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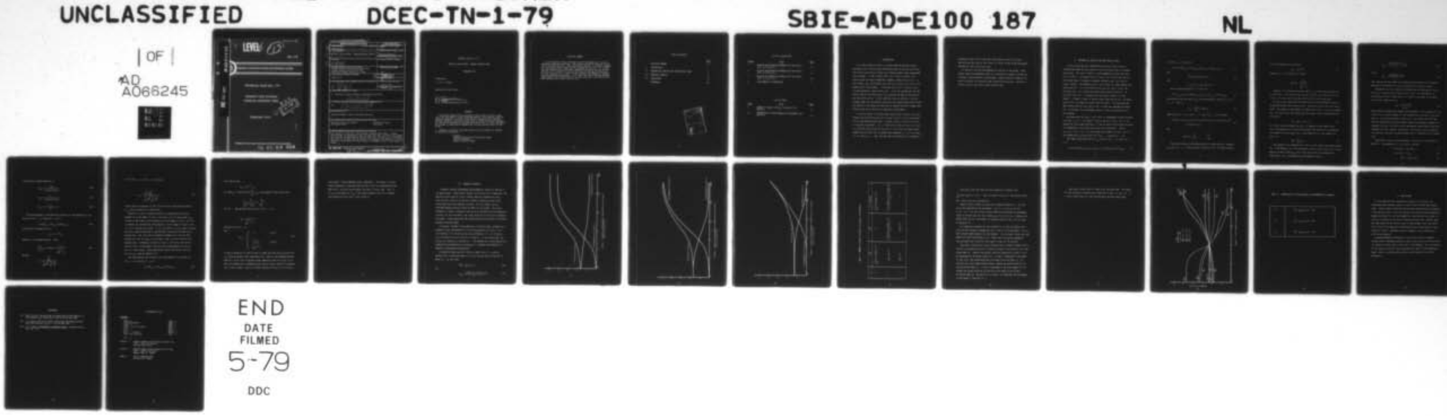

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DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 1-79

PRIORITY LOSS SYSTEMS - UNEQUAL HOLDING TIMES

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Two classes of voice calls use the same s-channel trunk group. A mathematical model is developed for the case where one class of traffic can preempt the other and the mean holding times of each class of traffic may be different. These results are then used to numerically investigate the behavior of various measures of performance for both classes of customers as well as the system in general.

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PRIORITY LOSS SYSTEMS - UNEQUAL HOLDING TIMES

FEBRUARY 1979

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FOREWORD

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EXECUTIVE SUMMARY

In this Technical Note, we analyze the link performance of a circuit-switched system with priorities. Two classes of traffic, high and low priority, are using the link. The high priority traffic is allowed to preempt the low priority traffic when initially blocked on the link. It is shown that the probabilities of blocking, preemption and total loss for the low priority calls significantly vary as the ratio of the mean holding times for each class of calls. From a system viewpoint two measures of performance are presented and analyzed. It is shown that the difference between these measures are significant and also very sensitive to the ratio of the mean holding time for each class of calls.

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I. INTRODUCTION

In a recent technical note [1], we described the modifications we have made to our circuit-switch performance model to consider priorities. Two priority disciplines, friendly and ruthless search, were incorporated into the model. In the ruthless case if all circuits are busy, the high priority call would immediately preempt a lower priority call. In the friendly case, the blocked high priority call would first try to find another path in the network. If one does not exist, it returns to the link and preempts a lower priority call. In [1] we assumed that the mean holding time of the high priority calls equaled the mean holding time of the low priority calls. In this technical note, we develop a link performance model for the ruthless discipline with unequal mean holding times. The results of this model are then used to examine the effect unequal mean holding times has on various link measures of performance.

To see the effect of different mean holding time, consider the single channel case with Poisson arrivals for each class of traffic and exponentially distributed holding times. Suppose a class 2 (low priority) call is occupying the channel. If λ_i is the arrival rate of class i calls and $1/\mu_i$ the mean holding time for the class i calls, then the steady state probability that the class 2 customer gets preempted is $\rho_1\alpha/(\rho_1\alpha+1)$ where $\rho_1=\lambda_1/\mu_1$ and $\alpha=\mu_1/\mu_2$. Thus, one sees that the probability of preemption is dependent on the ratio of the class 2 mean holding time to the class 1 mean holding time, and so, this ratio is a factor in the system performance measures for the low priority calls.

In section II we give the mathematical analysis of the system. Some special cases are presented as well as a discussion of several related, but different, system measures of performance. Several numerical examples are given in section III along with some interesting conclusions. Section IV contains overall conclusions drawn from this work.

