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ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW,(U)

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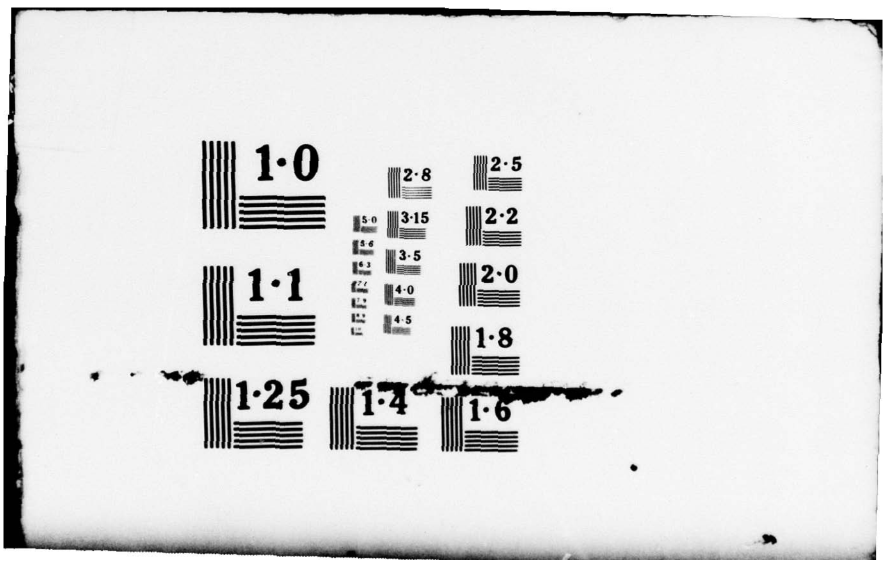
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ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW

by

Ye. V. Mamontov

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А а	А а	A, a	Р р	Р р	R, r
Б б	Б б	B, b	С с	С с	S, s
В в	В в	V, v	Т т	Т т	T, t
Г г	Г г	G, g	У у	У у	U, u
Д д	Д д	D, d	Ф ф	Ф ф	F, f
Е е	Е е	Ye, ye; E, e*	Х х	Х х	Kh, kh
Ж ж	Ж ж	Zh, zh	Ц ц	Ц ц	Ts, ts
З з	З з	Z, z	Ч ч	Ч ч	Ch, ch
И и	И и	I, i	Ш ш	Ш ш	Sh, sh
Й й	Й й	Y, y	Щ щ	Щ щ	Shch, shch
К к	К к	K, k	Ъ ъ	Ъ ъ	"
Л л	Л л	L, l	Ы ы	Ы ы	Y, y
М м	М м	M, m	Ь ь	Ь ь	'
Н н	Н н	N, n	Э э	Э э	E, e
О о	О о	O, o	Ю ю	Ю ю	Yu, yu
П п	П п	P, p	Я я	Я я	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α		Nu	Ν	ν
Beta	Β	β		Xi	Ξ	ξ
Gamma	Γ	γ		Omicron	Ο	ο
Delta	Δ	δ		Pi	Π	π
Epsilon	Ε	ε ϵ		Rho	Ρ	ρ ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ ς
Eta	Η	η		Tau	Τ	τ
Theta	Θ	θ ϑ		Upsilon	Υ	υ
Iota	Ι	ι		Phi	Φ	φ ϕ
Kappa	Κ	ϰ κ		Chi	Χ	χ
Lambda	Λ	λ		Psi	Ψ	ψ
Mu	Μ	μ		Omega	Ω	ω

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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ANALYTICAL PERTURBANCES IN AN UNSTEADY TRANSONIC FLOW

Ye. V. Mamontov

Small perturbances in an unsteady transonic gas flow are described by the equation [1], [2]

$$-\varphi_x \varphi_{xx} \pm \varphi_{yy} - 2\varphi_{xt} = 0 \qquad (1)$$

Here $\phi(x, y, t)$ (dimensionless) is the pertubance velocity potential. The variables x, y, t are also dimensionless.

The following boundary problem was formulated in [3], [4]. In the region $D = \{(x, y, t);\ t > 0,\ x > 0,\ -\infty < y < \infty\}$ it is necessary to find the solution to $\phi(x, y, t)$ of equation (1), if we know

$\phi(x,y,0) = \phi_0(x,y)$, $\phi(0,y,t) = \phi_0(y,t)$ and $\phi_x(0,y,t) = \phi_1(y,t) > 0$

In other words, potential ϕ is known at the initial point in time throughout the flow, while the values of both components of the velocity vector are known on straight line $x = 0$ at $t > 0$. Inequality $\phi_1 > 0$ means that this straight line is in the supersonic part of the flow. The uniqueness of the solution to this problem and its validity for the linear model of equation (1) were shown in these reports.

