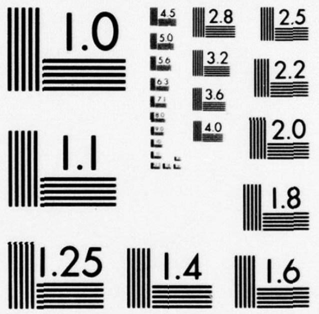
AD-A065 633   AIR FORCE FLIGHT DYNAMICS LAB WRIGHT-PATTERSON AFB OHIO   F/G 13/13
'ANALYZE' - ANALYSIS OF AEROSPACE STRUCTURES WITH MEMBRANE ELEM--ETC(U)
DEC 78 V B VENKAYYA, V A TISCHLER

UNCLASSIFIED   AFFDL-TR-78-170   NL

1 OF 2
AD
A065633


1.0
4.5
5.0
5.6
6.3
7.1
8.0
9.0
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

2

AFFDL-TR-78-170

LEVEL II

(2)

"ANALYZE" — ANALYSIS OF AEROSPACE STRUCTURES WITH MEMBRANE ELEMENTS

VIPPERLA B. VENKAYYA
VICTORIA A. TISCHLER
ANALYSIS AND OPTIMIZATION BRANCH
STRUCTURAL MECHANICS DIVISION

D D C
MAR 13 1979
C

DECEMBER 1978

TECHNICAL REPORT AFFDL-TR-78-170
Final Report for May – August 1978

Approved for public release; distribution unlimited.

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (IO) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

VIPPERLA B. VENKAYYA
Project Engineer
Design & Analysis Methods Gp.
Analysis & Optimization Branch

Charles A. Bair
CHARLES A. BAIR, Jr., Major, USAF
Chief, Analysis & Optimization Br.
Structural Mechanics Division

FOR THE COMMANDER

RALPH L. KUSTER, Jr., Colonel, USAF
Chief, Structural Mechanics Division

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

AIR FORCE/56780/23 February 1979 — 350

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 AFFDL-TR-78-170 /	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE [redacted] 6 'ANALYZE' - Analysis of Aerospace Structures with Membrane Elements.		5. TYPE OF REPORT & PERIOD COVERED Final Report. May - Aug[redacted]978
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Vipperla B./Venkayya Victoria A./Tischler		8. CONTRACT OR GRANT NUMBER(s) 16 2401
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Flight Dynamics Laboratory Analysis & Optimization Branch (FBR) Wright-Patterson Air Force Base, OH		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Proj. No. 2401 Task 240102 Work Unit 240102-08 17 02
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base, OH		12. REPORT DATE 11 Dec[redacted] 1978
		13. NUMBER OF PAGES 117 12 118p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Structural Analysis Finite Element Analysis Membrane Elements Aerospace Structures		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report contains documentation for the program 'ANALYZE'. The program library consists of a bar, a membrane triangle, a quadrilateral, and a shear panel. The equations of finite element analysis, element formulations, program organization, and subroutine descriptions provide a comprehensive theoretical background for the program. The input and output instructions together with the sample problem and the results should provide adequate information for the use of this program. (See reverse side)		

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

JOB

012 070 79 03 12 061

Block 20 (cont'd)

"ANALYZE" is an in-house program and can be used on INTERCOM for problems up to 150 to 200 degrees of freedom and a comparable number of elements. This program is extremely useful in training engineers in the use of finite element programs, in the development of finite element models of large aerospace structures and in research in structural analysis and optimization.

1473

FOREWORD

This report is prepared as part of an in-house effort under Project 2401, Task No. 240102, "Design and Analysis Methods for Aerospace Vehicle Structures," and Work Unit 24010208, "Automated Design of Advanced Aerospace Structures." The work was carried out in the Design and Analysis Methods Group of the Analysis & Optimization Branch (FBR), Structural Mechanics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.

The time period of the effort for this Technical Report is May 1978 - August 1978. The manuscript was originally released as AFFDL-FBR-TM-78-89 in August 1978.

TABLE OF CONTENTS

SECTION	TITLE	PAGE
1	INTRODUCTION	1
2	ANALYSIS	6
3	FINITE ELEMENTS	15
4	ORGANIZATION OF THE PROGRAM	27
5	DESCRIPTION OF THE SUBROUTINES	34
6	INPUT INSTRUCTIONS	47
7	OUTPUT DESCRIPTION	51
8	SAMPLE PROBLEM	55
	REFERENCES	58
APPENDIX A:	ESTIMATION OF CORE REQUIREMENTS	59
APPENDIX B:	LISTING OF THE PROGRAM	63
APPENDIX C:	LISTING OF THE SAMPLE DATA	91
APPENDIX D:	RESULTS OF THE SAMPLE PROBLEM ANALYSIS	95

LIST OF ILLUSTRATIONS

FIGURE		PAGE
1	CONTINUUM AND FINITE ELEMENT MODEL	2
2	ELEMENTS AND LOCAL COORDINATE SYSTEM	3
3	EXAMPLES OF UNIT DISPLACEMENT MODES	10
4	QUADRILATERAL OR SHEAR PANEL DIVIDED INTO FOUR TRIANGLES	10
5	FLOW CHART FOR PROGRAM ANALYZE	28
6	PARTITIONED ELEMENT STIFFNESS MATRIX AND ADDRESSES IN THE TOTAL STIFFNESS MATRIX	35
7	DISTRIBUTION OF NON-ZERO ELEMENTS IN THE STIFFNESS MATRIX	38
8	AERODYNAMIC PLANFORM AND PRIMARY STRUCTURAL ARRANGEMENT OF WING	56
9	FINITE ELEMENT REPRESENTATION OF A WING BOX	57

LIST OF TABLES

TABLE		
1	PROGRAM DESCRIPTION	46
2	RESULTS FROM ANALYZE AND NASTRAN	111

1. INTRODUCTION

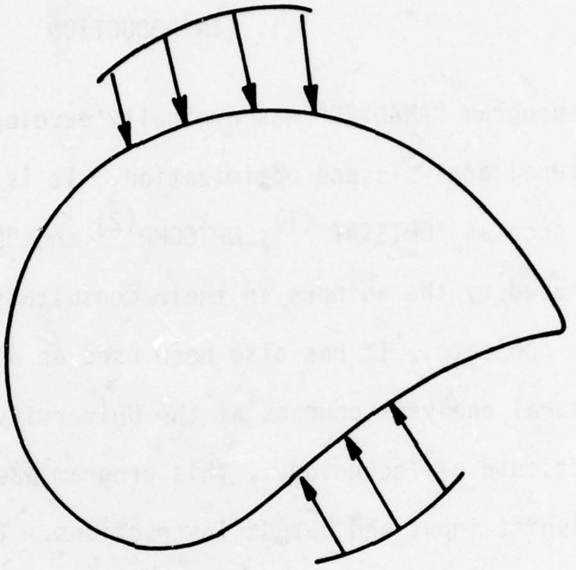
The program "ANALYZE" was orginally developed for in-house studies in structural analysis and optimization. It is the basis for a number of programs such as "OPTSTAT"(1), OPTCOMP(2) and "DANALYZ"(3). "ANALYZE" has been used by the authors in their consultation work on a number of Air Force projects. It has also been used as a demonstration program in structural analysis courses at the University of Dayton and the Air Force Institute of Technology. This program was distributed earlier with makeshift input and output instructions. These instructions did not include details of the theory nor the internal organization of the program. The purpose of this report is to generate comprehensive documentation for the "ANALYZE" program.

The program is based on the displacement method of finite element analysis(4-6). In such an analysis the continuum is replaced by a discrete model consisting of a finite number of nodes connected by elements (See Figure 1). This discretization reduces the original differential equations of the continuum to a set of algebraic equations which can be solved much more readily on digital computers.

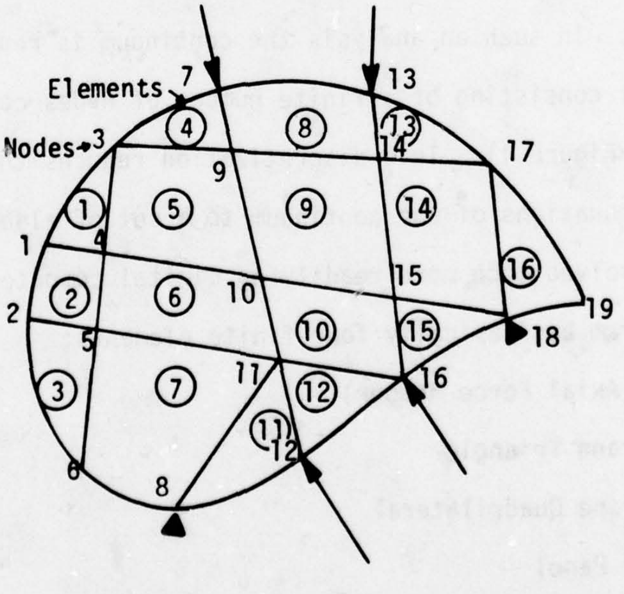
The program has basically four finite elements:

1. Bar (Axial Force Member)
2. Membrane Triangle
3. Membrane Quadrilateral
4. Shear Panel

The four elements and their local coordinate systems are shown in Figure 2. The bar is a constant strain line element and is equivalent to a rod

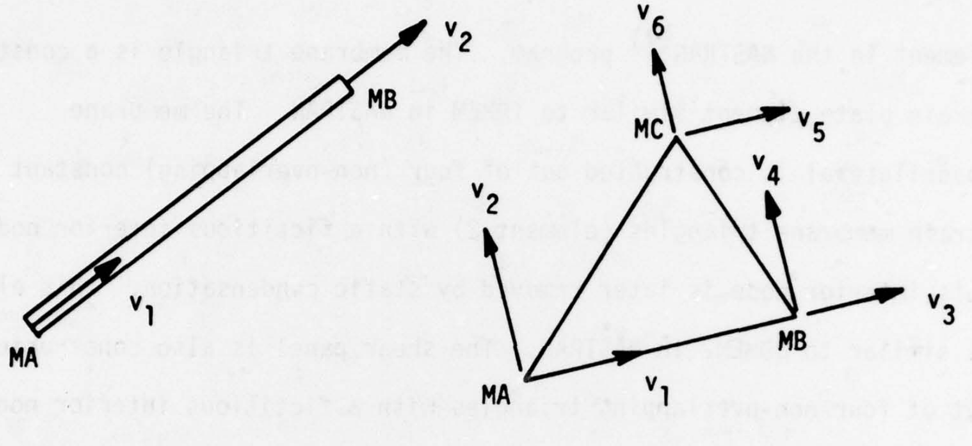


(a) Continuum



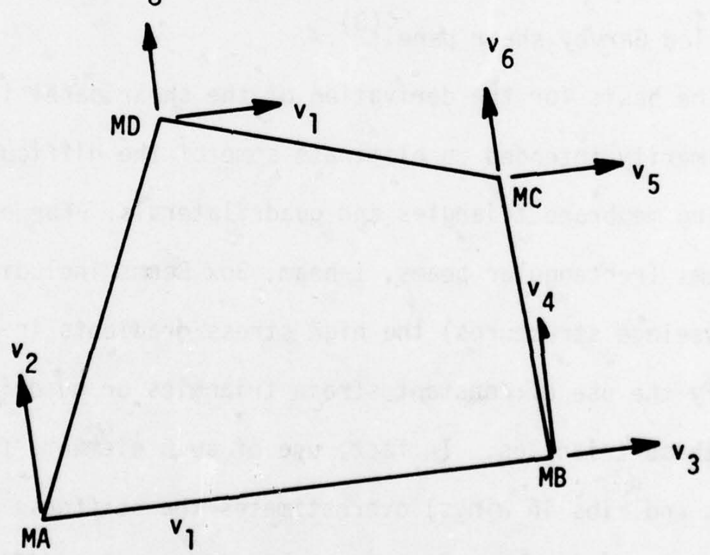
(b) Finite Element Model

FIG. 1: Continuum and Finite Element Model



(a) Bar Element

(b) Triangular Membrane Element



(c) Quadrilateral or Shear Panel

FIG. 2: Elements and Local Coordinate System

element in the NASTRAN(7) program. The membrane triangle is a constant strain plate element similar to TRMEM in NASTRAN. The membrane quadrilateral is constructed out of four (non-overlapping) constant strain membrane triangles (element 2) with a fictitious interior node. This interior node is later removed by static condensation. This element is similar to QDMEM2 in NASTRAN. The shear panel is also constructed out of four non-overlapping triangles with a fictitious interior node. However, only the shear energy is considered in determining the stiffness of this element. Although the formulation is somewhat different, this element gives comparable results to the NASTRAN SHEAR element or the so called Garvey shear panel(8).

The basis for the derivation of the shear panel is empirical, and it is primarily intended to eliminate some of the difficulties encountered in using membrane triangles and quadrilaterals. For example, in beam problems (rectangular beams, I-beam, Box Beams including multicell wings and fuselage structures) the high stress gradients in the webs do not justify the use of constant strain triangles or quadrilaterals derived from these triangles. In fact, use of such elements for the webs (spars and ribs in wings) overestimates the stiffness by an order of magnitude. Aerospace engineers have offset this difficulty to a large extent by judicious use of membrane elements in conjunction with the shear panels. In fact the early finite element models of wings and fuselages consisted primarily of bars and shear panels. However, the present practice of using membrane triangles and quadrilaterals for the top and bottom skins, bars for the posts, spar and rib caps, and shear panels for the spars and ribs eliminates to a large extent the need for determining the equivalent thicknesses and cross-sectional areas in the bars and shear panels model. The models consisting of these elements are most satisfactory for determining the primary load paths in built-up structures such as wings and fuselages. In addition the simplicity of these elements makes interpretation of the results easy and also keeps the analysis costs low because the stiffness matrices of these elements can be generated in a fraction of a second. The detailed formulation and additional information on these elements are given in Section 3.

In finite element analysis, a large proportion of the time is spent in the solution of the force displacement relations. The program uses standard Gaussian elimination with modifications to take into account the symmetry and sparseness characteristics of the stiffness matrix. The details of the solution scheme and storage of the stiffness matrix are given in Sections 2 and 5. "ANALYZE" is an incore program whose core requirements depend on the problem size, primarily measured in terms of the number of degrees of freedom and the size of the semi-bandwidth. However, the bandwidth per se is not considered in the program. With an available core of about $100K_8$ one can solve problems of up to 300 to 400 degrees of freedom. With the full core of a machine like the CDC 6600, it is possible to solve problems of up to 1500 degrees of freedom and a comparable number of elements. The details of core requirements are discussed in Appendix A.

The program is written in standard ANSI Fortran IV.

2. ANALYSIS

In the finite element analysis the continuum is replaced by a discrete model consisting of a finite number of nodes connected by elements (members). The rationale in such an approximation is that the response between the nodes (i.e., in the elements) can be expressed as a function of the response at the nodes. The functional relationship between the two responses is approximated by various interpolation functions or shape functions. The type of functions depends on the complexity of the problem at hand. This discretization reduces the original differential equations of the continuum to a set of algebraic equations which can be solved much more readily on digital computers.

The equations of the finite element analysis can be derived conveniently by considering the strain energy of the deformed system. For example, if the elastic body is idealized by m finite elements connecting q nodes (See Figure 1), the strain energy of the i^{th} element can be written as

$$\tau_i = \frac{1}{2}\int_{V_i} \underset{\sim}{\sigma}_i^{t*} \; \underset{\sim}{\varepsilon}_i \, dV \tag{1}$$

where $\underset{\sim}{\sigma}_i$ and $\underset{\sim}{\varepsilon}_i$ are the stress and strain vectors and V_i is the volume of the element. For a linearly elastic body the relation between stress and strain can be written as

$$\underset{\sim}{\sigma}_i = \underset{\sim}{E}_i \; \underset{\sim}{\varepsilon}_i \tag{2}$$

---

* Superscript t on a matrix represents transpose

where $\underset{\sim}{E}_i$ is the symmetric matrix of material elastic constants. For typical plane stress problems the elastic constants matrix is of dimension 3x3. For an isotropic material in plane stress problems the elements of $\underset{\sim}{E}$ are as follows:

$$\underset{\sim}{E} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\mu) \end{bmatrix} \tag{3}$$

where E and μ are the elastic modulus and poisson's ratio of the material respectively. For an orthotropic material the elastic constants matrix is given by

$$\underset{\sim}{E} = \frac{E_1}{1-\beta\mu^2} \begin{bmatrix} 1 & \mu\beta & 0 \\ \mu\beta & \beta & 0 \\ 0 & 0 & \frac{G}{E_1}(1-\beta\mu^2) \end{bmatrix} \tag{4}$$

where E_1 and E_2 are the longitudinal and transverse moduli, respectively, in the directions of the material property axes. β is the ratio of transverse to longitudinal modulus (E_2/E_1). G and μ are the shear modulus and poisson's ratio respectively.

