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by

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$sinh^{-1}$
cos	cos	ch	cosh	arc ch	$cosh^{-1}$
tg	tan	th	tanh	arc th	$tanh^{-1}$
ctg	cot	cth	coth	arc cth	$coth^{-1}$
sec	sec	sch	sech	arc sch	$sech^{-1}$
cosec	csc	csch	csch	arc csch	$csch^{-1}$

Russian	English
rot	curl
lg	log

MEASUREMENT OF RESONANCE VIBRATIONS OF TURBINE BLADES WITH THE ELURA DEVICE

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Resonance has been accepted as the name of the vibrations which develop during the movement of rotor blades in a flow with a field of velocities and pressures which is nonuniform over the circumference and which is caused by flow around any fixed obstacles (shafts of the engine input device, stator blades or nozzle devices, etc.). These vibrations are forced. Their frequency is strictly a multiple of the number of revolutions. The position of the exciting force in space is unchanged, therefore the blade arrives at any particular point of the housing above the wheel with each revolution of the rotor strictly in the same phase. This is why it is necessary to use the method with the ELURA * device for the measurement of vibrations of the resonance type, which differs from the methods for measuring other types of vibrations (cutoff, self-excited vibrations, etc.), the frequency of which is not a multiple of the number of revolutions [1]. [ЭЛУРА - electron-beam device for the recording of amplitude]. The principle of measurement, arrangement of the sensors and the layout for hooking them up to the device, which are described in [1], are the same for all types of vibrations. Thus for measuring the amplitude of shifting of blade tips (Figure 1), a cogged disk with an angle pitch of the teeth equal to the angle pitch of the blades of the investigated wheel is mounted on the rotor. Mounted opposite it is a fixed pulse sensor D_k [$Д_к$], generating an electric pulse when each tooth passes by it. This pulse, arriving in the ELURA device, triggers a generator of saw-tooth voltage GS [ГС], the signal of which shifts the beam on the screen of an electron beam tube in a vertical direction, forming a short line. The second pulse sensor D_0 [$Д_о$], generating one pulse per revolution, synchronizes another generator of saw-tooth voltage OR [ОР], shifting the beam on the horizontal with a frequency equal to the number of revolutions a second. As a result of the operation of the two generators the image on the screen represents a vertical raster, similar to television, with a number of lines equal to the number of rotor blades.

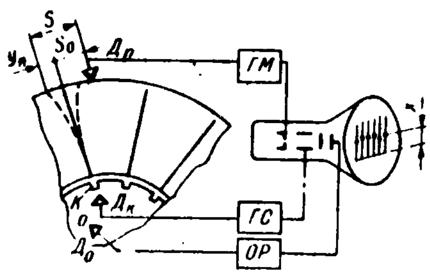

Figure 1. Layout for measuring the amplitudes of shifting of blade tips.

The third pulse sensor D_n [$Д_п$] is mounted in the housing above the wheel. After shaping, its pulses arrive at the tube modulator, illuminating the beam at the moment the blade tip goes by sensor D_p. Therefore with each revolution of the wheel a clear point-mark M appears on each line, the distance x of which from the beginning of the line is proportional to the distance s from the blade tip to the sensor at the moment the tooth passes the sensor D_k

$$x = M_c s, \qquad (1)$$

where M_c - scale of measurement.

If the blade does not vibrate, then with each revolution the mark appears in the same place on the line (Figure 2,a)

$$x = M_c S_0. \tag{2}$$

If there are vibrations the blade approaches the point of installation of the sensors with a certain deviation y_n. Therefore with each revolution the mark appears at a distance

$$x = M_c (S_0 + y_n). \tag{3}$$

In the case of vibrations with a frequency which is not a multiple of the number of revolutions, the deviation of y_n under the sensor with each revolution turns out to be different, and the marks, appearing at different distances from the beginning of the line, form a clear line, the length of which is proportional to the amplitude of vibrations of the blade tip. Having recorded the lengths of the clear lines on the rows with the help of a camera, it is possible thus to determine the amplitudes of vibrations of all the wheel blades (Figure 2, b and c).

In the case of resonance vibrations, as it was noted above, the deviation of y_n at the site of mounting of the sensors is the same with each revolution. Therefore the marks appear at the same distance from the beginning of the row, the magnitude of which depends not only on amplitude, but on phase.

