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STABILITY AND AMPLIFICATION OF SHOCK WAVES.(U)
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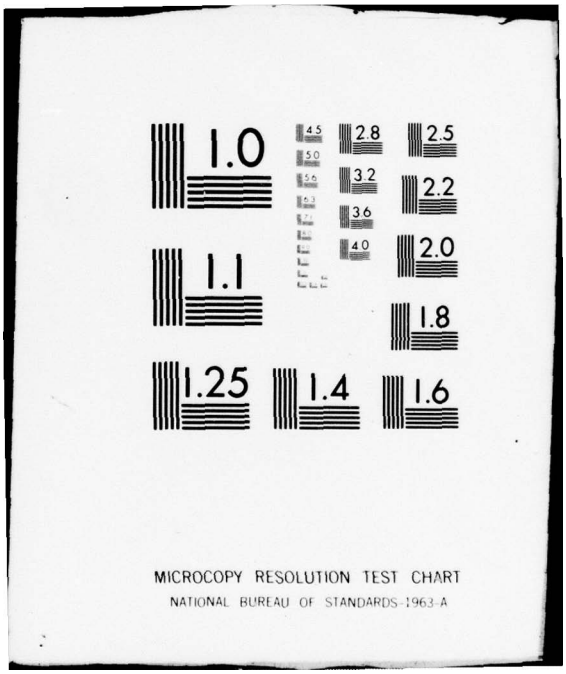
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STABILITY AND AMPLIFICATION OF SHOCK WAVES

PAUL HARRIS

DECEMBER 1978

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
LARGE CALIBER
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The concept of shock wave instability leading to shock pressure amplification is investigated with respect to materials properties and thermodynamic equilibrium. It is concluded that a materials property requirement (negative macroscopic Grüneisen parameter) associated with single-pass pressure amplification is inconsistent with the amplifying medium being in a state of thermodynamic equilibrium. A resulting interpretation, for inert media, is that such amplifying states are

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20. Abstract (continued)

metastable and only to be found in materials which are in the process of undergoing a phase transition. As a consequence, it is argued that practical shock pulse pressure amplification is not a possibility.

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INTRODUCTION

When a step-function shock wave is introduced into a medium, with the result that a multi-step function disturbance propagates within the medium, shock wave physicists refer to that medium as unstable.

A medium can also be unstable with respect to fluctuations (e.g., volume fluctuations) in the absence of an externally induced dynamic strain. This type of instability is called STATE INSTABILITY. In order for a multi-step shock structure to be observed, each step state must be state stable. This report will treat aspects of both shock and state stability and instability.

Shock wave stability studies usually involve transitions between known stable states. This study is interested in the possibility of shock transitions to new and relatively exotic media and is therefore concerned with both types of stability. In the process of this dual approach, it is shown that the requirements for shock instability and state stability (i.e., the longtime existence of a state achieved via a hypothetical shock transition) can be mutually exclusive in terms of materials properties.

One of the most interesting aspects of shock instability theory involves the possibility of shock amplitude amplification. Such amplification can occur (ref. 1) when small amplitude acoustic disturbances, originating behind a propagating shock front, catch up to and reflect from the impedance discontinuity of that front. If materials properties are correctly chosen (corresponding to shock instability criteria), the acoustic reflection amplitude can be larger than the amplitude of the prereflection acoustic signal. Such a phenomenon represents amplification of the acoustic signal and results in amplification of the amplitude of the shock front.

Acoustic signals originating behind the shock front can be obtained via a member of mechanisms:

1. Local exothermic chemical reaction behind the shock front can send out differential compressive disturbances via a thermal expansion coefficient.

2. Grading (i.e. varying) the impedance properties of an impacting projectile will produce a ramp disturbance which can be approximated by an acoustic disturbance riding on a step shock.

3. A second order (isovolumetric without latent heat) phase transition, which results in an elastically harder material, will lead to, for a given strain, a higher stress. Thus, if such transitions occur behind the shock front, the resulting higher stress regions will propagate as acoustic disturbances.

In order for the amplification to result in avalanching, for one incoming acoustic disturbance, the signal which is reflected from the shock front must lose less amplitude at the impacted surface than was gained upon reflection from the shock front. This is essentially a statement about the relative impedance of the driver.

Regardless of whether one has single-pass amplification or amplification with some avalanching, the amplification process represents the possibility of using materials properties to pulse-shape a shock front. This is especially exciting when the shocked medium is inert.

If, in an inert material, a shock of amplitude P with pulse width τ is introduced and pulse shaped to amplitude 2P and pulse width $\tau/2$, then the pulse-shaped shock disturbance would be more efficient than the original shock in initiating detonation in explosives. A commonly used criteria (ref. 2) for detonability is $P^2\tau$ = constant so that the pulse-shaped shock would, in a sense, be more efficient by a factor of two. Other applications exist.

SIMPLE SHOCK WAVE INSTABILITIES

Exceeding the Hugoniot elastic stress limit (ref. 3) or exceeding the stress necessary for a polymorphic (structural) phase transition (ref. 3 and 4) (e.g., α to ε iron at approximately 130 kbars) represent the two most common experimental methods of generating the multi-step shock structure characteristic of a medium exhibiting a shock instability. In such simple cases the P-V and P-u curves appear as shown in figures 1 and 2.

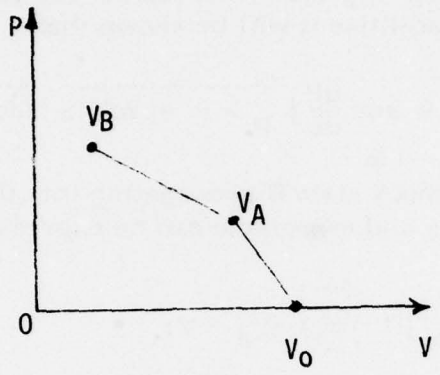

Figure 1. P-V Hugoniot segments for elastic limit or polymorphic transition occurring at V_A.

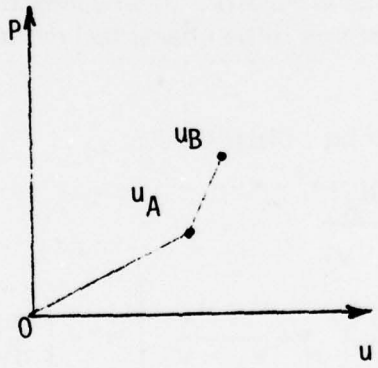

Figure 2. P-u Hugoniot segments for elastic limit or polymorphic transition occurring at V_A.

