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SENSITIVITY ANALYSIS IN LARGE SCALE MULTI-OBJECTIVE SYSTEMS.(U)

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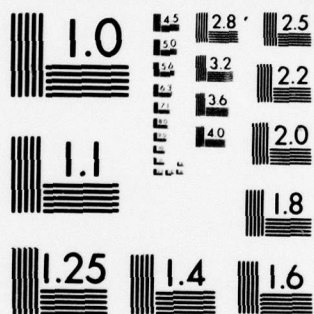
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allows the sensitivity problem to be solved through the use of mathematical programming techniques. The best of these techniques for the general array design case is shown to be Nonlinear Goal Programming. The sensitivity values for five planar symmetric array designs are determined and compared.

The results of this comparison indicate the potential of the proposed measure and solution technique for the development of a fast and inexpensive method for array design sensitivity analysis. Particularly, the resulting sensitivity values are free from the a priori assumptions and high costs inherent in simulation. Furthermore, the method is based on a general framework which is applicable to the analysis of several array design factors including size, geometry, pattern, and component reliability. The method may also be extended to multi-objective design problems outside the area of array design.

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ABSTRACT

No matter what method is employed in the programming of a transducer array, in practice, the performance predicted by the model will not be achieved due to error in construction and operation of the array. Furthermore, the degree to which a given amount of error degrades an array design (i.e., design sensitivity) will differ greatly from one design to the next. A measure of sensitivity is proposed, based on the maximum impact which a given amount of error can produce on a particular design. The adoption of this measure allows the sensitivity problem to be solved through the use of mathematical programming techniques. The best of these techniques for the general array design case is shown to be Non-linear Goal Programming. The sensitivity values for five planar symmetric array designs are determined and compared.

The results of this comparison indicate the potential of the proposed measure and solution technique for the development of a fast and inexpensive method for array design sensitivity analysis. Particularly, the resulting sensitivity values are free from the a priori assumptions and high costs inherent in simulation. Furthermore, the method is based on a general framework which is applicable to the analysis of several array design factors including size, geometry, pattern, and component reliability. The method may also be extended to multiobjective design problems outside the area of array design.

CHAPTER I

INTRODUCTION

1.1 Purpose of the Study

Mathematical programming techniques appearing during the past 10 years have provided a means for designing complex array patterns. From 1946 until the advent of these techniques, arrays were almost universally programmed using the Dolph Tschebbyscheff method, which involved setting the directional response function equal to a Tschebbyscheff polynomial of like degree.[1,9] This method applied only to linear arrays in the exact sense. However, by the use of the product theorem, patterns produced by planar arrays could be approximated. The Dolph-Tschebbyscheff method allowed main beam width and side lobe height to be specified. In contrast, it is now possible to achieve main beam shape and place nulls, as well as achieve other objectives. A concomitant to the increased demands on array performance is a decreased tolerance of the array pattern to small deviations in the values of certain key parameters. Thus, the potential impact of error has become of increased concern to the designer. This concern provides the motivation for the present study, the purpose of which is to find a practical means of assessing array pattern sensitivity.

This process entails the choice of some measure of the impact of

error on an array pattern to serve as an indicator of sensitivity, and concurrently, the development of a method by which this measure can be evaluated. The choice reflects a compromise between attaining as much information regarding sensitivity as possible, and keeping the time and expense involved in its acquisition within reasonable limits.

The design of this study encompasses five main objectives. These are:

1. To define a consistent measure of array pattern sensitivity, which may be applied to all classes of array, and to any type of array design.
2. To develop or, more correctly, to adapt an efficient method to determine array pattern sensitivity, based on the above measure.
3. To discuss and compare alternate means for measuring and determining sensitivity.
4. To apply the chosen method to a representative problem. Specifically, on an array of given size and geometry, the sensitivities of five different array patterns, displaying a wide range of possible design features, will be determined and compared.
5. To discuss the extension of the method to the general array design sensitivity analysis problem.

1.2 Organization of the Paper

The paper is organized as follows. Chapter Two includes an introduction to transducer arrays and array patterns, pertinent terminology, coordinate systems, and conventional representations.

Methods of array pattern design using mathematical programming techniques are summarized, and error sources are discussed, as well as their resolution into variables of the directivity function.

In Chapter Three, alternate methods of measuring sensitivity are discussed. These are the expected value and the maximum value of change in the array pattern. Then, methods of finding these values are compared, including simulation and a set of mathematical optimization techniques.

MAXIM, a program which measures the maximum impact of error on an array pattern is presented, and the use of MAXIM to develop an index of sensitivity is discussed.

Chapter Four is a description of an experimental application of MAXIM. Sensitivity values are generated for five directivity patterns. The results are presented in graphic and tabular form.

These results are discussed in Chapter Five, and extensions and alternate applications of the method are suggested.

CHAPTER II

ARRAY DESIGN AND ERROR

2.1 Transducer Arrays

A transducer is any device which converts a signal from one transmission medium into a signal in another. When operating as a receiver the transducer converts a signal incident to its surface into an electric signal, and converts an electric signal into a radiated signal when operating as a transmitter.

A transducer array consists of a set of transducers, termed elements, which are placed according to some fixed geometrical arrangement, and are electrically interconnected. Arrays fall into three categories, depending on the number of physical dimensions necessary to accommodate the element positions. These are: linear, planar, and conformal, corresponding to one, two, and three dimensions, respectively.

2.2 Array Patterns

One of the major objectives involved in acoustic array design is the achievement of a desired array pattern, or equivalently, directivity pattern. If the operating array is viewed from an aspect angle, expressed as a solid angle, θ, ϕ , where the θ component is measured from the positive Z axis, and the ϕ component from the positive X axis (see

fig.2.1), the power radiated along that angle (or the sensitivity of the array to a signal incident to it from that angle) is called the directional response of the array in the θ, ϕ direction. The directional response is measured in dB. Letting θ and ϕ vary to define some region in space (often a hemisphere), the directional response at all angles contained in that region is called an array pattern. The parameters which determine the array pattern are:

1. The number and placement of elements.
2. The wavelength at which the system operates.
3. The relative amplitude and phase shading coefficients of the excitation currents to the individual elements.

Figure 2.2 is a contour plot. It represents a typical array pattern defined on $0 < \theta < 180$ and $0 < \phi < 180$. This pattern is obtained from a 10 by 10 planar symmetric array with unshaded element amplitude and phase settings. The response of the array is given on a five-degree grid in negative decibels. Thus -0 dB represents the peak power radiated in any direction, and one half of the total gain (radiated power) of the array is in the region -3 dB or less off peak. In figure 2.2 the center of this region, called the main beam, or main lobe, is at $\theta = 90, \phi = 90$. In this case, the axis or boresight of the array falls along the positive Y axis. Any other lobe in the array pattern with a relatively high gain, for instance at $\theta = 60, \phi = 90$, is considered to be noise and is called a side lobe. Likewise, any region where the gain falls below some prespecified level, say -60 dB, is termed a null, e.g., $\theta = 15, \phi = 40$. Figure 2.3 is a simulated 3^d plot of the same pattern.

An array pattern may be defined in terms of three categories of

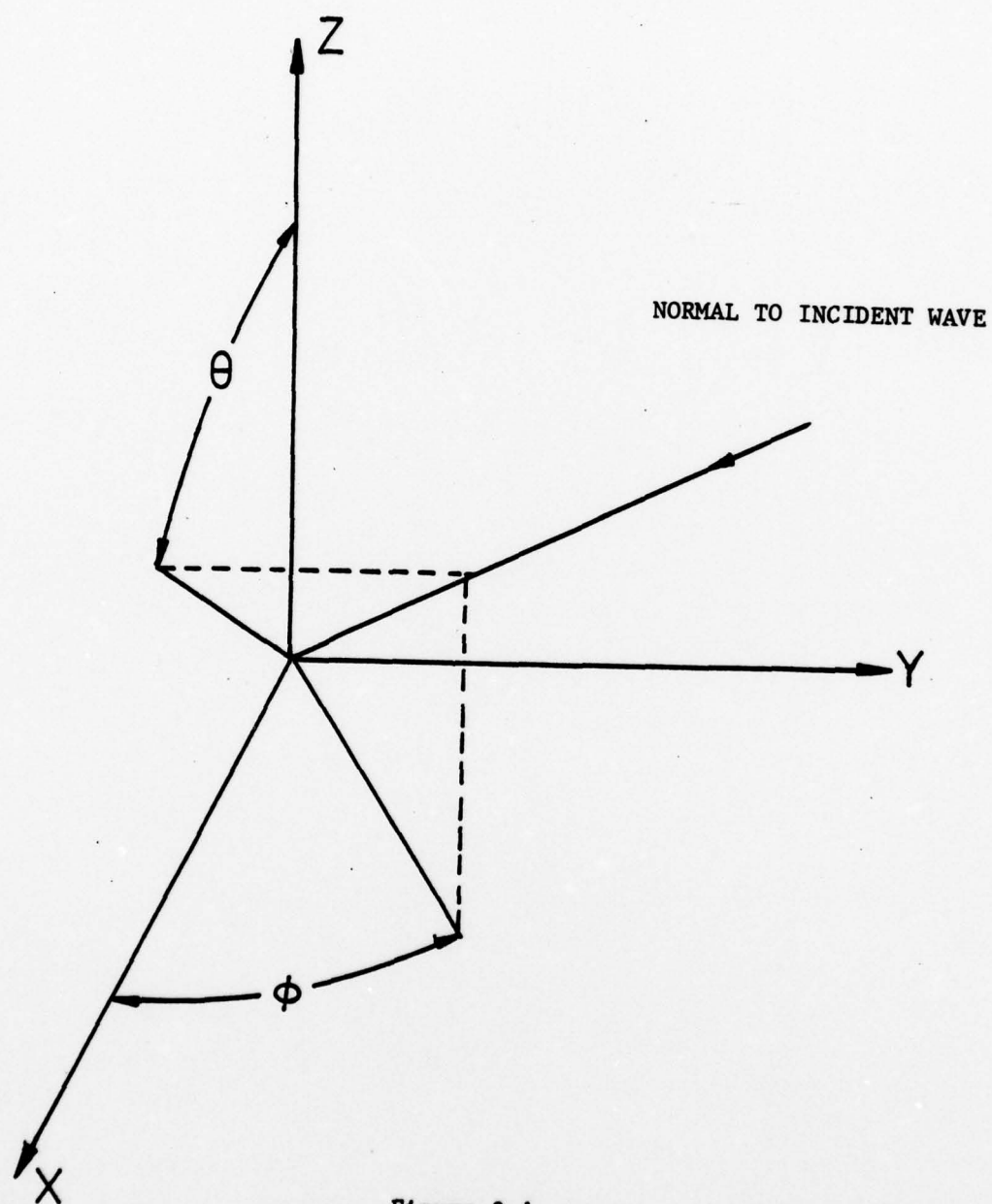


Figure 2.1
Coordinate System

BORESIGHT AT 90.90	
THETA	PHI
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80	
5	47 47 47 46 46 46 46 45 45 45 45 44 44 44 44 44 44 44 45 45 45 45 46 46 46 46 46 47 47 47
10	48 48 47 45 44 42 41 40 38 37 36 35 34 33 33 32 32 32 32 33 33 34 35 36 37 38 40 41 42 44 45 47 48 48
15	39 40 40 41 42 45 50 66 46 40 35 32 30 28 27 26 26 25 26 26 27 28 30 32 35 40 46 66 50 45 42 41 40 40 39
20	37 37 36 35 35 34 34 35 37 42 55 37 31 27 24 23 22 21 22 23 24 27 31 37 55 42 37 35 34 34 35 35 36 37 37
25	45 47 53 62 46 40 36 34 33 33 35 45 38 29 24 21 20 19 20 21 24 29 38 45 35 33 33 34 36 40 46 62 53 47 45
30	38 38 38 38 40 43 53 47 39 35 34 35 46 36 28 23 21 21 21 23 28 36 46 35 34 35 39 47 53 43 40 38 38 38 38
35	55 53 51 49 47 47 47 51 66 51 45 43 45 64 40 34 31 30 31 34 40 64 45 43 45 51 66 51 47 47 47 49 51 53 55
40	48 49 53 67 54 46 43 42 44 54 46 39 38 46 39 30 26 25 26 30 39 46 38 39 46 54 44 42 43 46 54 67 53 49 48
45	38 38 38 39 42 51 46 38 36 37 55 37 32 34 39 25 20 19 20 25 39 34 32 37 55 37 36 38 46 51 42 39 38 38 38
50	47 45 43 42 41 42 48 54 42 39 43 49 37 36 63 30 24 22 24 30 63 36 37 49 43 39 42 54 48 42 41 42 43 45 47
55	57 64 62 51 47 44 45 51 53 43 43 57 42 38 50 34 27 25 27 34 50 38 42 57 43 43 53 51 45 44 47 51 62 64 57
60	38 39 42 49 49 39 36 37 49 38 33 39 37 30 36 27 19 16 19 27 36 30 37 39 33 38 49 37 36 39 49 49 42 39 38
65	44 44 45 47 54 57 46 43 47 54 42 43 51 38 41 36 27 24 27 36 41 38 51 43 42 54 47 43 46 57 54 47 45 44 44
70	37 36 36 36 38 45 42 35 36 56 35 33 58 30 31 29 18 16 18 29 31 30 58 33 35 54 36 35 42 49 38 36 36 37
75	38 37 35 33 34 34 50 34 33 42 35 30 47 28 28 28 16 13 16 28 28 28 47 30 35 42 33 34 50 38 34 33 35 37 38
80	47 43 39 37 36 38 55 39 35 41 39 33 44 32 30 32 19 16 19 32 30 32 44 33 39 41 35 39 55 38 36 37 39 43 47
85	41 33 27 24 22 24 34 27 21 25 27 19 28 18 16 19 5 2 5 19 16 18 28 19 27 25 21 27 34 24 22 24 27 33 41
90	44 32 25 21 19 21 30 25 19 22 25 16 24 16 13 16 2 0 2 16 13 16 24 16 25 22 19 25 30 21 19 21 25 32 44
95	41 33 27 24 22 24 34 27 21 25 27 19 28 18 16 19 5 2 5 19 16 18 28 19 27 25 21 27 34 24 22 24 27 33 41
100	47 43 39 37 36 38 55 39 35 41 39 33 44 32 30 32 19 16 19 32 30 32 44 33 39 41 35 39 55 38 36 37 39 43 47
105	38 37 35 33 34 36 50 34 33 42 35 30 47 28 28 28 16 13 16 28 28 28 47 30 35 42 33 34 50 38 34 33 35 37 38
110	37 36 36 36 38 45 42 35 36 56 35 33 58 30 31 29 18 16 18 29 31 30 58 33 35 58 36 35 42 49 38 36 36 37
115	44 44 45 47 54 57 46 43 47 54 42 43 51 38 41 36 27 24 27 36 41 38 51 43 42 54 47 43 46 57 54 47 45 44 44
120	38 39 42 49 49 39 36 37 49 38 33 39 37 30 36 27 19 16 19 27 36 30 37 39 33 38 49 37 36 39 49 49 42 39 38
125	57 64 62 51 47 44 45 51 53 43 43 57 42 38 50 34 27 25 27 34 50 38 42 57 43 43 53 51 45 44 47 51 62 64 57
130	47 45 43 42 41 42 48 54 42 39 43 49 37 36 63 30 24 22 24 30 63 36 37 49 43 39 42 54 48 42 41 42 43 45 47
135	38 38 38 39 42 51 46 38 36 37 55 37 32 34 39 25 20 19 20 25 39 34 32 37 55 37 36 38 46 51 42 39 38 38 38
140	48 49 53 67 54 46 43 42 44 54 46 39 38 46 39 30 26 25 26 30 39 46 38 39 46 54 44 42 43 46 54 67 53 49 48
145	55 53 51 49 47 47 47 51 66 51 45 43 45 64 40 34 31 30 31 34 40 64 45 43 45 51 66 51 47 47 47 49 51 53 55
150	38 38 38 38 40 43 53 47 39 35 34 35 46 36 28 23 21 21 21 23 28 36 46 35 34 35 39 47 53 43 40 38 38 38 38
155	45 47 53 62 46 40 36 34 33 33 35 45 38 29 24 21 20 19 20 21 24 29 38 45 35 33 33 34 36 40 46 62 53 47 45
160	37 37 36 35 35 34 34 35 37 42 55 37 31 27 24 23 22 21 22 23 24 27 31 37 55 42 37 35 34 34 35 35 36 37 37
165	39 40 40 41 42 45 50 66 46 40 35 32 30 28 27 26 26 25 26 26 27 28 30 32 35 40 46 66 40 45 42 41 40 40 39
170	48 48 47 45 44 42 41 40 38 37 36 35 34 33 33 32 32 32 32 32 33 33 34 35 36 37 38 40 41 42 44 45 47 48 48
175	47 47 47 46 46 46 46 45 45 45 45 44 44 44 44 44 44 44 44 44 45 45 45 45 46 46 46 46 46 47 47 47

Figure 2.2
Contour Plot. 10 by 10 Unshaded Array

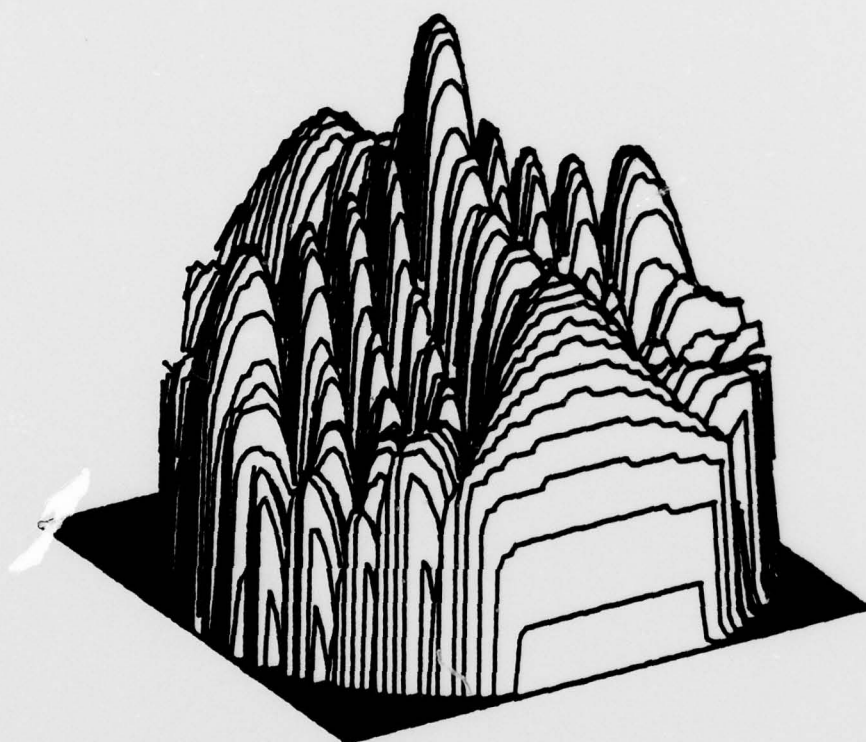


Figure 2.3
Simulated 3^d Plot. 10 by 10 Unshaded Array

design feature:

1. The size and shape of the main beam.
2. The "height" of the side lobes.
3. The placement of nulls.

2.3 Array Design Through Optimization Techniques

During the past ten years, problems in array pattern design have been successfully solved using mathematical optimization techniques. Among the methods which have been employed are Lagrangian multipliers, linear programming, method of steepest ascent, SUMT, and non-linear goal programming. Although these methods differ widely with regard to solution technique (and relative success), they usually share an underlying view of the array design problem as an optimization model in m objectives, and $2n$ variables. Thus an array of n independent elements, or sets of elements, is required to meet a set of specified directional responses, at m aspect angles $(\theta, \phi$ points), thereby defining an array pattern. In general, it is desired to maximize an objective function

$$Z(f_1(a_1, \dots, a_{2n}), \dots, f_m(a_1, \dots, a_{2n}), f'_1, \dots, f'_m)$$

where $f_1(a_1, \dots, a_{2n})$ is the achieved directional response at θ_1, ϕ_1 , $a_1, \dots, a_{2n}$ are the amplitude and phase shading coefficients of the elements of the array, and f'_1 is the specified response at θ_1, ϕ_1 . The function Z will take on increasingly greater values as

$$f_1(a_1, \dots, a_{2n}) \rightarrow f'_1, \quad i=1, \dots, m$$

Due to a somewhat unfortunate overlap in terminology, the set of values which Z takes on as $a_1, \dots, a_{2n}$ vary continuously, is called a response surface. The term "response surface" has to do with the response of the objective function and does not refer directly to the directional response of the array pattern. A response surface in 2 rather than $2n$ dimensions is shown in figure 2.4. The point (a'_1, a'_2) is called a global optimum. That is, there is no point (a_1, a_2) in the domain of Z such that $Z(a_1, a_2) > Z(a'_1, a'_2)$. The point (a''_1, a''_2) is a local optimum, since, although the conditions for global optimality do not hold, there exists a region given by:

$$|a_i - a''_i| < d_i, i=1,2, d_i > 0$$

such that $Z(a_1, a_2) \leq Z(a''_1, a''_2)$ holds for all a_1, a_2 within the region.

Of the optimization methods listed above, Lagrangian multipliers and linear programming are called indirect methods, since their solution techniques involve the simultaneous solution of a set of equations. The method of steepest ascent, the sequential unconstrained minimization technique, and non-linear goal programming, are direct search techniques relying on evaluation of response surface elevation, and some search movement logic to find an optimum.

Recently [10,15], Non-linear Goal Programming (NLGP), which is based on the modified Hooke and Jeeves pattern search discussed in appendix B, has been shown to provide good, robust solutions to the array design problem. The success of NLGP is due in part to the simplicity and directness of its search technique. All necessary information for search movement decisions is available directly from the objective function. Another factor in the success of NLGP derives from the goal programming problem formulation, which accommodates multiple

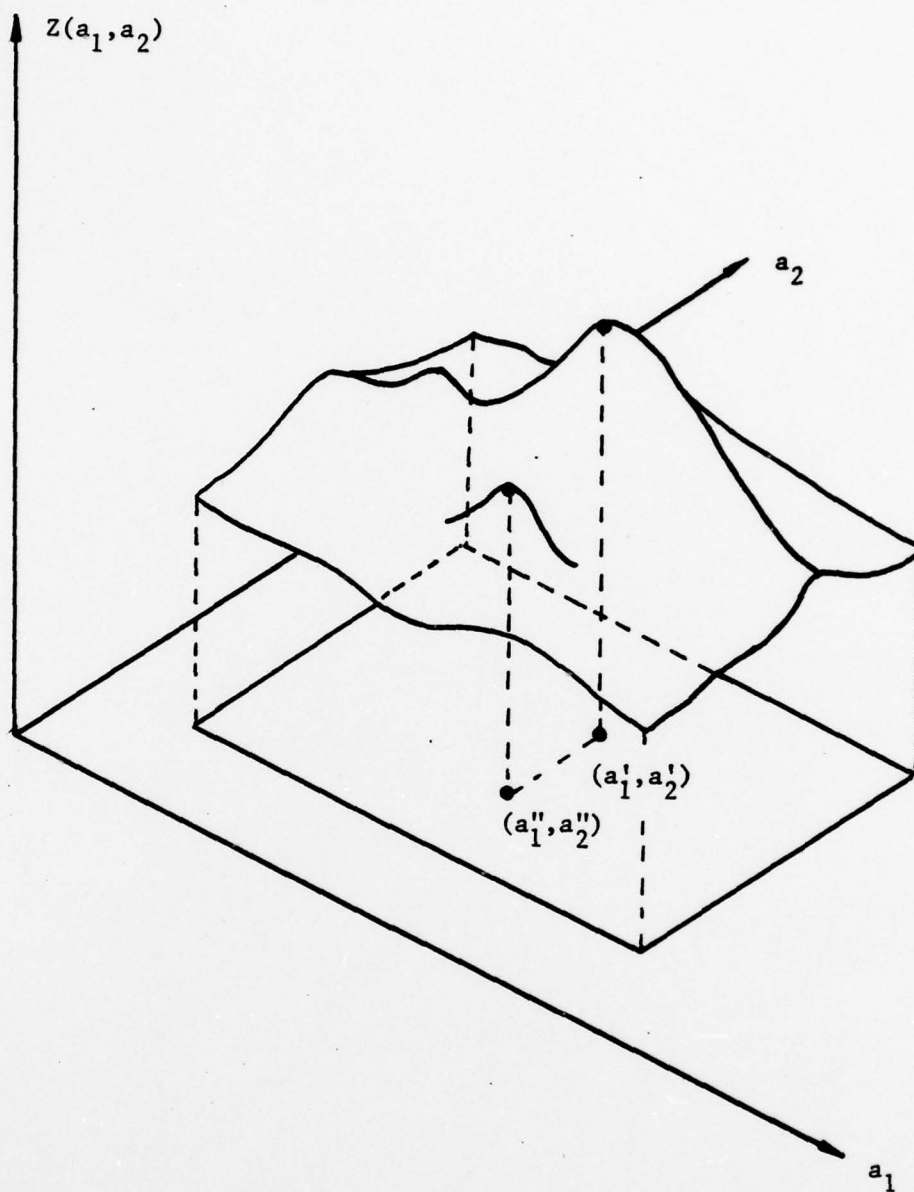


Figure 2.4
Generalized Response Surface
Showing Local and Global Optima

possibly conflicting, or even incommensurable objectives. This particular feature of NLGP is especially applicable to the array design problem, since the achievement of different array pattern features usually conflict. For example, it is difficult to achieve a narrow main beam, and at the same time maintain low side lobe levels.

2.4 Array Sensitivity and Error

Sensitivity analysis is the measurement of system response to deviations in the values of decision variables from those assumed in the optimal solution. The technique by which sensitivity is found is not dependent on the method originally used to obtain the solution.

If, as is often the case, the size, shape, and operating wavelength of the array have been specified in advance, the array design solution consists of a vector of element amplitude and phase shading coefficients. It happens that the values of these variables, specified in the solution are not precisely attained in practice. The reason for this may be traced to several sources of error, inherent in the construction and operation of a transducer array. The following list of error sources is not intended to be comprehensive, but to provide a representative sample.

Error sources are usually divided into two classes, depending on whether their effects on the array pattern are random or systematic.

The most important error source in the random category is control error in the excitation signals received by the individual elements. This error source is analogous to instrument drift, and will vary with time. Random construction errors in the manufacture and placement of elements, in the manufacture of phase shifters, or in the length and

fitting of feed lines will have a fixed but unpredictable effect on a given array.

Chief among the systematic errors is a phenomenon called mutual coupling. The signal emitted from any one element will induce a sympathetic excitation in every other element in the array. The amplitude and phase of this component of a given element's radiation depend on its position in the array, and the amplitude and phase setting of each other element. Systematic phase error may also be introduced by sensitivity of phase shifters, transmission lines, and other array components to frequency variations. Another source of phase error is the approximation inherent in discrete or digital phase shifters.

The effects of these and any other error source may be represented as a change in element amplitude and phase-shading coefficients. However, the error introduced by the mathematical model of directional response, which assumes elements with point apertures, and isotropic radiation patterns is not treated here. This error source may be resolved through refinements in the model if necessary.

CHAPTER III

THE MEASUREMENT OF SENSITIVITY

3.1 Sensitivity by Feature

In the array design problem the analyst is forced to consider the entire pattern at once, which results in a multiobjective problem formulation. This is because, as has been previously noted, the achievement of different array pattern features often conflict. While a pattern satisfying all design objectives would nominally be optimal, it probably won't be feasible. Optimality is then assigned to the "best" trade off which is both acceptable and feasible. The relative importance of different features may be reflected either in a set of weighting factors or in a preemptive priority structure. It is precisely this type of multiobjective problem which goal programming was designed to solve.

In the case where array pattern sensitivity is to be determined, however, it is more profitable to examine individual features of an array pattern separately. This is because the degradation of the different features will conflict in exactly the same manner as their achievement. By separating a pattern with, say, four important features into four different sensitivity problems, the question of deciding the relative "worst" between alternative examples of impact where different

features are affected is circumvented. With this separation, four new response surfaces may be created, each representing the achievement of one feature. Although the solution vector lies at the optimal point of the array pattern response surface, it does not follow that it lies at the optimal point of any of the individual feature response surfaces.

3.2 The Error Region and Error Surface

Consider the solution vector of an array design problem. Each component of this vector may be said to have an interval of error about it, representing the potential cumulative effects of all error sources which can be resolved into error in amplitude and phase-shading coefficients. The true value of the elements will lie somewhere along these intervals, which are expressed as plus or minus some percentage of the range over which the amplitude or phase can vary. Amplitudes are conventionally normalized to dimensionless values between 0 and 1, phase to values between 0 and 360 degrees. For an n -element array, these error intervals will form a hypercube in $2n$ space, with the solution vector at its center. This hypercube will be called the error region. A 2-dimensional error region is shown in figure 3.1.

If the error region is superimposed on a feature response surface, it will contain all values which that objective function which indicates the achievement of the desired feature will take on due to error. This part of the response surface will be called the error surface.