The essence of the finite element approximation is that the internal displacements of the elements are expressed as functions of the displacements of the discrete nodes to which they are connected. The local coordinate systems and the nodal degrees of freedom of the four elements are shown in Figure 2. The functional relationship between the element internal displacements and the discrete nodal displacements is given by

$$\underset{\sim}{w}_i = \underset{\sim}{\phi}_i \, \underset{\sim}{v}_i \tag{5}$$

where the matrix $\underset{\sim}{w}_i$ represents the displacements in the element which are functions of the spatial coordinates (x, y). The shape function $\underset{\sim}{\phi}_i$ is a rectangular matrix, and its elements are also functions of the spatial coordinates. The vector $\underset{\sim}{v}_i$ represents the nodal displacements in the direction of the element degrees of freedom in the local coordinate system (Figure 2). Now the strain-displacement relations can be written as

$$\underset{\sim}{\varepsilon}_i = \underset{\sim}{B}\,\underset{\sim}{w}_i \tag{6}$$

where $\underset{\sim}{B}$ is a differential operator. For a plane stress problem $\underset{\sim}{B}$ is given by

$$\underset{\sim}{B} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{7}$$

Substitution of Equations 2, 5 and 6 in 1 gives the expression for strain energy in the following form

$$\tau_i = \frac{1}{2}\,\underset{\sim}{v}_i^t\,\underset{\sim}{k}_i\,\underset{\sim}{v}_i \tag{8}$$

where $\underset{\sim}{k}_i$ is the element (member) stiffness matrix with respect to the discrete coordinates $\underset{\sim}{v}$ and is given by

$$\underset{\sim}{k}_i = \int_{V_i} \underset{\sim}{\phi}_i^t\,\underset{\sim}{B}^t\,\underset{\sim}{E}_i\,\underset{\sim}{B}\,\underset{\sim}{\phi}_i\,dV \tag{9}$$

An alternate but a convenient method of determining the elements of the member stiffness matrix is by invoking the principle of virtual work$^{(9)}$

which gives

$$1 \cdot k_{pq} = \int_{V_i} \underset{\sim}{\sigma}_i^{(p)^t} \underset{\sim}{\varepsilon}_i^{(q)} \, dV \qquad (10)$$

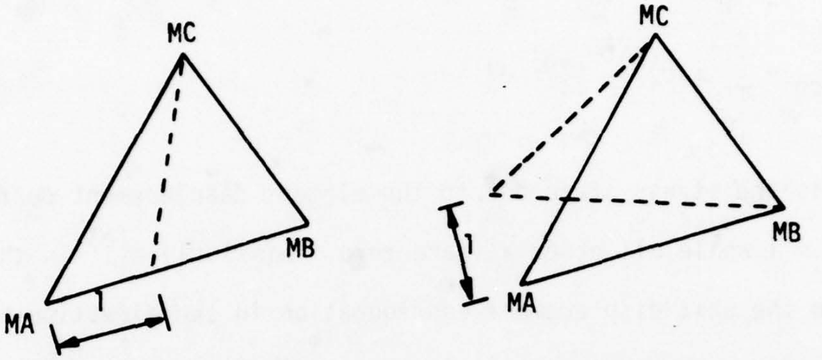
where $\underset{\sim}{\sigma}_i^{(p)}$ is the stress state due to the element displacement configuration in which $v_p = 1$ while all other v's are zero. Similarly $\underset{\sim}{\varepsilon}_i^{(q)}$ is the strain state due to the unit displacement configuration in the direction of the q^{th} degree of freedom. These two conditions are shown in Figure 3 for the degrees of freedom 1 and 2 of the membrane triangle. It should be noted that besides assuming appropriate shape functions, the integration in Equations 9 or 10 is one of the difficult tasks in the case of complex elements in finite element analysis. However, for membrane elements this integration does not present any difficulties as will be seen in the next section. For more complex elements the usual practice is to adopt numerical integration schemes.(10,11)

From Equation 8 and Castigliano's first theorem, the relation between the element nodal forces and the displacements may be written as

$$\underset{\sim}{s}_i = \left[\frac{\partial \tau_i}{\partial v_j} \right] = \underset{\sim}{k}_i \, \underset{\sim}{v}_i \qquad (11)$$

where $\underset{\sim}{s}_i$ is the element nodal force matrix corresponding to the displacement matrix $\underset{\sim}{v}_i$. Similar force-displacement relations for the total structure can be derived from the strain energy of the structure. The total strain energy Γ of the structure can be written as the sum of the energies of the individual components.

$$\Gamma = \sum_{i=1}^{m} \tau_i = \frac{1}{2} \sum_{i=1}^{m} \underset{\sim}{v}_i^t \, \underset{\sim}{k}_i \, \underset{\sim}{v}_i \qquad (12)$$



(a) First Unit Mode (b) Second Unit Mode

FIG. 3: Examples of Unit Displacement Modes

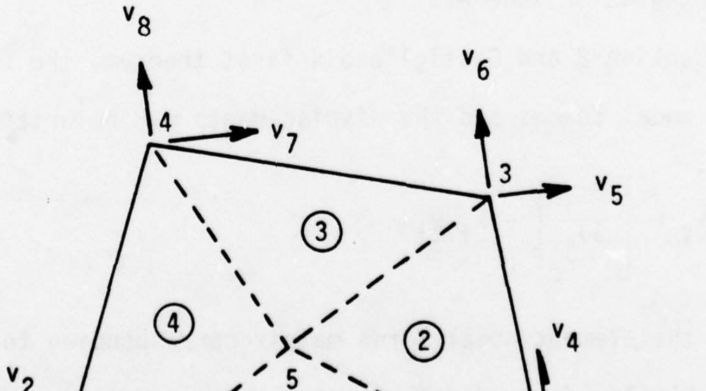


FIG. 4: Quadrilateral or Shear Panel Divided into Four Triangles

In general, for most structures, it is convenient to define a local coordinate system for each element and a global coordinate system for the total structure. In such a case the element and structure generalized coordinates can be related by

$$\underset{\sim}{v}_i = \underset{\sim}{a}_i \, \underset{\sim}{u} \tag{13}$$

where $\underset{\sim}{a}_i$ is the compatibility matrix. Its elements can be determined by kinematic reasoning alone provided the structure is kinematically determinate. The matrix $\underset{\sim}{u}$ is the generalized displacement vector of the structure in the global coordinate system. It is interesting to note that Equation 13 not only transforms element displacements from local to global coordinates but also gives information about how the elements are connected to the structure. From Equation 13 and the principle of virtual work it is easy to show that the transformation between the forces on the structure and the element internal forces is given by

$$\underset{\sim}{P} = \underset{\sim}{a}_i^t \, \underset{\sim}{s}_i \tag{14}$$

where $\underset{\sim}{P}$ is the force vector on the structure in the global coordinate system. The transformation given in Equation 14 is sometimes referred to as a contragradient transformation.(12)

Substitution of Equation 13 in 12 gives the expression for the total strain energy in the form

$$\Gamma = \frac{1}{2} \underset{\sim}{u}^t \, \underset{\sim}{K} \, \underset{\sim}{u} \tag{15}$$

where $\underset{\sim}{K}$, the total stiffness matrix of the structure, is written as the sum of the component stiffness matrices.

$$\underset{\sim}{K} = \sum_{i=1}^{m} \underset{\sim}{a}_i^t \, \underset{\sim}{k}_i \, \underset{\sim}{a}_i \tag{16}$$

Again using Castigliano's first theorem the relation between the generalized force matrix $\underset{\sim}{P}$ corresponding to the displacement matrix $\underset{\sim}{u}$ may be written as

$$\underset{\sim}{P} = \left[\frac{\partial \Gamma}{\partial u_j}\right] = \underset{\sim}{K}\,\underset{\sim}{u} \tag{17}$$

In most structural analysis problems the stiffness matrix $\underset{\sim}{K}$ is sparsely populated. It is essential to take advantage of this fact in solving the load deflection equations (Equation 17), particularly in the case of problems with a large number of degrees of freedom where the cost of computation can be prohibitive otherwise. The "ANALYZE" program uses Gaussian elimination with modifications to take into account the symmetry and sparseness of the stiffness matrix.

Basically Gaussian elimination involves decomposition of the stiffness matrix by

$$\underset{\sim}{K} = \underset{\sim}{L}\,\underset{\sim}{D}\,\underset{\sim}{L}^t \tag{18}$$

where $\underset{\sim}{L}$ is the unit lower triangular matrix and $\underset{\sim}{D}$ is a diagonal matrix. The advantage of this decomposition scheme is that the $\underset{\sim}{L}$ matrix retains some of the sparseness characteristics of $\underset{\sim}{K}$ which consequently reduces the numbers of computations. Also $\underset{\sim}{L}$ and $\underset{\sim}{D}$ can be assigned the same storage as $\underset{\sim}{K}$.

The next step is the forward substitution by

$$\underset{\sim}{L}\,\underset{\sim}{Y} = \underset{\sim}{P} \tag{19}$$

where the matrix $\underset{\sim}{Y}$ is given by

$$\underset{\sim}{Y} = \underset{\sim}{D}\,\underset{\sim}{L}^t\,\underset{\sim}{u} \tag{20}$$

In Equation 19 the solution of $\underset{\sim}{Y}$ can be accomplished by simple forward substitution. Once $\underset{\sim}{Y}$ is obtained, $\underset{\sim}{u}$ can be solved by back substitution using Equation 20. The last two steps together are generally referred to as Forward-Back Substitution (FBS). Solution of Equation 17 for multiple load vectors involves the decomposition of the stiffness matrix once and repetition of FBS as many times as there are load vectors.

With the help of these basic equations the steps in the finite element analysis can be outlined as follows:

1. Input information consists of
    a. Geometry of the structure
        Node Coordinates
        Element Connections
        Section Properties
    b. Material properties
    c. Boundary conditions
    d. Loading
    e. Clues for appropriate (desired) output.
2. Element information consists of
    a. Determination of the local coordinate system for each element.
    b. Selection of the appropriate shape functions (Equation 5).
    c. Determination of the element stiffness matrix (Equation 9 or 10).
3. Transformation of the element stiffness matrix to the global coordinate system (Equation 16 without summation).
4. Determination of the structure stiffness matrix by summation of the component stiffnesses (Summation in Equation 16).
5. Incorporation of the boundary conditions.
6. Solution of the load-deflection equations (Equations 17, 18, 19, and 20).

7. Determination of the element displacements in their local coordinate system (Equation 13).

8. Determination of the stresses in each element (Equations 6, 5, and 2).

9. Output the structure displacements, element stresses and other information such as element strain energies, etc.

The next section consists of the details of the stiffness matrix formulations for the four elements in this program.

3. FINITE ELEMENTS

The program "ANALYZE" has four elements as mentioned earlier. They are all membrane elements. These four elements are generally adequate for determining the primary load paths of most aircraft structures. However, for a detailed stress analysis of local areas, higher order elements may be necessary.

BAR (ROD) ELEMENT

Basically this element is an axial force member. Its primary use is in two and three dimensional truss structures. It is also used extensively as spar and rib caps, posts around shear panels, stiffners and other line elements in aircraft structures. The local coordinate system of this element is shown in Figure 2. The positive x-axis is directed along the line joining the two ends. v_1 and v_2 represent the element end displacements. The corresponding two end forces are s_1 and s_2. The displacement field in the element is assumed to be linear, which gives constant strain. For a linearly elastic material this assumption yields constant stress as well.

If w, the displacement at any point along the length of the bar, is given by

$$w = ax + b \tag{17}$$

where a and b are two undetermined coefficients and x is the coordinate of the point in the local coordinate system, then the end displacements v_1 and v_2 are given by

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{18}$$

where x_1 and x_2 are the coordinates of the two ends in the local coordinate system. Then the shape function (Equation 5) corresponding to this linear displacement field can be written as

$$\underset{\sim}{\phi} = \frac{1}{(x_1 - x_2)} \left[(x - x_2), \; -(x - x_1) \right] \tag{19}$$

From the strain-displacement relations, the axial strain in the element is given by

$$\varepsilon_x = \frac{\partial w}{\partial x} = a \tag{20}$$

From the principle of virtual work (Equation 10) the individual elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \sigma_x^{(i)} \, \varepsilon_x^{(j)} \, dV = (-1)^{i+j} \frac{AE}{L} \tag{21}$$

where A is the cross-sectional area, L is the length of the member, and E is the modulus of elasticity of the material. The member stiffness matrix is given by

$$\underset{\sim}{k} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{22}$$

The member force matrix is given by

$$\underset{\sim}{s} = \underset{\sim}{k} \, \underset{\sim}{v} \tag{23}$$

The stress in the member is given by

$$\sigma_x = E \, \varepsilon_x \tag{24}$$

or

$$\sigma = \frac{s_1}{A} = \frac{-s_2}{A} \tag{25}$$

The strain energy in the element is given by

$$\tau_i = \frac{1}{2} \underset{\sim}{s}^t \underset{\sim}{v} \qquad (26)$$

or

$$\tau_i = \frac{1}{2} \sigma_x \varepsilon_x A L \qquad (27)$$

TRIANGULAR MEMBRANE ELEMENT

The membrane triangle is the basic plate element in the program. It is used to construct the membrane quadrilateral as well as the shear panel with some modifications. The membrane triangle can be used effectively in all cases where the primary loading is inplane forces. These include top and bottom skins of aircraft wings, flanges of I and box beams when they are subjected to constant normal stresses (tension or compression) only and skins of sandwich construction. However, they are not suitable for situations where high stress gradients exist. For example, they are unsuitable for spars and ribs of wings and other lifting surfaces, webs of I and box beams and flat plates where the primary load is bending. If used in such cases, they overestimate the stiffness or generate singularity. Figure 2 shows the triangle elements with the local coordinate system. The generalized coordinates v_1, v_2, ---, v_6 represent the inplane displacements of the three nodes in the local coordinate system. The displacement field in the element is assumed to be linear. This gives constant strain in the element. For a linearly elastic material the stress in the element will also be constant.

The linear displacement field in the element can be represented by

$$\begin{aligned} w_x &= a_1\, x + b_1\, y + c_1 \\ w_y &= a_2\, x + b_2\, y + c_2 \end{aligned} \qquad (28)$$

where w_x and w_y are the x-y displacements in the plane of the plate in the local coordinate system. a_1, b_1 etc. are the six undetermined coefficients. Equation 28 can be written in matrix form as follows:

$$\underset{\sim}{w} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix} \qquad (29)$$

The six unknown coefficients can be uniquely determined by the six boundary conditions at the nodes.

$$\left[\begin{array}{c} v_1 \\ v_3 \\ v_5 \\ \hline v_2 \\ v_4 \\ v_6 \end{array}\right] = \left[\begin{array}{ccc|ccc} x_1 & y_1 & 1 & 0 & 0 & 0 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & x_1 & y_1 & 1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{array}\right] \left[\begin{array}{c} a_1 \\ b_1 \\ c_1 \\ \hline a_2 \\ b_2 \\ c_2 \end{array}\right] \qquad (30)$$

where x_1, y_1, ----, x_3 and y_3 are the coordinates of the three nodes of the triangle in the local coordinate system. It should be noted that the nodal displacements are grouped into x and y directions, so that the nodal coordinate matrix on the right hand side partitions into a diagonal matrix. The inversion of the partitioned diagonal matrix involves simply the inversion of the component matrix. Now the shape matrix $\underset{\sim}{\phi}$ is given by

$$\underset{\sim}{\phi} = \underset{\sim}{x}\, \underset{\sim}{Z}^{-1} \tag{31}$$

where the matrix $\underset{\sim}{x}$ is given by

$$\underset{\sim}{x} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \tag{32}$$

and the $\underset{\sim}{Z}$ matrix is given by

$$\underset{\sim}{Z} = \left[\begin{array}{c|c} \underset{\sim}{X} & 0 \\ \hline 0 & \underset{\sim}{X} \end{array}\right] \tag{33}$$

The coordinate matrix $\underset{\sim}{X}$ is given by

$$\underset{\sim}{X} = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \tag{34}$$

It is interesting to note that each column of $\underset{\sim}{Z}^{-1}$ represents a unit displacement mode: i.e. the j^{th} column of the inverse represents a displacement mode in which $v_j = 1$ while all other nodal displacements are zero (See Figure 3). This fact is used to advantage in determining the elements of the member stiffness matrix.

From linear strain-displacement relations the strains can be written as

$$\varepsilon_x = \frac{\partial w_x}{\partial x} = a_1 \tag{35}$$

$$\varepsilon_y = \frac{\partial w_y}{\partial y} = b_2 \tag{36}$$

$$\varepsilon_{xy} = \frac{\partial w_x}{\partial y} + \frac{\partial w_y}{\partial x} = b_1 + a_2 \tag{37}$$

From the principle of virtual work (Equation 10) the elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \underset{\sim}{\sigma}^{(i)^t} \underset{\sim}{\varepsilon}^{(j)} \, dV = \int_V \underset{\sim}{\varepsilon}^{(i)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(j)} \, dV \tag{38}$$

where $\underset{\sim}{\sigma}^{(i)}$ and $\underset{\sim}{\varepsilon}^{(j)}$ are the stress and strain matrices corresponding to the unit displacement modes explained under Equation 34. Since the linear displacement variation implies constant strain, the integral in Equation 38 can be replaced by the volume of the element:

$$k_{ij} = \frac{1}{2} |\underset{\sim}{X}| \; t \; \underset{\sim}{\varepsilon}^{(i)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(j)} \tag{39}$$

where $|\underset{\sim}{X}|$ is the determinant of the nodal coordinate matrix which represents twice the area of the element and t is the thickness of the element. Now the stiffness matrix of the element is given by

$$\underset{\sim}{k} = \frac{1}{2} |\underset{\sim}{X}| \; t \begin{bmatrix} \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \\ \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \\ \vdots & \vdots & & \vdots \\ \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \end{bmatrix} \tag{40}$$

The member force matrix is given by

$$\underset{\sim}{s} = \underset{\sim}{k} \, \underset{\sim}{v} \tag{41}$$

The stress matrix in the element is given by

$$\underset{\sim}{\sigma} = \underset{\sim}{E} \, \underset{\sim}{\varepsilon} \tag{42}$$

The strain energy in the element is given by

$$\tau_i = \frac{1}{2} \underset{\sim}{S}^t \underset{\sim}{v} \tag{43}$$

or

$$\tau_i = \frac{1}{4} |\underset{\sim}{X}| \; t \; \underset{\sim}{\sigma}^t \underset{\sim}{\varepsilon} \tag{44}$$

The next important step in the evaluation of the stress state in an element is the selection of a suitable failure criteria because of the combined stresses (σ_x, σ_y, and σ_{xy}) in plate elements. The modified energy of distortion or Von-Mises criteria is adopted to determine the effective stress in an element. The effective stress is given by

$$\sigma_{eff} = (\sigma_x^2 + \sigma_y^2 - \sigma_x \; \sigma_y + 3\sigma_{xy}^2)^{1/2} \tag{45}$$

The margin of safety is evaluated by first determining the effective stress ratio (ESR)

$$ESR = \left[\left(\frac{\sigma_x}{XX}\right)^2 + \left(\frac{\sigma_y}{YY}\right)^2 - \left(\frac{\sigma_x \sigma_y}{XXYY}\right) + \left(\frac{\sigma_{xy}}{ZZ}\right)^2 \right]^{1/2} \tag{46}$$

where XX and YY are the tension or compression allowable in the x and y directions, respectively, and ZZ is the shear allowable. Then the margin of safety (MS) is determined by

$$MS = \frac{1\text{-}ESR}{ESR} \tag{47}$$

If the user does not provide the allowable stress values, then default values of 60,000 psi for tension and compression allowables in both directions and 36,000 psi for the shear allowable are used.

QUADRILATERAL MEMBRANE ELEMENT

The quadrilateral element is most frequently used to represent membrane skins unless the corners etc. require the use of the triangular element.

Figure 4 shows the local coordinate system and the generalized coordinates (displacements) v_1 through v_8. The element is assumed to be a flat plate, and all nodes are assumed to lie on a plane connecting the first three nodes (1, 2, and 3). In effect the warping in the element is ignored. This approximation results in an overestimation of the stiffness of a truly warped quadrilateral element. In most cases the effect of the approximation is small, and it can be further reduced by reducing the mesh size of the model in the regions of high warping. However, if the warp is too large, the quadrilateral should be broken up into two or more triangles.

