       ELSE (freq OF t)[ ENTIER ((yy-min)/div) + 1 ]  PLUS weight;
         sum PLUS (yy*weight); npts PLUS 1;
         npts weighted PLUS weight
       FI
    OD
    END ;
```

```
PROC update twowaytable= ([ ] INT x,y,  REF  TWOWAYTABLE t) VOIC :
 #.(x and y arrays assumed same length).#
   BEGIN  INT minx =minx CF t, miny =miny OF t;
   INT maxx =minx + divx OF t * nfx OF t - 1,
       maxy =miny + divy CF t * nfy OF t - 1;
   FOR i  TO  UPB x
   DO  INT xx=x[ i ], yy=y[ i ];
      IF  xx>=minx  AND  xx<=maxx  AND yy>=miny  AND yy<=maxy
      THEN  INT kx = ENTIER ((xx-minx OF t)/divx OF t) + 1,
                ky = ENTIER ((yy-miny OF t)/divy OF t) + 1;
        (freqx OF t)[kx] PLUS 1; (freqy OF t)[ky] PLUS 1;
        (freq OF t)[kx,ky] PLUS 1;
        sumx OF t PLUS xx; sumy OF t PLUS yy; npts OF t PLUS 1
      ELSE (xx<minx | belowx OF t PLUS 1 |: xx>maxx | abovex OF t PLUS 1);
           (yy<miny | belowy OF t PLUS 1 |: yy>maxy | abovey CF t PLUS 1)
      FI
   OD
   END ;

PROC scale table freqs= ( TABLE t, INT prob dens fac) VOID :
 #..converts freqs to probability density times "prob dens fac"..#
   BEGIN
   IF npts OF t > 0
   THEN  REAL factor= prob dens fac/(div OF t * npts weighted CF t);
      FOR i  TU nf OF t
      DO (freq'of t)[i]:=  (freq CF t)[i] * factor  OD
   FI
   END ;

PROC output table= ( TABLE t) VOID :
   BEGIN  4 PRINT min OF t; 4 PRINT (min OF t + div OF t*nf OF t -1);
   4 PRINT div OF t; 8 PRINT npts OF t;
   8 PRINT prob dens fac;
   print(newline);
   FOR i  TO  UPB freq OF t
   DO 4 PRINT  RCUND (freq OF t)[i]  OD ;
   print("9999",newline)
   END
```


HASP-II***HASP-II.......END JOB 608....12.29.47 PM 8 JUN 78....POST RJH6...J0608RJH6....!

```
PROC rdrc =( INT n)[ ] CHAR :
  BEGIN
  [1:n] CHAR c;
  FOR i  TO n  DO read(c[ i])  OD ;
  c
  END ;

OP = =([ ] CHAR x,y) BOOL :
  BEGIN
  INT l= UPB x;  BOOL equal:=(l= UPB y);
  FOR i  TO l  WHILE equal  DO equal:=(x[ I ]=y[ I ])  OD ;
  equal
  END ;

PRIO  PLUS =1;
OP.  PLUS = ( REF  INT a, INT b) INT :
 (a:=a+b);

OP  PLUS = ( REF  REAL a,  REAL b) REAL :
  (a:=a+b);

OP  CLEAR = ( REF [ ] INT a) VOID :
 ( FOR i  TO  UPB a  DO a[ i ]:=0  OD );

OP  CLEAR =( REF [,] INT a) VOID :
 ( FOR i  TO 1 UPB a
   DO  FOR j  TO 2 UPB a  DO a[ i,j]:=0  OD  OD );

OP  CLEAR =( REF [ ] REAL a) VOID :
 ( FOR i  TO  UPB a  DO a[ i ]:=0  OD );

OP  CLEAR =( REF [,] REAL a) VOID :
 ( FOR i  TO 1 UPB a
   DO  FOR j  TO 2 UPB a  DO a[ i,j]:=0  OD  OD );

OP  RC =( STRING s)[ ] CHAR :
  ( INT l= UPB s; [1:l] CHAR c;
   FOR i  TO l  DO c[ i ]:=i ELEM s  OD ;
   c);

PROC ratio =( REAL a,b) REAL :
  BEGIN  REAL r= (b<=0 AND b>=0 | (a<=0 AND a>=0|0|2000000) | a/b); r
  END ;

PROC max =( INT a,b) INT :
 ((a>b|a|b));

PROC msg = ( STRING s1,s2) VOID :
 ( FILE f=standout; standout:=out; print(newline,s1,s2); standout:=f);

PROC msgval = ( STRING s1,s2, INT n) VOID :
 ( FILE f=standout; standout:=out; print(newline,s1,s2,n); standout:=f);

PROC valu =([ ] CHAR g) INT :
  BEGIN  INT res:=0;  INT fac:=1;  BOOL neg=(g[ 1 ]="-");
  FOR i  FROM  UPB g  BY -1  TO (neg|2|1)
  DO  INT j= ABS g[ i ]- ABS "0";
    (j<0  OR j>9 |  GOTO error);
```

```
      res:=res+j*fac; fac:=fac*10
   OD ;
   (neg | res:=-res);
   res  EXIT
  error:  msg(er,"non-digit found -");
    FOR i  TO  UPB g  DO print(g[i])  OD ;  -999
   END ;

 PROC rep int =( INT val,n)[ ] CHAR :
   BEGIN [1:n] CHAR c;  INT v:=(val<0 | -val | val);
   FOR i  FROM n  BY -1  TO (val<0|2|1)
   DO  INT w= ENTIER (v/10); c[i]:= REPR ( ABS "0"+v-w*10); v:=w  OD ;
   (v¬=0 | msgval(er,"rep int value too big",val));
   (val<0 | c[1]:="-");
   c
   END ;

 PRIO  PRINT =1;

 OP  PRINT = ([ ] CHAR c) VOID :
  ( FOR i  TO  UPB c  DO print(c[i])  OD );

 OP  PRINT = ( INT l, INT a) VOID :
  ( PRINT rep int(a,l));

 OP  PRINT = ( INT l,[ ] INT a) VOID :
  ( FOR i  TO  UPB a  DO l PRINT a[i]  OD );

 PROC prnt =( INT val, l) VOID :
   BEGIN #..prints an integer (right justified) without
          leading zeroes..#
   [ ] CHAR c =rep int (val,l);
   BOOL started := FALSE ;
   FOR i  TO l
   DO  IF c[i]="0"  AND  NOT started
       THEN print(" ")
       ELSE started:= TRUE ; print(c[i])
       FI
   OD
   END ;
```

HASP-II***HASP-II.......END JOB 606....12.28.52 PM 8 JUN 78....POST RJH6...J0606RJH6....

APPENDIX C. FULL NUMERICAL RESULTS

The numerical results from the foregoing program for all 27 sections are lengthy and have therefore been published separately as a Supplement to this Report. Readers who have not received a copy of this Supplement with their Report may obtain one by writing to the authors at Scott Polar Research Institute, Cambridge CB2 1ER, Great Britain.

ACKNOWLEDGEMENTS

The authors acknowledge with thanks the support of the Office of Naval Research under contract N00014-78-G-0003. The data transfer was arranged through the good offices of Robert S Pritchard and Professor Norbert Untersteiner. Our thanks are also due to Murray J Stateman, AIDJEX data manager, and to the staff of the Arctic Submarine Laboratory, San Diego, for furnishing and pre-processing of data. The original data can be consulted at the AIDJEX Data Bank, 4059 Roosevelt Way N E, Seattle, Wash. 98105.

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