Perfect information about the sensitivity of a feature would consist of two types of data. The first of these would be a complete map of the error surface. This is equivalent to complete enumeration of the response function value over $2n$ continuous intervals. It is easily

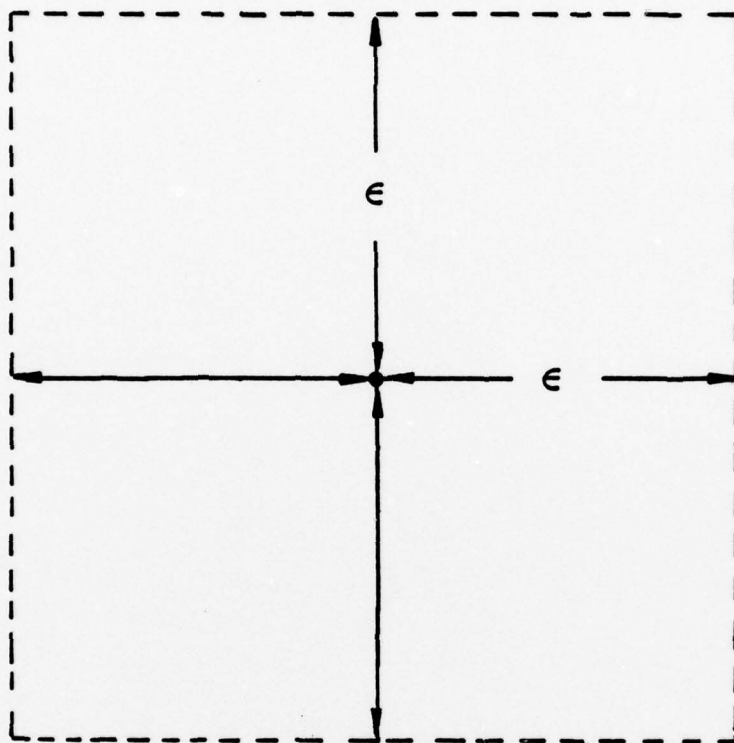


Figure 3.1
Generalized Error Region
About Design Solution Point

seen that even a low-resolution approximation would require that an astronomical number of points be evaluated as the array becomes large. For example, the array pattern of a 100 -element array depends on 200 decision variables. If the error interval of each variable is approximated by 50 equally spaced points, forming a 200 -dimensional lattice, total enumeration of the error surface response at the lattice points requires the array pattern to be evaluated 10,000 times. It should be noted as well, that as the array size increases, so does the cost of each evaluation. The second set of data would be composed of detailed knowledge of the effects of error sources. This would include such information as analysis of component interaction in various array constructions, probability distributions, the different random error types, and so forth. This kind of data is not available in practice.

Even though perfect sensitivity information is impractical or impossible to obtain, it is still desirable to find some characteristic of the error surface which will allow an assessment of sensitivity to be made. Candidates include the average elevation and the range (maximum elevation minus minimum elevation) of the error surface. The corresponding measures would be, respectively, the expected value of the feature's performance, and the worst value of the feature's performance in the error region.

3.3 Simulation

An estimate of the mean and standard deviation of the error surface elevation can be obtained by sampling points within the error region, and evaluating the objective function at these points. This amounts to a simulation of error impact. The sample will consist of a set of

points, constrained to lie within the error region, and randomly generated according to some distribution. For this purpose the distribution will usually be either normal about the solution vector, with a specified standard deviation, or uniform across the error region. Since the error sources discussed above are, in fact, partially systematic, and their random components partially fixed with regard to effect on a particular array, the assumptions inherent in these types of Monte Carlo simulations may seriously bias the analysis.

A second problem with simulation is the required sample size, and consequent expense. The number of degrees of freedom in the directivity function is twice the number of independent elements in the array. Reliable sample sizes tend to become quite large. It is noteworthy that in the 100-element case mentioned earlier (not a particularly large array), even a ten per cent sample amounts to 1000 points.

3.4 Optimization Models

Since simulation and enumeration are the only means of arriving at the expected value of array pattern feature sensitivity, and both prove to be impractical in light of present computer speeds and capacities, it remains to examine the error surface range. The maximum elevation on the error surface is assumed to be the solution vector for the array pattern. Quite a few optimization methods may be used to find the minimum elevation. These turn out to be the same techniques which have previously been brought to bear on the array design problem. It should not be surprising, considering the relationship between the two problems, that the relative success of a technique in solving one is a good predictor of performance on the other.

Representatives of three classes of optimization models will be discussed. These are:

1. Calculus-based methods.
2. Linear Programming.
3. Direct search techniques.

In order to restrict an indirect method, i.e. one based on the solution of a set of equations, to the error region, two inequality constraints must be added to the problem for each decision variable. A typical calculus-based technique for minimization subject to constraints is the use of Kuhn Tucker conditions. This method uses Lagrangian multipliers to create a set of equations which are solved simultaneously.

In the case of the general (non-symmetrical) n -element array these consist of:

1. $2n$ Lagrangians of the directivity function set equal to 0.
2. $4n$ inequality constraints.
3. $4n$ slack constraints, bounding the values which the multipliers may take on.

To find the minimum on a region, different combinations of the multipliers are set to 0, forcing the method to consider stationary points only along one or more boundaries of the error region. No multipliers set to 0 corresponds to the interior of the error region. This process is combinatorically explosive. At the worst case, up to 2^{4n} separate problems, each non-linear in $2n$ variables and $10n$ constraints, may have to be solved. Solving even one of these problems however, can be a somewhat monumental undertaking.

Two formulations which linearize the directivity function have been

developed. McMahon et al. [18] were the first to translate the array pattern design problem into a linear programming problem. This approach will solve for both amplitude and phase coefficients, but in order to do so, is forced to separate the real and imaginary components of the pattern. Because of this, it is impossible to reduce the problem size by taking symmetry into account.

Unlike calculus-based techniques, which are applied to the error surface, linear programming evaluates the directivity pattern. Thus a set of θ, ϕ points must be specified to define a feature which corresponds to some worst case. At each θ, ϕ point, four inequalities are generated, two each for the real and imaginary parts of the pattern. Two synthetic decision variables are developed for each element in the array, but as these are unrestricted, their number is functionally doubled. Also, for each element, four inequality constraints are added in order to define the error region.

Even before considering the number of θ, ϕ points required to define the worst case for a feature, there will be a problem in $4n$ variables, and $4n$ constraints. While this problem is orders of magnitude smaller than that generated by Kuhn Tucker conditions, its solution is still an expensive project. There will also be a certain amount of unavoidable round off error due to the subtraction and division required at each pivot.

In a later linear formulation, Wilson [20] created a linear programming problem which deals only with the real component of the pattern. Symmetry conditions may be incorporated into the directivity function in this model, and the decision variables are restricted to non-negative values. However, this drastic reduction in problem size is

accomplished at the expense of the ability to solve for phase settings. Wilson's model assumes that all elements are excited in phase, and solves only for the amplitude coefficients. Although this technique could be easily adapted to a sensitivity analysis of the restricted (planar symmetric and unphased array) class of problem which it was intended to solve, it is not applicable to the general case.

Direct search techniques have been the most effective means of solving the general array design problem. The sequential unconstrained minimization technique, (SUMT) [11, 12], gradient search (method of steepest ascent) , and Non-linear Goal Programming [4] have all been utilized to search for an optimum directivity pattern. Summaries and comparisons of these techniques have been made by Ignizio [15] and by Draus [10]. When applied to identical pattern synthesis problems, NLGP was found to equal or exceed (sometimes markedly) the performance of the other two methods, while at the same time displaying a clear superiority in computational efficiency. These results, plus the multiobjective capacity of the goal programming formulation make NLGP the most attractive direct search technique, and by extension, the best optimization model to adapt to the general directivity pattern sensitivity problem.

3.5 MAXIM/PS

NLGP/PS is the goal programming code specifically designed for planar symmetric array pattern design. This code was written by James P. Ignizio and Susan M. Draus at the Industrial Engineering Department, and Applied Research Laboratory, The Pennsylvania State University. Its accuracy and robustness have been extensively tested.

Only minor revisions were necessary in order to adapt NLGP to search for the minimum elevation of the error surface. These changes involve parameters of the pattern search, which is described in detail in appendix B.

1. The initial step sizes were reduced to compensate for the constrained region in which the search takes place.
2. Logic was added to the evaluation of provisional base points such that any point outside the error region registers no improvement. This effectively constrains the search to the error region.
3. Changes were made in the functional relationships which define the goals, that is, the response desired at given θ , ϕ points. In NLGP the achieved levels of response were set equal to the desired levels in the main region, and less than or equal in the side region. In the revised version, response in the main region is set less than or equal to the desired level, and in the side region, greater than or equal to the desired level.
4. The solution vector of the design problem is used as the initial base point for the pattern search.

The goal programming formulation, the directional response formulation, and the pattern search algorithm are identical in both codes. The revised version is called MAXIM/PS, as it measures the MAXimum IMPact of an error range on a feature.

Direct search techniques do not guarantee a global optimal solution on a multimodal response surface, and the Hooke and Jeeves pattern search is no exception. This is less of a problem in the array design

case, because only a satisfactory solution is required. The question of whether or not the global minimum of an error surface has been found is of greater concern, since a difference of several dB of sensitivity may be involved. In practice, however, error intervals are very small (on the order of ± 2 or 3 percent of the decision variable range), so that the chance of multimodality on any error interval, and hence on the error surface, is significantly reduced. If an extra measure of confidence is required, it is possible to make multiple search runs, commencing from different starting points.

3.6 The Index of Sensitivity

As noted previously, in the determination of pattern sensitivity important features should be considered separately. Thus, the θ , ϕ points corresponding to a high side lobe might be chosen and assigned an aspired value of -1 dB. MAXIM would then search within the error region for the point at which this side lobe was driven as high as possible. A measure on the θ , ϕ points used to define the side lobe, say the highest point or the mean height, could then be compared between the solution to the design problem and the maximum error impact solution. If the directional response at these θ , ϕ points had risen by an average of 15 dB, or if the maximum height of the feature had risen by 15 dB, the feature would be given a sensitivity value of 15 dB. The same procedure would then be repeated for all relevant features of the pattern. These would include other side lobes, nulls, and the main beam. The main beam θ , ϕ points would be given an aspired value of -60 dB to bring the response down as far as possible.

For a given error range, the result of this approach is a set of

sensitivity values associated with the important features of the array pattern. These are to be compared with minimum acceptable sensitivity values for these features. The maximum amount, in dB, by which the acceptable sensitivity value for any feature of the pattern is violated is the IOS, or index of sensitivity assigned to the pattern. Note that the IOS is specific to a given error range.

CHAPTER IV

APPLICATION: THE USE OF MAXIM TO GENERATE AN IOS

4.1 The Experiment

In order to evaluate the utility of the IOS measure of sensitivity, it was decided to test and compare a series of directivity patterns on a single array. The directivity patterns were chosen so as to reflect a broad sampling of objectives. Synthesis took place on a 10 x 10 planar symmetric array. The choice of array size and geometry was based on two considerations. The number of elements was to be large enough to produce patterns which were somewhat difficult, and hence interesting from the standpoint of sensitivity. At the same time, the number of decision variables involved was to be kept down to a reasonable level, allowing the analysis to be accomplished at a small to moderate cost in computation time.

4.2 Array Description

The 10 x 10 planar symmetric array is square and consists of 100 equally spaced elements. The elements are considered to lie in the X-Z plane, 25 elements to a quadrant, with symmetry about both the X and Z axes. The elements are placed and numbered as shown in figure 4.1. In the planar symmetric type of array, sets of four symmetrically placed

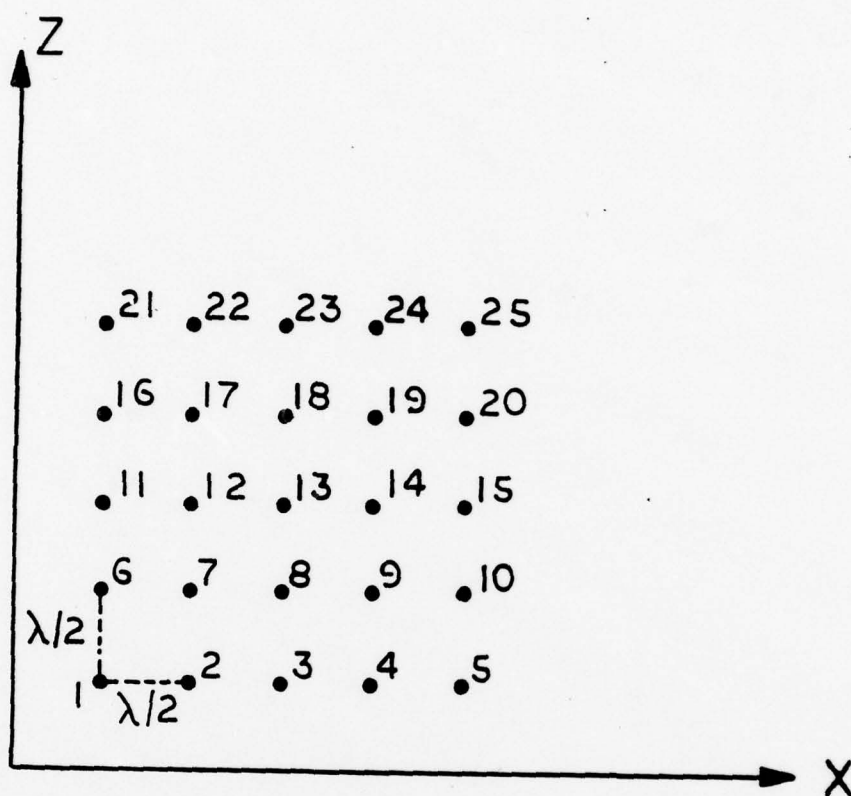


Figure 4.1
Element Positions in the First Quadrant.

elements receive identical excitation currents. This fact decreases both the amount of hardware necessary for array construction, and the difficulty in programming the array. In the 10 x 10 case, it is necessary to consider only 25 sets of elements, a reduction from 200 to 50 decision variables.

The directivity pattern is measured over a hemisphere defined by:

$$0 \leq \theta \leq 180 \text{ and } 0 \leq \phi \leq 180$$

See figure 2.1. Thus boresight falls along the positive Y axis.

The directivity function for this array is given by:

$$G_{\phi} = 10 \text{ Log } [16[(\sum_{i=1}^{25} A_i \cos \alpha_i \cos b_i \cos c_i)^2 + (\sum_{i=1}^{25} A_i \sin \alpha_i \cos b_i \cos c_i)^2]]$$

where:

α_i = the phase shift associated with element i.

$$b_i = 2\pi/\lambda x_i \sin\theta \cos\phi$$

$$c_i = 2\pi/\lambda z_i \cos\theta$$

A_i = the excitation amplitude of element i.

λ = the operating wavelength of the array.

See appendix A for the derivation of this result from the general directivity function.

4.3 Experimental Patterns

Five patterns were synthesized using NLGP. This process is documented in references 10 and 15. These are:

1. 20 by 30 degree elliptical main beam with uniform response. Side lobes ≤ -27 dB off peak. -61 dB nulls at (20,20), (160,20), (20,160), (160,160).
2. 30 degree circular main beam with uniform response. Side lobes ≤ -26 dB off peak.
3. 30 degree circular main beam with uniform response. Side lobes ≤ -26 dB off peak. Broad nulls ≤ -47 dB off peak, centered about (40,35), (40,165), (140,165), (140,35), (140,165).
4. 30 degree square main beam with uniform response. Side lobes ≤ -28 dB off peak.
5. 30 by 70 degree rectangular main beam with uniform response. Side lobes ≤ -27 dB off peak.

In the discussion to follow, the experimental patterns will be referred to as patterns 1 through 5. These five patterns are illustrated by 5 degree resolution contour plots, and 2 degree resolution simulated 3^d plots in figures 4.2 to 4.11. Tables 4.12 to 4.15 show the element set amplitude and phase-shading coefficients for the five design solutions.

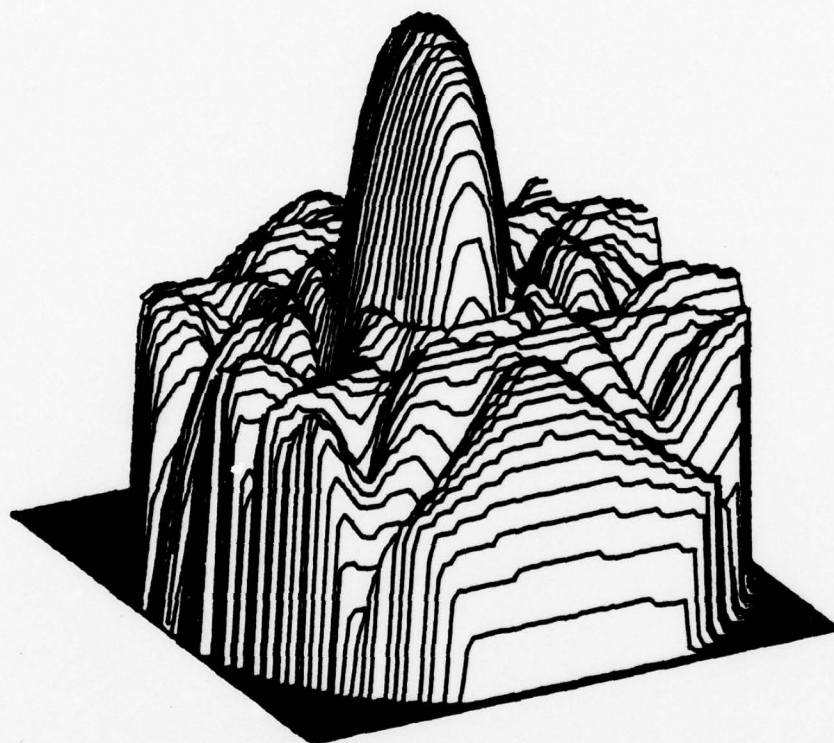


Figure 4.2
Pattern 1. Design Solution

Simulated 3^d Plot

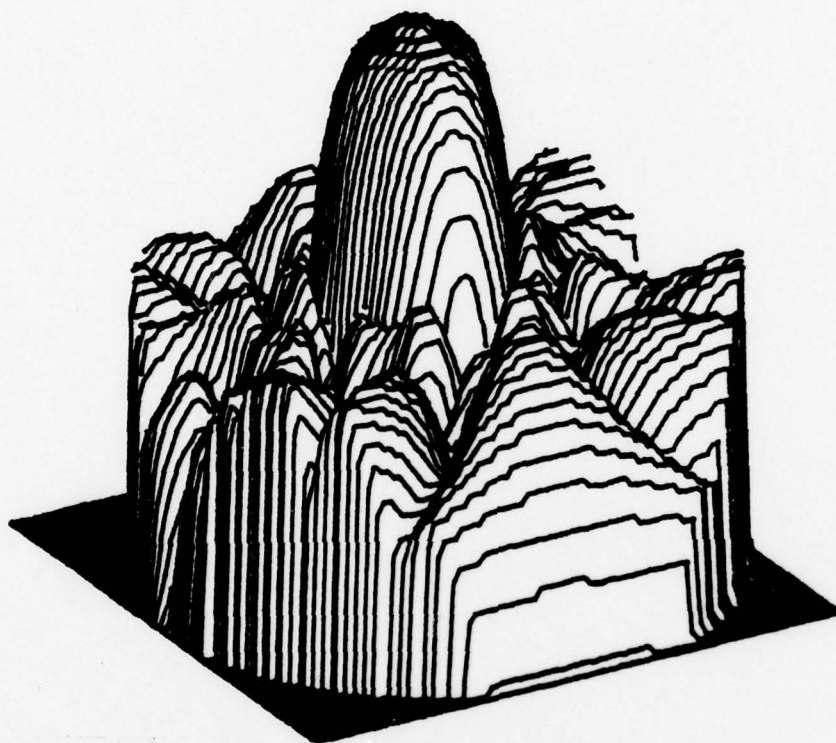


Figure 4.4
Pattern 2. Design Solution
Simulated 3^d Plot

Figure 4.5
Pattern 2. Design Solution
Contour Plot

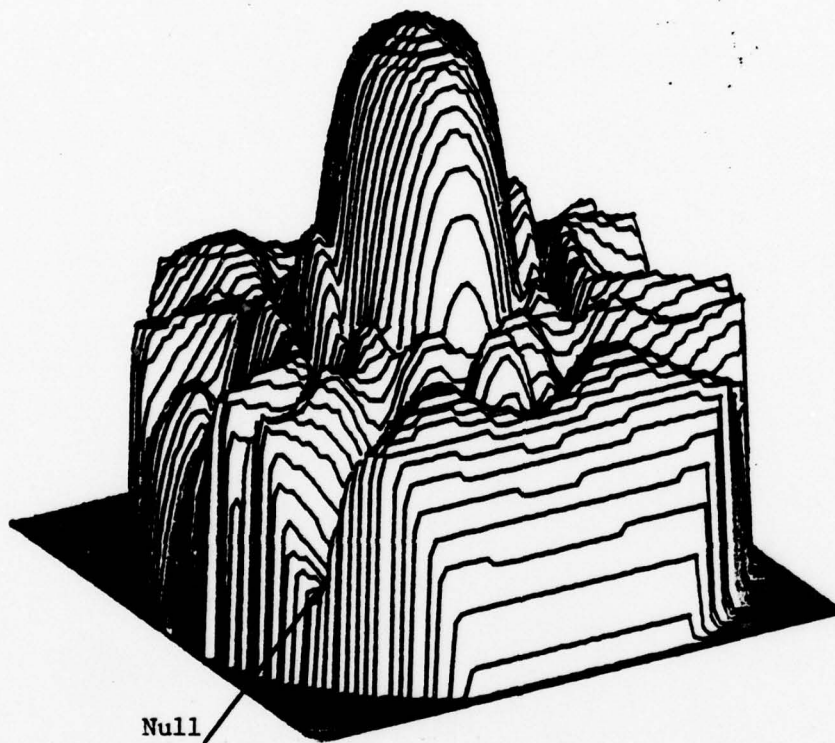


Figure 4.6
Pattern 3. Design Solution
Simulated 3^d Plot

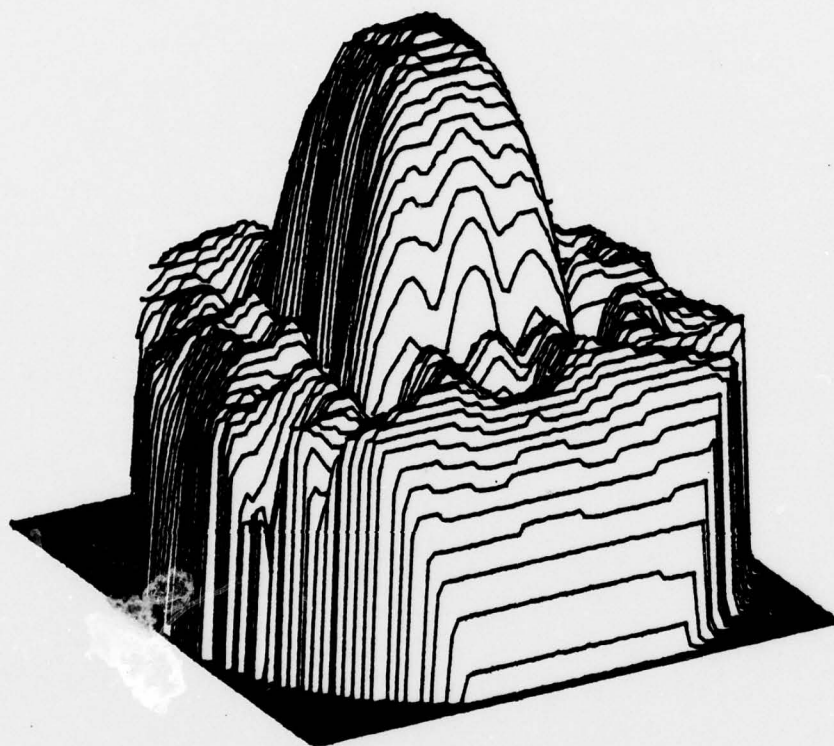


Figure 4.8
Pattern 4. Design Solution
Simulated 3^d Plot

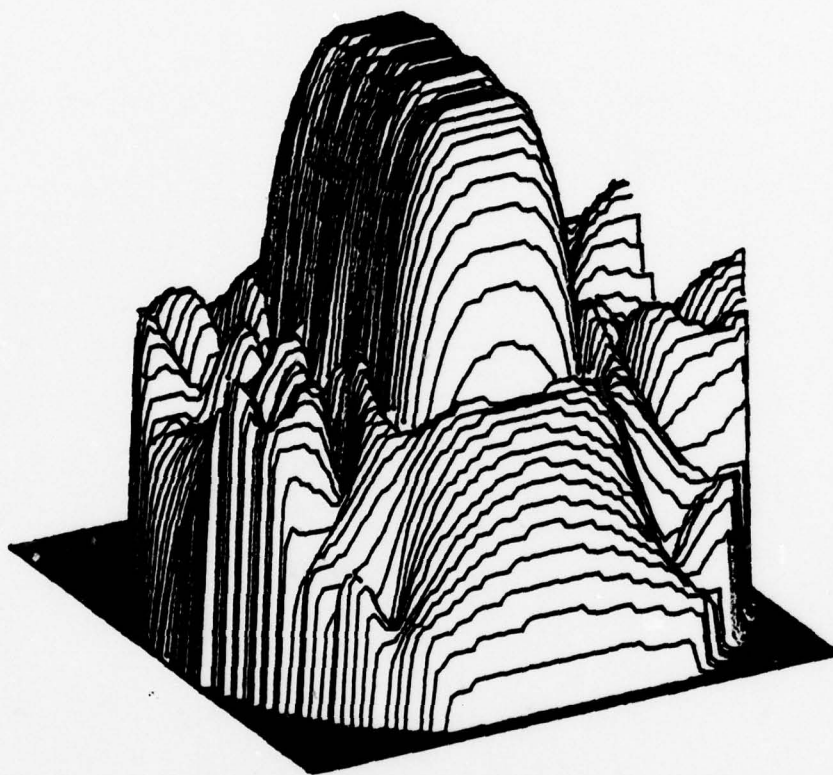


Figure 4.10
Pattern 5. Design Solution
Simulated 3^d Plot

GAIN DEGRADATION IS
-0.444

Figure 4.11
Pattern 5. Design Solution
Contour Plot

TABLE 4.1

Pattern 1. Amplitude and Phase settings.

Design Problem Solution Vector.

Element Number	A_i	α_i
1	1.00000	45.08598
2	0.83026	50.47632
3	0.55588	61.58244
4	0.28236	81.43579
5	0.14425	125.26320
6	0.73256	29.22868
7	0.55325	40.92334
8	0.36320	46.62827
9	0.17836	86.20399
10	0.00016	-23.47665
11	0.47729	5.06743
12	0.31261	4.35339
13	0.16010	7.01000
14	0.06544	-72.95085
15	0.03629	-110.93900
16	0.35335	-27.58368
17	0.24252	-58.43025
18	0.18708	-57.70381
19	0.00300	-96.97165
20	0.05679	-95.49803
21	0.19194	-79.03387
22	0.16094	-56.59669
23	0.09302	-116.47880
24	0.02788	-106.64440
25	0.01390	-153.24800

TABLE 4.2

Pattern 2. Amplitude and Phase settings.

Design Problem Solution Vector.

Element Number	A_i	α_i
1	1.00000	86.59134
2	0.72492	76.29446
3	0.40101	46.41832
4	0.21527	-16.98532
5	0.16095	-59.23582
6	0.67942	81.91975
7	0.49048	63.05440
8	0.23734	18.64299
9	0.24548	-33.18059
10	0.13429	-52.38556
11	0.24719	58.71460
12	0.21164	19.38586
13	0.27099	-8.95276
14	0.21564	-35.90080
15	0.13460	-62.06729
16	0.14507	-31.14095
17	0.14949	-28.63022
18	0.21673	-37.39592
19	0.14453	-54.42915
20	0.02817	-68.23637
21	0.13854	-65.68265
22	0.11909	-53.94441
23	0.06781	-47.95035
24	0.07849	-64.44748
25	0.00075	-78.08585

TABLE 4.3

Pattern 3. Amplitude and Phase settings.

Design Problem Solution Vector.

Element Number	A_i	α_i
1	1.00000	86.58992
2	0.72632	76.29305
3	0.34581	51.41776
4	0.23573	-21.98474
5	0.14923	-43.99036
6	0.69419	81.91833
7	0.47213	63.05298
8	0.24763	18.64278
9	0.23630	-30.62059
10	0.15778	-61.38362
11	0.24962	58.71346
12	0.19205	21.43350
13	0.26085	-8.95269
14	0.20757	-40.90024
15	0.10105	-52.45972
16	0.14143	-34.98846
17	0.14389	-26.19022
18	0.20862	-37.39572
19	0.13913	-49.42932
20	0.05563	-57.98967
21	0.11944	-68.87962
22	0.11464	-57.30469
23	0.06528	-43.59050
24	0.07555	-50.84010
25	0.00073	-73.46278

TABLE 4.4

Pattern 4. Amplitude and Phase settings.

Design Problem Solution Vector.