As mentioned earlier, the stiffness of the quadrilateral element is determined by breaking it into four component triangles as shown in Figure 4. A fictitious node in the quadrilateral is located by averaging the coordinates of the four nodes as given by

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4} \tag{48}$$

$$y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \tag{49}$$

The stiffness of the four triangles is then computed by Equation 40 in the local coordinate system shown in Figure 2c. Addition of the four stiffness matrices gives a 10 x 10 stiffness matrix with two degrees of freedom included for the fifth node. This fictitious node is later removed by static condensation before adding to the total structure. The procedure for static condensation is outlined next.

The force displacement relations of the 5 node quadrilateral are written as

$$\underset{\sim}{R}_Q = \underset{\sim}{k}_Q \, \underset{\sim}{r}_Q \qquad (50)$$

where the subscript refers to the quadrilateral element with 5 nodes. Equation 50, partitioned to isolate the degrees of freedom of the fifth node, can be written as

$$\left[\begin{array}{c} \underset{\sim}{R}_I \\ \hline \underset{\sim}{R}_{II} \end{array}\right] = \left[\begin{array}{c:c} \underset{\sim}{K}_{I,\,I} & \underset{\sim}{K}_{I,\,II} \\ \hdashline \underset{\sim}{K}_{II,\,I} & \underset{\sim}{K}_{II,\,II} \end{array}\right] \left[\begin{array}{c} \underset{\sim}{r}_I \\ \hline \underset{\sim}{r}_{II} \end{array}\right] \qquad (51)$$

Equation 51 can be written as two separate equations

$$\underset{\sim}{R}_I = \underset{\sim}{k}_{I,I} \, \underset{\sim}{r}_I + \underset{\sim}{k}_{I,II} \, \underset{\sim}{r}_{II} \qquad (52)$$

$$\underset{\sim}{R}_{II} = \underset{\sim}{k}_{II,I} \, \underset{\sim}{r}_I + \underset{\sim}{k}_{II,II} \, \underset{\sim}{r}_{II} \qquad (53)$$

Since the fifth node does not actually exist in the original model, no external forces can be applied to this node. This condition gives

$$\underset{\sim}{r}_{II} = -\underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I} \, \underset{\sim}{r}_I \qquad (54)$$

Substitution of Equation 54 in 52 gives

$$\underset{\sim}{R}_I = (\underset{\sim}{k}_{I,I} - \underset{\sim}{k}_{I,II} \, \underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I}) \, \underset{\sim}{r}_I \qquad (55)$$

From Equation 55 the stiffness matrix of the original quadrilateral can be written as

$$\underset{\sim}{k} = \underset{\sim}{k}_{II} - \underset{\sim}{k}_{I,II} \, \underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I} \qquad (56)$$

The stiffness as obtained by Equation 56 is added to the total structure after appropriate coordinate transformations to the global coordinate system.

When the structure displacements are determined, the fifth node displacements can be determined by Equation 54. Now the stresses in each triangle can be determined as before. The effective stress ratio is determined for each triangle separately (Equation 46), and then a weighted average is used in computing the effective stress ratio and the margin of safety. This weighted average is computed by

$$\mathrm{ESR} = \frac{(\mathrm{ESR})_1\,\Delta_1 + (\mathrm{ESR})_2\,\Delta_2 + (\mathrm{ESR})_3\,\Delta_3 + (\mathrm{ESR})_4\,\Delta_4}{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4} \qquad (57)$$

where $(\mathrm{ESR})_1$ thru $(\mathrm{ESR})_4$ are the effective stress ratios of the four triangles. Δ_1 thru Δ_4 are the respective planform areas of the triangles. Now the margin of safety MS is computed as before by Eq. 47.

SHEAR PANEL

As the name indicates, the shear panel is devised for the purpose of representing shear transmitting elements. For example in wing structures the top and bottom skins can be represented by membrane (triangle and quadrilateral) elements. If the same elements are used for spars and ribs, the resulting finite element model grossly overestimates the stiffness of the structure. What this means is that the displacements obtained by this model will be much smaller, or if this model is used for dynamic analysis, the frequencies of the structure will be much higher and cannot be matched with the results obtained from ground vibration tests. This behavior is due to the assumption of constant strain (stress) in the membrane element formulations. Most web elements in box or I-beams carry primarily shear and some normal stresses. In other words their deformation is primarily due to shear and not due to normal stresses. The normal stresses in webs usually have steep stress gradients, and the assumption of constant stress (or strain) is not justified. To offset this difficulty, and yet preserve the simplicity of the constant strain elements, a shear panel was formulated (Reference 8) with the assumption that it carries only shear stresses. The bars and other membrane elements that surround the shear panel are supposed to carry the normal stresses. Such a situation does not actually exist in reality, and thus the shear panel is an empirical element. However, the models built on such an assumption appear to produce satisfactory results.

Until recently it was a common practice in aircraft companies to model wings, fuselages, and empennage structures simply by bars and shear panels to obtain primary load path information. In such idealizations it was a common practice to assign a third of the cross-sectional area as spar and rib caps and the remainder for the shear panels. It should be pointed out that every shear panel must be surrounded on all four sides by normal stress carrying elements such as bars or membrane or bending elements. If the natural model does not contain such an element on any side of the shear panel, a nominal (or fictitious) bar (post) must be provided. Otherwise the model will have a singularity.

The shear panel in "ANALYZE" is constructed out of four triangles with the fictitious node inside as in the membrane quadrilateral discussed earlier. However, the stiffness matrices of the component triangles are determined by considering only the shear strain energy (Equation 39).

$$k_{ij} = \frac{1}{2} |\underset{\sim}{X}| \; t \; \varepsilon_{xy}^{(i)} \; G \; \varepsilon_{xy}^{(j)} \qquad (58)$$

where G is the shear modulus, and $\varepsilon_{xy}^{(i)}$ and $\varepsilon_{xy}^{(j)}$ are the shear strains due to the unit displacement modes discussed earlier. There is one point that must be made here. The shear stress (strain) in an element changes with the orientation of the reference axis. Thus the stiffness matrix of the element can be sensitive to the reference axis. For rectangular elements the shear strain energy would be the same regardless of which side is selected for the reference axis. However, for quadrilaterals the stiffness matrix does depend on the reference axis. The errors produced by such departures are usually not significant, but it is worthwhile to make note of the assumptions involved. The ANALYZE program has a provision for specifying any one of the four sides of the quadrilateral as the reference axis.

As in the quadrilateral element the shear stresses in all four triangles are determined separately but with respect to the same reference axis. Of course, the normal stresses in the shear panels have no meaning. The margin of safety is determined by a weighted average of the effective stress ratios(ESR) as in the quadrilateral. The strain energy is determined by considering only the shear stress and strain.

4. ORGANIZATION OF THE PROGRAM

The material presented in this section is intended either to help introduce changes into the program or to expand its scope for the specific needs of a researcher as the authors have done in the past ten years. The steps outlined at the end of Section 2 are summarized in the flow-chart in Figure 5. There are a total of 16 boxes in the flow-chart. Each of these boxes generally involves one or more subroutines. The subroutines that belong to each of these boxes are identified first, then the function of each subroutine will be discussed in the next section with the help of the equations given in Sections 2 and 3.

Box 1 - Input

Input in the present version of the "ANALYZE" program is not in subroutine form. However, the input statements are all at the beginning of the program, and thus they can be grouped into a single subroutine. Alternatively, one can generate an input routine of his own with provisions like one card per each element and a card for each node etc. For example, it is relatively easy to write a subroutine with NASTRAN type input. The description of the various arrays (See input instructions) and their dimension requirements given in Appendix A can be quite helpful in writing such an input routine.

Box 2 - Map Stiffness Matrix

This step involves a single subroutine called "POP". The purpose of this routine is simply to estimate the storage requirements of the stiffness matrix and to map its profile. The stiffness matrix is stored in a single array called SK. The elements of the matrix are stored columnwise

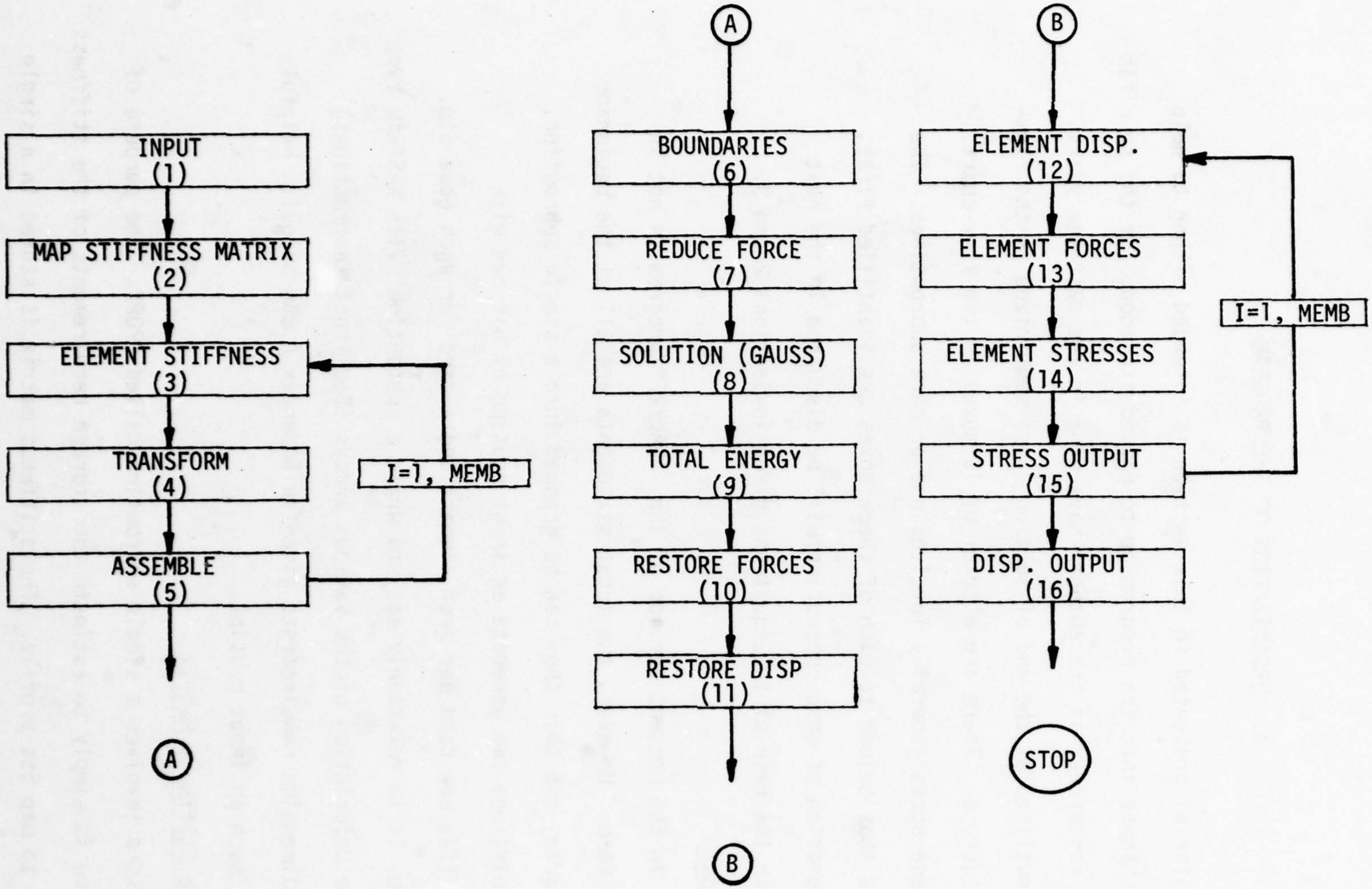


Figure 5. Flow Chart for Program "ANALYZE"

starting from the first non-zero element in the column to the diagonal element. Since the matrix is symmetric, only the upper triangle is stored.

Box 3 - Element Stiffness

There are four elements in the program. All of them require the subroutine "COORD". In addition all the plate elements require the routine "ELSTIC". The remaining subroutines are listed separately for each element.

i. Bar (Rod) Element:

The bar element is shown in Figure 2a with the local coordinate system and degrees of freedom. This element requires the subroutine "ELSTIF" which generates the bar stiffness matrix in the local coordinate system and also transforms it to the global coordinate system.

ii. Triangular Membrane Element:

The element and its local coordinate system are shown in Figure 2b. The subroutine "PLSTIF" is the only other routine required by this element. It generates the stiffness matrix of the triangle in the local coordinate system.

iii. Quadrilateral Membrane Element and Shear Panel

The elements and their local coordinate system are shown in Figure 2c. The subroutines "QDRLTL", "PLSTIF", "SUM", "CONDNS", "CHANGE" and "CRAMER" are the additional routines required by these elements. Together these subroutines generate the stiffness matrix of either the quadrilateral membrane or shear panel. The routine "QDRLTL" calls "PLSTIF", "SUM" and "CONDNS". The routine "PLSTIF" calls "CRAMER". Similarly "CONDNS" calls "CHANGE".

Box 4 - Transform

This step involves a single subroutine called "TRNSFM". It transforms the stiffness matrices of the triangles, quadrilaterals, and shear panels from the local to the global coordinate system.

Box 5 - Assemble

"ASEMBL" is the only subroutine used in this step. Its purpose is to add the element stiffness matrices to the total stiffness matrix of the structure. The steps 3 thru 5 form a loop in which all the element stiffness matrices are computed and assembled into the total stiffness matrix.

Box 6 - Boundaries

The routine called "BOUND2" eliminates the rows and columns of the stiffness matrix corresponding to the support degrees of freedom of the structure. In addition it also condenses the stiffness matrix.

Box 7 - Reduce Force

This step involves a routine called "REDUCE". It eliminates the rows of the force matrix corresponding to the support degrees of freedom.

Box 8 - Solution of the Force Deflection Equations

The routine "GAUSS" solves the load deflection equations by Gaussian elimination. A large percentage of the analysis time (80 to 90%) is spent in this routine, and its efficiency is extremely important in reducing the costs of the analysis. At the end of this step the displacements of the structure are available in condensed form (excluding boundary degrees of freedom) in the global coordinate system.

Box 9 - Total Energy

The total energy of the structure is computed by

$$W = \frac{1}{2} \underset{\sim}{R}^t \underset{\sim}{r}$$

The strain energy of the structure (U) is computed by adding the strain energies of the elements in step 14 (Box 14). A comparison of W and U provides an equilibrium check.

Boxes 10 and 11 - Restore Forces and Displacements

These two boxes use the same routine called "RESTOR". The purpose of this routine is to restore the force and deflection matrices to their original dimension to include the boundary degrees of freedom. Its purpose is essentially opposite to that of the routine "REDUCE" in Box 7.

Box 12 - Element Displacements

The routine "COORD" and "ELFORC" facilitate extraction and transformation of the element displacements from the global to the local coordinate system.

Box 13 - Element Forces

This step is not in all versions of "ANALYZE". Element forces are not necessary to compute stresses. However, this step can be restored if the element shear flows and other force information are necessary.

Box 14 - Element Stresses

The details of this step depend on the type of element.

i. Bar (Rod) Element:

The stress in this element is computed in the program itself. No additional routines are involved. At the same time the element strain energy is also computed.

ii. Triangular Membrane Element:

The subroutines "STRESS" and "CRAMER" are involved in this step. The routine "STRESS" calls "CRAMER". The purpose of this routine is to calculate stresses in the triangular element. In addition this routine calculates strain energy and the effective stress in the element (See Equations 44 and 45).

iii. Quadrilateral Membrane and Shear Panel

This step involves routines "ELSTIC", "QDRLTL", "PLSTIF", "SUM", "CONDNS", "CRAMER", "QLSTRS" and "STRESS". It should be noted that the routine "QDRLTL" calls "PLSTIF", "SUM and "CONDNS". "PLSTIF" in turn calls "CRAMER".

Box 15 - *Stress Output*

The instructions for the output of the table of stresses are in the main program. No subroutine is used for the output itself. The steps 12 thru 15 form a loop in which the stress information for all the elements is computed and printed in a table. This is one of the two main tables of output of this program. Explanation of this table is given in the section on output (Section 7).

Box 16 - Displacement Output

This step involves a single subroutine called "PRNTDR". This routine prints out the second important table of output which contains information about the nodes. This information includes the coordinates of the nodes, applied forces and the calculated displacements for each node. The detailed explanation of this table is given in the section on output (Section 7).

In addition to the above 16 steps there are instructions for weight computations and other details, and their purpose can be identified from the program. There are very few comment cards in the main body of the program and this omission is by design in order to avoid continuous updating. The user can incorporate his own comment cards with the help of the explanation given in this section.

5. DESCRIPTION OF THE SUBROUTINES

"ANALYZE" consists of the main program and 21 Subroutines. The main program has 260 cards. The length of the Subroutines varies from 15 to 62 cards. The total length of the program is under 1000 cards. A list of the Subroutines, the number of Cards in each Subroutine and other details are given in Table 1. The flow chart, Fig. 5, and the explanation in the previous section give details of the main program. The description of the Subroutines is given in the remainder of this section.

Subroutine "POP"

The purpose of Subroutine "POP" is to estimate the storage requirements of the stiffness matrix before actually determining it. This information can be generated from the element connections with the nodes. For example, if an element connects 4 nodes, and if each node has 3 degrees of freedom in the global coordinate system, then the stiffness matrix of the element would be of dimension 12 x 12. This matrix can be partitioned four ways, in both row and column directions as shown in Fig. 6. The location of these sixteen submatrices in the total stiffness matrix can be determined by the address of the nodes to which the element is connected. If the element is connected to the nodes MA, MB, MC, and MD, then the addresses of the element submatrices in the total stiffness matrix are shown in Fig. 6.