II. MATHEMATICAL ANALYSIS AND SOME SPECIAL CASES

In this section we give a mathematical analysis of the link performance when there are two classes of traffic trying to use an s-channel trunk group. The class 1 traffic is given preemptive priority over the class 2 traffic. This means that if a class 1 call arrives and finds all the channels busy it preempts a randomly selected class 2 call that is using a channel. The preempted class 2 call leaves the system without completing service. If all the channels are busy with class 1 calls, the arriving class 1 call leaves the system without receiving service. An arriving class 2 call is accepted into the system if there is a free channel. If there is none, it leaves without receiving service. No queueing of calls is allowed for either class of traffic. The case where the mean holding time for each class of call is equal was considered by Burke [2]. To the best of our knowledge, Burke's is the only related work on this problem.

We assume that the class i calls arrive in independent Poisson processes with rate $\lambda_i, i=1,2$. The length of time to service a class i call is exponentially distributed with mean μ_i^{-1}, and the service time random variables are independent of each other and of the arrival processes. Define $\rho_i=\lambda_i/\mu_i$ $(i=1,2)$, $\alpha=\mu_1/\mu_2$, Q_i the steady state number of class i calls in the system, and $P_{i,j}=Pr\{Q_1=i,Q_2=j\}$ for $i=0,1,\ldots,s$ and $j=0,1,\ldots,s-i$.

The steady state equations for $P_{i,j}$ are for $i=0,1,\ldots,s-1$ and $j=0,1,\ldots,$ $s-i-1$

$$(\rho_1\alpha+\rho_2+i\alpha+j)P_{i,j}=\rho_1\alpha P_{i-1,j}+\rho_2 P_{i,j-1}+(i+1)\alpha P_{i+1,j}+(j+1)P_{i,j+1}; \qquad (1)$$

for $i=0,1,\ldots,s-1$ and $j=s-i$

$$(\rho_1\alpha+i\alpha+s-i)P_{i,s-i}=\rho_1\alpha P_{i-1,s-i}+\rho_2 P_{i,s-i-1}+\rho_1\alpha P_{i-1,s-i+1} \qquad (2)$$

and

$$sP_{s,0}=\rho_1 P_{s-1,0}+\rho_1 P_{s-1,1} \qquad (3)$$

with

$$P_{-1,j}=P_{i,-1}=P_{s+1,j}=P_{i,s+1}\equiv 0.$$

One can rewrite equation (1) in the form

$$(j+1)P_{i,j+1}=(\rho_1\alpha+\rho_2+i\alpha+j)P_{i,j}-\rho_1\alpha P_{i-1,j}-\rho_2 P_{i,j-1}-(i+1)\alpha P_{i+1,j}; \qquad (4)$$

from which one can see that $P_{i,j+1}$ can be expressed in terms of $P_{i,0}$ $i=0,1,\ldots,s$. That is,

$$P_{i,j} = \sum_{k=0}^{s} A(i,j,k)P_{k,0} \qquad (5)$$

where for $k=0,1,\ldots,s-1$, $j=0,1,\ldots,s-1$ and $i=0,1,\ldots,s-j-1$ we have

$$(j+1)A(i,j+1,k)=-\rho_1\alpha A(i-1,j,k)-\rho_2 A(i,j-1,k)+(\rho_1\alpha+\rho_2+i\alpha+j)A(i,j,k) -(i+1)\alpha A(i+1,j,k) \qquad (6)$$

and

$$A(i,0,k) = \begin{cases} 1 & i=k \\ 0 & i\neq k. \end{cases}$$

Thus, the solution of the problem rests on finding the $(s+1)$ unknowns $P_{i,0}$ $i=0,1,2,\ldots,s$. Using equation (2) one can find s of these equations;

the final equation can be given by

$$P_{s,0} = E_B(\rho_1, s) \tag{7}$$

where $E_B(\rho_1,s)$ is Erlang's Loss Formula,

$$E_B(a,s) = \frac{a^s/_{s!}}{\sum_{r=0}^{s} a^r/_{r!}} .$$

Equation (7) follows from the fact that $P_{s,0}$ is the blocking probability for the class 1 calls and this has to equal Erlang's Loss Formula since the class 1 calls only have to contend with calls from their own class.

As one can see, there does not appear to be a straightforward solution to the problem and no simple results exist. One has to solve a set of s+1 equations. The measure of performance of interest for class i is its loss probability. For class 1 the loss probability, PL_1, equals blocking probability and is given by

$$PL_1 = E_B(\rho_1,s)=P_{s,0}. \tag{8}$$

For class 2 the loss probability, PL_2, is composed of two probabilities: first, the probability of blocking and second, the probability of preemption. The probability of blocking, PB_2, is the probability all the channels are busy; i.e.,

$$PB_2 = \sum_{i=o}^{s} P_{i,s-i} . \tag{9}$$

The probability of preemption for class 2 can be found as was done by Burke [2]. Since $PB_2-E_B(\rho_1,s)$ is the proportion of time a high priority call is preempting, we have $\lambda_1[PB_2-E_B(\rho_1,s)]$ is the rate at which the preemption is taken place. Thus, the probability of preemption, PP_2, is

$$PP_2 = \frac{\lambda_1[PB_2 - E_B(\rho_1,s)] \;/_{\mu_2}}{\rho_2} \tag{10}$$

and so

$$PP_2 = \frac{\rho_1\alpha[PB_2 - E_B(\rho_1,s)]}{\rho_2} . \tag{11}$$

Thus, one can see that there is no simple characterization of the desired measures of performance for class 2 because PB_2 is not easily found.