Element Number	A_1	α_1
1	1.00000	83.71129
2	0.58462	74.89516
3	0.14241	34.26509
4	0.19317	-61.69458
5	0.10087	-100.38940
6	0.55779	84.17863
7	0.34828	59.88213
8	0.23030	6.18118
9	0.18049	-23.08340
10	0.10228	-77.36714
11	0.00410	-174.76480
12	0.14119	-0.87991
13	0.26177	5.56305
14	0.19908	-13.63951
15	0.08626	-12.35343
16	0.15645	-99.23828
17	0.10658	-45.02731
18	0.16667	-6.84196
19	0.14051	1.34623
20	0.01905	-10.38139
21	0.08704	-100.32360
22	0.03587	-78.75728
23	0.03556	-40.04144
24	0.02226	115.26540
25	0.01850	89.00087

TABLE 4.5

Pattern 5. Amplitude and Phase settings.

Design Problem Solution Vector.

Element Number	A_i	α_i
1	0.66059	102.85650
2	0.38303	110.02400
3	0.01744	105.41250
4	0.09856	-85.83286
5	0.07641	-48.83804
6	1.00000	41.89236
7	0.65056	37.02228
8	0.17021	38.25259
9	0.08036	-93.32645
10	0.14957	-147.13670
11	0.81610	2.49893
12	0.47947	1.34707
13	0.17213	-20.41713
14	0.07275	-167.79430
15	0.06579	-165.56710
16	0.53865	-75.06853
17	0.36734	-77.70242
18	0.12291	-93.52895
19	0.00422	63.56360
20	0.08095	68.92059
21	0.27639	-101.25180
22	0.17685	-107.32960
23	0.05861	-94.70036
24	0.05308	43.36661
25	0.03110	53.67607

For each pattern, sensitivity values were determined for:

1. The main beam,
2. the two highest side lobes,
3. nulls if present,

at ± 1 percent and ± 2 percent error in A_1 and α_1 values, and at -2 percent, -1 percent, 1 percent and 2 percent error in λ . Because of the symmetry inherent in the pattern, a side lobe or null in one orthant will have an identical counterpart in the other three. For convenience, a set of four such symmetrical features will be referenced only in the orthant defined by:

$$0 \leq \theta \leq 90 \quad \text{and} \quad 0 \leq \phi \leq 90$$

The process by which the sensitivity values were obtained is described in detail for pattern 1. It holds for the other four patterns as well.

4.4 The MAXIM Runs

Four major features were identified. These are the main beam, the two highest side lobes, denoted $sl(1)$ and $sl(2)$, and the nulls. These features are outlined in figure 4.2.

The sensitivity of each of these features to error in A_1 and α_1 was analyzed separately, using MAXIM. Thus two MAXIM runs were made for each feature, with error ranges for A_1 and α_1 set first at 1 percent, and then at 2 percent. A_1 and α_1 vary independently, and the capability to model this is built into MAXIM. Setting these error ranges equal reflects a desire to keep the analysis as simple as possible.

Since MAXIM evaluates the directivity pattern, rather than the response surface, it is necessary to develop a set of goals which will

drive the solution to the minimum elevation point on the error surface. Directional response values which correspond to extreme degradation are therefore specified at a number of θ, ϕ points in the feature. It was felt that a 5-degree resolution was sufficient to define the features examined here. Smaller, irregularly shaped features may require a finer resolution. The goals are of the form:

$$G_{\theta\phi} + n_{\theta\phi} - p_{\theta\phi} = B_{\theta\phi}$$

where:

$G_{\theta\phi}$ is the directivity function for θ, ϕ .

$n_{\theta\phi}$ is a negative deviation variable.

$p_{\theta\phi}$ is a positive deviation variable.

$B_{\theta\phi}$ is a value representing the worst possible response at θ, ϕ in dB.

For θ, ϕ in the main beam, $B_{\theta\phi} = -60$ dB. For θ, ϕ in the side lobes or nulls, $B_{\theta\phi} = 1$ dB.

The objective function of MAXIM is of the form:

$$\text{MIN } Z = \sum_{\theta\phi} W_{\theta\phi} P_{\theta\phi}$$

for θ, ϕ in the main beam.

and

$$\text{MIN } Z = \sum_{\theta\phi} W_{\theta\phi} n_{\theta\phi}$$

for θ, ϕ in a side lobe or null

where the weighting factors, $W_{\theta\phi}$, are set to unity, indicating no difference in priority between θ, ϕ points in the same feature. This is the basic goal programming formulation in one priority level. There are, in fact, three priority levels in the problem, but two of these, excitation symmetry and error range, are implicit in the directivity function and pattern search logic respectively.

The solution vector of the design problem was used as the initial base point for the pattern search.

4.5 Sensitivity Values

MAXIM generates a contour plot, displaying the worst possible response of the given feature on a specified error region. This is then compared with the contour plot of the design solution directivity pattern, previously generated by NLGP. Two sensitivity values are taken for each MAXIM contour plot, i.e., for each feature at each error range. These are the average difference ($\overline{\Delta G}$), and the difference in the worst directional response values for any θ, ϕ point in the feature, ($\Delta G_{\max}$).

$\overline{\Delta G}$ is given by:

$$\sum_{\theta\phi} \frac{(G_{\theta\phi} - G'_{\theta\phi})}{n}$$

for θ, ϕ points in a side lobe or null. Or:

$$\sum_{\theta\phi} \frac{(G'_{\theta\phi} - G_{\theta\phi})}{n}$$

for θ, ϕ points in the main beam.

Where

$G_{\theta\phi}$ = the response at θ, ϕ for the design solution.

$G'_{\theta\phi}$ = the worst response at θ, ϕ in the error region.

n = the number of θ, ϕ points used to define the feature.

$\Delta G_{\max}$ is given by:

$$M - M'$$

Where

$M = \text{Max } [G_{\theta\phi}]$ for θ, ϕ in a side lobe or null.

$\text{Min } [G_{\theta\phi}]$ for θ, ϕ in the main beam.

$M' = \text{Max } [G'_{\theta\phi}]$ for θ, ϕ in a side lobe or null.

$\text{Min } [G'_{\theta\phi}]$ for θ, ϕ in the main beam.

A negative value for $\overline{\Delta G}$ or $\Delta G_{\max}$ indicates that the error has produced an improvement in the feature. This will not occur due to error in A_1 and α_1 , but may for error in λ .

4.6 Measurement of Operating Wavelength Error Impact

The designs were synthesized using an operating wavelength (λ) of 1 unit length. In order to determine sensitivity to error in λ , patterns were made for each design where all element amplitude and phase settings were maintained at the design solution values, and operating wavelength was set at .98, .99, 1.01, and 1.02 units of length. The two sensitivity measures defined above, $\overline{\Delta G}$ and $\Delta G_{\max}$, were then measured on each feature.

4.7 Results

Figures 4.12 to 4.45 are contour plots of patterns 1 through 5, showing the maximum impacts of 1 and 2 percent error in amplitude and phase settings on the features outlined.

Figures 4.46 to 4.50 are graphs of feature sensitivity to error in A_1 and α_1 , measured in terms of $\overline{\Delta G}$ for each pattern. Figures 4.51 to 4.55 are graphs of feature sensitivity to error in A_1 and α_1 , measured in terms of $\Delta G_{\max}$ for each pattern.

Figures 4.56 and 4.57 are graphs of main beam sensitivity to error in A_1 and α_1 , across the 5 patterns, measured in terms of $\overline{\Delta G}$, and $\Delta G_{\max}$ respectively. Figures 4.58 and 4.59 are graphs of aggregated side lobe sensitivity to error in A_1 and α_1 across the 5 patterns, measured in terms of $\overline{\Delta G}$, and $\Delta G_{\max}$ respectively.

Tables 4.6 to 4.39 show the amplitude and phase values for each pattern generated by MAXIM.

Table 4.40 summarizes the data for amplitude and phase error sensitivity. Table 4.42 presents the data for operating wavelength sensitivity. As the figures indicate that sensitivity to ≤ 2 percent absolute error in λ is quite low, it was felt that little information was to be gained by the presentation of the associated contour plots.

PRI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

WAIN DEGRADATION IS
-0.140

Figure 4.12
Pattern 1. 1 Percent Error
Maximum Impact on The Main Beam

Figure 4.14
Pattern 1. 1 Percent Error
Maximum Impact on Side Lobe 1

DOESIGHT AT 70, JO		PRI	
THETA			
	0	5	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70
	75	80	85
	90	95	00
	05	10	15
	20	25	30
	35	40	45
	50	55	60
	65	70	75
	80	85	90
	95	00	05
	10	15	20
	25	30	35
	40	45	50
	55	60	65
	70	75	80
	85	90	95
	00	05	10
	15	20	25
	30	35	40
	45	50	55
	60	65	70

GAIN DEGRADATION IS
-8.269

Figure 4.16
Pattern 1. 1 Percent Error
Maximum Impact on Side Lobe 2

DORESIGHT AT 90,90

THETA

PHI

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
0	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	
5	55	55	55	55	55	55	55	54	54	54	54	54	53	53	53	53	53	53	53	53	53	53	53	53	53	54	54	54	54	54	55	55	55	55	55	55	
10	50	50	50	50	49	49	48	47	46	45	45	44	43	42	42	41	41	41	41	41	41	41	41	42	42	43	44	45	45	46	47	48	49	49	50	50	50
15	52	52	52	52	51	51	50	49	47	45	43	41	39	38	36	35	35	34	34	34	35	35	36	38	39	41	43	45	47	49	50	51	51	52	52	52	
20	68	67	63	59	55	53	51	50	50	49	48	44	40	37	34	32	31	30	30	30	31	32	34	37	40	44	48	49	50	50	51	53	55	59	63	67	
25	38	38	39	40	42	45	51	55	50	47	47	52	52	41	35	32	29	28	28	28	29	32	35	41	52	52	47	47	50	55	51	45	42	40	39	38	
30	29	29	30	30	31	33	35	39	44	43	40	39	39	42	39	33	29	27	27	27	29	33	39	42	39	39	40	43	44	39	35	33	31	30	30	29	
35	26	26	26	26	27	28	29	32	36	41	37	34	32	31	32	31	29	27	27	27	29	31	32	31	32	34	37	41	36	32	29	28	27	26	26	26	
40	27	27	27	27	26	26	27	29	32	39	40	32	29	27	27	27	26	26	26	26	26	26	27	27	27	29	32	40	39	32	29	27	26	26	27	27	
45	28	28	28	27	26	26	26	27	30	35	50	34	29	27	26	26	26	26	26	26	26	26	26	26	27	29	34	50	35	30	27	26	26	27	28	28	
50	27	27	26	26	25	25	25	25	27	30	35	35	31	30	29	30	32	34	35	34	32	30	29	30	31	35	35	30	27	25	25	25	25	26	26	27	
55	31	31	30	29	28	27	25	25	25	27	30	33	35	43	38	34	34	34	34	34	34	34	34	38	43	35	33	30	27	25	25	25	27	28	29	30	
60	31	31	32	32	33	33	32	31	30	30	32	33	35	34	29	29	33	27	25	27	33	29	29	34	35	33	32	30	30	31	32	33	33	32	32	31	
65	30	30	29	29	29	29	30	30	31	34	40	33	29	27	26	31	21	14	13	14	21	31	26	27	29	33	40	34	31	30	30	29	29	29	29	30	
70	34	34	33	32	32	33	33	30	26	25	26	26	25	25	27	22	12	7	6	7	12	22	27	25	25	26	26	25	26	30	33	33	32	32	33	34	
75	41	40	38	36	35	35	35	33	29	27	26	26	25	25	40	17	8	4	2	4	8	17	40	25	25	26	26	27	29	33	35	35	35	36	38	40	
80	43	41	37	33	30	27	25	25	25	28	36	29	26	32	23	12	6	2	1	2	6	12	23	32	26	29	36	29	25	25	25	27	30	33	37	41	
85	50	44	37	31	27	25	23	22	21	21	21	22	24	40	16	8	3	1	0	1	3	8	16	40	24	22	21	21	21	22	23	25	27	31	37	44	
90	119	49	37	30	25	23	22	21	20	19	18	19	22	32	14	7	2	0	0	0	2	7	14	32	22	19	18	19	20	21	22	23	25	30	37	49	
95	50	44	37	31	27	25	23	22	21	21	21	22	24	40	16	8	3	1	0	1	3	8	16	40	24	22	21	21	21	22	23	25	27	31	37	44	
100	43	41	37	33	30	27	25	25	25	28	36	29	26	32	23	12	6	2	1	2	6	12	23	32	26	29	36	28	25	25	25	27	30	33	37	41	
105	41	40	38	36	35	35	35	33	29	27	26	26	25	25	40	17	8	4	2	4	8	17	40	25	25	26	26	27	29	33	35	35	35	36	38	40	
110	34	34	33	32	32	33	33	30	26	25	26	26	25	25	27	22	12	7	6	7	12	22	27	25	25	26	26	25	26	30	33	33	32	32	33	34	
115	30	30	29	29	29	29	30	30	31	34	40	33	29	27	26	31	21	14	13	14	21	31	26	27	29	33	40	34	31	30	30	29	29	29	30		
120	31	31	32	32	33	33	32	31	30	30	32	33	35	34	29	29	33	27	25	27	33	29	29	34	35	33	32	30	30	31	32	33	33	32	32	31	
125	31	31	30	29	28	27	25	25	25	27	30	33	35	43	38	34	34	34	34	34	34	34	34	38	43	35	33	30	27	25	25	25	27	28	29	30	
130	27	27	26	26	25	25	25	27	30	35	35	31	30	29	30	32	34	35	34	32	30	29	30	31	35	35	30	27	25	25	25	25	26	26	27		
135	28	28	28	27	26	26	26	27	30	35	50	34	29	27	26	26	26	26	26	26	26	26	26	26	27	29	34	50	35	30	27	26	26	27	28		
140	27	27	27	26	26	27	29	32	39	40	32	29	27	27	27	26	26	26	26	26	27	27	29	32	40	39	32	29	27	26	26	27	27	27	27		
145	26	26	26	26	27	28	29	32	36	41	37	34	32	31	32	31	29	27	27	27	29	31	32	31	32	34	37	41	36	32	29	28	27	26	26		
150	29	29	30	30	31	33	35	39	44	43	40	39	39	42	39	33	29	27	27	27	29	33	39	42	39	39	40	43	44	39	35	33	31	30	30		
155	38	38	39	40	42	45	51	55	50	47	47	52	52	41	35	32	29	28	28	28	29	32	35	41	52	52	47	47	50	55	51	45	42	40	39		
160	68	67	63	59	55	53	51	50	50	49	48	44	40	37	34	32	31	30	30	30	31	32	34	37	40	44	48	49	50	50	51	53	55	59	63		
165	52	52	52	52	51	51	50	49	47	45	43	41	39	38	36	35	35	34	34	34	35	35	36	38	39	41	43	45	47	49	50	51	51	52	52		
170	50	50	50	50	49	49	48	47	46	45	44	43	42	42	41	41	41	41	41	41	41	41	42	42	43	44	45	45	46	47	48	49	49	50	50		
175	55	55	55	55	55	54	54	54	54	54	54	53	53	53	53	53	53	53	53	53	53	53	53	53	54	54	54	54	54	54	55	55	55	55			
180	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	

GAZE DEGRADATION IS
-0.284

Figure 4.17
Pattern 1. 2 Percent Error
Maximum Impact on Side Lobe 2

DORSIGHT AT 90.90

THETA	PHI															
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
0	128	128	128	128	128	128	128	128	128	128	128	128	128	128	128	128
5	58	58	58	58	58	57	57	57	57	57	57	56	56	56	56	56
10	49	49	49	49	49	48	48	48	47	47	46	46	45	45	44	44
15	41	41	41	41	42	42	42	42	42	42	41	40	39	39	38	38
20	35	35	35	35	36	36	36	37	37	38	38	38	37	36	34	34
25	32	32	32	32	32	32	32	33	34	35	36	37	37	35	34	32
30	37	36	35	34	33	32	31	30	30	31	32	33	36	39	40	37
35	35	36	37	40	47	44	37	33	30	29	29	30	31	34	37	43
40	27	27	28	28	29	32	36	51	39	32	30	29	30	32	34	34
45	28	28	28	28	28	29	32	38	41	33	31	30	32	35	34	33
50	28	28	28	28	29	30	32	35	39	36	32	31	33	39	48	38
55	29	29	29	29	29	29	31	36	54	38	34	33	34	32	30	29
60	36	36	35	35	34	34	33	32	33	35	37	37	38	33	28	29
65	39	39	39	39	40	42	52	47	40	37	37	37	33	30	30	21
70	47	47	46	45	43	42	40	37	36	37	45	38	31	31	26	17
75	39	38	36	34	33	33	35	37	32	30	36	38	33	30	17	10
80	44	42	38	35	32	32	35	41	32	30	43	30	29	20	11	7
85	60	54	46	41	38	39	50	36	30	32	33	24	24	15	8	5
90	143	65	53	47	45	50	42	32	29	32	29	22	14	7	4	3
95	60	54	46	41	38	39	50	36	30	32	33	24	24	15	8	5
100	44	42	38	35	32	32	35	41	32	30	43	30	29	20	11	7
105	39	38	36	34	33	33	35	37	32	30	36	38	33	30	17	10
110	47	47	46	45	43	42	40	37	36	37	45	38	31	31	26	17
115	39	39	39	39	40	42	52	47	40	37	37	37	33	30	21	16
120	36	36	35	35	34	34	33	32	33	35	37	37	38	33	28	29
125	29	29	29	29	29	29	31	36	54	38	34	33	34	32	30	29
130	28	28	28	28	28	29	30	32	35	39	36	32	31	33	39	48
135	28	28	28	28	28	28	29	32	38	41	33	31	30	32	35	34
140	27	27	28	28	29	32	36	51	39	32	30	29	30	32	34	34
145	35	36	37	40	47	44	37	33	30	29	29	30	31	34	37	43
150	37	36	35	34	33	32	31	30	30	31	32	33	36	39	40	37
155	32	32	32	32	32	32	32	33	34	35	36	37	37	35	34	32
160	35	35	35	35	36	36	36	37	37	38	38	38	37	36	34	34
165	41	41	41	41	42	42	42	42	42	42	42	42	41	40	39	39
170	49	49	49	49	49	48	48	48	47	47	46	46	45	45	44	44
175	58	58	58	58	58	57	57	57	57	57	57	56	56	56	56	56
180	128	128	128	128	128	128	128	128	128	128	128	128	128	128	128	128

GAIN DEGRADATION IS
-9.131

Figure 4.20
Pattern 2. 1 Percent Error.
Maximum Impact on the Main Beam

DORESIGHT AT 90,90

THETA	PHI															
	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
0	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120
5	59	59	59	59	58	58	58	58	58	58	58	57	57	57	57	57
10	49	49	49	49	49	49	49	48	48	48	47	47	46	46	45	45
15	41	41	41	41	41	41	41	42	42	42	42	41	40	40	39	39
20	36	36	36	36	36	36	36	36	37	38	38	38	37	36	35	35
25	34	34	34	33	33	33	33	33	34	35	36	37	36	35	33	33
30	40	39	38	37	35	34	32	31	31	30	31	31	32	34	37	41
35	34	34	35	37	41	45	39	34	32	30	29	30	31	32	35	40
40	27	27	27	28	29	31	35	46	39	33	30	29	30	32	33	34
45	27	27	27	27	27	29	33	39	42	33	30	30	32	35	34	33
50	27	27	27	27	28	29	31	35	40	35	32	31	33	40	43	36
55	28	28	28	27	27	27	28	30	36	56	36	32	33	34	31	28
60	34	34	33	33	32	31	31	31	32	37	39	35	39	35	28	27
65	43	43	43	44	46	43	39	37	37	39	45	39	31	29	29	21
70	43	43	42	40	39	39	39	37	35	36	43	39	31	30	25	17
75	38	37	35	33	32	32	33	35	31	30	36	35	31	31	17	11
80	45	43	39	35	33	32	35	38	31	31	40	30	31	20	11	7
85	63	57	49	44	41	42	44	35	30	32	32	25	24	15	8	5
90	147	71	59	54	52	48	38	32	29	33	29	24	23	13	7	5
95	63	57	49	44	41	42	44	35	30	32	32	25	24	15	8	5
100	45	43	39	35	33	32	35	38	31	31	40	30	31	20	11	7
105	38	37	35	33	32	32	33	35	31	30	36	35	31	31	17	11
110	43	43	42	40	39	39	39	37	35	36	43	39	31	30	25	17
115	43	43	43	44	46	43	39	37	37	39	45	39	31	29	29	21
120	34	34	33	33	32	31	31	31	32	37	39	35	39	35	28	27
125	28	28	28	27	27	27	28	30	36	56	36	32	33	34	31	28
130	27	27	27	27	28	29	31	35	40	35	32	31	33	40	43	36
135	27	27	27	27	27	29	32	39	42	33	30	30	32	35	34	33
140	27	27	28	29	31	35	46	39	33	30	29	30	32	33	34	34
145	34	34	35	37	41	45	39	34	32	30	29	30	31	32	35	40
150	40	39	38	37	35	34	32	31	31	30	31	31	32	34	37	41
155	34	34	34	33	33	33	33	33	32	33	34	35	36	37	36	35
160	36	36	36	36	36	36	36	36	36	37	38	38	38	37	36	35
165	41	41	41	41	41	41	41	42	42	42	42	41	40	40	39	39
170	49	49	49	49	49	49	49	48	48	47	47	46	46	46	45	45
175	59	59	59	59	58	58	58	58	58	58	58	57	57	57	57	57
180	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120

GAIN DEGRADATION IS
-9.037

Figure 4.21
Pattern 2. 2 Percent Error
Maximum Impact on the Main Beam

BORESIGHT AT 30.30

THETA	PHI																			
	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112
5	43	43	43	43	43	43	43	43	43	42	42	42	42	42	41	41	41	41	41	41
10	39	39	39	38	38	37	36	35	35	34	33	32	31	31	30	30	29	29	29	30
15	54	53	52	50	47	44	41	39	36	34	31	29	26	25	24	23	23	23	23	23
20	46	46	47	48	49	50	52	56	52	43	36	31	26	25	23	21	20	19	19	19
25	32	32	33	33	34	36	38	42	46	49	61	41	32	26	23	20	18	17	16	17
30	32	32	31	31	30	30	31	34	39	46	50	45	31	24	20	17	16	15	15	16
35	38	39	42	50	43	36	32	30	30	31	35	45	46	39	29	23	19	17	16	17
40	28	28	28	29	30	34	41	46	35	32	32	36	59	38	32	26	22	19	18	19
45	29	29	29	29	29	30	32	35	42	39	35	35	41	39	31	26	23	21	21	23
50	27	28	28	28	29	30	32	36	42	41	37	36	40	35	29	25	23	22	21	22
55	28	28	28	27	28	28	29	32	36	40	38	37	40	32	26	23	22	22	21	22
60	30	30	30	30	30	30	31	32	33	33	34	35	34	28	25	23	24	26	27	26
65	35	35	34	34	33	34	35	39	45	41	36	32	28	26	26	22	17	16	17	22
70	39	39	38	36	35	34	34	33	33	34	38	33	28	24	16	11	8	7	8	11
75	35	34	32	30	29	28	29	31	31	29	33	37	33	29	16	9	6	4	3	4
80	40	38	34	31	28	27	28	33	32	30	42	29	26	19	10	5	3	2	2	3
85	54	48	40	34	31	30	33	39	30	30	34	22	21	14	7	3	2	2	1	2
90	128	55	43	37	33	32	38	36	28	29	29	20	19	13	6	3	2	1	0	1
95	54	48	40	34	31	30	33	39	30	30	34	22	21	14	7	3	2	2	1	2
100	40	38	34	31	28	27	28	33	32	30	42	29	26	19	10	5	3	2	2	3
105	35	34	32	30	29	28	29	31	31	29	33	37	33	29	16	9	6	4	3	4
110	39	39	38	36	35	34	34	33	33	34	38	33	28	24	16	11	8	7	8	11
115	35	35	34	34	33	34	35	39	45	41	36	32	28	26	26	22	17	16	17	22
120	30	30	30	30	30	30	31	32	33	33	34	35	34	28	25	23	24	26	27	26
125	28	28	28	27	28	28	29	32	36	40	38	37	40	32	26	23	22	21	22	22
130	27	28	28	28	29	30	32	36	42	41	37	36	40	35	29	25	23	22	21	22
135	29	29	29	29	29	30	32	35	42	39	35	35	41	39	31	26	23	21	21	23
140	28	28	28	29	30	34	41	46	35	32	32	36	59	38	32	26	22	19	18	19
145	38	39	42	50	43	36	32	30	30	31	35	45	46	39	29	23	19	17	16	17
150	32	32	31	31	30	30	31	34	39	46	50	45	31	24	20	17	16	15	16	17
155	32	32	33	33	34	36	38	42	46	49	61	41	32	26	23	20	18	17	16	17
160	46	46	47	48	49	50	52	56	52	43	36	31	28	25	23	21	20	19	19	20
165	54	53	52	50	47	44	41	39	36	34	31	29	28	26	25	24	23	23	23	23
170	39	39	39	38	38	37	36	35	35	34	33	32	31	31	30	30	29	29	29	30
175	43	43	43	43	43	43	43	43	43	42	42	42	42	42	41	41	41	41	41	42
180	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112	112

GAIN DEGRADATION IS
-9.077

Figure 4.23
Pattern 2. 2 Percent Error
Maximum Impact on Side Lobe 1

PNI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.268

Figure 4.24
Pattern 2. 1 Percent Error
Maximum Impact on Side Lobe 2

PRI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.295

Figure 4.25
Pattern 2. 2 Percent Error
Maximum Impact on Side Lobe 2

BORESIGHT AT 90,90

THETA	PHI																			
	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134
5	60	60	60	60	60	60	60	60	61	61	61	61	61	61	61	61	61	61	61	61
10	46	46	46	46	46	46	47	47	47	47	48	48	48	49	49	49	49	49	49	49
15	40	40	40	40	39	39	39	39	39	39	40	40	41	41	42	42	43	43	42	41
20	39	39	39	38	38	37	37	36	35	35	35	35	36	37	38	39	39	39	39	38
25	46	46	45	43	42	40	38	37	35	34	33	32	32	33	34	36	38	39	40	39
30	51	51	53	56	64	55	47	43	39	36	34	32	31	32	33	36	41	49	51	49
35	43	43	43	43	44	46	48	51	48	42	37	34	32	32	34	38	43	39	37	39
40	39	40	40	41	43	44	45	45	44	41	38	35	34	35	37	35	32	31	32	35
45	37	37	38	40	42	45	48	49	46	43	41	40	38	37	38	39	36	32	31	32
50	34	34	34	35	37	39	40	41	41	41	41	40	38	37	36	39	44	39	36	39
55	35	35	35	35	35	35	34	34	34	35	37	39	38	34	32	30	30	31	31	30
60	47	48	50	57	50	42	37	33	32	33	35	36	35	32	31	32	40	36	32	36
65	38	38	38	37	36	35	34	33	33	34	36	34	31	30	35	29	20	16	15	16
70	42	41	40	38	36	33	31	30	30	32	42	34	28	29	24	16	11	8	8	11
75	43	42	40	38	36	35	33	30	28	29	37	34	30	28	17	10	7	5	4	5
80	47	46	42	38	36	37	39	33	28	29	45	29	31	21	11	7	5	3	3	3
85	61	55	48	42	40	42	49	37	29	11	35	24	26	16	8	5	4	2	1	2
90	137	63	52	45	43	47	43	32	28	32	31	23	24	14	7	4	3	1	0	1
95	61	55	48	42	40	42	49	33	29	31	35	24	26	16	8	5	4	2	1	2
100	47	46	42	38	36	37	39	33	28	29	45	29	31	21	11	7	5	3	3	3
105	43	42	40	38	36	35	33	30	28	29	37	34	30	28	17	10	7	5	4	5
110	42	41	40	38	36	33	31	30	30	32	42	34	28	29	24	16	11	8	8	11
115	38	38	38	37	36	35	34	33	33	34	36	34	31	30	35	29	20	16	15	16
120	47	48	50	57	50	42	37	33	32	33	35	36	35	32	31	32	40	36	32	36
125	35	35	35	35	35	35	34	34	34	35	37	39	38	34	32	30	30	31	31	30
130	34	34	34	35	37	39	40	41	41	41	41	40	38	37	36	39	44	39	36	39
135	37	37	38	40	42	45	48	49	46	43	41	40	38	37	38	39	36	32	31	32
140	39	40	40	41	43	44	45	45	44	41	38	35	34	35	37	35	32	31	32	35
145	43	43	43	44	46	48	51	48	42	37	34	32	32	34	38	43	39	37	39	43
150	51	51	53	56	64	55	47	43	39	36	34	32	31	32	33	36	41	49	51	49
155	46	46	45	43	42	40	38	37	35	34	33	32	32	33	34	36	38	39	40	39
160	39	39	39	38	37	37	36	35	35	35	35	35	36	37	38	39	39	39	39	38
165	40	40	40	40	39	39	39	39	39	39	39	40	40	41	41	42	42	43	43	42
170	46	46	46	46	46	46	47	47	47	47	48	48	48	49	49	49	49	49	49	48
175	60	60	60	60	60	60	60	60	61	61	61	61	61	61	61	61	61	61	61	61
180	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134	134