The important point of figures 1 and 2 is the sign of $\frac{dP}{dV}\Big|_H$ and $\frac{dP}{du}\Big|_H$ (the subscript H denoting Hugoniot*) for each of the various segments. For these simple shock instabilities it will be shown that

$$\frac{dP}{dV}\Big|_H < 0 \text{ and } \frac{dP}{du}\Big|_H > 0 \text{ ALWAYS hold.}$$

For a steady state shock state B propagating into the preshocked state A, the conservation of mass and momentum can be expressed in the jump conditions below:

$$u - u_A = \rho_A (U - u_A)(V_A - V), \tag{1}$$

$$P - P_A = \rho_A (U - u_A)(u - u_A), \tag{2}$$

where only the one-dimensional strain configuration (particle or flow velocity parallel to the shock velocity), and all quantities are measured with respect to a fixed laboratory coordinate system. Additionally, the states A and B are assumed to be in thermodynamic equilibrium so that the only forces present are those associated with the equation of state pressures P_B (deonted by P) and P_A. Viscosity and thermal conductivity, while possibly contributing to the structure of the shock fronts, are assumed to be unimportant at the positions where (P, u) are measured. This allows the P values and their derivatives to be characterized as "Hugoniot" values. U denotes shock velocity.

Substituting Eq. (1) into Eq. (2) gives

$$P - P_A = \frac{(u - u_A)^2}{(V_A - V)} \tag{3}$$

$$\therefore \frac{dP}{dV}\Big|_H = \frac{u - u_A}{V_A - V}\left[2\frac{du}{dV}\Big|_H + \frac{u - u_A}{V_A - V}\right] \approx -\left(\frac{du}{dV}\Big|_H\right)^2 \tag{4}$$

*When the subscript H indicating Hugoniot appears it will indicate a quantity characterizing a state in stable thermodynamic equilibrium, and that the state is achieved via the shock jump conditions (ref. 1).

In the units of Eq. (1) specific volume is written as $V = \rho^{-1}$. Thus, upon rewriting Eq. (1)

$$U - u_A = \frac{u - u_A}{\rho_A (V_A - V)} , \tag{5}$$

$$U = \left[\frac{u - u_A}{V - V_A} \right] V_A + u_A \tag{6}$$

$$\therefore U \approx - V_A \left(\frac{du}{dV} \Big|_H \right) + u_A . \tag{7}$$

Equation (7) and the right hand side of Equation (4) assume that only small excursions from the state A into the state B are being observed. Thus, within the state B, for an elastic constant M_B,

$$dP = - M_B \left(\frac{dV}{V_A} \right) \tag{8}$$

Substituting Eq. (8) into Eq. (7) yields

$$\frac{dP}{du} \Big|_H = \frac{M_B}{u - u_A} \tag{9}$$

If the jump condition of Eq. (2) holds, then $\frac{dP}{dV} \Big|_H < 0$ for small excursions from the state A. Further, from Eq. (2), if $u > u_A$ and $P > P_A$, then $U > u_A$ so that $\frac{dP}{du} \Big|_H > 0$.

If, however, large excursions from a known state A to a new state B are of interest, then the ratio $(u - u_A)/(V - V_A)$ can not be replaced by first-order derivatives, and more physics becomes necessary in order to determine the sign of $\frac{dP}{dV} \Big|_H$ at the new state.

In addition to being able to make a similar statement for the sign of $\frac{dP}{du} \Big|_H$, it should be noted that even small excursions to a new state allow for a sign change if the new state is characterized by ($u < u_A$, $P > P_A$), or ($u > u_A$, $P < P_A$.

Among other considerations, the sign of $\frac{dP}{du}\Big|_H$ is important, because it bears directly upon the possibility of amplification.

AMPLIFICATION

A double wave structure with an infinitesimal amplitude acoustic disturbance overtaking (and reflecting from) the second shock is illustrated schematically in figure 3.

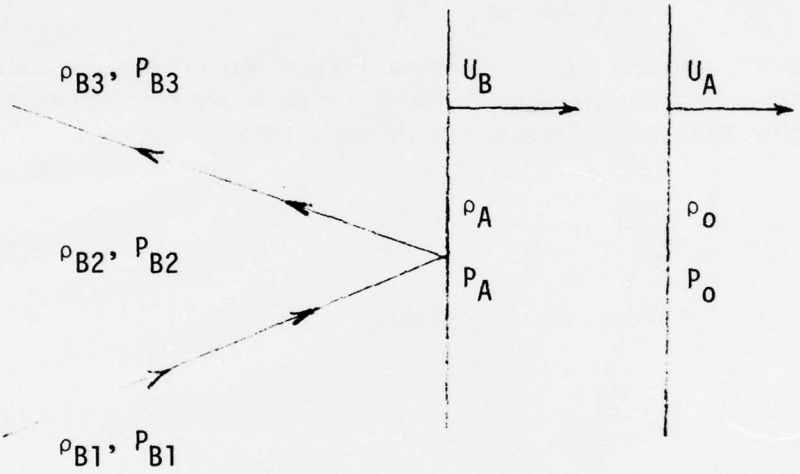

Figure 3. Reflection from a shock front.

Shock A moving into previously undisturbed material (subscript zero) is followed by shock B which places the medium into the Hugoniot state (ρ_{B1}, P_{B1}). A small amplitude (acoustic) compressive disturbance originating somewhere within (ρ_{B1}, P_{B1}) causes a state change to (ρ_{B2}, P_{B2}) and then reflects from the shock front B. The reflected wave causes a state change from (ρ_{B2}, P_{B2}) to (ρ_{B3}, P_{B3}). The states B2 and B3 are only infinitesimally removed from the state B1.

The following discussion of the amplification factor, $(P_{B3} - P_{B2})/(P_{B2} - P_{B1})$, is essentially due to the work of Fowles (ref. 1). For the incoming and reflected disturbances, away from the region of reflection,

$$P_{B2} - P_{B1} = \left(\frac{dP}{du} \Big|_{s+} \right) \left(u_{B2} - u_{B1} \right) , \tag{10}$$

$$P_{B3} - P_{B2} = \left(\frac{dP}{du} \Big|_{s-} \right) \left(u_{B3} - u_{B2} \right) , \tag{11}$$

where the infinitesimal amplitude of the disturbances allows the assumption that the propagation processes are isentropic (entropy changes are third order with respect to first-order pressure changes (ref. 5)). The subscript, s denotes isentropic, and the (+, -) denote "directionality" (see below).

The net change from the state B1 to the state B3 cannot be simply treated as an isentropic process because of a possible energy transfer from the shock front to the reflecting disturbance. It will be argued, however, that the state B3 can be achieved via a Hugoniot process with respect to the state B1.

Consider the infinitesimal amplitude acoustic disturbance to be in pulse form. A short time after the leading edge of the pulse is reflected, the situation is as shown in figure 4.