Although PL_1 and PL_2 ($=PB_2+PP_2$) characterizes the percentage of class 1 and class 2 offered erlangs that are lost, there is another measure of performance for both classes which is in terms of the number of customers who are lost, denoted by PLN;

$$PLN = \frac{\alpha\rho_1 PL_1 + \rho_2 PL_2}{\alpha\rho_1 + \rho_2} . \tag{12}$$

Some interesting comparisons between the percentage of lost erlange $PLE=(\rho_1 PL_1 + \rho_2 PL_2)/(\rho_1+\rho_2)$ and the number of lost customers PLN, are given in the next section. It should be pointed out that PLN is the results one would get from an event-by-event simulation where blocked and preempted customers are computed, or from a real world traffic measurement system where blocked and preempted calls are counted. By contrast, lost erlangs cannot be measured in a practical real world systems because this is a measure of call seconds lost.

Three special cases are now considered. The first is the condition in which s=1. From equations (1), (2), and (3), we have

$$(\rho_1\alpha+\rho_2)P_{0,0} = \alpha P_{1,0} + P_{0,1} \tag{13}$$

$$(\rho_1\alpha+1)P_{0,1} = \rho_2 P_{0,0} \tag{14}$$

$$P_{1,0} = \rho_1 P_{0,0} + \rho_1 P_{0,1} . \tag{15}$$

The solution to these equations is

$$P_{0,0} = \frac{1+\alpha\rho_1}{(1+\alpha\rho_1+\rho_2)(1+\rho_1)} \tag{16}$$

$$P_{0,1} = \frac{\rho_2}{(1+\alpha\rho_1+\rho_2)(1+\rho_1)} \tag{17}$$

$$P_{1,0} = \frac{\rho_1}{1+\rho_1} \ . \tag{18}$$

The second example is the condition in which $\alpha \to 0$; from equation (1), we have for $i=0,1,\ldots,s-1$ and $j=0,1,\ldots,s-i-1$

$$(\rho_2+j)P_{i,j} = \rho_2 P_{i,j-1} + (j+1)P_{i,j+1}; \tag{19}$$

the solution to equation (19) is

$$P_{i,j} = C_i \, \rho_2^{\,j}/j!$$

where C_i is an unknown constant. Since

$$\sum_{j=0}^{s-i} P_{i,j} = \Pr\{Q_1=i\} = \frac{\rho_1^{\,i}/i!}{\sum_{r=0}^{s} \rho_1^{\,r}/r!} \ , \tag{20}$$

we have

$$C_i = \frac{\dfrac{\rho_1^{\,i}}{i!}}{\sum_{r=0}^{s} \dfrac{\rho_1^{\,r}}{r!} \sum_{t=0}^{s-i} \dfrac{\rho_2^{\,t}}{t!}}$$

or for $i=0,1,\ldots,s$, $j=0,1,\ldots,s-i$ and $\alpha=0$,

$$P_{i,j} = \frac{\frac{\rho_1^i}{i!}\,\frac{\rho_2^j}{j!}}{\sum_{r=0}^{s}\frac{\rho_1^r}{r!}\sum_{t=0}^{s-i}\frac{\rho_2^t}{t!}} \,. \tag{21}$$

A quick check of equations (2) and (3) with $\alpha=0$ will show that the form of $P_{i,j}$ given by equation (21) holds there.

Equation (21) has an interesting physical interpretation; the joint probability of the number of class 1 and class 2 calls in the system, $P_{i,j}$, is equal to the product of the probability of the number of class 1 calls in a s-channel loss system times the probability of the number of class 2 calls in an (s-i)-channel loss system. As $\alpha=\mu_1/\mu_2\to 0$ while ρ_1 and ρ_2 remain constant, the class 2 mean holding time is getting small relative to the mean class 1 holding time. Thus, the class 2 preemption probability is going to zero and the only loss for class 2 occurs when a class 2 arrival finds all the channels busy. Furthermore, relative to class 1, the arrival and service rates of class 2 are extremely fast and so the system appears to class 2 as a (s-i)-loss system. These observations explain the form of the solution for $P_{i,j}$ given by equation (21).