From the physical standpoint, it is logical to anticipate that a solution will exist at small values of time t. This report studies the problem of the existence of analytical solutions.

This is based on the theorem proven by L. V. Ovsyannikov [5] about the solvability of differential equations in the scale of Banach spaces. We will write the necessary equation, we will select the scale of Banach spaces, and we will check the solvability of the conditions of L. V. Ovsyannikov's theorem.

1. We will consider equation (1) in region $D = \{(x, y, t)\ T > 0, x > 0, -\infty < y < \infty\}$ and we will assume that the values of solution $\phi(x, y, t)$ are assigned in planes $t = 0$ and $x = 0$. Equation (1) can be replaced by the system

$$u_t = -\frac{1}{2}uu_x + \frac{1}{2}v_y$$
$$u_y - v_x = 0 \quad , \tag{2}$$

where $u = \varphi_x,\ v = \varphi_y$ are the perturbation velocities on axes x and y, respectively. The values $u(x, y, 0) = u_0(x, y)$ and $v(0, y, t) = h(y, t)$ are found from the assigned values of the potential. Then

$$v(x,y,t) = h(y,t) + \frac{\partial}{\partial y}\int_0^x u(\xi,y,t)d\xi .$$

Substituting this expression in the first equation of the system, we will obtain the equation for function u

$$u_t = -\frac{1}{2}uu_x + \frac{1}{2}\frac{\partial^2}{\partial y^2}\int_0^x u(\xi,y,t)d\xi + \frac{1}{2}\frac{\partial h}{\partial y} \tag{3}$$

with the initial condition

$$u(x,y,0) = u_0(x,y). \tag{4}$$

If function u(x, y, t) is found, the potential can obviously be reconstructed. Using the substitution

$$w = u - u_0$$

we will give the initial condition for zero and we will consider the problem

$$w_t = -\frac{1}{2} ww_x - \frac{1}{2} u_0 w_x + lw - \frac{1}{2} u_{0x} w + lu_0 - \frac{1}{2} u_0 u_{0x} + \frac{1}{2}\frac{\partial h}{\partial y}, \qquad (3')$$

$$(x > 0, \quad t > 0, \quad -\infty < y < \infty),$$

$$w(x,y,0) = 0, \qquad (4')$$

where operator l acts as follows

$$lw = \frac{1}{2}\frac{\partial^2}{\partial y^2}\int_0^x w(\xi,y,t)d\xi .$$

2. The solvability of this problem will be proven in the analytical scale of the Banach spaces $B = \bigcup_{0<\rho\le\rho_0} B_\rho$. Space B_ρ consists of of function $w(x, y, t)$, for which the norm

$$\|w\|_\rho = \sum_{n=0}^{\infty} \frac{1}{n!} |^n w| \rho^n \qquad (5)$$

is finite at

$$|^n w| \equiv \max_{0\le k\le n} |w_{k,n-k}| \equiv \max_{0\le k\le n} \left|\frac{\partial^n w}{\partial x^k \partial y^{n-k}}(0,0,t)\right|.$$

These norms have the necessary properties [5]. We will also point out that the following inequalities are valid:

$$\|w_1 w_2\|_\rho \le \|w_1\|_\rho \|w_2\|_\rho, \quad \|\frac{\partial}{\partial x} w\|_\rho \le \frac{\partial}{\partial \rho}\|w\|_\rho \qquad (6)$$

3. Problem (3'), (4') can be considered to be the problem of searching for the solution to equation

$$\frac{dw}{dt} = f(w,t) \qquad (7)$$

with the initial condition

$$w(0) = 0 \qquad (8)$$

The nonlinear operator acts in the scale of Banach spaces if we require that functions u_0, u_{0x}, lu_0 and $\frac{\partial h}{\partial y}$ (at $0 < t < T$) belong to space B_{ρ_0}. Suppose there is a constant C such that

$$\|\frac{1}{2} u_0\|_\rho \le C, \qquad \|\frac{1}{2} u_{0x}\| \le C. \qquad (9)$$

If values $|^{*} h|$ are continuous with respect to t at $0 < t < T$, operator f is continuous with respect to t. Now we will prove that operator f is quasidifferential, i.e., the following inequality is satisfied at $w_1, w_2 \in B_\rho$, $\rho < \rho_0$

$$\|f(w_1) - f(w_2)\|_\rho \le \left(1 + \frac{\partial}{\partial\rho}\right)\left[F(\|w_1\|_\rho + \|w_2\|_\rho)\|w_1 - w_2\|_\rho\right] \tag{10}$$

with function $F(y)$ of the substantial variable $y \geqslant 0$ such that $F \geqslant 0$, $F' \geqslant 0$, $F'' \geqslant 0$. We will have

$$f(w_1) - f(w_2) = \left(-\frac{1}{2} w_1 w_{1x} + \frac{1}{2} w_2 w_{2x}\right) + \left(-\frac{1}{2} u_0 w_{1x} + \frac{1}{2} u_0 w_{2x}\right) + (l w_1 - l w_2) + \left(-\frac{1}{2} u_{0x} w_1 + \frac{1}{2} u_{0x} w_2\right).$$

