Transonic Shock-Turbulent Boundary Layer Interaction with Suction and Blowing

G.R. Inger, Virginia Polytechnic Institute & State University, Blacksburg, Va.

17th AEROSPACE SCIENCES MEETING
New Orleans, La./January 15-17, 1979
Transonic Shock-Turbulent Boundary Layer Interaction with Suction and Blowing.

G. R. Inger

Virginia Polytechnic Institute and State University

Blackburg, VA

Dept. of Aerospace and Ocean Engineering

Office of Naval Research

Arlington, VA 22217

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Shock-boundary layer interaction, boundary layers with mass transfer, shock turbulent boundary layer interaction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An approximate non-asymptotic theory of weak normal shock-unseparated turbulent boundary layer interactions is given which includes the effect of surface mass transfer. The results of a parametric study of Reynolds number effects on various interaction properties without mass transfer, including skin friction are first presented along with detailed comparisons with experiment including supercritical airfoil data. The extension and application of the theory to include moderate amounts of either suction or blowing through the surface is then discussed, especially as regards the influence of mass transfer on the
20. ABSTRACT (continued)

Skin friction behavior. As a consequence of its influence on the boundary layer profile shape away from the wall, small suction ($-\dot{m}_w/\rho_e u_e \leq 5 \times 10^{-4}$) appreciably reduces the upstream influence and thickening effects of the interaction but hastens the onset of separation in the shock foot region.
TRANSONIC SHOCK-TURBULENT BOUNDARY LAYER INTERACTION WITH SUCTION AND BLOWING

G. R. Inger*  
Virginia Polytechnic Institute and State University  
Blacksburg, VA/USA

Abstract

An approximate non-asymptotic theory of weak normal shock-unseparated turbulent boundary layer interactions is given which includes the effect of surface mass transfer. The results of a parametric study of Reynolds number effects on various interaction properties without mass transfer, including skin friction, are first presented along with detailed comparisons with experiment including supercritical airfoil data. The extension and application of the theory to include moderate amounts of suction or blowing through the surface is then discussed, especially as regards the influence of mass transfer on the skin friction behavior. As a consequence of its influence on the boundary layer profile shape away from the wall, small suction (\(-\dot{m}_w/\rho_{0}u_w < 5 \times 10^{-4}\)) appreciably reduces the upstream influence and thickening effects of the interaction but hastens the onset of separation in the shock foot region.

Nomenclature

\begin{tabular}{ll}
B & Mass transfer parameter, \(\dot{m}_w/\rho_{0}u_w\) \\
C_f & Skin friction coefficient, \(2\mu/\rho_{0}u_w^2\) \\
L & Distance to undisturbed shock location  \\
\(\dot{m}_w\) & Mass flux rate (per sec.) across surface (= \(\rho_{0}u_w\))  \\
M & Mach number  \\
P' & Pressure rise across incident normal shock  \\
Re_{L,Re_{\delta}} & Reynolds numbers \(\rho_{0}u_L/\nu\), \(\rho_{0}u_{\delta}/\nu\)  \\
T & Absolute static temperature  \\
u,v & Flow velocity components along x,y respectively  \\
x,y & Streamwise and normal coordinates, respectively  \\
\gamma & Ratio of specific heats  \\
\delta & Undisturbed boundary layer thickness  \\
\delta^{*} & Interfacial displacement growth, \(\delta^{*}/\delta\)  \\
\mu & Coefficient of viscosity  \\
\nu & Kinematic viscosity coefficient, \(\mu/\rho\)  \\
\rho & Density  \\
\tau & Shear stress  \\
\theta & Momentum thickness
\end{tabular}

Subscripts

1,2,3 Interaction regions, Fig. 2  \\
e & Edge of boundary layer  \\
0 & Undisturbed solid wall boundary layer conditions

* Professor of Aerospace and Ocean Engineering

Copyright © American Institute of Aeronautics and Astronautics, Inc., 1979. All rights reserved.
effects are given. Section 3 then discusses typical numerical results: first for zero mass transfer where unpublished Reynolds number effects including those on upstream influence and skin friction are presented plus several comparisons with experimental data; results showing the mass transfer effects on important interaction properties are then given. Section 4 concludes with a discussion of the limitations of the theory and recommendations for further studies.

