AD-A065661

FTD-ID(RS)T-1035-78

1

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

INTERIOR BALLISTICS OF BARREL SYSTEMS AND
SOLID-PROPELLANT ROCKETS
(Chapter XII)

by

M. Ye. Serebryakov

FOREIGN TECHNOLOGY DIVISION

Approved for public release;
distribution unlimited.

ACCESSION for

NTIS White Section X

DDC Buff Section ☐

UNANNOUNCED ☐

JUSTIFICATION..............

BY..............

DISTRIBUTION/AVAILABILITY CODES

Dist. AVAIL. and/or SPECIAL

A

FTD -ID(RS)T-1035-78

EDITED TRANSLATION

FTD-ID(RS)T-1035-78 7 July 1978

MICROFICHE NR: FTD-78-C-000943

INTERIOR BALLISTICS OF BARREL SYSTEMS AND SOLID-PROPELLANT ROCKETS (Chapter XII)

By: M. Ye. Serebryakov

English pages: 38

Source: Vnutrennyaya Ballistika Stvol'nykh Sistem i Porokhovykh Raket, Moscow, 1962, pp. 665-696

Country of origin: USSR

Translated by: Linguistics Systems, Inc.
F 33657-76-D-0389
Olga Karkalas

Requester: Ballistic Research Laboratories

Approved for public release; distribution unlimited

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION DIVISION FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

Chapter XII

Bore Systems with ~~Emission~~ Escape of Gases during ~~Combustion~~ Burning of Powder

Depending on the design and construction of the system gases can flow either from a chamber of constant volume with a nozzle or from the bore of the gun; gases can flow through small or large vents of varying shape (one or several round vents, perpendicular ~~parallel to~~ or at angles to the axis of the system, narrow circular aperture).

Depending on the shape, vents will vary in size and in the gas flow coefficient characterizing the compression of the flow.

We list the following as examples:

1) the flow proceeds from a chamber of constant volume with a nozzle in a gas operated gun;

2) the flow proceeds from a chamber of a recoilless gun in the direction opposite to the motion of the projectile concurrent with an increase in the volume of the bore resulting from the movement of the projectile along the bore of the barrel;

3) the flow proceeds from the bore of a barrel of mortar through a narrow circular space between the mine and the smooth bore of the barrel in direction of the motion of the mine; in this case some of the gas overtakes the mine and bursts forward.

The firing in the second case appears the most complex. In all cases at high pressures the law of the burning rate is expressed by the relation $u = u_1 p$.

12.1. Concept on the Characteristics of Interior Ballistics of Gas operated Guns

All equations derived in Chapter III for the combustion of powder in a bomb with a vent at high pressures are also applicable to separated combustion chambers in gas operated guns. The foundations of theories for these guns and the order of their construction was developed by V.M. Trofimov in 1923-1925.

The diagram for such a gun is presented in Fig. 12.1. Powder gases in a separate combustion chamber flow through a nozzle into the bore of the barrel; the projectile is some distance $l_{0\ bore}$ from the anterior section of the nozzle (a type of fore-chamber). The highest pressure in the combustion chamber reaches 2000-2500 kg/cm^2; the presence of the nozzle between the chamber and the bottom of the projectile enables the regulation of the gradual flow of gas into the barrel bore and the obtention of a low and almost constant pressure. This in turn permits the use of projectile with thinner walls, an increase in the charge coefficient for the explosive, and a significant decrease in the thickness of the bore walls, which also decreases the weight of the barre .

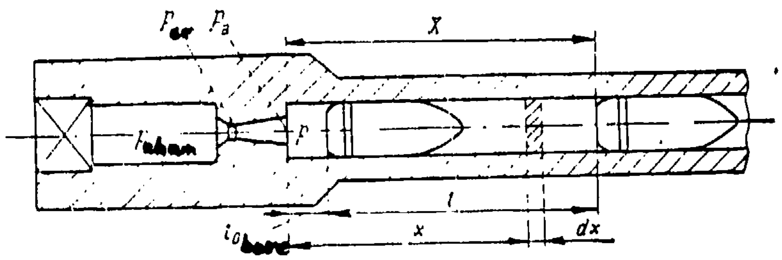


Fig. 12.1 Diagram for Gas Dynamic Gun

Powder combustion and the laws of pressure change of powder gases will be essentially identical under the same charge conditions (W_0, F_{min}, ω, Δ, I_f) in a bomb with a nozzle, described in Chapter III, and in a chamber of a gas operated gun. However, in experiments with a bomb with a nozzle, gases flow through the nozzle into a space with a constant atmospheric pressure; during combustion of the charge in a chamber of a gas operated gun, the gases flow through the nozzle into a closed space which gradually increases with the motion of the projectile. In this post-projectile space gas pressure is close to constant. This pressure appears as counterpressure with respect to the pressure within the chamber. Usually, during combustion of the powder charge the pressure within the barrel bore is several times less than in the chamber, so that it will not affect powder combustion and the supercritical nature of gas flow. After combustion of the charge the pressure within the combustion chamber only falls, but there can be a moment when it is equal to the pressure within the bore. Prior to this moment, the flow regime is already subcritical and the flow pattern depends on the counterpressure.

As a basis for the problem of interior ballistics for gas-operated gun, V.M. Trofimov proposed the following assumptions:

1. Processes occuring within the barrel bore do not affect powder burning within the chamber and gas flow in the nozzle ($P_{bore} << P_{cham}$).

2. Burning of powder follows the geometric law of burning.

3. Powder has a constant burning surface.

4. The law of the burning rate is given by the formula $u = u_1 p$.

5. The pressure within the chamber is highest at the end of combustion $p_{max} = p_f$

6. For the first period after firing, the process in the chamber is isothermal and the flow through the nozzle adiabatic. However, the gas temperature within the chamber T_{cham} is less than the combustion temperature T_1.

Subsequent research indicated that $T_{cham\,a} / T_1 = \tau_{cham\,a}$ was close to 0.90.

7. The pressure at the bottom of the projectile results from shock waves formed in the gas flow during its escape from the nozzle and propagated in the bore with the speed of sound in stationary ~~still~~ gas.

Later (in 1950) the following were assumed:

1. The composition of the combustion products is constant; c_p and c_w are average and constant during powder burning.

2. Heat emission is evidently not considered; however, this can be accomplished by conventional procedure by decreasing the

force of the powder f or increasing the index $\theta = k-1$.

3. Gas in the chamber is motionless; consequently, the pressure p, gas temperature T, the specific volume w and the gas density ρ are identical in a given moment in the entire chamber.

Hence, the general relation follows:

$$p(w-a) = RT_{cham}$$

4. The gas motion within the nozzle is steady and one dimensional, so that for each cross section at a distance x from the entrance cross section, this equality is correct:

$$p_x w_x = RT_x.$$

The assumptions presented above for deriving theoretical functions and the functions themselves adequately reflect the processes occuring during powder burning in a chamber with a nozzle.

The pressure within the chamber reaches a maximum at the end of burning not only for powder with a constant burning surface but also for slightly regressive powder (ribbon); for powder with seven perforations p_{max} is reached at the moment of disintegration ψ_s when σ_s has the highest value ~ 1.37.

Gas temperature in the chamber during the initiation of burning decreases to $T_{cham\ av} \approx 0.92\ T_1$ and thereafter remains fairly constant to the end of powder burning or the disintegration.

The theoretical relations used are applicable to gas-operated guns when the pressure within the bore p_{bore} is less than the critical pressure p_{cr} with respect to the pressure within the chamber.

During firing from a gas-operated gun the division into periods is somewhat different than for conventional guns.

1. The period before the cutting of the driving band in the groove (typical).

2. The second period lasts up to the end of powder burning and the emission of gases into the barrel bore; gas pressure in the chamber significantly exceeds gas pressure in the bore which increases gradually.

3. The first phase of the second period extends from the end of powder burning to the moment of pressure equalization in the chamber and in the bore; at this time, the pressure at the bottom of the projectile p_{pr} is greater than the average pressure in the bore under the impact of the gas on the projectile bottom (the reverse of that which is observed with conventional guns).

4. The second phase of the second period is the period of adiabatic expansion of gases to the ejection of the projectile from the barrel bore.

5. Period of the aftereffect of the gas (in the conventional sense).

Some Relations for Ballistic Elements

The force acting in the bore of the gas-operated gun on the bottom of the projectile has the rate v (according to V.M. Trofimov).

$$\Pi = s\rho_{bore}\, a_0 (u_a - v), \qquad (12.1)$$

where U_a =rate of gase flowing from the nozzle;

a_0 =speed of sound of the escaping gas

ρ_{bore} =mass density of the gases in the bore:

$$\rho_{bore} = \frac{\omega\eta}{gs(l_{0\,bore} + l)};$$

$\omega\eta$ =weight of gas escaping from the chamber into the bore.