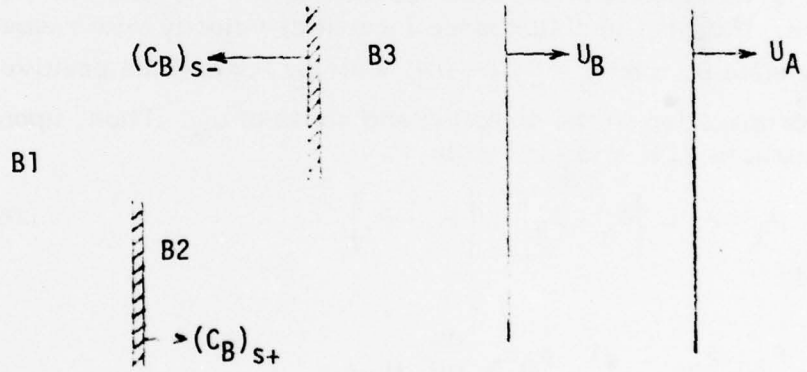

Figure 4. Acoustic pulse reflected from the shock front B. C_B is the isentropic acoustic velocity with respect to the state B.

The argument is simply that the shaded boundaries represent acoustic fronts, and, that if the state B3 is thermodynamically stable, it must be achievable via a dynamic (P, V, T) process. Thus,

$$P_{B3} - P_{B1} = \left(\frac{dP}{du} \Big|_H \right)_B \left(u_{B3} - u_{B1} \right) . \qquad (12)$$

Using equations (10) through (12), with the subscript B understood, forms

$$(u_3 - u_1) - (u_3 - u_2) - (u_2 - u_1) = 0 \qquad (13)$$

which results in

$$\left(\frac{du}{dP} \Big|_H \right)\left(P_3 - P_1 \right) - \left(\frac{du}{dP} \Big|_{s-} \right)\left(P_3 - P_2 \right) - \left(\frac{du}{dP} \Big|_{s+} \right)\left(P_2 - P_1 \right) = 0. \qquad (14)$$

For infinitesimal amplitude disturbances propagating with respect to the preexisting B state, equation (2) becomes

$$dP_{\pm} = \rho_B \left(U_{\pm} - u_B \right) du = \pm \rho_B c_B \, du, \qquad (15)$$

where $U_{\pm}$ is the velocity of the disturbance front in the laboratory reference frame. If c_B is the disturbance (acoustic) velocity with respect to a stationary state B, the $U_{\pm} = \pm c_B + u_B$, with $c_B > 0$ and the positive sign denoting propagation in the direction and sense of u_B. Thus, upon combining equations (14) and (15) with

$$P_3 - P_1 = \left(P_3 - P_2 \right) + \left(P_2 - P_1 \right) \qquad (16)$$

yields

$$\frac{P_3 - P_2}{P_2 - P_1} = \frac{1 - \rho_B c_B \frac{du}{dP} \Big|_H}{1 + \rho_B c_B \frac{du}{dP} \Big|_H} . \qquad (17)$$

It is understood that the B1 state (B state for short) properties are to be used in evaluating the right hand side of equation (17).

If, in equation (17), $\frac{du}{dP}\Big|_H > 0$, as is almost universally the case, then $\{(P_3 - P_2)/(P_2 - P_1)\} < 1$ and amplification upon reflection does not occur*. On the other hand, if $\frac{du}{dP}\Big|_H < 0$, then the right hand side of equation (17) is greater than unity, and amplification upon reflection occurs.

There is a question of the full implication of the gain predicted by equation (17). Whether or not one reflected pulse continues to grow in amplitude depends upon the details of a further reflection from an imagined boundary far to the left of the shock B in figure 4. If, however, a great number of pulses are generated within the bulk of the B1 state, then the question may become not one of single-pass amplification but one of a net increase in pressure behind the shock front B. The net increase in pressure criteria would be the more interesting requirement if the subject were explosives modeled with pulse generation (via local energy release) probability proportional to the pressure.

The net increase in the pressure requirement translates to $\{(P_3 - P_2)/(P_2 - P_1)\} > -1$, which from equation (17) is equivalent to**,

$$\rho_B c_B \left(\frac{du}{dP}\Big|_H \right) > -1. \qquad (18a)$$

clearly equation (18a) overlaps the single-pass amplification requirement

$$\frac{du}{dP}\Big|_H < 0 . \qquad (18b)$$

---

*The reflected pulse, which eventually propagates into the state B1 with relative velocity c_{B-} (fig. 4), has an amplitude relative to the B1 state given by $(P_3 - P_1)$.

**From Eq. (17) we see that $\{(P_3 - P_2)/(P_2 - P_1)\} > -1$ can be written as $\{(1 - X)/(1 + X)\} > -1$ with $x \equiv \rho_B c_B \frac{du}{dP}\Big|_H$. This last inequality is satisfied by

(a) $1 - X > -1 - X$ when $1 + X > 0$. $\therefore X > -1$.
(b) $1 - X < -1 - X$ when $1 + X < 0$. $\therefore$ No solution.

It is of interest at this point to anticipate some of the following discussion in this report and briefly inquire into the material properties consistent with equations (18 a and b). We write the shock velocity - particle velocity relationship (ref. 6 and 7)

$$U = \alpha + \beta u$$

And, in a form consistent with figure 3

$$(U_B - u_A) = \alpha + \beta\ (u_B - u_A)\,. \tag{19}$$

In equation (19), when $u = u_A$, the shock velocity U_B corresponds to an infinitesimal disturbance propagating into the precompressed state A. Thus, $\alpha = c_A$ (the local sound speed in state A). Substituting Eq. (19) into Eq. (2) yields

$$P_H = \rho_A c_A\ (u - u_A) + \rho_A \beta\ (u - u_A)^2 + P_A\,. \tag{20}$$

$$\therefore\ \left.\frac{dP}{du}\right|_H < 0 \rightarrow \rho_A c_A + 2\,\rho_A\ \beta\ (u - u_A) < 0 \tag{21}$$

$$\therefore\ \frac{c_A}{\beta} < -2\ (u - u_A)\,. \tag{22}$$

Thus, for $u > u_A$, β must be less than 0 in order to satisfy Eq. (18 b).

Huang (ref. 8) shows, subject to the restrictions of the Slater (ref. 9) model,

$$\left\{ \frac{\partial}{\partial p} \left[-V \left(\left.\frac{\partial p}{\partial V}\right|_T \right) \right] \right\}_T = 2\gamma + \frac{1}{3}\,, \tag{23}$$

where the subscript T denotes the isothermal derivative, that*

$$\beta = \frac{\gamma_A}{2} + \frac{1}{3} \tag{24}$$

Thus, negative β implies a negative Grüneisen parameter, γ_A, for the state A.

*There are actually a number of different forms in the literature for the relationship between β and γ_A (see references 6 and 7 for examples which differ from Eq. (24)). Each of those forms, however, leads to the conclusion that negative β implies negative γ_A.

