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ON DISCRETE VORTEX SYSTEM OF WING OF FINITE SPAN.(U)

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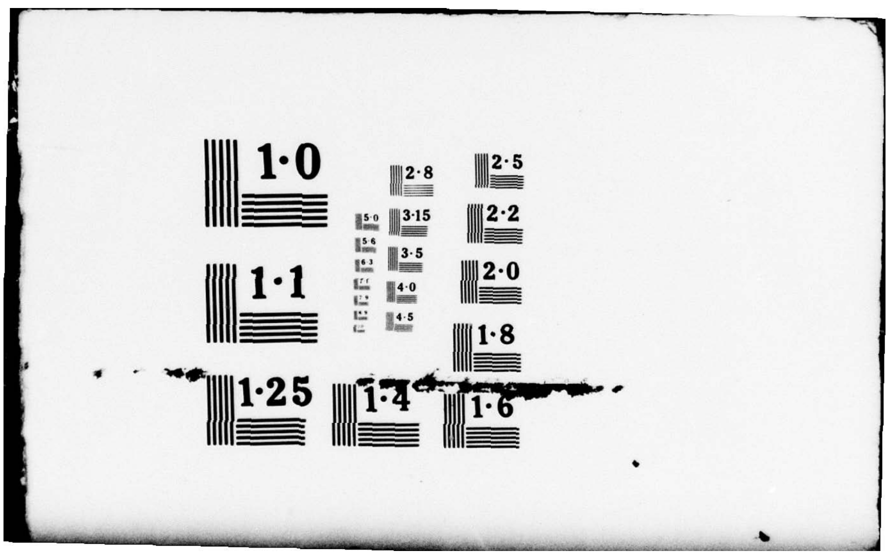
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FOREIGN TECHNOLOGY DIVISION

ON DISCRETE VORTEX SYSTEM OF WING OF FINITE SPAN

by

N. F. Vorob'yev

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

ON DISCRETE VORTEX SYSTEM OF WING OF FINITE SPAN

N. F. Vorob'yev

Here we investigate the problem of an inviscid, incompressible flow past the lifting surface of a wing. The wing surface itself is replaced by vortex surface S, while the sheet of vortices flowing from the trailing edge and, in the general case, from the lateral and leading edges of the wing, is represented as vortex surface Σ. This surface consists of vortices whose axes in the case of steady motion are directed along the flow line. Vortex density $\bar{\rho}$ on surfaces S and Σ are determined from the condition of nonpassage (nonpenetration). Moreover, the conditions of shed on the edges of the wing must be met. The presence of a vortex sheet flowing from the edges of the wing makes it possible to assure the condition of velocity limitation on the edges from which the vortex sheet flows [1-3]. In the case of a linear system, when the vortex sheet flows only from the trailing edge, on the leading and lateral edges of the wing, as we know, the speed of the inflow (penetration), determined within the framework of an ideal fluid, is infinitely great. In the nonlinear case, where the shape of the surface is unknown, solution to integral equations for wings of complex plan shape is difficult [1, 2].

There exists a method for calculating the aerodynamic characteristics of a flat wing of arbitrary plan shape, where the vortex layer simulating the wing surface is replaced by a system of discrete vortices, the intensity of which is determined from the conditions of nonpassage (nonpenetration) [4]. The vortex sheet outside of the wing is also simulated by discrete vortex lines, which represent a continuation of vortices on the surface of the wing itself. Each of the vortex lines outside of the wing consists of rectilinear segments which take the direction of velocity at the corresponding point in space. The position of the vortex lines outside of the wing is determined by the method of successive approximations in the calculation process. The solution to the problem of the flow past a wing of finite span according to the system of discrete vortices is reduced to solving a system of algebraic equations. This method can be conveniently used on the computer for wings of arbitrary plan shape. The wing can also be represented by a system of discrete vortices in the case where it is cambered. In the discrete system of a wing of finite span the problem of conversion in the case of an increase in the number of discrete vortices replacing the wing and the problem of satisfying the shed conditions on the edges of the wing remain open.

In the present study we show that with proper selection of the discrete vortices which replace the wing surface and points at which the conditions of nonpassage are satisfied, when the number of vortices is increased, the algebraic sums used to represent the velocity induced on the wing surface by discrete vortices will be transformed into integrals whose convergence can be proven, while the introduction of additional vortices near the edges will assure that the shed condition is met.