GAIN DEGRADATION IS
-9.214

Figure 4.26
Pattern 3. 1 Percent Error
Maximum Impact on the Main Beam

2MI

1 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.115

Figure 4.27
Pattern 3. 2 Percent Error
Maximum Impact on the Main Beam

SORSIGHT AT 70, 70

PHETA

PHZ

051015202530354045505560657075808590950005101520253035404550556065707580

0133

GAIN DEGRADATION IS
-9.268

Figure 4.28
Pattern 3. 1 Percent Error
Maximum Impact on Side Lobe 1

PRI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.216

Figure 4.29
Pattern 3. 2 Percent Error
Maximum Impact on Side Lobe 1

PMT

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.351

Figure 4.30
Pattern 3. 1 Percent Error
Maximum Impact on Side Lobe 2

PRI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-9.390

Figure 4.31
Pattern 3. 2 Percent Error
Maximum Impact on Side Lobe 2

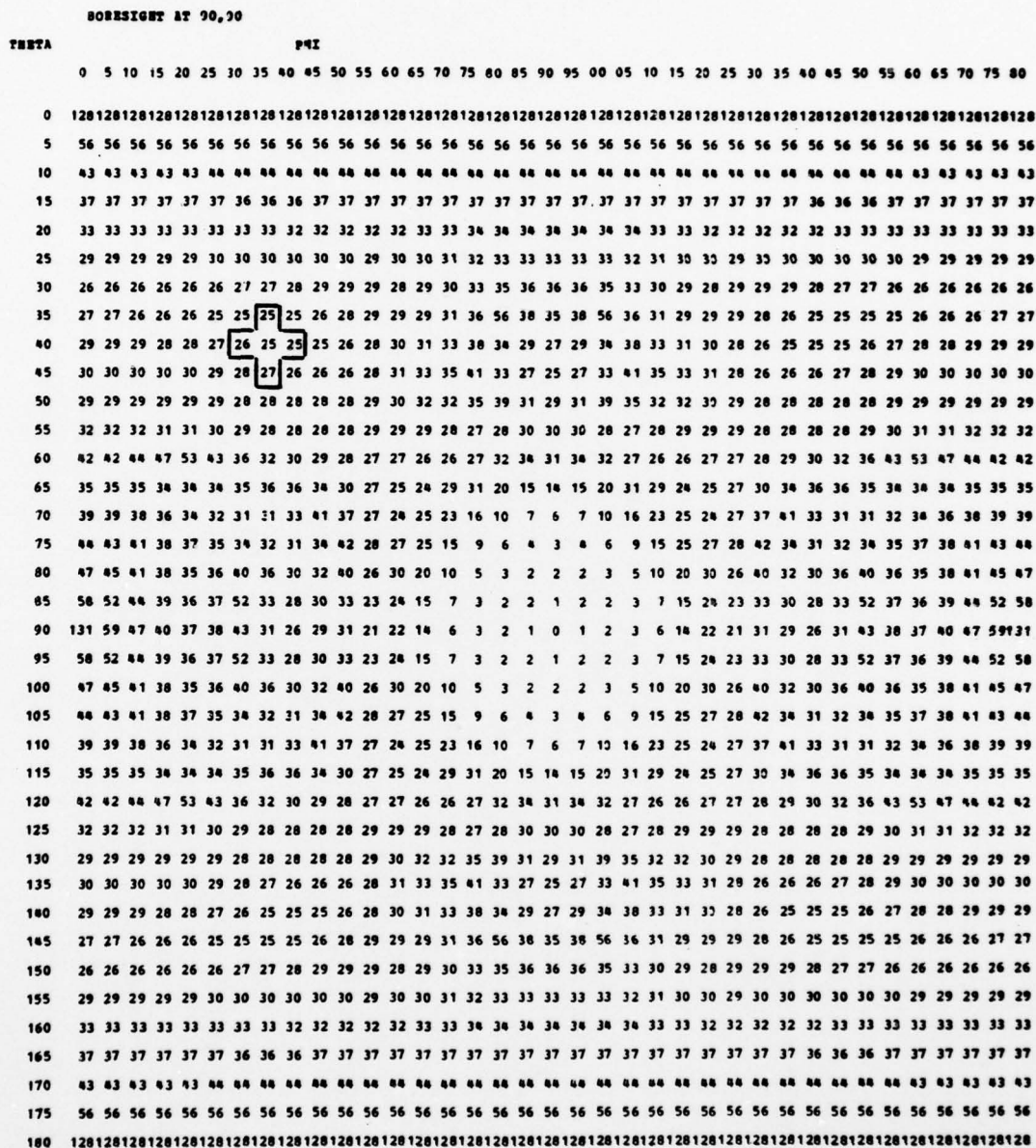


Figure 4.32
Pattern 3. 1 Percent Error
Maximum Impact on the Null

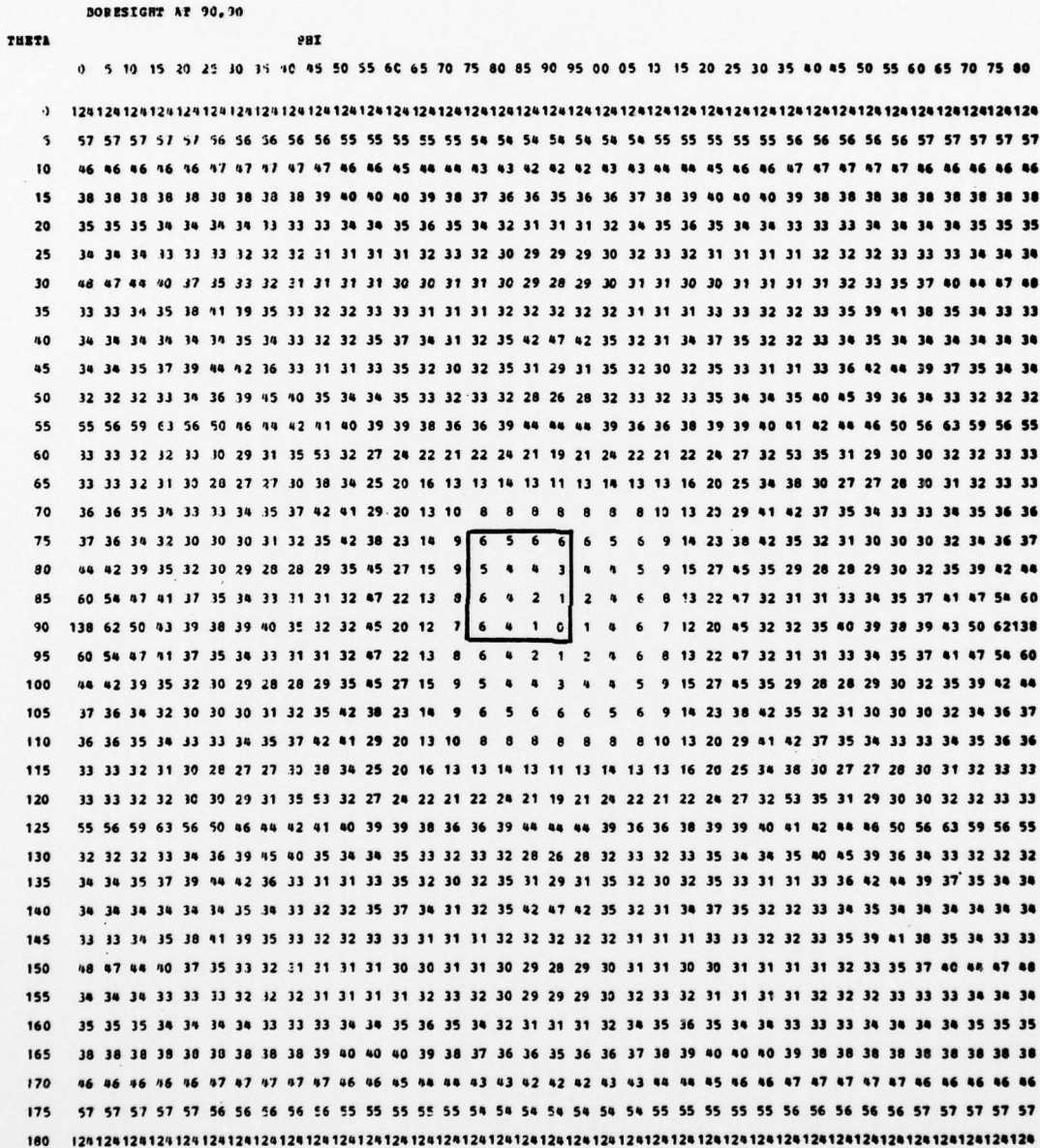


Figure 4.35
Pattern 4. 2 Percent Error
Maximum Impact on the Main Beam

DOORSIGHT AT 20.90

THEIA

PRI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-10.503

Figure 4.36
Pattern 4. 1 Percent Error
Maximum Impact on Side Lobe 1

BORESIGHT AT 90, 90

PHETA

PHX

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
0	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122
5	56	56	56	56	56	56	55	55	55	54	54	53	53	52	52	52	52	52	52	52	52	52	53	53	54	54	55	55	55	56	56	56	56	56	56	56
10	34	34	34	34	35	35	36	37	38	39	41	43	44	44	43	41	41	40	40	40	41	41	43	44	44	43	41	39	38	37	36	35	35	34	34	34
15	24	24	24	24	24	24	25	26	26	28	29	32	35	37	36	35	33	33	33	35	36	37	35	32	29	28	26	26	25	24	24	24	24	24	24	
20	21	21	21	21	20	20	20	20	20	20	21	23	25	29	33	32	30	29	30	32	33	29	25	23	21	20	20	20	20	20	20	20	21	21	21	
25	24	24	23	23	22	21	20	19	18	18	17	18	19	20	23	28	31	28	27	28	31	28	23	20	19	18	17	18	18	19	20	21	22	23	23	24
30	31	31	30	29	27	25	23	21	20	18	17	16	16	17	20	23	31	29	27	29	31	23	20	17	16	16	17	18	20	21	23	25	27	29	30	31
35	35	35	35	36	35	32	29	26	23	20	19	17	16	17	18	21	29	38	31	38	29	21	18	17	16	17	19	20	23	26	29	32	35	36	35	35
40	36	36	37	38	39	38	33	29	26	23	21	20	19	19	20	21	26	31	31	31	26	21	20	19	19	20	21	23	26	29	33	38	39	38	37	36
45	32	32	33	34	35	34	32	29	26	25	24	23	23	22	23	24	25	24	23	24	25	24	23	22	23	23	24	25	26	29	32	34	35	34	33	32
50	33	33	33	34	34	33	30	27	25	24	24	24	25	27	28	28	27	25	23	22	23	25	27	28	28	27	25	24	24	25	27	30	33	34	33	33
55	42	42	40	38	34	31	29	27	25	25	25	25	25	26	27	30	36	40	36	30	27	26	25	25	25	25	25	25	27	29	31	34	38	40	42	42
60	37	36	35	33	30	28	26	25	25	26	27	24	20	18	18	20	24	20	17	20	24	20	18	18	20	24	27	26	25	25	26	28	30	33	35	36
65	36	35	34	32	30	28	26	25	25	27	30	24	18	13	11	12	13	11	9	11	13	12	11	13	18	24	30	27	25	25	26	28	30	32	34	35
70	36	35	34	33	32	31	30	29	28	28	30	25	18	12	8	6	6	6	5	6	6	6	8	12	18	25	30	28	28	29	30	31	32	33	34	
75	36	36	34	32	31	31	35	35	28	26	27	27	21	12	6	3	3	3	3	3	3	3	6	12	21	27	27	26	28	35	35	31	31	32	34	
80	41	39	36	32	30	29	32	40	28	24	24	27	28	13	6	3	1	2	2	2	1	3	6	13	28	27	24	24	28	40	32	29	30	32	36	
85	53	47	39	34	30	29	31	37	28	23	22	27	21	11	5	3	2	1	1	1	2	3	5	11	20	27	22	23	28	37	31	29	30	34	39	
90	126	53	42	35	31	29	30	34	27	22	21	27	19	10	5	3	2	1	0	1	2	3	5	10	19	27	21	22	27	34	30	29	31	35	42	
95	53	47	39	34	30	29	31	37	28	23	22	27	21	11	5	3	2	1	1	1	2	3	5	11	20	27	22	23	28	37	31	29	30	34	39	
100	41	39	36	32	30	29	32	40	28	24	24	27	28	13	6	3	1	2	2	2	1	3	6	13	28	27	24	24	28	40	32	29	30	32	36	
105	36	36	34	32	31	31	35	35	28	26	27	27	21	12	6	3	3	3	3	3	3	3	6	12	21	27	27	26	28	35	35	31	31	32	34	
110	36	35	34	33	32	31	30	29	28	28	30	25	18	12	8	6	6	6	5	6	6	6	8	12	18	25	30	28	28	29	30	31	32	33	34	
115	36	35	34	32	30	28	26	25	25	27	30	24	18	13	11	12	13	11	9	11	13	12	11	13	18	24	30	27	25	25	26	28	30	32	34	
120	37	36	35	33	30	28	26	25	25	26	27	24	20	18	18	20	24	20	17	20	24	20	18	18	20	24	27	26	25	25	26	28	30	33	35	
125	42	42	40	38	34	31	29	27	25	25	25	25	25	26	27	30	36	40	36	30	27	26	25	25	25	25	25	25	27	29	31	34	38	40		
130	33	33	33	34	34	33	30	27	25	24	24	25	27	28	28	27	25	23	22	23	25	27	28	28	27	25	24	24	25	27	30	33	34	34	33	
135	32	32	33	34	35	34	32	29	26	25	24	23	23	22	23	24	25	24	23	24	25	24	23	22	23	23	24	25	26	29	32	34	35	34	33	
140	36	36	37	38	39	38	33	29	26	23	21	20	19	19	20	21	26	31	31	31	26	21	20	19	19	20	21	23	26	29	33	38	39	38	37	
145	35	35	35	36	35	32	29	26	23	20	19	17	16	17	18	21	29	38	31	38	29	21	18	17	16	17	19	20	23	26	29	32	35	36	35	
150	31	31	30	29	27	25	23	21	20	18	17	16	16	17	20	23	31	29	27	29	31	23	20	17	16	16	17	18	20	21	23	25	27	29	30	
155	24	24	23	23	22	21	20	19	18	18	17	18	19	20	23	28	31	28	27	28	31	28	23	20	19	18	17	18	19	20	21	22	23	23	24	
160	21	21	21	21	20	20	20	20	20	20	21	23	25	29	33	32	30	29	30	32	33	29	25	23	21	20	20	20	20	20	20	21	21	21	21	
165	24	24	24	24	24	24	25	26	26	28	29	32	35	37	36	35	33	33	33	35	36	37	35	32	29	28	26	26	25	24	24	24	24	24		
170	34	34	34	34	35	35	36	37	38	39	41	43	44	44	43	41	41	40	40	40	41	41	43	44	44	43	41	39	38	37	36	35	35	34	34	
175	56	56	56	56	56	56	55	55	55	54	54	53	53	52	52	52	52	52	52	52	52	52	52	53	53	54	54	55	55	55	56	56	56	56	56	
180	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	

GAIN DEGRADATION IS
-10.505

Figure 4.37
Pattern 4. 2 Percent Error
Maximum Impact on Side Lobe 1

BORNSIGHT AT 90,90

THETA

PNI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-10.681

Figure 4.38
Pattern 4. 1 Percent Error
Maximum Impact on Side Lobe 2

THETA

PNI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-10.706

Figure 4.39
Pattern 4. 2 Percent Error
Maximum Impact on Side Lobe 2

THETA

PBI

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 00 05 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

[illegible]

GAIN DEGRADATION IS
-0.512

Figure 4.40
Pattern 5. 1 Percent Error
Maximum Impact on the Main Beam

BORESIGHT AT 90,90

PHETA

PHX

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
0	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	
5	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	
10	41	41	41	41	41	41	41	41	41	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	41	41	41	41	41	41	41	41	
15	35	35	35	35	34	34	34	34	34	34	34	33	33	33	33	33	33	33	33	33	33	33	33	33	33	34	34	34	34	34	34	34	35	35	35	35	
20	34	34	33	33	32	32	31	31	30	29	29	29	29	28	28	28	28	28	28	28	28	28	28	29	29	29	29	29	30	31	31	32	32	33	33	34	34
25	37	37	36	36	35	33	32	30	29	28	26	26	25	25	25	25	25	25	25	25	25	25	25	25	25	26	26	28	29	30	32	33	35	36	36	37	
30	37	37	37	37	36	35	34	32	30	28	26	24	23	22	22	22	22	22	22	22	22	22	22	22	23	24	26	28	30	32	34	35	36	37	37	37	
35	40	40	40	39	39	37	35	33	31	28	26	24	22	22	21	21	22	22	22	22	22	21	21	22	22	24	26	28	31	33	35	37	39	39	40	40	
40	38	38	40	43	48	49	42	37	33	30	27	25	23	22	22	22	22	22	22	22	22	22	22	23	25	27	30	33	37	42	49	48	43	40	38	38	
45	30	30	31	31	32	33	34	37	38	32	29	28	27	25	22	19	18	17	17	17	18	19	22	25	27	28	29	32	38	37	34	33	32	31	31	30	30
50	29	30	30	30	30	28	28	30	37	36	29	30	32	21	15	12	11	10	10	10	11	12	15	21	32	30	29	36	37	30	28	28	30	30	30	29	
55	28	29	30	33	34	30	27	27	31	42	27	28	30	16	10	7	6	6	6	6	6	7	10	16	33	28	37	42	31	27	27	30	34	33	30	29	28
60	28	28	29	31	37	40	31	28	29	41	27	26	32	15	8	5	4	4	4	4	4	5	8	15	32	26	27	41	29	28	31	40	37	31	29	28	28
65	33	33	33	34	36	43	40	34	34	48	30	28	32	16	9	6	4	4	4	4	4	6	9	16	32	28	30	48	34	34	40	43	36	34	33	33	33
70	35	34	33	33	32	34	39	53	38	38	40	42	28	16	10	6	5	4	4	4	5	6	10	16	28	42	40	38	38	53	39	34	32	33	33	34	35
75	35	34	32	30	29	29	32	39	33	31	43	33	27	16	9	5	4	3	4	3	4	5	9	16	27	33	43	31	33	39	32	29	29	30	32	34	35
80	42	41	37	33	30	29	30	35	37	34	41	38	28	15	8	4	3	3	3	3	3	4	8	15	28	38	41	34	37	35	30	29	30	33	37	41	42
85	59	53	45	39	35	33	32	34	39	51	51	33	22	14	8	5	3	2	1	2	3	5	8	14	22	33	51	51	39	34	32	33	35	39	45	53	59
90	130	62	50	43	39	36	34	34	34	37	53	29	20	13	8	5	2	1	0	1	2	5	8	13	20	29	53	37	34	34	34	36	39	43	50	62	130
95	59	53	45	39	35	33	32	34	39	51	51	33	22	14	8	5	3	2	1	2	3	5	8	14	22	33	51	51	39	34	32	33	35	39	45	53	59
100	42	41	37	33	30	29	30	35	37	34	41	38	28	15	8	4	3	3	3	3	3	4	8	15	28	38	41	34	37	35	30	29	30	33	37	41	42
105	35	34	32	30	29	29	32	39	33	31	43	33	27	16	9	5	4	3	4	3	4	5	9	16	27	33	43	31	33	39	32	29	29	30	32	34	35
110	35	34	33	33	32	34	39	53	38	38	40	42	28	16	10	6	5	4	4	4	5	6	10	16	28	42	40	38	38	53	39	34	32	33	33	34	35
115	33	33	33	34	36	43	40	34	34	48	30	28	32	16	9	6	4	4	4	4	6	9	16	32	28	30	48	34	34	40	43	36	34	33	33	33	
120	28	28	29	31	37	40	31	28	29	41	27	26	32	15	8	5	4	4	4	4	5	8	15	32	26	27	41	29	28	31	40	37	31	29	28	28	
125	28	29	30	33	34	30	27	27	31	42	27	28	30	16	10	7	6	6	6	6	6	7	10	16	30	28	27	42	31	27	27	30	34	33	30	29	28
130	29	30	30	30	30	28	28	30	37	36	29	30	32	21	15	12	11	10	10	10	11	12	15	21	32	30	29	36	37	30	28	28	30	30	30	29	
135	30	30	31	31	32	33	34	37	38	32	29	28	27	25	22	19	18	17	17	17	18	19	22	25	27	28	29	32	38	37	34	33	32	31	31	30	30
140	38	38	40	43	48	49	42	37	33	30	27	25	23	22	22	22	22	22	22	22	22	22	22	22	23	25	27	30	33	37	42	49	48	43	40	38	38
145	40	40	40	39	39	37	35	33	31	28	26	24	22	22	21	21	22	22	22	22	22	21	21	22	24	25	28	31	33	35	37	39	39	40	40	40	
150	37	37	37	37	36	35	34	32	30	28	26	24	23	22	22	22	22	22	22	22	22	22	22	22	23	24	26	28	30	32	34	35	36	37	37	37	
155	37	37	36	36	35	33	32	30	29	28	26	26	25	25	25	25	25	25	25	25	25	25	25	25	26	26	28	29	30	32	33	35	36	36	37	37	
160	34	34	33	33	32	32	31	31	30	29	29	29	28	28	28	28	28	28	28	28	28	28	28	28	29	29	29	29	30	31	31	32	32	33	33	34	34
165	35	35	35	35	34	34	34	34	34	34	34	34	33	33	33	33	33	33	33	33	33	33	33	33	34	34	34	34	34	34	34	34	34	35	35	35	35
170	41	41	41	41	41	41	41	41	41	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	41	41	41	41	41	41	41	41	41	
175	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52
180	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122	122

GAIN DEGRADATION IS
-0.561

Figure 4.41
Pattern 5. 2 Percent Error
Maximum Impact on the Main Beam

BORESIGHT AT 90.90

THETA

PHI

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
0	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	
5	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	46	46	46	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	
10	37	37	37	37	37	37	36	36	36	36	36	35	35	35	35	35	34	35	35	35	35	35	35	35	35	36	36	36	36	36	37	37	37	37	37	37	
15	34	34	34	34	33	33	32	32	31	30	30	29	29	29	28	28	28	28	28	28	28	28	28	29	29	29	30	30	31	32	32	33	33	34	34	34	
20	38	38	38	37	35	34	33	31	30	28	27	26	25	24	24	23	23	23	23	23	23	24	24	25	26	27	28	30	31	33	34	35	37	38	38	38	
25	50	50	49	47	44	41	38	34	31	28	26	24	23	22	21	20	20	20	20	20	20	21	22	23	24	25	28	31	34	38	41	44	47	49	50	50	
30	41	41	42	44	47	49	46	40	35	30	27	24	22	20	20	19	19	19	19	19	19	20	20	20	22	24	27	30	35	40	46	49	47	44	42	41	
35	36	37	37	37	38	39	39	38	36	32	28	25	22	20	20	20	19	19	19	19	19	19	20	20	20	22	25	28	32	36	38	39	39	38	37	37	36
40	40	41	43	44	44	43	42	39	35	31	28	26	24	23	22	22	24	26	26	26	24	22	22	23	24	26	28	31	35	39	42	43	44	44	43	41	
45	29	29	29	29	29	30	33	39	33	28	28	29	24	20	18	17	17	17	17	17	18	20	24	29	28	28	33	39	33	30	29	29	29	29	29		
50	27	28	29	29	28	26	24	25	30	42	29	30	28	17	12	9	8	8	8	8	9	12	17	28	30	29	42	30	25	24	26	28	29	29	28	27	
55	24	24	25	28	33	30	25	24	26	41	28	29	25	13	7	5	3	3	3	3	5	7	13	25	29	28	41	26	24	25	30	33	28	25	24	24	
60	23	23	24	25	29	39	32	27	27	36	28	26	26	12	6	3	2	2	2	2	2	3	6	12	26	26	28	36	27	27	32	39	29	25	24	23	
65	28	28	27	28	29	34	58	35	33	44	30	28	28	14	7	4	2	2	2	2	2	4	7	14	28	28	30	44	33	35	58	34	29	28	27	28	
70	33	33	32	31	31	33	39	38	33	37	40	34	27	14	8	4	2	2	2	2	2	4	8	14	27	34	40	37	33	38	39	33	31	31	32	33	
75	37	37	35	33	32	33	38	36	30	32	42	29	27	14	6	2	1	1	1	1	1	2	6	14	27	29	42	32	30	36	38	33	32	33	35	37	
80	45	43	39	36	34	34	40	42	31	31	43	32	29	14	6	2	1	1	1	1	1	2	6	14	29	32	43	31	31	42	40	34	36	36	39	43	
85	55	49	41	36	32	31	32	40	37	31	31	35	24	13	6	3	1	1	1	1	1	3	6	13	24	35	31	31	37	40	32	31	32	36	41	49	
90	128	54	42	35	31	29	29	35	40	29	28	29	21	13	6	3	1	0	0	0	1	3	6	13	21	29	28	29	40	35	29	29	31	35	42	54	
95	55	49	41	36	32	31	32	40	37	31	31	35	24	13	6	3	1	1	1	1	1	3	6	13	24	35	31	31	37	40	32	31	32	36	41	49	
100	45	43	39	36	34	34	40	42	31	31	43	32	29	14	6	2	1	1	1	1	1	2	6	14	29	32	43	31	31	42	40	34	36	39	43		
105	37	37	35	33	32	33	38	36	30	32	42	29	27	14	6	2	1	1	1	1	1	2	6	14	27	29	42	32	30	36	38	33	32	33	35	37	
110	33	33	32	31	31	33	39	38	33	37	40	34	27	14	8	4	2	2	2	2	2	4	8	14	27	34	40	37	33	38	39	33	31	31	32	33	
115	28	28	27	28	29	34	58	35	33	44	30	28	28	14	7	4	2	2	2	2	2	4	7	14	28	28	30	44	33	35	58	34	29	28	27	28	
120	23	23	24	25	29	39	32	27	27	36	28	26	26	12	6	3	2	2	2	2	2	3	6	12	26	26	28	36	27	27	32	39	29	25	24	23	
125	24	24	25	28	33	30	25	24	26	41	28	29	25	13	7	5	3	3	3	3	3	5	7	13	25	29	28	41	26	24	25	30	33	28	25	24	
130	27	28	29	29	28	26	24	25	30	42	29	30	28	17	12	9	8	8	8	8	9	12	17	28	30	29	42	30	25	24	26	28	29	29	28	27	
135	29	29	29	29	29	30	33	39	33	28	28	29	24	20	18	17	17	17	17	17	18	20	24	29	28	28	33	39	33	30	29	29	29	29	29		
140	40	41	43	44	44	43	42	39	35	31	28	26	24	23	22	22	24	26	26	26	24	22	22	23	24	26	28	31	35	39	42	43	44	44	43	41	
145	36	37	37	37	38	39	38	36	32	28	25	22	20	20	20	19	19	19	19	19	19	20	20	20	22	25	28	32	36	38	39	39	38	37	37	36	
150	41	41	42	44	47	49	46	40	35	30	27	24	22	20	20	19	19	19	19	19	19	20	20	20	22	24	27	30	35	40	46	49	47	44	42	41	
155	50	50	49	47	44	41	38	34	31	28	26	24	23	22	21	20	20	20	20	20	20	21	22	23	24	26	28	31	34	38	41	44	47	49	50	50	
160	38	38	38	37	35	34	33	31	30	28	27	26	25	24	24	23	23	23	23	23	23	24	24	25	26	27	28	30	31	33	34	35	37	38	38	38	
165	34	34	34	34	33	33	32	32	31	30	30	29	29	28	28	28	28	28	28	28	28	28	29	29	29	30	30	31	32	32	33	33	34	34	34		
170	37	37	37	37	37	36	36	36	36	36	36	35	35	35	35	35	34	35	35	35	35	35	35	35	35	35	36	36	36	36	37	37	37	37	37		
175	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	46	46	46	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47		
180	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117	

GAIN DEGRADATION IS
-0.317

Figure 4.42
Pattern 5. 1 Percent Error
Maximum Impact on Side Lobe 1

AD-A066 098

PENNSYLVANIA STATE UNIV UNIVERSITY PARK APPLIED RESE--ETC F/G 17/1
SENSITIVITY ANALYSIS IN LARGE SCALE MULTI-OBJECTIVE SYSTEMS.(U)
SEP 78 J H PERLIS N00017-73-C-1418