If all the elements are connected to all the nodes, then the stiffness matrix of the structure will be fully populated. The non-zero elements in the matrix are considered as population. Since most of the elements connect only a few nodes, the stiffness matrices are usually sparsely populated. Determining the profile of the stiffness matrix population is the essential function of the routine "POP".

$$\begin{array}{c|ccc:ccc:ccc:ccc|}
 & \text{3MA-2} & & & \text{3MB-2} & & & \text{3MC-2} & & & \text{3MD-2} & & \\
\hline
\text{3MA-2} & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
\hdashline
\text{3MB-2} & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
\hdashline
\text{3MC-2} & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
\hdashline
\text{3MD-2} & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
 & + & + & + & + & + & + & + & + & + & + & + & + \\
\hline
\end{array}$$

Fig. 6 Partitioned Element Stiffness Matrix and Addresses in the Total Stiffness Matrix

The distribution of the nonzero elements is dependent upon the way the nodes of the finite element model are numbered. Because of the symmetry of the stiffness matrix, only the lower or upper triangular matrix is considered. For the purpose of this discussion definitions of the following terms are in order. The gross population (P_{gross}) of the stiffness matrix is defined as the total number of elements in the upper triangle of the matrix. The net population (P_{net}) is the total number of non-zero elements in the upper triangle. Zeros resulting from transformations are not excluded from the net population. The apparent population ($P_{apparent}$) is the actual number of elements considered as nonzeros by a given solution scheme. From these definitions

$$P_{net} \leq P_{apparent} \leq P_{gross} \qquad (59)$$

For a given structure P_{gross} and P_{net} are invariant and are given by

$$P_{gross} = \frac{N\,(N+1)}{2} \qquad (60)$$

and

$$P_{net} = \frac{n\,(n+1)}{2}\,(\text{number of nodes}) + \sum_{i=1}^{m} \frac{n^2[k_i(k_i-1)]}{2} - n^2\,(NR) \qquad (61)$$

where N is the total number of degrees of freedom of the structure, n is the number of degrees of freedom of each node (all the nodes are assumed to have the same number of degrees of freedom; when this is not true the necessary modification is simple), k_i is the number of nodes to which the i^{th} element is connected, and m is the number of elements in the structure. The quantity NR is given by

$$NR = \sum_{i=1}^{p} (b_i - 1) \qquad (62)$$

where b_i is the number of elements connecting the same pair of nodes and p is the total number of pairs of directly connected nodes. If the structure consists of bar and/or beam elements only, NR is zero. For the example shown in Figure 6a, the value of NR is 3.

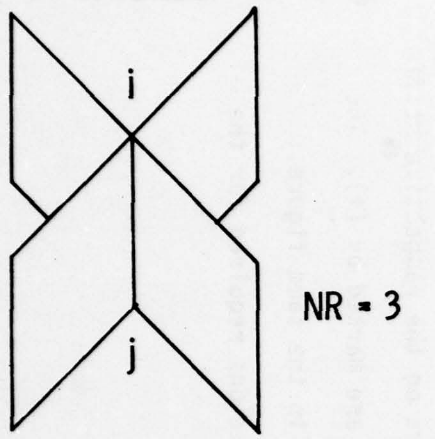
The quantity $P_{apparent}$ is dependent on the nature of the solution scheme used. For Gaussian elimination with no pivoting (LDL^T), $P_{apparent}$ may be defined as

$$P_{apparent} = \sum_{j=1}^{N} Q_j \tag{63}$$

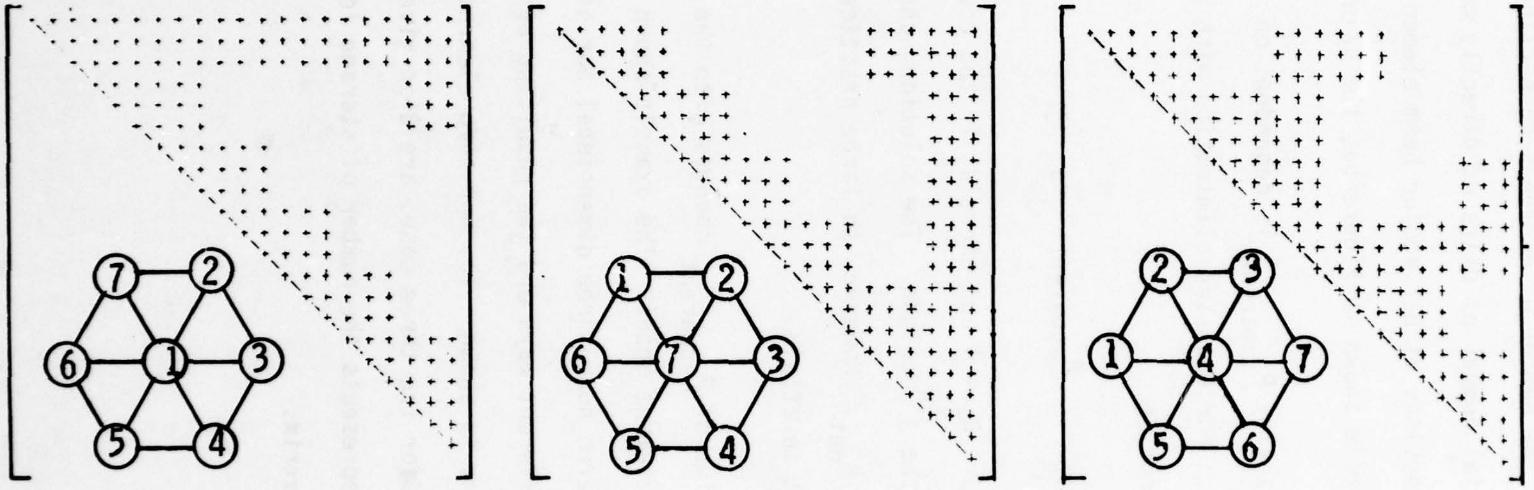
where $Q_j = j - R_j + 1$ and where R_j is the row number of the first nonzero element in the j^{th} column. The solution scheme is most efficient when $P_{apparent} = P_{net}$. However, in large practical structures this condition is difficult to attain.

The value of $P_{apparent}$ changes with the node numbering scheme of the finite element model. The example shown in Figure 7 illustrates this point. A seven node three dimensional bar structure (n = 3) is numbered in three different ways and the resulting effect on the respective stiffness matrices is shown. The non-zero elements are marked by (+). The populations for the three cases are also given in the same figure. $P_{apparent}$ represents the number of storage locations required for the stiffness matrix.

a. INTERSECTING PLATES



SCHEME NO.	P_{GROSS}	P_{NET}	$P_{APPARENT}$
1	231	150	231
2	231	150	177
3	231	150	177



SCHEME 1 SCHEME 2 SCHEME 3

FIGURE 7: DISTRIBUTION OF NONZERO ELEMENTS IN THE STIFFNESS MATRIX

Subroutine "ELSTIC"

This routine generates the 3 x 3 elastic constants matrix for a given material (see Eq. 3).

Subroutine "COORD"

This routine establishes the local coordinate system for all the elements and also determines the nodal coordinates in the local system. It generates the direction cosine matrix which will be used to transform the element stiffness matrices to the global coordinate system (see Eqs. 13 and 16).

i. Bar Element

The local coordinate system of the bar element is established by drawing a line between the two nodes MA and MB (see Fig. 2) connecting the bar. The direction cosines are determined by

$$
\begin{aligned}
X_{Comp} &= X_{MA} - X_{MB} \\
Y_{Comp} &= Y_{MA} - Y_{MB} \qquad (64) \\
Z_{Comp} &= Z_{MA} - Z_{MB}
\end{aligned}
$$

$$L = (X^2_{Comp} + Y^2_{Comp} + Z^2_{Comp})^{1/2} \qquad (65)$$

$$\ell_1 = \frac{X_{Comp}}{L} \quad m_1 = \frac{Y_{Comp}}{L} \quad n_1 = \frac{Z_{Comp}}{L} \qquad (66)$$

where X_{MA}, Y_{MA} and Z_{MA} are the three coordinates of the node MA in global coordinate system. The direction cosines ℓ_1, m_1, and n_1 become the first row of the 3 x 3 matrix $\underset{\sim}{A}$.

ii. Triangular Membrane Element

The local coordinate system of the triangular membrane element is established by assigning the local x-axis to the line joining nodes MA and MB. The direction cosines of this line are determined as in the case of the bar element. The plane of the plate is established by two unit vectors in the directions of the lines joining nodes MA-MB and MA-MC. If $\hat{a}$ and $\hat{b}$ are these two unit vectors, then the normal to the plane is obtained by

$$\hat{a} \times \hat{b} = \vec{c} \tag{67}$$

Since $\hat{a}$ and $\hat{b}$ are not orthogonal vectors, $\vec{c}$ is not a unit vector. The unit vector in this direction is given by

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} \tag{68}$$

The local z-axis is in the direction of the unit vector $\hat{c}$. Now the local y-axis is established by

$$\hat{c} \times \hat{a} = \hat{d} \tag{69}$$

The direction cosines of x and y become the first two rows of matrix $\underset{\sim}{A}$.

iii. Quadrilateral Membrane and Shear Panel

The local coordinate system of the quadrilateral membrane and the shear panel are established by a procedure similar to that of the triangle. The plane of the triangle connecting the three nodes MA, MB, and MC becomes the reference plane. Any warping in the quadrilaterals and shear panels is ignored. If there is too much warping in the quadrilaterals, it is better to divide them into two or more triangles or reduce the mesh size. In the case of excessively warped shear panels, the size of the grid must be reduced. "ANALYZE" does not have a provision for determining the warp and the consequent kick forces.

The node MA of the element becomes the origin of the element local coordinate system and the coordinates of the remaining nodes are determined by expressions similar to the following:

$$x_3 = (X_{MC} - X_{MA})\ell_1 + (Y_{MC} - Y_{MA})m_1 + (Z_{MC} - Z_{MA})n_1$$

$$y_3 = (X_{MC} - X_{MA})\ell_2 + (Y_{MC} - Y_{MA})m_2 + (Z_{MC} - Z_{MA})n_2$$

This subroutine also determines the coordinates of the fictitious node needed to break the quadrilateral and shear panels into four triangles. This interior node is established by

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4} \qquad y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \tag{70}$$

where x_1, x_2 ....x_5 and y_1, y_2 ....y_5 are the coordinates of the five nodes (including the fictitious interior node) of the quadrilaterals and shear panels in the local coordinate system.

Subroutine "ELSTIF"

This subroutine determines the stiffness matrix of the bar by Eq. 22. It also transforms the bar stiffness matrix to the global coordinate system by

$$\underset{\sim}{K}_i = \underset{\sim}{a}_i^t \; \underset{\sim}{k}_i \; \underset{\sim}{a}_i \tag{71}$$

Subroutines "PLSTIF" and "CRAMER"

The routine "PLSTIF" determines the element stiffness matrix of the triangle in the local coordinate system. This is also the basic routine for determining the stiffness matrices of the four triangles of the quadrilateral and the shear panel.

"PLSTIF" first calls the routine "CRAMER", which determines the inverse of the matrix $\underset{\sim}{X}$ by Cramer's rule. The matrix $\underset{\sim}{X}$ is given by Eq. 34. The determinant of $\underset{\sim}{X}$ represents twice the area of the triangle.

Then the "PLSTIF" subroutine determines the element stiffness matrix by Eq. 40. In determining the matrices $\underset{\sim}{\varepsilon}^{(i)}$ and $\underset{\sim}{\varepsilon}^{(j)}$, it takes advantage of the fact that the columns of Z^{-1} (see Eq. 33) represent unit displacement modes (see explanation under Eq. 34).

In computing the stiffness matrices of the triangles of the shear panels, "PLSTIF" considers only the shear strain energy. For example, in such a case, Eq. 40 becomes

$$\underset{\sim}{k} = \frac{1}{2}\left|\underset{\sim}{X}\right| t \begin{bmatrix} \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(1)} & \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(2)} \text{-----} & \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(6)} \\ \vdots & & \vdots \\ \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(1)} & \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(2)} \text{-----} & \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(6)} \end{bmatrix} \qquad (72)$$

Subroutine "QDRLTL"

This subroutine simply manages the routines "PLSTIF", "SUM", and "CONDNS" in computing the stiffness matrix of the quadrilateral membrane and shear panel. This routine also makes provision for assigning different sides as reference axis for the shear panels.

Subroutine "SUM"

This subroutine adds the four triangle stiffness matrices computed by "PLSTIF" to produce a 10 x 10 stiffness matrix (including two degrees of freedom for the interior node) for the quadrilateral or shear panel.

Subroutine "CONDNS"

This routine condenses the 10 x 10 quadrilateral or shear panel stiffness matrix to an 8 x 8 matrix. The condensation is done by using Eq. 56.

Subroutine "CHANGE"

This routine interchanges the rows and columns of the quadrilateral (or shear panel) stiffness matrix so that the element degrees of freedom are in ascending order before addition to the structure stiffness matrix. This step is necessary because the routine "ASEMBL" assumes that the element degrees of freedom are in ascending order.

Subroutine "TRNSFM"

This routine transforms the plate element stiffness matrices from the local to the global coordinate system by (see Eq. 16)

$$\underset{\sim}{K}_i = \underset{\sim}{a}_i^t \ \underset{\sim}{k}_i \ \underset{\sim}{a}_i \tag{73}$$

where $\underset{\sim}{K}_i$ is the transformed element stiffness matrix of the i^{th} element.

Subroutine "ASEMBL"

This routine adds the element stiffness matrices to the total stiffness matrix.

$$\underset{\sim}{K} = \sum_{i=1}^{m} \underset{\sim}{K}_i \tag{74}$$

For an explanation of the rules of this addition see the description of subroutine "POP". It should be noted that only the upper half of the stiffness matrix is stored. This storage is columnwise starting with the first non-zero element above the diagonal.

Subroutine "PRINTK"

The purpose of this routine is to print the stiffness matrix (if desired) rowwise starting with the first non-zero element and proceeding to the diagonal.

Subroutine "BOUND2"

This routine eliminates the rows and columns corresponding to the constrained degrees of freedom and condenses the stiffness matrix.

Subroutine "REDUCE"

This routine eliminates the rows of the applied force matrix corresponding to the constrained degrees of freedom. It is assumed that each column of the force matrix represents an independent load condition.

Subroutine "GAUSS"

"GAUSS" solves the load deflection equations (Eq. 17) by Gaussian elimination. The first step of the solution is the decomposition of the stiffness matrix by Eq. 18. The next two steps represent forward and back substitution using Eqs. 19 and 20 respectively. For the solution of additional load vectors only the steps FBS have to be repeated. If "GAUSS" is entered with any value other than 0 for the parameter NDCOMP, only the last two steps will be executed. The matrices $\underset{\sim}{L}$ and $\underset{\sim}{D}$ are stored in place of the original stiffness matrix.

Subroutine "RESTOR"

This routine restores the displacement or force matrix to full size by assigning zero values to boundary degrees of freedom.

Subroutine "ELFORC"

This routine extracts the element displacements from the global coordinate system and transforms them to the local coordinate system by Eq. 13.

Subroutine "STRESS"

The purpose of the "STRESS" routine is to compute strains and stresses in the triangular element. It first calls the routine "CRAMER" which computes $\underset{\sim}{X}^{-1}$ (Eq. 34) by Cramer's rule. The strains in the element are then calculated by Eqs. 30 and 35 thru 37. The stresses in the element are computed by Eq. 2. Also it computes the strain energy and the effective stress in the element by Eqs. 1 and 45 respectively.

Subroutine "QLSTRS"

This routine prepares the data for computing stresses in the four triangles of the quadrilateral or shear panel elements. First it determines the interior node displacements from the corner node displacements using Eq. 54. Then it calls subroutine "STRESS" to compute the stresses in the four triangles. It adds the strain energy of the four triangles to obtain the total strain energy. It identifies the triangle with the largest effective stress and normalizes the effective stress of the three remaining triangles with respect to this largest value.

Subroutine "PRNTDR"

This subroutine prints out the table of node information. This includes the node number, its coordinates, applied forces, and the displacements.

NAME	NUMBER OF CARDS	CALLED FROM
ANALYZE	315	Main Program
POP	62	ANALYZE
ELSTIC	15	ANALYZE
COORD	44	ANALYZE
ELSTIF	21	ANALYZE
PLSTIF	46	ANALYZE, QDRLTL
CRAMER	19	PLSTIF, STRESS
QDRLTL	32	ANALYZE
SUM	23	QDRLTL, QLSTRS
CONDNS	36	QDRLTL, QLSTRS
CHANGE	25	CONDNS
TRNSFM	36	ANALYZE
ASEMBL	41	ANALYZE
PRINTK	15	ANALYZE
BOUND2	35	ANALYZE
REDUCE	18	ANALYZE
GAUSS	57	ANALYZE
RESTOR	28	ANALYZE
ELFORC	22	ANALYZE
STRESS	33	ANALYZE, QLSTRS
QLSTRS	65	ANALYZE
PRNTDR	39	ANALYZE
TOTAL	1027	

Table 1: Program Description

6. INPUT INSTRUCTIONS

Input for the programs is divided into a number of card sets. Each card set will consist of one or more cards. Only three Formats are used for input. An integer Format (14I5), a floating point Format (6F10.0) and a mixed Format 3(F10.0,2I5). The first four card sets will each have one card regardless of the size of the problem. The number of cards required for the remaining card sets depends on the problem size. The first card set indicates the number of problems (structures) to be analyzed. If this number is more than one, the program assumes that the remaining card sets will be supplied for each problem one after the other. The next card set is for the title of the problem. Card set three defines the basic parameters like the number of elements, nodes etc. And set 4 defines the properties of a reference material. This material can be any one of the materials used. The remaining card sets define material properties (5 and 6), type of elements (7), element connections (8, 9, 10, 11), sizes of the elements (12), element-material identification (13), node coordinates (14), boundaries (15) and loading information (16 and 17).

INPUT INSTRUCTION DETAILS

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
1 (14I5)	NSTR	Number of data sets
2 (8A10)	TITLE	An alphanumeric description of the problem to be solved.
3 (14I5)	MEMBS	Number of elements
	JOINTS	Number of nodes
	NBNDRY	Number of restrained degrees of freedom
	LOADS	Number of loading conditions
	MM	MM =2 Two dimensional problem MM =3 Three dimensional problem
	NR	Variable used only for calculating the net population of the stiffness matrix. It has no other role in the program. See Section 5.
	INCHES	INCHES =1 Coordinate data is in inches INCHES ≠1 Coordinate data is in feet
	KIPS	KIPS =1 Applied forces are in kips KIPS ≠1 Applied forces are in pounds
	NMAT	Number of materials
	MSSTRS	MSSTRS =0 Margin of safety calculated from default allowable stresses. MSSTRS ≠0 Margin of safety calculated from input allowable stresses.
4 (6F10.5)	EEE	YOUNG'S modulus/10^6 of one of the elements in psi.
	PMU	POISSON'S ratio of one of the elements.
	RHO	Density of one of the elements in lbs/in^3.

CARD SET (FORMAT)	PARAMETER	DESCRIPTION

IF MSSTRS = 0, skip CARD SET 5.

5 (6F10.5)	ALSTRS(I) I=1,...,3*NMAT	Allowable stresses/10^3 in tension, compression and shear for the Ith material.

IF NMAT ≠ 1, CARD SET 4 parameters can be for any of the materials.
IF NMAT = 1, skip CARD SET 6.

6 (6F10.5)	YOUNGM(I)	YOUNG'S modulus/10^6 for the Ith material in psi.
	POISON(I)	POISSON'S ratio for the Ith material.
	RHO1(I) I=1,...,NMAT	Density for the Ith material in lbs/in^3.
7 (14I5)	NNODES(I),I=1,...,MEMBS	Element Type NNODE(I) =2 BAR =3 TRIANGLE =4 QUADRILATERAL MEMBRANE =5 SHEAR PANEL
8 (14I5)	MA(I),I=1,...,MEMBS	First node number of each element.
9 (14I5)	MB(I),I=1,...,MEMBS	Second node number of each element.
10 (14I5)	MC(I),I=1,...,MEMBS	Third node number of each element.
11 (14I5)	MD(I),I=1,...,MEMBS	Fourth node number of each element.

