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APPLICATION OF LEARNING CURVES OF AIRCRAFT PRODUCED AT MORE THAN ONE LOCATION TO THE F-16 LIGHTWEIGHT FIGHTER

THESIS

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Preface

Thanks are due to Dr. Keith Womer for providing the thesis topic and for advising this thesis. Also, thanks are due to Lt Col William C. Letzkus for his efforts as reader of this thesis.

Anthony Mlinar

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Abstract

This thesis analyzed the effect of coproduction of the F-16 fighter on the learning curve of the F-16 aircraft purchased by the United States. This analysis was performed by studying the effect of coproduction of previously coproduced aircraft. The aircraft that were analyzed are the B24, B29, B47E, B52F, F84F, F86F, and F100C.

The F-16 is being produced in the United States and Europe. However, since the aircraft purchased by the USAF will be a combination of parts from both the United States and Europe, the learning curve of the USAF purchased aircraft will be a combination of two production lines. The combination of two production lines result in increased average cost per unit when compared to the average cost per unit of aircraft produced on only one line.

Also, the analysis of previously coproduced aircraft showed that the coproducer's learning curve had a higher first unit cost and steeper slope than would be expected if all learning was initially transferred and utilized by the coproducer. These results imply that the European Consortium production learning curve will have a higher first unit cost, steeper slope, and average cost per unit than

the United States production learning curve.

Therefore, the USAF purchased F-16 learning curve will have a higher first unit cost, steeper slope, and higher average cost per unit than a single production line in the United States.

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I. Introduction

This thesis research is concerned with determining the effect of the coproduction agreement with the European Consortium on the learning curve of the F-16 fighter aircraft purchased by the United States. Since part of the aircraft purchased by the United States is to be coproduced in Europe and part of the European purchased aircraft is to be coproduced in the United States, the learning curve estimates become more complex. This thesis attempts to quantify the effects on the learning curve presented by the F-16 coproduction effort.

Prior to and during the acquisition of aircraft systems, cost estimation is accomplished on a continuing basis. This cost estimation provides analysts with data which can be used to maintain cost control, determine cost overruns, determine costs of engineering changes and provide agencies such as Congress and the Department of Defense with an information base upon which acquisition decisions are made.

When aircraft are produced at two different locations and parts produced at one location are used at the other location (even though the same part may be produced at both locations), the complexity of cost estimation increases. Also, when one production plant is located in the United States and the second plant is located outside the United States under foreign ownership, cost estimation takes on political overtones and necessitates an added effort on the part of the estimator to determine specific data associated with the foreign production.

The production of the F-16 lightweight fighter aircraft is an example of such a production situation. The F-16 fighter will be produced in the United States by General Dynamics Corporation and in four European countries who are members of the F-16 Consortium: Belgium, Denmark, Netherlands, and Norway. Since 10% of the value of the first 650 United States Air Force (USAF) purchased F-16 aircraft will be produced by the European Consortium, the effect of coproduction on the USAF aircraft must be determined.

Also, United States companies will manufacture 60% of the value of the 348 aircraft to be purchased by the European Consortium. These manufactured parts will have an effect on the average cost per unit due to increased production requirements of a multi-national effort. Therefore, this production will tend to reduce the average cost per aircraft purchased by the USAF because of the learning curve effect. A study by the Congressional Budget Office concluded that in the case of the F-14, Foreign Military Sales resulted in a 20.6 million dollars saved because of the learning curve effect. (Ref 14:8)

Because of the international nature of the F-16 coproduction effort and the possibility of future coproduction efforts where Foreign Military Sales (FMS) are involved, it is necessary to understand the effect on cost of coproduced weapon systems as opposed to weapon systems manufactured by only one producer.

The purpose of this thesis is to look at only one aspect of cost estimation of a coproduction effort: the learning curve, or progress curve, of the USAF F-16 aircraft purchase and the effect of coproduction on this learning curve. Since the learning curve is a significant factor in the estimation of the direct manhours required to manufacture complex weapon systems, the effect on the learning curve of the coproduced system should be known.

Background

The history of the learning curve in the aircraft industry started with T. P. Wright of the Curtiss-Wright Corporation. His theory is that the equation $y = ax^b$ can be used to predict direct labor costs (y) in terms of cumulative units of production (x). Wright concluded, based on empirical data available to him, that "b" had a fixed value of -.322, or an 80% slope on a log-log scale. In Wright's formulation of the theory, which is now known as the "cumulative average curve theory," the dependent variable "y" represents the cumulative average direct labor per aircraft. Wright also stated that raw material costs had a 95% slope and an 88% slope applied to "purchased" materials. (Ref 18)

In an undated booklet prepared for Lockheed Aircraft Corporation personnel, J. R. Crawford stated, based on 200 jobs in the airframe manufacturing process, that the equation $y = ax^b$ describes the relationship between the direct man-hours (y) required to produce "each unit" and the cumulative number of units produced (x). Unlike Wright, Crawford believed that different learning curve slopes applied to different types of aircraft. (Ref 6) This method of plotting unit costs is known as the "unit curve" hypothesis.

The slopes of the unit curve and cumulative average curve are not the same when identical cost data are used. In fact, a unit curve that is derived from the cumulative average curve is not the same as the unit curve derived from the data. Conversely, a cumulative average curve derived from the unit curve is not the same as the cumulative curve derived from the cost data. This inconsistency between the unit curve and cumulative curve has caused confusion among analysts when performing cost estimation, since each hypothesis produces a curve with a distinct slope.

A new hypothesis has been formulated by Karl Berend, called the "unified linear progress curve," where he demonstrates a method for resolving the dilemma of different slopes for the cumulative average curve and unit curve. This hypothesis states that the cumulative average curve is plotted correctly but that the unit cost values for each unit should be plotted at a midpoint between that unit and the previous unit. Simply stated, Berend proposes that the unit curve should be plotted at different points than J. R. Crawford hypothesized, but with the same form of equation, $y_m = ax_m^b$, where cost values are plotted at the midpoint. The advantage of this method of plotting the unit curve is that the slope "b" for the unit curve is the same as the slope of the cumulative average curve. (Ref 4)

Karl Berend states that this new method of plotting learning curve, the unified linear progress curve, makes the continued use of the present hypotheses (cumulative average and unit curve) not only undesirable but also completely unnecessary. He also gives the following advantages of the unified linear progress curve:

1. It provides the same standard of linearity to everybody.

2. Only one set of mathematics is required.

3. Except for deriving midpoints on the linear unit curve, all of the mathematics related to the linear cumulative average (Wright) hypothesis is correct.

4. Only one Linear Progress Curve need be taught. Analysis of similarities and differences between hypotheses is not required--only of historical interest.

5. Communication among analysts and organizations is greatly simplified.

6. This approach clarifies the function of the different curves; i.e., cumulative total for estimating and unit curve for analysis.

7. Computer models can be simplified--some use the two current hypotheses in the same model.

8. Previous research based on either of the two hypotheses is not necessarily invalidated since there is now a clear distinction between the two Y-intercepts: (a) for the unit curve and (A) for the cumulative curves.

9. It removes a considerable bottleneck and will undoubtedly help in the understanding and solution of non-linear relationships. (Ref 4:42)

If the unified linear progress curve does provide such improvements claimed by Karl Berend, then its application in estimating future costs and simultaneously providing an analytical tool to determine cost deviations would appear to be useful.

Coproduction

Following WW II the United States had the enviable position, because of economic strength and technical expertise, of being able to direct which countries would receive arms and how many arms they would receive. The United States was able to decide which arms were in the best interests of the receiving country and in the best interest of the United States.

However, according to Catledge and Knudsen, as the various nations of the free world progressively strengthened their economies, they began to compete with the United States in the arms market. The nations of Western Europe were in a position to assume a more significant and responsible role in actually developing military forces, rather than merely relying on the United States to furnish the necessary military equipment. (Ref 4:11) On 14 December 1956, Secretary of Defense Charles Wilson announced a policy whereby the United States offered to supply designs and technical assistance on newer and more sophisticated weapon systems to certain Western European countries. McArdle states that the purpose of the policy was "to develop a coordinated production base in Europe for modern weapon systems." (Ref 12:6) This marked the beginning of a new policy in the Foreign Military Sales (FMS) program called coproduction. Some coproduction efforts that evolved from early ventures included the Hawk, the Bullpup, the Sidewinder, and the F-104 weapon systems.

Since 1961, as is evident in the provisions of the Foreign

Assistance Act of 1961, the orientation of the military assistance program has shifted from grant aid to foreign countries toward FMS. (Ref 9:5) Included in the FMS are coproduction efforts which are becoming more desirable to foreign nations. Because the United States must now compete for FMS, the weapon systems' manufacturers must be able to provide not only a better weapon system, but must be willing to share technical knowledge with the customer. In fact, international competition during 1974 to produce a lightweight fighter as a replacement for European nations' aging F104G Super Starfighter inventories (i.e., the Swedish Vigger and French F-1 Mirage) resulted in the decision by General Dynamics (YF-16) and Northrop (YF-17) to "intend to farm out the manufacture of elements of their aircraft in the event that one or the other is selected." (Ref 15:4)

F-16 Lightweight Fighters. The F-16 is the product of specific requirements set forth by the Department of Defense for a lightweight air combat fighter. It was designed to take advantage of current technology, and was priced so that large numbers could be purchased. With 78 percent inexpensive aluminum alloy construction, the single-seat F-16 can sustain 7.3g maneuvers with a full load of fuel and weapons. It has flown in excess of mach 2.0 in level flight. The F100-PW-100 turbofan engine (same engine as used on the F-15) develops 25,000 pounds of thrust, providing an attractively high thrust-to-weight ratio. The F-16 was first flown in 1974.

The F-16 is nuclear capable and weighs less than one-half as much, accelerates twice as fast, and requires one-third the maintenance hours as the F-4. The F-4 is the aircraft that the F-16 will eventually replace in the active Air Force fighter inventory. (Ref 9:299-302)

F-16 Coproduction. The F-16 is now programmed to replace the F-4 in the active Air Force inventory and replace the aging F-106G Super Starfighters in European Air Forces. The countries that are going to purchase the F-16 in Europe, known as the European Participating Governments (EPG) or the "European Consortium," are Belgium, Denmark, Netherlands, and Norway. They will coproduce the F-16 as agreed to in the June 1975 Memorandum of Understanding (MOU). The coproduction agreement includes 650 United States Air Force (USAF) aircraft, 348 European Participating Government (EPG) aircraft, and currently 160 Foreign Military Sales (FMS) aircraft.

The total USAF F-16 purchase is 1388 aircraft, with the first 650 to be coproduced. The EPG industry will produce 10% of the procurement value of the first 650 USAF aircraft, 40% of the procurement value of the EPG aircraft, and 15% of future FMS aircraft (160 are presently planned). Therefore, the production base contains 1388 USAF aircraft, 348 EPG aircraft and at present

160 FMS aircraft. The current coproduction plan includes production of some of the parts of USAF aircraft in Europe and production of some of the parts of EPG aircraft in the United States. (Ref 7:7)

The major F-16 contractors are:

Prime Contractor:	General Dynamics Corporation Fort Worth Division, Texas
Engine Contractor:	Pratt & Whitney Division of United Aircraft Corporation East Hartford, Connecticut

Statement of the Problem

If past trends continue, the competition in FMS will increase as more countries obtain the necessary technical expertise in manufacturing modern, complex weapon systems. Also, it appears that more countries desire to obtain technical expertise through a transfer of technology when they purchase such weapon systems. With competition in FMS rising and technical expertise becoming a desired good, coproduction of complex and highly technical weapon systems will probably continue to account for a significant portion of the United States FMS.

If future coproduction of FMS is similar to the coproduction of the F-16 aircraft, determining the effect of the coproduction on the learning curve of the USAF purchased aircraft would be useful for future cost estimation. Although aircraft have been produced at more than one location, the F-16 provides a unique situation where a percentage of the United States production will be performed by foreign corporations. In fact, the F-16 will have at least two producers, domestic and foreign, for each part of the aircraft. This situation will result in two learning curves: one for the developer and the other for the coproducer. The learning rate for the USAF purchased aircraft may be the result of two different learning curves. This situation leads to the following question: What is the final or resulting learning curve and how does it differ from a learning curve where the aircraft is manufactured by one producer?

Objectives.

1. Determine the effect on the learning curve of the USAF purchased and coproduced F-16 fighter aircraft by analyzing the data of other aircraft which have been previously produced at more than one location.

2. Apply the Unified Linear Progress Curve as hypothesized by Karl Berend to the data for aircraft produced at more than one location.

3. Determine whether the Unified Linear Progress Curve resolves the dilemma of whether the unit curve or the cumulative average curve should be used.