It follows that

$$\Pi = \frac{\omega}{g}\eta_{\kappa}\frac{a_0(U_a - v)}{(l_{0\,bore} + l)}\frac{p_{cham} - p_{в}}{p_{cham\,max} - p_{в}}, \qquad (12.2)$$

where

$$\eta = \eta_{\kappa}\frac{p_{cham} - p_{в}}{p_{cham\,max} - p_{в}} = \eta_{\kappa}\frac{I}{I_{\kappa}}.$$

After some conversions V.M. Trofimov gives the final expression for the force Π:

$$\Pi = \frac{\omega}{l_{0_{cham}} + l} \frac{a_0^2}{g} \Phi(l), \qquad (12.3)$$

where $\phi(l) = \eta_K \frac{p_{cham} - p_B}{p_{cham\,max} - p_B}\left(1 + \frac{a p_{cham}}{f\sqrt{\varepsilon}}\right)$ for the period of powder burning; $\phi(l) = \left[1 - (1-\eta_k) - \frac{p_{cham}}{p_{cham\,max}}\right]\left(1 + \frac{a p_{cham}}{f\sqrt{\varepsilon}}\right)$ after powder burning.

The equation for the motion of the projectile is:

$$\frac{\varphi q}{g} \cdot \frac{dv}{dt} = \Pi.$$

The formula for the pressure of gasses within the bore is:

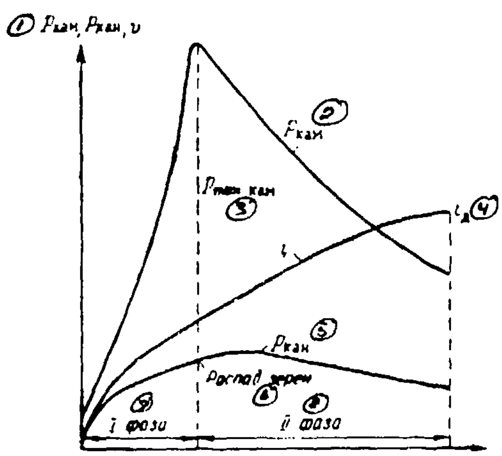
$$p_{cham} = \frac{f\omega\eta_k\tau}{s(l_{0_{bore}} + l)} \cdot \frac{p_{cham} - p_B}{p_{cham}^{max} - p_B}. \qquad (12.4)$$

The distribution of gases along the barrel bore is given by the equation:

$$p_{bore^x} = p_{pr}\left\{1 + \frac{1}{2}\frac{\omega}{\varphi_1 q}\eta\left(1 - \frac{x^2}{X^2}\right) - \right.$$

$$\left. - \frac{1}{2}\frac{\omega}{g}\frac{\eta}{s}\left(1 - \frac{x^2}{X^2}\right)\left[\frac{U_a}{X}(U_a - v) - \frac{dU_u}{dt}\right]\right\}. \qquad (12.5)$$

The pressure p_{pr} is greater than p_{av} in the bore.



1) Pcham; Pbore > V
2) Pcham
3) Pmax cham
4) V_D
5) Pbore
6) Grain disintegration
7) Phase I
8) Phase II

Fig. 12.2. Curves p, l and v, l in the bore of a gas-operated gun.

Fig. 12.2 illustrates the curves of the pressure within the chamber p_{cham}, the pressure within the bore p_{bore}, and the velocity of the projectile v as a function of the trajectory of the projectile. The graph shows that the pressure within the barrel bore increases slowly and after reaching maximum pressure also decreases very slowly; the curve p_{bore}, l is similar to the curve for constant pressure; therefore, the curve for projectory veolocity v, l resembles a straight line for a substantial section of the trajectory (under the effect of almost constant pressure the increase in velocity is also constant). The curve of the chamber pressure at corresponding moments varies withing much greater limits.

12.2. Characteristics of Interiror Ballistics for a Recoilless Gun (RG)

The lack of recoil or total balance of the barrel gun is achieved by the action of the force of the reaction of a fraction of the poder gases escaping from the chamber through the nozzle at the bottom of the sleeve or breech in a direction opposite to the motion of the projectile. This also relieves the load on the gun carriage which exists during recoil with the gun carriage becoming only a directional mount supporting the barrel. This pronouncedly decreases the weight of the system as a whole and significantly increases the metal use coefficient $\eta_b = E_b/Q_b = mv_b^2/2Q_b$ to 130-150 in place of the conventional 70-130.

However, this is attained primarily because of the significant increase in the weight of the charge and the volume of the chamber for a given initial velocity of the projectile v_d and at the highest pressure of the propellant gases p_{max}. Actually, in addition to the amount of gases necessary to obtain the given ballistics in the given barrel (active part of the charge), a certain weight of charge is required to form gases which are propelled in the reverse direction through the nozzle and create the force of the reaction to balance the barrel (passive part of the charge).

As a rule, for the given v_D and p_{max}, the weight of the charge and the volume of the chamber of the recoilless guns are 2 to 3 times greater than the weight of the charge and the volume of the chamber in conventional guns. Consequently, recoilless guns are used to obtain relatively low projectile velocities (v_D=350-500 m/sec).

Because of the increase in the total charge weight, the output coefficient for the charge $\eta_\omega = \varepsilon_D/\omega = mv_D^2/2\omega$ in recoilless guns is significantly less than for conventional guns η_ω =20-50 t·m/kg rather than 120-140.

In order that the process during the escape of gas through relatively large cross sections of the nozzle (F_{cr}: s $\approx$ 0.6) should proceed relatively intensely, the surface area of the charge should be very large with the powder curve relatively thin, since it is known that $S_1/\Lambda_1 = \varkappa/e_1$.

Therefore, the end of burning occurs early and the curve p, l is ~~sharp~~ peaked; η_D =0.15-0.45 rather than 0.40-0.70 as for conventional guns. Recoilless guns have an essential tactical flaw: during firing they are decamouflaged by the gases escaping from the nozzle at an angle to the ground which lift up large puffs of dust and thereby disclose the location of the gun. These gases create the impact wave which increases the pressure around the gun and the gun team should be either to the side of the gun or at a sufficiently large distance from it.

Because of their relatively light weight, the recoilless guns are also used as direct support for the infantry. These guns were widely used by the U.S. Army in the Korean War. According to reports, the U.S.A. and England have 120 mm recoilless antitank weapons.

Types of Recoilless Guns

Weapons with open-ended cylindrical barrels (Fig. 12.3). This type of recoilless gun on a light weight tripod with 70 mm caliber was developed and tested in 1916 by the engineer D. Ryabushinskii. In this case the weight of the barrel was 7 kg and the weight of the projectile 3 kg, weight of gunpowder was 0.3000–0.4000 kg; v_b=60 m/sec; a distance of 320 m was abhieved during firing.

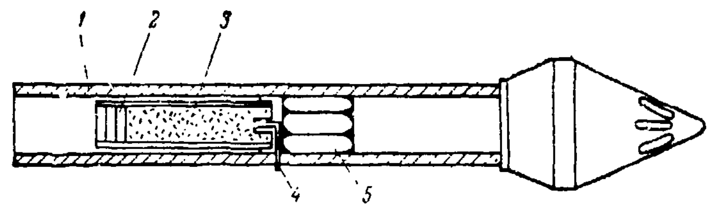


Fig. 12.3. Diagram for an open-ended barrel
1) barrel; 2) base plate 3) charge; 5) stabilizer.

The German Army in World War II used this type of weapon with supercaliber charge known as the antitank weapon Faust-patrone; later the U.S.A used the "bazooka". Many countries use this design for their grenade throwers with hollow-charge shells.

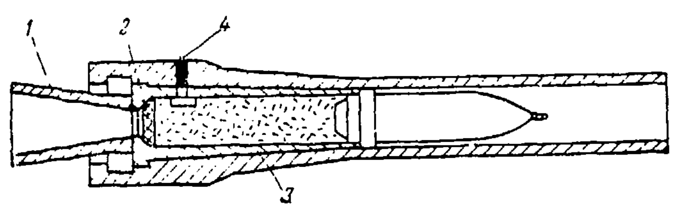


Fig. 12.4. Diagram for a central nozzle and base plate
1) nozzle; 2) base plate 3) sleeve 4) ignition

Recoilless guns with axial withdrawal of gases have vents at the bottom of the barrel sleeve and single-flow widening nozzle (Fig. 12.4). Ignition occurs at the side through a special channel. To increase the initial pressure to ensure uniform burning of the charge, the nozzle at the bottom of the barrel sleeve is closed by a base plate (wooden or plastic) which is ejected at a pressure close to the pressure for boosting the projectile p_0.

The drawback of this type of design is that certain amounts of still unburned grains fo the powder charge are drawn in through the opening large of the nozzle by the gases; this causes variations in the firing.

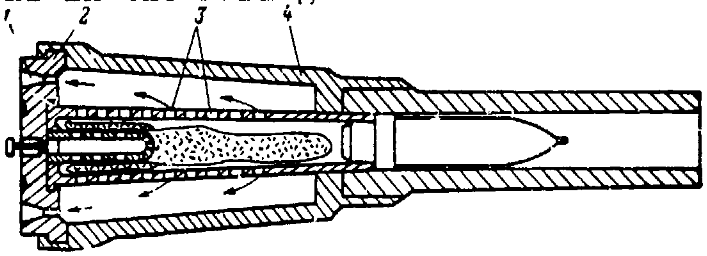


Fig. 12.5. Diagram for a perforated barrel sleeve
1) nozzle; 2) bolt; 3) perforations; 4) chamber.