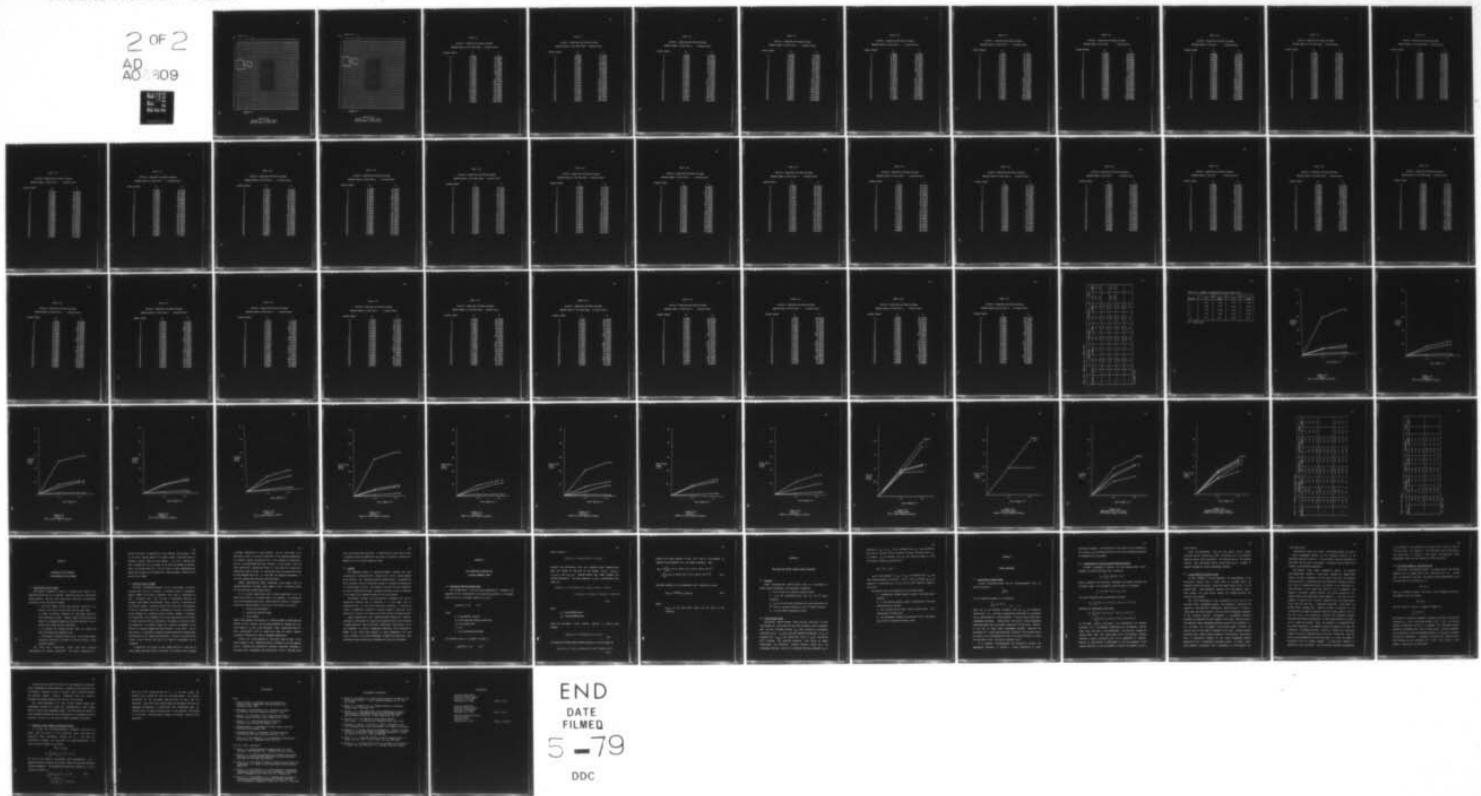
UNCLASSIFIED

ARL/PSU/TM-78-302

NL

2 OF 2

AD
A066098



DOBSIGHT AT 90,30

THETA

PHE

0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
0	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	
5	50	50	50	50	50	50	50	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	57	50	50	50	50	50	50	50	
10	52	52	52	51	51	50	50	49	48	48	47	47	46	46	45	45	45	45	45	45	45	45	46	46	47	47	48	48	49	50	50	51	51	52	52	52	
15	53	53	54	56	57	56	53	50	48	45	44	42	41	40	39	38	38	37	38	38	38	39	40	41	42	44	45	48	50	53	56	57	56	54	53	53	
20	37	37	38	38	39	40	42	45	49	49	44	41	38	36	35	34	33	33	33	33	33	34	35	36	38	41	44	49	49	45	42	40	39	38	38	37	
25	32	32	32	32	33	33	34	35	37	40	42	40	36	34	32	30	30	29	29	29	30	30	32	34	36	40	42	40	37	35	34	33	33	32	32	32	
30	32	32	31	31	31	31	31	31	32	33	35	36	34	32	30	28	28	27	27	27	28	28	30	32	34	36	35	33	32	31	31	31	31	31	31	32	
35	29	30	30	30	30	30	30	30	30	30	30	32	32	32	31	31	30	30	30	30	30	31	31	32	32	32	30	30	30	30	30	30	30	30	30	29	
40	23	23	24	24	25	26	27	28	29	29	29	29	31	34	32	29	28	27	27	28	29	32	34	31	29	29	29	29	28	27	26	25	24	24	23	23	
45	20	20	20	20	20	21	22	24	27	29	29	29	27	21	18	15	14	14	14	14	15	18	21	27	29	29	29	27	24	22	21	20	20	20	20	20	
50	21	21	21	20	20	19	19	20	23	27	28	31	23	15	10	8	7	6	6	6	7	8	10	15	23	31	28	27	23	20	19	19	20	20	21	21	
55	21	21	21	23	24	23	21	20	22	26	27	30	22	12	6	3	2	2	2	2	3	6	12	22	30	27	26	22	20	21	23	24	23	21	21	21	
60	19	19	19	20	22	25	29	26	25	28	26	27	24	11	5	2	1	0	1	0	1	2	5	11	24	27	26	28	25	26	29	25	22	20	19	19	
65	22	22	22	21	22	23	27	15	60	39	29	28	23	12	6	2	1	1	1	1	2	6	12	23	28	29	39	60	35	27	23	22	21	22	22	22	
70	32	32	31	30	29	29	29	20	27	29	34	40	22	12	6	3	1	1	1	1	3	6	12	22	40	34	29	27	28	29	29	29	30	31	32	32	
75	37	37	36	33	33	35	54	32	27	27	36	32	22	12	5	1	0	0	0	0	1	5	12	22	32	36	27	27	32	54	35	33	33	35	37	37	
80	41	39	36	32	29	29	31	40	34	32	51	31	24	12	5	1	0	0	0	0	1	5	12	24	31	51	32	34	40	31	29	29	32	36	39	41	
85	55	49	41	35	31	30	30	35	38	32	33	45	23	13	6	2	1	1	1	1	2	6	13	23	45	33	32	38	35	30	30	31	35	41	49	55	
90	130	56	44	37	33	31	31	34	32	28	29	42	21	12	6	3	1	0	0	0	1	3	6	12	21	42	29	28	32	34	31	31	33	37	44	55	130
95	55	49	41	35	31	30	30	35	38	32	33	45	23	13	6	2	1	1	1	1	2	6	13	23	45	33	32	38	35	30	30	31	35	41	49	55	
100	41	39	36	32	29	29	31	40	34	32	51	31	24	12	5	1	0	0	0	0	1	5	12	24	31	51	32	34	40	31	29	29	32	36	39	41	
105	37	37	35	33	33	35	54	32	27	27	36	32	22	12	5	1	0	0	0	0	1	5	12	22	32	36	27	27	32	54	35	33	33	35	37	37	
110	32	32	31	30	29	29	28	27	29	34	40	22	12	6	3	1	1	1	1	1	3	6	12	22	40	34	29	27	28	29	29	29	30	31	32	32	
115	22	22	22	21	22	23	27	35	60	39	29	28	23	12	6	2	1	1	1	1	2	6	12	23	28	29	39	60	35	27	23	22	21	22	22	22	
120	19	19	19	20	22	25	29	26	25	28	26	27	24	11	5	2	1	0	1	0	1	2	5	11	24	27	26	28	25	26	29	25	22	20	19	19	
125	21	21	21	23	24	23	21	20	22	26	27	30	22	12	6	3	2	2	2	2	3	6	12	22	30	27	26	22	20	21	23	24	23	21	21	21	
130	21	21	21	20	20	19	19	20	23	27	28	31	23	15	10	8	7	6	6	7	8	10	15	23	31	28	27	23	20	19	19	20	20	21	21	21	
135	20	20	20	20	21	22	24	27	29	29	29	27	21	18	15	14	14	14	14	14	15	18	21	27	29	29	29	27	24	22	21	20	20	20	20	20	
140	23	23	24	24	25	26	27	28	29	29	29	31	34	32	29	28	27	27	28	29	32	34	31	29	29	29	29	28	27	26	25	24	24	23	23		
145	29	30	30	30	30	30	30	30	30	30	30	32	32	32	31	31	30	30	30	30	31	31	32	32	32	30	30	30	30	30	30	30	30	30	29		
150	32	32	31	31	31	31	31	32	33	35	36	34	32	30	28	28	27	27	27	28	28	30	32	34	36	35	33	32	31	31	31	31	31	31	32	32	
155	32	32	32	32	33	33	34	35	37	40	42	40	36	34	32	30	30	29	29	29	30	32	34	36	40	42	40	37	35	34	33	33	32	32	32		
160	37	37	38	38	39	40	42	45	49	49	44	41	38	36	35	34	33	33	33	33	34	35	36	38	41	44	49	49	45	42	40	39	38	38	37	37	
165	53	53	54	56	57	56	53	50	48	45	44	42	41	40	39	38	38	37	38	38	38	39	40	41	42	44	45	48	50	53	56	57	56	54	53	53	
170	52	52	52	51	51	50	50	49	48	48	47	46	46	45	45	45	45	45	45	45	45	45	46	46	47	47	48	48	49	50	50	51	51	52	52	52	
175	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50		
180	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	120	

JAIN DEGRADATION IS
-0.537

Figure 4.44
Pattern 5. 1 Percent Error
Maximum Impact on Side Lobe 2

PMI

	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
0	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130
5	56	56	56	56	56	56	56	56	57	57	57	57	57	57	57	58	58	58	58	58	57	57	57	57	57	57	57	57	57	56	56	56	56	56	56	56	
10	43	43	43	43	43	43	43	43	43	43	44	44	44	45	45	45	45	45	45	45	45	44	44	44	43	43	43	43	43	43	43	43	43	43	43	43	
15	40	40	40	39	39	39	38	37	37	36	36	36	36	36	37	37	38	38	38	38	37	37	36	36	36	36	36	36	37	37	38	39	39	39	40	40	40
20	30	30	31	31	32	33	34	35	35	34	33	32	31	31	31	32	32	33	33	33	32	32	31	31	31	32	33	34	35	35	34	33	32	31	31	30	30
25	24	24	24	24	25	25	26	27	28	30	31	30	28	27	27	28	29	29	29	28	27	27	27	28	30	31	30	28	27	26	25	25	24	24	24	24	
30	23	23	23	23	22	22	22	23	23	25	26	27	27	26	25	25	26	26	26	25	25	25	26	27	27	26	25	23	23	22	22	22	23	23	23	23	
35	21	21	21	21	21	21	21	21	21	22	23	24	25	26	26	26	27	27	27	27	26	26	26	25	24	23	22	21	21	21	21	21	21	21	21	21	
40	18	18	18	18	18	18	19	20	21	21	22	23	24	25	25	24	24	24	24	24	25	25	24	23	23	22	21	21	20	19	18	18	18	18	18		
45	15	15	15	15	15	16	16	18	19	21	22	23	22	18	15	13	12	13	13	13	12	13	15	18	22	23	22	21	19	18	16	16	15	15	15	15	
50	17	17	16	16	15	15	15	15	17	19	23	25	20	13	8	6	5	5	5	5	5	6	8	13	20	25	23	19	17	15	15	15	15	16	17	17	
55	18	18	18	18	18	18	17	16	17	19	23	28	25	10	5	2	1	1	1	1	2	5	10	20	28	23	19	17	16	17	18	18	18	18	18	18	
60	16	16	16	17	18	19	22	22	21	22	24	27	22	10	4	1	0	0	0	0	1	4	12	22	27	24	22	21	22	22	19	18	17	16	16	16	
65	18	18	18	18	18	18	20	24	29	30	26	26	21	11	5	1	0	0	0	0	1	5	11	21	26	26	30	29	24	20	18	18	18	18	18	18	
70	28	28	27	26	24	28	24	23	22	22	24	27	29	19	11	5	2	0																			

GAIN DEGRADATION IS
-0.546

Figure 4.45
Pattern 5. 2 Percent Error
Maximum Impact on Side Lobe 2

Table 4.6

Pattern 1 Amplitude and Phase Settings

Maximum Impact on the Main Beam. 1 percent error.

Element Number	A_i	α_i
1	0.99001	41.49132
2	0.81989	46.88094
3	0.54476	57.98431
4	0.29153	77.83815
5	0.13375	121.66500
6	0.74183	25.63405
7	0.56344	37.32784
8	0.37299	43.03300
9	0.18821	82.60611
10	0.01031	-19.88239
11	0.48664	8.66110
12	0.32213	7.94737
13	0.16964	10.60483
14	0.05548	-69.35330
15	0.02564	-107.34150
16	0.36302	-23.98924
17	0.23218	-54.83496
18	0.17712	-54.10857
19	-0.00683	-100.55660
20	0.04657	-91.89992
21	0.18199	-75.43584
22	0.15151	-53.00188
23	0.08324	-112.88140
24	0.01772	-103.04570
25	0.00364	-155.93350

Table 4.7

Pattern 1. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 2 Percent Error.

Element Number	A_i	α_i
1	0.98007	37.91867
2	0.80996	43.30829
3	0.53490	54.41331
4	0.30084	74.26247
5	0.12382	118.08930
6	0.75111	22.06139
7	0.57337	33.75526
8	0.38292	39.46039
9	0.19814	79.03043
10	0.02023	-16.30980
11	0.49657	12.23315
12	0.33206	11.51942
13	0.17957	14.17688
14	0.04555	-65.77925
15	0.01571	-103.76740
16	0.37295	-20.41682
17	0.22225	-51.26228
18	0.18361	-50.53598
19	-0.01676	-103.90100
20	0.03664	-88.32588
21	0.17206	-71.86179
22	0.14157	-49.42924
23	0.07331	-109.30740
24	0.00779	-99.47165
25	-0.00629	-153.67570

Table 4.8

Pattern 1. Amplitude and Phase settings.

Maximum Impact on Side lobe 1. 1 Percent Error.

Element Number	A_1	α_1
1	1.00994	48.68059
2	0.83987	54.07021
3	0.54873	65.17307
4	0.27155	85.02690
5	0.15368	128.85380
6	0.72184	25.63399
7	0.54346	37.32780
8	0.35301	43.03294
9	0.18796	82.72293
10	0.01031	-19.90404
11	0.46666	8.66104
12	0.30215	7.94732
13	0.14966	10.60477
14	0.07143	-76.54237
15	0.04562	-114.53050
16	0.36302	-31.17850
17	0.25216	-62.02150
18	0.19710	-61.29488
19	0.01315	-93.37694
20	0.06655	-98.86388
21	0.20197	-82.62491
22	0.17149	-60.18817
23	0.08324	-120.07050
24	0.01772	-110.23470
25	0.01760	-156.83910

Table 4.9

Pattern 1. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 2 Percent Error.

Element Number	A_1	α_1
1	1.01985	52.25250
2	0.84979	57.63838
3	0.53490	68.74329
4	0.26162	88.59712
5	0.12382	132.42400
6	0.71192	22.06139
7	0.53353	33.75526
8	0.36832	39.46039
9	0.19814	89.43881
10	-0.01318	-19.15211
11	0.45673	12.23315
12	0.29222	11.51942
13	0.13973	14.17688
14	0.08538	-80.11287
15	0.05555	-118.10100
16	0.37295	-34.75050
17	0.26209	-65.59200
18	0.20703	-64.86537
19	0.02307	-100.20710
20	0.07647	-102.61650
21	0.21190	-86.19540
22	0.18141	-63.75867
23	0.07331	-123.64100
24	0.00779	-113.80520
25	0.02713	-160.40960

Table 4.10

Pattern 1. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 1 Percent Error.

Element Number	A_i	α_i
1	1.00961	41.74815
2	0.82021	53.95192
3	0.56436	58.09955
4	0.29120	84.90733
5	0.13407	128.73420
6	0.74150	32.70503
7	0.54379	44.39886
8	0.37266	43.15051
9	0.18788	89.67531
10	-0.00059	-26.80592
11	0.48631	8.54297
12	0.30248	0.87618
13	0.16931	3.53352
14	0.05580	-76.27386
15	0.04529	-114.38990
16	0.36269	-24.10683
17	0.24270	-61.90031
18	0.19677	-54.22615
19	-0.00651	-93.50945
20	0.06622	-98.96780
21	0.18231	-75.55025
22	0.15183	-60.06697
23	0.10313	-112.99580
24	0.01804	-110.11350
25	0.02329	-149.76440

Table 4.11

Pattern 1. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 2 Percent Error.

Element Number	A_i	α_i
1	1.01988	52.25273
2	0.80996	57.64056
3	0.57462	54.41354
4	0.30146	88.59930
5	0.12382	132.42620
6	0.75175	36.39546
7	0.53353	33.75549
8	0.38292	39.46065
9	0.19814	93.36726
10	0.02024	-30.64375
11	0.49657	12.23329
12	0.29222	-2.81394
13	0.17957	-0.15656
14	0.04555	-80.11456
15	0.05555	-103.83400
16	0.36733	-20.41698
17	0.26209	-65.59369
18	0.20703	-50.53621
19	-0.01676	-93.19521
20	0.07647	-100.87590
21	0.17206	-71.86348
22	0.18135	-63.76036
23	0.07893	-109.30900
24	0.00779	-113.80690
25	0.03355	-146.07760

Table 4.12

Pattern 1. Amplitude and Phase settings.

Maximum Impact on the Null.

1 Percent Error.

Element Number	A_1	α_1
1	0.99021	41.49115
2	0.81994	52.09497
3	0.56463	65.17279
4	0.29153	85.02663
5	0.15373	128.85350
6	0.72342	32.82320
7	0.54346	44.51698
8	0.35323	43.03284
9	0.16823	82.72266
10	0.01031	-27.07143
11	0.48664	1.47198
12	0.30764	0.75828
13	0.14966	10.60474
14	0.05548	-76.54156
15	0.02570	-114.52970
16	0.34304	-23.98907
17	0.23218	-62.02069
18	0.19710	-59.32150
19	0.01315	-93.49113
20	0.06655	-92.01680
21	0.20197	-75.55272
22	0.17149	-60.11172
23	0.08329	-120.06970
24	0.01772	-110.23390
25	0.02362	-156.83830

Table 4.13

Pattern 1. Amplitude and Phase settings.

Maximum Impact on the Null.

2 Percent Error.

Element Number	A_i	α_i
1	0.98647	37.91867
2	0.81008	53.71478
3	0.57450	68.74329
4	0.30146	88.59712
5	0.16365	132.42400
6	0.71232	36.39523
7	0.53353	48.08904
8	0.34950	39.46039
9	0.15830	79.03043
10	0.02023	-30.64362
11	0.49657	-2.10023
12	0.30314	-2.81393
13	0.13973	14.17688
14	0.04555	-80.11287
15	0.01583	-118.10100
16	0.33311	-20.41682
17	0.22225	-65.59200
18	0.20703	-60.94119
19	0.02307	-89.80022
20	0.07647	-88.32588
21	0.21190	-71.86179
22	0.18141	-61.45319
23	0.07343	-123.64100
24	0.00779	-113.80520
25	0.03355	-160.40960

Table 4.14

Pattern 2. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 1 Percent Error.

Element Number	A_1	α_1
1	0.99001	82.99451
2	0.71493	72.69763
3	0.41100	42.82335
4	0.22526	-13.39055
5	0.15096	-55.64073
6	0.66943	78.32292
7	0.50047	59.45757
8	0.24733	17.08455
9	0.25547	-29.58571
10	0.12430	-48.79063
11	0.25718	55.11913
12	0.22163	16.72641
13	0.28098	-5.35801
14	0.22563	-32.30592
15	0.12461	-58.47069
16	0.15506	-27.54604
17	0.15947	-25.03529
18	0.22672	-33.80103
19	0.13454	-50.83426
20	0.01818	-64.63976
21	0.12854	-62.08603
22	0.10943	-50.34950
23	0.05815	-44.35547
24	0.06849	-60.85086
25	-0.00923	-81.58513

Table 4.15

Pattern 2. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 2 Percent Error.

Element Number	A_1	α_1
1	0.98008	79.42007
2	0.70500	69.12321
3	0.42092	39.25085
4	0.23519	-9.81820
5	0.14103	-52.06812
6	0.65950	74.74850
7	0.51040	55.88519
8	0.25725	15.54277
9	0.26540	-26.01320
10	0.12097	-45.21815
11	0.26710	51.54634
12	0.23156	14.19118
13	0.29091	-1.78578
14	0.23555	-28.73344
15	0.11468	-54.89915
16	0.16498	-23.97356
17	0.16940	-21.46283
18	0.23665	-30.22852
19	0.12461	-47.26175
20	0.00825	-61.06621
21	0.11861	-58.51248
22	0.11317	-46.77702
23	0.06899	-40.78296
24	0.05857	-57.27869
25	-0.01916	-85.18558

Table 4.16

Pattern 2. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 1 Percent Error.

Element Number	A_1	α_1
1	0.99001	82.99374
2	0.71493	72.69687
3	0.39101	42.82324
4	0.22526	-20.57953
5	0.17094	-62.80807
6	0.68941	85.51195
7	0.50047	66.64658
8	0.22735	22.23720
9	0.23549	-29.58563
10	0.12430	-55.97975
11	0.23719	55.11902
12	0.22163	15.79092
13	0.28098	-12.54722
14	0.22563	-39.49507
15	0.14459	-62.30493
16	0.13508	-27.54596
17	0.13949	-25.03519
18	0.20674	-33.80095
19	0.13454	-58.02119
20	0.01818	-71.82802
21	0.14852	-62.08505
22	0.12908	-57.53702
23	0.07780	-51.54457
24	0.08847	-61.31598
25	0.01075	-74.48824

Table 4.17

Pattern 2. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 2 Percent Error.

Element Number	A_1	α_1
1	0.98007	79.41789
2	0.70500	69.12102
3	0.38108	39.25060
4	0.23518	-24.15146
5	0.18087	-66.35463
6	0.69933	89.08096
7	0.51039	70.21561
8	0.21741	25.80904
9	0.22556	-26.01297
10	0.11437	-59.54799
11	0.22726	51.54608
12	0.23156	12.21849
13	0.29091	-16.11913
14	0.23555	-43.06696
15	0.15451	-65.30235
16	0.12515	-23.97336
17	0.12956	-21.46260
18	0.19681	-30.22830
19	0.12461	-61.59137
20	0.00825	-75.39813
21	0.15845	-66.00323
22	0.13901	-61.10658
23	0.08773	-55.11650
24	0.09840	-67.68253
25	0.02067	-71.09834

Table 4.18

Pattern 2. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 1 Percent Error.

Element Number	A_1	α_1
1	1.00995	83.11012
2	0.71493	79.88518
3	0.41099	42.82304
4	0.20528	-13.39043
5	0.17094	-62.82626
6	0.66943	85.51047
7	0.50047	59.57318
8	0.22734	15.04794
9	0.25547	-36.77461
10	0.13350	-48.79034
11	0.25717	55.19592
12	0.20165	15.79082
13	0.28098	-5.35789
14	0.20837	-32.30568
15	0.14459	-65.65773
16	0.15505	-34.65994
17	0.13949	-32.22432
18	0.22672	-33.80075
19	0.13454	-58.02000
20	0.01818	-71.82681
21	0.14852	-69.27309
22	0.12908	-50.34926
23	0.05782	-51.54437
24	0.08847	-68.03792
25	-0.00923	-81.67628

Table 4.19

Pattern 2. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 2 Percent Error.

Element Number	A_i	α_i
1	1.01983	79.41620
2	0.70500	83.45398
3	0.42092	39.25041
4	0.19535	-9.81802
5	0.18086	-66.39648
6	0.65949	89.07927
7	0.51039	55.88481
8	0.21741	11.47547
9	0.26539	-40.34660
10	0.14813	-45.21776
11	0.26710	52.10831
12	0.19172	12.21839
13	0.29090	-1.78575
14	0.22138	-28.73311
15	0.15451	-69.22795
16	0.16498	-37.74136
17	0.12956	-35.79630
18	0.23664	-30.22812
19	0.12461	-61.59027
20	0.00825	-75.39703
21	0.15845	-72.84331
22	0.13901	-46.77669
23	0.04789	-55.11636
24	0.09840	-71.60814
25	-0.01916	-85.24651

Table 4.20

Pattern 3. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 1 Percent Error.

Element Number	A_i	α_i
1	0.99033	83.10712
2	0.71665	72.81026
3	0.35546	47.94011
4	0.24539	-18.50732
5	0.15889	-40.51294
6	0.68453	78.43555
7	0.48179	59.57018
8	0.25729	16.45805
9	0.24596	-27.14305
10	0.14811	-57.90044
11	0.25928	55.23537
12	0.20170	17.95587
13	0.27051	-5.47562
14	0.21723	-37.42282
15	0.09138	-48.98233
16	0.15109	-31.51103
17	0.15355	-22.71280
18	0.21828	-33.91832
19	0.12946	-45.95184
20	0.04597	-54.51227
21	0.10977	-65.39644
22	0.10497	-53.82712
23	0.07493	-40.11311
24	0.06588	-47.36270
25	-0.00894	-76.95766

Table 4.21

Pattern 3. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 2 Percent Error.

Element Number	A_1	α_1
1	0.98008	79.41866
2	0.70639	69.12178
3	0.36572	44.25023
4	0.25565	-14.81753
5	0.16914	-36.82297
6	0.67427	74.74707
7	0.49204	55.88377
8	0.26755	15.53319
9	0.25621	-23.45320
10	0.13786	-54.21576
11	0.26953	51.54518
12	0.21196	15.02099
13	0.28077	-1.78571
14	0.22749	-33.73288
15	0.08113	-45.29236
16	0.16135	-27.82106
17	0.16381	-19.02280
18	0.22854	-30.22832
19	0.11921	-42.26193
20	0.03571	-50.82227
21	0.09952	-61.70944
22	0.09588	-50.13727
23	0.07958	-36.42317
24	0.05563	-43.67273
25	-0.01919	-80.56252

Table 4.22

Pattern 3. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 1 Percent Error.

Element Number	A_i	α_i
1	0.99001	90.18063
2	0.71632	79.88376
3	0.33603	55.01173
4	0.22574	-18.38959
5	0.13924	-40.39520
6	0.70418	85.48949
7	0.48212	66.64369
8	0.23764	15.04776
9	0.24629	-34.21460
10	0.16777	-64.97406
11	0.23963	55.11768
12	0.18205	17.83820
13	0.25108	-5.35782
14	0.19758	-37.30504
15	0.09105	-48.86459
16	0.13144	-31.39325
17	0.13390	-22.59503
18	0.21861	-40.98973
19	0.14912	-53.02333
20	0.06562	-61.58011
21	0.12943	-72.47006
22	0.12463	-60.89513
23	0.05528	-39.99532
24	0.06556	-47.24496
25	-0.00926	-77.05322

Table 4.23

Pattern 3. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 2 Percent Error.

Element Number	A_i	α_i
1	0.98007	93.75113
2	0.70639	83.45425
3	0.33230	58.57985
4	0.21581	-14.81742
5	0.12931	-36.82277
6	0.71411	89.03709
7	0.49204	70.21419
8	0.22771	11.47540
9	0.25621	-37.78667
10	0.17770	-68.54538
11	0.22970	51.54495
12	0.17212	14.26602
13	0.24735	-1.78570
14	0.18765	-33.73273
15	0.08113	-45.29216
16	0.12151	-27.82083
17	0.12397	-19.02257
18	0.22854	-44.56186
19	0.15904	-56.59546
20	0.07554	-65.15143
21	0.13935	-76.04137
22	0.13455	-64.46645
23	0.04536	-42.61943
24	0.05563	-43.67253
25	-0.01919	-79.54712

Table 4.24

Pattern 3. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 1 Percent Error.

Element Number	A_1	α_1
1	1.00995	90.18057
2	0.71632	72.81178
3	0.33581	55.01170
4	0.24572	-25.57875
5	0.15922	-47.58437
6	0.70418	85.50900
7	0.46214	59.57170
8	0.23764	22.23676
9	0.24629	-34.21460
10	0.16777	-63.57092
11	0.25961	62.30411
12	0.18205	17.83817
13	0.27084	-5.35781
14	0.19758	-44.49431
15	0.11104	-56.05379
16	0.15142	-31.39322
17	0.13390	-29.78421
18	0.21861	-33.80057
19	0.12913	-53.02332
20	0.06552	-61.57962
21	0.12943	-71.98157
22	0.10470	-60.89464
23	0.06820	-39.99530
24	0.08554	-54.43413
25	-0.00926	-69.98135

Table 4.25

Pattern 3. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 2 Percent Error.

Element Number	A_i	α_i
1	1.01987	93.75259
2	0.70639	69.12108
3	0.32588	58.58133
4	0.25565	-29.15099
5	0.16914	-51.03870
6	0.71411	89.08102
7	0.45221	55.88370
8	0.22771	25.80901
9	0.25621	-37.78683
10	0.17770	-68.54456
11	0.26953	65.87613
12	0.17213	14.26612
13	0.28077	-1.78570
14	0.18765	-48.06648
15	0.12096	-59.62334
16	0.16135	-27.82098
17	0.12397	-33.35643
18	0.22854	-30.22826
19	0.11920	-56.59561
20	0.06434	-65.15274
21	0.13935	-70.39001
22	0.12109	-64.46776
23	0.05657	-36.42313
24	0.09547	-58.00421
25	-0.01919	-66.29224

Table 4.26

Pattern 3. Amplitude and Phase settings.