2. Theoretical Formulation

2.1 Basic Features of the Interaction Flow Model

It is well-known experimentally that when separation occurs, the disturbance flow pattern associated with normal shock-boundary layer interaction is a very complicated one involving a bifurcated shock pattern\(^\text{14}\), whereas the unseparated case pertaining to turbulent boundary layers up to \(M_1 = 1.3\) has instead a much simpler type of interaction pattern which is more amenable to analytical treatment (see Fig. 1). With some judicious simplifications, it is possible to construct a fundamentally-based approximate analytical theory of the problem in this latter case. For the sake of orientation and completeness, a brief summary of this theory will now be given (full details can be found in Ref. 11).

The flow consists of a known incoming isobaric turbulent boundary layer profile \(M_2(y)\) subjected to small transonic perturbations due to an impinging weak normal shock. In the practical Reynolds number range of interest here \(\text{Re}_L > 10^6\) we purposely employ a non-asymptotic disturbance flow model in the turbulent boundary layer patterned after the Lighthill-Stratford double-deck approach\(^\text{13-15}\) that has proven highly successful in treating a variety of other problems involving turbulent boundary layer response to strong adverse pressure gradients\(^\text{16,17}\). This was done because of the large body of turbulent boundary layer-shock interaction data that strongly supports such a model in this Reynolds number range\(^\text{18-20}\) (including the transonic regime\(^\text{21-23}\)) and because of the findings of a separate general theoretical study\(^\text{32}\) showing that asymptotic theory results for very high Reynolds numbers \(5 \times 10^5\), although rigorous in this limit, do not extrapolate down to the present Reynolds number range. The resulting flow model includes the viscous disturbance flow surrounding a non-linear shock discontinuity and underlayed by a thin viscous disturbance sublayer as schematically illustrated in Fig. 2. The introduction of some further simplifications (including the assumption of small linearized disturbances ahead of and behind the nonlinear shock jump\(^\text{31}\)) is neglect of the detailed shock structure within the boundary layer\(^\text{35}\), which give accurate results for the overall properties of engineering interest provided \(M_1\) is not too close the unity \(M_1 \leq 1.05\), then yields an approximate analytical solution consisting of linearized potential supersonic disturbance flow in region 1 plus the subsonic disturbance flow in quadrant 3 caused by the interaction-generated interface displacement \(\eta_2(x)\) and the postshock perturbations along \(x = 0^+\) due to the impingement of region 1 Mach wave disturbances on the shock. This solution contains the essential global features of the mixed transonic character of the non-separating normal shock-turbulent boundary layer interaction problem including the significant lateral pressure gradient effects, upstream

\[C_f(x) = C_{f_0} - \sqrt{\left[C_f(x) - C_{f_0}\right]/\left(1 - \left(C_{f_0}/C_{f_{\infty}}\right)^2\right)} \left(C_{f_{\infty}}\right)^{1/3} \text{Pr}_{\infty} F(x/R_0) \]

where the non-dimensional function \(F\) is essentially unity ahead of the shock \(x < 0\) and then decreases behind it, decaying slowly like \(F \sim (x/L_o)^{-1/3}\) far downstream. Eq. 1 indicates that, depending on Reynolds number \(C_{f_0}\), a sufficiently strong interactive pressure rise can cause incipient separation \(C_f < 0^+\) near the shock. This solution contains the essential global features of the mixed transonic character of the non-separating normal shock-turbulent boundary layer interaction problem including the significant lateral pressure gradient effects, upstream

\[C_f(x) = C_{f_0} - \sqrt{\left[C_f(x) - C_{f_0}\right]/\left(1 - \left(C_{f_0}/C_{f_{\infty}}\right)^2\right)} \left(C_{f_{\infty}}\right)^{1/3} \text{Pr}_{\infty} F(x/R_0) \]

where the non-dimensional function \(F\) is essentially unity ahead of the shock \(x < 0\) and then decreases behind it, decaying slowly like \(F \sim (x/L_o)^{-1/3}\) far downstream. Eq. 1 indicates that, depending on Reynolds number \(C_{f_0}\), a sufficiently strong interactive pressure rise can cause incipient separation \(C_f < 0^+\) near the shock. This solution contains the essential global features of the mixed transonic character of the non-separating normal shock-turbulent boundary layer interaction problem including the significant lateral pressure gradient effects, upstream

As far as the overall interaction solution for \(10^5 < \text{Re}_L < 10^8\) is concerned, these nonlinear shock jump conditions plus the various non-uniform viscous flow effects within the boundary layer reduce the lower Mach No. limit otherwise pertaining to the linearized supersonic theory in purely inviscid potential uniform flow-see the AIAA paper version of Ref. 11 for more detailed discussion.