The final special case is that of $\alpha\to\infty$; from equation (1) we have for $i=0,1,\ldots,s-1$ and $j=0,1,\ldots,s-1-i$

$$(\rho_1+i)P_{i,j} = \rho_1 P_{i-1,j}+(i+1)P_{i+1,j}; \tag{22}$$

this implies that

$$P_{i,j} = D_j \, {\rho_1}^i / i!$$

for some D_j. Since $\Pr\{Q_1=i\} = \sum_{j=0}^{s-i} P_{i,j}$, using equation (20) we must have

$$\sum_{j=0}^{s-i} D_j = \left[\sum_{r=0}^{s} {\rho_1}^r / r! \right]^{-1}$$

for all i. One possible choice for D_j, j=0,1,...,s is

$$D_0 = \frac{1}{\sum_{r=0}^{s} {\rho_1}^r / r!},$$

and $D_j=0$ for j=1,2,...,s;

then

$$P_{i,j} = \begin{cases} 0 & i \geq 0, j \geq 1 \\ \dfrac{{\rho_1}^i / i!}{\sum_{r=0}^{s} {\rho_1}^r / r!} & i \geq 0, j=0. \end{cases} \quad (23)$$

A check of equations (2) and (3) with $\alpha=\infty$ shows that the form of solution for $P_{i,j}$ given by equation (23) holds there also. When $\alpha \to \infty$ the expected holding time for a class 2 call is getting large compared to that of a class 1 call. Thus, the probability of preemption, given the call gets a channel, is approaching 1, and so either a class 2 call gets blocked on arrival, or if it gets a free channel it gets preempted almost immediately. The reason it is preempted immediately is the mean holding time is very long compared with the mean class 1 holding time and hence, the class 1 arrival rate. Thus, if $j \geq 1$ $P_{i,j} \equiv 0$ and when $j=0$, $P_{i,0}$ is the state probabilities of an s-channel loss system with only class 1 calls using it.

III. NUMERICAL EXAMPLES

A computer program implementing the mathematical results of section II has been written. Those results require the solution of s+1 equations. Depending on the values of s and α, severe numerical problems can arise. Even for small values of s(s=3) and α greater 1,000, the accuracy (even using double precision) becomes a problem. For this reason, we also developed an event-by-event simulation model for the system. The curves presented in figures 1 through 4 were basically generated by the mathematical solution; for the case where α was large, equation (23) was used to generate the trends. These trends were then checked against the results of the event-by-event simulation model.

In figures 1 through 3 the probability of blocking (PB_2), probability of preemption (PP_2) and probability of lost ($PL_2=PB_2+PP_2$) for class 2 calls are presented. For s=8 three cases are considered: $\rho_1=7$, $\rho_2=1$ (figure 1), $\rho_1=\rho_2=4$ (figure 2) and $\rho_1=1$ and $\rho_2=7$ (figure 3). In each figure PB_2, PP_2 and PL_2 are plotted as a function of α. The probability of blocking and loss probability are monotonically increasing in α; whereas the preemption of blocking is monotonically decreasing.

We summarize these results in table I, where $Pr\{Q_1=i\}$ is given by equation (20). Noting that when $\alpha=1$, we can use the results contained in Burke [2]. For this case,