Maximum Impact on the Null

1 Percent Error.

Element Number	A_i	α_i
1	0.99033	83.10838
2	0.71665	79.76546
3	0.33614	54.89377
4	0.24539	-18.50745
5	0.15889	-40.51306
6	0.70385	85.39075
7	0.48179	59.57143
8	0.25729	15.16562
9	0.22663	-34.09666
10	0.14812	-64.85544
11	0.25928	55.23553
12	0.18238	24.90955
13	0.25119	-5.47570
14	0.21723	-37.42291
15	0.11071	-48.98245
16	0.13176	-38.46460
17	0.15355	-22.71289
18	0.21828	-33.91841
19	0.12946	-52.90540
20	0.04597	-61.46149
21	0.12910	-72.35143
22	0.11508	-60.90169
23	0.05561	-47.04689
24	0.06588	-54.31622
25	0.01039	-69.98114

Table 4.27

Pattern 3. Amplitude and Phase settings.

Maximum Impact on the Null

2 Percent Error.

Element Number	A_1	α_1
1	0.98007	79.41647
2	0.70639	83.45425
3	0.32588	58.57985
4	0.25565	-25.22350
5	0.16914	-44.31465
6	0.71411	89.07954
7	0.49204	55.88351
8	0.26754	11.47540
9	0.21637	-37.78667
10	0.13786	-68.54538
11	0.26953	51.54495
12	0.17212	28.59955
13	0.24093	-1.78570
14	0.22749	-33.73273
15	0.12096	-45.29216
16	0.12151	-42.15465
17	0.16381	-24.37172
18	0.22854	-30.22809
19	0.11920	-56.59546
20	0.03571	-65.15143
21	0.13935	-76.04137
22	0.11554	-62.16019
23	0.04536	-50.71310
24	0.05563	-58.00290
25	0.02064	-76.69783

Table 4.28

Pattern 4. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 1 Percent Error.

Element Number	A_i	α_i
1	0.99000	80.22981
2	0.57462	71.41367
3	0.15240	30.66972
4	0.14752	-58.21255
5	0.09088	-96.90738
6	0.54779	80.69716
7	0.35826	56.28606
8	0.24028	9.77547
9	0.19048	-21.48817
10	0.09229	-73.88510
11	-0.00589	-171.35440
12	0.15118	2.71470
13	0.27176	9.15734
14	0.20907	-10.04460
15	0.09625	-8.75852
16	0.14645	-95.75626
17	0.11651	-41.43205
18	0.17666	-3.24723
19	0.15050	4.94055
20	0.02904	-6.78648
21	0.07705	-96.84164
22	0.02588	-75.27524
23	0.04555	-36.44624
24	0.03225	111.78390
25	0.02849	85.51938

Table 4.29

Pattern 4. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 2 Percent Error.

Element Number	A_i	α_i
1	0.98007	76.53784
2	0.56469	67.72171
3	0.16233	27.09732
4	0.13759	-54.52652
5	0.08095	-93.21754
6	0.53786	77.00519
7	0.36819	52.71344
8	0.25021	13.34765
9	0.20040	-17.91579
10	0.08236	-70.19528
11	-0.01582	-170.89520
12	0.16111	6.28663
13	0.28168	12.72952
14	0.21899	-6.47231
15	0.10617	-5.18623
16	0.13653	-92.06642
17	0.12637	-37.85968
18	0.18658	0.32496
19	0.16043	8.51276
20	0.03897	-3.21438
21	0.06712	-93.15181
22	0.01596	-71.58540
23	0.05548	-32.87387
24	0.04218	108.09200
25	0.03842	81.82742

Table 4.30

Pattern 4. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 1 Percent Error.

Element Number	A_i	α_i
1	0.99001	80.23009
2	0.57462	78.48587
3	0.13242	37.85910
4	0.14752	-65.28502
5	0.09088	-103.97980
6	0.56778	87.76935
7	0.35826	63.47284
8	0.24029	2.56845
9	0.19048	-28.67743
10	0.11227	-80.95758
11	-0.00589	-178.27770
12	0.15097	-4.47435
13	0.25178	9.15739
14	0.18909	-10.04464
15	0.07627	-8.75856
16	0.14646	-102.82870
17	0.09658	-41.43213
18	0.17666	-10.43635
19	0.15050	-2.24838
20	0.02904	-13.97578
21	0.09703	-96.84274
22	0.04586	-82.34772
23	0.02557	-43.63550
24	0.03225	118.85610
25	0.02849	92.59158

Table 4.31

Pattern 4. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 2 Percent Error.

Element Number	A_1	α_1
1	0.98007	76.53615
2	0.56469	82.05467
3	0.12249	41.43098
4	0.13759	-68.85524
5	0.10301	-107.55000
6	0.57770	91.33815
7	0.36819	67.04164
8	0.25021	-0.98577
9	0.20040	-32.24942
10	0.12219	-82.19569
11	-0.01582	-181.36070
12	0.15954	-8.04643
13	0.24185	12.72942
14	0.17916	-6.47224
15	0.06634	-5.18616
16	0.13652	-106.39890
17	0.08665	-37.85950
18	0.18658	-14.00842
19	0.16043	-5.82033
20	0.03897	-17.54759
21	0.10695	-93.15073
22	0.05579	-82.00270
23	0.01564	-47.20738
24	0.04218	122.42490
25	0.03842	96.16039

Table 4.32

Pattern 4. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 1 Percent Error.

Element Number	A_i	α_i
1	1.00995	87.30200
2	0.57462	71.41594
3	0.13242	37.85910
4	0.16751	-58.21365
5	0.11086	-103.97980
6	0.56778	87.76935
7	0.33828	56.28613
8	0.22031	9.77551
9	0.19048	-21.48825
10	0.11227	-80.95758
11	0.01409	-178.35520
12	0.15118	-4.47435
13	0.25178	9.15739
14	0.20907	-10.04464
15	0.07627	-15.94782
16	0.16644	-102.82870
17	0.11651	-48.62131
18	0.15668	-10.43635
19	0.15050	4.94060
20	0.00906	-13.97578
21	0.09703	-103.83660
22	0.02588	-82.34772
23	0.04555	-43.63550
24	0.03225	118.83660
25	0.00851	92.59158

Table 4.33

Pattern 4. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 2 Percent Error.

Element Number	A_1	α_1
1	1.01985	90.87250
2	0.56469	67.72171
3	0.12249	41.43115
4	0.17743	-54.52652
5	0.12078	-107.55110
6	0.57770	61.33984
7	0.32835	52.71344
8	0.21038	13.34765
9	0.20040	-17.91579
10	0.12219	-84.52890
11	0.02401	-177.61450
12	0.16111	-8.04650
13	0.24185	12.72952
14	0.21899	-11.57497
15	0.06634	-19.51967
16	0.17636	-106.40000
17	0.12637	-52.19350
18	0.14675	-14.00849
19	0.16043	7.43505
20	-0.00087	-11.92710
21	0.10696	-105.17790
22	0.03318	-85.91902
23	0.05548	-47.20755
24	0.04218	122.38420
25	-0.00141	91.33383

TABLE 4.34

Pattern 5. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 1 Percent Error.

Element Number	A_i	α_i
1	0.65060	99.26013
2	0.37304	106.42760
3	0.00745	101.81600
4	0.08857	-82.23674
5	0.08640	-45.24315
6	1.00998	38.29744
7	0.66055	33.42734
8	0.18020	34.65767
9	0.07037	-89.73033
10	0.13958	-143.54060
11	0.82609	6.09347
12	0.48946	4.94162
13	0.18212	-16.82225
14	0.06276	-164.23010
15	0.05580	-161.97100
16	0.52866	-71.47241
17	0.37535	-74.10629
18	0.11292	-89.93283
19	0.01421	59.96715
20	0.09094	65.32414
21	0.26640	-97.65575
22	0.16686	-103.73350
23	0.04862	-91.10423
24	0.06307	39.77171
25	0.04109	50.08115

TABLE 4.35

Pattern 5. Amplitude and Phase settings.

Maximum Impact on the Main Beam. 2 Percent Error.

Element Number	A_1	α_1
1	0.64067	95.68428
2	0.36311	102.85170
3	-0.00248	103.27540
4	0.07864	-78.66121
5	0.09632	-41.67044
6	1.01987	34.72470
7	0.67048	29.85460
8	0.19013	31.08493
9	0.06044	-86.15480
10	0.12964	-139.96510
11	0.83602	9.66548
12	0.49939	8.51364
13	0.19205	-13.24991
14	0.05283	-160.64750
15	0.04587	-158.39550
16	0.51873	-67.89690
17	0.34742	-70.53078
18	0.10299	-86.35732
19	0.02414	56.39423
20	0.10087	61.74829
21	0.25647	-94.08023
22	0.15693	-100.15800
23	0.03870	-87.52872
24	0.07300	36.19897
25	0.05101	46.50844

TABLE 4.36

Pattern 5. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 1 Percent Error.

Element Number	A_1	α_1
1	0.67058	99.37532
2	0.39302	106.54280
3	0.02743	108.96940
4	0.10855	-89.40341
5	0.08640	-52.43205
6	0.99001	38.29704
7	0.64057	33.46292
8	0.16022	41.81230
9	0.07037	-96.91640
10	0.15956	-150.72670
11	0.82609	6.09326
12	0.48946	4.94142
13	0.18212	-16.82198
14	0.06276	-164.31280
15	0.07572	-162.08570
16	0.54864	-78.65848
17	0.37733	-81.29237
18	0.13289	-97.11890
19	-0.00577	59.99971
20	0.09094	72.51126
21	0.26640	-97.77045
22	0.16685	-103.84820
23	0.04862	-91.21893
24	0.06307	39.77130
25	0.02111	50.08075

TABLE 4.37

Pattern 5. Amplitude and Phase settings.

Maximum Impact on Side Lobe 1. 2 Percent Error.

Element Number	A_1	α_1
1	0.68051	106.09150
2	0.40294	113.25900
3	0.03736	110.26660
4	0.11847	-92.95166
5	0.07910	-56.00414
6	0.98007	34.72459
7	0.63064	29.85451
8	0.15029	43.11046
9	0.07766	-100.48820
10	0.16948	-154.29850
11	0.83602	9.66543
12	0.49939	8.51360
13	0.17931	-13.24987
14	0.07365	-160.62240
15	0.08558	-158.39530
16	0.55856	-82.23029
17	0.38725	-84.86417
18	0.14282	-100.69070
19	-0.01570	58.69946
20	0.10086	76.08180
21	0.25647	-94.08002
22	0.15692	-100.15770
23	0.03870	-87.52849
24	0.07300	44.78825
25	0.01378	46.98734

TABLE 4.38

Pattern 5. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 1 Percent Error.

Element Number	A_1	α_1
1	0.65060	106.44720
2	0.39302	113.61470
3	0.00745	101.93120
4	0.10855	-89.42329
5	0.06642	-45.24286
6	1.00995	45.48639
7	0.64057	33.42696
8	0.16022	41.84660
9	0.07037	-89.84552
10	0.15956	-143.65580
11	0.80611	-1.09571
12	0.48946	4.94144
13	0.16214	-24.01114
14	0.06276	-171.38470
15	0.07578	-165.52790
16	0.54864	-78.65897
17	0.35735	-81.29286
18	0.13290	-90.04802
19	-0.00577	60.04353
20	0.09094	72.43388
21	0.26640	-104.84230
22	0.18683	-110.92000
23	0.04862	-91.21942
24	0.04309	46.96063
25	0.03351	50.08078

TABLE 4.39

Pattern 5. Amplitude and Phase settings.

Maximum Impact on Side Lobe 2. 2 Percent Error.

Element Number	A_i	α_i
1	0.64067	110.01770
2	0.40294	117.18520
3	-0.00248	104.43720
4	0.11847	-92.99461
5	0.05649	-41.67038
6	1.00713	49.05843
7	0.63064	29.85451
8	0.15029	45.41866
9	0.06044	-86.15459
10	0.16948	-139.96490
11	0.79618	-4.66772
12	0.49939	8.51360
13	0.15221	-27.58324
14	0.05283	-174.95600
15	0.08570	-162.32360
16	0.55856	-82.23029
17	0.34742	-84.86417
18	0.14282	-86.35709
19	-0.01570	58.69946
20	0.10086	73.77478
21	0.25647	-108.41360
22	0.19676	-114.49130
23	0.03870	-87.52849
24	0.03317	50.53267
25	0.01147	46.50833

Table 4.40. Summary of A_1 , α_1 Impact Data.

Pattern	Error	Main Beam		Side Lobe 1		Side Lobe 2		Null	
		$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}
1	1%	0.87	1.0	4.80	5.0	5.10	6.0	29.00	29.0
	2%	0.91	2.0	7.90	8.0	5.60	9.0	34.0	34.0
2	1%	0.92	1.0	7.78	9.0	5.51	6.0		
	2%	1.15	1.0	11.78	13.0	9.12	11.0		
3	1%	0.92	1.0	8.36	8.0	6.00	4.0	24.80	21.0
	2%	1.10	1.0	11.64	12.0	10.36	9.0	29.60	26.0
4	1%	0.81	1.0	7.10	8.0	7.27	9.0		
	2%	1.81	2.0	10.50	12.0	11.64	14.0		
5	1%	0.97	1.0	10.85	11.0	7.53	5.0		
	2%	1.97	2.0	15.50	16.0	11.02	9.0		

 $\Delta\bar{G}$, ΔG_{MAX} in dB.

Table 4.41. Summary of Aggregated Side Lobe Impact Data.

Pattern	1%		2%	
	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}
1	4.95	5.50	6.75	8.50
2	6.65	7.50	10.45	12.00
3	7.18	6.00	11.00	10.50
4	7.23	8.50	11.07	13.00
5	9.19	8.0	13.56	12.5

$\Delta\bar{G}$, ΔG_{MAX} in dB.

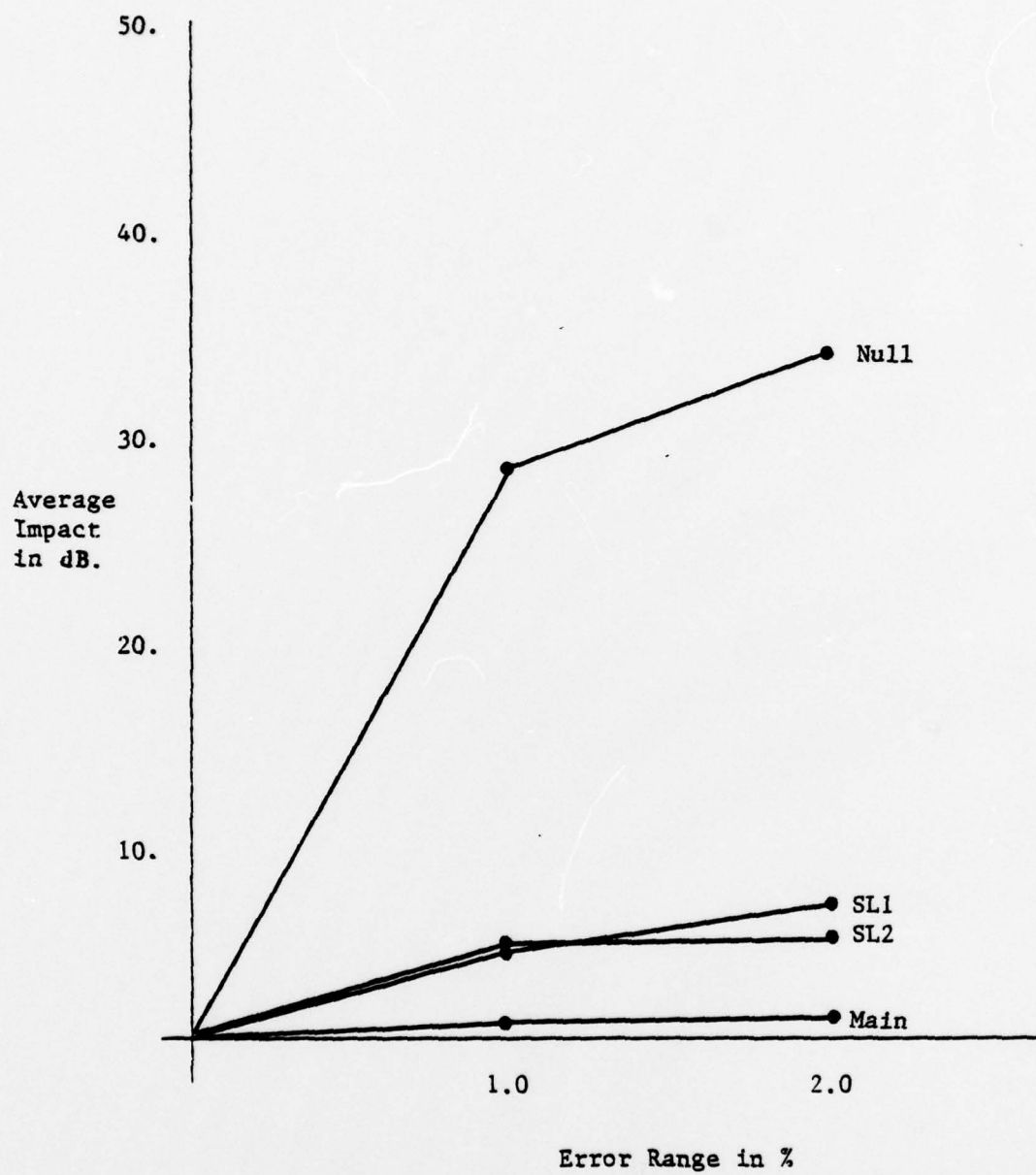


Figure 4-46
Pattern 1
 $\overline{\Delta G}$ vs. Error Range by Feature.

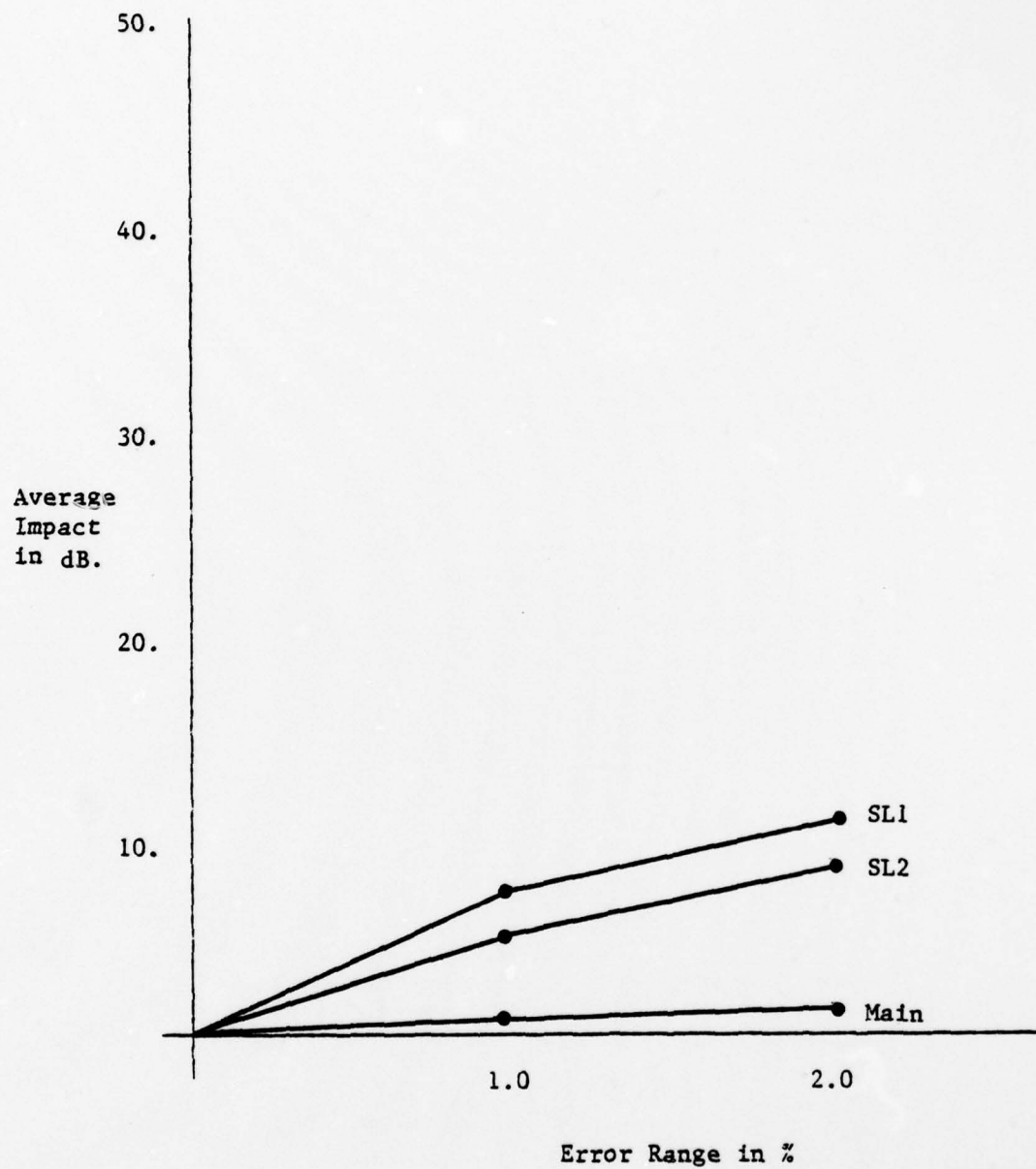


Figure 4.47
Pattern 2
 ΔG vs. Error Range by Feature.

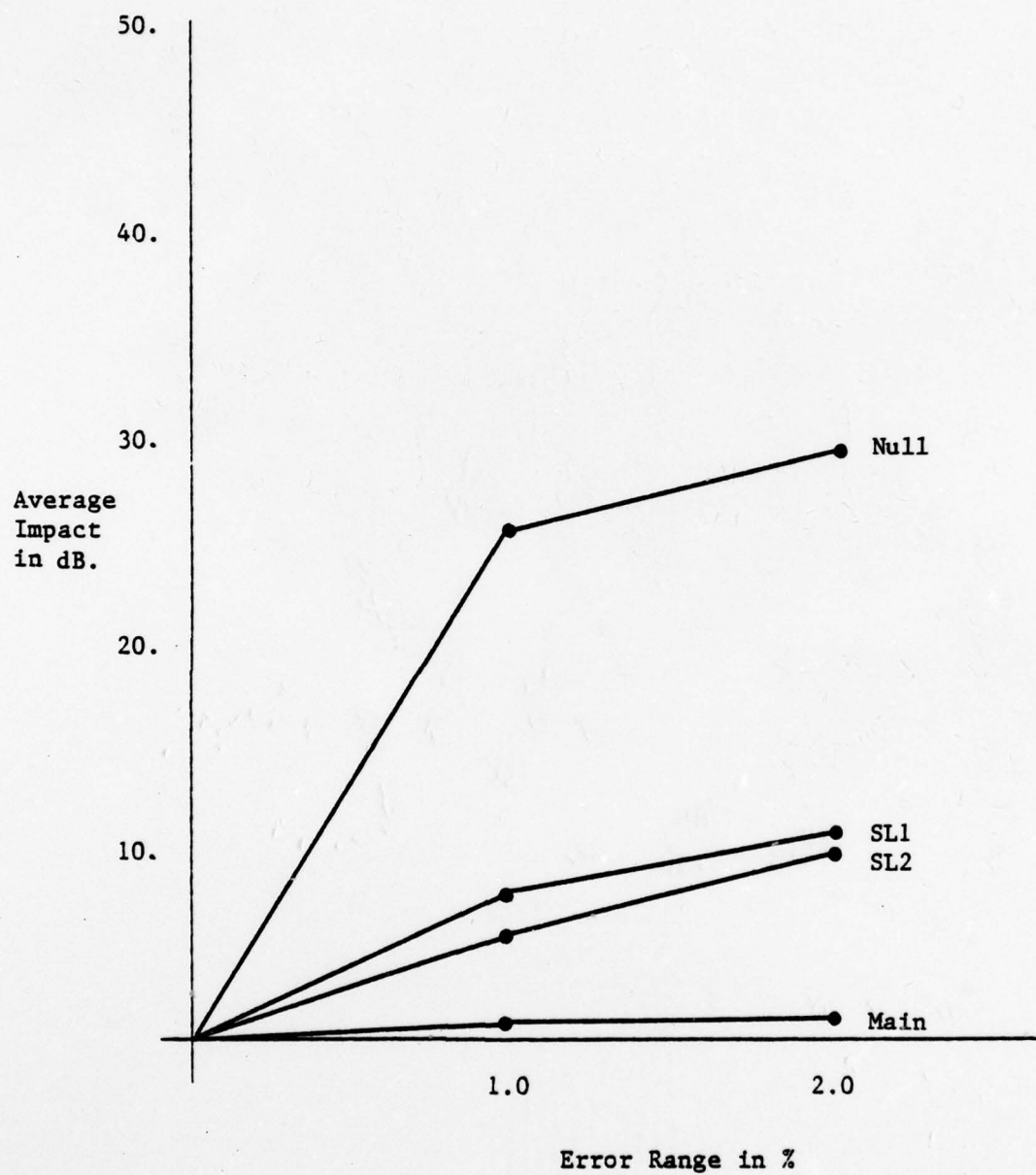


Figure 4.48
Pattern 3
 $\overline{\Delta G}$ vs. Error Range by Feature.

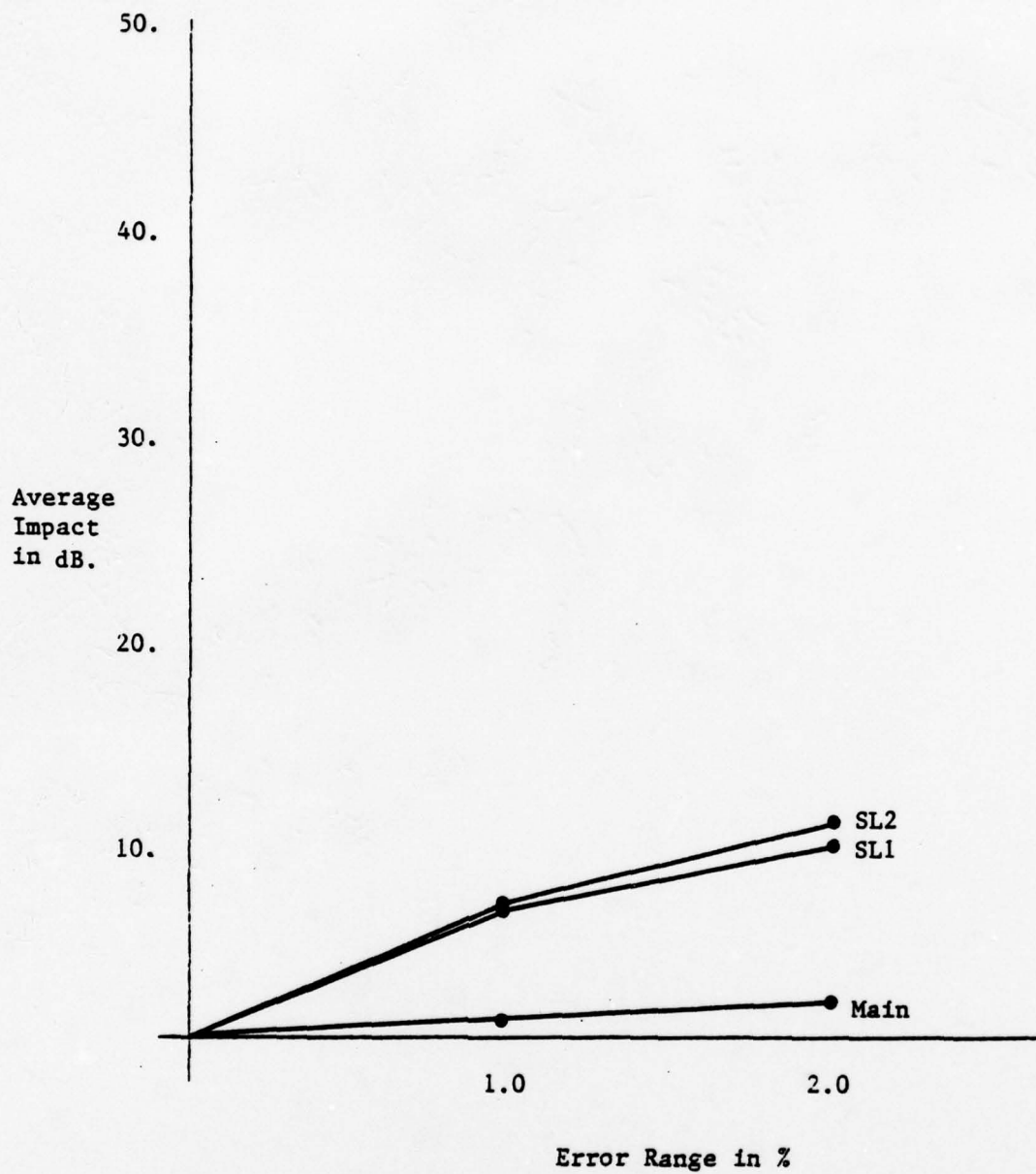


Figure 4.49
Pattern 4
 $\overline{\Delta G}$ vs. Error Range by Feature.