Recoilless guns with a perforated barrel sleeve (Fig. 12.5) have lateral surfaces with many small vents to hinder the ejection of powder. After escaping initially into the side chamber exterior the gases subsequently flow through several nozzles at the bottom of the chamber. Ignition is of a conventional type. This type of recoilless gun was used by the U.S. Army in the Korean War.

Conditions for Balancing the System

Assume the recoilless gun is secured to the pins of the mount; the force of the reaction in the supports of the pins is designated as P (for each support P/2).(Fig. 12.6).

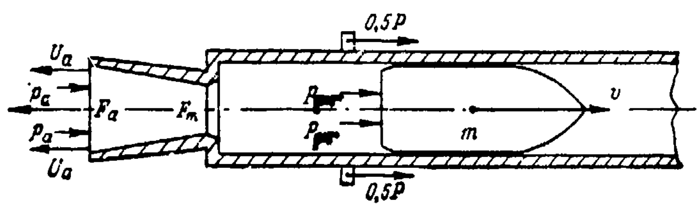


Fig. 12.6. Diagram of the forces acting on a recoilless gun

Let's consider the moment when the projectile with the mass m has the velocity v; gases flowing under pressure into the chamber p through the critical cross section of the nozzle F_{cr} expand and passing through the external section of the nozzle F_a, they have the pressure p_a, which is usually significantly greater than atmospheric pressure (p_a of the order 5-7 kg/cm^2); the velocity of gas flow in the outgoing cross section is U_a.

Let's use the theorem of mechanics: the momentum of a particle of the system under the action of internal forces is equal to the sum of momenta of external forces.

1. The momentum under the effect of internal forces: projectile, $+m\,dv$; gases flowing through the nozzle $-\frac{G_{sec}}{g}\,dt\,U_a$.

2. External forces: momentum of the reaction of the support $\pm P dt$; momentum of the force of gas pressure in the external cross section of the nozzle is $\pm p_a F_a\,dt$. We obtain

$$m\,dv - \frac{G_{sec}}{g}\,dt\,U_a = p_a F_a\,dt \pm P dt. \quad (12.6)$$

But $\frac{G_{sec}}{g} U_a + F_a p_a = R$ is the force of the reaction of gases on the barrel; $R = \zeta F_{cr}\, p$,

where $\zeta = f\left(\frac{d_a}{d_{cr}}\right) = f_1\left(\frac{F_a}{F_{cr}}\right)$; this is presented in the table in chapter II (cf. page 100).

To calculate the loss, the factor is introduced $\chi < 1$:

$$\chi = \varphi_a \varphi',$$

where $\varphi_a \approx 0.95$, is the coefficent of gas expansion;

$\varphi' \approx 0.90$ is the velocity coefficient;

$\chi \approx 0.85$.

Then

$$R = \chi \zeta F_{cr}\, p. \quad (12.7)$$

From the equation for the motion of the projectile $\varphi m \frac{dv}{dt} = sp$

we have

$$m\,dv = \frac{s}{\varphi}\, p\,dt. \quad (12.8)$$

By substituting the expression (12.7) and (12.8) into (12.6) and cancelling for dt, we obtain

$$\frac{s}{\varphi} p = \chi \zeta F_{cr} p \pm P. \qquad (12.9)$$

For a balanced gun P should equal zero. Then, the conditions for equilibrium are written as follows (after cancelling p):

$$\frac{s}{\varphi} = \zeta \chi F_{cr},$$

from which we find the ratio $\frac{F_{cr}}{s}$, which is the critical cross section of the nozzle divided by the cross section of the bore of the gun to obtain a recoilless barrel:

$$\frac{F_{cr}}{s} = \frac{1}{\varphi \chi \zeta}. \qquad (12.10)$$

Since $\zeta = 1.6 - 1.7$ depends on the ratio $\frac{F_a}{F_{cr}}$; then $\frac{F_{cr}}{s}$ depends on $\frac{F_a}{F_{cr}}$, φ and χ (loss factor).

In calculating the coefficient φ for recoilless guns using the formula $\varphi = \varphi_1 + \frac{1}{3}\frac{\omega_{dv}}{q}$ one must account for only that fraction of the weight of the gas powder mixture which results in the motion of the projectile. To calculate this, one must first determine the fraction of gases and charge which escape through the nozzle in the opposite direction.

At the end of the first period, i.e., at the end of the burning of the powder, this fraction is calculated by the existing formula:

$$\eta_K = \frac{\varphi_2 F_{cr} K_0 I_K}{\omega \sqrt{f \tau_{av}}}.$$

where τ_{av} is determined using the formula of P.N. Shkvornikov:

$$\tau_{av} = \frac{1}{1 + \theta \eta_K},$$

where τ_{av} with the variable η_K can be 0.9; $\eta_K = 0.5 - 0.6$ is typical for recoilless guns.

After burning of the powder the escape of gases continues. Since the pressure in recoilless guns in the second period decreases rapidly and the muzzle velocity of the projectile is v_D only slightly greater than v_f, then in the first approximation at the moment of ejection of the projectile the fraction of gases flowing through the nozzle from the chamber will be $\eta_D = \eta_K \frac{v_D}{v_f}$;

with $v_\Delta / v_\phi \approx 1.05 - 1.10$ $\quad \eta_\Delta \approx 0.65 \approx \frac{2}{3}$.

In this case a fraction of the charge $1-\eta_\Delta$ will follow the projectile and we obtain the expression for the coefficient

$$\varphi = \varphi_1 + \frac{1}{3}\frac{\omega}{q}(1-\eta_{\Delta\,av}).$$

With special efforts a more detailed calculation of secondary work with recoilless guns can be obtained.

In this regard an expression in analogy with the expression for conventional weapons can be written as a ratio for p_{dn} and p_{pr}

$$p_{dn} = p_{pr}\left[1 + \frac{1}{2}\frac{\omega}{\varphi_1 q}(1-\eta)\right],$$

$$p_{av} = p_{pr}\left[1 + \frac{1}{3}\frac{\omega}{\varphi_1 q}(1-\eta)\right],$$

where $\eta = \frac{Y}{\omega} = \frac{\int_0^t G_{loss}\,dt}{\omega}$ is the relative gas loss through the nozzle.

Therefore, the state recoilless produces the relation

$$\frac{F_{cr}}{s} = \frac{1}{\varphi \chi \zeta}.$$

Assuming, for instance, $\frac{\omega}{q} = 0.30$, $\zeta = 1.65$, $\chi = 0.85$, $\eta_\Delta = 0.65$, $1-\eta_\Delta = 0.35$

we obtain $\varphi = 1.02 + \frac{1}{3} \cdot 0.30 \cdot 0.35 = 1.055$.

Then the ratio of the area of the critical cross section of the nozzle F_{cr} to the cross section of the bore is s

$$\frac{F_{cr}}{s} = \frac{1}{1.055 \cdot 0.85 \cdot 1.65} = \frac{1}{1.479} = 0.676 \approx 0.68.$$

Characteristics of the Interior Ballistics of Recoilless Guns

Basic features of firing taking into account characteristics of ballistics of recoilless guns. The features of firing from recoilless guns are more complex than the features of firing from a conventional gun due to the reasons described above.

Approximately 2/3 of the weight of the charge escapes in the form of gas through the nozzle; together with the propellant gases a fraction of the grain powder is also ejected through the nozzle; these are usually small-grained (with seven perforations or short grains with one perforation) and are easily carried off with the escaping propellant gases. With a relatively large nozzle diameter the temperature of the propellant gases in the chamber and in the bore falls signficantly. In this case, a fraction of the gases moving the projectile and the fraction of gases escaping with some of the powder through the nozzle, strictly speaking, act under differing conditions; undoubtedly, there is a zone of relative stillness of gas layers, on the one side of which the gases move in the direction of the projectile, and on the other side of which, the flow is oriented in the direction of the nozzle.

Due to the complexity of the features discussed in solving the problems of interior ballistics, processes discussed are outlined and simplified by introducing into the solution of the basic problem the following assumptions.

1. The process of the flow of propellant gases through the nozzle is assumed as stationary and in this case all formulas of steady-state motion are used with the introduction of experimental correlation coefficients.

2. It is assumed that there is no the ejection of unburned powder grains through the nozzle, although this does take place. Therefore, in fact, the consideration of the movement of gas powder mixtures is replaced by the consideration of the movement of only propellant gases of the same weight.

3. In case of a large opening for the nozzle, a significant drop in gas temperature is taken into account:

$$\tau = \frac{T}{T_1} < 1.$$

In addition, all previous assumptions for conventional guns remain in force (geometric law of powder burning and its derived relations $\psi = f(z)$ and $\sigma = f_1(z)$, law of the rate of burning $u = u_1 p$, $\theta = \frac{c_p}{c_w} - 1 = \text{const}$ and others are assumed).

The system of equations for processes occuring during firing in recoilless guns is presented below.

The Compilation of an Essential System of equations

Equations for gas formation and the inflow of gases are

$$\frac{d\psi}{dt} = \frac{\varkappa\sigma}{I_\kappa} p,$$

$$\psi = \varkappa z + \varkappa\lambda z^2,$$

$$\sigma = 1 + 2\lambda z = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi}.$$

Equation for gas loss through the nozzle

$$\eta = \frac{\varphi_2 F_{cr} K_0}{\omega\sqrt{f}} \int_0^t \frac{p\,dt}{\sqrt{\tau}},$$

where φ_2 is the loss factor for the correlation of calculated and experimental values of η: $\varphi_2 \approx 0.95$.