Using (6) and (9), we will estimate the norms of the terms on the right:

$$\left\| \frac{1}{2} u_{0x} w_2 - \frac{1}{2} u_{0x} w_1 \right\|_\rho \le C \|w_1 - w_2\|_\rho ,$$

$$\left\| \frac{1}{2} u_0 w_{2x} - \frac{1}{2} u_0 w_{1x} \right\|_\rho \le C \frac{\partial}{\partial\rho} \|w_1 - w_2\|_\rho ;$$

further, we will have

$$\|w_2 w_{2x} - w_1 w_{1x}\|_\rho \le \|w_2 w_{2x} - w_2 w_{1x}\|_\rho + \|w_2 w_{1x} - w_1 w_{1x}\|_\rho$$

$$\le \|w_2\|_\rho \frac{\partial}{\partial\rho} \|w_1 - w_2\|_\rho + \|w_1 - w_2\|_\rho \frac{\partial}{\partial\rho} \|w_1\|_\rho$$

and

$$\| w_2 w_{2x} - w_1 w_{1x} \|_\rho \le \| w_1 \|_\rho \frac{\partial}{\partial \rho} \| w_1 - w_2 \|_\rho \dotplus \| w_1 - w_2 \|_\rho \frac{\partial}{\partial \rho} \| w_2 \|_\rho$$

whence

$$\| \frac{1}{2} w_2 w_{2x} - \frac{1}{2} w_1 w_{1x} \|_\rho \le \frac{1}{4} \frac{\partial}{\partial \rho} [(\| w_1 \|_\rho \dotplus \| w_2 \|_\rho) \| w_1 - w_2 \|_\rho].$$

We will show that

$$\| l w_1 - l w_2 \|_\rho \le \frac{1}{2} \frac{\partial}{\partial \rho} \| w_1 - w_2 \|_\rho .$$

Actually, if

$$\tilde{w} = w_1 - w_2 = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{1}{k!(n-k)!} \tilde{w}_{k,n-k} x^k y^{n-k},$$

then

$$l\tilde{w} = \frac{1}{2} \int_0^x \tilde{w}_{yy} dx = \frac{1}{2} \sum_{n=2}^{\infty} \sum_{k=0}^{n-2} \frac{1}{(k+1)!(n-k-2)!} \tilde{w}_{k,n-k} x^{k+1} y^{n-k-2}$$

Whence

$$|^{n-1} l w| = \max_{0 \le k \le n-2} \frac{1}{2} |\tilde{w}_{k,n-k}| \le \frac{1}{2} |^{n} \tilde{w}|$$

and

$$\| l\tilde{w} \|_{\rho} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n!} |^{n}\, l\tilde{w} | \rho_{n}$$

$$\leq \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n!} |^{n+1}\, \tilde{w} | \rho_{n} = \frac{1}{2} \frac{\partial}{\partial \rho} \| \tilde{w} \|_{\rho}.$$

The estimates obtained show that in our case, inequality (10) is valid with the function

$$F(y) = C + \frac{1}{2} + \frac{1}{4} y.$$

All the conditions of L. V. Ovsyannikov's theorem are satisfied, and it guarantees that problem (7), (8), and that means also problem (3), (4), have a single solution from space B_{ρ} for any $\rho < \rho_0$ and sufficiently small $t \geq 0$. More precisely, there exists a $k > 0$, so that the solution belongs to B_{ρ} for the values of ρ, t from the region

$$\Delta = \{ (\rho, t): \ \rho + kt < \rho_0, \ t \geq 0, \ 0 < \rho < \rho_0 \}$$

In conclusion, the author is indebted to L. V. Ovsyannikov for his helpful advice.

Bibliography

[1] C. C. Lin, E. Reissner, H. S. Tsien. J. Math. and Phys., 27, No. 3, 220, 1948.

[2] O. S. Ryzhov. Study of Transonic Flows in Laval Nozzles. M. VTs AN SSSR, 1965.

[3] Ye. V. Mamontov, DAN, 185, 3, 538, 1969.

[4] Ye. V. Mamontov, Coll. "Solid Medium Dynamics," Iss. 1, Novosibirsk, 1969.

[5] L. V. Ovsyannikov, DAN, 200, 4, 789, 1971.

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