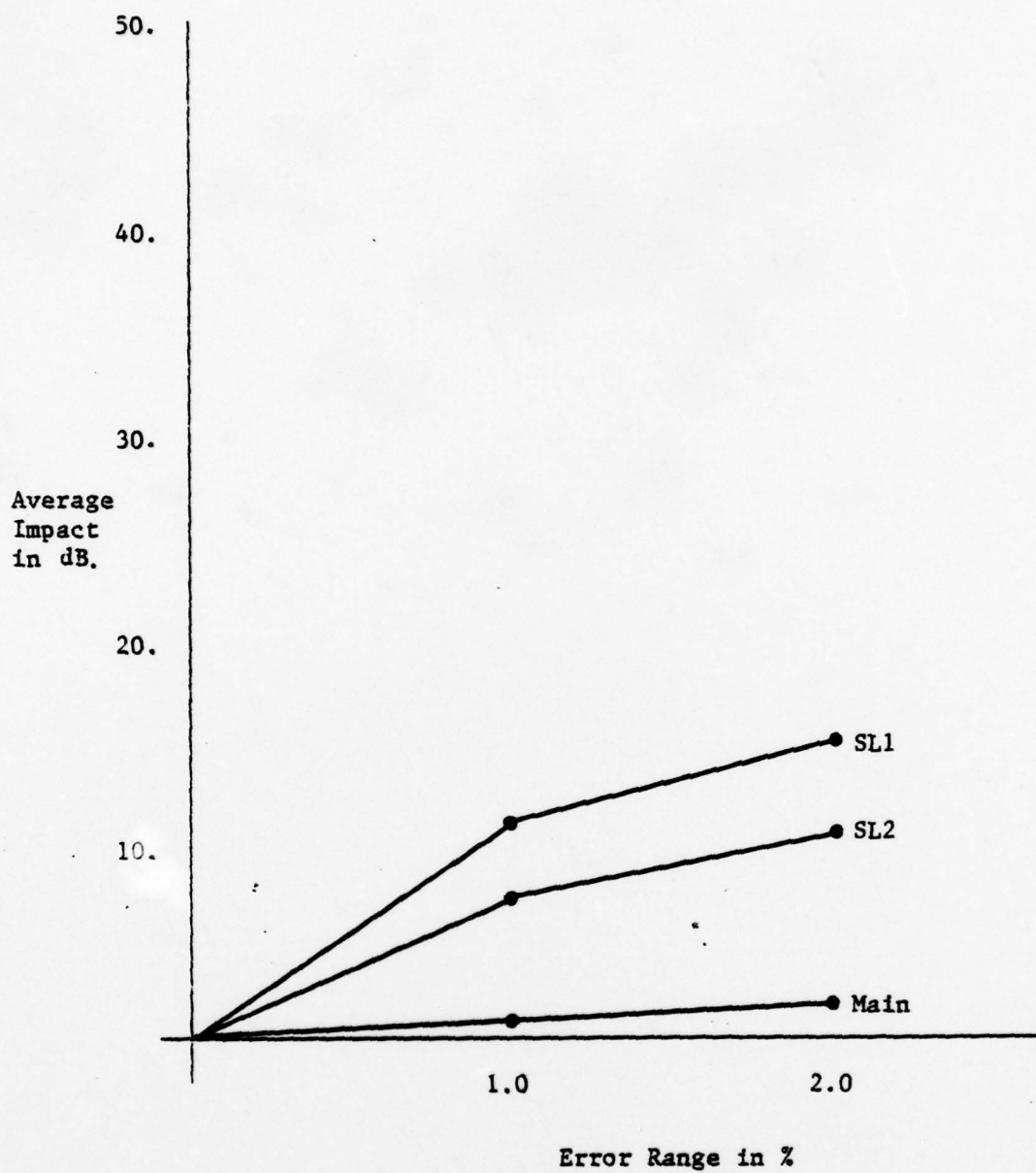


Figure 4.50
Pattern 5
 $\overline{\Delta G}$ vs. Error Range by Feature.

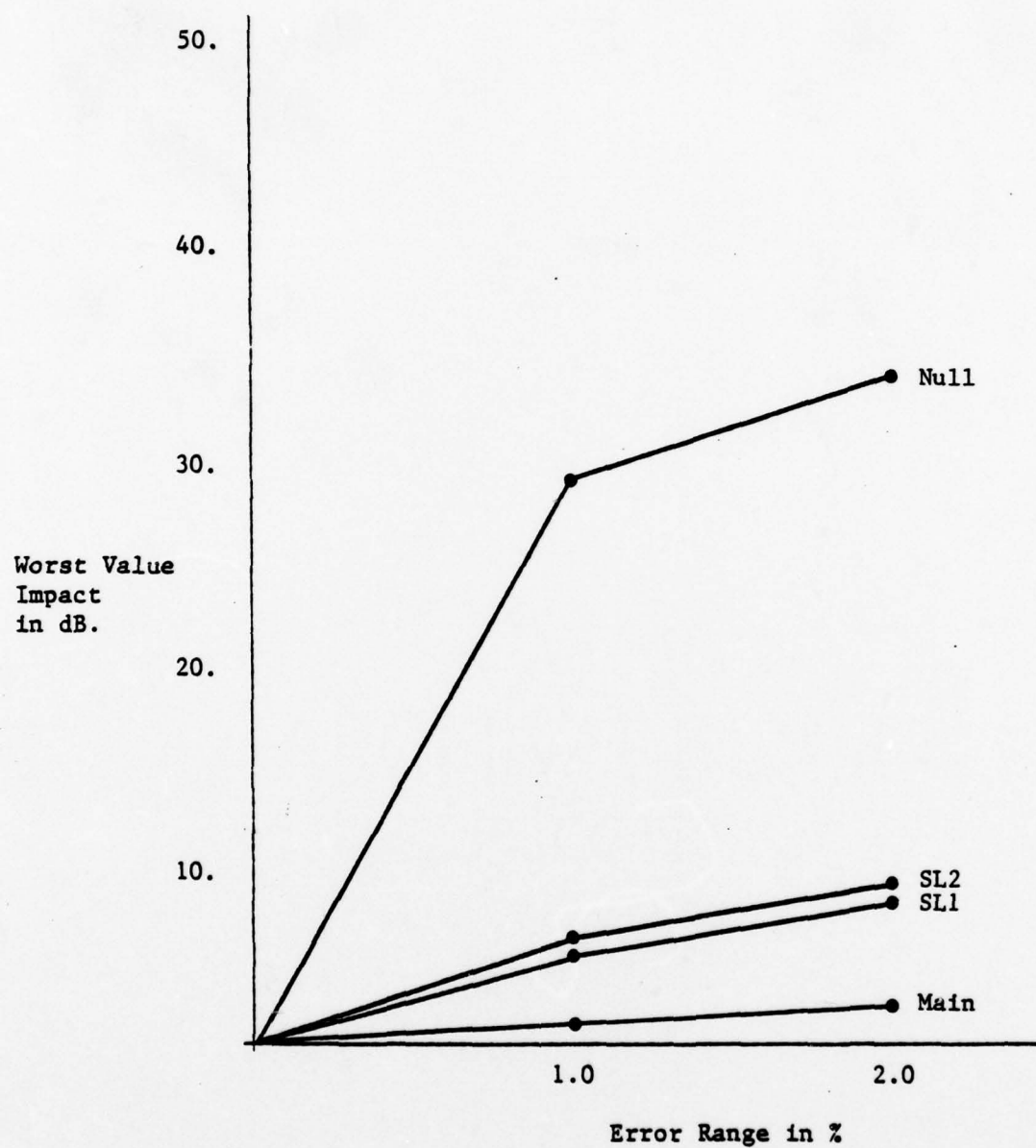


Figure 4.51
Pattern 1
 ΔG_{max} vs. Error Range by Feature.

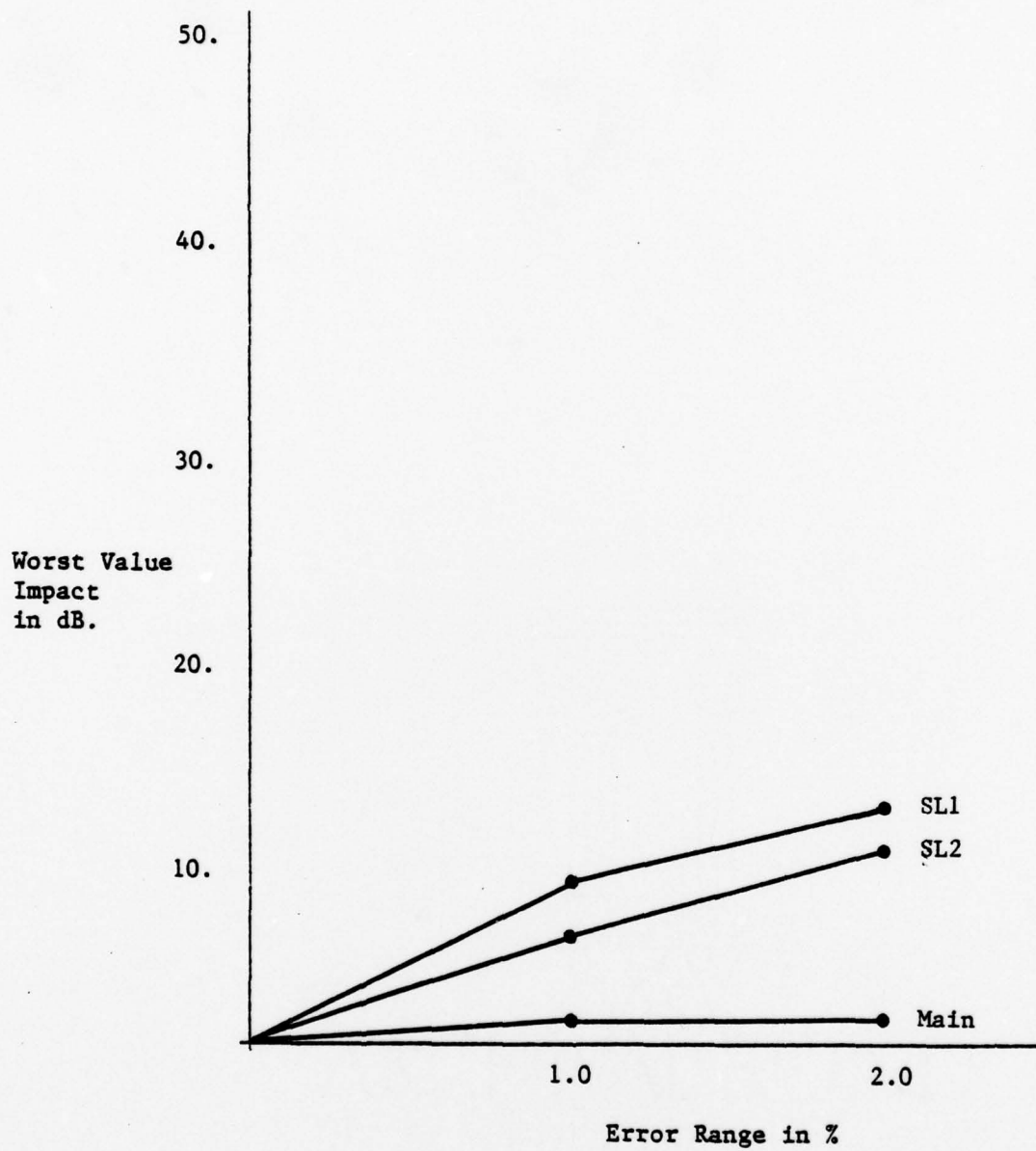


Figure 4.52
Pattern 2
 $\Delta G_{\max}$ vs. Error Range by Feature.

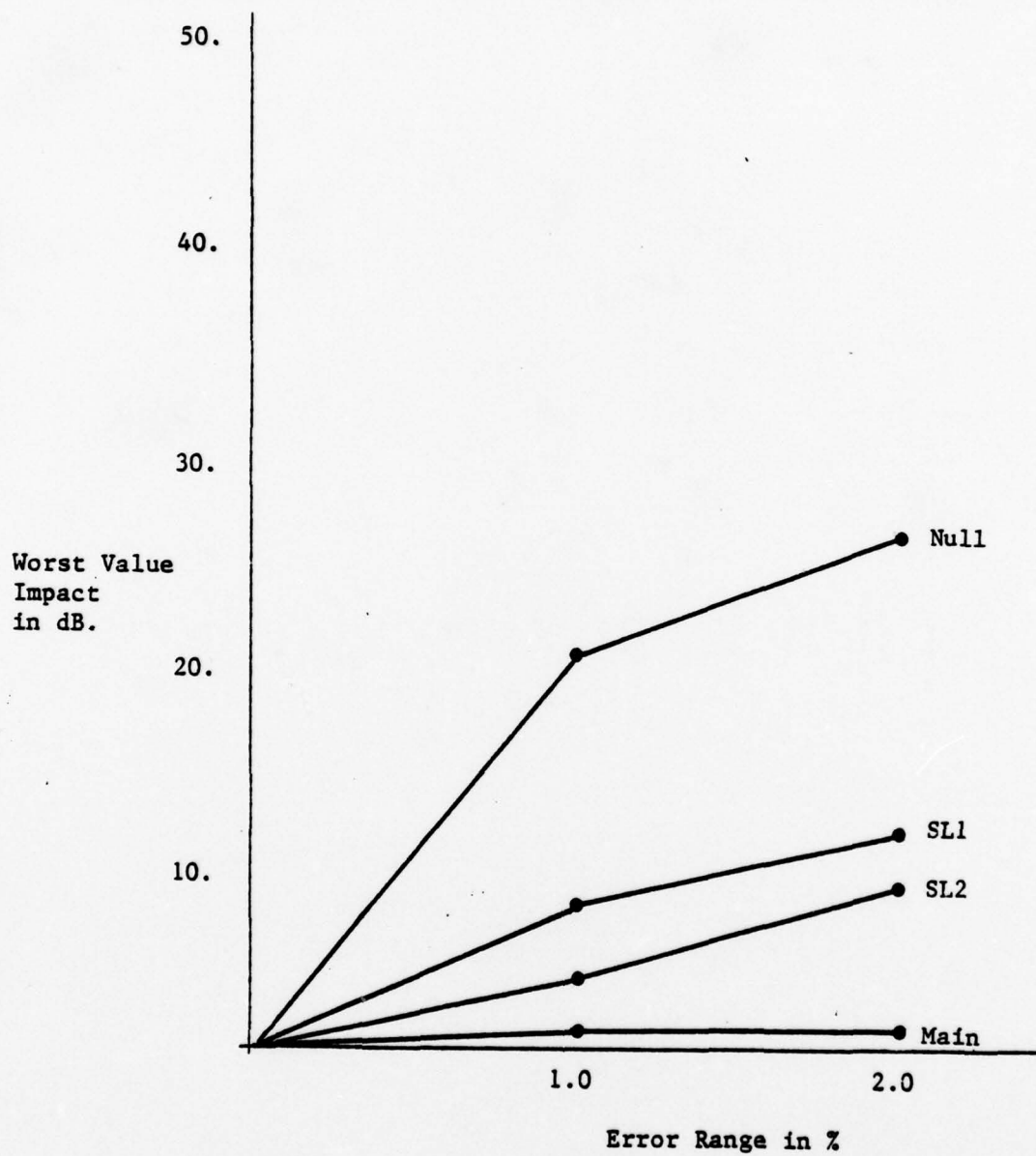


Figure 4.53
Pattern 3
 Δ Gmax vs, Error Range by Feature.

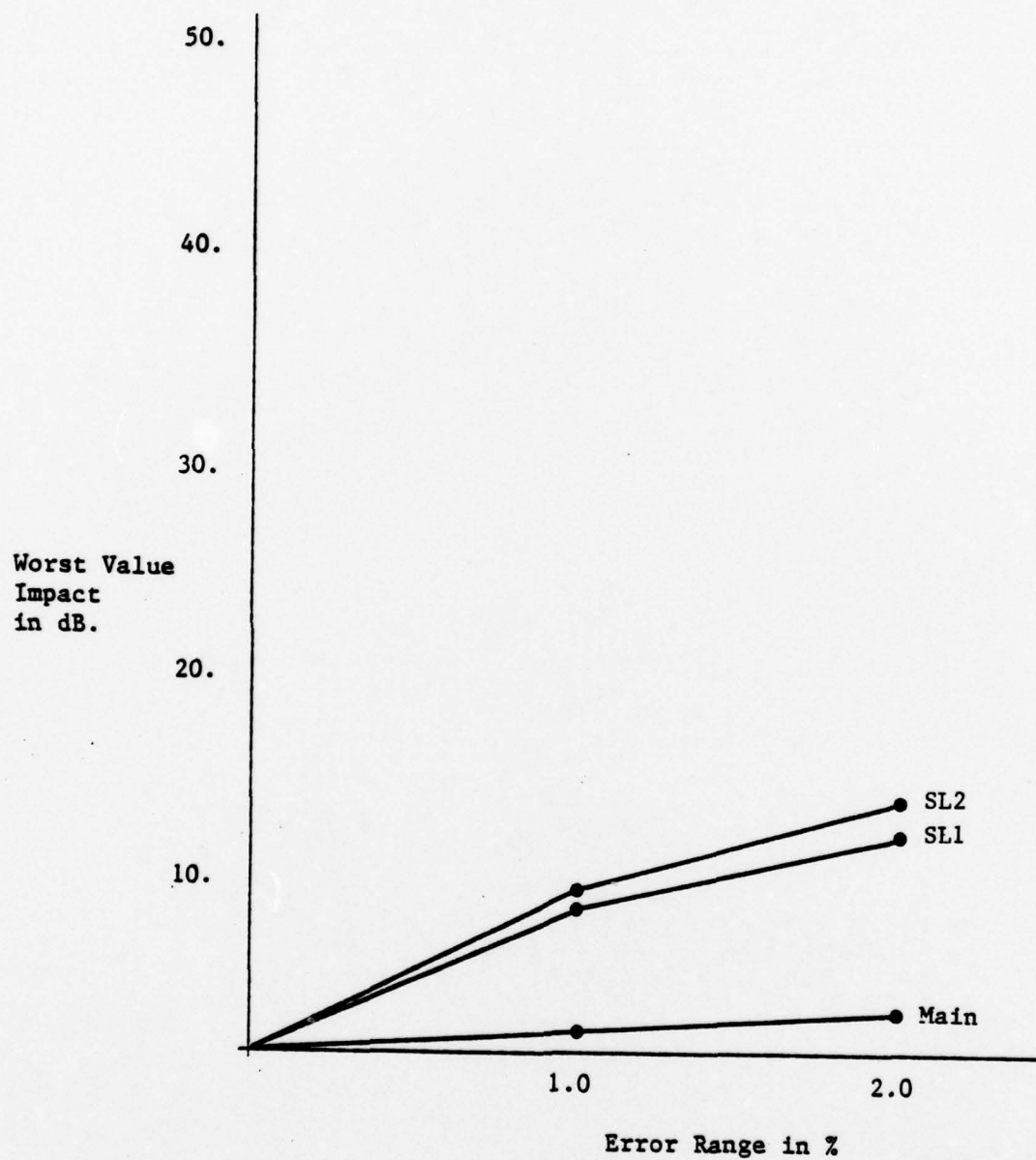


Figure 4.54
Pattern 4
 ΔG_{max} vs. Error Range by Feature.

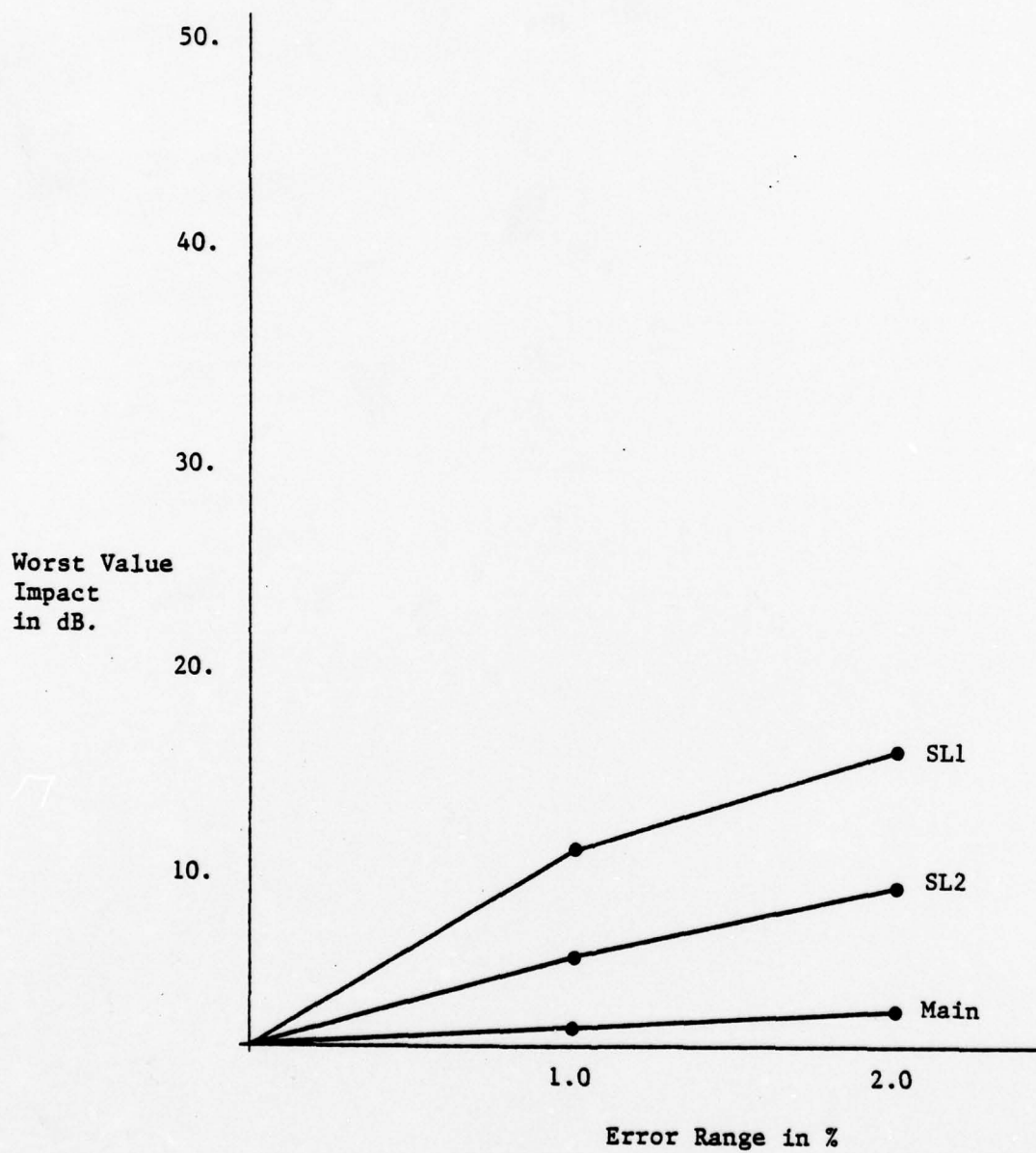


Figure 4.55
Pattern 5
 $\Delta G_{\max}$ vs. Error Range by Feature.

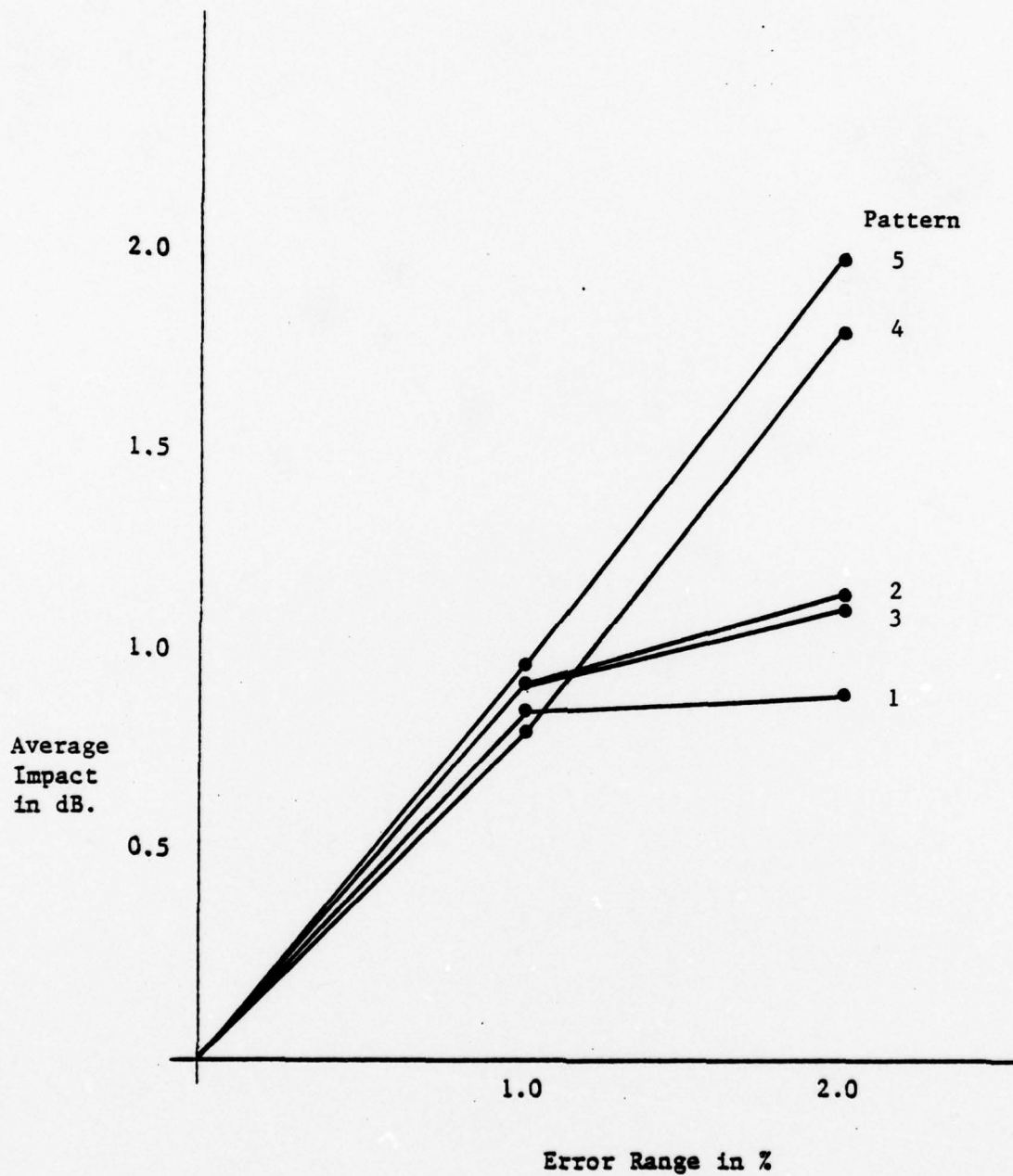


Figure 4.56
Main Beam Impact
 $\overline{\Delta G}$ vs. Error Range by Pattern.