In the first period the temperature falls slowly and $\tau = \tau_{av} = \text{const}$ can be assumed. Then

$$\eta = \frac{\varphi_2 F_{cr} K_0}{\omega\sqrt{f\tau_{av}}} \int_{t_0}^t p\,dt,$$

where

$$\int_{t_0}^t p\,dt = I_\kappa (z - z_0).$$

The nozzle is usually closed with a base plate which is calculated in such a way that it is ejected at the beginning of the motion of the projectile at pressure p_0; in the closed chamber the pressure p_0 corresponds to the values ψ_0 and z_0. The nozzle is opened and the flow of gases begins at $p = p_0$, $\psi = \psi_0$ and $z = z_0$

Then $\eta = \frac{\varphi_2 F_{cr} K_0 I_\kappa}{\omega\sqrt{f\tau_{av}}}(z - z_0) = \eta_\kappa x.$

Equation for the state of gases during the presence of flow. Granting that the free space of the chamber during gas escape (as in mortar) is expressed by the relation

$$W_{\psi_\eta} = W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha\omega(\psi-\eta),$$

the equation for the state of gases is written as follows:

$$p(W_{\psi_\eta} + sl) = RT\omega(\psi-\eta)$$

or

$$ps(l_{\psi_\eta} + l) = f\tau\omega(\psi-\eta),$$

where

$$\tau = \frac{T}{T_1}.$$

Essential Equation for Interior Ballistics. Energy Equation.

First we establish the energy equilibrium during firing.

At a given moment t, the amount of charge ψ was burned, and the amount of energy liberated (energy inflow) is expressed by the relation $EQ\omega\psi = \frac{RT_1}{\theta}\omega\psi = \frac{f\omega}{\theta}\psi.$

This energy is expended:

a) for providing the projectile with the critical energy $\frac{\varphi m v^2}{2}$;

b) to alter the internal energy of the mass of gas in the barrel bore at the given moment which has the temperature $T < T_1$;

$$\frac{RT}{\theta}\omega(\psi-\eta) = \frac{f\omega}{\theta}\tau(\psi-\eta).$$

This value can be replaced by the expression from the equation of state $\frac{ps(l_{\psi\eta}+l)}{\theta}$;

c) to provide information on the movement of gases escaping through the nozzle at a given moment; in this case one should consider that the energy expended for the transfer and pushing of gases will be greater than the internal energy of the motionless gases if they were to remain in the barrel.

The energy expended for pushing the gases is expressed by the equation

$$Ec_p T\omega\eta = kEc_w T\omega\eta,$$

or

$$\frac{f\omega}{\theta}(1+\theta)\tau_{av}\eta_K x = \frac{f\omega}{\theta} D'x,$$

where

$$D' = (1+\theta)\eta_K \tau_{av}.$$

In comparing the inflow and expenditure of energy and transferring θ in the last term on the rightside, we obtain the equation for energy transformation

$$f\omega\psi = ps(l_{\psi_\eta} + l) + f\omega D'x + \frac{\theta\varphi m v^2}{2}.$$

This can be rewritten in the general form

$$ps(l_{\psi_\eta} + l) = f\omega(\psi - D'x) - \frac{\theta}{2}\varphi m v^2. \qquad (12.11)$$

The value $D'x$ represents the fraction of energy used to eject gases through the nozzle.

5. Equation for the motion of the projectile is

$$ps\,dl = \varphi m v\,dv \qquad (12.12)$$

or

$$ps\,dt = \varphi m\,dv. \qquad (12.12')$$

The system of these equations is solved in a general form using the parameters and functions of Prof. N.F. Drozdov..

Solution of the Essential Problem of Interior Ballistics for Recoilless Guns

Preiliminary period (general formulas).
It is assumed that the escape of gases through the nozzle occurs simultaneously with the beginning of the motion of the projectile at the pressure p_0:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_n = \frac{\psi_0}{\varkappa}.$$

The first period is $(x = z - z_0)$

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2, \qquad (12.13)$$

$$v = \frac{sI_K}{\varphi m} x. \qquad (12.14)$$

By dividing the equation (12.12) by the expression (12.11), we obtain:

$$\frac{dl}{l_{\psi\eta}+l}=\frac{\varphi m v\,dv}{f\omega(\psi-D'x)-\frac{\theta\varphi m v^2}{2}}=\frac{Bx\,dx}{\psi_0+(k_1-D')x-\left(\frac{B\theta}{2}-x\lambda\right)x^2}=$$

$$=-\frac{B}{B_1}\frac{x\,dx}{x^2-\frac{k_1-D'}{B_1}x-\frac{\psi_0}{B_1}}.$$

The solution to this differential equation at $l_{\psi\eta}=l_{\psi\eta_{av}}=l_c$, where $\psi_{av}=\frac{\psi_0+\psi}{2}$ results in the form

$$\ln\left(1+\frac{l}{l_c}\right)=-\frac{b}{B_1}\ln Z_x$$

or

$$l=l_c(Z_x^{-B/B_1}-1), \qquad (12.15)$$

where $Z_x=\int_0^x\frac{x\,dx}{x^2-\frac{k_1-D'}{B_1}x-\frac{\psi_0}{B_1}}$ is the integral of Prof. N.F. Drozdov, in which the escape of gases through the nozzle is taken into account by subtracting D' from the usual variable k_1.

As is evident $Z_x=f(\gamma,\beta)$; for the given case taking into account the escape of gases through the nozzle the expression for γ and β will differ from analogous values given in chapter VII:

$$\gamma=\frac{B_1\psi_0}{(k_1-D')^2} \text{ and } \beta=\frac{B_1}{(k_1-D')}x,$$

i.e., γ and β at a given x have other values in comparison with the variables γ and β for conventional guns under the same charge conditions since the measure of gas loss D' is subtracted from k_1.

Gas pressure is calculated from the basic equation

$$p=\frac{f\omega}{s}\frac{\psi-D'x-\frac{B\theta}{2}x^2}{l_{\psi\eta}+l}. \qquad (12.16)$$

In this case also the value D'x is subtracted from ψ which makes this formula different from the basic one.

The value χ_m is found using the formula, which is analogous to the basic one: $x_m = \frac{k_1 - D'}{2B_1 + B}$.

By using χ_m and the formulas (12.13)-(12.16), ψ_m, v_m, l_m and p_{max} are calculated. Using the value of $\chi_k = 1 - z_0$ and the same formulas the elements for the first end of the period can be found $(\psi_K = 1)$: v_f, l_K, p_K. These values are also the initial ones for the second period.

The second period is the adiabatic expansion of propellant gases concurrent with the escape of gases through the nozzle. This period is characterized by the fact that all the powder has burned($\psi = 1$) but the escape of gases continues simultaneously with the movement of a fraction of gases after the projectile: the relative fraction of gases escaping at a given moment $\eta = Y/\omega$ continues to increase and $\eta > \eta_K$; the velocity of the projectile also continues to increase: $v > v_f$.

For the solution we have a system of two equations:

$$ps\,dl = \varphi m v\,dv, \qquad (12.12)$$

$$ps(l_{1\eta} + l) = f\omega(1 - D'x) - \frac{\theta\varphi m v^2}{2}. \qquad (12.11)$$

Since the loss of gases through the nozzle

$$D'x = (1+\theta)\eta_K x = (1+\theta)\frac{\varphi_2 F_{cr} K_0}{\omega\sqrt{f\tau_{av}}} I_K x$$

is proportional to the momentum of the pressure $I - I_0$, counted from the beginning of the motion of the projectile, and the velocity of the projectile v is also proporitonal to the value $I - I_0 = \int_{\psi_0}^{\psi}$ regardless of whether the powder burned, then the value of v is proportional to x in the second period also. Therefore, the system of equations (12.12) and (12.11) for the second period can be solved using the method for the first period; only a slightly different expression for the integral of the trajectory is obtained. By dividing the equation (12.12) by (12.11), we obtain:

$$\frac{dl}{l_{1\eta} + l} = \frac{\varphi m v\,dv}{f\omega\left[1 - D'x - \frac{v^2}{v_{pr}^2}\right]} = B\frac{x\,dx}{1 - D'x - \frac{B\theta}{2}x^2},$$

since

$$\frac{v^2}{v^2_{np\,pr}} = \frac{B\theta}{2} x^2.$$

In this case $B_1 = \frac{B\theta}{2}$ and $\frac{B}{B_1} = \frac{2}{\theta}$,

$$\frac{dl}{l_{1\eta} + l} = -\frac{2}{\theta} \frac{x\,dx}{x^2 + \frac{D'}{B_1} x - \frac{1}{B_1}}.$$

Designating $\frac{B_1}{D'} x = \beta$ and $\frac{B_1}{D'^2} = \gamma$ and multiplying the numerator and denominator on the right hand side by $\frac{B_1^2}{D'^2}$, we obtain

$$\frac{dl}{l_{1\eta} + l} = -\frac{2}{\theta} \cdot \frac{\beta\,d\beta}{\beta^2 + \beta - \gamma} = -\frac{2}{\theta} d \ln Z.$$

By integrating the preceding equation, we obtain

$$\frac{l_{1\eta_{av}} + l}{l_{1\eta_{av}} + l_k} = \left(\frac{Z}{Z_K}\right)^{-\frac{2}{\theta}},$$

where

$$Z = \int_0^{\beta} \frac{\beta\,d\beta}{\beta^2 + \beta - \gamma}; \quad Z_b = \int_0^{\beta_x} \frac{\beta\,d\beta}{\beta^2 + \beta - \gamma}.$$

Finally we have an association between l and x (or v) in the form of

$$l = (l_{1\eta_{av}} + l_k)\left(\frac{Z}{Z_k}\right)^{-\frac{2}{\theta}} - l_{1\eta_{av}}. \qquad (12.17)$$

The formula obtained is of the same type as the formulas for the first period but the function Z differs from the integral of Prof. N.F. Drozdov Z_x in that the input values γ and β differ from the integral Z_x and in addition, in the integrand expression in the numerator the value β has a "+" and not a "-" sign. This integral can be calculated by the same method as the integral Z_x.