$$PB_2 = E_B(\rho_1+\rho_2,s) \tag{24}$$

and

$$PP_2 = \frac{\rho_1[E_B(\rho_1+\rho_1,s)-E_B(\rho_1,s)]}{\rho_2}. \tag{25}$$

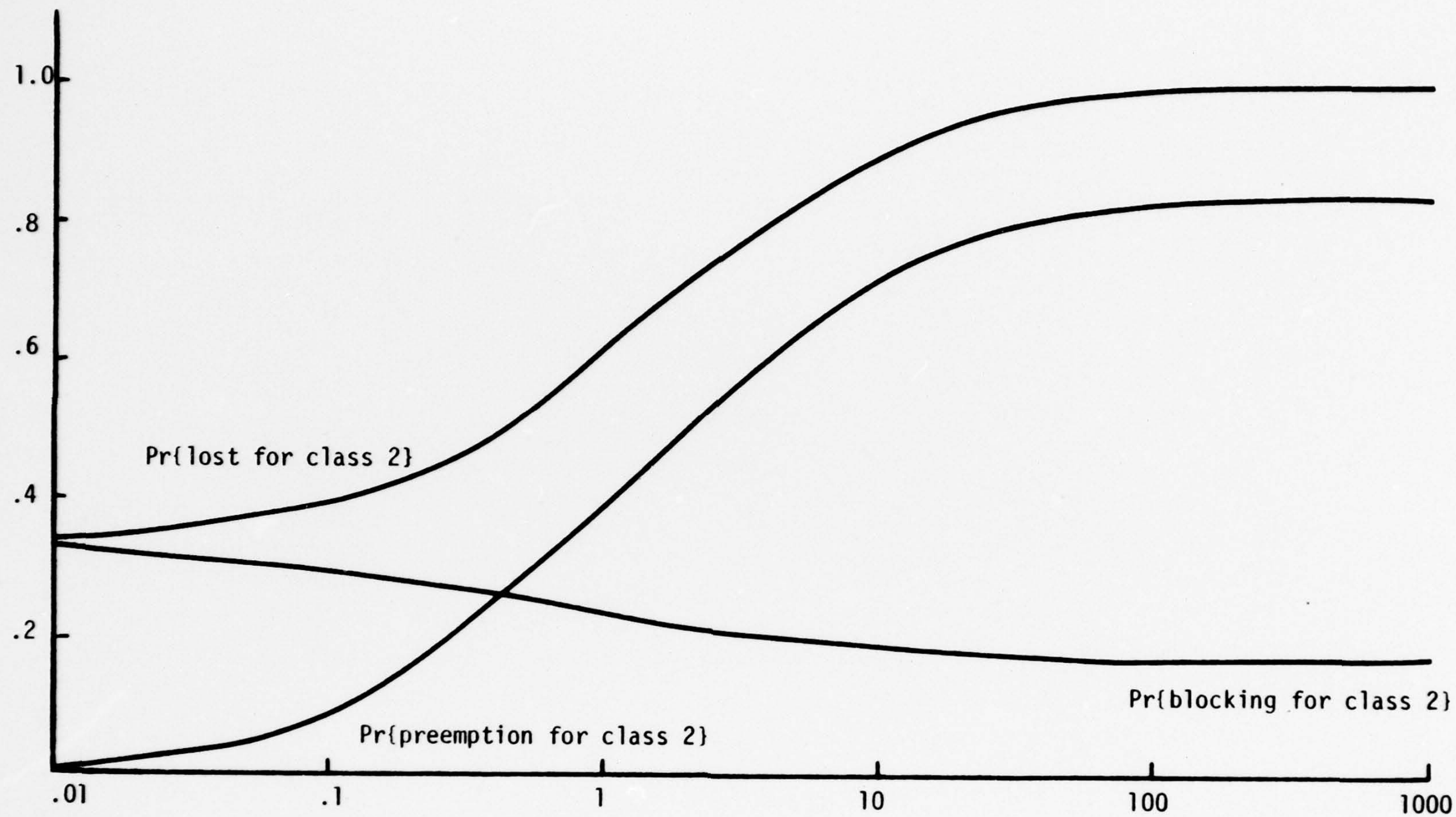

Figure 1. Blocking and Preemption Probabilities for Class 2 ($s=8$, $\rho_1=7$ and $\rho_2=1$)