Generally a wing of arbitrary plan shape is a certain smooth surface S, which must be covered by an orthogonal grid of curvilinear coordinates related to the wing surface. The coordinate system is selected such that the line ζ = const connects the leading and trailing edges of the wing (Fig. 1). Applied to the surface is the discrete coordinate grid ξ = const, ζ = const, which breaks the wing down into rectangles with sides $\Delta\xi_i$, $2\Delta\zeta_j$. One of the coordinate lines passes through point $N(\xi_i, \zeta_j)$ and is the coordinate line of the grid

$\xi=\xi_i$. The two other coordinate lines of the grid, which are the closest to point $N(\xi_i, \zeta_j)$ and are orthogonal in relation to line $\xi=\xi_i$, are selected such that point $N(\xi_i, \zeta_j)$ represents the middle of the side of the coordinate grid. This is line $\zeta=\zeta_j-\Delta\zeta_j$, $\zeta=\zeta_j+\Delta\zeta_j$.

Selection of the direction of vortex lines which replace the wing can be arbitrary. These vortices are not subject to the laws of behavior of a vortex line in the flow of an ideal fluid. It is assumed that the segment of the coordinate line whose middle is point $N(\xi_i, \zeta_j)$ is the segment of a vortex line of constant intensity. Points $N(\xi_i, \zeta_j)$, through which the Π-shaped vortices pass, can be numbered by row and column: every point has a number (m, n). At points $(\xi_i, \zeta_j-\Delta\zeta_j)$, $(\xi_i, \zeta_j+\Delta\zeta_j)$ the vortex line has a break and continues along coordinate lines $\zeta=\zeta_j-\Delta\zeta_j$, $\zeta=\zeta_j+\Delta\zeta_j$ to the trailing edge of the wing.

Beyond the Π-shaped vortex of intensity $\Delta\Gamma_{ij}$ on the wing on coordinate line segment $\xi=\xi_i+\Delta\xi_i=\xi_{m+1}$, the middle of which is point $\xi_i+\Delta\xi_i, \zeta_j$, we find the following vortex line of constant intensity $\Delta\Gamma_{m+1,n}$. At points $(\xi_i+\Delta\xi_i, \zeta_j-\Delta\zeta_j)$, $(\xi_i+\Delta\xi_i, \zeta_j+\Delta\zeta_j)$ this vortex line also has a break and continues along coordinate lines $\zeta=\zeta_j-\Delta\zeta_j$, $\zeta=\zeta_j+\Delta\zeta_j$ to the trailing edge of the wing.

Thus, the entire wing S is covered by a system of discrete rectangular Π-shaped vortices, which are related to the wing. The

Π-shaped vortices arranged on the wing S, and also a certain additional number of discrete vortices, which can be introduced later for meeting the conditions of shed on the edges of the wing, descend from the wing and continue outside of the wing to infinity, simulating vortex sheet Σ. The vortex lines representing vortex sheet Σ consist of finite rectilinear segments which lie in the direction of the velocity at corresponding points outside of the wing [4]. The intensity of the vortices is determined from the conditions of nonpassage on the wing surface and the shed conditions on the edges of the wing. When the cell dimensions are decreased, in the expression for the velocity induced at points on the wing surface, we get a peculiarity related to sections of attached vortices. Related to the sections of free vortices representing vortex sheet Σ is the regular part of the expression for velocity induced at points on the wing surface S. For proof of convergence of the process for reduced cell values of the coordinate grid we must select point positions on the wing surface at which the conditions of nonpassage are satisfied. As such we select the points M(x, z) with coordinates $x=\xi_i+\frac{\Delta\xi_i}{2}$, $z=\zeta_j$, which lie in the geometrical centers of the coordinate cells (see Fig. 1).

Henceforth considerations related to passage to the limit with a decrease in the coordinate cell $\Delta\xi_i$, $2\Delta\zeta_j$ without limitation of generality will be done for a flat wing of arbitrary plan shape.

Everything that has been said above also applies to arbitrary smooth surfaces without curvature separation lines. In the case of a flat wing the orthogonal coordinate system is rectilinear on wings, and, consequently, each П-shaped vortex related to the wing consist of three rectilinear segments.

The velocity induced by elementary vortex $d\bar{l}$ of intensity Γ at a certain point separated by distance r from the middle of the elementary vortex is determined by the Biot-Savart formula

$$d\bar{V} = \frac{\Gamma}{4\Pi} \cdot \frac{[d\bar{l} \times \bar{r}]}{r^3}.$$

According to this formula, for points on a flat wing the velocities induced by vortex lines lying in the same wing plane are directed along the normal to the surface. Here the value of the velocity induced at point M(x, z) on the wing by a П-shaped vortex of intensity $\Delta\Gamma_{ij}$ passing through point $N(\xi_i, \zeta_j)$ and consisting of rectilinear segments of finite length $2\Delta\zeta_j$, $\xi_i - x_a(\zeta_j - \Delta\zeta_j)$, $\xi_i - x_a(\zeta_j + \Delta\zeta_j)$, where $\xi = x_a(\zeta)$ is the equation for the trailing edge of the wing, can be represented in the form of

$$\Delta V_{ij} = -\frac{\Delta\Gamma_{ij}}{4\Pi}[F(x, z, \xi_i, \zeta_j + \Delta\zeta_j) - F(x, z, \xi_i, \zeta_j - \Delta\zeta_j)] \quad \text{when} \quad (\xi_i, \zeta_j \neq z), \qquad (1)$$