After analogous transformations, we obtain

$$Z = \left(1 - \frac{\beta}{b-1}\right)^{\frac{b-1}{b}} \left(1 + \frac{\beta}{b+1}\right)^{\frac{b+1}{b}},$$

where $b = \sqrt{1 + 4\gamma}$ and $\gamma = \frac{B_1}{D'^2} = \frac{B\theta}{2D'^2}$.

Pressure in the second period is calculated by the formula

$$p=\frac{f\omega}{s}\frac{1-D'x-\frac{B\theta}{2}x^2}{l_{1\eta}+l}, \qquad (12.18)$$

where $$x=x_{\kappa}\frac{v}{v_{\phi}}.$$

The typical curve for gas pressure p, l in the bore of a recoilless gun is peaked; the powder burns rapidly at a low value and in the second period the curve of adiabatic expansion with loss falls rapidly; muzzle pressure is insignificant (Fig. 12.7).

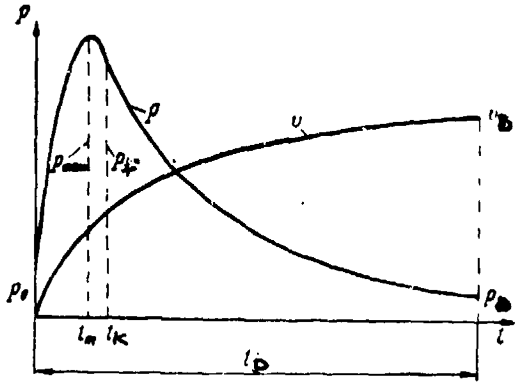


Fig. 12.7. Curves p, l and v, l for recoilless weapon

12.3 Features of Interior Ballistics for Mortar

Features of Firing from a Mortar

In comparison with firing from a conventional artillery weapons, firing from mortar has several features.

1. The first characteristic is the design and the placement of the charge. A diagram for the design of mortar is presented below (Fig. 12.8).

The primary charge of the mortar is placed in a cardboard cartridge (barrel sleeve), placed in the tube of the stabilizer 1 (tail end of the projectile). The tube has 4 or 6 vents circular 2, through which the propellant gases formed within the barrel sleeve after piercing of the cardboard should escape into the space behind the mine and ignite the booster charge ω.

During ignition the projectile is lowered in the bore expelling the air through the gap 3. The capsule of the cartridge with the primary charge hits the fuse 4 fastened to the bottom of the mortar bore; the capsule and the powder charge are ignited; at this time the powder burns initialy in the closed space of the cartridge at fairly high density of ignition Δ_0=0.50-0.60. At a certain moment the gas pressure pierces the wall of the cardboard sleeve and the gases escape through the vents 2 in the tube of the stabilizer into the space of the chamber W_0 (projectile space).

Under such conditions of very rapid burning of the very fine, small powder the greatest pressure of gases within the tube of the stabilizer, as experiments showed, depends to a significant extent on the size of the vents in the tube, the thickness of the barrel sleeve wall, and on the temperature of the charge. As a result of slight difference in pressure at which the sleeve walls are penetrated, the values of the greatest pressure within the tube of the stabilizer can be widely scattered.

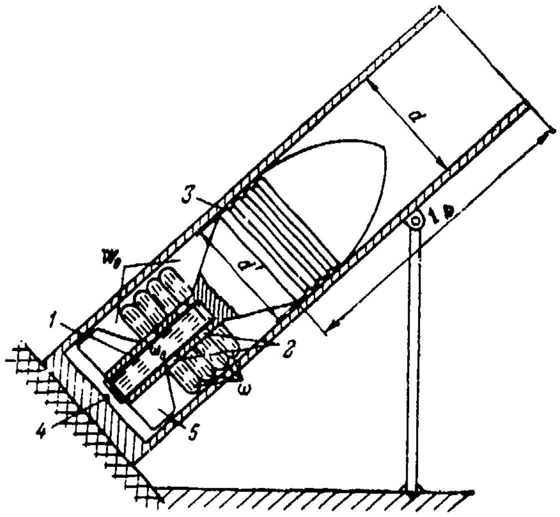


Fig. 12.8. Diagram of the design of mortar

Consequently, the value of the composition, weight of the capsule igniter, and the rapidity of powder burning are more significant in the mortar than in guns the greater the igniting momentum is, the more uniform is the ignition of the powder.

2. The following feature is the fact that the gases of the primary charge ω_0, burning initially within the chamber of the stabilizer at Δ=0.50-0.60, escape into the projectile space, expand rapidly, and cool. Since the surface of the fin stabilizers 5 and the bottom part of the mortar shell is large and the density of the ~~ammunition~~ ~~charge of~~ the primary charge with respect to the entire volume of the chamber W_0 ($\Delta \approx$ 0.01) is small; there is a great loss for heat transfer to the walls of the bore and projectile. This loss is even greater as a result of the slow movement of the ~~shell~~ projectile and the long intermediate time during which the gases are in contact with the walls of the mortar.

If there are booster charges ω, then the powder within them is ignited by the gases of the primary charge and the movement of the projectile proceeds under the effect of the total pressure of gases of the primary and booster charges.

3. The third feature of firing from mortar is the bursting through of the gases through the gap between the projectile and the bore. Because of this gap 3 between the projectile and the bore walls, a fraction of the gases will burst through this gap at the beginning of projectile movement and subsequently their energy is not used.

In valve mortars with open remote valve a significant amount of gases also escaped through the valve. The loss of gases through the gap and the valve are included in the foundations of general relations in gas dynamics.

As high-speed photography shows, a signficant fraction of the gases escape from the barrel bore of the mortar before the projectile appears from the bore, accompanied by the release of most of the gases. This fraction of the gases which bursts through the gap and is not responsible for creating speed for the shell, composes from 10-15% of the entire quantity, while in conventional guns the fraction of gases escaping from the gap between the driving band and the perforations is insignificant.

4. The fourth feature is that the pressure of the ejection is almost equal to zero. Just so in a smooth bore, there is no loss of energy for overcoming the friction and the rotation of the shell.

Thus, the solution of the problem of interior ballistics, on the one hand, is simplified by the fact that the pressure of the ejection and some of the secondary work is taken as equal to zero, and on the han. other, it is complicated as a consequence of the necessity to include the large loss for heat transfer and loss of gases through the gap.

While in firing a conventional mortar with support in an end plate recoil is essentially nonexistant and the relative weight of the charge $\frac{\omega}{q}$ is very small (0.01-0.02 during full charge), then in practice the coefficient $\varphi=1$ can be used.

To maintain unity of procedure and designation of parameters and functions further presentations on the solution of basic problems of interior ballistics for mortar are given using the designations of Prof. P.F. Drozdov for conventional artillery weapons.

Analytical Solutions of Basic Problems of Smoothbore Mortars (Simplified Method of Prof. M.E. Serebryakov)

The following assumptions are made as the basis for the analytical solution.

1. The pressure of the ejection is absent. A mortar has a circular gap between the shell and the bore; its area is s_{gap}.

2. The burning of the primary charge within the tube of the stabilizer is not considered.

Gases of the primary charge escaping from the tube of the stabilizer into the projectile space, creates the pressure p_0, at which the powder of the booster charge ignites. In this way the

the primary charge is the igniter of the booster charge.

3. The ignition of the booster charge is instantaneous and simultaneous for all grains and at all points on the surface of each grain.

4. The burning of the grains of the booster charge proceeds in parallel layers according the geometric law of burning and is expressed by **existing formulas:**

$$\psi = \varkappa z + \varkappa \lambda z^2;$$
$$\sigma = 1 + 2\lambda z.$$

5. The rate of the burning of the powder is proportional to the pressure (to the power of one): $u = \frac{de}{dt} = u_1 p,$

where u_1 is the rate of burning at p=1.

6. The motion of the projectile begins at pressure p_0 concurrently with the initiation of burning of the booster charges (at $p=p_0$, $\psi=0$; $l=0$; $v=0$).

7. The escape of gases through the gap begins with the initiation of burning of the booster charges and with the movement of the projectile.

8. The total momentum of the rise in pressure $\int_0^{t_b} p\,dt = \frac{e_1}{u_1}$ does not depend on the density of the charge Δ and on the value of the initial pressure p_0 at which the powder ignites.