As shown below, this gives a good results for \(\text{Re}_L = 10^5 - 10^8\), however, it begins to significantly break down for \(\text{Re}_L > 10^8\) where the effect of the logarithmic portion of the profile on the sublayer solution becomes significant\(^\text{32}\).
influence and interactive skin friction for an arbitrary input turbulent boundary layer profile. Moreover, many detailed comparisons with all available experimental data (Ref. 31 and below) plus recent streamlining of the computational cost. Hence the theory provides a sound basis for interpreting experimental data on unseparated flows and for further extensions, in particular to allow mass transfer through the surface.

2.2 The influence of Mass Transfer

The influence of surface mass transfer on the interaction solution is two-fold: (1) it alters the incoming undisturbed flow (on which the disturbance solution depends) by changing $\rho_u - \rho_o$ and the profile shape away from the wall; (2) it further introduces new mass transfer-induced terms in the disturbance equations. Now the secondary effects (2) may in fact be neglected with good approximation under the assumed conditions of small-to-moderate normal mass transfer rates ($\rho_u / \rho_o$ well $\leq 10^{-3}$) typical of practical applications, according to the following considerations. Under the continued assumption that the viscous disturbance sublayer lies within the laminar sublayer region of a turbulent boundary layer, the mass transfer effect in the leading approximation does not introduce any curvature into the $M_0(y)$ profile but only alters its slope ($\gamma$); consequently in the viscous disturbance sublayer perturbation equations those terms proportional to $d^2M_0 / dy^2$ and $W_0$ (which in turn is proportional to $d^2$ of momentum equation near the wall) can be neglected, leaving the form of these equations unchanged. Likewise, under the continued assumption that turbulent fluctuations are uncorrelated with the interactive disturbances, the explicit new terms in the overlying rotational-inviscid perturbation equations that are proportional to $W_0$ can be neglected also, since detailed studies of the hydrodynamic stability equations have shown that these terms have an altogether negligible effect throughout a high Reynolds number parallel shear flow boundary layer unless the surface mass transfer is quite large (approaching blow-off). Thus to a consistent degree of first approximation, the form of both the viscous and inviscid disturbance equations is unchanged by moderate blowing or suction provided their effect on the mean flow-based coefficients in these equations is included. It is reemphasized that the primary mass transfer effects on the viscous sublayer field and its thickness are thus fully accounted for.

The present interactive perturbation solution in principle may be used with any mean turbulent boundary layer profile input and hence could be coupled with either an experimental measurement or any desired state-of-the-art numerical prediction code. Here, to bring out clearly and efficiently where the various mass transfer effects enter, we have chosen an accurate analytical profile model that has proven especially well-suited to such non-uniform flow perturbation problems. We assume for simplicity that the blowing or suction is on the average uniform and normal to the wall, that its streamwise extent is large compared to the short interaction range, and that it extends far enough upstream to have established a well-defined local equilibrium profile in the incoming boundary layer. Then the mass transfer effect on turbulent skin friction in terms of the basic parameter $B = u'_w / \rho_u u'_0$ can be described by the relation

$$C_f = 1 - \frac{23}{2} \left( \frac{T_{ref} T_e}{1.4} \right) \frac{1}{B}$$

where $T_{ref} / T_e = 1 + 0.038 \rho_u / \rho_o$ and $0.5 \left( \frac{1}{T_w} - 1 \right)$ for $\gamma = 1.4$, and $C_f$ is the zero-blowing value (we used a reference temperature-based Schulz-Grünewald relation). According to Eq. (2) suction, for example, increases the skin friction.