where

$$F(x, z, \xi, \zeta) = \frac{1}{z-\zeta}\left[\frac{\sqrt{(x-\xi)^2+(z-\zeta)^2}}{x-\xi} - \frac{[x-x_3(\zeta)]}{\sqrt{[x-x_3(\zeta)]^2+(z-\zeta)^2}}\right],$$

$$\Delta V_j^0 = -\frac{\Delta\Gamma_i}{4\Pi}[F(x, z, \xi_i, z+\varepsilon_j) - F(x, z, \xi_i, z-\varepsilon_j)], \quad (2)$$

$$\text{when } (\xi_i, \zeta_j = z),$$

where

$$\Delta\Gamma_i = \Delta\Gamma_{ij}(\xi_i, z), \ \Delta\zeta_j = \varepsilon_j.$$

Point M(x, z) is selected in the center of the coordinate cell, so that in the case of a finite number of discrete vortices $\xi_i \neq x$, while $\zeta_j = z$ only for a certain column of coordinate cells. The velocity induced at point M(x, z) by all discrete Π-shaped vortices associated with the wing S is represented by the sum of velocities induced by each of the Π-shaped vortices. If we perform summation in a fixed row, in this case discarding the term which correspond to value ζ = z, and then summation for all rows, the velocity at point (x, z) can be represented in the form of

$$V = -\frac{1}{4\Pi}\left\{\sum_m \sum_n{}' \rho(\xi_i, \zeta_j)\frac{F(x, z, \xi_i, \zeta_j+\Delta\zeta_j) - F(x, z, \xi_i, \zeta_j-\Delta\zeta_j)}{2\Delta\zeta_j} \times \right.$$
$$\left. \times\, 2\Delta\zeta_j\Delta\xi_i + \sum_m \rho(\xi_i, z)[F(x, z, \xi_i, z+\varepsilon_j) - F(x, z, \xi_i, z-\varepsilon_j)]\Delta\xi_i\right\}, \quad (3)$$

where the intensity $\Delta\Gamma_{ij}$ of the Π-shaped vortex passing through point $\xi_i, \zeta_j\,(\xi_m, \zeta_n)$ is represented in the form of $\Delta\Gamma_{ij} = \rho(\xi_i, \zeta_j)\,\Delta\xi_i$, here $\Delta\xi_i$ is the distance along axis ξ from point $\xi_i, \zeta_j(\xi_m, \zeta_n)$ of this vortex to point $\xi_i+\Delta\xi_i, \zeta_j(\xi_{m+1}, \zeta_n)$, through which the following Π-shaped vortex passes.

In formula (3) summation by column is excluded in the double sum, where $\zeta_n = z$. Summation of this column is done separately.

For an infinite increase in the number of discrete vortices, when first $2\Delta\zeta_j \to 0$, and then $\Delta\zeta_j \to 0$, the velocity induced at point (x, z) by all attached vortices can be represented in the form of

$$V = -\frac{1}{4\Pi}\left\{\iint\limits_{S-2\epsilon} \rho(\xi, \zeta) F'_{\zeta}(x, z, \xi, \zeta)\, d\zeta d\xi + \int\limits_{x_{\Pi}(z)}^{x_3(z)} \rho(\xi, z)\,[F(x, z, \xi, z+\varepsilon) - \right.$$
$$\left. - F(x, z, \xi, z-\varepsilon)]\, d\xi\right\}, \qquad (4)$$

where S-2ε is the area of the wing with the excluded flat width of 2ε near point $\zeta = z$; $\xi = x_3(\zeta)$, $\xi = x_{\Pi}(\zeta)$ represents the equation of the trailing and leading edges of the wing, respectively, while the derivative of the function F(x, z, ξ, ζ) takes the form of

$$F'_{\zeta}(x, z, \xi, \zeta) = \frac{1}{(z-\zeta)^2}\left\{\frac{x-\xi}{\sqrt{(x-\xi)^2 + (z-\zeta)^2}} - \right.$$
$$\left. - \frac{[x - x_3(\zeta)]^3 + 2\,[x - x_3(\zeta)]^2 (z-\zeta)^2 - x'_3(\zeta)\,(z-\zeta)^3}{\{[x - x_3(\zeta)]^2 + (z-\zeta)^2\}^{3/2}}\right\}.$$