9. The loss of gases through the gap according to general formulas of gas dynamics is proportional to the momentum of the increase in pressure:

$$Y = \omega\eta = \zeta' A s_{gap} \int_0^t p\,dt = \zeta' A s_{gap} I,$$

where η is the fraction of gases escaping through the gap;

A is the loss factor: $A = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2gk}{k+1}\frac{1}{f}};$

ξ' =loss coefficient characterizing the form of the vent or gap; for a round vent ξ =0.95; for a crescent shaped slot $\xi \approx$ 0.66;

S_{gap} =area of the cross section of the gap.

10. Heat transfer was considered in experiments using a special installation in which the primary charge burns under the same conditions as in mortar.

By determining the greatest gas pressure of the primary charge p_{max0} using this experimental installation, we find the force of the powder f_0 of the primary charge, taking into account the cooling of the gases due to their escape from the tube of the stabilizer into the shell volume and due to the heating of the bore walls and the fins of the stabilizer:

$$f_0 = p_{max\,0}\left(\frac{1}{\Delta_0} - \alpha_0\right),$$

where Δ_0=density of the charge of the primary charge with respect to the entire projectile space W_0 (chamber).

The value of f_0 is much less than the force f of the booster charges from powder of the same nature, which is determined by experiments in a conventional manometric bomb.

Additional heat transfer during the movement of the shell is negligible (since the bore walls and shell are heated by gases escaping through the circular gap and ~~out~~distancing the shell) or is calculated indirectly. Since the gases do not perform work to rotate the shell, to overcome friction against the grooves, and for the recoil, and $\frac{\omega}{q}$ =1%, then the coefficient for calculating secondary work may be assumed equal to one.

During firing from the mortar the following periods may be delineated:

1) burning of the primary charge up to the piercing of the

opening into the sleeve and the escape of gases into the chamber; this phase with the given assumptions is analogous to the preliminary period;

2) the first period corresponds to the burning of the booster charges concurrent with the escape of a fraction of the gases through the gap (l varies from 0 to l_b);

3) the second period is the expansion of gases formed in the first period concurrent with the escape their through the gap s_{gap}.

The movement of the projectile and the escape of gases through the gap (on the basis of the given assumptions) begins with the pressure p_0, created by the gases of the primary charge; p_0 can be calculated in the experimental installation:

$$p_0 = \frac{f_0 \omega_0}{W_0 - \frac{\omega}{\delta} - \alpha \omega_0} = \frac{f \Delta_0}{1 - \frac{\Delta}{\delta} - \alpha_0 \Delta_0},$$

where $\Delta = \frac{\omega}{W_0}$

If s designates the cross section of the mortar bore:

$$s = \frac{\pi}{4} d^2,$$

and s' designates the area of the cross section of the projectile at its central largest diameter; $s' < s$, then the area of the gap between the projectile and the bore walls is $s_{gap} = s - s'$.

The velocity of the projectile is determined by the motion equation

$$\varphi m \, dv = s' p \, dt,$$

$$v = \frac{s'}{\varphi m} \int_0^t p \, dt = \frac{s'}{\varphi m} I = \frac{s' I_K}{\varphi m} z,$$

where

$$z = \frac{I}{I_K} = \frac{e}{e_1}.$$

To solve the problem the following system of equations can be used.

1. The basic equation for pyrodynamics taking into account

the escape of a fraction of the gases through the gap and loss for heat transfer

$$sp(l'_\psi + l) = f_0\omega_0 + f\omega\psi - f'Y - \frac{\theta}{2}\varphi m v^2,$$

where

$$l'_\psi = \frac{1}{s}\left[W_0 - \frac{\omega}{\delta}(1-\psi) - \alpha(\omega\psi - Y) - \alpha_0\omega_0\right]$$

includes the loss of gases through the gap s-s'.

The value f' is the force of the gas mixture of the primary and booster charges; essentially, it varies with burning time of the booster charges from f_0 to $\frac{f_0\omega_0 + f\omega}{\omega_0 + \omega} < f$, with the intermediate value $f' = \frac{f_0\omega_0 + f\omega\psi}{\omega_0 + \omega\psi}$.

Since the heat transfer to the walls during the first period is not yet directly accounted for, it can be indirectly included by taking the value f of the booster charges as greater than f'. In this case the basic equation is rewritten as follows:

$$sp(l'_\psi + l) = f_0\omega_0 + f(\omega\psi - Y) - \frac{\theta}{2}\varphi m v^2. \qquad (12.19)$$

2. The equation for the movement of the is projectile

$$s' p\, dl = \varphi m v\, dv \qquad (12.20)$$

3. The law for the burning of powder (geometric) for small flake powder is $\psi = \chi z + \chi\lambda z^2$.

4. The formula for the velocity of the projectile is

$$v = \frac{s' I_b}{\varphi m} z.$$

5. The relative loss of gases is

$$\eta = \frac{Y}{\omega} = \frac{\zeta' A s_{gap} I_K}{\omega} z = \eta_K z, \qquad (12.21)$$

where

$$\eta_K = \frac{\zeta' A s_{gap} I_K}{\omega} = \frac{\zeta' A s_{gap}}{\omega}\frac{e_1}{u_1};$$

$\zeta' < 1$ = coefficient for the shape of the vents;

η_K = relative loss of gases at the end of the burning of the powder.

Let us introduce the designation:

$$B' = \frac{s' I_k^2}{f\omega\varphi m} = \left(\frac{s'}{s}\right)^2 \frac{s^2 I_k^2}{f\omega\varphi m} = \left(\frac{s'}{s}\right)^2 B;$$

$\chi_0 = \frac{f_0 \omega_0}{f\omega}$ =relative energy of the primary charge.

By substituting in equation (12.19) the variables ψ, ν and Y (or η) by their expression through z, we obtain a basic equation of interior ballistics in the following form:

$$sp(l'_\psi + l) = f\omega\left[\chi_0 + \varkappa z + \varkappa\lambda z^2 - \eta_k z - \frac{B'\theta}{2} z^2\right] =$$

$$= f\omega\left[\chi_0 + (\varkappa - \eta_k) z - \left(\frac{B'\theta}{2} - \varkappa\lambda\right) z^2\right]. \qquad (12.22)$$

From equation (12.20) we have

$$s' p\, dl = \frac{s'^2 I_k^2}{\varphi m} z\, dz. \qquad (12.20')$$

By dividing (12.20') term by term by (12.22), we obtain

$$\frac{s'}{s} \frac{dl}{l'_\psi + l} = B' \frac{z\, dz}{\chi_0 + k'_1 z - B'_1 z^2} = -\frac{B'}{B'_1} \frac{z\, dz}{z^2 - \frac{k'_1}{B'_1} z - \frac{\chi_0}{B'_1}} = -\frac{B'}{B'_1} d \ln Z,$$

where $k'_1 = \varkappa - \eta_k$; $B'_1 = \frac{B'\theta}{2} - \varkappa\lambda$; $A_s = \frac{s}{s'} \frac{B'}{B'_1}$.

Z is the known function of Prof. N.F. Drozdov. Thus, we obtain

$$\frac{dl}{l'_\psi + l} = -A_s \frac{z\, dz}{\xi_1(z)} = -A_s d \ln Z. \qquad (12.23)$$

The equation (12.23) can be solved exactly by the method of Prof. N.F. Drozdov by reducing it to a linear differential equation of the first order, but taking into account that the density of the charge in the projectile is small ($\Delta < 0.15$) and that consequently, l'_ψ varies little, one can assume an average value for $l'_\psi = l'_{\psi_{av}}$. After integration a simple expression for the shell projectile is obtained in the form

$$\frac{l'_{\psi_{av}} + l}{l'_{\psi_{av}}} = 1 + \frac{l}{l'_{\psi_{av}}} = Z^{-A_s},$$

from which follows

$$l = l'_{\psi_{av}}(Z^{-A_s} - 1). \qquad (12.24)$$

lg Z^{-1} is calculated according the to the table of N.F. Drozdov (cf. page 417) with the initial values $\gamma = \frac{B'_1 \chi_0}{k_1'^2}$; $\beta = \frac{B'_1}{k'_1} z$.

Consequently, the solution to the problem for the first period for the mortar differs from the solution for artillery weapons only in that at z there is the coefficient $k_1 = \varkappa - \eta_K$ in place of $k_1 = \varkappa\sigma_0$, the value $\chi_0 = \frac{f\omega_0}{f\omega}$ in place of ψ_0, and the value $B' = B\left(\frac{S'}{S}\right)^2$ in place of B. These features of firing from mortar express themselves in the values $\gamma = \frac{B_1'\chi_0}{k_1'^2}$ and $\beta = \frac{B_1'}{k_1'}z$, which in turn increases the value Z^{-1} and the path l and decreases the value of the pressure p in comparison with the pressure during the absence of gas escape through the gap.