Scope and Limitations. In order to predict the effect on the learning curve of the USAF purchased F-16 aircraft, cost data from previous aircraft that were produced at more than one location were analyzed. The aircraft whose cost data were analyzed were the B24, B29, B47E, B52F, B84F, and the F100C. The data for the B24 and B29 are obtained from the <u>Source Book of World War II Data: Airframe Industry</u> by the Air Material Command. (Ref 17) The data for the remaining aircraft were obtained from Project Backfill. (Ref 13) Because of the limited amount of data available for aircraft that were coproduced, i.e., produced by different manufacturers, some of the aircraft analyzed were produced at different plants but by the same manufacturer.

This thesis will also apply the "loss" of learning equations, as proposed by Thomas R. Hoffman. Hoffman derived and developed a learning curve which takes into account a break in production. (Ref 10:412) Since coproduced aircraft are constructed at some time after development, there is some learning that goes into the layout of the coproducer's factory from the prior experience of the developer. Therefore, an application of the loss of learning equations can be used to further model the coproduction situation of the F-16.

A limitation of this thesis is the number of coproduced aircraft that are analyzed. Some coproduced aircraft that are not analyzed are the F-5E (coproduced by the Republic of China), F-104J (Japan), F-104G (European Consortium), CF-104 (Canada), and the F-104S (Italy). These aircraft were not analyzed due to an unavailability of learning curve data.

Also, it is not considered a limitation that some of the data that were analyzed were not produced with today's technology or manufacturing methods. The effect of coproduction on the learning curve is assumed to be the same for aircraft produced during World War II and aircraft produced today.

This thesis is limited to the above mentioned aircraft and the F-16. The learning curves used are limited to the unit curve, cumulative total curve, cumulative average curve, and the loss of learning curve as developed by Hoffman.

Organization of the Thesis

Chapter I discussed in general the F-16 coproduction effort. A background of learning curves was presented with an introduction to the Unified Linear Progress Curve. Also presented was a statement of the problem, objectives of the thesis, and the scope and limitations of the thesis.

Chapter II developes and describes the unit curve, the cumulative average curve, the cumulative total curve, and the loss of learning equations.

Chapter III applies the Unified Linear Progress Curve and the loss of learning equations to data on previously coproduced aircraft. The results of the data analysis are then applied to the F-16 coproduction effort.

Chapter IV summarizes the thesis and provides specific conclusions with regard to the F-16 coproduction effort. Also, the use of the Unified Linear Progress Curve is summarized and specific conclusions regarding its use are provided.

II. Types of Learning Curves

The purpose of this chapter is to review the basic types of learning curves (unit, cumulative average, and cumulative total) used today in comparison with the Unified Linear Progress Curve, and to introduce and derive the loss of learning curve equation. For a more comprehensive insight into the subject of learning curves, the following references provide an excellent background: Cost-Quantity Relationships in the Airframe Industry by Asher (Ref 2); Planning Production Costs: Using the Improvement Curve by Cochran (Ref 5); and Unified Linear Progress Curve Formulation by Berend (Ref 3).

The Learning Curve

The linear learning curve (linear when plotted on log-log scale) is based on the assumption that as the total quantity of units produced is doubled, the cost of production (such as direct man-hours per unit or direct man-hours per pound) declines by a constant percentage. It is further assumed that this decline remains constant over the entire range of production. The constant rate of decline is the slope of the learning curve.

In the airframe industry the learning curve is most widely used for estimating the production manhour requirements for a given airframe. Therefore, the two variables used in plotting a learning curve are direct manhours (DMH) or direct manhours per pound (DMH/lb) versus the cumulative airframe number. The relationship that exists between the variables is found by fitting a curve to the historical data points and then deriving an equation that describes the curve.

The linear unit, cumulative total, and cumulative average learning curves will be derived. It will be shown that the form of the equation for the unit curve is the same as the form of the equation for the cumulative average curve. Then the unit curve will be derived from the cumulative curves, be shown to be non-linear when plotted on a log-log scale, and not equivalent to the linear unit curve. Also, the cumulative curves will be derived from the linear unit curve and be shown to be non-linear and not equivalent to the linear cumulative curves.

The Unified Linear Progress Curve is a technique intended to resolve the dilemma of which learning hypothesis, unit or cumulative, should be used. Therefore, the Unified Linear Progress Curve will be derived and an example developed to provide further clarification.

Unit Learning Curve

The unit learning curve is a plot on a logrithmic scale of cost per unit (Y_u) versus the cumulative unit number (X) of output produced. The unit curve is represented by the equation

$$Y_u = aX^b \tag{1}$$

where "b" is the slope ($-1 \leq b \leq 0$), and "a" is the Y intercept at X = 1.

In learning theory "a" is referred to as the first unit cost. "Percent slope," derived from "b," represents the percent change in cost between two cumulative unit number "X" and "2X." For example, if the unit costs are 10.0 DMH and 8.0 DMH for cumulative unit number 100 and 200, respectively, then an 80% learning curve is represented. In general the following equations are used to derive percent slope or "b":

$$\text{Percent Slope} = 100(2)^b \tag{2}$$

$$b = \frac{\log\ (\text{percent slope}/100}{\log\ 2} \tag{3}$$

The factor "2" in equation (2) is arbitrary but is the usual standard among analysts. An example of the unit curve can be seen in Figure 1.

Cumulative Curves

The cumulative total curve is defined

$$Y_t = \sum_{1}^{n} x_n \tag{4}$$

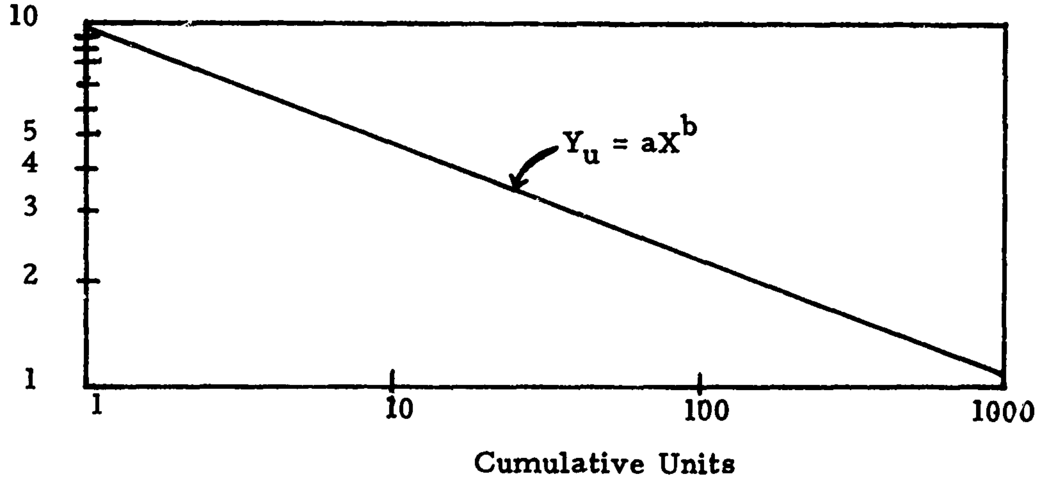

Fig. 1: 80% Unit Learning Curve

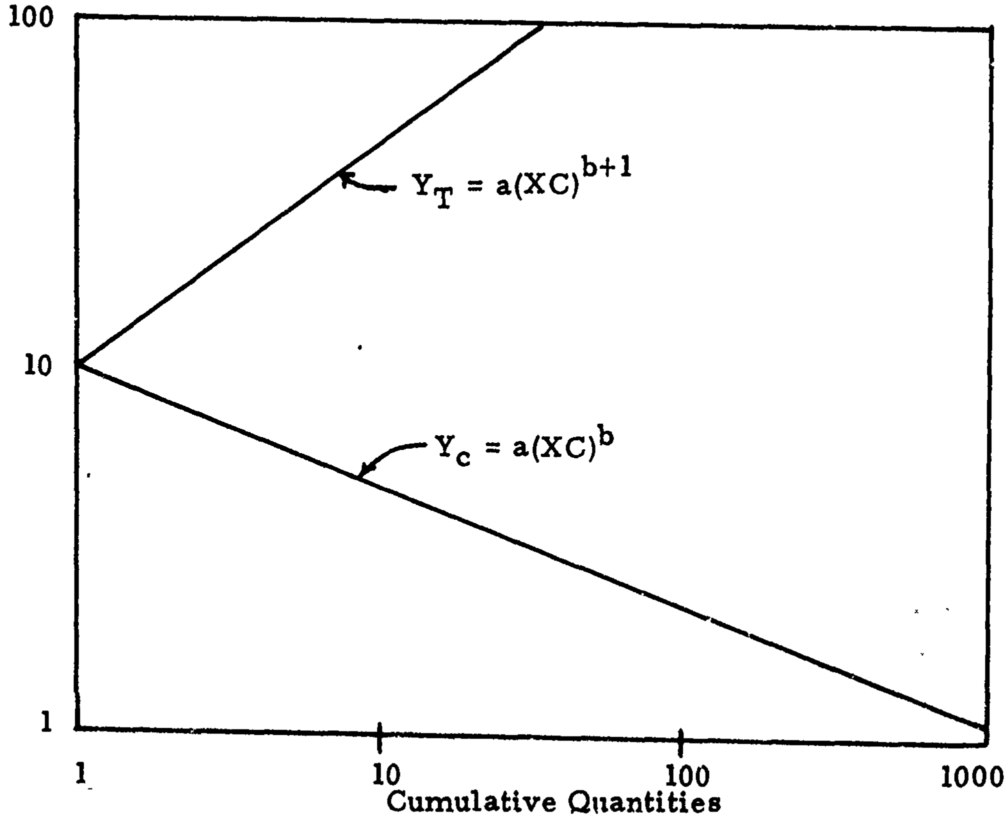

Fig. 2: 80% Cumulative Average and Cumulative Total Learning Curves

where Y_t is the total cost of n units and X_n is the cost of unit X.

The cumulative average curve is the cumulative total curve divided by n:

$$Y_c = \frac{Y_t}{n} = \frac{\sum_{1}^{n} X_n}{n} \tag{5}$$

The continuous equations for the cumulative average and cumulative total learning curves can be shown to be:

cumulative total

$$Y_t = a(XC)^{b+1} \tag{6}$$

cumulative average

$$Y_c = a(XC)^{b} \tag{7}$$

where Y_c is the average cost of cumulative quantity XC, Y_t is the total cost of cumulative quantity XC, "a" is the first unit cost, and "b" is the slope of the cumulative average curve, which ranges from -1 to 0. An example of the cumulative curves can be seen in Fig. 2. Comparing the unit curve equation (1) and the cumulative curve equations (6) and (7), it is seen that the equations are of the same form.

<u>Unit Curve Versus Cumulative Curves</u>

As can be seen from the previous discussion, the unit and cumulative curves have the same form of equation and are derived from the same data. Therefore, it would seem that the unit curve equation could be used to derive the cumulative average curve equation, or vice versa. This is not the case, however, and some confusion has resulted regarding which learning curve to use.

It is this inconsistency between curves that led to the development of the Unified Linear Progress Curve. The unit curve will now be derived from the linear cumulative curves and the cumulative curves derived from the linear unit curve. The inconsistencies between the curves will be noted and followed by the development of the Unified Linear Progress Curve which is hypothesized to resolve the inconsistencies.

Unit Curve from the Cumulative Curve. Individual unit cost can be obtained from the cumulative curves by subtracting the cost of cumulative quantity (XC-1) from cumulative quantity (XC):

$$Y_u = a\left((XC)^{b+1} - (XC-1)^{b+1} \right) \tag{8}$$

When this equation is plotted at X on a log-log scale, the result is nonlinear and has been found to asymptotically approach the equation

$$Y = a(b+1)X^b \tag{9}$$

Two aspects of this development are noted at this point:

1. The equation for the unit curve (derived from the cumulative curve) is non-linear and is not the same as the linear unit curve, equation (1).

2. The asymptote's (equation (9)) Y intercept at X = 1 is "a(b+1)," which is not the same as the linear unit curve's Y intercept, "a," equation (1).

An example of these curves can be seen in Fig. 3.