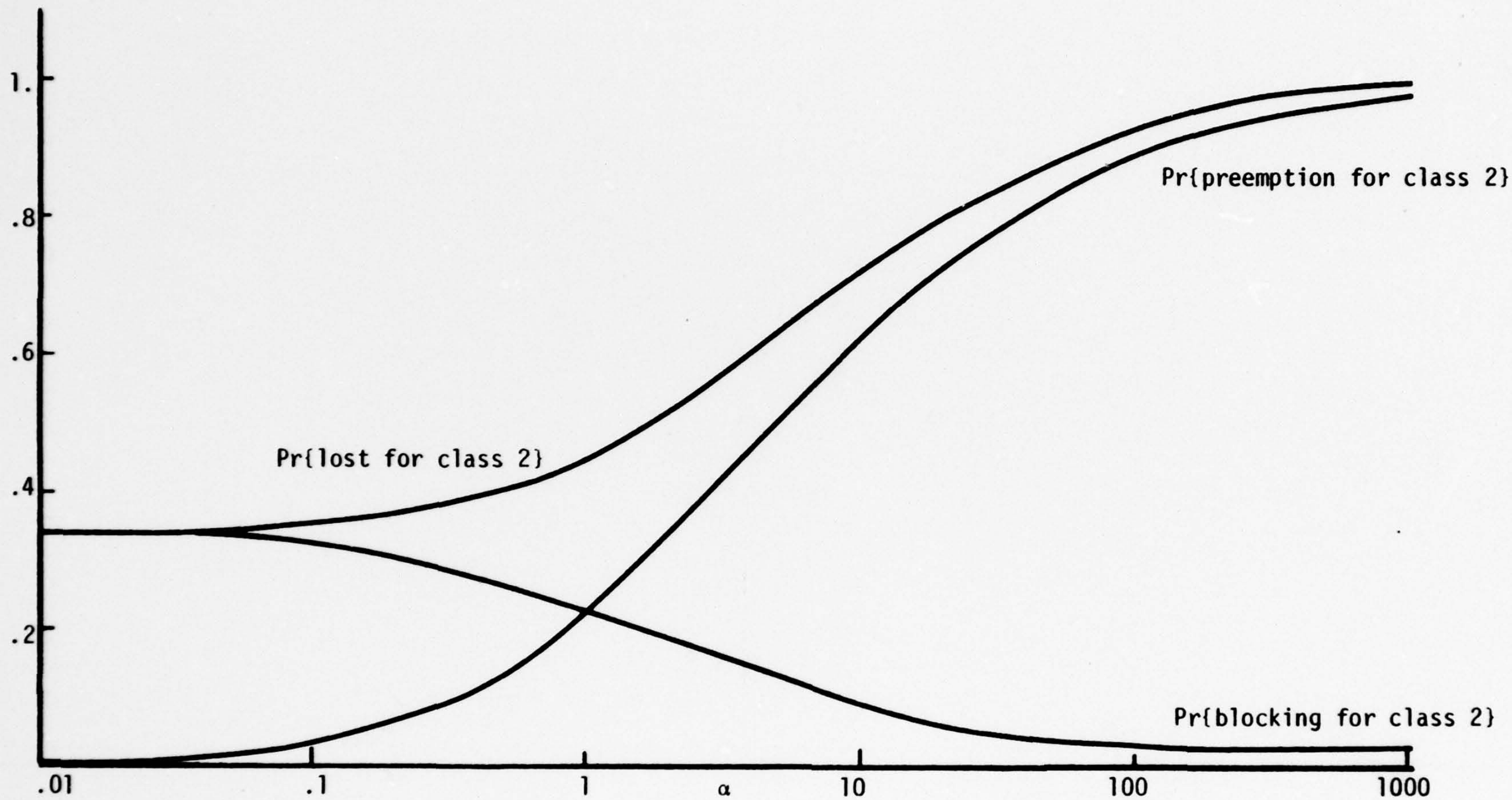

Figure 2. Blocking and Preemption Probabilities for Class 2
($s=8$, $\rho_1=4$ and $\rho_2=4$)