The pressure p is calculated using the formula obtained from equation (12.19):

$$p = \frac{f\omega}{s}\,\frac{\chi_0+\psi-\eta-\frac{\theta}{2}\,\frac{\varphi m v^2}{f\omega}}{l'_\psi+l} = \frac{f\omega}{s}\,\frac{\chi_0+k_1'z-B_1'z^2}{l'_\psi+l}. \quad (12.25)$$

To determine z_m, corresponding to the greatest gas pressure p_{max}, we differentiate this expression with respect to z and using the equation (12.23), after a series of transformations we obtain

$$z_m = \frac{\varkappa\left(1+\frac{p_{max}}{f\delta_1}\right)-\eta_k\left(1+\frac{\alpha p_{max}}{f}\right)}{B'\left(\frac{s}{s'}+\theta\right)-2\varkappa\lambda\left(1+\frac{p_{max}}{f\delta_1}\right)} = \frac{\varkappa-\eta_k\left(\dfrac{1+\frac{\alpha p_{max}}{f}}{1+\frac{p_{max}}{f\delta_1}}\right)}{\dfrac{B'\left(\frac{s}{s'}+\theta\right)}{1+\frac{p_{max}}{f\delta_1}}-2\varkappa\lambda},$$

where $\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}$.

At $\eta_K = 0$ and $s = s'$, this formula becomes the general formula for z_m.

If $z_m \leq 1$, we have a real maximum pressure; if $z_m > 1$, the maximum is unreal, and in this case, the greatest actual pressure will occur at the end of burning p_f:

$$p_f = \frac{f\omega}{s}\,\frac{1+\chi_0-\eta_k-\frac{B'\theta}{2}}{l_1'+l_k},$$

where $l_1' = l_0[1 - \alpha\Delta(1-\eta_\kappa)]$ and $\Delta_\kappa = \frac{\omega}{W_0}(1-\eta_\kappa) = \Delta(1-\eta_\kappa)$.

The remaining factors at the end of the first period will be $v_f = \frac{s' I_\kappa}{\varphi m}$; $l_\kappa = l'_{\psi_{av}}(Z_\kappa^{-A_\kappa} - 1)$,

where $\beta_\kappa = \frac{B_1'}{k_1'}$.

For solving the problem of the second period, when $\psi = 1$, there is the following system of equations:

$$sp(l_1' + l) = f_0\omega_0 + f\omega(1 - \eta_\kappa z') - \frac{\theta}{2}\varphi m v^2, \qquad (12.26)$$

$$s'p\,dl = \varphi m v\,dv, \qquad (12.20)$$

where

$$z' = \frac{I}{I_\kappa} = \frac{\int_0^t p\,dt}{\int_0^{t_\kappa} p\,dt} = \frac{v}{v_f},$$

in this case z' is greater than one.

The value $\eta_\kappa z'$ is also accounted for in the second period of gas emission from the gap. As in the first period, the total loss is proportional to the momentum of the gas pressure, which in turn, is proportional to the velocity of the projectile.

Equation (12.26) can be rewritten as:

$$sp(l_1' + l) = f\omega\left[\chi_0 + 1 - \frac{\eta_\kappa}{v_\kappa}v - \frac{v^2}{v_{pr}^2}\right], \qquad (12.26')$$

where

$$\eta_\kappa = \frac{\zeta' A s_{gap} I_\kappa}{\omega}; \quad v_f = \frac{s' I_\kappa}{\varphi m},$$

$$v_{pr}^2 = \frac{2f\omega}{\varphi\theta m};$$

$$\frac{\eta_\kappa}{v_f} = \frac{\zeta' A s_{gap} I_\kappa}{\omega}\frac{\varphi m}{s' I_\kappa} = \zeta' A \frac{\varphi}{g}\frac{q}{\omega}\frac{s_{gap}}{s'} = \eta_\kappa'.$$

Dividing (12.20) by (12.26'), we obtain:

$$\frac{dl}{l_1' + l} = \frac{s}{s'}\frac{\varphi m}{f\omega}\frac{v\,dv}{1 + \chi_0 - \eta_\kappa' v - \frac{v^2}{v_{pr}^2}} = -\frac{s}{s'}\frac{2}{\theta}\frac{v\,dv}{v^2 + \eta_\kappa' v_{pr}^2 v - (1+\chi_0)v_{pr}^2}$$

or

$$\frac{dl}{l_1' + l} = -\frac{s}{s'}\frac{2}{\theta}\frac{v\,dv}{v^2 + \eta_2 v - \eta_3}, \qquad (12.27)$$

where

$$\eta_2 = \eta_\kappa' v_{pr}^2 = 2\zeta' A\frac{f}{\theta}\frac{s_{gap}}{s'} = \frac{\eta_\kappa}{\frac{v_f}{v_{pr}^2}} = \frac{\eta_\kappa}{v_f}v_{pr}^2,$$

$$\eta_3=(1+\chi_0)\,v^2_{\text{пр}}=(1+\chi_0)\,\frac{2f\omega}{\varphi\theta m}=v'^2_{\text{пр}}.$$

By integrating (12.27) we obtain

$$\int\limits_{l_\kappa}^{l}\frac{dl}{l_1'+l}=-\frac{s}{s'}\,\frac{2}{\theta}\int\limits_{v_\kappa}^{v}\frac{v\,dv}{v^2+\eta_2 v-\eta_3},\qquad (12.28)$$

$$\int\limits_{l_\kappa}^{l}\frac{dl}{l_1'+l}=\ln\frac{l_1'+l}{l_1'+l_\kappa}.\qquad (12.29)$$

The integral of the right side is found by expanding the integrand function to a partial fraction by the method proposed by Prof. N.F. Drozdov.

Solve for the roots of the equation $v^2+\eta_2 v-\eta_3=\xi'(v)=0$:

$$v=-\frac{\eta_2}{2}\left(1\pm\sqrt{1+4\frac{\eta}{\eta_2^2}}\right)=-\frac{\eta_2}{2}(1\pm b),$$

where

$$\frac{\eta_2}{2}=\frac{f}{\theta}\,\frac{s_{\text{кн}}}{s'}\,\zeta' A;\quad b=\sqrt{1+4\frac{\eta_3}{\eta_2^2}}=\sqrt{1+4\gamma};$$

$$\gamma=\frac{\eta_3}{\eta_2^2}=\frac{(1+\chi_0)\,v^2_{\text{пр}}}{\eta'^2(v^2_{\text{пр}})^2}=\frac{1+\chi_0}{v^2_{\text{пр}}\eta_\kappa'^2},\quad \text{где}\ \eta_\kappa'=\frac{\eta_\kappa}{v_\kappa};$$

$$\gamma=\frac{(1+\chi_0)}{\eta_\kappa}\left(\frac{v_\kappa^2}{v^2_{\text{пр}}}\right)=\frac{(1+\chi_0)}{\eta_\kappa}\,\frac{B'\theta}{2};$$

$$v_1=-\frac{\eta_2}{2}(1+b);\quad v_2=-\frac{\eta_2}{2}(1-b)=\frac{\eta_2}{2}(b-1);$$

$$\eta_\kappa=\frac{\zeta' A s_{\text{кн}} l_\kappa}{\omega};$$

$$v_2-v_1=\eta_2 b;\quad \frac{v}{\xi'(v)}=\frac{A_1}{v-v_1}+\frac{A_2}{v-v_2};$$

$$A_1=\frac{b+1}{2b};\quad A_2=\frac{b-1}{2b};$$

$$\int\limits_{v_\kappa}^{v}\frac{v\,dv}{\xi'(v)}=\frac{b+1}{2b}\int\limits_{v_\kappa}^{v}\frac{dv}{v-v_1}+\frac{b-1}{2b}\int\limits_{v_\kappa}^{v}\frac{dv}{v-v_2}=$$

$$=\ln\left(\frac{v-v_1}{v_\kappa-v_1}\right)^{\frac{b+1}{2b}}\left(\frac{v-v_2}{v_\kappa-v_2}\right)^{\frac{b-1}{2b}}=\ln\frac{Z_v'}{Z_{v_\kappa}'}.\qquad (12.30)$$

By substituting the expressions (12.29) and (12.30) into (12.28), we obtain

$$\left(\frac{l_1'+l}{l_1'+l_\kappa}\right)^{-\frac{s'}{s}\frac{\theta}{2}}=\left(\frac{l_1'+l_\kappa}{l_1'+l}\right)^{\frac{s'}{s}\frac{\theta}{2}}=\left(\frac{v-v_1}{v_f-v_1}\right)^{\frac{b+1}{2b}}\left(\frac{v-v_2}{v_f-v_2}\right)^{\frac{b-1}{2b}}=\frac{Z_v'}{Z_{v_f}'}$$

or we finally have

$$\left(\frac{l_1'+l_\kappa}{l_1'+l}\right)^{\frac{s'}{s}\theta}=\left(\frac{v-v_1}{v_f-v_1}\right)^{\frac{b+1}{b}}\left(\frac{v-v_2}{v_f-v_2}\right)^{\frac{b-1}{b}},$$

$$l=l_1'\left[\left(1+\frac{l_\kappa}{l_1'}\right)\left(\frac{Z_v'}{Z_{v_f}'}\right)^{-\frac{s}{s'}\frac{2}{\theta}}-1\right]. \qquad (12.31)$$

Using this equation, given the values $v > v_f$, we first find the value of the left hand side and then the corresponding value of the trajectory of the projectile l; by constructing a graph, we find the value v_b corresponding to the value l_b by interpolating or graphically, and as a check control we once again at the value $v = v_b$.