Cumulative Curves from the Unit Curve. Similarly, a cumulative average curve and a total cumulative curve can be derived from the unit curve. The cumulative total curve is the sum of each unit cost. Therefore,

$$Y_t = \sum_1^n Y_u = \sum_1^n aX^b = a\sum_1^n X^b \tag{10}$$

It is noted that this cumulative total curve is also non-linear and can be shown to asymptotically approach the equation

$$Y = \left(\frac{a}{b+1}\right)(XC)^{b+1} \tag{11}$$

The cumulative average curve is simply the cumulative total curve divided by n, the number of units.

$$Y_c = \frac{Y_t}{n} = \frac{a}{n}\sum_1^n X^b \tag{12}$$

This curve is also non-linear and asymptotically approaches the line

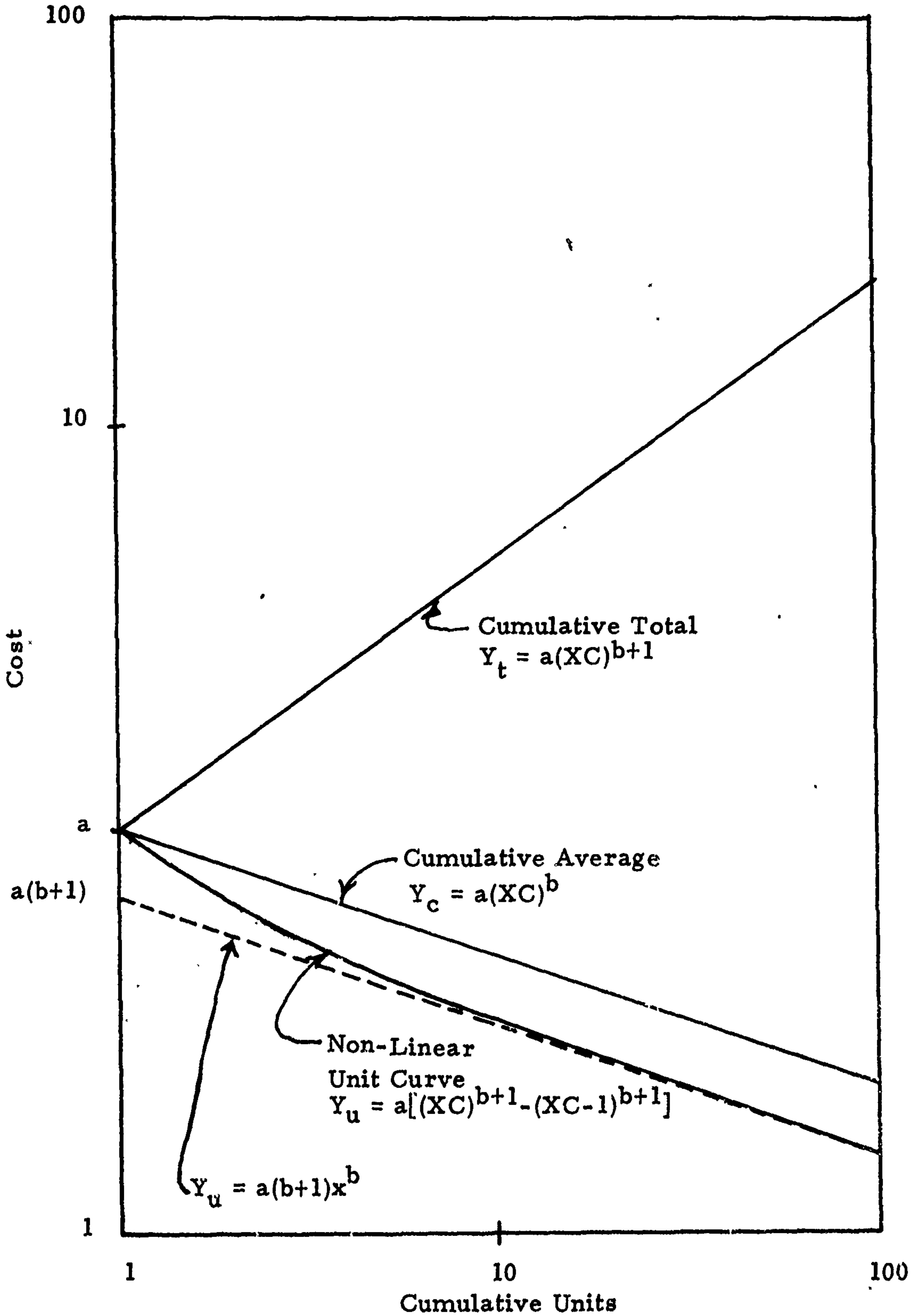

Fig. 3: Unit Curve Derived from an 80% Linear Cumulative Average Curve

$$Y = \frac{\left(\frac{a}{b+1}\right)(XC)^{(b+1)}}{n} \tag{13}$$

Again, it is noted that:

1. The equations for the cumulative curves (10 and 12) are non-linear and are not equal to the linear cumulatives curves (6 and 7).

2. The Y intercept of both asymptotes (11 and 13) at XC = 1 is $\frac{a}{b+1}$, and is not the same as the linear cumulative curve's Y intercept, "a" (6 and 7).

An example of these curves can be seen in Fig. 4.

Unified Linear Progress Curve

The Unified Linear Progress Curve will now be developed and applied to resolve the inconsistencies between the unit and cumulative curves.

The following discussion and derivation of the Unified Linear Progress Curve is based on the cost research report by Karl Berend, Unified Linear Progress Curve Formulation. (Ref 3) The reader who is interested in a more detailed discussion of the Unified Linear Progress Curve is referred to this report.

One way to approach the development of the Unified Linear Progress Curve is to study how "lot data" are plotted on a unit curve. When aircraft are produced in lots and therefore only the

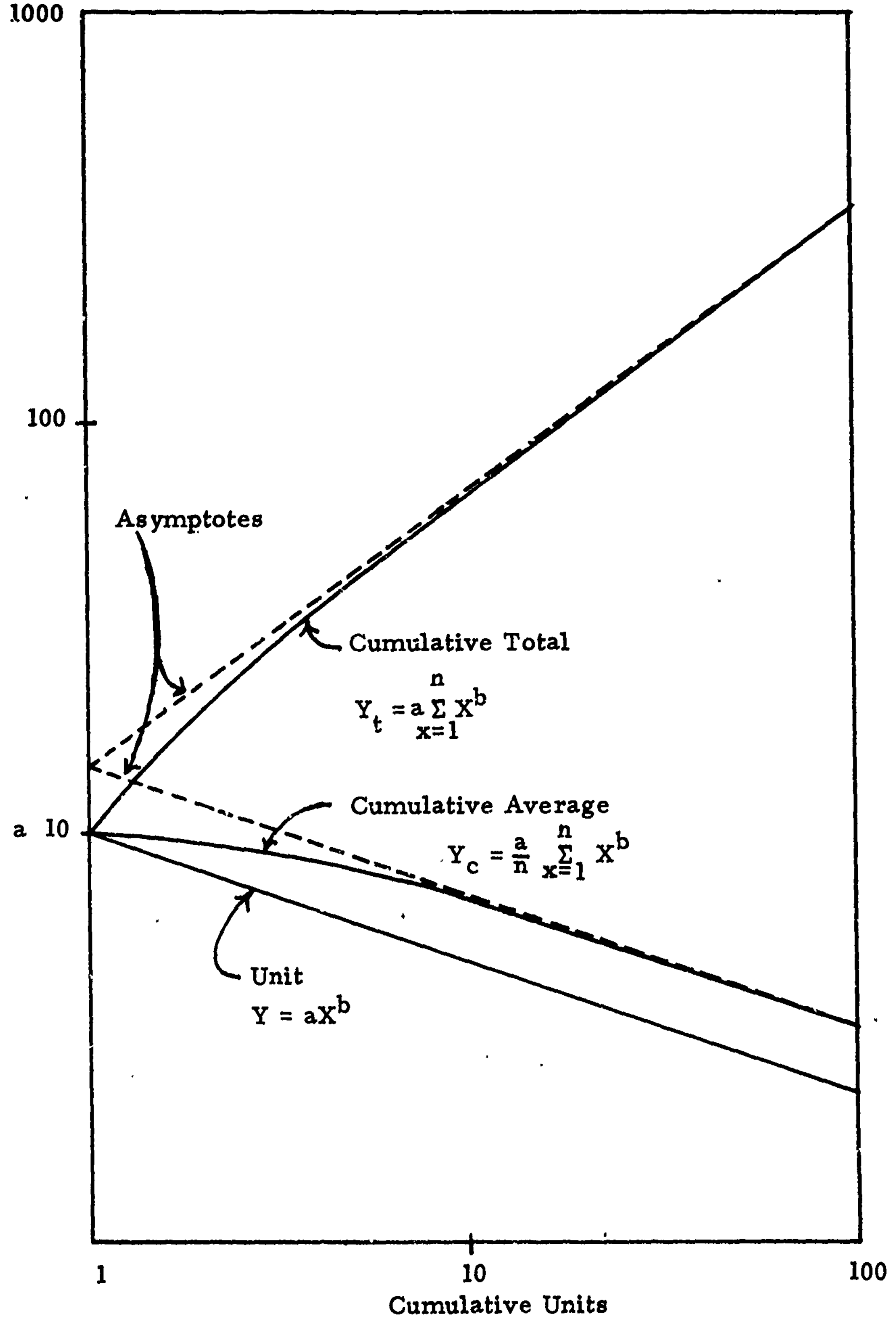

Fig. 4: Cumulative Curves Derived from 80% Linear Unit Curve

cost per lot is available, the practice is to plot the data at the lot midpoint on the unit curve. The arithmetic lot midpoint is determined by adding the unit number of the first unit in a lot to the unit number of the last unit in a lot and dividing by two. However, there is a problem that occurs when plotting at the midpoint: the plot points do not fall on the linear unit curve or the non-linear unit curve (as derived from the cumulative average curve). Also, the midpoint plots will fall closer to the asymptote of the non-linear unit curve than does the non-linear unit curve. The resulting implication is that the asymptote of the non-linear unit curve, equation (9), is a likely candidate for the unit curve, and a unit value plotted on the asymptote is plotted at its true midpoint.

It is therefore implied that unit costs should not be plotted at the cumulative unit number, but at some point between one unit and one half unit less. A good way to test this hypothesis is to integrate the asymptote, equation (9), and then calculate the error, the difference between the actual cost of "n" units and the cost computed by integration

As an example, consider an 80% cumulative average curve which has the following values

$$b = \log (.80/100)/\log 2$$

$$= -.321928$$

$$a = 10 \text{ first unit cost}$$

therefore,

$$Y_c = a(XC)^b$$

$$= 10(XC)^{-.321928} \qquad (14)$$

and,

$\underline{XC}$	$\underline{Y_c}$
1	10
2	8
3	7.021
4	6.4

Unit costs can now be found by applying the equation

$$a(XC)^{b+1} - a(XC-1)^{b+1} = Y_u \qquad (15)$$

therefore,

$\underline{X}$	$\underline{Y_u}$
1	10
2	6
3	5.063
4	4.537

Therefore, the total cost of four units is 10 + 6 + 5.063 + 4.537 = 25.6.

The equation of the asymptote for a unit curve derived from a

80% cumulative average curve is

$$Y_u = a(b+1)^b \tag{16}$$

Integrating equation (16) from 0 to 4 yields

$$\begin{aligned} Y_u &= a(b+1)X^b dx \\ &= \left.\frac{a(b+1)X^{b+1}}{(b+1)}\right|_0^4 \\ &= \left. aX^{b+1}\right|_0^4 \\ &= 10(4)^{.67807} \\ &= 25.6 \end{aligned}$$

Therefore, integrating the asymptote over 4 units yields the actual cost of 4 units with no error. Further testing the asymptote as the unit curve, an integration is now performed from 3 to 4 results in

$$\begin{aligned} &\int_3^4 a(b+1)X^b dx \\ &= \left. 10X^{.67807}\right|_3^4 \\ &= 10(4^{.67807} - 3^{.67807}) \\ &= 4.537 \end{aligned}$$

which is the cost of the fourth unit. The implication of the above calculations is that if the individual unit values are plotted at their "true" midpoints, then the unit curve will be the same as the asymptote. In fact, the true midpoints of the first four units can be determined by reversing equation (16).

$$\log X_m = \frac{\log Y_u - \log (a(b+1))}{b}$$

where X_m is the midpoint.

$\underline{X}$	$\underline{X}_m$
1	.29915
2	1.46224
3	2.4779
4	3.4839

Note that the true midpoints are different from the arithmetic average midpoints: .5, 1.5, 2.5, and 3.5.

The Unified Linear Progress Curve is now defined to be composed of three related curves:

$Y_t = A(XC)^{b+1}$	cumulative total curve	(17)
$Y_c = A(XC)^b$	cumulative average curve	(18)
$Y_u = A(b+1)X_m^b$	unit curve	(19)

<u>Glossary of Terms for Unified Linear Progress Curve</u>

b = exponent for cumulative average and unit curves for slopes greater than -1 (50%) and normally less than 0 (100%) = Log (percent slope/100)/Log 2

$b+1$ = exponent for cumulative total curve

XC = total cumulative quantity = $P+L$

X = cumulative unit

P = prior quantity

L = lot quantity following prior quantity P, $L = XC - P$

Quantities not limited to integers

A = the theoretical first unit cost (and therefore the Y-intercept for the cumulative average and total curves) = $a/(b+1)$

a = the Y-intercept on the unit curve at $X = 1$ (it represents the cost per unit achieved at the end of production of the first unit) = $A(b+1)$

T_{xc} = total cost for cumulative quantity XC

$T_{L(P)}$ = total cost for lot quantity L following a prior quantity P

Y_{xc} = cumulative average cost for cumulative quantity XC

$Y_{L(P)}$ = average cost for a lot quantity L following a prior quantity P

U_x = unit cost of cumulative unit X

X_m = X-coordinate of midpoint on unit curve for cost of Y_m for any size lot (first or follow-on for quantities less than, equal to, or greater than 1 unit

Y_u = cost of X_m = average cost of lot

Unified Linear Progress Curve Equations. In order to demonstrate the unique nature of the unified linear progress curves, four separate areas will be discussed using an example: (1) The derivation of the estimating equations from the cumulative total curve.