NOTE: For bars leave MC(I) and MD(I) blank. For triangles leave MD(I) blank. For each element let MA(I) be the lowest node number and MB(I) be the next lowest. For Quadrilaterals and Shear Panels, MC(I) and MD(I) are determined by continuing in the direction defined by MA(I) and MB(I).

12 (6F10.5)	TH(I),I=1,...,MEMBS	Thickness of each element. For a bar thickness is cross-sectional area.

IF NMAT =1, skip CARD SET 13.

13 (14I5)	MYOUNG(I), I=1,...,MEMBS	Material number of each element.

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
14 (6F10.5)	X(I)	X coordinate of the I^{th} node.
	Y(I)	Y coordinate of the I^{th} node.
	Z(I) I=1,...,JOINTS	Z coordinate of the I^{th} node.
	IF MM=2, only X(I) and Y(I) are input.	
15 (14I5)	IBND(I),I=1,...,NBNDRY	Degree of freedom numbers of those nodes which are restrained. For node K the degree of freedom numbers are 3*K-2, 3*K-1, and 3*K for MM=3 and 2*K-1, 2*K for MM=2.
16 (14I5)	NJLODS(I),I=1,...,LOADS	Number of load components in the I^{th} loading condition.
17 3(F10.0,2I5)	TFR(J)	Value of the load.
	IM(J)	Direction of the load $IM(J)\begin{cases}=1 \text{ x direction}\\ =2 \text{ y direction}\\ =3 \text{ z direction}\end{cases}$
	JM(J) J=1,...,NJLODS(I)	Number of the node where the load is applied.

7. OUTPUT DESCRIPTION

The primary output of the program ANALYZE consists of two tables (items 6 and 8 of the output description details). The first table gives element information and the second table gives information about the nodes. The element information includes member number, thickness (cross-sectional area of the bars), planform area (length of a bar), element type, stress information, strain energy, and margin of safety. The information about the nodes includes node (joint) number, node coordinates, applied forces, and the resulting displacements. In addition to these two tables output 3a (coming from subroutine POP) gives important information about the population distribution of the stiffness matrix. The value of the apparent population is crucial in determining the dimension of the stiffness matrix (SK). This dimension must be at least as big as or bigger than this value.

Item 7 gives information about the total strain energy (U) and the work of the external forces (W) for the structure. This information can be very useful for an equilibrium check.

Item 4 gives the weight of the structure. The remaining information is not really very important to the user.

OUTPUT DESCRIPTION DETAILS

Output for Program ANALYZE consists of the following:

1) Untitled echo of all input data except boundaries and applied loads.

2) Boundary data, i.e. contents of array IBND (CARD SET 15)

3) Output from Subroutine POP concerning the distribution of elements in the stiffness matrix. This information is generated before the stiffness matrix of the structure is assembled.

(a) Gross Population - total number of elements in the upper triangle of the matrix.

Net Population = actual population of possible non-zero elements in the upper triangle of the stiffness matrix. This number would be correct only if NR is correct in CARD SET 3.

Apparent Population = actual number of elements considered as non-zero by a given solution scheme. Thus the apparent population represents the number of storage locations required for the stiffness matrix.

(b) Starting Row Numbers for each column - the number of the row where the first non-zero element occurs in each column.

(c) Number of Diagonal Elements in Single Array Stiffness Matrix. For each Column I the actual number of elements, ID(I), in the upper triangular matrix up to and including that column, i.e.

$$ID(I) = \frac{I(I+3)}{2} - \sum_{j=1}^{I} b_j$$

where b_j is the row number given for Column I in (b). Thus for the last column, ILAST,

ID(ILAST) = Apparent Population

4) Weight of the structure

5) Boundary conditions applied to the stiffness matrix alter the arrays defined by (b) and (c) above, and thus they are reprinted.

6) Output for each element after analysis.

(a) MEMB - Element Number

(b) THICK - Thickness of the element. For a bar thickness is cross-sectional area.

(c) AREA - Area of the element. For a bar area is length.

(d) TYPE - Type is a composite number which describes the element type and material number. Type is defined as
TYPE = NNODES(I)x10 + MYOUNG(I).
See CARD SETS 7 and 13.
Note: If the number of materials is greater than 10, TYPE is meaningless. If the number of materials is 1, MYOUNG(I)=1 for all I.

(e) MA, MB, MC, MD - defined in CARD SETS 8,9,10,and 11.

(f) SIGMA-X (σ_x), SIGMA-Y (σ_y), SIGMA-XY (σ_{xy}).
Stresses in the x-y local coordinates of the element.
EFSTR-1, EFSTR-2, EFSTR-3, EFSTR-4 - Effective stresses in the element determined by the Von Mises Criterion.

The stress output varies per element type.

(i)	BAR	SIGMA-X only
(ii)	TRIANGLE	SIGMA-X, SIGMA-Y, SIGMA-XY, EFSTR-1
(iii)	QUADRILATERAL MEMBRANE	

The Quadrilateral membrane element is divided into 4 triangles for analysis. SIGMA-X, SIGMA-Y, SIGMA-XY are for that triangle with the maximum effective stress. This maximum effective stress is given as EFSTR-i for some i, i=1,...,4. Then EFSTR-j, j≠i, are defined as the ratio of the effective stress for triangle j to the maximum effective stress.

(iv) SHEAR PANEL

The Shear Panel is also divided into 4 triangles for analysis. SIGMA-XY (τ_{xy}) is for that triangle with the maximum effective stress. Then EFSTR-i, i=1,...,4 are as defined in (iii).

(g) ENERGY - Total strain energy in the element.

(h) MS - Margin of Safety for the element.

NOTE: If the number of loading conditions is greater than 1, output (f) and (h) are given continuously for each load case.

7) The total strain energy (U) of the structure and the work (W) of the external forces for each loading condition.

8) Output for each node after analysis.

   (a) JOINT - Node Number

   (b) X, Y, Z - x, y, and z coordinate of the node

   (c) FORCE-X, FORCE-Y, FORCE-Z - applied forces in the x, y, and z direction.

   (d) DISPL-X, DISPL-Y, DISPL-Z - Displacements in the x, y, and z direction.

<u>NOTE</u>: If the number of loading conditions is greater than 1, output (c) and (d) are given continuously for each load case.

8. SAMPLE PROBLEM

A three spar wing shown in Figure 8 is idealized by membrane quadrilaterals, shear panels, and bars (axial force members). The top and bottom skins are represented by membrane quadrilaterals and the spars and ribs by shear panels. In addition, the top and bottom nodes are connected by bar elements or posts. The root section of the wing is assumed to be fixed. The finite element model of the wing box is shown in Figure 8.

The wing is analyzed for two independent loads. These loading conditions are generated by simplified pressure distributions representative of a subsonic, forward-center-of-pressure loading, and a supersonic near-uniform-pressure loading. The material properties (aluminum) are given in Figure 9.

The results of the analysis are given in Appendix D. They are given in two tables. The first table gives element information. This includes sizes, connections, stresses, energies, and margins of safety. The second table gives information about nodes. This includes coordinates, applied loads, and displacements. For a detailed description of the complete output, see Section 7.

The wing was also analyzed by the NASTRAN program. Table 2 compares ANALYZE and NASTRAN z-displacements.

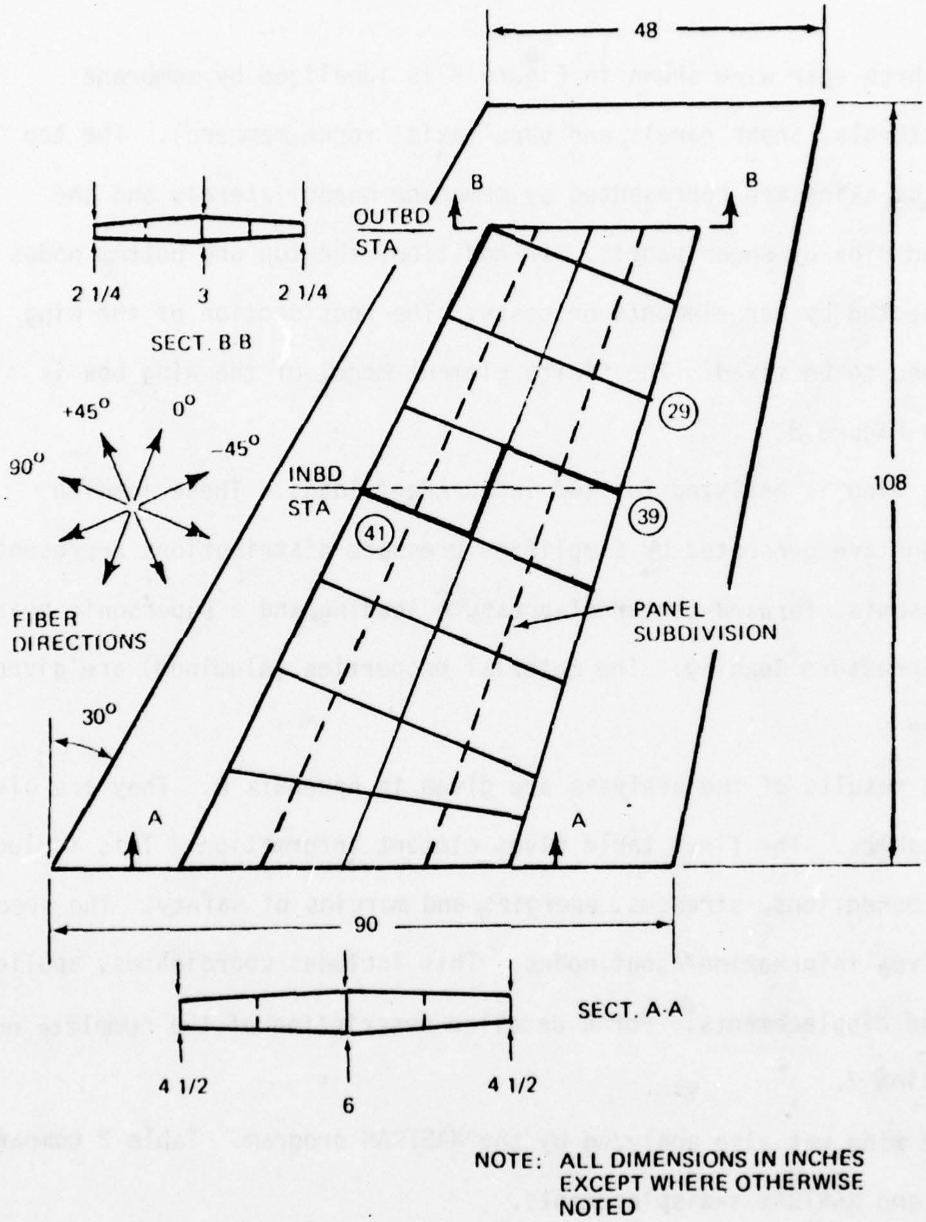


Figure 8. Aerodynamic Planform and Primary Structural Arrangement of Wing

Notes:

Even numbered nodes are on bottom surface

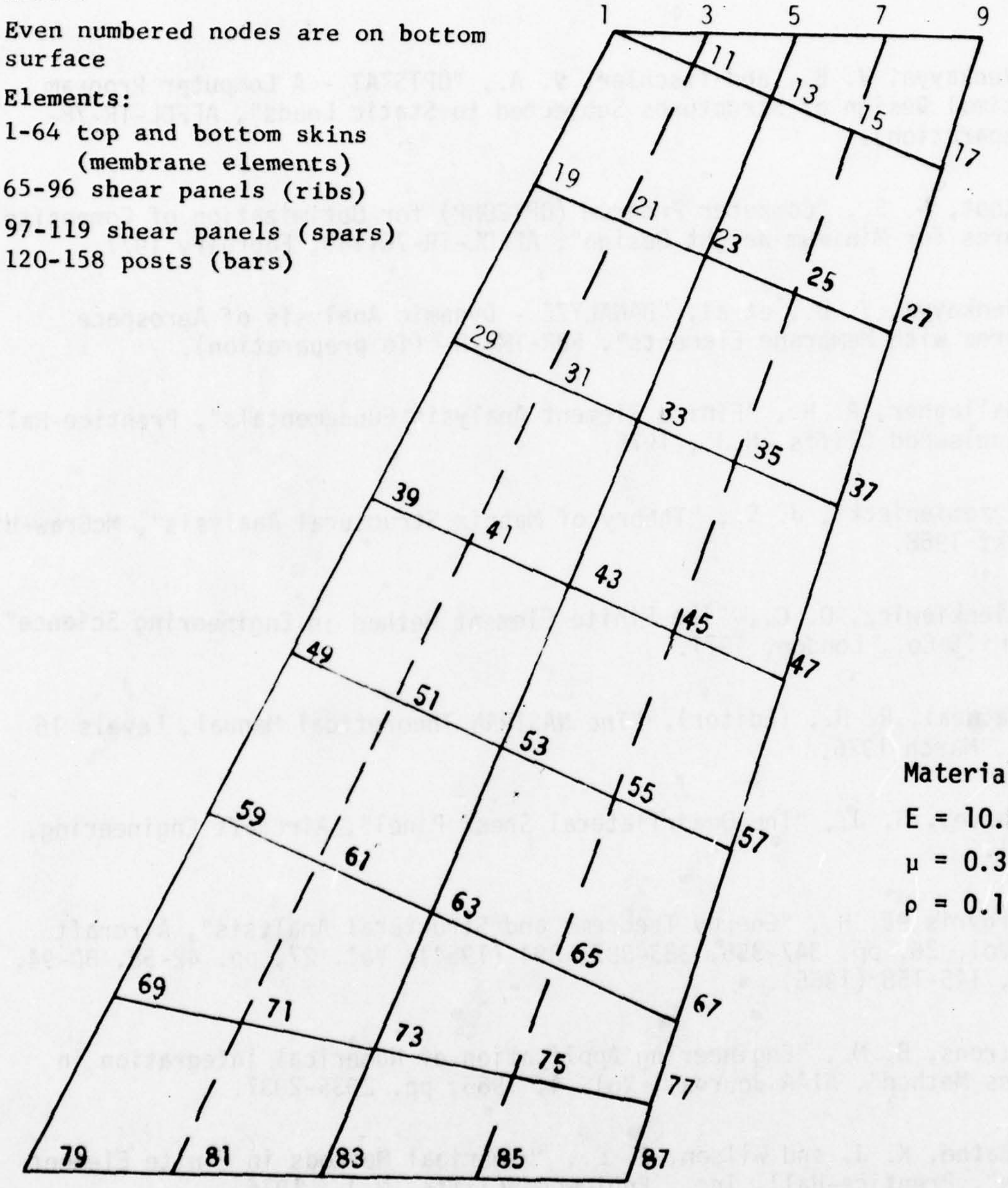
Elements:

1-64 top and bottom skins (membrane elements)

65-96 shear panels (ribs)

97-119 shear panels (spars)

120-158 posts (bars)



Material - Aluminum

$E = 10.5 \times 10^6$ psi

$\mu = 0.3$

$\rho = 0.1$ lbs/in^3

Figure 9. Finite Element Representation of Wing Box

REFERENCES

[1] Venkayya, V. B., and Tischler, V. A., "OPTSTAT - A Computer Program for Optimal Design of Structures Subjected to Static Loads", AFFDL-TR-78- (in preparation).

[2] Khot, N. S., "Computer Program (OPTCOMP) for Optimization of Composite Structures for Minimum Weight Design", AFFDL-TR-76-149, February 1977.

[3] Venkayya, V. B., et al, "DANALYZE - Dynamic Analysis of Aerospace Structures with Membrane Elements", FBR-TM-78- (in preparation).

[4] Gallagher, R. H., "Finite Element Analysis Fundamentals", Prentice-Hall Inc., Englewood Cliffs, N.J., 1975.

[5] Przemieniecki, J. S., "Theory of Matrix Structural Analysis", McGraw-Hill, New York, 1968.

[6] Zienkiewicz, O. C., "The Finite Element Method in Engineering Science", McGraw Hill Co., London, 1971.

[7] MacNeal, R. H., (Editor), "The NASTRAN Theoretical Manual, Levels 16 and 17", March 1976.

[8] Garvey, S. J., "The Quadrilateral Shear Panel", Aircraft Engineering, May 1951.

[9] Argyris, J. H., "Energy Theorems and Structural Analysis", Aircraft Engr., Vol. 26, pp. 347-356, 383-387, 394 (1954); Vol. 27, pp. 42-58, 80-94, 125-134, 145-158 (1955).

[10] Irons, B. M., "Engineering Application of Numerical Integration in Stiffness Method", AIAA Journal, Vol. 4, 1966, pp. 2035-2037.

[11] Bathe, K. J. and Wilson, E. L., "Numerical Methods in Finite Element Analysis", Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976.

[12] Hohn, F. E., "Elementary Matrix Algebra", The McMillan Company, New York, 1958.

APPENDIX A: ESTIMATION OF CORE REQUIREMENTS

The purpose of this appendix is to aid in the approximate estimation of core requirements for the program. These change with the problem size. For example with $100k_8$ (120K bytes or 30K decimal) it is possible to solve problems of the size 250 to 300 degrees of freedom assuming that the nodes are numbered with reasonable care for an optimum stiffness profile (See the discussion under subroutine "POP" in Section 5). The dimensional requirements of various arrays are explained by comment cards at the beginning of the program. However, this section reiterates the importance of adjusting the dimensions of some important arrays.

The arrays can be grouped into nine types. The number of elements, degrees of freedom, loading conditions, the number of boundaries and the number of materials are some of the variables that affect the size of the arrays. The arrays must be dimensioned at least as big or bigger than the number of these variables in the problem. The arrays with fixed sizes (not affected by problem size) are dimensioned first. The total core requirement of these arrays is relatively small. Next, the arrays that depend on the number of materials are dimensioned. The third group consists of a single array IBND which is dimensioned according to the number of boundary conditions. The fourth group varies with the number of loading conditions. In this case the dimension of the single arrays is equal to the number of loading conditions. For rectangular arrays, the first dimension is fixed, and the second dimension represents the number of loading conditions. The fifth group varies with the number of elements. The sixth group depends on the number of nodes.

The number of degrees of freedom determines the dimensions of the seventh group. The number of degrees of freedom and the loading conditions determine the size of the eighth group of arrays. The first dimension of these arrays represents the degrees of freedom, and the second dimension represents the loading conditions. The SK matrix in the last group depends on the number of degrees of freedom and the profile of the total structure stiffness matrix which in turn depends on the ordering of the node numbers (See the discussion under subroutine "POP" in Section 5).