Function $F'_{\zeta}(x, z, \xi, \zeta)$ exists in the range of S-2ε. In the range of S-2ε within the internal integral of the double integral of formula (4) we can integrate by parts. After setting z > 0, without limiting the generality we get

$$V = -\frac{1}{4\pi}\left\{-\iint\limits_{S-2\epsilon} \rho'_{\zeta}(\xi, \zeta) F(x, z, \xi, \zeta)\, d\zeta d\xi - \int\limits_{0}^{x_3(0)} \rho\,[\xi, z_{\text{л}}(\xi)] \times \right.$$
$$\times F[x, z, \xi, z_{\text{л}}(\xi)]\, d\xi + \int\limits_{0}^{x_3(0)} \rho(\xi, 0)\, F(x, z, \xi, 0)\, d\xi + \int\limits_{0}^{x_{\Pi}(z-\epsilon)} \rho\,[\xi, z_{\text{пр}}(\xi)] \times$$
$$\times F[x, z, \xi, z_{\text{пр}}(\xi)]\, d\xi + \int\limits_{x_3(z-\epsilon)}^{x_3(0)} \rho\,[\xi, z_{\text{пр}}(\xi)]\, F[x, z, \xi, z_{\text{пр}}(\xi)]\, d\xi -$$

$$- \int\limits_{0}^{x_{\text{п}}(z-\varepsilon)} \rho(\xi,0) F(x,z,\xi,0)\, d\xi - \int\limits_{x_3(z-\varepsilon)}^{x_3(0)} \rho(\xi,0) F(x,z,\xi,0)\, d\xi +$$
$$+ \int\limits_{x_{\text{п}}(z-\varepsilon)}^{x_3(z-\varepsilon)} \rho(\xi,z,-\varepsilon) F(x,z,\xi,z-\varepsilon)\, d\xi - \int\limits_{x_{\text{п}}(z+\varepsilon)}^{x_3(z+\varepsilon)} \rho(\xi,0) F(x,z,\xi,0)\, d\xi + \tag{5}$$
$$+ \int\limits_{x_{\text{п}}(z-\varepsilon)}^{x_3(z+\varepsilon)} \rho[\xi, z_{\text{пр}}(\xi)] F[x,z,\xi,z_{\text{пр}}(\xi)]\, d\xi - \int\limits_{x_{\text{п}}(z+\varepsilon)}^{x_3(z+\varepsilon)} \rho(\xi, z+\varepsilon) F(x,z,\xi,z+\varepsilon)\, d\xi +$$
$$+ \int\limits_{x_{\text{п}}(z)}^{x_3(z)} \rho(\xi,z)[F(x,z,\xi,z+\varepsilon) - F(x,z,\xi,z-\varepsilon)]\, d\xi \Big\}.$$

For proof of the existence of velocity on the wing determined by formula (5), we must make assumptions on the shape of the wing contour and on the nature of vortex density on the wing. For contour L we assume the continuity of equations $\xi = x_{\text{п}}(\zeta)$, $\xi = x_3(\zeta)$ of the leading and trailing edges of the contour in the range of $a < \zeta < b$ (can be the lateral edges, parallel to axis ξ when $\zeta = a$, $\zeta = b$). We also assume that on the wing, including the edges of the wing, the value of vortex density $\rho(\xi, \zeta)$ and derivative $\rho'_{\zeta}(\xi,\zeta)$, from whose value we determine the maximal density of discrete vortex lines on wing S coinciding with the direction of axes ξ, satisfy the Holder boundary condition.

Now let us prove the existence of velocity on a wing determined by formula (5) for internal wing points. The double integral in formula (5) can be represented in the form of

$$\iint\limits_{S-2\varepsilon} \frac{\rho'_{\zeta}(\xi,\zeta)\, \Phi(x,z,\xi,\zeta)\, d\zeta\, d\xi}{(x-\xi)(z-\zeta)},$$

where expression

$$\Phi(x,z,\xi,\zeta) = \frac{\sqrt{(x-\xi)^2+(z-\zeta)^2}\sqrt{[x-x_3(\zeta)]^2+(z-\zeta)^2} - (x-\xi)[x-x_3(\zeta)]}{\sqrt{[x-x_3(\zeta)]^2+(z-\zeta)^2}} \tag{6}$$

represents the continuous function for wing points (x, z) which do not lie on the trailing edge of the wing. This integral represents the main value of the Cauchy type iterated integral, which exists for points which do not lie on the wing contour and which because of this represents the internal points for each of the iterated integrals [5].