The Grüneisen parameter, γ, is defined by

$$\gamma = V \left.\frac{dP}{dE}\right|_V , \tag{25}$$

where E is specific internal energy and has the equivalent thermodynamic definition

$$\gamma = \chi V J / c_p . \tag{26}$$

χ is the thermal expansion coefficient, S the entropy, c_p the specific heat at constant pressure, and J the isentropic bulk modulus,

$$J = - V \left.\frac{\partial P}{\partial V}\right|_S . \tag{27}$$

J is related to the isothermal modulus by (ref. 8)

$$\left.\frac{\partial P}{\partial V}\right|_S = (1 + \gamma \chi T) \left.\frac{\partial P}{\partial V}\right|_T . \tag{28}$$

Because $\left.\frac{\partial P}{\partial V}\right|_T < 0$ is required (ref. 11) for thermodynamic stability (with respect to volume fluctuations), we have $J > 0$. Thus, from Eq. (26), a negative γ is associated with a negative thermal expansion coefficient.

Thermal expansion measurements have found very few materials which exhibit negative χ. The most common examples of this unusual property are silicon (ref. 12 and 13) and indium antimonide (InSb) (ref. 12) at approximately 20°K and atmospheric pressure and fused (vitreous) silica (ref. 12, 14, and 15) (SiO_2) at perhaps room temperature.

Silicon and InSb are not practical shock amplification candidates, as their χ values are only known to be negative at the inconvenient temperature of 20°K (at atmospheric pressure). Similarly, fused SiO_2 has, at best, a small negative χ at room temperature. What is more important than known candidates for practical shock amplification is the simple existence of these materials, and the physics relating to that existence.

The physics picture (ref. 10) associated with a negative χ involves a transverse phonon (acoustic or optic) of relatively low frequency. By analogy with a stretched violin string (ref. 10) compression further lowers the frequency (by reducing the violin string tension). The single mode Grüneisen parameter is defined by

$$\gamma_i = - \frac{\partial \ln \omega_i}{\partial \ln V} = - \frac{V}{\omega_i} \left(\frac{\partial \omega_i}{\partial V} \right) \quad . \tag{29}$$

Clearly, decreasing ω_i in compression (decreasing V) gives negative γ.

As a general rule (ref. 16), phase transitions are associated with $\omega_i \rightarrow 0$ as the transition is approached. The thought then is to utilize the material properties associated with an incipient phase transformation in order to achieve shock amplification. While the idea of so utilizing the phase transformation processes was discussed previously (ref. 17 and 18), the approach as expressed in the combination of Eqs. (17) and (29) represents a new approach to the physics of amplification.

A negative mode, γ_i, of Eq. (29) does not guaranty that the macroscopic γ of Eq. (26) is negative. The relationship between γ and γ_i is complicated; it depends upon the detailed phonon (mode) spectrum and the degree to which each mode is occupied (i.e. the degree to which the various lattice vibrational states are filled). Thus, the possibility of a negative γ_i associated with phase transitions is a hint on where to look for candidate amplification materials, rather than a prescription for choosing a specific material. It is possible for γ_i to be negative and at the same time to have the macroscopic γ positive. In a later section of this report the question of candidate selection will be treated in more detail.

SHOCK STABILITY LIMITS

Analytical studies (ref. 19) have indicated that a shock disturbance which is outside the limits given by

$$-1 \leqslant j^2 \left(\frac{dV}{dP} \Big|_H \right) \leqslant 1 + 2\bar{M} \quad , \tag{30}$$

is subject to an exponential temporal growth* of behind the shock perturbations of the hydrodynamic parameters.

*The analysis assumes an irrotational scalar fluid with the growth restricted to the region immediately behind the shock front.

In Eq. (30), $\bar{M}$ is the local Mach number defined by

$$\bar{M} = \left| \frac{U - u}{c} \right| , \tag{31}$$

where c is the local infinitesimal disturbance sound speed and j is the mass flux relative to a coordinate system attached to the shock front*. It is also true, as shown by Fowles (ref. 1) that those same limits of Eq. (30) correspond to the existence of a shock instability leading to the possibility of a two wave structure. That dual occurrence of exponential growth and shock instability represents an interesting observation in that it begins to connect a double wave structure with the individual mode physics** ultimately responsible for its existence.

Equation (18) shows that $\frac{du}{dP}\Big|_H < 0$ is sufficient for either single pass amplification or bulk pressure growth. Thus, a relationship between $\frac{du}{dP}\Big|_H$ and $\frac{dV}{dP}\Big|_H$ is desired so that the limits of Eq. (30) can be connected with Eq. (18).

From Equations (1) and (2)

$$(u - u_A)^2 = (P - P_A)\ (V_A - V), \tag{32}$$

$$\therefore 2\ (u - u_A)du = (V_A - V)\ dP - (P - P_A)\ dV. \tag{33}$$

But, Equations (1) and (2) can also be combined to give

$$P - P_A = \rho_A^{\,2}\ (U_B - u_A)^2\ (V_A - V) \tag{34}$$

so that j_{BA} becomes $j_{BA}^2 = \frac{P - P_A}{V_A - V}$, (35)

---

*For the unperturbed state B of figure 3, $\bar{M}$ and j become

$$\bar{M}_B = \left| \frac{U_B - u_B}{c_B} \right| , \qquad j_{BA} = \rho_A\ (U_B - u_A).$$

**Swan and Fowles (ref. 19) treat a linearized system so that their results may be thought of as pertaining to a single mode of Fourier analysis.

where it is understood that $(P, V, u) = (P_B, V_B, u_B)$. Upon substituting Eq. (35) into Eq. (32)

$$(u - u_A)^2 = j_{BA}^2 \ (V_A - V)^2 . \tag{36}$$

Thus, Eq. (33) becomes

$$\pm 2\, j_{BA} (V_A - V)\, du \approx (V_A - V)\, dP - (P - P_A)\, dV, \tag{37}$$

or

$$j_{BA} \left(\frac{du}{dP} \Big|_H \right) = \pm \frac{1}{2} \left[1 - j_{BA}^2 \left(\frac{dV}{dP} \Big|_H \right) \right] . \tag{38}$$

Eq. (38) gives the desired connection between $\frac{du}{dP}\Big|_H$ and $\frac{dV}{dP}\Big|_H$.

For a shock moving to the right, $j_{BA} > 0$. Thus, the comparison between Eqs. (38) and (30) leads to the matrix shown in table 1.

Table 1

Correspondence between limit violations* for $\frac{du}{dP}\Big|_H$ and $\frac{dV}{dP}\Big|_H$

Sign in Eq. (38).	$j^2 \frac{dV}{dP}\Big|_H > 1 + 2\bar{M}$	$j^2 \frac{dV}{dP}\Big|_H < -1$
+	$\frac{du}{dP}\Big|_H < -\frac{\bar{M}}{j} < 0$	$\frac{du}{dP}\Big|_H > j^{-1}$
-	$\frac{du}{dP}\Big|_H > \frac{\ddot{M}}{j} > 0$	$\frac{du}{dP}\Big|_H < j^{-1} < 0$

*j and $\bar{M}$ again denote j_{BA} and $\bar{M}_B$, respectively.

The implications of the sign contained in Eq. (38) are easily seen from Eq. (35) and the square root of Eq. (36) with $j_{BA} > 0$. Those implications are shown in table 2.