Actually, assume that a blade, excited by a harmonic of force $p=P \cos\omega t$, is vibrating according to the law $y=A \cos(\omega t+\psi)$. Then

$$y_n = A \cos(\varphi_n - \psi), \tag{4}$$

where A - amplitude of vibrations; ϕ_n - phase angle between the point of the housing in which the exciting harmonic reaches a maximum, and the mounting site of the sensor D_n; ψ - phase shift between the harmonic of the exciting force and the shift of the blade tip.

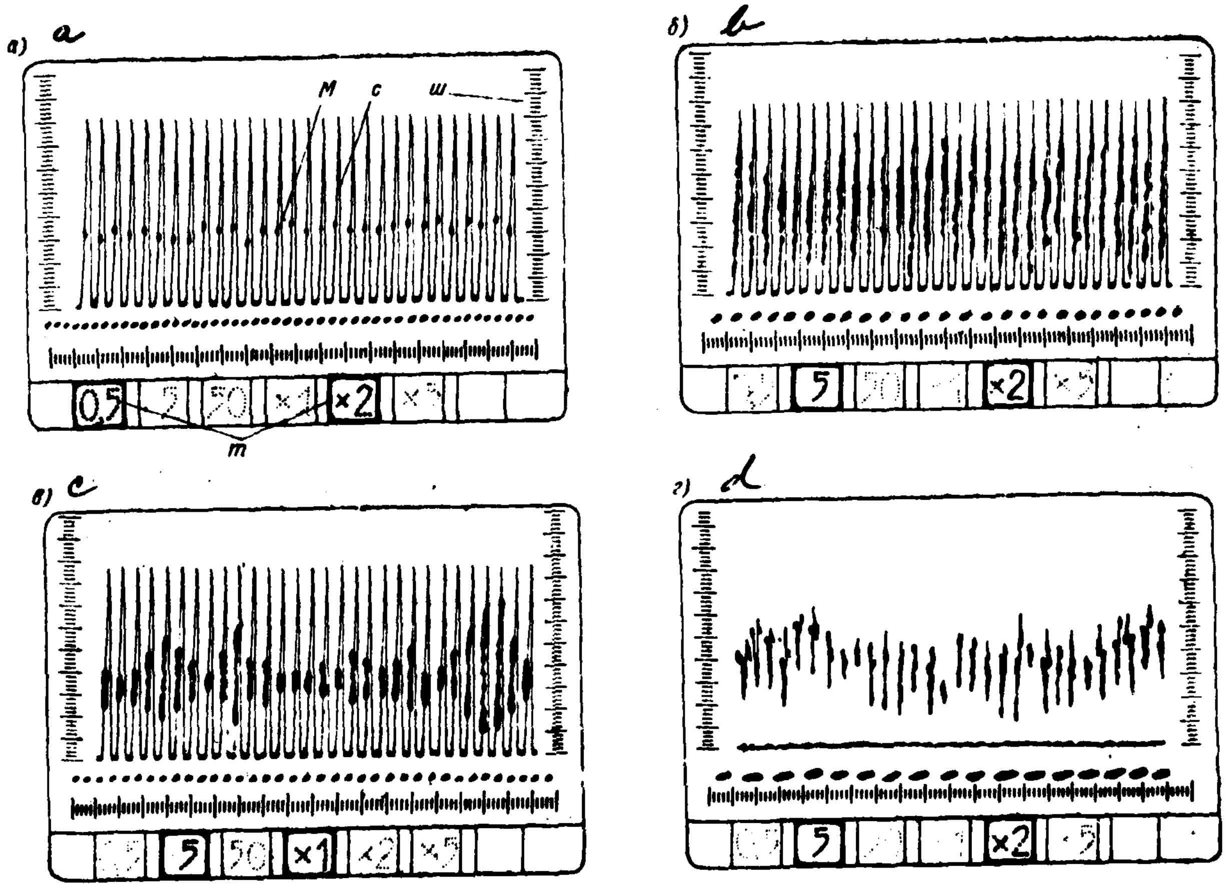

Figure 2. Image on the screen of the ELURA-3 device with constant revolutions of the wheel: a - in the absence of vibrations; b and c - cutoff and self-induced vibrations; d - resonance vibrations, measured "on the passage" by revolutions: М - marks; с - rows; м - scale of measurements; ш - scale for interpretation.