After adding the single integrals in which we have the expression $\rho(\xi, 0)\ F(x, z, \xi, 0)\ d\xi$ under the integral sign, there remain two terms, which on the basis of the main-value theorem can be represented in the form of

$$\int_{x_{\text{п}}(z-\varepsilon)}^{x_{\text{п}}(z+\varepsilon)} \rho(\xi, 0)\, F(x, z, \xi, 0)\, d\xi + \int_{x_{\text{з}}(z+\varepsilon)}^{x_{\text{з}}(z-\varepsilon)} \rho(\xi, 0)\, F(x, z, \xi, 0)\, d\xi =$$

$$= \rho\,[x_{\text{п}}(z), 0]\, F\,[x, z, x_{\text{п}}(z), 0]\, 2\varepsilon + \rho\,[x_{\text{з}}(z), 0]\, F\,[x, z, x_{\text{з}}(z), 0]\, 2\varepsilon,$$

where

$$F\,[x, z, x_{\text{п,з}}(z), 0] = \frac{1}{z}\left\{\frac{\sqrt{[x - x_{\text{п,з}}(z)]^2 + z^2}}{x - x_{\text{п,з}}(z)} - \frac{[x - x_{\text{з}}(0)]}{\sqrt{[x - x_{\text{з}}(0)]^2 + z^2}}\right\}$$

is the function limited to points which do not lie on the edges of the wing (value $z > 0$). Under the above assumptions on the finiteness of the values of vortex density on the wing edges, the value of each of these terms when $\varepsilon \to 0$ reverts to zero.

The sum of single integrals which under the integral sign contain function

$$F(x,z,\xi,z\pm\varepsilon)=\frac{1}{(\mp\varepsilon)}\left\{\frac{\sqrt{(x-\xi)^2+\varepsilon^2}}{x-\xi}-\frac{[x-x_3(z\pm\varepsilon)]}{\sqrt{[x-x_3(z\pm\varepsilon)]^2+\varepsilon^2}}\right\}=$$
$$=\frac{H(x,z,\xi,z\pm\varepsilon)}{(\mp\varepsilon)},$$

where

$$\lim_{\varepsilon\to 0}H(x,z,\xi,z\pm\varepsilon)=0,$$

also reverts to zero when $\varepsilon\to 0$. Let us show this using function $F(x, z, \xi, z+\varepsilon)$ as an example.

$$\int\limits_{x_{\Pi}(z)}^{x_3(z)}\rho(\xi,z)F(x,z,\xi,z+\varepsilon)d\xi-\int\limits_{x_{\Pi}(z+\varepsilon)}^{x_3(z+\varepsilon)}\rho(\xi,z+\varepsilon)F(x,z,\xi,z+\varepsilon)d\xi=$$
$$=-\int\limits_{x_{\Pi}(z+\varepsilon)}^{x_3(z+\varepsilon)}[\rho(\xi,z+\varepsilon)-\rho(\xi,z)]F(x,z,\xi,z+\varepsilon)d\xi+\int\limits_{x_{\Pi}(z)}^{x_{\Pi}(z+\varepsilon)}\rho(\xi,z)\times$$
$$\times F(x,z,\xi,z+\varepsilon)d\xi+\int\limits_{x_3(z+\varepsilon)}^{x_3(z)}\rho(\xi,z)F(x,z,\xi,z+\varepsilon)d\xi=\int\limits_{x_{\Pi}(z+\varepsilon)}^{x_3(z+\varepsilon)}[\varepsilon\rho_z'(\xi,z)+$$
$$+0(\varepsilon^2)]\frac{H(x,z,\xi,z+\varepsilon)}{\varepsilon}d\xi-\frac{1}{\varepsilon}\int\limits_{x_{\Pi}(z)}^{x_{\Pi}(z)+\varepsilon x_{\Pi}'(z)}\rho(\xi,z)H(x,z,\xi,z+\varepsilon)d\xi-$$
$$-\frac{1}{\varepsilon}\int\limits_{x_3(z)+\varepsilon x_3'(z)}^{x_3(z)}\rho(\xi,z)H(x,z,\xi,z+\varepsilon)d\xi.$$

In the first term the integrand, because of the property of function H, reverts to zero when $\varepsilon\to 0$, while the last two terms are represented on the basis of the mean-value theorem in the form of

$$-\rho\left[x_{\Pi}(z)+\frac{\varepsilon}{2}x_{\Pi}'(z),z\right]H\left[x,z,x_{\Pi}(z)+\frac{\varepsilon}{2}x_{\Pi}'(z),z+\varepsilon\right]x_{\Pi}'(z)-$$
$$-\rho\left[x_3(z)+\frac{\varepsilon}{2}x_3'(z),z\right]H\left[x,z,x_3(z)+\frac{\varepsilon}{2}x_3'(z),z+\varepsilon\right]x_3'(z)$$

and, on the strength of function H, also revert to zero when $\varepsilon \to 0$.

The remaining single integrals in the right part of formula (5) represent, when $\varepsilon \to 0$ the contour integral

$$\oint_L \rho[\xi, f(\xi)] F[x, z, \xi, f(\xi)] d\xi,$$

where $\zeta = f(\xi)$ is the equation of the contour L, integration with respect to which is done counterclockwise. The contour integral can be written in the form of

$$\oint_L \rho[\xi, f(\xi)] \frac{\Phi[x, z, \xi, f(\xi)]}{(x-\xi)[z-f(\xi)]} d\xi,$$

where function Φ is determined by dependence (6). For points (x, z) which do not lie on contour L with the condition of continuity of equations for the leading and trailing edges, two situations can be encountered in the contour integral: 1) $x = \xi$, $z \neq f(\xi)$, 2) $x \neq \xi$, $z = f(\xi^0)$. The case where $x = \xi$, $z = f(\xi)$ for internal points (x, z) of a wing cannot exist.