The corresponding mass transfer effect on $\delta_0$ can be estimated as follows. Since it is well-known that the momentum thickness to boundary layer thickness ratio and the Crocco energy equation solution are both insensitive to moderate amounts of mass transfer, we can use the following approximate zero blowing relationship based on a power-law $(u_o - y/\delta)$ profile:

$$\delta_0 \approx \frac{(N+1)(N+2)}{N} \left\{ \frac{1}{2} + \frac{N(N+1)M_{\infty}}{T_w - T_e} \right\}$$

where $\delta_0$ is found from the incompressible form of the momentum equation including mass transfer:

$$\frac{d\delta}{dx} = \frac{C_f}{2} + B$$

and $x$ here is the running length from some upstream reference point. Thus under the aforementioned assumption as $B$ is a constant over some region $x_1 < x < x_2$ (and zero outside) with $x_1, x_2$ far from the interaction zone, the yieldssurface mass transfer is quite large (approaching blow-off). Thus to a consistent degree of first approximation, the form of both the viscous and inviscid disturbance equations is unchanged by moderate blowing or suction provided their effect on the mean flow-based coefficients in these equations is included. It is reemphasized that the primary mass transfer effects on the viscous sublayer field and its thickness are thus fully accounted for.

In addition to $T_w$, mass transfer also influences the profile shape away from the wall as given by the following turbulent boundary layer shear stress profile relation recommended by Conrad and Donaldson (similar expressions also have been proposed by others):

$$\frac{T_w}{T_e} = 1 - 3.9^2 + 3 \eta^3 + (1 - \eta^2)(u'_w u'_e)(2B/C_f^0)$$

where $\eta = y/\delta$. Further, Eq. 4 is to be used with the basic turbulent shear stress definition that
\[
\frac{d(u/u_e)}{dh} = \frac{C_f^2 \text{Re}_{\theta}}{2} \left( \frac{T}{T_e} \right) \left( \frac{\tau/\nu}{w_e} \right)\]

Regarding the turbulent kinematic eddy viscosity distribution \(v_{\text{eff}}\), the available experimental evidence implies that its functional form is significantly affected only by relatively large surface mass transfer rates provided the mass transfer effect on the value of \(w_e \) is taken into account, to a consistent degree of approximation, then, the two-layer piecewise-continuous viscosity formulation developed for use in interaction problems without mass transfer by Inger and Williams may be applied also to the weak-to-moderate blowing or suction cases studied here. Thus we have for \( w_T \):

\[
\begin{align}
\frac{v_{\text{eff}}}{e} \Bigg|_{\text{INNER}} &= (T/T_e)^{1+\omega} \quad \text{for } n = n_s^* \\
\frac{v_{\text{eff}}}{e} \Bigg|_{\text{OUTER}} &= (T/T_e)^{1+\omega} + 0.51 \text{Re}_e^{1/4} \quad \text{for } n = n_s^* + 0.16
\end{align}
\]

where \( \omega \) is the Klebanoff intermittency factor and \( n_s^* \) is found by requiring that the no slip condition \( U_0(0) = 0 \) be satisfied upon inward integration of Eq. 5 from the outer initial condition \( U_0(1) = U_e \). Thus the substitution of Eqs. (4) and (5) plus the Crocco integral temperature profile \( T(u) \) into Eq. 5 and subsequent integration yields accurate yet fundamentally-based incoming turbulent boundary layer velocity and Mach number profiles including compressibility, heat transfer and moderate amounts of wall suction or injection. The results satisfy the proper boundary conditions including vanishing gradients at the boundary layer edge, conform to the Law of the Wall near the surface, are continuous across the entire boundary layer with a velocity defect-type behavior in the outer part, and are in good agreement with experiment over a wide range of transonic-to-moderately supersonic Mach numbers including the effects of surface mass transfer.

3. Numerical Results and Discussion

3.1 Zero Mass Transfer

To provide a basis for appreciating and scaling the influence of mass transfer on the interaction it is desirable to examine first some theoretical results for the impermeable wall case. Some general results for this case emphasizing the influence of Mach number have already been given in Ref. 11 and thus need not be repeated; we concentrate here on more recent and heretofore-unpublished results for the effect of Reynolds number and comparisons with experimental data.