(2) Derivation of the theoretical first unit cost from cumulative total, cumulative average, and unit values by assuming a slope. (3) Derivation of the midpoints for the unit curve from unit and lot quantities. (4) Conversion from the unit curve equation to the estimating equation, cumulative total.

The example used will assume a first unit cost of 10 and an 80% slope. Therefore, first unit cost A = 10 and for an 80% slope

$b = \dfrac{\log (80/100)}{\log 2} = -0.32193.$

Cost Estimation Using the Cumulative Total Curve

Cost Estimate	General Equation	Example (80%)
Cumulative total 300 = 478.262	$A = \dfrac{T_{300}}{300^{b+1}}$	$A = \dfrac{478.262}{300^{.67807}} = 10.00$
Lot total 200 after prior quantity of 100 = 251.201	$A = \dfrac{T_{200(100)}}{300^{b+1} - 100^{b+1}}$	$A = \dfrac{251.201}{300^{.67807} - 100^{.67807}} = 10.00$
Cumulative average 300 = 1.5942	$A = \dfrac{Y_{300}}{300^{b}}$	$A = \dfrac{1.5942}{300^{-.32193}} = 10.00$
Lot average 200 after prior quantity of 100 = 1.2560	$A = \dfrac{[Y_{200(100)}][200]}{300^{b+1} - 100^{b+1}}$	$A = \dfrac{(1.2560)(200)}{300^{.67807} - 100^{.67807}} = 10.00$
100th unit = 1.5421	$A = \dfrac{U_{100}}{100^{b+1} - 99^{b+1}}$	$A = \dfrac{1.5421}{100^{.67807} - 99^{.67807}} = 10.00$

100th unit = 1.5421	$A \cong \frac{U_{100}}{(b+1)(99.5^{b})}$	$A \cong \frac{1.5421}{(.67807)99.5^{-.32193}}$ = 10.00

As can be seen from the above equations, the cumulative total curve can be used to derive values for cumulative total, cumulative average, and unit curves. The important point is that all of these curves are related, and each curve can be used to derive values for the remaining curves.

Derivation of First Unit Cost

Cost Estimate	General Equation	Example (80%) (A=10 and b=-.32193)
Cumulative total	$T_{xc}=A(XC)^{b+1}$	$T_{300}=10.0(300)^{.67807}$ =478.262
Lot total	$T_{L(P)}=A((XC)^{b+1} -P^{b+1})$	$T_{200(100)}=10(300^{.67807} -100^{.67807})$ =251.201
Cumulative average	$Y_{xc}=\frac{T_x}{XC}= A(XC)^b$	$Y_{300}=10.0(300)^{-.32193}$ =1.5942
Lot average	$Y_{L(P)}=\frac{T_{L(P)}}{L}$	$Y_{200(100)}=\frac{T_{200(100)}}{200}$ =1.2560
Nth unit	$U_x=A(X^{b+1}-(X-1)^{b+1})$	$U_{100}=10.0(100^{.67807} -99^{.67807})$ =1.5421
Approx. Nth unit	$U_x \cong a(X-.5)^b$	$U_{100} \cong 6.7807(99.5)^{-.32193}$ =1.5421

These first unit cost derivations require only that the slope and one representative cost estimate be known. This cost estimate can be a lot, unit, cumulative average, or cumulative total value. Again, the important point is that all of the curves are related.

Because the unit curve is plotted on log-log paper, the mid-point values for unit X falls between X-1 and X-.5 instead of at X-.5. The calculation of a mid-point for a unit or lot value requires a slope estimation. It is therefore necessary when deriving the unit curve and determining midpoint values to use an iterative process. This is because the estimated slope must be adjusted to the data. Therefore, a "b" value must be assumed, then the midpoints, X_m, calculated. A regression using the X_m values is then performed and a new "b" value calculated. The iterative process is then continued by substituting the new "b" value to calculate new mid-points, X_m. When a "b" value is calculated which is the same as the previously calculated "b" value, the iterative process is complete. Examples of midpoint calculations are now shown.

<u>Midpoint for:</u>	<u>General Equation</u>	<u>Example (80%)</u> (b=-.32193; b+1=.67807)
Lot quantity (L) of 16 after prior qty (P) of 4	$X_m(L,P)=\left(\frac{X^{b+1}-P^{b+1}}{L(b+1)}\right)^{1/b}$	$X_m(16,4)=\left(\frac{20^{b+1}-4^{b+1}}{16(b+1)}\right)^{1/b}$ $=10.662$
20th unit	$X_{u(x)}=\left(\frac{X^{b+1}-(X-1)^{b+1}}{b+1}\right)^{1/b}$	$X_{u(20)}=\left(\frac{20^{b+1}-19^{b+1}}{b+1}\right)^{1/b}$ $=19.497$
First-Lot quantity of 20	$X_{m(x)}=X(b+1)^{-1/b}$	$X_{m(20)}=20(.29915)$ $=5.983$

The following equations are presented as the derivation of the estimating equation (cumulative total curve) from the unit curve and thus demonstrate the interrelationship of the two curves.

General Unit Curve Regression Equation	$Y_m = a(X_m)^b$	$Y_m = 6.7807(X_m)^{-.32193}$
Conversion to Estimating Equation	$T_x = \left(\frac{a}{b+1}\right)(XC)^{b+1}$	$T_{300} = \left(\frac{6.7807}{.67807}\right)(300)^{.67807}$
Estimating Equation	$T_x = A(XC)^{b+1}$	$T_{300} = 10.00(300)^{.67807}$

In summary, certain facts about plotting the unit curve on the asymptote should be noted. (1) The new unit curve has the same slope as the cumulative average slope. (2) The new unit curve is plotted at a value for cumulative unit X between X-1 and X-.5. This means that the average cost of cumulative unit X is plotted at its midpoint on the unit curve instead of at cumulative unit X. (3) The Y intercept at X = 1 for the new unit curve has a value of "a(b+1)" instead "a."

Prior Experience, or Loss of Learning

A paper by Thomas R. Hoffman entitled "Effect of Prior Experience on Learning Curve Parameters" offers a new view of the learning curve of a product after a break in production. (Ref 10) Hoffman's paper illustrates how prior experience on highly similar products affect both the time needed for the "first" unit of subsequent production and the "apparent" percent learning. Figures 5 and 6 illustrate the meaning of "apparent" percent learning. Figure 5 represents an 85 percent learning curve plotted on rectilinear graph paper. Using the conventional learning curve equation

$$Y = aX^b \qquad (20)$$

the plots in both figures begin at X = 1, since equation (20) assumes that the cumulative X begins from the first unit. If some other cumulative origin were used, the "apparent" percent learning would be changed even though the basic phenomenon were the same. For example, if the learning pattern shown in Figure 5 were followed, but the origin for the new order were at X = 10 units, the graph would be as in Figure 6. The apparent percent learning arrived at by calculating the ratio of the cost to produce thirty units to the cost to produce fifteen units of the new order would be about 91.5 percent, although the "true" or underlying percent is 85. (Ref 13:412) The "apparent" learning factor is not a constant but it is a function of the origin shift and the point at which it is measured.

Derivation of Loss of Learning Equations. This section derives the expressions of the relationship between "apparent" and "true" percent learning rates. Starting with the general form of the learning curve equation

$$Y_x = aX^b \qquad (21)$$

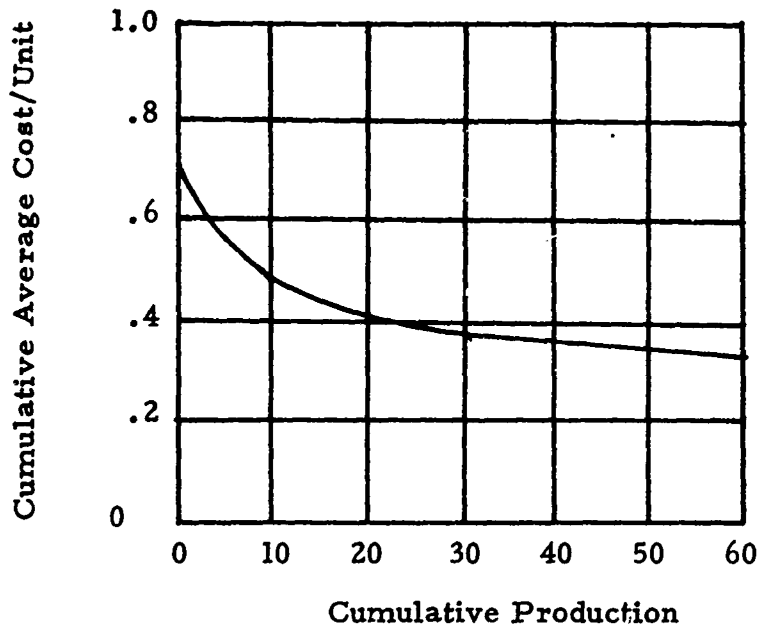

Fig. 5: Eighty-Five Percent Learning Curve

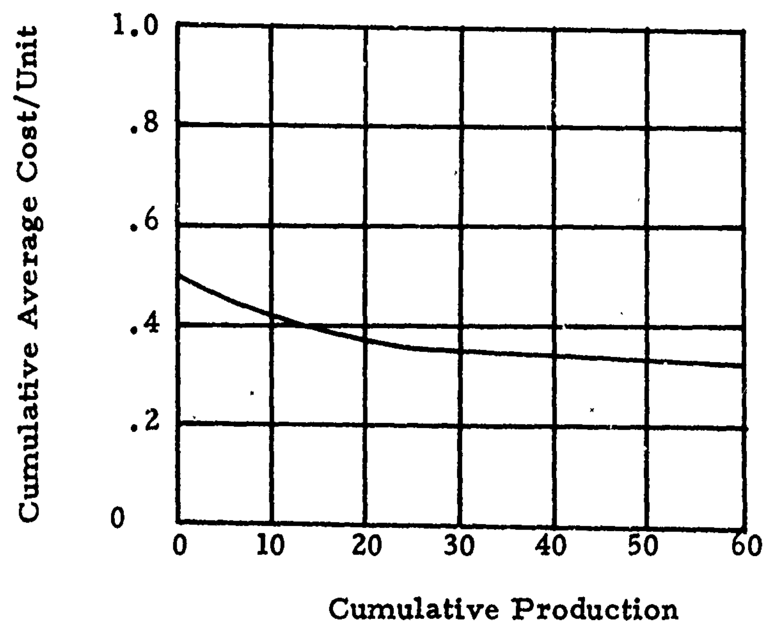

Fig. 6: Eighty Percent Learning Curve with Ten-Unit Shift

and assuming that some displacement of the origin is made, that is, a scale transformation is made such that

$$Z = X - C \tag{22}$$

then, for the Z scale

$$Y_z = dZ^s \tag{23}$$

where

Y_z^* = cumulative average cost per unit

d = cost of unit one

x = total units produced = Z+C

c = prior units produced

Z = cumulative number of units produced after C

S = slope of the learning curve

C = prior units

If $Z = 1, \ Y_1^* = d$

but, on the X-scale where X = Z + C

$$Y_x = Y_{1+c} = a(1+C)^b \tag{24}$$

therefore,

$$Y_1^* = Y_{1+c} = d = a(1+C)^b \tag{25}$$

and

$$Y_{z-c} = a(Z+C)^b = dZ^s \tag{26}$$

Now, substituting the previous expression for "d," eq. (25),

$$a(Z+C)^b = a(1+C)^b Z^s \tag{27}$$

Therefore,

$$Z^s = \left(\frac{Z+C}{1+C}\right)^b \tag{28}$$

Taking the logarithms of eq (28)

$$S \log Z = b \log \left(\frac{Z+C}{1+C}\right) \tag{29}$$

and substituting the expressions for b and s,

where $b = \log (\% \text{ learning})/\log 2$

and $s = \log (\%^* \text{ learning})/\log 2$

we have

$$\left(\frac{\log \%^*}{\log 2}\right) \log Z = \left(\frac{\log \%}{\log 2}\right) \log \left(\frac{Z+C}{1+C}\right)$$

or

$$\log \%^* = \log \% \left(\frac{\log\left(\frac{Z+C}{1+C}\right)}{\log Z}\right) \tag{30}$$

Log %* represents the "apparent" percent learning and log % is the "true" percent learning. It is evident from equation (30) that the effect of having produced "C" units would be to raise the "apparent" percent learning. Therefore, equation (30) should be able to be used to incorporate the effect of retained learning on subsequent work.