In the first case the integral $\oint_L \frac{F_1(\xi) d\xi}{x-\xi}$, where $F_1(\xi) = \rho[\xi, f(\xi)] \cdot \frac{\Phi[x, z, \xi, f(\xi)]}{z-f(\xi)}$ represents a continuous function, where $F_1(x) = \rho[x, f(x)]$ is a Cauchy type integral and exist in the sense of the main value.

In the second case the integral $\oint_L \frac{F_2(\xi) d\xi}{z-f(\xi)}$, where

$F_2(\xi)=\rho[\xi,f(\xi)]\cdot\frac{\Phi[x,z,\xi,f(\xi)]}{x-\xi}$ is a continuous function. Here $F_2(\xi^0) = 0$ for Π-shaped vortices which end on the trailing edge is reduced to the form of the Cauchy type integral by substitution of variables $f(\xi) = t$

$$\oint_L \frac{F_2[\xi(t)]\,dt}{f'[\xi(t)](z-t)},$$

where when $\xi \to \xi^0$ $t \to z$. This integral exists in the sense of the main value when $f'(\xi^0) \neq 0$. For contour L with the condition of continuity of the equations of the leading and trailing edges value $f'(\xi) = 0$ can occur only for end points on the contour $\zeta = a$, $\zeta = b$, which cannot be points $\xi = \xi^0$ for points (x, z), which do not belong to contour L. If there are lateral edges parallel to axis ξ when $\zeta = a$, $\zeta = b$, where $f'(\xi) = 0$, then neither can the points on the lateral edges be points $\xi = \xi^0$ for points (x, z) which do not belong to the wing contour. This means that in the second case the contour integral exists in the sense of the main value.

Thus, the velocity induced by the Π-shaped vortices on wing S with transition to the limit from the discrete system to the system of the vortex surface, is determined for internal wing points by the formula

$$V=\frac{1}{4\pi}\left\{\iint_S \rho'_\zeta(\xi,\zeta)\,F(x,z,\xi,\zeta)\,d\zeta\,d\xi+\oint_L \rho[\xi,f(\xi)]\,F[x,z,\xi,f(\xi)]\,d\xi\right\}. \qquad (7)$$

The integrals which determine velocity are integrals of the Cauchy type, and for internal wing points they exist in the sense of the main value. Essential for representing velocity in the sense of the main value of Cauchy type integrals (including iterated integrals) is the breakdown of the integration range into parts within the neighborhood of a certain point: $0 \leqslant \xi \leqslant x - \delta$, $x + \delta \leqslant \xi \leqslant x_3(0)$ and $a \leqslant \zeta \leqslant z - \varepsilon$, $z + \varepsilon \leqslant \zeta \leqslant b$. The selection of the point $M(x, z)$ in the center of the coordinate system, carried out in this work for the discrete case, assures convergence of the integrals in the sense of the main value with transition to the limit.

The end (nonzero) density values of the vortex lines on portions of the wing contour where the vortex sheet does not descend (flow off), give us infinite velocity values within the framework of an ideal fluid. On wing contour L the conditions for the existence of the integrals through which the velocity induced by the vortices is expressed will be met in the case where contour L is a line which lies entirely within the vortex surface $S + \delta S$. This means that the vortex surface of the wing must continue unbroken beyond wing S. Then contour L becomes a line whose points are the internal points of the vortex surface $S + \delta S$ which lie within the contour $L + \delta L$. For the internal points of the surface the existence of velocity in the sense of the main value is proven. Here the velocities at points on contour L are determined by formula (7), where integration is done with respect to the surface S + δS and the contour L + δL. The vortex sheet lying within the contour L + δL, whose shape in the nonlinear case is known in advance, will, as already mentioned, give us the regular velocity component for points on the wing S and its edges L.

In the studied case, where the wing is a lifting surface without thickness, for the existence of finite velocity on the edges we must impose the condition of the smooth joining of the wing surface S and the vortex sheet Σ which flows off of it and the condition of continuous transition of the vortex density of these surfaces on their boundary - contour L. On contour L satisfaction of the condition of finite velocity does not generally require that the vortex density on the edge of the wing revert to zero, provided the vortex sheet flows from it. The vanishing of the density of the vortex lines which coincide with the direction of axis ξ on the trailing edge of the wing in the case of the linear system is caused by the form of the vortex sheet beyond the wing and comes from the condition of shed from the edges of the wing formulated above. In the linear system, where the vortex lines beyond the wing take the direction of velocity at infinity, on surface Σ there are no vortex line components which are perpendicular to axis ξ (axis ξ on the wing as the direction of velocity at infinity), and from the condition of the continuous transition of the vortex surface S into surface Σ it follows that on the trailing edge of the wing S the intensity of the vortex density of the last of the П-shaped vortices should be equal to zero. For the vortex line (wing of infinite span) from the condition of finite velocity we also get the values for zero vortex density on the trailing edge of the vortex segment. In the case of the end point of a vortex line the condition of finite velocity, determined by the Cauchy type integral, can be satisfied only when vortex density vanishes at the end of the line [5]. However, in the case of a vortex shedding from the edges of a wing of finite span, the edges of the wing do not represent the end points of the vortex lines, and vortex density on the shed line is generally not equal to zero.