Table 2

Implications of $\pm$ sign in Eq. (38).

	+ Sign	- Sign
$u > u_A$	$v < v_A$ $P > P_A$	$v > v_A$ $P < P_A$
	normal compression	abnormal rarefaction
$u < u_A$	$v > v_A$ $P < P_A$	$v < v_A$ $P > P_A$
	normal rarefaction (expansion)	abnormal compression

The "normal compression" and "normal rarefaction" labels in table 2 are obvious and correspond to the usually encountered shock wave experiments. The states labeled "abnormal" (corresponding to the minus sign in table 2) are easily seen to be associated with

$$\frac{\Delta u}{\Delta P} = \frac{u_B - u_A}{P_B - P_A} < 0. \tag{39}$$

taking the limit of Eq. (39) as B approaches A shows that the abnormal B states possess (at least) the one sided derivative

$$\left(\frac{du}{dP} \Big|_H \right)_{BA} < 0. \tag{40}$$

Thus, the minus sign choice in Eq. (38) presupposes the physics necessary for amplification as discussed in Eq. (18), specifically in Eq. (18 b). Thus, the minus sign solutions in Eq. (38) will be neglected.*

The (P, u) and (P, V) plane possibilities corresponding to the plus sign choice are illustrated in figures 5 and 6.

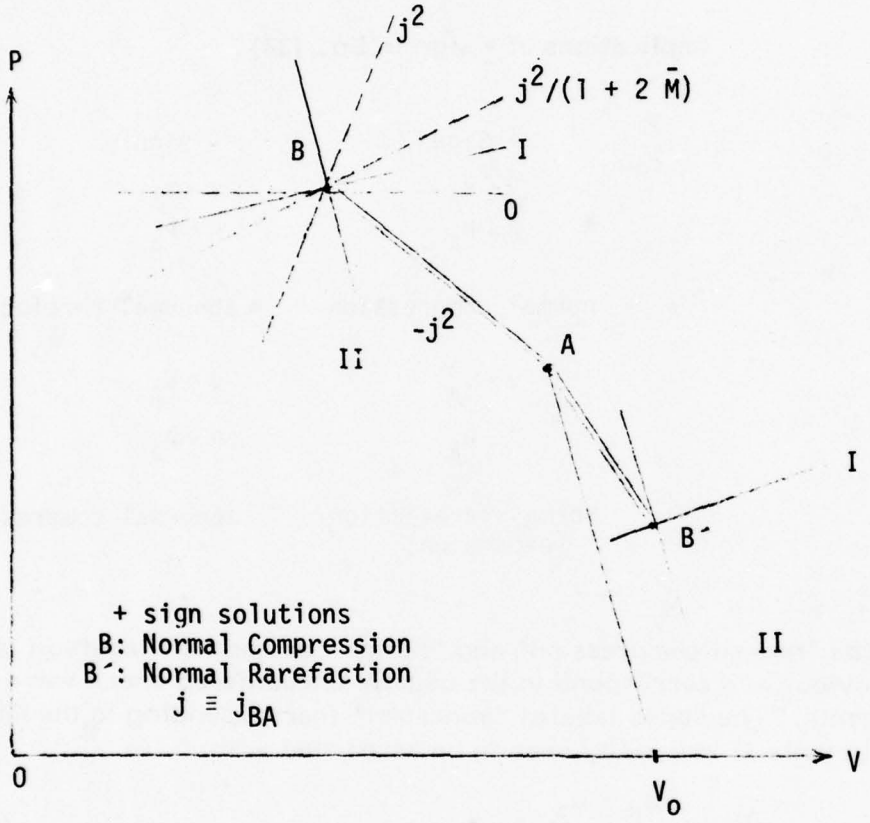

Figure 5. (P,V) plane possibilities corresponding to limit violations. The chords (of slope -j) connecting the states A, B, and B' are drawn with double solid lines. The solid lines through B, B' states represent limit violating derivatives (slopes) from Eq. (30). Lines I and II correspond to lines I and II, respectively, in figure 6.

*By concentrating on the plus sign solutions of Eq. (38) our investigation will be restricted to positive chord B states (i.e., $\frac{\Delta u}{\Delta P} > 0$) which concurrently admit negative local $\frac{du}{dP}\Big|_H$ derivatives (and thus result in amplification).

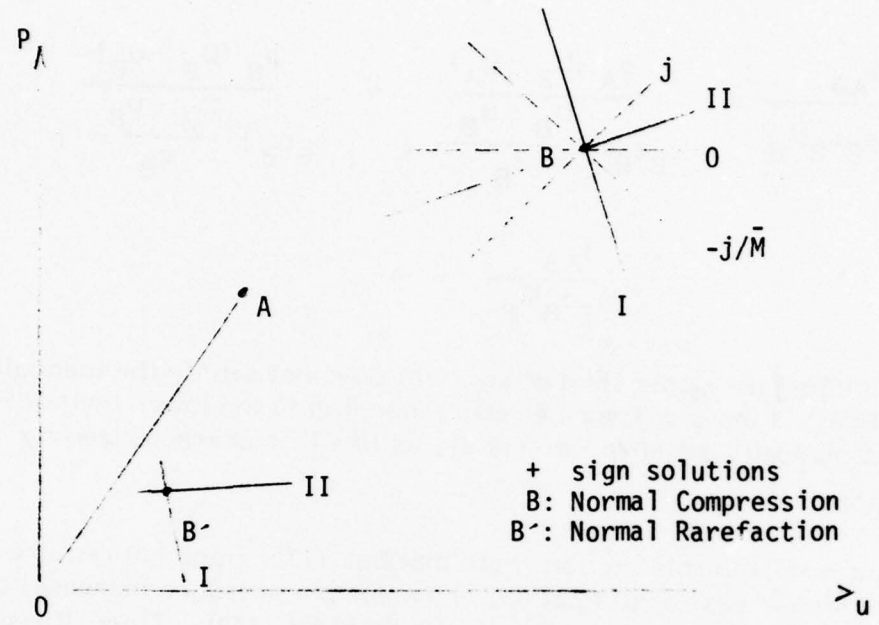

Figure 6. (P, u) plane possibilities corresponding to limit violations. The solid lines through the B, B' states represent limit violating derivatives from Eqs. (30) and (38). Lines I and II correspond to lines I and II, respectively, in figure 5.

While the slope I states obviously satisfy the $\frac{du}{dP}\Big|_H < 0$ requirement of Eq. (18 b) for single-pass amplification, the situation with respect to the gross pressure generation requirement of Eq. (18 a) necessitates analysis.