Summary

This chapter attempted to provide the reader with a basic understanding of the unit curve, the cumulative curves, the Unified Linear Progress Curve, and the loss of learning curve. The relationship and discrepancies between the unit and cumulative curves was examined. The proposed resolution of the discrepancies between the unit and cumulative curves, the Unified Linear Progress Curve, was explained. The mathematical development of the loss of learning curve was also demonstrated.

III. Data Analysis

The purpose of this chapter is to provide an analysis of the learning curves of previously coproduced aircraft. This analysis includes a determination of distinctive features of learning curves of coproduced aircraft and an application of these features to the learning curve of the F-16 fighter aircraft. This analysis of coproduced aircraft was performed using the Unified Linear Progress Curve and, therefore, the use of the Unified Linear Progress Curve will also be analyzed.

Methodology

The methodology used to analyze the data and apply it to the F-16 coproduction situation is separated into five sections. The learning curve parameters of previously coproduced aircraft are computer calculated. The learning curve parameters are calculated by using the method proposed by Berend in his report, Unified Linear Progress Curve Formalation. This application of the Unified Linear Progress Curve is then analyzed with a determination of which learning curve, the unit or cumulative average, is the correct one.

Then the first unit cost of the coproducer is compared to the first unit cost that would be expected if no loss of learning occurred.

The slope of the coproducer's learning curve is compared to the slope one would expect if no loss of learning occurred. The slope that is expected is calculated using the "loss of learning" equations as proposed by Hoffman. The effect of parallel production on the learning curve is then shown.

Finally the data analysis is applied to the F-16 coproduction situation. Specific conclusions of what effect coproduction will have on the learning curve of the USAF purchased F-16 fighter are provided.

Previously Coproduced Aircraft

In order to provide some measure to predict the effect of coproduction on the F-16 fighter aircraft, an analysis of previously coproduced aircraft learning curves was performed. The aircraft whose learning curves are analyzed are the B24, B29, B47E, B52F, B84F, and the F100C. The data for the B24 and B29 are obtained from the Source Book of World War II Data: Airframe Industry by the Air Material Command. (Ref 17) The data for the remaining aircraft were obtained from Project Backfill. (Ref 13) However, it is noted that not all of the aircraft were coproduced in the strict sense, i.e., by different manufacturers. Rather, some of the aircraft were produced by the same manufacturer at different plants. These data were included due to the limited amount of available coproduction data. It is not meant to imply, however, that all aircraft which have been coproduced are included in this thesis.

Simple linear regression was applied to the aircraft data to obtain the learning curves. The learning curves that were computed were the following:

$$Y_U = aX_m^b \qquad \text{Unit} \tag{31}$$

$$Y_C = A(XC)^b \qquad \text{Cumulative Average} \tag{32}$$

$$Y_T = A(XC)^{b+1} \qquad \text{Cumulative Total} \tag{33}$$

The cumulative curves were calculated as they have historically been accomplished, while the unit curve was calculated as proposed by Berend. (Ref 4) Using Berend's method of deriving the unit curve results in the slope value "b" of the unit curve being the same "b" for the cumulative curves. Each curve was calculated by taking the logarithm of the data and performing a simple linear regression to obtain "b," "a," and "A."

$$\log Y_U = \log a + b \log X_m \tag{34}$$

$$\log Y_C = \log A + b \log (XC) \tag{35}$$

$$\log Y_T = \log A + (b+1) \log (XC) \tag{36}$$

The regression equations used to fit the general equation

$$y = C_1 + C_2X \tag{37}$$

are

$$C_1 = \frac{(\Sigma Y)(\Sigma X^2)-(\Sigma X)(\Sigma XY)}{N\Sigma X^2-(\Sigma X)^2} \tag{38}$$

$$C_2 = \frac{N\Sigma XY-(\Sigma X)(\Sigma Y)}{N\Sigma X^2-(\Sigma X)^2} \tag{39}$$

However, for the unit curve calculation, a lot midpoint, X_m, must be calculated in addition to "a," "b," and "A" according to the equation

$$X_m(L, P) = \left(\frac{X^{b+1}-P^{b+1}}{L(b+1)}\right)^{1/b} \tag{40}$$

where L is the lot size and P is the prior quantity produced. But equation (40) requires a slope value, "b," which is not known. Therefore an iterative method must be used to calculate "b" and X_m. By initially assuming a "b" value equivalent to an 80% slope the regression equation (39) was used to calculate a new "b" value. This calculated "b" value was then inserted in equation (40) and the regression was again performed using equation (39). This iterative process was continued until the final calculated "b" was less than $\pm.0001$ different from the previously calculated "b."

It is noted at this point that the initial "b" value for an 80% curve is arbitrary and any "b" value could have been used. Only the first few lot midpoints are significantly affected by the choice of "b." The later lots will have true midpoints close to their arithmetic midpoint value for a wide range of "b" values. Therefore the choice of the initial "b" value will not affect the final "b" value that was calculated iteratively.

The coefficient of determination, r^2, was calculated to show the ratio of the amount of variance explained to the total variance

$$r^2 = \left[\frac{\Sigma XY}{((\Sigma x^2)(\Sigma y^2))^{1/2}}\right]^2 \tag{41}$$

Also, the standard error of estimate was also calculated for each curve adjusted for sample size according to the equation

$$SSE = \left[\frac{\Sigma Y^2 - C_1 \Sigma Y - C_2 \Sigma XY}{N-2}\right]^{1/2} \tag{42}$$

The computer performed three separate regressions to obtain the unit curve, the cumulative average curve, and the cumulative total curve. Each regression output provided values of "a," "b," A, R^2, and SSE (Standard Error of Estimate). The results of the three regression are seen in Tables I through V.

Table I displays the results of the three regressions performed to obtain learning curve parameters for the B24 aircraft. The B24 was developed by Consol-Voltee, San Diego, and coproduced by Consol-Voltee, Fort Worth, North American, and Ford. Table I shows the results of the regression calculator for each producer.

As an example, the regression to derive unit curve parameters resulted in the "b" value of the slope, the slope, the intercept of the unit curve at x = 1 (a), the theoretical first unit cost (A), the coefficient of determination (R^2), and the Standard Error of Estimate (SSE). Table I also shows the same calculations for the cumulative average, and cumulative total curves.

From Table I a comparison of the slopes of the developer and coproducers is possible. The slope from the unit curve calculation is 78.6% for the developer and 74.1%, 76.2%, and 71.6% for the coproducers. Another comparison is the theoretical first unit cost between the developer and coproducers. The developer's first unit cost is 16.6 DMH/LB while the coproducers' are 27.09 DMH/LB, 18.66 DMH/LB, and 43.00 DMH/LB. The point to note here is that these coproducers had higher first unit costs and steeper slopes than the developer. The implications of these facts are discussed in the analysis sections.

Table I also provides the coeficient of determination, R^1, and Standard Error of Estimate (SSE) for each regression. The R^2 for the cumulative curves are greater than the R^2 for the unit curve. Also the SSE is smaller for the cumulative curves than the SSE for the unit curve.

From Table I it is seen that the slope value "b," the slope, the first unit cost value "A," and unit intercept "a" are not the same for the unit and cumulative curves. This is also the case for all the other aircraft learning curves (see Tables II through V). This inconsistency causes a dilemma of which curve to use when using learning curves, the unit or cumulative. The dilemma is addressed further in the section, Unified Linear Progress Curve Analysis.

Finally, Table I provides the approximate number of units produced by the developer before coproducer began production. As an example, the coproducer, Ford, began production after the developer, Consol-Voltee, San Diego, had produced 2262 units. This information is used in the following sections, Coproducer's First Unit Cost and Loss of Learning.

Table II displays the learning curve parameters calculated for the B29 aircraft. Table II shows that the developer had a steeper slope and higher first unit cost than the coproducers. Also, the R^2 were all above the .9 level with the cumulative curves having higher R^2 than the unit. The cumulative curves for the B29 also had lower SSE than the unit curve SSE. Also displayed is the number of units produced by the developer before the coproducer began production. Again, the unit curve calculated "b," "A," and "a" values are different from the cumulative calculated values.

Table III displays the learning curve parameters for the B47E and B52F aircraft. The lowest R^2 value is greater than .68 and unit curve R^2 are lower than cumulative curve R^2. Also displayed are the SSE and the prior number of units produced. Both developers for the B47E and B52F have steeper slopes than their respective coproducer.

Table IV displays the learning curve parameters for the F84F and F86F aircraft. From Table IV it is seen that the developers have steeper slopes and higher first unit cost than the producers. Also, displayed are the R^2, SSE values, and prior number of units produced. The lowest R^2, .1610, was for the unit curve of the developer of the F86F, North American, Los Angeles. Again the unit curve parameters "b," "A," and "a" are different from the cumulative curve parameters.

Table V provides learning curve parameters, R^2 and SSE for the developer of the F100C, North American, Los Angeles. However, only the prior number of units produced are available for the coproducer, North American, Columbus, since only one lot of 25 was produced. However, this lot will be used for estimating and comparing the first unit cost to the expected first unit cost and is therefore included.

Tables I through V all show that the unit curve derived parameters "b," "A," and "a" are not equivalent to the cumulative curve derived parameters. This dilemma of which curve is the correct is discussed in the section, Unified Linear Progress Curve Analysis.

Also, Tables I and IV show that the coproducers of the B24, F84F, and F86F all have steeper slopes and higher first unit cost than their respective developers. This result is discussed in the section on Loss of Learning.

Unified Linear Progress Curve Analysis

Unified Linear Progress Curve theory states that the slope of the cumulative average learning curve is the same as the unit learning curve when the unit cost values are plotted at their true midpoints. The true midpoint application was shown in the example in Chapter II. The true midpoint is determined by the equation

$$X_m(L, P) = \left(\frac{X^{b+1} - P^{b+1}}{L(b+1)}\right)^{1/b} \tag{43}$$

where

b = log (percent slope/100/log 2

L = lot size (may be 1 or a fraction)

P = prior quantity

As shown in Chapter II, if the cumulative average equation is

$$Y_c = A(XC)^b \tag{44}$$

then

$$Y_u = A(b+1)X^b \tag{45}$$

$$= aX^b \tag{46}$$

Table I

B24 Learning Curve Parameters

	Developer Consol-Voltee San Diego	Consol-Voltee Fort Worth	North American Dallas	Ford Willow Run
Unit				
b	-.3467	-.4312	-.3918	-.4827
slope	78.6%	74.1%	76.2%	71.6%
a	10.84	15.41	11.35	22.246
A	16.60	27.09	18.66	43.00
R^2	.9021	.9057	.9713	.9656
SSE	.0546	.0737	.0683	.0958
Cum Ave				
b	-.3002	-.3261	-.3539	-.4761
slope	79.8%	81.2%	78.2%	71.9%
a	7.74	8.42	10.26	21.57
A	11.06	12.50	15.88	41.17
R^2	.9952	.9952	.9978	.9962
SSE	.0207	.0262	.0267	.0317
Cum Total				
b+1	.6998	.6739	.6461	.5239
slope	.9987	.9971	.9991	.9999
a	7.74	8.42	10.26	21.57
A	11.06	12.50	15.88	41.17
R^2	.9741	.9686	.9926	.9956
SSE	.0207	.0262	.0207	.0317
Approx Prior Units Produced	N/A	1292	1897	2262

Table II

B29 Learning Curve Parameters

	Developer Boeing Wichita	Bell Marietta	Boeing Renton	Martin Omaha
Unit				
b	-.4528	-.3842	-.3119	-.3432
slope	73.1%	78.9%	80.6%	78.8%
a	19.26	13.89	7.918	7.54
A	35.20	22.56	11.50	11.47
R^2	.9543	.9539	.9206	.9852
SSE	.0700	.0935	.0778	.0411
Cum Ave				
b	-.3879	-.3608	-.2586	-.3324
slope	76.4%	77.9%	83.6%	79.4%
a	14.94	13.07	6.54	7.27
A	24.40	20.46	8.82	10.89
R^2	.9822	.9783	.9680	.9912
SSE	.0325	.0496	.0368	.0273
Cum Total				
b+1	.6121	.6392	.7414	.6676
slope	152.8%	155.7%	167.2%	158.8%
a	14.94	13.07	6.54	7.27
A	24.40	20.46	8.82	10.89
R^2	.9950	.9912	.9960	.9978
SSE	.0335	.0496	.0368	.0273
Approx Prior Units Produced	N/A	56	133	267