Pressure is calculated by the formula

$$p=\frac{f\omega}{s}\frac{\left(1+\chi_0-\frac{\eta_\kappa}{v_f}v-\frac{v^2}{v_{f}^2}\right)}{l_1'+l}. \qquad (12.32)$$

Example. The calculation of ballistice elements for an 82 mm mortar

Input Data

$W_0=0{,}720$; $l_b=10{,}20$, $s=0{,}5277$; $s_{gap}=0{,}0082$; $f=1120\cdot10^3$;
$q=3{,}4$; $l_0=1{,}363$; $\omega_0=0{,}0072$; $\omega=0{,}0366$; $\alpha=0{,}85$;
$f_0=679\cdot10^3$; $\Lambda_b=7{,}48$; $\varkappa\lambda=-0{,}255$; $l_\kappa=55$; $A\xi_1'=0{,}004$.
$\theta=0{,}15$; $c_q=6{,}16$; $A=0{,}006$; $\xi_1'=0{,}666$;
$\varkappa=1{,}255$; $\Delta=0{,}0608$; $\varphi=1$;
$\delta=1{,}64$;

ξ_1' is the coefficient for the shape of the vents; its value 0.666 is from data from experiments in a special device.

Computation of Constants

$\chi_0=0{,}1192$; $\eta_\kappa=0{,}04923$; $B'=0{,}5923$;
$B_1'=0{,}2994$; $\frac{B'}{B_1'}=1{,}979$; $\gamma=0{,}02452$; $\beta_\kappa=0{,}2483$.

Computational Results

Elements of firing: $l_m = l_K$ = 0.700 dm; p_D =48 kg/cm^2; $p_f = p_{max}$=398 kg/cm^2; v_D =205.5 m/sec.

These results are similar to experimental data p_{max}=380 to 390, v_D=202 to 205.

With the same constants and θ=0.20, $p_f = p_{max}$=392; =201.0.

At θ=0.18 a better correlation between calculated and experimental results would have been obtained.

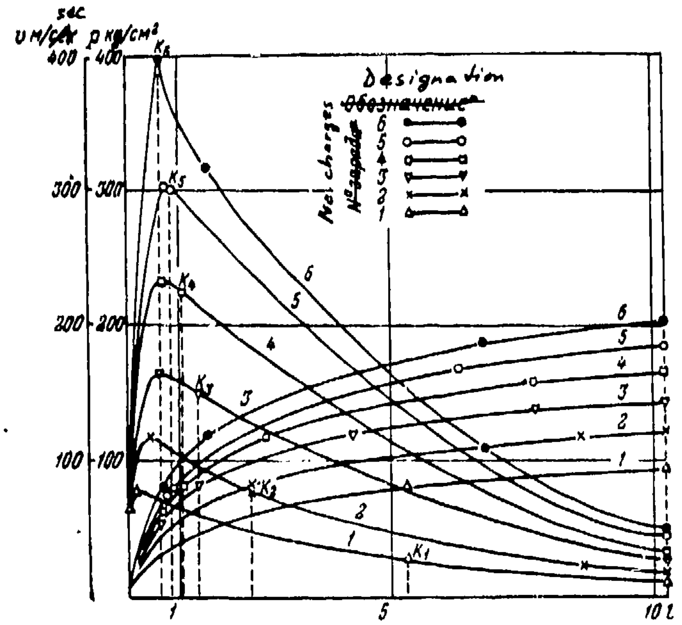


Fig. 12.9. Curves p, l and v, l calculated for a mortar with different charges

The results presented indicate that the analytical formulas derived above enable with good accuracy the calculation of ballistic elements of firing from mortar ($p_f, p_{max}, l_f, l_m, v_f, v_m, p_D, v_D$) and the construction of curves for the pressure of propellant gases and the velocity of the shell as a function of its trajectory.

If there is only a primary charge ω_0 and no booster charges ω, the problem is solved as in cases of instant burning of the charge taking into account heat transfer due to the decrease in the force of the powder f to f_0, determined by using a special installation or by theoretical computations.

Fig. 12.9 presents curves p, l and v, l calculated for a mortar with different charges. With the decrease in the charge, the cessation of burning of the powder is shifted to the muzzle face and p_{max} and v_0 decrease.

Concepts on Ballistic Design of Mortars

The purpose of the ballistic design of mortar is the calculation of the dimensions of the barrel bore and the conditions of ignition necessary to fire a projectile of a given caliber and weight at a specific speed in the muzzle face. It is necessary to find the volume of the chamber W_0, the length of the trajectory in the bore ℓ_∂, weight of the charge ω, the thickness of powder $2e_1$, and the value of the maximum pressure p_{max}. In addition, additional conditions can be created, for instance the value of the pressure p_{max} and others.

The problem of mortar desing, as is the case for grooved bores, is indefinite and allows many solutions.

Prof. G.V. Oppokov recommends* the following procedure for the ballistic design of mortars.

Initially, the values of p_{max} and Δ are calculated based on the values of the power factor $C_e = \frac{q v_0^2}{2 g d^3} = c_q \frac{v_0^2}{2g}$

$$p_{max} = 30 C_e; \quad \Delta = 0{,}0045 C_e \text{ or } \Delta = 0{,}00015 p_{max} \text{ kg/cm}^2,$$

where C_e is given in m.m/dm^3.

*G.V. Oppokov. Ballistika gladkostvol'nykh sistem [Ballistics of Smoothbore Systems] Izd. Artakademii im. Dzerzhinskogo, 1943

The ballistic calculations for the bore are derived for the greatest charge. The value η_ω for full charges is small, since the pressure curve for a full charge is peaked and the burning ends near p_{max}; $\psi \approx 1$. For a 120 mm mortar η_ω=132 t·m/kg, for a 107 mm 130 t·m/kg, and for a 82 mm 166 t·m/kg.

In most cases in mortar design the volume of the chamber W_0 has an already previously assigned volume, since it is determined by the dimensions and shape of the tail end of the projectile during its lowering to the bottom, when the capsule touches the firing pin

After collection of data on existing shapes and their analysis further calculations are made in the following sequence:

1) $E_D = \frac{mv_D^2}{2}$;

2) $C_e = \frac{E_D}{d^3} = q\frac{v_D^2}{2g}$;

3) $p_{max} = 30C_e$ кг/см^2;

4) $\Delta = 0{,}0045C_e$ кг/дм^3;

Then

5) W_0 is known.

6) $\omega = \Delta W_0$;

7) $\eta_\omega = \frac{2v_D^2}{2g\omega} = \frac{E_D}{\omega}$;

8) $v_{pr} = \sqrt{\frac{2gf}{\varphi\theta}\frac{\omega}{q}}$;

9) $p_{pr} = \frac{f\Delta}{1-\frac{\Delta}{\delta}}$*.

Subsequently, being given the value p_{max}/p_{pr} one uses special tables** and finds the values B, $\Lambda_\delta = \frac{\ell_\delta}{\ell_0}$, $\Lambda_v = \frac{v_{m\,Tab}}{v_{f\,Tab}}$, $\eta = \frac{p_{av}}{p_{max}}$ and other ballistic elements. Having obtained in the results of the calculations of the first variant at several values of p_{max} and Δ, specific values ℓ_δ and other characteristics, one must in additional calculations use 2-3 more values of Δ, near Δ of the first variant, and again calculate all characteristics of

*Pressure $p_{pr} = \frac{f\Delta}{1-\Delta/\delta}$ is analogous to pressure $p_1 = \frac{f\Delta}{1-\alpha\Delta}$, but with respect to the initial free space $W_0(1-\Delta/\delta)$ and not to the volume at the end of burning $W_0(1-\alpha\Delta)$.

**G.V. Oppokov. Ballistika gladkostvol'nykh sistem /Ballistics of smoothbore Systems/, Izd. Artakademii im. Dzerzhinskogo, 1943 of the mortar and the conditions of ignition; after comparing the obtained data one should find that density of charge at which the entire trajectory will be the shortest (without an great decrease in the coefficient η_{ω}). After this, p_{max} is converted into $\pm$ 30-50 kg/cm^2; the characteristics of another series of variants are calculated; and the largest given value for p_{max} is determined. The curves p, l and v, l are calculated for the final variant selected. The thickness of the powder $2e_1$ is determined only for the final variant selected:

$$2e_1 = \frac{2u_1}{s} \sqrt{Bf\omega\varphi m}.$$

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION		MICROFICHE
A205	DMATC	1
A210	DMAAC	2
B344	DIA/RDS-3C	9
C043	USAMIIA	1
C509	BALLISTIC RES LABS	1
C510	AIR MOBILITY R&D LAB/FIO	1
C513	PICATINNY ARSENAL	1
C535	AVIATION SYS COMD	1
C591	FSTC	5
C619	MIA REDSTONE	1
D008	NISC	1
H300	USAICE (USAREUR)	1
P005	ERDA	1
P005	CIA/CRB/ADB/SD	1
NAVORDSTA (50L)		1
NASA/KSI		1
AFIT/LD		1

ORGANIZATION		MICROFICHE
E053	AF/INAKA	1
E017	AF/RDXTR-W	1
E403	AFSC/INA	1
E404	AEDC	1
E408	AFWL	1
E410	ADTC	1
E413	ESD	2
	FTD	
	CCN	1
	ASD/FTD/NICD	3
	NIA/PHS	1
	NICD	2