With a change in the number of revolutions, i.e., a change in the frequency of excitation ω, there is a change in amplitude A and phase ψ of vibrations, and, consequently, a change in deviation y_n under the sensor, i.e., the position of the mark on the row.

This process is shown in Figure 3. At a frequency ω_1 the blade vibrates with an amplitude A_1, and the phase shift is equal to ψ_1. The shifting of the blade tip y in different points on the

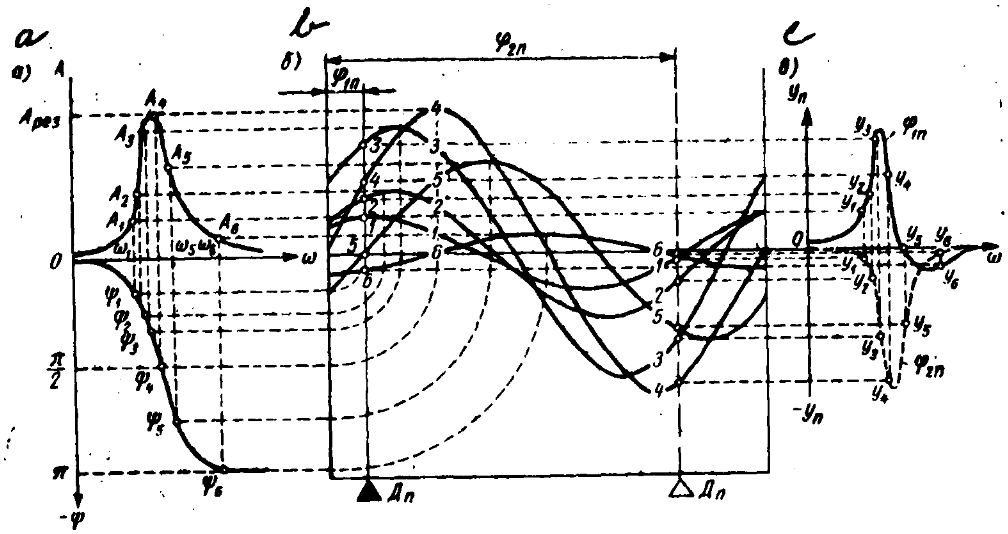

Figure 3. Amplitude- and phase-frequency characteristics of a blade (a); deviation of the blade tip along the circumference over the wheel at different frequencies of excitation (b); deviation of blade tip at sites of mounting of the sensor D_n, designated by ▲ or △ (c).
Numbers 1-6 correspond to an increase of frequency.

circumference of the housing over the wheel at this frequency is depicted in Figure 3,b, curve 1, and the deviation y_n at the site of mounting of the sensor D_n - by point 1. With an increase in the number of revolutions the amplitude and phase shift increase (curves 2 and 3), and there is an increase in the deviation of the blade under the sensor (points 2 and 3). Then, in spite of the increase in amplitude, the deviation y_n is decreased (point 4), and with $\omega=\omega_5$, in spite of the quite high amplitude, the deviation $y_n=0$. Then the deviation changes sign (point 6), and, finally, vibrations practically cease and the deviation again turns out to be equal to zero. The change in the magnitude of y_n depending on the frequency of excitation (revolutions) is shown in Figure 3,c. According to such a law the mark on the row on the screen of the device will shift with a change in revolutions of the rotor.

If the camera shutter is open during revolutions which are close to resonance, and then the number of revolutions is changed until the marks cease to move over the rows, then on the film the trajectories of movement of the marks will be recorded in the form of clear lines as is shown in Figure 2,d. It is evident that the measured deviation y_n and its change by revolutions depends on which site of the housing the sensor D_n is mounted in. Actually, with the installation of the sensor at the site noted in Figure 3,b by the triangle on the right the law of change for y_n has another form, represented by the broken curve in Figure 3,c. However, a further analysis shows that regardless of the site of installation of the sensors (angle ϕ_n), the length of the trajectory of a mark turns out to be practically precisely proportional to the maximum amplitude of vibrations of the blade.