The predicted influence of Reynolds number on the interaction pressure field for a typical Mach number case is shown in Fig. 3. It is seen that there is a moderate Reynolds number effect even without unseparation: the extent of the interaction upstream and downstream of the incident shock decreases with increasing Reynolds number, tending toward a solution typical of the response to a simple step pressure rise at very high Reynolds numbers, in agreement with both experimental observations and Navier-Stokes numerical simulation of turbulent boundary layer shock wave interactions. Moreover, at the boundary layer edge the strengths of the local shock jump and post-shock expansion increase and decrease, respectively, with increasing Reynolds number; at sufficiently high \( \text{Re}_e \) the post-shock expansion region predicted by present theory becomes very small and weak and hence probably difficult to detect experimentally.

The corresponding upstream influence (defined as the distance \( x_{up} \) ahead of the shock where the local interaction-induced pressure rise is only 5% of the overall total) at various shock strengths as a function of Reynolds number is shown in Fig. 4, plotted in ratio to \( \delta_0 \) (which also of course experiences Mach and Reynolds effects). These values are of order unity \( (x_{up}/\delta_0) \), as we should indeed expect for the short-range type of interactions characteristic of turbulent boundary layers, and decrease markedly with both the shock strength and Reynolds number, at moderate Reynolds numbers, \( x_{up}/\delta_0 \) decreases monotonically with \( \text{Re}_e \) approximately as a power law. It is emphasized that in addition to being in full qualitative agreement with many experimental observations, both the magnitude and parametric trends of these theoretical results are completely concordant with several detailed correlation studies and of upstream influence data on interacting turbulent boundary layers that directly verify the present non-asymptotic triple deck flow model.

The scale effect on the corresponding interactive displacement thickness growth (Fig. 5) is also of practical interest since this thickening often has a significant back-effect on the inviscid flow and shock position on airfoils or in channel flows. It is seen that the predicted displacement growth decreases significantly with increasing Reynolds number as would be expected (again, this trend agrees with experiment).

The typical distribution of interactive skin friction along the interaction is shown in Fig. 6 and illustrates how \( C_f \) typically decreases toward the shock owing to the adverse pressure gradient disturbance induced by the shock-boundary layer interaction (increasing shock Mach number enhances this owing to the stronger local interaction pressure gradient involved). When the interaction is strong enough, the present theory predicts vanishing skin friction and a very short separation bubble slightly behind the shock foot, which is directly confirmed by several detailed studies of the skin friction behavior across trans-
sonic turbulent boundary layer interactions, \( \text{Re} \), Reynolds number has the expected influence on the skin friction and incipient separation behavior: the relative effect of the interaction at a given shock strength decreases with \( \text{Re} \), incipient separation occurring more readily at lower Reynolds number as observed experimentally.

To further validate and illustrate the theory, it is desirable to make some direct comparisons with experiment under the assumed transonic flow conditions; this is difficult, however, because most of the existing transonic shock-boundary layer interaction data on airfoils involve high local Mach numbers (\( M > 1.3-1.4 \)) with a distinct lambda-shock interaction pattern and pronounced boundary layer separation which cannot be compared meaningfully with the present theory. Nevertheless, two suitable non-separated cases were found in some published MAE wind tunnel tests of supercritical airfoil sections; the measured pressure distributions and corresponding theoretical predictions (based on the local pre-shock Mach number and Reynolds number conditions at the experimentally-observed shock location) are shown in Fig. 7. The theory is seen to predict the upstream influence well, whereas it overestimates the pressure recovery downstream. This is typical of such airfoil tests and has been shown \( \text{Re} \) to be caused by the fact that, in contrast to the normal incident shock theoretically assumed, the actual shock occurring in airfoil experiments is usually oblique (albeit still with subsonic post-shock flow) owing to the interactive displacement thickness back-effect on the surrounding inviscid flow; this lowers the actual overall shock pressure rise in transonic flow 20–30% below the normal shock value at the same incoming flow Mach number. As illustrated by the good comparison with some recent DFVLR-Göttingen interaction data \( \text{Re} \) on a supercritical wing section shown in Fig. 8, when this obliquity effect is incorporated the present theory gives a satisfactory account of the interaction downstream as well as upstream of the shock.