Table III

B47E and B52F Learning Curve Parameters

	B47E		B52F	
	Developer Boeing Wichita	Douglas Tulsa	Developer Boeing Seattle	Boeing Wichita
Unit				
b	-.2131	-.0666	-.0914	-.0757
slope	86.1%	95.5%	93.86%	94.9%
a	3.65	2.47	2.319	2.26
A_2	4.64	2.64	2.553	2.44
R^2	.6870	.7229	.9626	.9024
SSE	.0649	.0199	.0099	.0117
Cum Ave				
b	-.1185	-.0471	-.0907	-.0713
slope	92.0%	96.8%	93.91%	95.2%
a	2.46	2.34	2.317	2.24
A_2	2.80	2.46	2.548	2.41
R^2	.7738	.9260	.9868	.9928
SSE	.0246	.0042	.0043	.0018
Cum Total				
b+1	.8815	.9529	.9093	.9287
slope	184.2%	193.6%	187.8%	190.4%
a	2.46	2.34	2.32	2.24
A_2	2.80	2.46	2.55	2.41
R^2	.9948	.9998	.9998	1.0
SSE	.0246	.0042	.0043	.0018
Approx Prior Units Produced	N/A	183	N/A	6

Table IV

F84F and F86F Learning Curve Parameters

	F84F		F86F	
	Developer Republic Farmingdale	General Motors Kansas City	Developer North American Los Angeles	North American Columbus
Unit				
b	-.3614	-.4611	-.0784	-.3028
slope	77.8%	72.6%	94.7%	81.1%
a	26.65	42.71	2.66	13.51
A	41.73	79.26	2.89	19.37
R^2	.9188	.9554	.1610	.9825
SSE	.0827	.0849	.0999	.0311
Cum Ave				
b	-.3065	-.4230	-.0935	-.3182
slope	80.8%	74.6%	93.7%	80.2%
a	21.55	38.37	2.84	14.1
A	31.08	66.51	3.14	20.7
R^2	.9920	.9890	.8595	.9994
SSE	.0181	.0333	.0181	.0051
Cum Total				
b+1	.6935	.5770	.9065	.6818
slope	161.7%	149.2%	187.4%	160.4%
a	21.55	38.37	2.84	14.1
A	31.08	66.51	3.14	20.7
R^2	.9984	.9940	.9982	.9980
SSE	.0181	.0333	.0181	.0051
Approx Prior Units Produced	N/A	48	N/A	113

Table V

F100C Learning Curve Parameters

	Developer North American Los Angeles	North American Columbus
Unit		
b	-.2706	N/A
slope	82.9%	
a	8.45	
A	11.58	
R^2	.9826	
SSE	.0330	
Cum Ave		
b	-.2871	N/A
slope	81.9%	
a	8.76	
A	12.30	
R^2	.9932	
SSE	.0178	
Cum Total		
b+1	.7129	N/A
slope	163.9%	
a	8.76	
A	12.30	
R^2	.9988	
SSE	.0178	
Approx Prior Units Produced	N/A	173

It is noted here that XC, cumulative quantity, and X, cumulative unit, have the same numerical value although they describe different concepts. Therefore, the cumulative and unit curves can be plotted on the same scale with no loss of meaning.

The question that now arises is which curve should be used to determine the slope. Karl Berend states that "If an equation is to be derived for the purposes of projection of cost based on historical data points, the cumulative total estimating equation will more than likely be based on an analysis of the unit curve which, of course, involves midpoint calculations." (Ref 3:28)

To test which curve provides the more accurate slope, a separate regression was performed on the data for unit, cumulative average, and cumulative total curves. For the cumulative curves, the same "b" value was calculated as were the same "A" and "a" values. However, the unit curve calculations resulted in "b," "A," and "a" values which are different from the corresponding values of the cumulative curves (see Tables I through V).

The objective of determining which curve the unit or cumulative average, should be used is now addressed. If the cumulative curves are better cost predictors, then it follows that the cumulative curves should also be better predictors of the total cost. Therefore, a comparison of actual total cost to the total cost predicted by the unit and cumulative curves was performed. Table VI shows the actual total cost, the total cost predicted by the unit curve, and the total cost predicted by the cumulative curves for each production line.

Using Table VI, an analysis of the total cost of the B24 produced by Consol-Voltee, San Diego, results in the unit curve prediction of total cost (5107.6) being closer to the actual cost (4960.0) than the cumulative curve's prediction (5116.5). Total cost comparisons for all 17 aircraft results in the unit curve being the better total cost predictor 13 times. Therefore, the cumulative curves predict the total cost better only 4 times.

To further study the dilemma of whether the unit curve or cumulative curves should be used, Figures 7 and 8 are displayed. Figure 7 shows the unit and cumulative average data and the learning curves for the B47E produced by Boeing, Wichita. Figure 8 shows the unit and cumulative average data and learning curves for the F84F produced by Republic. Figures 7 and 8 show that the cumulative average data trend is toward the cumulative average curve derived from the unit curve, and not toward the estimated cumulative average curve. This result and the result of the total cost was predicted better when derived from the unit curve 14 of 17 times leads to the conclusion that the unit curve should be used to find the slope and corresponding "A," "a," and "b" values.

The conclusion which is drawn from the application of the Unified Linear Progress Curve in this thesis is that the unit curve

Table VI

Predicted Versus Actual Cumulative Total DMH/LB

	Cumulative Total From Unit Curve	Cumulative Total From Cumulative Curves	Actual Cumulative Total
B24			
Consol-Voltee, San Diego	5107.6	5116.5	4960.0
Consol-Voltee, Fort Worth	2097.3	2161.6	2028.3
North American	1168.9	1512	1189
Ford	4561	4635	4648
B29			
Boeing, Wichita	2023.7	2267.4	1980.4
Bell	1155.7	1217.0	1170.0
Boeing, Renton	1379.4	1532.0	1361.0
Martin	707.1	718.3	702.9
B47E			
Boeing, Seattle	812.0	912.0	841.0
Douglas	159.8	162.0	160.0
B52F			
Boeing, Seattle	79.49	79.54	79.42
Boeing, Wichita	82.37	82.60	82.35
F84F			
Republic	5541.5	6282.8	6086.9
General Motors	2487.2	2663.7	2399.0
F86F			
North Amer., Los Angeles	2637.7	2564.4	2551.0
North Amer., Columbus	1865.5	1802.0	1809
F100C			
North Amer., Los Angeles	999.2	959.5	1000.2

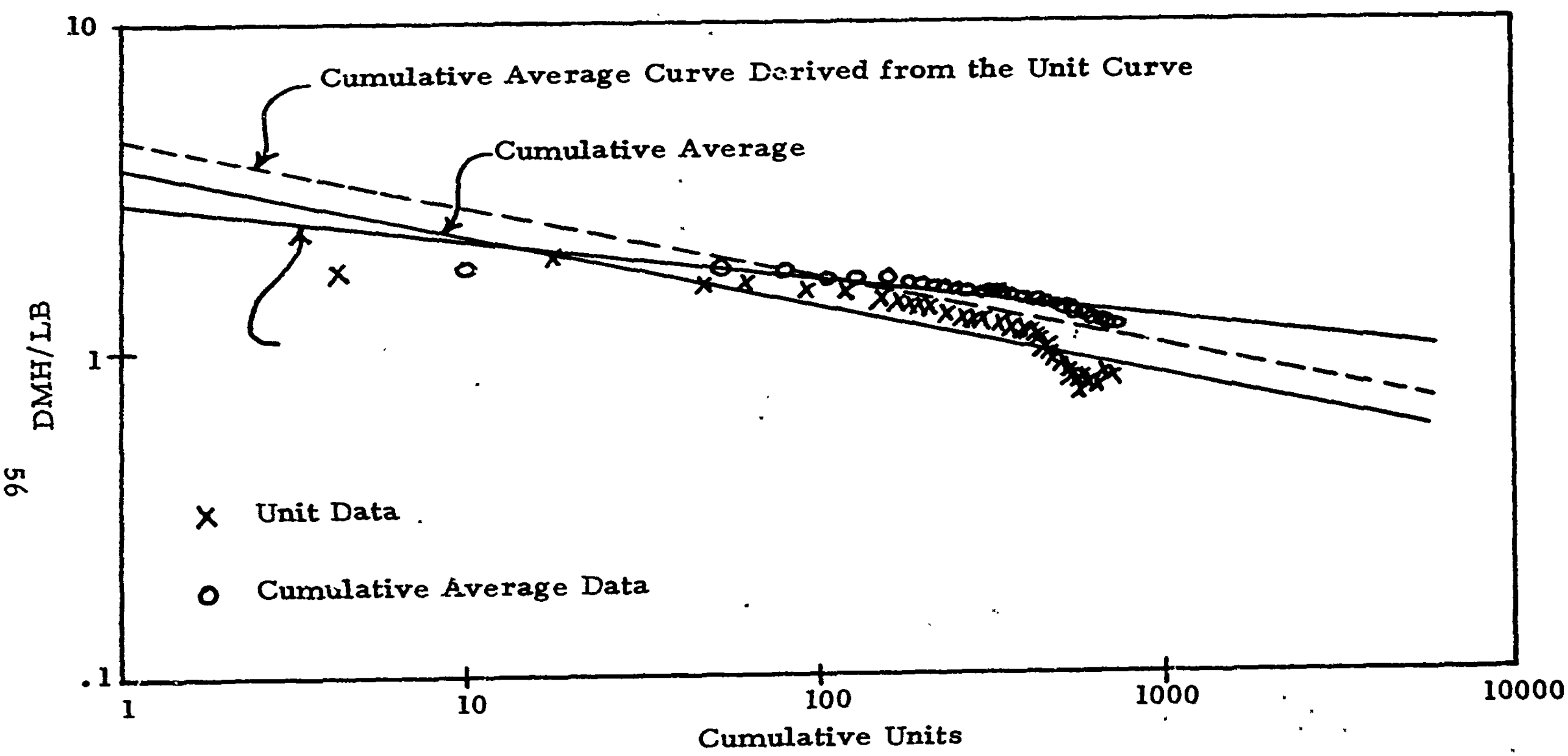

Fig. 7: B47E Boeing, Wichita

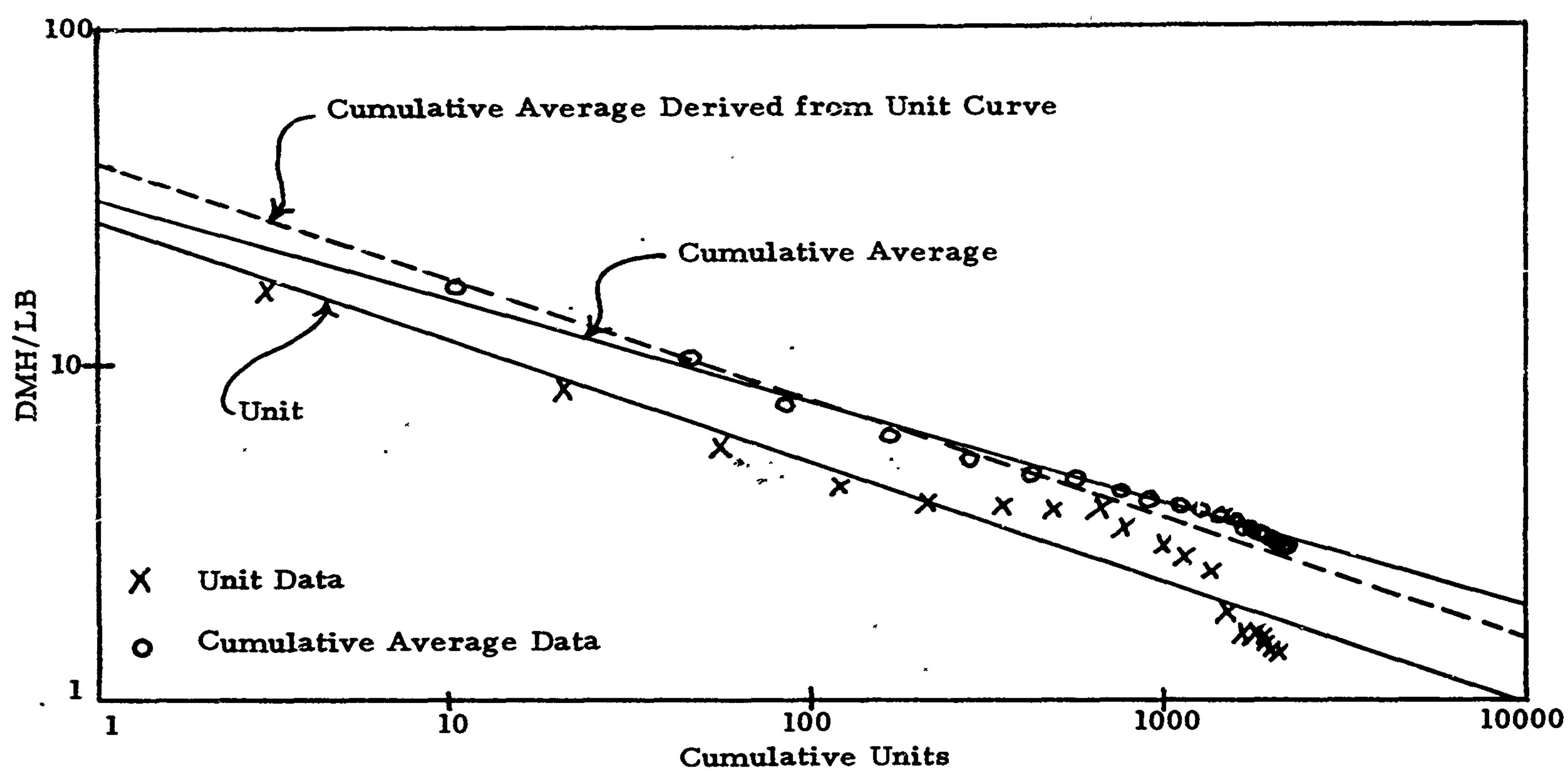

Fig. 8: F84F, Republic

plotted at its true midpoints provides the correct slope value "b." Also, the "b" value is the same for the cumulative curves. Therefore, the unit curve would be used for analytic purposes which the cumulative curves, derived from the unit curve, would be used for estimating.