In order for line I of figure 6 to fit within the inequality of Eq. (18 a) it is necessary that $(j_{AB}/\rho_B c_B \bar{M}_B) < 1$. But*,

*While Eq. (39) predicts $(j_{AB}/\rho_B c_B \bar{M}_B) < 0$ (and thus also < 1) for a shock moving to the left, it is not obvious that such behavior is to be expected. The change in sign for a wave moving to the left is a consequence of the absolute value operator contained in the definition of $\bar{M}_B$. That absolute value appears in the derivation of Eq. (30) when (see Eq. (38) of reference 19) an equation of the form $x < 1$ is replaced by $\left| x\, e^{i\theta} \right| < 1$, and consequently may be forcing a directionality related sign difficulty.

$$\frac{j_{AB}}{\rho_B c_B \bar{M}_B} = \frac{\rho_A (U_B - u_A)}{\rho_B c_B \left| \frac{U_B - u_B}{c_B} \right|} = \frac{\rho_B (U_B - u_B)}{\rho_B c_B \left| \frac{U_B - u_B}{c_B} \right|} .$$

$$\therefore \quad \frac{j_{AB}}{\rho_B c_B \bar{M}_B} = 1$$

Thus, violating the upper limit of Eq. (30) does not satisfy the inequality of Eq. (18 a). Line II of figure 6, corresponding to the lower limit of Eq. (30), automatically satisfies Eq. (18 a), as line II is characterized by $\frac{dP}{du}\Big|_H > 0$.

In summarizing this section, note that Eq. (17), Hugoniot requirements for single-pass amplification, or for simple pressure increase, are each separately satisfied by one of the limits of Eq. (30). Thus, there is agreement between the detailed analytic derivation (ref. 19) leading to Eq. (30) and the less detailed, but conceputally satifying picture associated with Eq. (18 a and b).

THERMODYNAMIC STABILITY

The single-pass amplification requirement, $\frac{dP}{du}\Big|_H < 0$, corresponds to $\frac{dP}{dV}\Big|_H > 0$. But, $\frac{\partial P}{\partial V}\Big|_T < 0$ is required (ref. 11) for thermodynamic stability with respect to volume fluctuations. It is, therefore, necessary to investigate the relationship between $\frac{\partial P}{\partial V}\Big|_T$ and $\frac{dP}{dV}\Big|_H$.

Should both $\frac{\partial P}{\partial V}\Big|_T < 0$ and $\frac{dP}{dV}\Big|_H > 0$ hold simultaneously, the implication would be the existence of a stable shock state B which is capable of amplifying behind the shock signals WHICH COULD ORIGINATE VIA VOLUME FLUCTUATIONS. Thus, we have a contradiction and, in turn, the implication that such a state B (in figs. 1. 2, and 3) could not exist (in the steady state sense), and that a new stable state C, characterized by $\frac{\partial P}{\partial V}\Big|_T < 0$ and $\frac{dP}{dV}\Big|_H < 0$, would be the final result. Figure 7 illustrates the expected possibilities in the (P, V) plane.

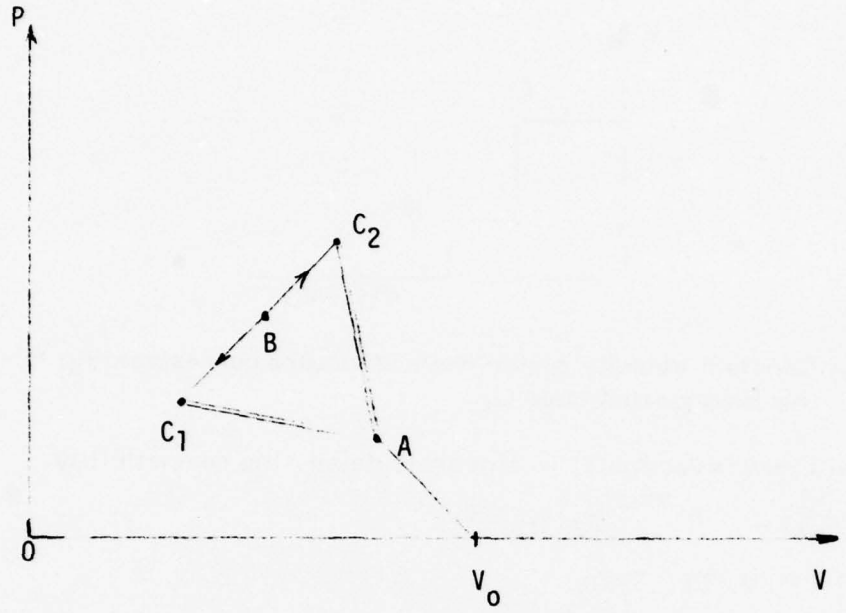

Figure 7. Possible final stable states C_1 or C_2 achievable via $\frac{dP}{dV}\Big|_H > 0$ with respect to the unstable state B.

If, as illustrated by state C_2 in the exaggerated figure 7, the cord C_2A is steeper than chord AV_0, then the resulting constant velocity piston induced shock structure is as shown in figure 8. The shock structure for the exaggerated C_1 state is illustrated in figure 9. Both C states are characterized by $\frac{dP}{dV}\Big|_H < 0$.

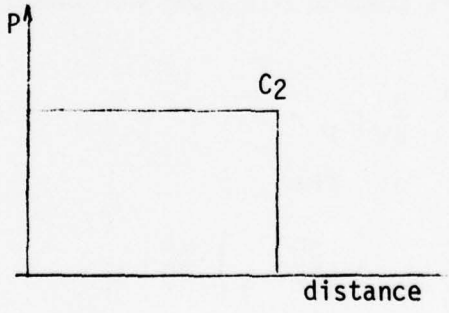

Figure 8. Constant velocity piston wave structure corresponding to the exaggerated state C_2.

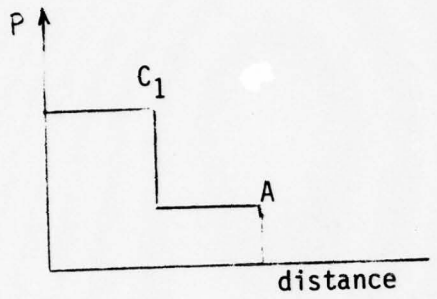

Figure 9. Constant velocity piston wave structure corresponding to the exaggerated state C_1.

We will now investigate, in algebraic detail, the compatibility between $\frac{\partial P}{\partial V}\Big|_T < 0$ and $\frac{dP}{dV}\Big|_H > 0$.