The preliminary estimate of the size of the SK array can be based on the estimation of the semi-bandwidth. This would be an upper bound for the dimension of SK. The actual dimension of SK can be determined after passing through the subroutine "POP". This routine gives a number for the apparent population of the stiffness matrix from the information of the element connections. SK must be dimensioned at least as big or bigger then the apparent population in order to solve the problem. Usually SK is the largest array in the program, and its size can be reduced by numbering the nodes for the optimum profile of the stiffness matrix. In absence of an adequate procedure for optimization of this profile, some sort of bandwidth optimization is acceptable. It should be noted that the value of the variable MAXSK (defined in the beginning of the program) should be the same as the dimension of SK. When the dimension of SK is changed, the value of MAXSK should also be changed.

The next largest arrays are FR and DR. They represent the applied force and the computed displacement matrix respectively. The dimension of the arrays depends on the number of degrees of freedom and the independent loading conditions. The first dimension should be at least as big or bigger than the number of degrees of freedom of the problem. Similarly the second dimension is determined by the number of loading conditions. In addition the first dimension should be the same as the variable NNMAX defined in the beginning of the program. Whenever the dimensions of FR and DR are changed, NNMAX must also be changed accordingly.

The arrays ICOL and IDIAG depend on the number of degrees of freedom of the problem. Together they identify the profile of the stiffness matrix. For instance, ICOL(I) gives the row number of the first non-zero element in the Ith column of the stiffness matrix. IDIAG(I) gives the address of the diagonal element of the Ith column of the stiffness matrix in the single array SK.

The arrays MA, MB, MC and MD are assigned for element connections. NNODES is for the type of elements. The array TH is for the sizes (thickness of plate elements and cross-sectional area of bars) of the elements. The array MYOUNG identifies the material type of the elements. The remaining arrays are small and have minor influence on the core requirements.

Frequent Errors Encountered in Using "ANALYZE"

1. The element connections MA, MB, MC and MD must be specified by starting with the lowest node number for MA and the next lowest, but adjacent node number, for MB. MC and MD are then defined by continuing in the direction established by MA and MB. See the description of card sets 8, 9, 10, and 11 in the input instructions, Section 6.

2. The boundary degrees of freedom (IBND) must be in ascending order. See the description of card set 15 in Section 6, Input Instructions.

3. The first dimension of FR and DR must be the same as the value of the variable NNMAX (defined at the beginning of the program).

4. The value of MAXSK (defined at the beginning of the program) must be equal to or greater than the value of the apparent population given by the routine "POP". The dimension of the array SK must be equal to the value given for MAXSK.

5. The sides of the shear panels must be attached to one or more normal stress carrying elements such as posts (bars), membrane quadrilaterals or triangles.