Ackeret, Feldman and Rott's famed experimental study of shock-boundary layer interaction on a plate and wall in the choked transonic flow of a slightly-curved wind tunnel nozzle provided further examples of unseparated turbulent flow that can be rendered suitable for comparison with the present theory; we have chosen those for which both wall and inviscid pressure distributions, as well as displacement thickness, are given. It should be emphasized, however, that direct comparisons with their data involve numerous uncertainties: (a) the inviscid flow edge is only approximately defined, (b) the shock location and shape are uncertain to within 25 to \( \pm 0.5 \times d_s \), (c) error in reading the curves, (d) a significant background inviscid pressure gradient beclouds interpretation of the outer interaction zone and the incoming turbulent layer profile, (e) the upstream boundary layer history is only partially understood, especially following forced transition cases, and (f) a significant channel blockage effect occurs from the interactive boundary layer thickening, which reduces the effective theoretical shock strength and hence the downstream interaction pressure level. \( \text{Re} \), A typical non-separating inter-

### 3.2 Mass Transfer Effects

Preliminary study indicated that the dominant influence of mass transfer comes from the effect on the profile shape away from the wall; including only the \( \tau_w \) effect gives a significant error in both magnitude and sign of the profile changes. The typical consequences of this on the interaction solution itself are illustrated in Fig. 10, which shows how the various contributions to the suction/blowing effect on the Mach number profile influence the wall pressure distribution (analogous results were obtained for the interactive displacement thickness and skin friction). Whereas the contribution of the mass transfer effect on \( d_s \) is negligible compared to that on \( \tau_w \), the influence on overall profile shape is large and overwhelmingly opposite to the \( \tau_w \) effect. This conclusion was found to apply over a wide range of conditions and is concordant with the fact that transonic interactions are known to be very sensitive to the upstream turbulent boundary layer profile form factor.

Referring hereafter to the complete mass transfer model we observe that suction, because of its predominant effect in decreasing the Mach number gradient and hence enhancing the profile "fullness" away from the wall, reduces the streamwise extent and thickening of the interaction, making it appear more inviscid-like in character with a steeper adverse pressure gradient. Thus suction is qualitatively equivalent to an increase in Reynolds number. Blowing has the expected opposite effects of spreading out the interaction pressure field and increasing the displacement thickness.

Results of a systematic study of suction/blowing effects on the interaction pressure field for a typical case are presented in Fig. 11. In Fig. 11A it is seen that moderate amounts of suction significantly reduce the upstream influence distance and overall streamwise extent of the interaction and steepen the adverse wall pressure gradient, whereas blowing has equally the opposite
The non-dimensional upstream influence distance this can be eliminated by small blowing rates ex-
also strengthens th.loc al shock jump at the theory indeed shows a small separation region ap-
effect. Concordant with these trends, suction and blow-
ing on the interactive displacement thickness
distribution is shown in Fig. 13, while the total downstream thickness is plotted vs. B in Fig. 14.
It is seen (as expected) that this thickening is significantly influenced by mass transfer; for example, the moderate suction value \( B = -0.0003 \) reduces \( \Delta \delta^* \) nearly three-fold. It is noted that the influence of Reynolds number on the interaction in the presence of surface mass transfer effects described below was also studied; suffice it here to state that over the range of values \( |B| < 0.0005 \), its relative effect was found to be quite similar to that shown above in Section 3.1.

The interactive local skin friction is of particular interest since it identifies possible flow separation and provides re-initialization data for continuing boundary layer calculations downstream of the interaction zone. We note in this regard that although the present theory is no longer valid for separated flow \( \left( \frac{x}{\delta_0} > 0 \right) \) over some portion of the wall \( \left( \frac{x}{\delta_0} > \right) \) it is still useful to indicate trends toward this situation, i.e., where and when incipient separation \( \left( \frac{x}{\delta_0} = 0 \right) \) first occurs. As indicated in Fig. 15, the influence of mass transfer involves two opposing effects: far upstream or downstream, the skin friction-increasing effect of suction dominates (which tends to delay separation), whereas in the neighborhood of the shock foot \( \left( \frac{x}{\delta_0} \approx 1 \right) \) the suction-induced steepening of the local adverse pressure gradient becomes of controlling importance and \( \frac{x}{\delta_0} \) is actually reduced. Thus in con-
trast to what occurs in non-interacting boundary layers, slight suction here actually has a locally-
advective effect on the incoming incipient separation, reducing the downstream interaction pressure distribution and hence the local skin friction. The present theory provides a useful analytical framework for the evaluation and parametric study of the interaction mass transfer effects in a variety of practical applications. Moreover, it provides the basis for further improvement: extension of the analytical model to larger mass transfer rates by including their explicit effect on the viscous disturbance sublayer and mean turbu-
ent boundary layer eddy viscosity equations.