Let $P = P(V, T)$. Thus,

$$\frac{dP}{dV}\Big|_H = \frac{\partial P}{\partial V}\Big|_T + \frac{\partial P}{\partial T}\Big|_V \frac{dT}{dV}\Big|_H . \tag{40}$$

For a quasi-static process within the state B

$$TdS = dE + PdV. \tag{41}$$

Rewriting Eq. (41) as

$$T \frac{\partial S}{\partial T}\Big|_P \left(dT\Big|_p\right) = \left(dE\Big|_p\right) + P\left(dV\Big|_p\right),$$

where S, E, and V are specific (i.e., per unit mass) quantities, immediately leads to

$$c_p \frac{\partial T}{\partial V}\Big|_P = \frac{\partial E}{\partial V}\Big|_P + P. \tag{42}$$

Now, let $E = E(P, V)$. Then,

$$\frac{dE}{dV}\Big|_H = \frac{\partial E}{\partial V}\Big|_P + \left(\frac{\partial E}{\partial P}\Big|_V\right) \frac{dP}{dV}\Big|_H . \tag{43}$$

Combining Eqs. (42), (43) and (25), yields

$$\frac{dE}{dV}\Big|_H = \left[C_p \left(\frac{\partial T}{\partial V}\Big|_P\right) - P\right] + \frac{V}{\gamma}\left(\frac{dP}{dV}\Big|_H\right) . \qquad (44)$$

Let $T = T(P, V)$, then

$$\frac{dT}{dV}\Big|_H = \frac{\partial T}{\partial V}\Big|_P + \left(\frac{\partial T}{\partial P}\Big|_V\right)\frac{dP}{dV}\Big|_H . \qquad (45)$$

Because $\frac{dE}{dV}\Big|_H$ is a known function of the shock structure (see below), Eqs. (44) and (45) can be utilized to eliminate $\frac{dT}{dV}\Big|_H$ from Eq. (40). $\frac{dP}{dV}\Big|_H$ is then given in terms of $\frac{\partial P}{\partial V}\Big|_T$ and other thermodynamic derivatives and the observational parameters associated with the shock state. Combining Eqs. (44) and (45) gives

$$\frac{dT}{dV}\Big|_H = \frac{1}{c_p}\left(\frac{dE}{dV}\Big|_H\right) - \left(\frac{\partial T}{\partial P}\Big|_V + \frac{V}{c_p\gamma}\right)\frac{dP}{dV}\Big|_H + \frac{P}{c_p} . \qquad (46)$$

Upon substituting Eq. (46) into Eq. (40),

$$\left(2 + \frac{V}{c_p\gamma}\frac{\partial P}{\partial T}\Big|_V\right)\frac{dP}{dV}\Big|_H =$$

$$\frac{\partial P}{\partial V}\Big|_T + \frac{1}{c_p}\left(\frac{\partial P}{\partial T}\Big|_V\right)\frac{dE}{dV}\Big|_H + \frac{P}{c_p}\left(\frac{\partial P}{\partial T}\Big|_V\right) . \qquad (47)$$

The Hugoniot energy relation, for the state B propagating into the state A, has the form (ref. 1, 7)

$$E - E_A = \frac{1}{2}(P + P_A)(V_A - V) . \qquad (48)$$

$$\therefore \frac{dE}{dV}\Big|_H = -\frac{P + P_A}{2} + \frac{V_A + V}{2}\left(\frac{dP}{dV}\Big|_H\right) . \qquad (49)$$

Substituting Eq. (49) into Eq. (47) yields, after minor rearrangement,

$$\left.\frac{dP}{dV}\right|_H = \left.\frac{\partial P}{\partial V}\right|_T \left[\frac{c_p\left(\left.\frac{\partial P}{\partial T}\right|_V\right)^{-1} + \frac{P - P_A}{2}\left(\left.\frac{\partial P}{\partial V}\right|_T\right)^{-1}}{c_p\left(\left.\frac{\partial P}{\partial T}\right|_V\right)^{-1} + \frac{V}{\gamma} - \left(\frac{V_A - V}{2}\right)}\right] \quad (50)$$

Eq. (50) is the desired expression for the relationship between $\left.\frac{dP}{dV}\right|_H$ and $\left.\frac{\partial P}{\partial V}\right|_T$. With the exception of (P_A and V_A), all quantities on the right hand side of Eq. (50) are to be evaluated at the state B.

A sign analysis of Eq. (50) is performed next, assuming $\left.\frac{\partial P}{\partial V}\right|_T < 0$ for thermodynamic stability. From reference 20

$$\left(\left.\frac{\partial P}{\partial T}\right|_V\right)\left.\frac{\partial T}{\partial V}\right|_P = -\left.\frac{\partial P}{\partial V}\right|_T, \quad (51)$$

and Eqs. (26), (27), and (28), it follows that

$$c_p\left(\left.\frac{\partial P}{\partial T}\right|_V\right)^{-1} = -\frac{\chi V J}{\gamma}\left[\frac{\left.\frac{\partial T}{\partial V}\right|_P}{\left.\frac{\partial P}{\partial V}\right|_T}\right] = -\frac{J}{\gamma}\left(\left.\frac{\partial P}{\partial V}\right|_T\right)^{-1}. \quad (52)$$

$$\therefore c_p\left(\left.\frac{\partial P}{\partial T}\right|_V\right)^{-1} = \frac{V(1 + \gamma\chi T)}{\gamma} \quad (53)$$

Substituting Eq. (53) into Eq. (50) for ΔP and ΔV, each being small and of arbitrary sign, yields

$$\therefore \left.\frac{dP}{dV}\right|_H \approx \left.\frac{\partial P}{\partial V}\right|_T \left[\frac{1 + \gamma\chi T}{2 + \gamma\chi T}\right] . \quad (54)$$

Thus, for small ΔP and ΔV, $\left.\frac{dP}{dV}\right|_H < 0$ if $\left.\frac{\partial P}{\partial V}\right|_T < 0$ and $|\gamma\chi T| < 1$ REGARDLESS of the sign of γ. Thus, thermodynamic stability requires $\left.\frac{dP}{dV}\right|_H < 0$ regardless of the sign of γ for small pressure and volume excursions from the stable state A.

It is possible to take the sign analysis a step further. By imposing $J > 0$ as a requirement*, and employing the extreme right hand side of Eq. (52) in Eq. (50), the numerator within the square brackets of Eq. (50) will be negative if $\gamma < 0$, $P > P_A$, and $\frac{\partial P}{\partial V}\Big|_T < 0$. Since the denomenator within the square brackets of Eq. (50) can be written

$$-\frac{J}{\gamma}\left(\frac{\partial P}{\partial V}\Big|_T\right)^{-1} - \left[\frac{V_A - V\,(1 + 2\gamma^{-1})}{2}\right],$$

certainly $\frac{dP}{dV}\Big|_H < 0$ will hold if $\left[V_A - V\,(1 + \frac{2}{\gamma})\right] > 0$. This last inequality implies

$$\frac{1}{\gamma} < \frac{V_A - V}{2V} \quad . \tag{55}$$

But, we have already chosen $\gamma<0$ so that Eq. (55) becomes

$$|\gamma| > \frac{2V}{V_A - V} \, , \tag{56}$$

which requires γ to be large and negative.

The above paragraph shows that even for a large negative γ, $\frac{dP}{dV}\Big|_H < 0$ if $\frac{\partial P}{\partial V}\Big|_T < 0$ holds. Thus, the requirement of thermodynamic stability (with respect to volume fluctuations) appears to restrict $\frac{dP}{dV}\Big|_H$ to values such that the upper inequality of Eq. (30) is not violated. The discussion associated with Eq. (14) showed that $\frac{dP}{du}\Big|_H < 0$ is associated with $\gamma< 0$. The implication is clear; $\frac{dP}{dV}\Big|_H < 0$ is inconsistent with a state of thermodynamic equilibrium. Consequently, a state characterized by $\frac{dP}{dV}\Big|_H < 0$ is expected to be metastable.