In the discrete system we can impose a distribution of vortex lines which at the transition to the limit from the discrete system to the system of the vortex surface would cause a break in the transition of vortex density on the wing contour L. This is assured if a point which in the discrete system is considered to be a wing edge point is limited on the vortex sheet side by a vortex line of the same intensity as on the inner side of the wing (Fig. 2). Since on wing S the vortex lines parallel to axis ζ and axis ξ have two directions, for the selected system of rectangular П-shaped vortices, the limiting density value of discrete vortices on the wing in the direction of axis ζ equals $\rho(\xi = \text{const}, \zeta)$, while the limiting density value of discrete vortices on the wing in the direction of axis ξ equals $\int_{x_{\text{П}}(\zeta)}^{\xi} \rho'_{\zeta}(\xi, \zeta =$ const) $d\xi$, for each point on the edge at which the condition of nonpassage is satisfied there are generally two additional vortices. On the side of the vortex sheet one vortex runs parallel to axis ξ, the other - parallel to axis ζ.

The wing is broken down by a discrete grid of coordinate lines into a coordinate cells. In the center of each of these at point (m, n) the condition of nonpassage is satisfied. Here, on contour segments which do not coincide with the direction of the coordinate lines, the contour is replaced by a broken line, which is located outside of the wing (the wing surface is taken with an excess). The rectangular П-shaped vortex lines which are bound to the wing and whose intensity is determined from the conditions of nonpassage, are shown in the left side of Fig. 2 as continuous lines. The arrows indicate the positive direction selected. The figure also shows the part of these lines which lies in a single cell. Here each of the bound vortex lines begins and ends on the trailing edge of the wing. Continuation of these vortex lines outside of the wing, beginning with the leading edge, coincides with the direction of the flow velocity. On the left side of the figure their continuation, now as free (vortex lines), is shown by solid lines which begin at the corresponding points on the trailing edge. The boundary points at which the conditions of nonpassage are satisfied and which for the discrete system represent the contour points, are marked by x's. At these points the shed conditions must be met, i.e., conditions imposed which assure continuous transition of the vortex surface of the wing S into the vortex surface of the sheet Σ.

For a trailing edge which is not parallel to coordinate axis ζ, point (m, 1) which lies in the center of rectangle $B_{m,1}F_{m,1}C_{m,1}D_{m,1}$ is limiting. Here the boundary of the wing S and of the vortex sheet Σ are the sides $B_{m,1}D_{m,1}$ and $D_{m,1}C_{m,1}$. To assure an unbroken vortex surface in the direction of axis ξ (vortex lines coincide with direction of axis ζ) along the side $D_{m,1}C_{m,1}$, vortex line of intensity $\Delta\Gamma_{m,1}$ is introduced, where $\Delta\Gamma_{m,1}$ represents the intensity of the Π-shaped vortex line bound to the cell whose center is the point (m, 1). At points $D_{m,1}$, $C_{m,1}$ the vortex line breaks and thereafter behaves as a free vortex line. To assure the continuity of the vortex surface in the direction of axis ζ along side $B_{m,1}D_{m,1}$, we introduce the vortex line of intensity $2\Delta\Gamma_{m,1}-\Delta\Gamma_{m,2}$. At points $B_{m,1}$, $D_{m,1}$ the introduced vortex line breaks and thereafter behaves as a free vortex line. In Fig. 2 the introduced vortices are shown as dashed lines. There selected positive direction is indicated by arrows.

For a trailing edge parallel to axis ζ point (m, n), which lies in the center of the rectangle $B_{m,n}F_{m,n}D_{m,n}C_{m,n}$ is the boundary point. Here the boundary of the wing S and of the vortex sheet Σ is the side $B_{m,n}C_{m,n}$. To assure the continuity of the vortex surface along side

$B_{m,n}C_{m,n}$, we introduce a vortex line of intensity $\Delta\Gamma_{m,n}$. At points $B_{m,n}C_{m,n}$ the introduced line breaks, and thereafter behaves as a free vortex line.