APPENDIX B: LISTING OF THE PROGRAM

```
      PROGRAM ANALYZE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)            ANALYZE   2
C                                                                       ANALYZE   3
C     THE FOLLOWING DIM ARE FOR INTERNAL USE                            ANALYZE   4
      DIMENSION AA(3,3),EK(12,12),B(12,12),C(12,12),XI(5),ETA(5),EE(3,3) ANALYZE  5
     1          ,MAA(4),MBB(4),MCC(4),EKK(12,12),TRANG(4),TFR(10),      ANALYZE   6
     2          IM(10),JM(10),ALS(3),TITLE(8)                           ANALYZE   7
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF MATERIALS              ANALYZE   8
      DIMENSION YOUNGM(20),POISON(20),RHO1(20)                          ANALYZE   9
C     THE FOLLOWING DIM ARE THREE TIMES THE NUMBER OF MATERIALS         ANALYZE  10
      DIMENSION ALSTRS(60)                                              ANALYZE  11
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF BOUND COND (NBNDRY)    ANALYZE  12
      DIMENSION IBND(50)                                                ANALYZE  13
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF LOADING CONDITIONS (L) ANALYZE  14
      DIMENSION EDDR(12,5),SSX(4,5),SSY(4,5),SSXY(4,5),SXY(5),KTR(5),   ANALYZE  15
     1          EFSTRS(5),EFFSTR(4,5),EDR(12,5),SX(5),SY(5),NJLOBS(5),  ANALYZE  16
     2          ELEENG(5),ENGTOT(5),ENGSTR(5),ESR(5),SFTM(5)            ANALYZE  17
C     IF THE NUMBER OF LOADING CONDITIONS EXCEED 10, THEN CHANGE THE    ANALYZE  18
C     DIMENSION OF EX, EY, EXY IN SUBROUTINE STRESS, ENGG IN SUBROUTINE ANALYZE  19
C     OLSTRS AND TDR1, TDR2 IN SUBROUTINE RESTOR                        ANALYZE  20
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF ELEMENTS               ANALYZE  21
      DIMENSION MA(160),MB(160),MC(160),MD(160),TH(160),NNODES(160),    ANALYZE  22
     1          MYOUNG(160)                                             ANALYZE  23
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF JOINTS                 ANALYZE  24
      DIMENSION X(90),Y(90),Z(90)                                       ANALYZE  25
C     THE FOLLING DIM PERTAIN TO THE NUMBER OF DEG OF FREEDOM (NN)      ANALYZE  26
      DIMENSION IDIAG(270),ICOL(270)                                    ANALYZE  27
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF DEG OF FREEDOM (NN)    ANALYZE  28
C     AND THE NUMBER OF LOADING CONDITIONS (L)                          ANALYZE  29
      DIMENSION DR(270,5),FR(270,5)                                     ANALYZE  30
C     THE FOLLOWING DIM PERTAINS TO THE TOTAL STIFFNESS MATRIX (SK)     ANALYZE  31
      DIMENSION SK(9110)                                                ANALYZE  32
C                                                                       ANALYZE  33
C*****************************************************************       ANALYZE  34
C                                                                       ANALYZE  35
C     THIS PROGRAM WAS DEVELOPED                                        ANALYZE  36
C                                                                       ANALYZE  37
C               DR. VIPPERLA B. VENKAYYA                                ANALYZE  38
C               AIR FORCE FLIGHT DYNAMICS LABORATORY (AFFDL/FBR)        ANALYZE  39
C               WRIGHT-PATTERSON AIR FORCE BASE,DAYTON,OHIO             ANALYZE  40
C                                                                       ANALYZE  41
C*****************************************************************       ANALYZE  42
C                                                                       ANALYZE  43
      INTEGER TYPE                                                      ANALYZE  44
C     NNMAX MUST BE THE DIMENSION OF FR,DR,IDIAG,ICOL                   ANALYZE  45
      NNMAX = 270                                                       ANALYZE  46
C     MAXSK MUST BE EQUAL OR GREATER THAN THE DIM OF SK                 ANALYZE  47
      MAXSK = 9110                                                      ANALYZE  48
      READ(5,2)    NSTR                                                 ANALYZE  49
    1 READ(5,76)  (TITLE(I), I = 1,8)                                   ANALYZE  50
   76 FORMAT(8A10)                                                      ANALYZE  51
      WRITE(6,77)  (TITLE(I), I = 1,8)                                  ANALYZE  52
   77 FORMAT(5X,8A10)                                                   ANALYZE  53
      KSTR=1                                                            ANALYZE  54
      READ(5,2) MEMBS,JOINTS,NBNDRY,LOADS,MM,NR,INCHES ,KIPS,NMAT,MSSTRS ANALYZE  55
      WRITE(6,2) MEMBS,JOINTS,NBNDRY,LOADS,MM,NR,INCHES,KIPS,NMAT,MSSTRS ANALYZE  56
      READ(5,3)FEE,PMU,RHO                                              ANALYZE  57
      IF (RHO .LT. .00001) RHO=0.1                                      ANALYZE  58
```

```
      DO 7783 I = 1,NMAT
      KX = 3*(I-1) + 1
      ALSTRS(KX) = 60000.
      ALSTRS(KX+1) = 60000.
 7783 ALSTRS(KX+2) = 36000.
      IF (MSSTRS .EQ. 0) GO TO 7782
      KX = 3*NMAT
      READ(5,3) (ALSTRS(I), I = 1,KX)
      DO 7781 I = 1,KX
 7781 ALSTRS(I) = 1000.*ALSTRS(I)
 7782 CONTINUE
      ALS(1) = ALSTRS(1)
      ALS(2) = ALSTRS(2)
      ALS(3) = ALSTRS(3)
      WRITE(6,333) EEE,PMU,RHO,ALS(1),ALS(2),ALS(3)
  333 FORMAT(6F15.3)
      IF (NMAT .LE. 1) GO TO 7777
      READ(5,3) (YOUNGM(I),POISON(I),RHO1(I), I = 1,NMAT)
      DO 7784 I = 1,NMAT
      KX = 3*(I-1) + 1
      WRITE(6,333) YOUNGM(I),POISON(I),RHO1(I),ALSTRS(KX),
     1ALSTRS(KX+1),ALSTRS(KX+2)
 7784 CONTINUE
 7777 READ(5,2)(NNODES(I),I=1,MEMBS)
      READ(5,2)(MA(I),I=1,MEMBS)
      READ(5,2)(MB(I),I=1,MEMBS)
      READ(5,2)(MC(I),I=1,MEMBS)
      READ(5,2)(MD(I),I=1,MEMBS)
      READ(5,3)(TH(I),I=1,MEMBS)
      IF (NMAT .LE. 1) GO TO 7778
      READ(5,2) (MYOUNG(I), I =  1,MEMBS)
 7778 DO 5464 I=1,MEMBS
      IF (NMAT .EQ. 1) MYOUNG(I) = 1
      WRITE(6,33)I,NNODES(I),MYOUNG(I),MA(I),MB(I),MC(I),MD(I),TH(I)
 5464  CONTINUE
   33 FORMAT(7I5,4F10.5)
    2 FORMAT(14I5)
      EEE = EEE*(10.0**6)
      E = EEE
      E1=1.0
      IF (MM .LT. 3) GO TO 4
      READ(5,3)(X(I),Y(I),Z(I),I=1,JOINTS)
      GO TO 6
    4  READ (5,3)  (X(I),Y(I),I=1,JOINTS)
      DO 11 I=1,JOINTS
   11  Z(I)=0.0
    3 FORMAT (6F10.5)
    6  CONTINUE
      IF(INCHES .EQ. 1)GO TO 9
 7000  FORMAT (20X, 3F10.4)
      DO 7 I=1,JOINTS
      X(I)=X(I)*12.0
      Z(I)=Z(I)*12.0
    7  Y(I)=Y(I)*12.0
    9  CONTINUE
      WRITE(6,7000) (X(I),Y(I),Z(I), I = 1,JOINTS)
      NN=MM*JOINTS
```

```
      NM=NN-NBNDRY
      READ(5,2) (IBND(I),I=1,NBNDRY)
      WRITE(6,5)
    5 FORMAT(1H1,///2X,10HBOUNDARIES ///)
      WRITE(6,1009)(IBND(I),I=1,NBNDRY)
      DO 10 I=1,NN
      DO 10 J=1,LOADS
      DR(I,J)=0
   10 FR(I,J)=0
      READ(5, 2) (NJLODS(I),I=1,LOADS)
      DO 21 J=1,LOADS
      KH=NJLODS(J)
   12 IF(KH-3) 13,13,14
   13 KX=KH
      GO TO 15
   14 KX=3
   15 READ(5,16) (TFR(I),IM(I),JM(I),I=1,KX)
   16 FORMAT (3(F10.0,2I5))
      DO 19 I=1,KX
      KY=MM*JM(I)-MM+IM(I)
   19 FR(KY,J)=FR(KY,J)+TFR(I)
      KH=KH-KX
      IF(KH)21,21,12
   21 CONTINUE
      IF(KIPS .NE. 1)GO TO 666
      DO 17 I=1,NN
      DO 17 J=1,LOADS
   17 FR(I,J)=1000.0*FR(I,J)
  666 CONTINUE
      CALL POP(MEMBS,JOINTS,MM,MA,MB,MC,MD,NNODES,ICOL,IDIAG,NONZRO,NR)
      IF(NONZRO .GT. MAXSK)GO TO 1000
      DO 8 I=1,NONZRO
    8 SK(I)=0
      CALL ELSTIC(1.0,PMU,EE)
      DO 120 I=1,4
      MAA(I)=I
      MBB(I)=I+1
  120 MCC(I)=5
      MAA(4)=1
      MBB(4)=4
      WEIGHT = 0.0
      DO 400 L = 1,MEMBS
      IF (NMAT .LE. 1) GO TO 20
      KX = MYOUNG(L)
      E = YOUNGM(KX)*10**6
      PMU = POISON(KX)
      E1 = E/EEE
      CALL ELSTIC(E1,PMU,EE)
   20 CALL COORD(MA(L),MB(L),MC(L),MD(L),X,Y,Z,AA,XI,ETA,AL,NNODES(L),0)
      IF(NNODES(L) -3)102,100,124
  124 CALL ODRLTL(EK,EKK,TH(L),QUAD,MA(L),MB(L),MC(L),MD(L),MAA,MBB,MCC,
     1XI, ETA,NNODES(L),EE,TRANG,0)
      GO TO 101
  100 CONTINUE
      CALL PLSTIF(EK ,TH(L),TRIANG, 1,2,3                    ,XI,ETA,EE,0.,0)
      QUAD = TRIANG
  101 CALL TRNSFM(EK,AA,B,C,MM,NNODES(L),12)
```

```
      GO TO 103
  102 CALL ELSTIF(AA,B,C,TH(L),MM,AL,E1)
      QUAD = AL
  103 CALL ASEMBL(SK,C,MA(L),MB(L),MC(L),MD(L),MM,IDIAG,NNODES(L),12)
   30 FORMAT(/1X,6E15.5/)
      IF (NMAT .LE. 1) GO TO 405
      KX = MYOUNG(L)
      RHO = RHO1(KX)
      IF(RHO1(KX) .LE. .00001) RHO=0.1
  405 WEIGHT = WEIGHT + TH(L)*QUAD*RHO
  400 CONTINUE
      WRITE(6,410) WEIGHT
  410 FORMAT(1H0,10X,25HWEIGHT OF THE STRUCTURE =,E15.5)
   35 CONTINUE
C     CALL PRINTK(SK,IDIAG,NN)
      CALL BOUND2(SK,IBND,NN,NBNDRY,IDIAG,ICOL)
      WRITE(6,1009)(ICOL(I),I=1,NM)
      WRITE(6,1009)(IDIAG(I),I=1,NM)
      NONZRO=IDIAG(NM)
 1009 FORMAT(1X,10I13)
      NDCOMP=0
      CALL REDUCE(FR,IBND,NN,NBNDRY,LOADS,NNMAX)
      CALL GAUSS(SK,FR,DR,ICOL,IDIAG,LOADS,NM,NNMAX,NDCOMP)
      IF(NDCOMP.EQ.10) GO TO 2000
      CALL RESTOR(DR,IBND,NN,NBNDRY,LOADS,NNMAX)
      CALL RESTOR(FR,IBND,NN,NBNDRY,LOADS,NNMAX)
      DO 112 I=1,NN
      DO 112 J=1,LOADS
  112 DR(I,J)=DR(I,J)/EEE
      DO 180 I = 1,LOADS
      ENGSTR(I) = 0.0
      DO 179 J = 1,NN
      ENGSTR(I) = ENGSTR(I) + FR(J,I)*DR(J,I)
  179 CONTINUE
      ENGSTR(I) = .5*ENGSTR(I)
  180 CONTINUE
      NPAGE=1
      LINES = 1
      DO 1501 I=1,LOADS
 1501 ENGTOT(I)=0.
      DO 300 L=1,MEMBS
      IF (NMAT .LE. 1) GO TO 85
      KX = MYOUNG(L)
      E = YOUNGM(KX)*10**6
      PMU = POISON(KX)
      E1 = E/EEE
      CALL ELSTIC(E1,PMU,EE)
      TYPE = NNODES(L)*10 + KX
      IF (MSSTRS .EQ. 0) GO TO 86
      KY = 3*(KX-1) + 1
      ALS(1) = ALSTRS(KY)
      ALS(2) = ALSTRS(KY+1)
      ALS(3) = ALSTRS(KY+2)
      GO TO 86
   85 TYPE = NNODES(L)*10 + 1
   86 IF((LINES+LOADS) .LT. 54 .AND. L .GT. 1)GO TO 84
      LINES=1
```

```
      WRITE(6,98)NPAGE
      NPAGE=NPAGE+1
      WRITE(6,83)
   84 CONTINUE
      CALL COORD(MA(L),MB(L),MC(L),MD(L),X,Y,Z,AA,XI,ETA,AL,NNODES(L),0)
      CALL ELFORC(AA,DR,EDR,MM,MA(L),MB(L),MC(L),MD(L),NNODES(L),LOADS,
     1NNMAX)
      IF(NNODES(L) .LE. 3)GO TO 110
      CALL QDRLTL(EK,EKK,TH(L),QUAD,MA(L),MB(L),MC(L),MD(L),MAA,MBB,MCC,
     1XI, ETA,NNODES(L),EE,TRANG,1)
      CALL QLSTRS(EDR,EDDR,XI,ETA,MAA,MBB,MCC,SX,SY,SXY,EFSTRS,E,PMU,
     1LOADS,SSX,SSY,SSXY,EFFSTR,KTR,EKK,ELEENG,SFTM,ALS,ESR,NNODES(L))
      KX=KTR(1)
      ELEENG(1)=ELEENG(1)*0.5*TH(L)
      ENGTOT(1)=ENGTOT(1)+ELEENG(1)
      IF (NNODES(L) .EQ. 5) GO TO 220
      WRITE(6,87) L,TH(L),QUAD,TYPE,MA(L),MB(L),MC(L),MD(L),SSX(KX,1),
     1SSY(KX,1),SSXY(KX,1),(EFFSTR(I,1),I=1,4),ELEENG(1),SFTM(1)
  222 IF(LOADS .EQ. 1)GO TO 300
      DO 211 K=2,LOADS
      KX=KTR(K)
      ELEENG(K)=ELEENG(K)*0.5*TH(L)
      ENGTOT(K)=ENGTOT(K)+ELEENG(K)
      IF (NNODES(L) .EQ. 5) GO TO 225
      WRITE(6,95)SSX(KX,K),SSY(KX,K),SSXY(KX,K),(EFFSTR (I,K),I=1,4),
     1ELEENG(K),SFTM(K)
      GO TO 211
  225 WRITE(6,82) SSXY(KX,K),(EFFSTR(I,K),I=1,4),ELEENG(K),SFTM(K)
  211 CONTINUE
      GO TO 300
  220 WRITE(6,81) L,TH(L),QUAD,TYPE,MA(L),MB(L),MC(L),MD(L),
     1SSXY(KX,1),(EFFSTR(I,1),I=1,4),ELEENG(1),SFTM(1)
      GO TO 222
  110 IF(NNODES(L) .LT. 3)GO TO 213
      CALL STRESS(EDR,XI,ETA,1,2,3,SX,SY,SXY,EFSTRS,E,PMU,ALS,ESR,
     1LOADS,ELEENG,TRIANG,3)
      ELEENG(1)=ELEENG(1)*0.5*TH(L)
      ENGTOT(1)=ENGTOT(1)+ELEENG(1)
      SFTM(1) = (1.0 - ESR(1))/ESR(1)
      WRITE(6,88) L,TH(L),TRIANG,TYPE,MA(L),MB(L),MC(L),SX(1),SY(1),
     1SXY(1),EFSTRS(1),ELEENG(1),SFTM(1)
      IF(LOADS .EQ. 1)GO TO 300
      DO 212 K=2,LOADS
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
      ELEENG(K)=ELEENG(K)*0.5*TH(L)
      ENGTOT(K)=ENGTOT(K)+ELEENG(K)
  212 WRITE(6,94)SX(K),SY(K),SXY(K),EFSTRS(K),ELEENG(K),SFTM(K)
      GO TO 300
  213 DO 215 K=1,LOADS
      SX(K)=E*(EDR(2,K)-EDR(1,K))/AL
      ELEENG(K)=(0.5*SX(K)**2/E  )*AL*TH(L)
      ESF(K) = SQRT((SX(K)/ALS(1))**2)
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
      IF (SX(K) .GE. 0.0) GO TO 215
      ESF(K) = SQRT((SX(K)/ALS(2))**2)
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
  215 ENGTOT(K)=ENGTOT(K)+ELEENG(K)
```

```
      WRITE(6,89) L,TH(L),AL,TYPE,MA(L),MB(L),SX(1),ELEENG(1)
      IF(LOADS .EQ. 1)GO TO 300
      DO 214 K=2,LOADS
  214 WRITE(6,93)SX(K),ELEENG(K),SFTM(K)
  300 LINES=LINES+LOADS+1
      DO 1503 KL=1,LOADS
 1503 WRITE(6,1502)KL,ENGTOT(KL),ENGSTR(KL)
 1502 FORMAT(///,20X,39HTHE TOTAL ENERGY FOR LOADING CONDITION ,I2,4H IS
     1 ,F12.4,2X,3H(U),10X,E12.4,3H(W))
   90 LINES=1
      CALL PRNTDR(FR,OF,X,Y,Z,NN,MM,LOADS,JOINTS,NPAGE,NNMAX)
   83 FORMAT(1X,4HMEMB,2X,5HTHICK,3X,4HAREA,2X,4HTYPE,1X,2HMA,2X,2HMB,
     12X,2HMC,2X,2HMD,3X,7HSIGMA-X,4X,7HSIGMA-Y,3X,8HSIGMA-XY,3X,
     27HEFSTR-1,3X,7HEFSTR-2,3X,7HEFSTR-3,3X,7HEFSTR-4,4X,6HENERGY,
     36X,2HMS)
   81 FORMAT(/I5,F7.3,F9.2,5I4,22X,E11.4,5E10.4,E10.3)
   82 FORMAT(63X,E11.4,5E10.4,E10.3)
   87 FORMAT(/I5, F7.3,F9.2,5I4,3E11.4,5E10.4,E10.3)
   88 FORMAT(/I5, F7.3,F9.2,4I4,4X,3E11.4,E10.4,30X,E10.4,E10.3)
   89 FORMAT(/I5, F7.3,F9.2,3I4,8X,E11.4,62X,E10.4,E10.3)
   93 FORMAT(41X,E11.4,62X,E10.4,E10.3)
   94 FORMAT(41X,3E11.4,E10.4,30X,E10.4,E10.3)
   95 FORMAT(41X,3E11.4,5E10.4,E10.3)
   98 FORMAT(1H1,120X,5HPAGE ,I3/)
 2000 IF(KSTR.EQ.NSTR) GO TO 1000
      KSTR=KSTR+1
      GO TO 1
 1000 CONTINUE
      STOP
      END
```

```
      SUBROUTINE POP(MMB,JN,MM,MA,MB,MC,MD,KTYPE,IC,ID,NZ,NR)
      DIMENSION MA(1),MB(1),MC(1),MD(1),IC(1),ID(1),KTYPE(1)
      IX(I,J)=I*(J-1)+1
      NZ=0
      NN=MM*JN
      NET=0
      DO 10 I=1,NN
   10 IC(I)=NN
      DO 50 L=1,MMB
      NNODES=2
      ITRI=0
      KX=IX(MM,MA(L))
      KY=IX(MM,MB(L))
   15 IF(IC(KY) .LT. KX) GO TO 18
      DO 19 I=1,MM
      IC(KY)=KX
   19 KY=KY+1
   18 IF(KTYPE(L)-3)20,16,17
   16 IF(ITRI .EQ. 1)GO TO 20
      KY=IX(MM,MC(L))
      ITRI=1
      NNODES=3
      GO TO 15
   17 IF(ITRI .EQ. 2)GO TO 20
      IF(ITRI .EQ. 1)GO TO 14
      KY=IX(MM,MC(L))
      ITRI=ITRI+1
      NNODES=4
      GO TO 15
   14 KY=IX(MM,MD(L))
      ITRI=ITRI+1
      GO TO 15
   20 NET=NET+(MM**2)*((NNODES*(NNODES-1))/2)
   50 CONTINUE
      NET=NET-(MM**2)*NR
      DO 30 I=1,NN,MM
      IF(IC(I) .LT. I)GO TO 30
      KX=I
      DO 25 J=1,MM
      IC(KX)=I
   25 KX=KX+1
   30 CONTINUE
      DO 40 I=1,NN
      NZ=NZ+(I-IC(I)+1)
   40 ID(I)=NZ
      KX=(NN*(NN+1))/2
      NET=NET+(MM*(MM+1)*JN)/2
      WRITE(6,2)
      WRITE(6,3) KX,NET,NZ
      WRITE(6,4)
      WRITE(6,5)(IC(I),I=1,NN)
      WRITE(6,6)
      WRITE(6,5)(ID(I),I=1,NN)
    2 FORMAT(1H1,////20X,16HGROSS POPULATION,4X,14HNET POPULATION,
     14X,19HAPPARENT POPULATION///)
    3 FORMAT(18X,I14,I18,I22//)
    4 FORMAT(//2X,36HSTARTING ROW NUMBERS FOR EACH COLUMN///)
    5 FORMAT(5X,10I12)
    6 FORMAT(//2X,61HNUMBERS OF DIAGONAL ELEMENTS IN SINGLE ARRAYSTIFFNE
     1SS MATRIX ///)
      RETURN
      END
```

```
SUBROUTINE ELSTIC       74/74    OPT=1      FTN 4.6+446          08/21/78  10.12.39          PAGE      1

                       SUBROUTINE ELSTIC(E,PMU,EE)                                          ELSTIC       2
                       DIMENSION EE(3,3)                                                    ELSTIC       3
                       PMU1 = 1.0 - PMU**2                                                  ELSTIC       4
                       EE(1,1) = E/PMU1                                                     ELSTIC       5
                       EE(2,1) = E*PMU/PMU1                                                 ELSTIC       6
                       EE(3,1) = 0.0                                                        ELSTIC       7
                       EE(2,2) = EE(1,1)                                                    ELSTIC       8
                       EE(3,2) = 0.0                                                        ELSTIC       9
                       EE(3,3) = E/(2.*(1.0 + PMU))                                         ELSTIC      10
                       DO 18 I = 1,2                                                        ELSTIC      11
                       IP = I + 1                                                           ELSTIC      12
                       DO 18 J = IP,3                                                       ELSTIC      13
                 18    EE(I,J) = EE(J,I)                                                    ELSTIC      14
                       RETURN                                                               ELSTIC      15
                       END                                                                  ELSTIC      16
```

```
SUBROUTINE COORD       74/74    OPT=1    FTN 4.6+446        08/21/78  10.12.39          PAGE      1

      SUBROUTINE COORD(K1,K2,K3,K4,X,Y,Z,AA,XI,ETA,AL,NND,NO)            COORD      2
      DIMENSION X(1),Y(1),Z(1),AA(3,3),AB(3),XI(5),ETA(5)                COORD      3
      XCOMP=X(K2)-X(K1)                                                  COORD      4
      YCOMP=Y(K2)-Y(K1)                                                  COORD      5
      ZCOMP=Z(K2)-Z(K1)                                                  COORD      6
      AL=SQRT(XCOMP**2+YCOMP**2+ZCOMP**2)                                COORD      7
      AA(1,1)=XCOMP/AL                                                   COORD      8
      AA(1,2)=YCOMP/AL                                                   COORD      9
      AA(1,3)=ZCOMP/AL                                                   COORD     10
      IF(NND .LT. 3)RETURN                                               COORD     11
      XCOMP=X(K3)-X(K1)                                                  COORD     12
      YCOMP=Y(K3)-Y(K1)                                                  COORD     13
      ZCOMP=Z(K3)-Z(K1)                                                  COORD     14
      AL=SQRT(XCOMP**2+YCOMP**2+ZCOMP**2)                                COORD     15
      AB(1)=XCOMP/AL                                                     COORD     16
      AB(2)=YCOMP/AL                                                     COORD     17
      AB(3)=ZCOMP/AL                                                     COORD     18
      AL=SQRT((AA(1,2)*AB(3)-AA(1,3)*AB(2))**2+(AA(1,3)*AB(1)            COORD     19
     1-AA(1,1)*AB(3))**2+(AA(1,1)*AB(2)-AA(1,2)*AB(1))**2)               COORD     20
      AA(2,1)=((AA(1,3)**2)*AB(1)-AA(1,1)*AA(1,3)*AB(3)-AA(1,1)*         COORD     21
     1AA(1,2)*AB(2)+(AA(1,2)**2)*AB(1))/AL                               COORD     22
      AA(2,2)=((AA(1,1)**2)*AB(2)-AA(1,1)*AA(1,2)*AB(1)-AA(1,2)*         COORD     23
     2AA(1,3)*AB(3)+(AA(1,3)**2)*AB(2))/AL                               COORD     24
      AA(2,3)=((AA(1,2)**2)*AB(3)-AA(1,2)*AA(1,3)*AB(2)-AA(1,1)*         COORD     25
     3AA(1,3)*AB(1)+(AA(1,1)**2)*AB(3))/AL                               COORD     26
      IF(NO .EQ. 1)RETURN                                                COORD     27
      XI(1)=0.0                                                          COORD     28
      ETA(1)=0.0                                                         COORD     29
      XI(2)=(X(K2)-X(K1))*AA(1,1)+(Y(K2)-Y(K1))*AA(1,2)+(Z(K2)-Z(K1))*AA COORD     30
     1(1,3)                                                              COORD     31
      ETA(2)=0.0                                                         COORD     32
      XI(3)=(X(K3)-X(K1))*AA(1,1)+(Y(K3)-Y(K1))*AA(1,2)+(Z(K3)-Z(K1))*AA COORD     33
     1(1,3)                                                              COORD     34
      ETA(3)=(X(K3)-X(K1))*AA(2,1)+(Y(K3)-Y(K1))*AA(2,2)+(Z(K3)-Z(K1))*A COORD     35
     1A(2,3)                                                             COORD     36
      IF(NND .LE. 3)RETURN                                               COORD     37
      XI(4)=(X(K4)-X(K1))*AA(1,1)+(Y(K4)-Y(K1))*AA(1,2)+(Z(K4)-Z(K1))*AA COORD     38
     1(1,3)                                                              COORD     39
      ETA(4)=(X(K4)-X(K1))*AA(2,1)+(Y(K4)-Y(K1))*AA(2,2)+(Z(K4)-Z(K1))*A COORD     40
     1A(2,3)                                                             COORD     41
      XI(5)=(XI(2)+XI(3)+XI(4)) /4.0                                     COORD     42
      ETA(5)=(ETA(3)+ETA(4))/4.0                                         COORD     43
      RETURN                                                             COORD     44
      END                                                                COORD     45
```

```
SUBROUTINE ELSTIF       74/74    OPT=1      FTN 4.6+446          08/21/78  10.12.39           PAGE     1

                  SUBROUTINE ELSTIF(A,B,C,AE,MM,AL,E)                                     ELSTIF       2
                  DIMENSION A(3,3),B(12,12),C(12,12)                                      ELSTIF       3
                  EK=AE*E/AL                                                              ELSTIF       4
                  DO 25 I=1,MM                                                            ELSTIF       5
                  J=I+MM                                                                  ELSTIF       6
                  B(1,I)=EK*A(1,I)                                                        ELSTIF       7
                  B(1,J)= -B(1,I)                                                         ELSTIF       8
                  B(2,I) =-B(1,I)                                                         ELSTIF       9
               25 B(2,J)=B(1,I)                                                           ELSTIF      10
                  DO 26 I=1,MM                                                            ELSTIF      11
                  DO 26 J=1,MM                                                            ELSTIF      12
               26 C(I,J)=A(1,I)*B(1,J)                                                    ELSTIF      13
                  DO 36 I=1,MM                                                            ELSTIF      14
                  I1=I+MM                                                                 ELSTIF      15
                  DO 36 J=1,MM                                                            ELSTIF      16
                  J1=J+MM                                                                 ELSTIF      17
                  C(I,J1)=-C(I,J)                                                         ELSTIF      18
                  C(J1,I)=-C(I,J)                                                         ELSTIF      19
               36 C(I1,J1)=C(I,J)                                                         ELSTIF      20
                  RETURN                                                                  ELSTIF      21
                  END                                                                     ELSTIF      22
```

```
      SUBROUTINE PLSTIF(EKK,TH,TRIANG,MA,MB,MC,X,Y,EE,SHR,NONORM)
      DIMENSION EKK(12,12),X(1),Y(1),EE(3,3),
     1     U(6),               A(3,3),E1(3),E2(3),AX(3)
      CALL CRAMER(A,TRIANG,X,Y,MA,MB,MC)
      DO 20 I = 1,6
      DO 15 II = 1,6
   15 U(II) = 0.0
      U(I) = 1.0
      E1(1) = A(1,1)*U(1) + A(1,2)*U(3) + A(1,3)*U(5)
      E1(2) = A(2,1)*U(2) + A(2,2)*U(4) + A(2,3)*U(6)
      E1(3) = A(1,1)*U(2) + A(1,2)*U(4) + A(1,3)*U(6) + A(2,1)*U(1) +
     1                 A(2,2)*U(3) + A(2,3)*U(5)
      DO 20 J = I,6
      DO 16 II = 1,6
   16 U(II) = 0.0
      U(J) = 1.0
      E2(1) = A(1,1)*U(1) + A(1,2)*U(3) + A(1,3)*U(5)
      E2(2) = A(2,1)*U(2) + A(2,2)*U(4) + A(2,3)*U(6)
      E2(3) = A(1,1)*U(2) + A(1,2)*U(4) + A(1,3)*U(6) + A(2,1)*U(1) +
     1                 A(2,2)*U(3) + A(2,3)*U(5)
      EKK(I,J) = 0.0
      IF (NONORM .EQ. 0) GO TO 14
      AX(1)=SHR**2
      AX(2)=2.*AX(1)-1.
      AX(1)=2.*SQRT((1.-AX(1))*AX(1))
      E1(3)=(E1(2)-E1(1))*AX(1)+E1(3)*AX(2)
      E2(3)=(E2(2)-E2(1))*AX(1)+E2(3)*AX(2)
      E1(1) = 0.0
      E1(2) = 0.0
      E2(1) = 0.0
      E2(2) = 0.0
   14 DO 18 K = 1,3
      AX(K) = 0.0
      DO 17 L = 1,3
   17 AX(K) = AX(K) + EE(K,L)*E2(L)
   18 CONTINUE
      DO 19 K = 1,3
   19 EKK(I,J) = EKK(I,J) + E1(K)*AX(K)
      EKK(I,J) = EKK(I,J)*TH*TRIANG
   20 CONTINUE
      DO 30 I = 1,5
      IX = I + 1
      DO 30 J = IX,6
   30 EKK(J,I) = EKK(I,J)
      RETURN
      END
```

```
SUBROUTINE CRAMER      74/74    OPT=1      FTN 4.6+446         08/21/78  10.12.39          PAGE      1

                 SUBROUTINE CRAMER(A,TRIANG,X,Y,MA,MB,MC)                                  CRAMER       2
                 DIMENSION A(3,3),X(1),Y(1)                                                CRAMER       3