Let us consider a classical case. Assume the blade is a vibrational system with one degree of freedom and damping, proportional to velocity, and the exciting harmonic has a constant amplitude and phase in the investigated range of revolutions. Then the shifting of the blade tip is described by the equation

$$\ddot{y} + \frac{\delta}{\pi}\omega_0\dot{y} + \omega_0^2 y = P\cos\omega t, \tag{5}$$

the known solution of which has the form which was given, for example, in work [2]

$$y = A(\omega)\cos[\omega t - \psi(\omega)], \tag{6}$$

where

$$A(\omega) = \frac{y_{cm}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2+\left(\frac{\delta}{\pi}\frac{\omega}{\omega_0}\right)^2}}; \tag{7}$$

$$\psi(\omega) = \operatorname{arctg}\frac{\frac{\delta}{\pi}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}. \tag{8}$$

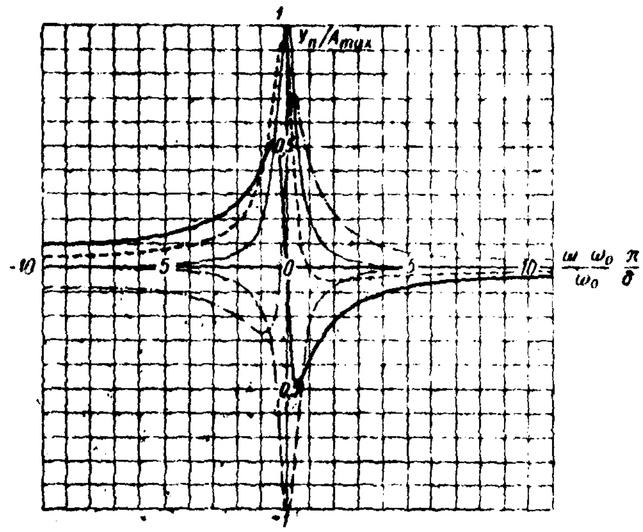

Figure 4. Relative deviations of a blade tip under a sensor when it is installed at different angles

Table

φ_n, град (1)	0	60	90	150	270
Обозначение (2)	- - - -	— — —	——	—·—·—	— · —

Key: (1) deg; (2) Designation.

Here δ - logarithmic decrement of damping; ω_0 - natural frequency; y_{cm} - static shifting of the blade tip under the action of force p.

Formulas (7) and (8) represent analytical expressions of the amplitude- and phase-frequency characteristics of the blade, which we used above (Figure 3,a).

The analytical expression for the curves depicted in Figure 3,c can be obtained by substituting in expression (6) instead of the argument ωt the coordinate of the site of installation of the sensor D_n

$$y_n = A(\omega)\cos[\varphi_n - \psi(\omega)]. \tag{9}$$

This expression is not new. It was obtained already stemming from simple geometric considerations - expression (4). Having substituted in it the values A (ω) and ψ (ω) from formulas (7) and (8), and having made the transformations, we have

$$y_n = y_{cm}\frac{\pi}{\delta}\,\frac{\cos\varphi_n + \dfrac{\frac{\delta}{\pi}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\sin\varphi_n}{\dfrac{\omega}{\omega_0}\left[\dfrac{1-\left(\frac{\omega}{\omega_0}\right)^2}{\frac{\delta}{\pi}\frac{\omega}{\omega_0}} - \dfrac{\frac{\delta}{\pi}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right]}. \qquad (10)$$

Before moving on to an analysis of the expression obtained, let us note that based on numerous experimental data the decrement of damping δ for compressor blades in the case of their vibrations based on the lowest forms is determined mainly by aerodynamic damping and is usually found within the limits of 0.01-0.1, never exceeding a value of 0.3. In the case of high-frequency vibrations, where damping in the material is determining, damping is even less. With such values of δ the amplitude reaches maximum values at a frequency of excitation close to natural ($\omega \approx \omega_0$), and, according to formula (7)

$$A_{max} = y_{cm}\frac{\pi}{\delta}. \qquad (11)$$

Substituting value A_{max} from expression (11) into (10), we obtain

$$\frac{y_n}{A_{max}} = \frac{\cos\varphi_n + \dfrac{\frac{\delta}{\pi}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\sin\varphi_n}{\dfrac{\omega}{\omega_0}\left[\dfrac{1-\left(\frac{\omega}{\omega_0}\right)^2}{\frac{\delta}{\pi}\frac{\omega}{\omega_0}} - \dfrac{\frac{\delta}{\pi}\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right]}. \qquad (12)$$

The results of the calculations * made using formula (12) are given in Figure 4 for several values of the angle ϕ_n. As it should be expected, the dependences of y_n on frequency have a different nature with different angles ϕ_n. However, the amplitude of any curve does not depend on ϕ_n and is equal to unity, i.e., the length of the mark trajectory is proportional to the maximum amplitude of vibrations.