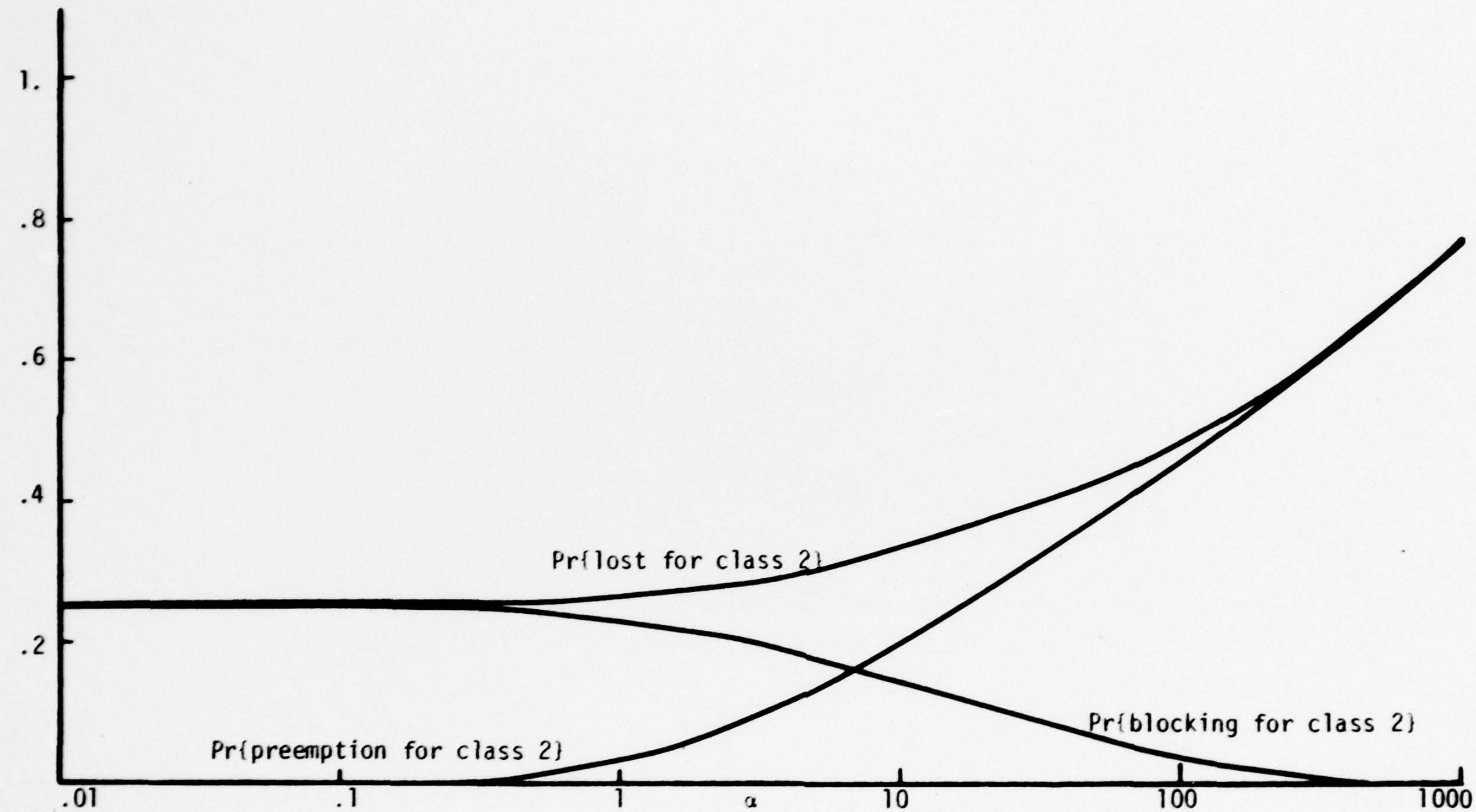

Figure 3. Blocking and Preemption Probabilities for Class 2
($s=8$, $\rho_1=1$ and $\rho_2=7$)

TABLE I. SUMMARY OF RESULTS FOR PB_2, PP_2 AND PL_2 FOR VARYING α

α	PB_2	PP_2	$PL_2 = PB_2+PP_2$
0	$\sum_{i=0}^{s} Pr\{Q_1=i\}E_B(\rho_2,s-i)$	0	$\sum_{i=0}^{s} Pr\{Q_1=i\}E_B(\rho_2,s-i)$
1	$E_B(\rho_1+\rho_2,s)$	$\frac{\rho_1(E_B(\rho_1+\rho_2,s)-E_B(\rho_1,s))}{\rho_2}$	$\frac{(\rho_1+\rho_2)E_B(\rho_1+\rho_2,s)-\rho_1 E_B(\rho_1,s)}{\rho_2}$
∞	$E_B(\rho_1,s)$	$1-E_B(\rho_1,s)$	1

From table I, one sees that the loss probability increases from

$\sum_{i=0}^{s} Pr\{Q_1=i\}E_B(\rho_2,s-i)$ to 1. Thus, as α varies from 0 to ∞, the values of PB_2, PP_2 and PL_2 can vary significantly.

Figure 4 gives a family of curves for increasing values of α. For s=8 three load combinations are considered: A-ρ_1=7, ρ_2=1; B-ρ_1=ρ_2=4; and C-ρ_1=1, ρ_2=7. The solid lines for each combination represent the percentage number of erlangs that are lost, $PLE=(\rho_1 E_B(\rho_1,s)+\rho_2 PL_2)/(\rho_1+\rho_2)$; whereas the dashed lines are the percentage of customers that are lost, PLN, as given by equation (17).

For comparative purposes the loss probability for the case where class 1 calls are not allowed to preempt the class 2 calls is also presented. That is, both classes compete equally for the channels. For this case it turns out (see Cooper [3]) that $PLE=PLN=E_B(\rho_1+\rho_2,s)$. Hence, the line $E_B(8,8)$ represents the case where both classes of calls equally fight for the channels.

Since PLN is the portion of calls that are lost in terms of number of calls and PLE is the portion of calls in terms of erlangs of lost traffic, PLN $\neq$ PLE except when α=1. Several conclusions, which are summarized in table II, can be drawn based on different values of α. If one is interested in the number of lost calls, the system should be run without priorities when $\alpha \leq 1$ (i.e., priority calls have longer holding times), whereas the system should be run with priorities when $\alpha \geq 1$. If one is interested in the total number of lost erlangs the system should be run with priorities when $\alpha \leq 1$ and without priorities when $\alpha \geq 1$. We note as far as class 1 is concerned, the performance of the system is invariant in α.

The B and C curves cross in figure 4 for α=85 and α=350. The reason for the crossing is the particular selection of load, $\rho_1=1$ and $\rho_2=7$. If ρ_1 and ρ_2 were closer to 4 the crossing would not have taken place.