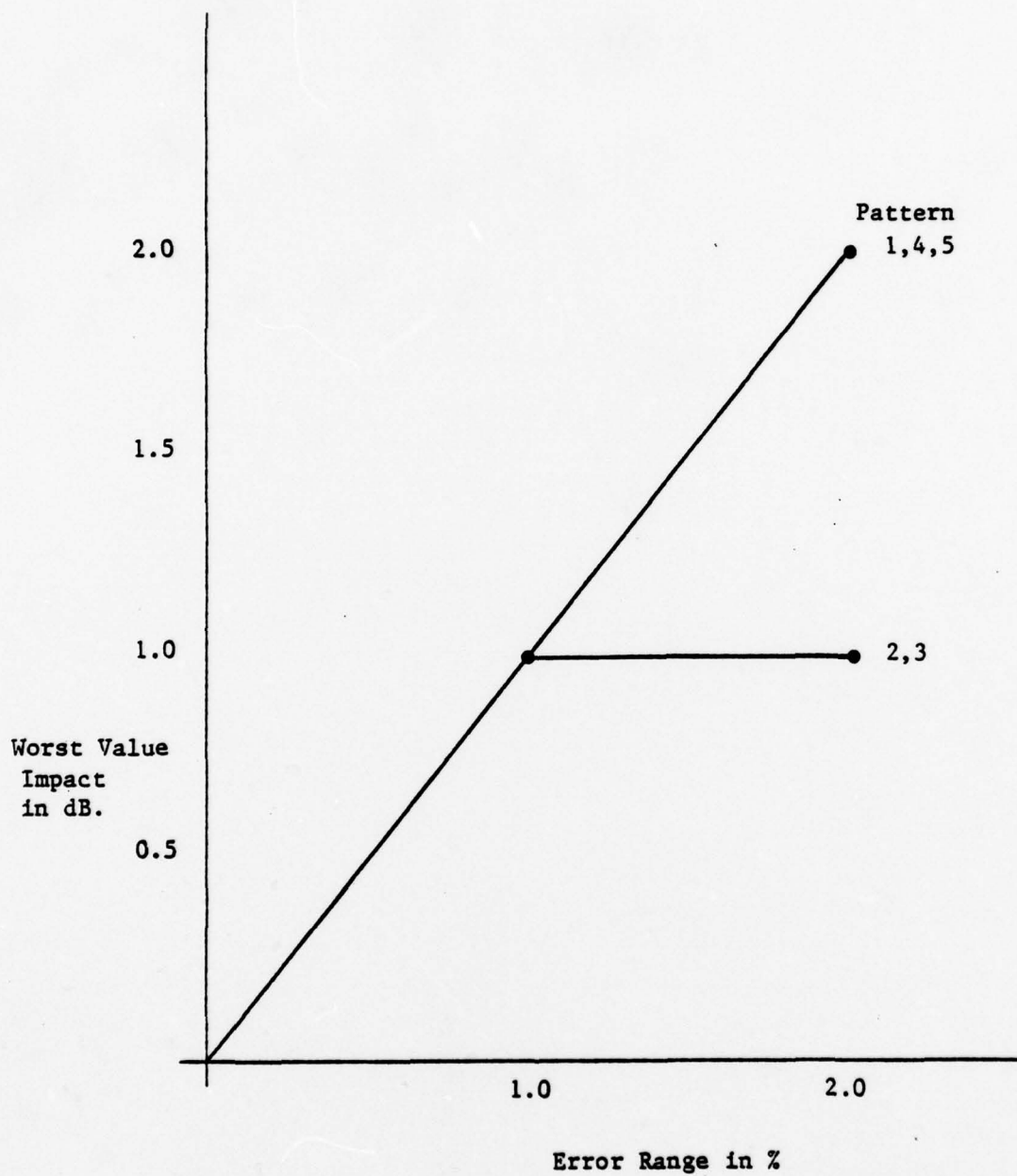


Figure 4.57
Main Beam Impact
 ΔG_{max} vs. Error Range by Pattern.

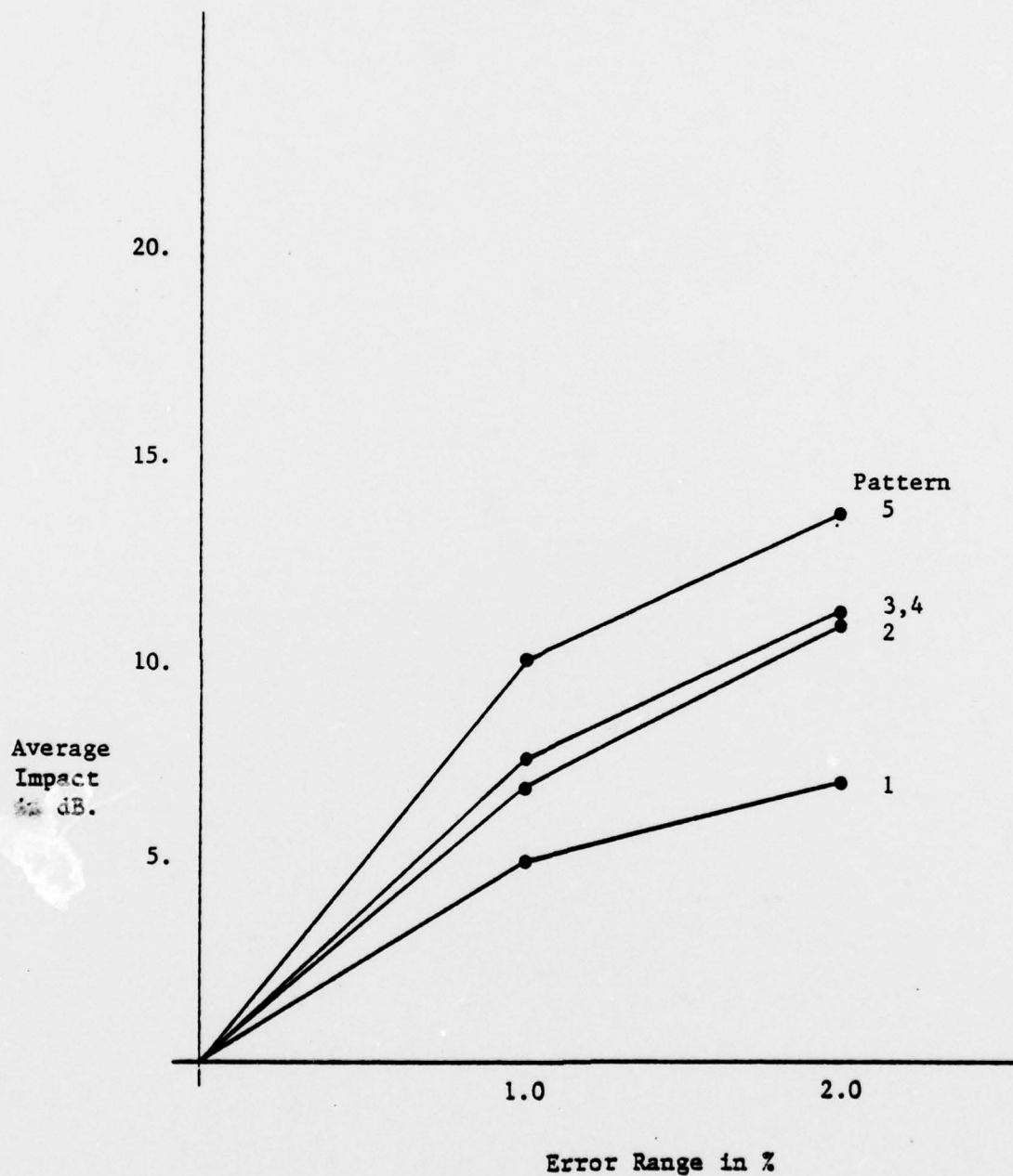


Figure 4.58
Aggregated Side Lobe Impact.
 ΔG vs. Error Range by Pattern.

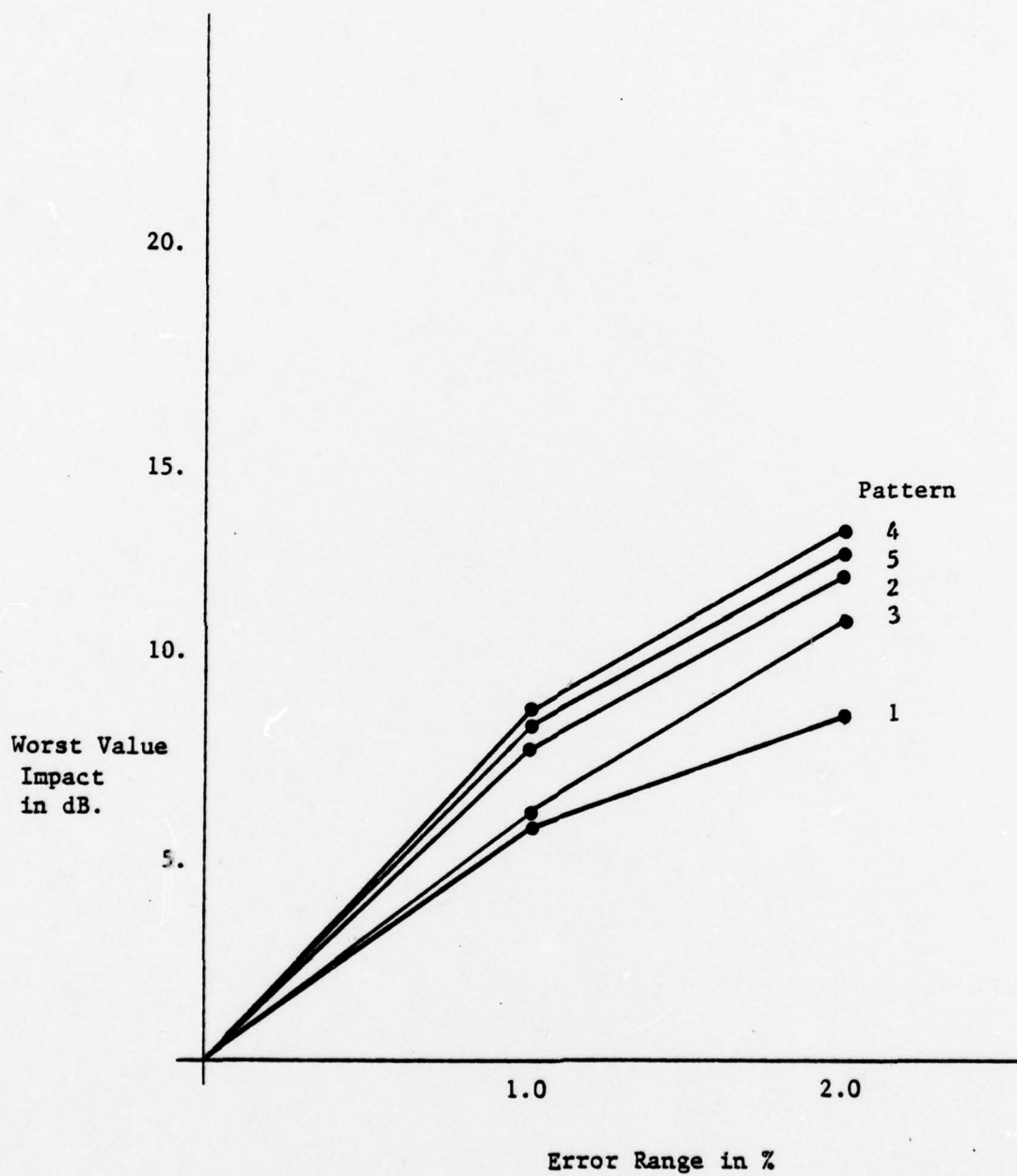


Figure 4.59
Aggregated Side Lobe Impact.
 $\Delta G_{\max}$ vs. Error Range by Pattern.

Table 4.42. Summary of λ Error Impact Data.

Pattern	λ	Main Beam		Side Lobe 1		Side Lobe 2		Null	
		$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}
1	.98	0.333	1.0	-0.571	0.0	0.0	1.0	-1.0	-1.0
	.99	0.110	0.0	-0.214	0.0	0.167	1.0	0.0	0.0
	1.01	0.0	0.0	0.357	0.0	0.333	1.0	1.0	1.0
	1.02	0.0	0.0	0.143	-1.0	0.500	1.0	3.0	3.0
2	.98	0.077	0.0	-0.667	0.0	0.462	0.0		
	.99	0.0	0.0	-0.222	0.0	0.346	0.0		
	1.01	0.0	0.0	0.0	0.0	-0.308	0.0		
	1.02	0.0	0.0	-0.222	0.2	0.577	0.0		
3	.98	0.077	0.0	-0.727	0.0	-0.111	-1.0	-1.0	-1.0
	.99	0.077	0.0	-0.182	0.0	0.0	-1.0	-1.6	-3.0
	1.01	0.0	0.0	-0.454	0.0	-0.111	0.0	-0.6	0.0
	1.00	0.0	0.0	-0.273	0.0	-0.222	0.0	-0.2	1.0

Table 4.42 (continued).

Pattern	λ	Main Beam		Side Lobe 1		Side Lobe 2		Null	
		$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}	$\Delta\bar{G}$	ΔG_{MAX}
4	.98	0.187	1.0	-0.737	-1.0	-0.187	-1.0		
	.99	0.0	0.0	-0.818	-2.0	0.187	-1.0		
	1.01	-0.134	0.0	-0.818	-1.0	0.50	-1.0		
	1.02	-0.20	0.0	-0.737	-1.0	0.50	-1.0		
5	.98	0.158	1.0	-0.128	0.0	0.815	1.0		
	.99	0.031	0.0	-0.428	0.0	0.259	1.0		
	1.01	0.031	0.0	0.214	1.0	-0.593	0.0		
	1.02	-0.094	-1.0	0.143	0.0	-1.185	-1.0		

 $\Delta\bar{G}$, ΔG_{MAX} in dB.

CHAPTER V

DISCUSSION OF THE RESULTS AND EXTENSIONS OF THE METHOD

5.1 Implications of the results

The analysis presented is meant to illustrate the power of the MAXIM-IOS method, and not to generate sensitivity data for a particular design problem. None the less, some trends which may be tentatively identified in the data deserve comment.

1. The nulls appear as the most sensitive feature in the pattern by a factor of about three over the side lobes.
2. Among the features examined, the main beams showed the least sensitivity overall. However, among the main beams, the square or rectangular shapes were much more sensitive at the 2 percent error range than the round beam.
3. In general, the broader the main beam, the greater the side lobe sensitivity appeared to be.
4. The minimum elevation points on the error surface almost universally occurred at the edge of the error interval of each decision variable.

The first three observations accord well with intuitive expectations for feature sensitivity. The fourth observation is

exactly what would be expected for a well behaved error surface. That is, the error regions appear to be tightly enough constrained about an optimum to contain only one local minimum. It is to be expected that such a minimum will lie on an edge, as far from the maximum as possible. Thus it can be said that for a 10 by 10 array at least, MAXIM-generated sensitivity values can be accepted with some confidence within the two percent error region.

5.2 Potential uses for MAXIM

The worst performance possible for an array design or feature, deriving from error is, in itself, a valuable piece of information. However, MAXIM can be used to generate other types of information as well. In Chapter four, only one type of experiment which may be designed around MAXIM was described. There, a method to determine the relationship between directivity pattern and sensitivity was examined. If such an experiment were to be performed on a series of alternative pattern designs, the procedure could be refined somewhat. Presumably, the alternatives to be evaluated in an actual application are not likely to differ from each other as radically as the sample in the above study. The sensitivity of a pattern might, for instance, be examined as one feature in a pattern changes over some interval. It may prove that in some cases, it is possible to greatly decrease sensitivity through small modifications in the design specifications. It should be noticed that in such a case the IOS will tend to be based on a meaningful set of standards.

It would also be of value to take MAXIM values on a finer set of error ranges than those used in this study. The relative rate of change

in maximum degradation of given features over an error range is of importance, since it can give an indication of the expected degradation. For instance, suppose two features were to be compared for sensitivity, and the following MAXIM data were obtained. At one percent error the first feature had a sensitivity value of 5 dB, while the second had a sensitivity value of 10 dB. At two percent error the sensitivity value of both features was 15 dB. In this case, the expected performance of the first feature would be worse than the second.

Element amplitude and phase coefficient error ranges could be analyzed separately, allowing these ranges to take on realistic values for the particular system being studied.

If, in a given application, array pattern sensitivity is to be taken into account at the design stage, it is possible, by holding any two of the following three array characteristics constant, to perform a sensitivity analysis similar to the one in this study on the third.

1. Array size and geometry.
2. Directivity pattern design.
3. Error ranges.

Number three assumes the existence of a reliable means of converting the effects of error sources, such as those mentioned in Chapter two, into error ranges in A_1 and α_1 and conversely. Then, sensitivity value vs. error range graphs, such as figure 4.56 could be used to specify acceptable tolerances for array or component manufacture.

Finally, the arrays examined in this study have all been of the planar symmetric type. MAXIM may easily be extended to the more general case of conformal array sensitivity analysis, since NLGP Conformal, a non-linear goal programming code specifically for the conformal array

case, has already been developed. A modification of this code to yield a conformal version of MAXIM would also make it possible to analyze non-symmetric error in planar symmetric arrays.

5.3 Summary

The MAXIM-IOS method of sensitivity/impact analysis has been presented and illustrated with a sample study on five rather complex array patterns. The resulting measure, maximum impact, represents the worst realization which a particular feature of a pattern can assume due to a given amount of error. The change in the value of this realization as error in amplitude and phase settings increases is also an indicator of the shape of the response surface in the error region.

Since MAXIM is based on an optimization technique, this kind of sensitivity analysis may be completed at a very small cost in computation time. It will now be practical, therefore, to include the results of MAXIM-IOS analyses of specific proposed arrays and array patterns as a factor to be taken into account during the design process.

Maxim compares well with simulation-based sensitivity analysis. Although at first glance the sensitivity measures which can be estimated through simulation seem to give more information, these values are in fact dependent on a priori assumptions concerning error distribution. MAXIM, on the other hand, requires no such assumptions. The cost factor, however, is the chief advantage of MAXIM over simulation. This advantage becomes steadily more pronounced as array size increases.

APPENDIX A

THE DIRECTIVITY FUNCTION FOR A PLANAR SYMMETRIC ARRAY

A.1 Derivation From the General Case

For a planar array in the X-Z plane consisting of n elements, the magnitude of the contribution to the directivity pattern, at an aspect angle of θ, ϕ , of an element located at x_1, z_1 is:

$$A_1 \exp[j(u_1 + \alpha_1)] \quad (A.1)$$

where:

$$u_1 = kx_1 \sin\theta \cos\phi + kz_1 \cos\theta$$

A_1 is the amplitude shading coefficient.

α_1 is the phase shift.

$$k = 2\pi/\lambda$$

λ is the operating wavelength.

The combined output of n elements is given by:

$$A_1 \exp[j(u_1 + \alpha_1)] \quad (A.2)$$

which is equal to

$$A_1(\cos \alpha_1 + j \sin \alpha_1)(\cos u_1 + j \sin u_1)$$

Consider the contributions from four elements, placed symmetrically about the center of the array at the points: (x_1, z_1) , $(-x_1, z_1)$, $(-x_1, -z_1)$, and $(x_1, -z_1)$. Suppose further that these elements are excited identically (i.e., have identical A_1 and α_1 coefficients). This result is:

$$A_1(\cos \alpha_1 + j \sin \alpha_1)[\cos u_1 + j \sin u_2 + \cos u_2 + j \sin u_1 \\ + \cos(-u_1) + j \sin(-u_1) + \cos(-u_2) + j \sin(-u_2)]$$

(A.4)

where:

$$u_1 = kx_1 \sin \theta \cos \phi + kz_1 \cos \theta \\ u_2 = -kx_1 \sin \theta \cos \phi + kz_1 \cos \theta$$

Since, for any angle r , $\cos r = \cos(-r)$, and $\sin r = -\sin(-r)$, this becomes:

$$2A_1(\cos \alpha_1 + j \sin \alpha_1)[\cos u_1 + \cos u_2] \quad (A.5)$$

By applying the double angle formula, equation A.5 can be stated as:

$$4A_1(\cos \alpha_1 + j \sin \alpha_1) \cos(kx_1 \sin \theta \cos \phi) \cos(kz_1 \cos \theta) \quad (A.6)$$

Finally, the power response at θ, ϕ , for m sets of four elements is obtained by multiplying A.6 by its complex conjugate. Thus:

$$P_{\theta\phi} = 16 \left[\left(\sum_{i=1}^n A_i \cos \alpha_i \cos(\kappa x_i \sin \theta \cos \phi) \cos(\kappa z_i \cos \theta) \right)^2 + \left(\sum_{i=1}^n A_i \sin \alpha_i \cos(\kappa x_i \sin \theta \cos \phi) \cos(\kappa z_i \cos \theta) \right)^2 \right] \quad (\text{A.7})$$

The power response is to be expressed in dB. Therefore we write:

$$G_{(\theta, \phi)} = 10 \log [P_{(\theta, \phi)} / P_{(0,0)}] \quad (\text{A.8})$$

Where:

$P_{(0,0)}$ is the peak power output for the array in any direction.

APPENDIX B

THE HOOKE AND JEEVES PATTERN SEARCH ALGORITHM

B.1 Notation

Before describing the pattern search logic, it is necessary to define several parameters and notational conventions.

1. $\bar{a}$ is a vector of n decision variable values.
2. d_j is the perturbation step size for the j^{th} search iteration.
3. t_{jk} is the trial point obtained after perturbing about the k^{th} , ($k=1,n$), decision variable in the j^{th} search iteration.
4. α is the search acceleration factor.

B.2 Pattern Search Logic

The pattern search proceeds from an initial base point a , which then becomes the trial point for the first iteration when no variables have yet been perturbed, written t_{10} . Each variable is successively perturbed by $\pm d_1$. t_{10} and d_1 are user specified parameters. If t_{1k} is preferred over $t_{1(k-1)}$, the search will move to t_{1k} and perturb the $k+1^{\text{th}}$ variable. The algorithm evaluates trial points by simply substituting the associated decision variable values into the achievement function. After all n variables have been perturbed, t_{1n} is

compared to t_{10} . If t_{1n} is not preferred over t_{10} , the perturbation step size is reduced, and the variables are again perturbed about t_{10} . If, however, t_{1n} is preferred over t_{10} , the search proceeds to a new trial point, denoted t_{20} and given by:

$$t_{20} = (t_{1n} - t_{10}) \cdot \alpha$$

t_{20} is then compared to t_{1n} . If t_{20} is preferred over t_{1n} , the next iteration begins, from there. If not, then t_{1n} becomes t_{20} , d_2 is reduced to a fractional part of d_1 , and the next iteration begins at that point.

The pattern search can terminate in the following ways:

1. A prespecified maximum number of search iterations may be equalled.
2. A user defined maximum number of perturbation step size reductions may be reached.
3. The solution may be within some tolerance factor of the theoretically optimal point.
4. The improvement between two successive trial base points may be below a specified tolerance limit.

APPENDIX C

LINEAR TECHNIQUES

C.1 Properties of Linear Models

A linear optimization model has the following general form. An objective function

$$\sum_{i=1}^m d_i x_i$$

is to be maximized subject to n constraints:

$$\sum_{i=1}^m c_{ij} x_i \begin{matrix} (\leq) \\ (\geq) \end{matrix} b_j \quad j=1, \dots, n$$

where the x_i are decision variables, and the c_{ij} are numerical coefficients. If a non-linear optimization problem may be transformed into or approximated by a linear problem, it can be solved using linear programming techniques. Theoretically, the various linear programming algorithms always find the optimal solution(s) if they exist. However, in the case of large linear programming problems requiring the performance of a great many operations, round-off error accrued during the course of the calculations may lead to an invalid solution, or cause the algorithm to fail and find no solution at all.

It is possible, by manipulating the directivity function and aggregating variables, to achieve a linear formulation of array

directional response. The derivation of this result will be reviewed in this section, and the characteristics of the linear programming problems so obtained will be discussed.

C2. Linearization of the Directional Response Function

As shown in Appendix A, equation A.2, the combined output from n elements in a given direction, θ, ϕ , is given by:

$$P_{\theta\phi} = \sum_{i=1}^n A_i \exp[j(u_i + \alpha_i)]$$

where u_i depends on the operating wavelength, the element position, and the aspect angle. For an element in the X-Z plane, for instance,

$$u_i = \kappa x_i \sin \theta \cos \phi + \kappa z_i \cos \theta$$

The output expression may be equivalently written:

$$P_{\theta\phi} = \sum_{i=1}^n (\cos u_i + j \sin u_i) A_i (\cos \alpha_i + j \sin \alpha_i)$$

Expanding and rearranging terms gives:

$$P_{\theta\phi} = \sum_{i=1}^n [\cos u_i (A_i \cos \alpha_i) - \sin u_i (A_i \sin \alpha_i)] \\ + j \sum_{i=1}^n [\cos u_i (A_i \sin \alpha_i) + \sin u_i (A_i \cos \alpha_i)]$$

If the terms $A_i \cos \alpha_i$ and $A_i \sin \alpha_i$ are understood to be decision variables (i.e., the shading coefficients are now complex), and the terms $\sin u_i$ and $\cos u_i$ to be coefficients, this function assumes the general linear form. The values taken on by the synthetic decision variables, $A_i \sin \alpha_i$ and $A_i \cos \alpha_i$, are not in themselves of interest. However, as long as both of them appear in the real component of the response function, it will be possible to recover the separate A_i and α_i

upon solution.

Since the coefficients $\cos u_1$ and $\sin u_1$ depend only on element positions and the θ, ϕ points at which the pattern is to be evaluated, numerical values may be determined and substituted into the response function. Then the problem may be solved using any of a number of computer packages for linear programming problems.

C3. Linear Models for General Arrays

In 1961, McMahon et al. [18] presented the transformation of the directivity function into a linear form, as reviewed above, and presented results for several linear and planar arrays in the 12 to 70 element range. This formulation applies to the most general case of array design. The array may be phased, and element positions may conform to any surface.

The chief disadvantage of this formulation is the size of the resultant linear programming problem. Four inequality constraints are required at each θ, ϕ point defining the desired pattern in order to create upper and lower bounds on both the real and imaginary components of the response function. Furthermore, 4 non-negative decision variables are generated for each element of the array. Thus the size of the resultant linear programming problem is given by the polynomial, $Y = 4n \times 4m$, where n is the number of θ, ϕ points needed to define the pattern, and m is the number of elements in the array. Adequate pattern definition will usually require upwards of 65 θ, ϕ points per orthant. Thus, a conservative estimate for the linear programming problem generated by just a 100-element (conformal, planar, or linear) array, whose response is specified over a hemisphere, is 400 variables and

1,040 constraints.

Unfortunately, there is no direct relationship between the size of a linear programming problem and the complexity function of the algorithm required to solve it. It has, however, been the experience of the author that the use of this method to program arrays as small as 64 elements is prohibitively expensive.

The general array linear formulation suffers its severest shortcomings when applied to arrays whose elements are symmetrically placed. As shown in Appendix A for the planar array case, such symmetry conditions may be incorporated into the directional response function. This type of reformulation allows the pattern to be specified on a fractional part of the region over which it is to be defined. The number of θ, ϕ points needed to specify the pattern produced by an n element planar array displaying two-way symmetry is half that of a non-symmetric n -element array. Four-way symmetry halves this number again. Since each θ, ϕ point has 4 associated constraints, this method can greatly reduce problem size. However, as shown in equation A.5, the incorporation of symmetry conditions causes the $j \sin u_1$ terms to sum to 0 and drop out of the directivity function. The synthetic decision variables then become indeterminate in that unique values for A_1 and α_1 can no longer be recovered. Thus, the technique is incompatible with the linear formulation, and symmetric arrays will generate the same size problems as non-symmetric arrays. There is, however, one case where symmetry can reduce problem size for the linear formulation. When the array is planar-symmetric and relatively small ($n < m/2$), the number of constraints will be reduced somewhat if directional response is specified in only one orthant, and the decision variables corresponding

to the elements in one quadrant of the array are set equal to those in the other three. For instance, if the 100-element array, 260 θ, ϕ point case given above is treated in this manner, the associated linear programming problem has 400 variables and 860 constraints.

C.4 The Planar-symmetric, Non-phased Case

Wilson [20] has presented results for planar arrays with two-way symmetry, and all element phase shift coefficients set to 0. In this case, the response at θ, ϕ from two elements placed symmetrically in the X-Z plane and excited identically is:

$$2A_1 \cos u_1$$

where u_1 is defined as above. This result can be extended to the four-way symmetry case, and becomes:

$$4A_1 \cos u_1$$

Then the output of s sets of 4 symmetric elements is:

$$\sum_{i=1}^s 4A_1 \cos u_1$$

This function contains no imaginary component, and since the decision variables, A_1 , lie by definition between 0 and 1, only one decision variable is generated per set of 4 elements. The 100-element planar-symmetric, non-phased array with the same pattern as earlier examples, has an associated linear programming problem of 25 decision variables, and about 65 constraints. In general, the size of the problem in this case increases as the polynomial, $n/4 \times (m/4 + d/4)$ where d is the number of θ, ϕ points in the main beam.

The most general objective function for both McMahon's and Wilson's linear programming problems minimizes a weighted sum of deviations from the specified responses at the θ , ϕ points. Other objective functions are possible, however. Wilson's formulation [20], for instance, minimizes the maximum deviation over the set of θ , ϕ points.

The recent development of a code to solve massive linear goal programming problems [17] allows the reformulation of these linear models as linear goal programming models. The multi-objective nature of array synthesis problems may now be represented in a preemptive priority structure, as well as in the relative weights assigned to the goals.

C.5 MAXIM for Planar Symmetric Non-Phased Arrays

It is only for the planar-symmetric non-phased array that the linear model has proved to be of practical value, since here the associated linear programming problems are of a size which is economically feasible. The same holds for MAXIM applications. The linear version of MAXIM is as follows:

$$\begin{aligned} \text{MIN } z &= \sum_i w_i d_i \\ \text{St } : \sum_{j=1}^n \cos u_{ij} A_j + n_i - p_i &= b_i \end{aligned}$$

for the i θ , ϕ points in the feature under consideration. d is a negative deviation variable for the main beam, and a positive deviation variable elsewhere. The minimization takes place subject to a set of constraints given by:

$$\begin{aligned} \sum_{j=1}^n \cos u_{ij} A_j + n_i - p_i &= b_i & i=1, p \\ \left. \begin{aligned} A_j &\leq A_j^* + \epsilon_j \\ A_j &\geq A_j^* - \epsilon_j \end{aligned} \right\} & j=1, n/4 \end{aligned}$$

where A_1^* is the design solution, and 2ϵ is the error range. The analysis then proceeds as with the non-linear model. All results established for the non-linear case hold here as well, with two exceptions. Only those error sources which are resolvable into error in amplitude settings may be treated with this formulation. Thus, for instance, error in element placement could not be analyzed. With regard to the solution technique itself, however, the optimal solution is now guaranteed.

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