* The calculations were made with several values $\delta \leqslant 0.3$. However, the influence of the magnitude of δ is so small that the curves for all δ practically coincide.

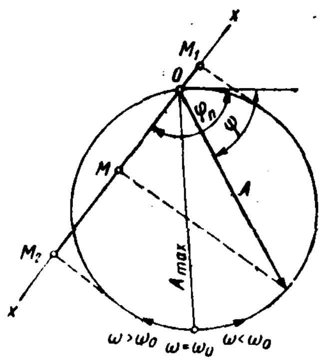

Figure 5. Amplitude-phase characteristic of a vibrational system.

For a stricter proof of this fact let us consider the amplitude-phase characteristic of the blade in a polar system of coordinates (Figure 5), where the length of vector A is equal to the amplitude of vibrations, at a certain frequency ω, and its angle of rotation - to the phase shift ψ. The projection of vector A onto a straight line x-x, made at an angle ϕ_n, then represents the measured deviation y_n. Actually the segment $OM = A \cos(\phi_n - \psi) = y_n$ [Compare with expressions (4) and (9)].

If we consider point O as the position of the mark in the absence of vibrations, then point M is the position of the mark when the blades are vibrating with an amplitude A and phase ψ, when the position of the sensor is determined by the angle ϕ_n.

With a change in the frequency of excitation of the tips the vector A describes an almost closed curve, and the mark M is shifted on the line x-x (row). In order that at any position of the sensor D_n this length would be equal to the maximum amplitude, it is necessary that the projection of the hodograph on any direction would be the same and equal to A_{max}. This condition is satisfied by a circumference with diameter A_{max}.

Let us estimate how close the hodograph is to a circle. We will write its equation in polar coordinates. From formulas (7) and (8) after conversion we obtain

$$A^2 + A\cos\psi - \left(\frac{\pi}{\delta}\sin\psi\right)^2 = 0. \qquad (13)$$

The diameter of this curve, determined as the distance between two mutually perpendicular vectors A, depends on the angle ϕ_n in the following manner:

$$\Big\{\left(\frac{\pi}{\delta}\right)^2 + \frac{1}{2}\Big[1 + \cos^2\varphi_n\Big]\sqrt{1 + 2\left(\frac{\pi}{\delta}\operatorname{tg}\varphi_n\right)^2} - \sin^2\varphi_n\Big]\sqrt{1 + \left(2\frac{\pi}{\delta}\operatorname{ctg}\varphi_n\right)^2}\Big\}^{\frac{1}{2}}. \qquad (14)$$

In expression (14) the sum of all the members, except the first, at any value of ϕ_n does not exceed unity in absolute value. Taking into account that $\delta \leqslant 0.3$, i.e. $(\pi/\delta)^2 > 100$, it is possible to write

$$D \approx y_{cm}\frac{\pi}{\delta} = A_{max}. \qquad (15)$$

The error of such an approximation does not exceed $\pm 0.5\%$. Consequently the hodograph of vector A is actually sufficiently close to a circle with a diameter equal to the maximum amplitude of vibrations.

Up until now we have been talking about an ideal isolated blade. Strictly speaking the wheel represents a bound system of oscillators. In the case of excitation of vibrations in such a system, the amplitude-phase characteristic of each blade will differ from the circumference more, the greater the coefficient of coherence and the difference in the natural frequencies of the blades.

Furthermore, a blade can be excited simultaneously in several forms. And, finally, in contrast to the accepted assumption, the amplitude and phase of the exciting harmonic do not remain strictly constant even in the narrow range of resonance revolutions. All of this lowers the accuracy of measurement and, sometimes, in the case of the necessity of a more detailed investigation, it is required to apply more complex methods, connected with setting up the turbine with several sensors. However, operational experience with a series of ELURA-3 devices shows that the results of measurement of resonance vibrations "on the passage" by revolutions make it possible, as a rule, to determine with sufficient accuracy for practice both the overall vibration strain of the rim and the scattering of amplitudes in a set of blades.

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