Coproducer's First Unit Cost

In order to compare a coproducer's aircraft cost to the aircraft cost of the developer, a comparison of the first unit DMH cost of the coproduced aircraft to the DMH cost of the simultaneously produced aircraft unit of the developer is needed. This is done by forming the ratio of the cost of the first unit of the coproducer to the cost of the developer's simultaneously produced aircraft. The approximate number of prior units produced by the developer before the aircraft was coproduced were obtained from the Source Book of World War II Data: Airframe Industry (Ref 17) and a paper by Fred D. Arditti (Ref 1). The coproducer's first unit cost is the "A" value of the unit curve calculation. The developer's simultaneous cost is computed from the developer's unit curve by substituting in the prior units produced before coproduction plus one:

$$y = aX^b \tag{47}$$

$$\text{DMH/LB Cost} = a\ (\text{prior units} + 1)^b \tag{48}$$

These ratios are displayed in Table VII. For the calculation of the ratios, the computed unit curve parameters are used. It is noted here that the inclusion of the F100C aircraft coproduction data only provides the ratio since the data for North American, Columbus, the coproducer's F100C aircraft is a single lot of 25.

Table VII displays each coproducer's first unit cost, "A," the expected first unit cost, and the ratio of actual/expected. The expected cost is the developer's DMH/LB cost of a aircraft that was produced at the time the coproducer's first unit was produced. The number of units that the developer produced prior to each coproducer's production is displayed in Tables I through V. Therefore, the coproducer, Consol-Voltee, Fort Worth, has a first unit cost of 27.09 DMH/LB, an expected first unit cost of .9044 DMH/LB and a ratio of 27.09/.9044 = 29.95. This shows that Consol-Voltee, Fort Worth, has a first unit DMH/LB cost of 29.95 times the expected DMH/LB cost.

From Table VII it is seen that in every case of coproduction presented the coproduced aircraft first unit DMH/LB cost was greater than the expected first unit DMH/LB cost. This cost ranged from 1.25 to 57.73 times as much as the aircraft produced by the developer. In the case of the B52F, where the ratio 1.25 is the smallest; "This is explained by the fact that Boeing of Seattle transferred a good part of its development team to Wichita to supervise

Table VII

Ratio of Actual/Expected First Unit DMH/LB

	Coproducer 1st Unit DMH/LB c	Expected DMH/LB d	Ratio c/d
B24			
Consol-Voltee, San Diego	N/A	Developer	N/A
Consol-Voltee, Fort Worth	27.09	.9044	29.95
North American	18.66	.7915	23.57
Ford	43.00	.7448	57.73
B29			
Boeing, Wichita	N/A	Developer	N/A
Bell	22.56	3.369	6.69
Boeing, Renton	11.50	2.09	5.48
Martin	11.47	1.531	7.48
B47E			
Boeing	N/A	Developer	N/A
Douglas	2.64	1.20	2.19
B52F			
Boeing, Seattle	N/A	Developer	N/A
Boeing, Wichita	2.44	1.94	1.25
F84F			
Republic	N/A	Developer	N/A
General Motors	79.26	6.53	12.14
F86F			
North Amer., Los Angeles	N/A	Developer	N/A
North Amer., Columbus	19.37	1.83	10.55
F100C			
North Amer., Los Angeles	N/A	Developer	N/A
North Amer., Columbus	5.83*	2.05*	2.84

*average DMH for lot of 25 A/C

the first stages of production. Consequently this does not represent a case of separation in the strict sense of the word." (Ref 1:16)

Loss of Learning

During the development of this thesis, the loss of learning curve was derived in Chapter II to show the effect of having produced a previous number of units on the learning curve, equation (30). However, a comparison of the loss of learning curve derived and the actual curve, when plotted on a log-log scale, shows that the loss of learning equations developed by Hoffman do not accurately describe the data. Figure 9 illustrates an original 80% learning curve, the actual nonlinear curve which results when there is a shift of 10 units, and the loss of learning curve as proposed by Hoffman. However, the learning curve proposed by Hoffman does provide a good estimate of the expected slope of a coproduced aircraft would be if there were no loss of learning.

Table VIII displays the coproducer's actual slope and expected slope as derived by Hoffman. The expected slope is based on the developer's slope and the number of units produced by the developer prior to the coproducer's production (see Tables I through V for number of prior units produced). As an example, the developer's slope for the B24 is 78.6% (Consol-Voltee, San Diego, --under expected slope) and the coproducer's expected slopes are 97.0%, 93.6%, and 95.98%. Under the heading of actual slope, the coproducer's actual slopes are displayed.

It is seen from Table VIII that the coproducer's actual slope is steeper in every case than the expected slope. In fact, the coproducer's of the B24, F84F, and F86F had actual slopes that were steeper than the expected slope and the developer's slope. Also the coproducers of these three aircraft had first unit costs "A" greater than the developer's first unit cost (see Table I and Table IV).

The difference in % slope ranged from 1.6% to 24.4%. The 1.6% slope difference is for the B52F produced by Boeing, Wichita (expected slope of 96.5% minus the actual slope of 94.9% equals 1.6%). The 24.4% slope difference is for the B24 produced by Ford (expected slope of 95.98% minus the actual slope of 71.6% equals 24.4%).

The implication of the coproducer's first unit cost being higher than the simultaneous unit cost of the producer and the coproducer's slope being steeper than expected, is that either a loss of learning did occur, or that some learning can not be transferred. In those cases where the coproducer's first unit cost was not only higher than the producer's simultaneous produced unit cost but was also higher than the producer's first unit cost, the implication is that some learning already existed at the producer's plant at the start of the production run and was not transferred. It is inferred from this result that not all learning is transferable and must therefore be

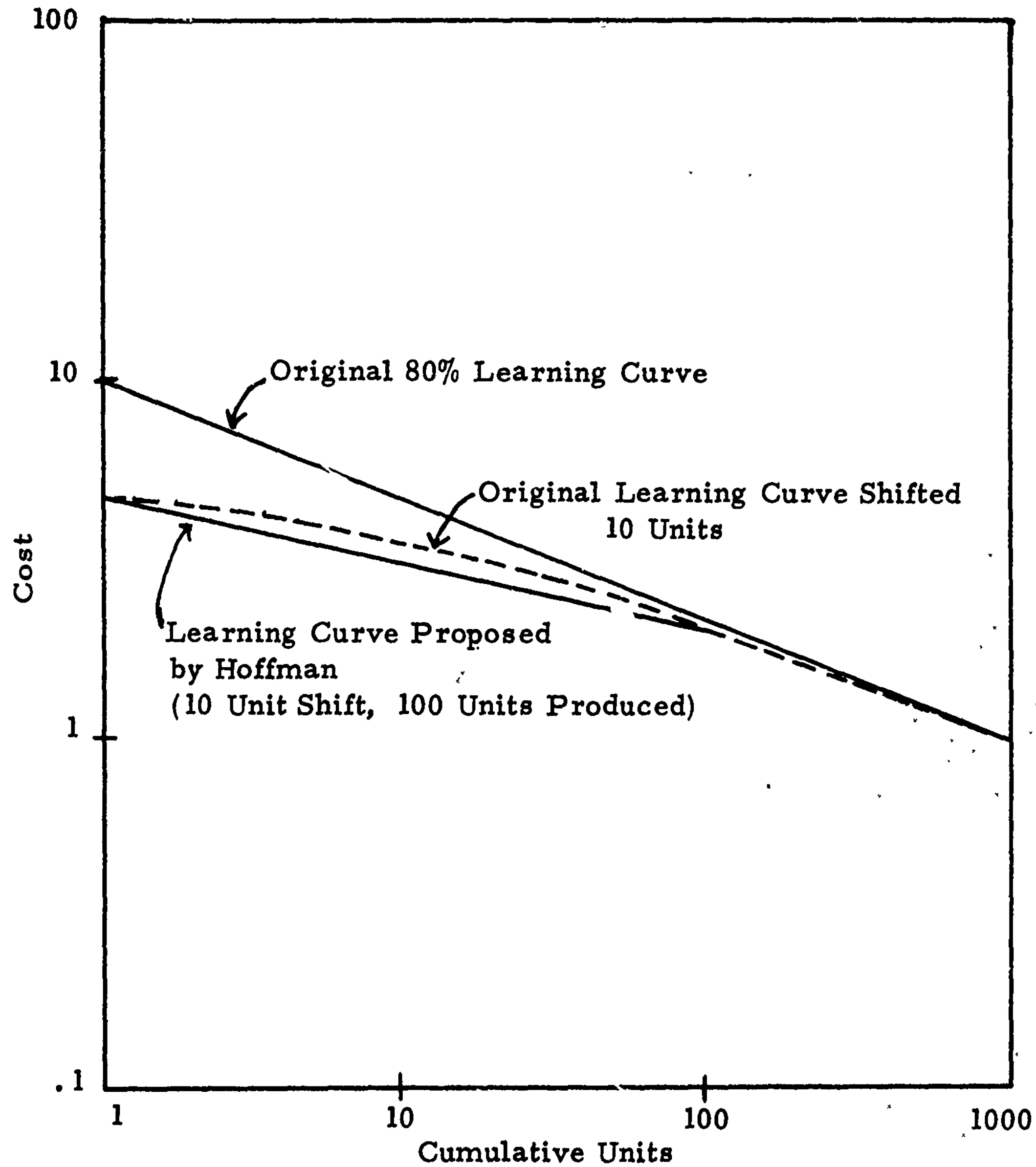

Fig. 9: Comparison of Loss of Learning Curve and Actual Learning Curve

Table VIII

Coproducer's Expected Learning Curve Slope

Manufacturer	Actual Slope	Expected Slope
<u>B24</u>		
Consol-Voltee, San Diego	Developer	78.6%
Consol-Voltee, Fort Worth	74.1%	97.0%
North American	76.2%	98.6%
Ford	71.6%	95.98%
<u>B29</u>		
Boeing, Wichita	Developer	73.1%
Bell	78.9%	88.6%
Boeing, Renton	80.6%	90.6%
Martin	78.8%	94.7%
<u>B47E</u>		
Boeing	Developer	86.1%
Douglas	95.5%	98.8%
<u>B52F</u>		
Boeing, Seattle	Developer	93.9%
Boeing, Wichita	94.9%	96.5%
<u>F84F</u>		
Republic	Developer	77.8%
General Motors	72.6%	90.3%
<u>F86F</u>		
North American, Los Angeles	Developer	94.7%
North American, Columbus	81.1%	98.4%

learned.

The reason that the coproducer's learning curve is steeper than the producer's curve can be understood by recognizing that some learning that was transferred may not have been initially applied in the most efficient manner. When an aircraft is coproduced, the coproducer has the advantage of laying out a more effective production line based on the producer's experience. The machines used to manufacture parts and construct the aircraft may also have the potential to be utilized more effectively. However, the coproducer may not be able to initially apply this transferred learning until the personnel become familiar with their tasks. Therefore, as the coproducer's personnel familiarize themselves with their tasks, the coproducer's learning rate can be expected to be greater than the producer's learning rate. The increased learning rate would result from previously transferred knowledge being utilized.

An element of confusion may result from the fact that some coproducers have a learning curve which is steeper than the developer's. This fact implies that the coproducer could eventually produce the aircraft at a lower cost than the developer. In fact, this has occurred once according to data that was analyzed: the B24 was eventually produced at less cost by Ford than by the developer, Consol-Voltee. In all other cases, either the coproducer production run was too short to determine whether the aircraft could be produced at less cost, or the coproducer's learning curve eventually approached the developer's learning curve without crossing the developer's learning curve and therefore producing at less cost. It is therefore assumed that in a situation where the coproducer has a steeper learning curve than the developer, the coproducer's learning curve is not linear on a log-log scale but starts with a steeper slope than the developer and then approaches the developer's slope.