With such added features, it should be possible to examine the basic question of how mass transfer influences incipient separation over a very wide range of suction or blowing rates as well as Mach and Reynolds numbers without the need for present-day empirisms.

4. Concluding Remarks

This non-asymptotic study of weak normal shock-turbulent layer interactions for two-dimen-
sional non-separating flows including mass transfer has shown that even small amounts of suction \( \left( -\frac{\Delta \theta}{\delta_0} \right) \) appreciably reduce both the streamwise scale and thickening effect of the interaction but hasten the onset of separation slightly behind the shock foot. Equal amounts of weak blowing on the other hand can completely eliminate interaction-induced separation. These results were found to be mainly a consequence of the mass transfer effect on the incoming boundary layer Mach number profile shape which in turn significantly affects the interaction pressure distribution and hence the local skin friction. The present theory provides a useful analytical framework for the evaluation and parametric study of the interaction mass transfer effects in a variety of practical applications. Moreover, it provides the basis for further improvement: extension of the analytical model to larger mass transfer rates by including their explicit effect on the viscous disturbance sublayer and mean turbu-
ent boundary layer eddy viscosity equations. With such added features, it should be possible to examine the basic question of how mass transfer influences incipient separation over a very wide range of suction or blowing rates as well as Mach and Reynolds numbers without the need for present-day empirisms.

Acknowledgment

This work was supported by the Office of Naval Research under contract N00014-75-C-0456.

\* Thus in the large scale, suction always has the beneficial influence of promoting a more rapid equilibration of the turbulent skin friction downstream.

\* It should be noted that the often-important blockage corrections in such tests due to interactive thickening of the wall boundary layers can be significantly reduced by suction (see Fig. 13).
References


Fig. 2 Theoretical Model of Non-Separating Interaction (Schematic)

Fig. 3 Reynolds Number Influence on Interaction Pressure Field

Fig. 4 Upstream Influence vs. Mach and Reynolds Number

Fig. 5 Scale Effect on Interactive Displacement Thickness Growth

Fig. 6 Reynolds Number Influence on Interactive Skin Friction Distribution
A. Comparison of Predicted Local Interaction Pressures with NAE Experiments for a Supercritical NACA 64A410 Airfoil: $M_a = .70, \text{ Re}_{\infty} = 8 \times 10^6$

Fig. 7
Comparison of Theory with NAE Supercritical Airfoil Data (Ref. 52)

B. Comparison of Predicted and Experimental Pressures for the NACA 64A410 Airfoil: $M_a = .751, \text{ Re}_{\infty} = 35 \times 10^6$

Fig. 8
Comparison of Present Theory with DFVLR-Göttingen Flow Measurements on Supercritical Wing Section (Ref. 51)
(a) Typical Pressure Data

(b) Wall and Boundary Layer Edge Pressure Comparisons

(c) Interactive Displacement Thicknesses

Fig. 9 Comparison with Wind Tunnel Data of Ackeret, Feldman and Rott

Fig. 10 Influence of the Mass Transfer Effect-Model on the Interaction Solution
Fig. 11 Parametric Effect of Mass Transfer on the Interaction Pressure Field

Fig. 12 Upstream Influence vs. Mass Transfer Parameter

Fig. 13 Interactive Displacement Thickness Distributions with Suction or Blowing
Fig. 14
Influence of Mass Transfer on Downstream Interactive Thickening of the Boundary Layer

Fig. 15, Suction and Blowing Effects on Interactive Skin Friction, $M_1 = 1.20$

Fig. 16
Blowing Effect on Skin Friction and Incipient Separation, $M_1 = 1.30$