For a lateral edge parallel to axis ξ, the point (m, 1), which lies in the center of rectangle $B_{m,1}C_{m,1}D_{m,1}F_{m,1}$, is the boundary point. Here the boundary of wing S and of the vortex sheet Σ is side $B_{m,1}C_{m,1}$. Along side $B_{m,1}C_{m,1}$ we introduce a vortex line of intensity $2\Delta\Gamma_{m,1}-\Delta\Gamma_{m,2}$, which continues to the end of the lateral edge, where it sheds as a free vortex line together with all attached vortex lines coinciding with the lateral edge. At point $B_{m,1}$ the vortex line behaves as a free vortex line. The selected positive direction of the bound and introduced vortex lines is indicated by arrows.

For a leading edge which is not parallel to axis ζ the point (m, 0), which lies in the center of rectangle $D_{m,0}B_{m,0}F_{m,0}C_{m,0}$, is the boundary point. Here the boundary of the wing S and of the vortex sheet Σ consists of the sides $D_{m,0}B_{m,0}$ and $D_{m,0}C_{m,0}$. Along side $D_{m,0}B_{m,0}$ $(D_{m,0}C_{m,0})$ we introduce a vortex line of intensity $\Delta\Gamma_{m+1,1}-\Delta\Gamma_{m,0}$ $(\Delta\Gamma_{m+1,1}-\Delta\Gamma_{m,1}+\Delta\Gamma_{m,0})$, which from point $D_{m,0}$ behaves as a free vortex line, and from point $B_{m,0}$ $(C_{m,0})$ continues along the wing parallel to axis ξ to the trailing edge of the wing, where it sheds as a free vortex line together with the corresponding attached vortices. The positive direction of the introduced and attached vortex line is indicated by arrows. The different signs for the value of intensity in the case of corresponding arrows, and also identical signs in the case of arrows of opposite directions, indicate the mutual obliteration of the effect of the vortex lines. As we see in Fig. 2, the intensities of the introduced vortex lines which assure the continuous transition of vortex surface S into surface Σ in the direction of axis ξ and axis ζ for point (m, 0), are such that they mutually obliterate the effect of the attached vortex bound to point (m, 0) and the effect of the sums of introduced vortices of intensity $\Delta\Gamma_{m,0}$. In the discrete wing system it is possible to not immediately introduce the attached vortex of intensity $\Delta\Gamma_{m,0}$ which corresponds to (m, 0), but to select the intensities of the introduced vortices equal to $\Delta\Gamma_{m-1,1}$ and $\Delta\Gamma_{m+1,1}-\Delta\Gamma^{\cdot}_{m,1}$, respectively.

Thus, the intensity of all newly introduced vortices on the wing edges which assure finite velocity at the points on the edges, is expressed as the intensity of the attached Π-shaped vortices plotted earlier, whose intensity is determined from the conditions of nonpassage on the wing surfaces.

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Fig. 1.

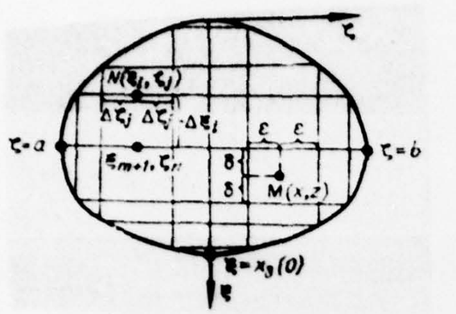
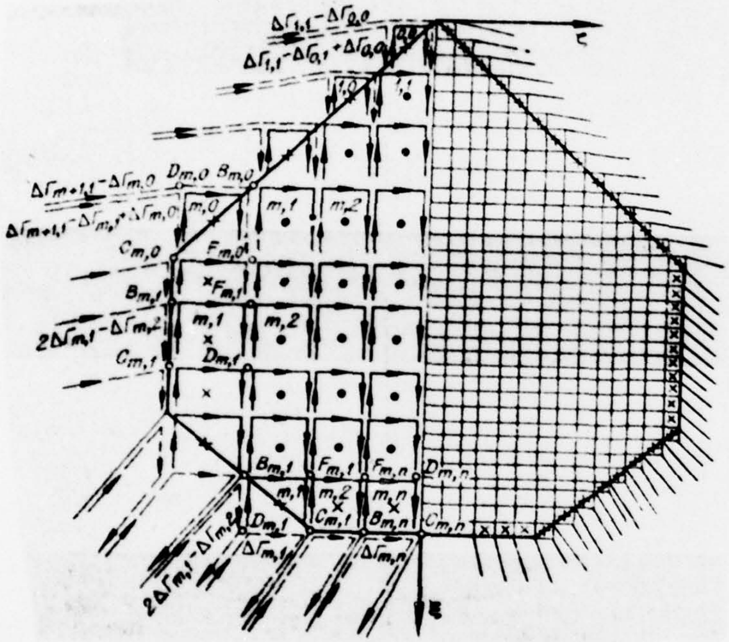


Fig. 2.



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