Parallel Production

Since the F-16 will be produced in both the United States and Europe, a discussion of the effect of parallel production on the learning curve is necessary. A study of parallel production was performed by John H. Russell in an article, Progress Function Models and Their Deviations. (Ref 16) This article shows the effect of increasing production lines on the average cost to produce a unit of output or the cumulative average learning curve. This effect is demonstrated in Figure 10 and Table IX.

The theorem of parallel production lines states: doubling the number of lines doubles the quantity, but the cumulative average remains the same. (Ref 16:5) Table IX shows that the cumulative average cost of 100,000 units increases as more parallel operations are added. Also, the time required to produce the 100,000 units decreases. This example is based on an 80 percent learning curve

Table IX

Parallel Production Lines:
Time and Man-Days to Produce 100,000 Units

Lines	Years	Man-Days	Cumulative Average	(1st Unit) Constant
1	20.5	4,913	.049	2.00
2	12.8	6,142	.061	2.50
4	8.0	7,677	.077	3.13
8	5.0	9,597	.096	3.91
16	3.1	11,996	.120	4.88

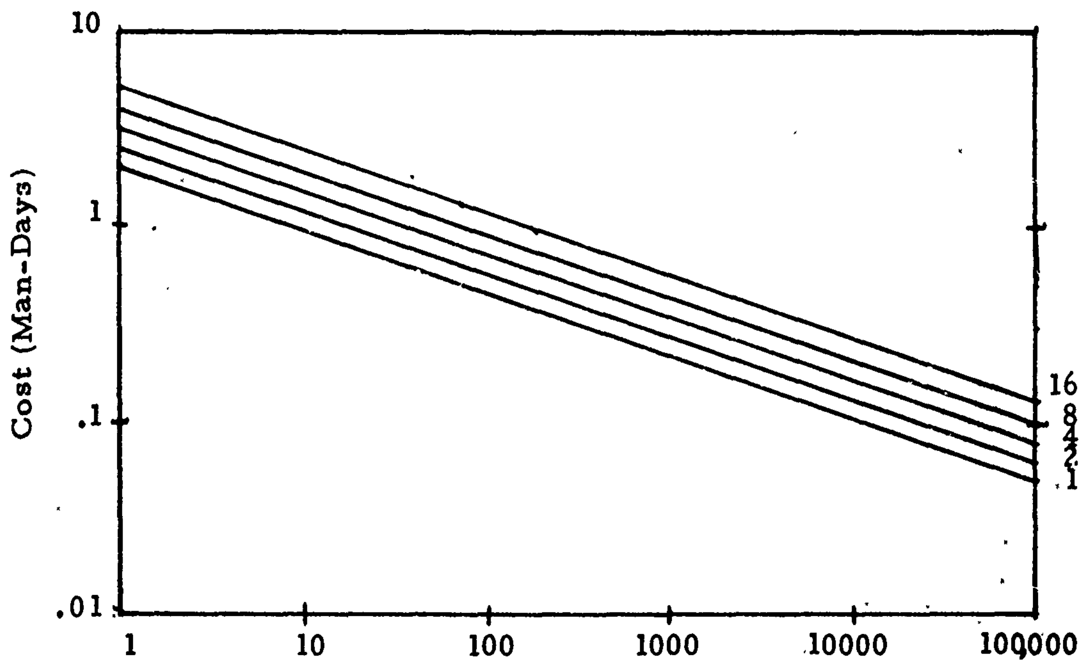

Fig. 10: Cumulative Average 80% Learning Curve for Parallel Production Lines

and a first unit value of 2.0 man-days effort.

The following situation from a GAO report to Congress of the F-16 provides a good example of the effect of multiple production lines.

> The F-16 attack radar contract was awarded to Westinghouse Corporation in November 1975. The original coproduction plan called for extensive radar coproduction, with six major components being manufactured by at least eight EPG producers. It was originally planned that all radar coproduction contracts would be awarded by May 1976. Initial EPG coproduction proposals in February 1976 quoted prices that were much higher than domestic prices and considered unacceptable by program officials.
>
> As a result, the radar coproduction plans were considerably revised, and a plan involving large production runs of a single component by one manufacturer in each EPG nation was proposed. This plan was accepted by the Steering Committee and contracts were signed in February 1977. Although this plan lowers the total dollar value of EPG radar coproduction, SPO officials stated that this approach results in acceptable U.S. and EPG radar prices and offset. (Ref 8:12)

Two observations are now made from the development of parallel production lines. First, the average cost of a unit increases as the number of parallel production lines increase. Second, the combined cumulative average learning curve of two or more parallel production lines result in a learning curve with the same slope as the original production lines but a higher first unit cost.

However, this combined learning curve is the result of two or more identical learning curves, which implies that subsequent production lines were initiated with no loss of learning. If loss of learning had occurred on subsequent production lines, as in the case of the coproduced aircraft; the resulting learning curve would have a steeper slope, higher first unit cost, and a greater average cost per unit than equivalent production lines.

The following results of the data analysis are now applied to the coproduction effect on the USAF purchased F-16 learning curve.

1. The coproducer's learning curve is steeper than the learning curve "expected" if perfect learning were transferred from developer to coproducer.

2. The coproducer's theoretical first unit cost "A" is higher than a comparable unit cost of the developer.

3. The use of parallel production lines increases the average unit cost for a given quantity produced as opposed to the quantity being produced on only one production line.

F-16 Learning Curve

The present plan for the coproduction of the F-16 as described in the F-16 Independent Cost Analysis is for the EPG to produce 10% of the procurement value of the first 650 USAF purchased F-16, 40% of the procurement value of the 348 EPG purchased F-16, and 15% of the procurement value of other FMS; currently 160 F-16. (Ref 7:5)

Therefore, a total 1158 aircraft will be coproduced with the EPG effectively producing 228.2 aircraft and the United States 929.8 aircraft.

It is noted that the data analvsis of coproduced aircraft was performed with learning curve data for airframes. Also, the F-16 analysis is for the complete aircraft. However, it is assumed that the results of the data analysis are sufficiently general and therefore apply to the complete F-16 aircraft; airframe, engines, and avionics.

The effect of coproduction on the learning curve of the USAF purchased aircraft is now studied by applying the effect of parallel production lines, the coproducer's first unit cost, and the coproducer's learning curve slope. The use of two production lines to produce the F-16, one in the United States and one in Europe, will increase the average cost of a unit as compared to having only one production line. Also, because the procurement cost is shared among the purchasers, the average cost per aircraft to the USAF will be higher than if the F-16 were produced on only one production line.

Although the average cost per aircraft will increase due to two production lines, the data analysis of previously coproduced aircraft adds the dimension of a higher than expected coproducer's first unit cost, i.e., that associated with a perfect transfer of learning. The data analysis of first unit cost, as displayed in Table VII provides first unit cost increases ranging from 1.25 to 57.73 times the producer's simultaneously produced cost. The implication of the data analysis is that the EPG's first unit cost will be higher than the first unit cost of the domestically produced F-16, which will also increase the average unit cost.

The other cost dimension provided by the data analysis of previously coproduced aircraft is that the coproducer's learning curve slope is steeper than expected due to loss of learning. From Table VIII, the increase in steepness of the coproducer's % slope ranged from 1.6% to 24.4%. Therefore, if the F-16 is simultaneously produced in Europe and the United States, the implication of the data analysis is that the European learning curve % slope will be 1.6% to 24.4% steeper than the United States production learning curve slope.

Since the USAF purchased F-16 aircraft are a mix of the two production runs in the United States and Europe, the learning curve of the USAF purchased aircraft will be a combination of the two learning curves. Therefore, the learning curve of the USAF aircraft will be steeper and have a higher first unit cost than would be expected with a perfect transfer of learning.

IV. Summary and Conclusions

Summary

This research effort analyzed the effect of coproduction of the F-16 fighter on the learning curve of the aircraft purchased by the United States. This analysis was performed by studying the historical effects of the coproduction of other aircraft on learning. The previously coproduced aircraft that were analyzed were the B24, B29, B47E, B52F, F84F, F86F, and F100C. Therefore, there were seven developers and a total of eleven coproducers.

The Unified Linear Progress Curve, as proposed by Berend, was used to derive the learning curves. The justification for using the Unified Linear Progress Curve is that it removes the question of which is the proper curve to use: the cumulative average or the unit curve. Berend claims that the proper plotting of unit data at true midpoints results in the unit and cumulative average curve being linear on a log-log scale and having the same slope.

In this study, both the unit and cumulative curves were derived. The inconsistencies which result when the unit curve is derived from the cumulative curves, as well as similar inconsistencies resulting from the derivation of cumulative curves from the unit curve, were demonstrated. The Unified Linear Progress Curve was derived and then used in an example to demonstrate the method Berend hypothesized to resolve the inconsistencies.

Three separate regressions were then performed on the data of the previously coproduced aircraft, resulting in a unit curve (as proposed by Berend), a cumulative average curve, and a cumulative total curve for each aircraft. According to the Unified Linear Progress Curve, the unit curve should be plotted at the true midpoints of the data and not at the cumulative unit, and the cumulative curves subsequently plotted as they have historically been accomplished. The analysis of the Unified Linear Progress Curve showed that the slope should be derived from the unit curve and the slope of the unit curve should be used to derive the cumulative curves. This results in the unit curve being used for production analysis, and the cumulative curves that were derived from the unit curve being used for cost estimation. The unit curve was therefore used to investigate the effect of coproduction.

The results of applying the Unified Linear Progress Curve are:

1. Calculation of true midpoints of unit curve data can be accomplished using an iterative process.

2. The cumulative curves, when derived from the unit curve, were better predictors of cumulative costs than cumulative curves derived by historical methods.

3. Since the cumulative curves are better predictors when derived from the unit curve, then the historical discrepancy between the unit and cumulative curves is resolved by using the Unified Linear Progress Curve.

4. The unit curve can be used for analytic purposes and the cumulative curves (when derived from the unit curve) can be used in cost estimation.

The F-16 is being produced in the United States and Europe. However, since the aircraft purchased by the USAF will be a combination of parts from both the United States and Europe, the learning curve of the USAF purchased aircraft will be a combination of two production lines. This will result in an increased average cost per unit when compared to the average cost per unit of aircraft produced on only one line.

In order to determine the effect of coproduction on the learning curve, the two parameters of the coproducers' learning curves were studied--first unit cost and slope. If an aircraft were coproduced with no loss of learning, the first unit cost to the coproducer would equal the cost of a simultaneously produced unit of the developer. However, the analysis of previously coproduced aircraft showed that the coproducer's first unit cost ranged from 1.25 to 57.73 times the producer's simultaneously produced cost.

Since the coproducers began production after the developer had produced a number of units, a direct comparison of slopes is not possible. However, by applying Hoffman's loss of learning equations it is possible to estimate the coproducer's expected slope if no loss of learning occurred. The data analysis showed that all of the coproducers had slopes steeper than those expected with no loss in learning. The % slopes of the coproducers ranged from 1.6% to 24.4% steeper than expected, which therefore implies a loss of learning.

From the analysis of the coproducer's learning curves it is proposed that the F-16 coproducer, EPG, will have a steeper slope and higher first unit cost than that expected with no loss in learning. Also, since the USAF purchased aircraft will be a combination of two production lines, one of which is probably steeper than expected, it is proposed that the USAF purchased F-16 aircraft will have a higher first unit cost and steeper slope than expected with no loss in learning.

Therefore, if F-16 coproduction follows a similar pattern of learning to previously coproduced aircraft, the following results are expected:

1. The learning curve of the EPG production will be steeper than the United States learning curve.

2. The theoretical first unit cost will be higher for the EPG learning curve than the United States learning curve.

3. Because of the mix of aircraft that will enter the USAF inventory (part United States production and part EPG production), the learning curve of the USAF purchased aircraft will be steeper, have a higher theoretical first unit value, and a higher average cost per unit than if the aircraft were only produced in the United States.

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Vita

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This thesis analyzed the effect of coproduction of the F-16 fighter on the learning curve of the F-16 aircraft purchased by the United States. This analysis was performed by studying the effect of coproduction on previously coproduced aircraft. The aircraft that were analyzed are the B24, B29, B47E, B52F, F84F, and F100C.

The F-16 is being produced in the United States and Europe.

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However, since the aircraft purchased by the USAF will be a combination of parts from both the United States and Europe, the learning curve of the USAF purchased aircraft will be a combination of two production lines. The combination of two production lines result in increased average cost per unit when compared to the average cost per unit of aircraft produced on one only one line.

Also, the analysis of previously coproduced aircraft showed that the coproducer's learning curve had a higher first unit cost and steeper slope than would be expected if all learning was initially transferred and utilized by the coproducer. These results imply that the European Consortium production learning curve will have a higher first unit cost, steeper slope, and higher average cost per unit than the United States production learning curve.

Therefore, the USAF purchased F-16 learning curve will have a higher first unit cost, steeper slope, and higher average cost per unit than a single production line in the United States.