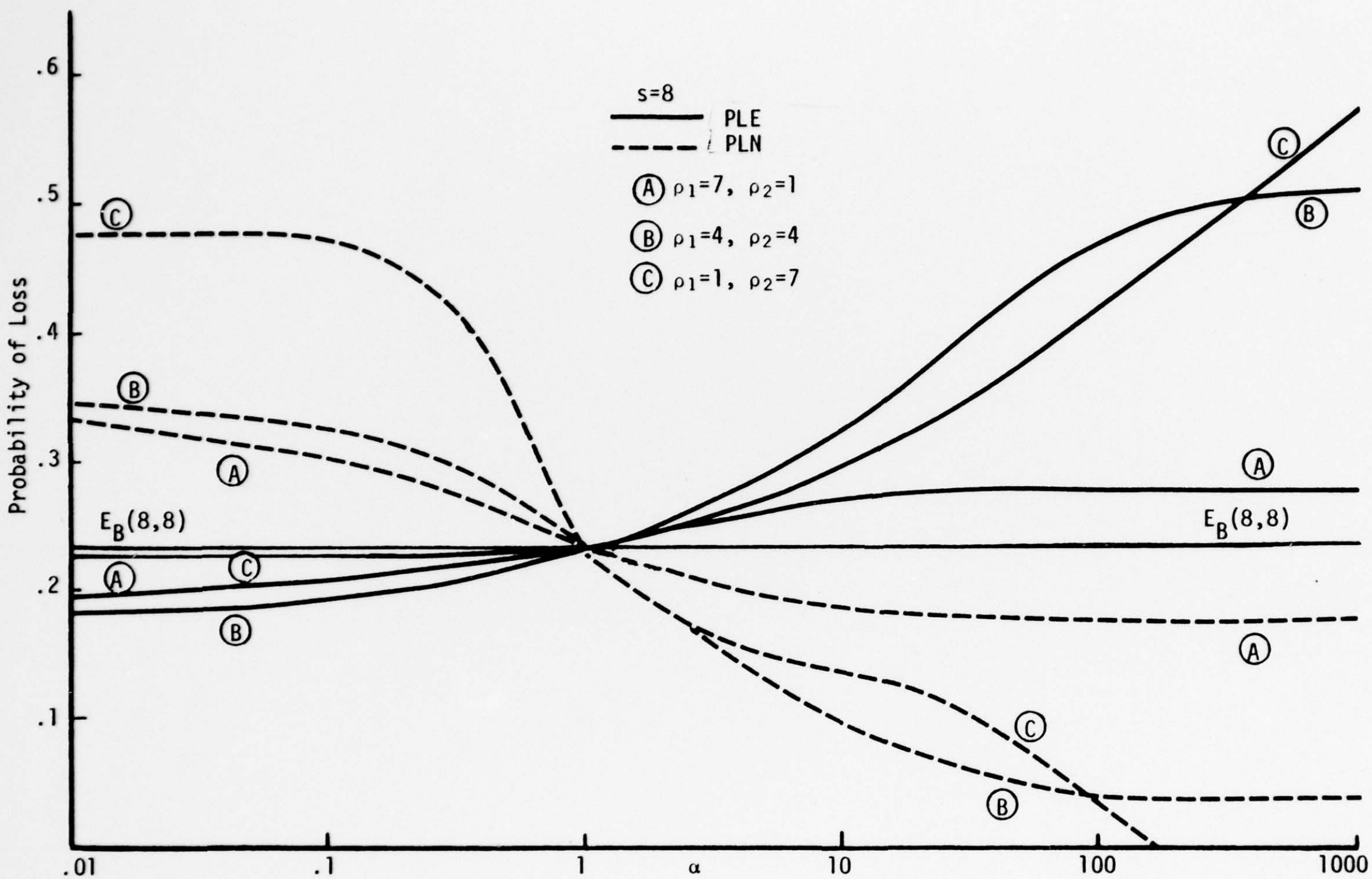

Figure 4. Loss Probability Comparisons

TABLE II. COMPARISONS OF SYSTEM MEASURES OF PERFORMANCE FOR VARYING α

$\alpha<1$	$PLE < E_B(\rho_1+\rho_2,s) < PLN$
$\alpha=1$	$PLE = E_B(\rho_1+\rho_2,s) = PLN$
$\alpha>1$	$PLE > E_B(\rho_1+\rho_2,s) > PLN$

IV. CONCLUSIONS

In this technical note we present an analysis of a priority loss system where the different classes of traffic have different mean holding times. Several general conclusions can be drawn from the results presented in the technical note. First, the non-priority traffic blocking probability, preemption probability and loss probability, significantly vary either indirectly or directly as the ratio of the mean holding time. Second, upper and lower bounds can be given for each of these probabilities, see table I. Third, there are two measures of performance that are considered for both classes of traffic. Depending on one's viewpoint, vastly different results can be obtained.

Although AUTOVON was developed for use of high priority customers, current traffic engineering practices require the trunks to be sized so that all classes of traffic meet a given level of performance. Thus, the results of this technical note will be incorporated into DCEC's voice performance model, which is currently being used to perform some of this traffic engineering.

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[1] DCEC TN 15-78, "An Algorithm for Predicting the Performance of a Voice Network with Priorities," DCEC TN 15-78, June 1978.

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