*In order that a real local adiabatic sound velocity given by $c^2 = -V^2 \frac{\partial P}{\partial V}\Big|_S$ exist.

What does all of this mean? If the sign analyses were correct, it would imply that $\gamma < 0$ materials do not exist in equilibrium, and that a medium capable of supporting single pass amplification must be formed into a metastable state (with some finite lifetime) before that single-pass pressure amplification could be expected to occur. Correspondingly, if one were to identify the $\frac{dP}{dV}\Big|_H > 0$ limit violation of Eq. (30) with a detonation (as does Fowles, reference 1), then the implication would be that there is little utility in studying the physics of explosives which are in an equilibrium state A. The physical properties of the $\gamma < 0$ non-equilibrium state B must be so radically removed from the state A that in effect one would be dealing with two radically different (isomorphic) materials.

However, it is known that states of macroscopic $\gamma < 0$ (e.g., Si and InSb at low temperatures) exist in apparent thermodynamic equilibrium! This is a puzzling paradox.

MATERIAL SELECTION

In this section, the possibility of finding non-energetic materials with $\gamma < 0$ in the vicinity of room temperature is investigated. The association between an individual mode ($\gamma_i < 0$) and phase transitions was already briefly discussed. Now the γ-phase transition relationship will be considered from a macroscopic point of view.

Two classes of phase transitions can be identified as depending upon V vs T on either side of the transition. The situations are illustrated in figures 10 and 11, along with the corresponding thermal expansion coefficients χ.

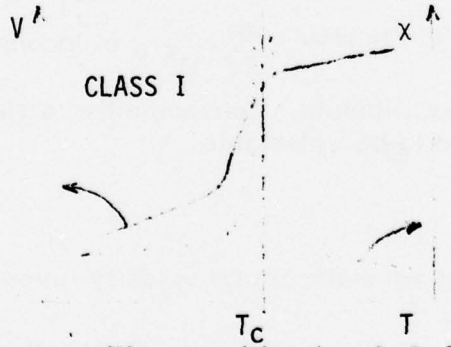

Figure 10. Phase transition resulting in $\chi > 0$. T_c denotes the transition temperature. The thermal expansion coefficient is displayed by the dashed curve.

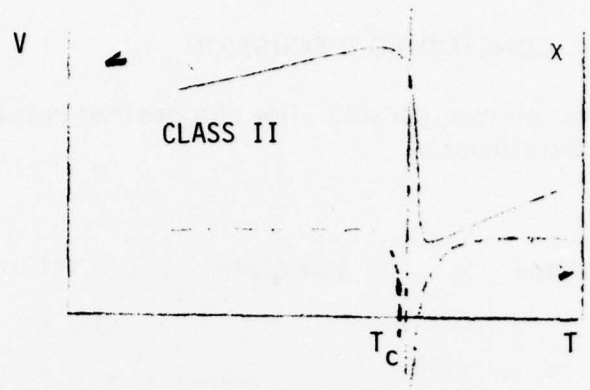

Figure 11. Phase transition resulting in $\chi < 0$. T_c denotes the transition temperature. The thermal expansion coefficient is displayed by the dashed curve.

Ti_xNi_{1-x}, depending upon heat treatment and composition, is capable of exhibiting either Class I or Class II behavior in the process of undergoing the TiNi (II $\leftrightarrow$ III) phase transition (ref. 21). Class I behavior is exhibited (ref. 22) by approximately stoichiometric TiNi and Class II behavior by (ref. 22) TiNi compounds containing approximately 55 percent nickel.

The important observation about the possibly negative macroscopic γ associated with the Class II behavior illustrated in figure 11 is that $\gamma < 0$ is to be found only in a narrow temperature band surrounding T_c. That observation supports the tentative conclusions (see the discussion associated with the inequality of Eq. (56)) in that a $\gamma < 0$ state implies metastability rather than a long-lived state in thermodynamic equilibrium.

Thus, a Class II material could not be used for continuous single-pass amplification (in the sense of $\frac{dP}{du}\Big|_H < 0$), because volume fluctuations behind the shock front B would tend to trigger the phase transition to a $\gamma > 0$ state on one side of T_c.

CONCLUDING DISCUSSION

Figure 12 illustrates an exaggerated view of a desired result of single-pass pressure amplification.

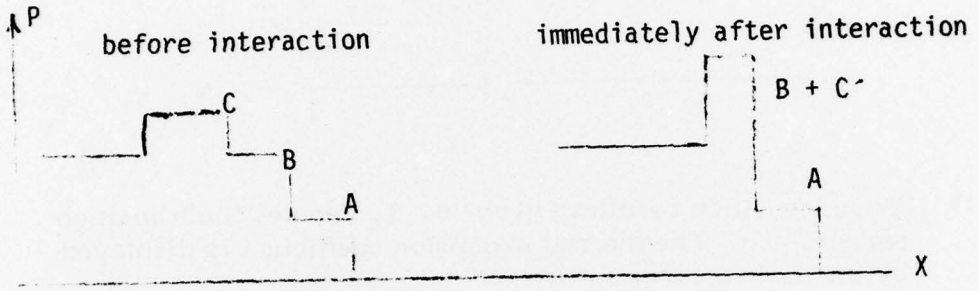

Figure 12. A finite amplitude pulse C introduced into the state B and overtakes the shock front B. Upon total reflection from the front B its amplitude is increased, (C'). The width of C' is shown as less than the width of C in order to conserve strain energy.

Where it is possible to extract the pulse C' of figure 12 and introduce it into an explosive, one would have a more efficient means of explosive initiation in the sense of the $P^2\tau$ criteria (ref. 2), as discussed earlier.

Unfortunately, previous arguments appear to require that an inert amplifying state B (macroscopic $\gamma < 0$) be metastable and possibly associated with an occurring phase transition (microscopic, or mode, $\gamma_i < 0$). Because the state B is metastable, it cannot be maintained while waiting for the introduction of the pulse C. Thus, practical single-pass pressure amplification does not seem to be possible.

The above argument on inert state inaccessibility due to metastability does not hold for an energetic (e.g. explosive) medium*. Indeed, it is known (ref. 23) that an ongoing energetic chemical reaction can serve as an amplifying medium for acoustic signals. Such an effect can be thought of as being allowed basically because thermal energy (release) per unit

*Equation (48) must be modified to take energy release into account in treating energetic materials.

volume is dimensionally equivalent to pressure (increase). Thus, if propagation conditions (e.g. Chapman - Jouguet conditions (ref. 24)) are fulfilled, an induced metastability leading to energy release (pressure increase) finds itself in phase with an amplified propagating disturbance.

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