                 TRIANG = X(MA)*(Y(MB) - Y(MC)) -Y(MA)*(X(MB) - X(MC)) +                   CRAMER       4
                1          (X(MB)*Y(MC) - X(MC)*Y(MB))                                     CRAMER       5
                 A(1,1) = Y(MB) - Y(MC)                                                    CRAMER       6
                 A(2,1) = X(MC) - X(MB)                                                    CRAMER       7
                 A(3,1) = X(MB)*Y(MC) - X(MC)*Y(MB)                                        CRAMER       8
                 A(1,2) = Y(MC) - Y(MA)                                                    CRAMER       9
                 A(2,2) = X(MA) - X(MC)                                                    CRAMER      10
                 A(3,2) = X(MC)*Y(MA) - X(MA)*Y(MC)                                        CRAMER      11
                 A(1,3) = Y(MA) - Y(MB)                                                    CRAMER      12
                 A(2,3) = X(MB) - X(MA)                                                    CRAMER      13
                 A(3,3) = X(MA)*Y(MB) - X(MB)*Y(MA)                                        CRAMER      14
                 DO 10 I = 1,3                                                             CRAMER      15
                 DO 10 J = 1,3                                                             CRAMER      16
              10 A(I,J) = A(I,J)/TRIANG                                                    CRAMER      17
                 TRIANG =(ABS(TRIANG))/2.0                                                 CRAMER      18
                 RETURN                                                                    CRAMER      19
                 END                                                                       CRAMER      20
```

```
SUBROUTINE QDRLTL       74/74   OPT=1     FTN 4.6+446       08/21/78  10.12.39        PAGE    1

      SUBROUTINE QDRLTL(EK,EKK,TH,QUAD,MA,MB,MC,MD,MAA,MBB,MCC,XI,ETA,
     1NNODES,EE,TRANG,NO)
      DIMENSION EK(12,12),EKK(12,12),MAA(1),MBB(1),MCC(1),XI(5),ETA(5)
     1,EE(3,3),TRANG(1)
      DO 125 I=1,12
      DO 125 J=1,12
  125 EK(I,J)=0.
      NNRM=0
      SHR=1.0
      IF(NNODES     .LE. 4)GO TO 108
      NNRM=1
      IF(NNODES     .EQ. 5)GO TO 108
      IF(NNODES     - 7)104,105,106
  104 XCOMP=XI(3)-XI(2)
      YCOMP=ETA(3)-ETA(2)
      GO TO 107
  105 XCOMP=XI(4)-XI(3)
      YCOMP=ETA(4)-ETA(3)
      GO TO 107
  106 XCOMP=XI(4)-XI(1)
      YCOMP=ETA(4)-ETA(1)
  107 ALL=SQRT(XCOMP**2+YCOMP**2)
      SHR=XCOMP/ALL
  108 QUAD=0.
      DO 130 I=1,4
      CALL PLSTIF(EKK,TH,TRIANG,MAA(I),MBB(I),MCC(I),XI,ETA,EE,SHR,NNRM)
      QUAD=QUAD+TRIANG
      TRANG(I)=TRIANG
  130 CALL SUM(EK,EKK,MAA(I),MBB(I),MCC(I))
      CALL CONDNS(EK,EKK,MA,MB,MC,MD,NO)
      RETURN
      END
```

```
SUBROUTINE SUM      74/74   OPT=1     FTN 4.6+446       08/21/78  10.12.39        PAGE    1

            SUBROUTINE SUM(EK,EKK,MA,MB,MC)
            DIMENSION EK(12,12),EKK(12,12),NA(3)
            M=2
            NA(1)=2*(MA-1)+1
            NA(2)=2*(MB-1)+1
            NA(3)=2*(MC-1)+1
            IH=0
            DO 100 I=1,6
            JH=0
            IF(I .LE. IH)GO TO 30
            IH=IH+M
            IHH=IH/M
            KX=NA(IHH)
         30 DO 90 J=1,6
            IF(J .LE. JH)GO TO 60
            JH=JH+M
            IHH=JH/M
            KY=NA(IHH)
         60 EK(KX,KY)=EK(KX,KY)+EKK(I,J)
         90 KY=KY+1
        100 KX=KX+1
            RETURN
            END
```

```
      SUBROUTINE CONDNS(EK,EKK,MA,MB,MC,MD,NO)
      DIMENSION EK(12,12),EKK(12,12)
      DO 5 I=1,8
      DO 5 J=1,8
    5 EKK(I,J)=0.
      DET=EK(9,9)*EK(10,10)-EK(9,10)**2
      AX=EK(9,9)
      EK(9,9)=EK(10,10)/DET
      EK(10,10)=AX/DET
      EK(9,10)=-EK(9,10)/DET
      EK(10,9)=EK(9,10)
      KX=0
      DO 10 I=9,10
      KX=KX+1
      DO 10 J=1,8
      DO 10 K=9,10
   10 EKK(KX,J)=EKK(KX,J)+EK(I,K)*EK(K,J)
      IF(NO .EQ. 1)RETURN
      KX=0
      DO 20 I=9,10
      KX=KX+1
      DO 20 J=1,8
      EK(I,J)=EKK(KX,J)
   20 EKK(KX,J)=0
      DO 30 I=1,8
      DO 30 J=1,8
      DO 30 K=9,10
   30 EKK(I,J)=EKK(I,J)+EK(I,K)*EK(K,J)
      DO 40 I=1,8
      DO 40 J=1,8
   40 EK(I,J)=EK(I,J)-EKK(I,J)
      IF(MC .LT. MB)CALL CHANGE(EK,3,5,4,12,12,0)
      IF(MD .LT. MB)CALL CHANGE(EK,3,7,4,12,12,0)
      IF(MD .LT. MC)CALL CHANGE(EK,5,7,4,12,12,0)
      RETURN
      END
```

```
SUBROUTINE CHANGE     74/74    OPT=1       FTN 4.6+446         08/21/78  10.12.39           PAGE     1

              SUBROUTINE CHANGE(EK,IX,IY,NND,M,L,IR)                              CHANGE      2
              DIMENSION EK( M, L)                                                 CHANGE      3
              KX=IX                                                               CHANGE      4
              KY=IY                                                               CHANGE      5
              M2=2*NND                                                            CHANGE      6
              IF(IP .EQ. 1)M2=L                                                   CHANGE      7
              DO 10 I=1,2                                                         CHANGE      8
              DO 5 J=1,M2                                                         CHANGE      9
              AX=EK(KX,J)                                                         CHANGE     10
              EK(KX,J)=EK(KY,J)                                                   CHANGE     11
          5   EK(KY,J)=AX                                                         CHANGE     12
              KX=KX+1                                                             CHANGE     13
         10   KY=KY+1                                                             CHANGE     14
              IF(IR .EQ. 1)RETURN                                                 CHANGE     15
              KX=KX-2                                                             CHANGE     16
              KY=KY-2                                                             CHANGE     17
              DO 20 I=1,2                                                         CHANGE     18
              DO 15 J=1,M2                                                        CHANGE     19
              AX=EK(J,KX)                                                         CHANGE     20
              EK(J,KX)=EK(J,KY)                                                   CHANGE     21
         15   EK(J,KY)=AX                                                         CHANGE     22
              KX=KX+1                                                             CHANGE     23
         20   KY=KY+1                                                             CHANGE     24
              RETURN                                                              CHANGE     25
              END                                                                 CHANGE     26
```

```
SUBROUTINE TRNSFM      74/74   OPT=1      FTN 4.6+446        08/21/78  10.12.39                 PAGE     1

                  SUBROUTINE TRNSFM(EK,AA,B,C,MM,NND,M)                          TRNSFM      2
                  DIMENSION EK(12,12),AA(3,3),B( M, M),C( M, M)                  TRNSFM      3
                  M2=2*NND                                                       TRNSFM      4
                  IF(NND .GT. 4)M2=8                                             TRNSFM      5
                  M3=MM*NND                                                      TRNSFM      6
                  IF(NND .GT. 4)M3=4*MM                                          TRNSFM      7
                  DO 100 I=1,M2                                                  TRNSFM      8
                  JA=MM                                                          TRNSFM      9
                  KA=0                                                           TRNSFM     10
                  IA=0                                                           TRNSFM     11
                  DO 100 J=1,M3                                                  TRNSFM     12
                  B(I,J)=0.0                                                     TRNSFM     13
                  IF(J-JA)90,90,80                                               TRNSFM     14
           80     JA=JA+MM                                                       TRNSFM     15
                  KA=KA+2                                                        TRNSFM     16
                  IA=IA+MM                                                       TRNSFM     17
           90     JAA=J-IA                                                       TRNSFM     18
                  DO 100 K=1,2                                                   TRNSFM     19
                  KAA=K+KA                                                       TRNSFM     20
          100     B(I,J)=B(I,J)+EK(I,KAA)*AA(K,JAA)                              TRNSFM     21
                  DO 200 J=1,M3                                                  TRNSFM     22
                  JA=MM                                                          TRNSFM     23
                  KA=0                                                           TRNSFM     24
                  IA=0                                                           TRNSFM     25
                  DO 200 I=1,M3                                                  TRNSFM     26
                  C(I,J)=0.0                                                     TRNSFM     27
                  IF(I-JA)190,190,180                                            TRNSFM     28
          180     JA=JA+MM                                                       TRNSFM     29
                  KA=KA+2                                                        TRNSFM     30
                  IA=IA+MM                                                       TRNSFM     31
          190     JAA=I-IA                                                       TRNSFM     32
                  DO 200 K=1,2                                                   TRNSFM     33
                  KAA=K+KA                                                       TRNSFM     34
          200     C(I,J)=C(I,J)+AA(K,JAA)*B(KAA,J)                               TRNSFM     35
                  RETURN                                                         TRNSFM     36
                  END                                                            TRNSFM     37
```

```
      SUBROUTINE ASEMBL(A,B,MA,MB,MC,MD,MM,ID,NNODES,M)
      DIMENSION A(1),B(M,M),ID(1),NA(4),NAA(3)
      IX(I,J)=I*(J-1)+1
      NND=NNODES
      IF(NND .GT. 4)NND=4
      M2=NND*MM
      NA(1)=IX(MM,MA)
      NA(2)=IX(MM,MB)
      IF(NNODES .GE. 3)NA(3)=IX(MM,MC)
      IF(NNODES .GE. 4)NA(4)=IX(MM,MD)
      IF(NNODES .LE. 3)GO TO 5
      DO 4 I=1,3
      KX=I/3
      KY=I/2
      IF(NA(KX+2) .LT. NA(KY+3))GO TO 4
      KH=NA(KX+2)
      NA(KX+2)=NA(KY+3)
      NA(KY+3)=KH
    4 CONTINUE
    5 DO 10 I=2,NND
   10 NAA(I-1)=NA(I)-NA(I-1)-MM
      KH=MM
      IAA=NA(1)
      KHH=1
      DO 30 J=1,M2
      IF(J .LE. KH)GO TO 15
      KHH=KHH+1
      IAA=NA(KHH)
      KH=KH+MM
   15 JX=ID(IAA)-IAA+NA(1)
      KY=MM
      DO 25 I=1,J
      IF(J .LE.KY .OR. I .LE. KY)GO TO 20
      KX=I/MM
      JX=JX+NAA(KX)
      KY=KY+MM
   20 A(JX)=A(JX)+B(I,J)
   25 JX=JX+1
   30 IAA=IAA+1
      RETURN
      END
```

```
SUBROUTINE PRINTK     74/74    OPT=1      FTN 4.6+446            08/21/78  10.12.39            PAGE     1

                  SUBROUTINE PRINTK(SK,IDIAG,NN)                               PRINTK       2
                  DIMENSION SK(1),IDIAG(1)                                     PRINTK       3
                  DO 80 I=1,NN                                                 PRINTK       4
                  IF(I .GT. 1) GO TO 65                                        PRINTK       5
                  KX=1                                                         PRINTK       6
                  KY=1                                                         PRINTK       7
                  GO TO 70                                                     PRINTK       8
              65  KX=IDIAG(I-1)+1                                              PRINTK       9
                  KY=IDIAG(I)                                                  PRINTK      10
              70  WRITE(6, 3)I                                                 PRINTK      11
              80  WRITE(6,2)(SK(K),K=KX,KY)                                    PRINTK      12
               3  FORMAT(I4)                                                   PRINTK      13
               2  FORMAT(10X,10E12.4)                                          PRINTK      14
                  RETURN                                                       PRINTK      15
                  END                                                          PRINTK      16
```

```
SUBROUTINE BOUND2       74/74    OPT=1      FTN 4.6+446         08/21/78  10.12.39         PAGE      1

                          SUBROUTINE BOUND2(A,IB,N,NB,ID,IC)                                  BOUND2       2
                          DIMENSION A(1),IB(1),ID(1),IC(1)                                    BOUND2       3
                          IH=NB                                                               BOUND2       4
                          NH=N                                                                BOUND2       5
                          DO 30 JA=1,NB                                                       BOUND2       6
                          IA=IB(IH)                                                           BOUND2       7
                          IF(IA .GE. NH) GO TO 20                                             BOUND2       8
                          KH=IA+1                                                             BOUND2       9
                          IF(IA .GT. 1) GO TO 5                                               BOUND2      10
                          KX=1                                                                BOUND2      11
                          JX=1                                                                BOUND2      12
                          GO TO 6                                                             BOUND2      13
                        5 JX=ID(IA)-ID(IA-1)                                                  BOUND2      14
                          KX=ID(IA-1)+1                                                       BOUND2      15
                        6 DO 10 I=KH,NH                                                       BOUND2      16
                          KY=1                                                                BOUND2      17
                          IF(IC(I) .LE. IA) GO TO 7                                           BOUND2      18
                          IC(I-1)=IC(I)-1                                                     BOUND2      19
                          I1=I                                                                BOUND2      20
                          KY=0                                                                BOUND2      21
                          GO TO 8                                                             BOUND2      22
                        7 IC(I-1)=IC(I)                                                       BOUND2      23
                          I1=I-1                                                              BOUND2      24
                        8 K=IC(I)                                                             BOUND2      25
                          ID(I-1)=ID(I)-JX-KY                                                 BOUND2      26
                          DO 10 J=K,I1                                                        BOUND2      27
                          IF(J .EQ. IA) JX=JX+1                                               BOUND2      28
                          KXX=KX+JX                                                           BOUND2      29
                          A(KX)=A(KXX)                                                        BOUND2      30
                       10 KX=KX+1                                                             BOUND2      31
                       20 NH=NH-1                                                             BOUND2      32
                          IH=IH-1                                                             BOUND2      33
                       30 CONTINUE                                                            BOUND2      34
                          PETURN                                                              BOUND2      35
                          END                                                                 BOUND2      36
```

```
SUBROUTINE REDUCE        74/74    OPT=1       FTN 4.6+446          08/21/78  10.12.39          PAGE      1

                        SUBROUTINE REDUCE (F,IB,N,NB,L,NN)
                        DIMENSION F(NN,L)  ,IB(1)
                        DO 5 J=1,L
                        IH=NB
                        NH=N
              1         I=IB(IH)
                        IF(I-NH) 2,4,4
              2         NH1=NH-1
                        DO 3 K=I,NH1
                        K1=K+1
              3         F(K,J) =F(K1,J)
              4         IH=IH-1
                        NH=NH-1
                        IF(IH.EQ.0) GO TO 5
                        GO TO1
              5         CONTINUE
                        RETURN
                        END
```

```
SUBROUTINE GAUSS       74/74    OPT=1      FTN 4.6+446          08/21/78  10.12.39         PAGE      1

                 SUBROUTINE  GAUSS(A,F,D,IC,ID,L,N,NN,NDCOMP)                              GAUSS       2
                 DIMENSION A(1),IC(1), ID(1),F(NN,L),D(NN,L)                               GAUSS       3
                 IF(NDCOMP .EQ. 1)GO TO 15                                                 GAUSS       4
                 DO 10 I=1,N                                                               GAUSS       5
                 I1=I-1                                                                    GAUSS       6
                 DO  9 J=I,N                                                               GAUSS       7
                 IF(IC(J) .GT. I)GO TO 9                                                   GAUSS       8
                 IX=ID(J)-J+I                                                              GAUSS       9
                 IF(I1 .EQ. 0)GO TO 8                                                      GAUSS      10
                 DO  7 K=1,I1                                                              GAUSS      11
                 IF(IC(J) .GT. K .OR. IC(I) .GT. K)GO TO 7                                 GAUSS      12
                 KX=ID(I)-I+K                                                              GAUSS      13
                 KY=ID(J)-J+K                                                              GAUSS      14
                 KZ=ID(K)                                                                  GAUSS      15
                 A(IX)=A(IX)-(A(KX)*A(KZ)* A(KY))                                          GAUSS      16
               7 CONTINUE                                                                  GAUSS      17
               8 IF(I .EQ. J)GO TO 9                                                       GAUSS      18
                 KZ=ID(I)                                                                  GAUSS      19
                 IF(A(KZ))5,6,5                                                            GAUSS      20
               5 A(IX)=A(IX)/A(KZ)                                                         GAUSS      21
               9 CONTINUE                                                                  GAUSS      22
              10 CONTINUE                                                                  GAUSS      23
              15 DO 40 K=1,L                                                               GAUSS      24
                 DO 30 I=1,N                                                               GAUSS      25
                 D(I,K)=F(I,K)                                                             GAUSS      26
                 I1=I-1                                                                    GAUSS      27
                 IF(I1 .EQ. 0) GO TO 30                                                    GAUSS      28
                 DO 20 J=1,I1                                                              GAUSS      29
                 IF(IC(I) .GT. J)GO TO 20                                                  GAUSS      30
                 IX=ID(I)-I+J                                                              GAUSS      31
                 D(I,K)=D(I,K)-A(IX)*D(J,K)                                                GAUSS      32
              20 CONTINUE                                                                  GAUSS      33
              30 CONTINUE                                                                  GAUSS      34
              40 CONTINUE                                                                  GAUSS      35
                 DO 70 I=1,N                                                               GAUSS      36
                 KX=ID(I)                                                                  GAUSS      37
                 DO 70 K=1,L                                                               GAUSS      38
              70 D(I,K)=D(I,K)/A(KX)                                                       GAUSS      39
                 DO 90 K=1,L                                                               GAUSS      40
                 IX=N                                                                      GAUSS      41
                 DO 90 I=2,N                                                               GAUSS      42
                 IX=IX-1                                                                   GAUSS      43
                 I1=I-1                                                                    GAUSS      44
                 KX=IX                                                                     GAUSS      45
                 DO 80 J=1,I1                                                              GAUSS      46
                 KX=KX+1                                                                   GAUSS      47
                 IF(IC(KX) .GT. IX)GO TO 80                                                GAUSS      48
                 KY=ID(KX)-KX+IX                                                           GAUSS      49
                 D(IX,K)=D(IX,K)-A(KY)*D(KX,K)                                             GAUSS      50
              80 CONTINUE                                                                  GAUSS      51
              90 CONTINUE                                                                  GAUSS      52
             110 RETURN                                                                    GAUSS      53
               6 NDCOMP=10                                                                 GAUSS      54
                 WRITE(6,120)I                                                             GAUSS      55
             120 FORMAT(///2X,46HSTRUCTURE IS UNSTABLE, THE DEGREE OF FREEDOM =,I5)        GAUSS      56
                 RETURN                                                                    GAUSS      57
                 END                                                                       GAUSS      58
```

```
      SUBROUTINE RESTOR( D,IB,N,NB,L,NN)
      DIMENSION D(NN,L),IB(1),TDR1(10),TDR2(10)
      NH=N-NB
      IH=1
    1 I=IB(IH)
      IF(I.GT.NH) GO TO 7
      DO 2 K=1,L
      TDR1(K)=D(I,K)
    2 D(I,K)=0.
    3 J=I+1
      IF(J.GT.NH) GO TO 5
      DO 4 K=1,L
    4 TDR2(K)=D(J,K)
    5 DO 6 K=1,L
      D(J,K) =TDR1(K)
    6 TDR1(K)=TDR2(K)
      IF(I.GE.NH) GO TO 9
      I=I+1
      GO TO 3
    7 DO 8 K=1,L
    8 D(I,K)=0.
    9 IF(IH.GE.NB) GO TO 10
      IH=IH+1
      NH=NH+1
      GO TO 1
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ELFORC(AA,DR,EDR,MM,MA,MB,MC,MD,NNODES,LOADS,NN)
      DIMENSION AA(3,3),DR(NN,LOADS),EDR(12,LOADS),NCON(4)
      NCON(1)=MM*(MA    -1)+1
      NCON(2)=MM*(MB    -1)+1
      IF(NNODES    .GE. 3)NCON(3)=MM*(MC    -1)+1
      IF(NNODES    .GE. 4)NCON(4)=MM*(MD    -1)+1
      NND=NNODES
      IF(NND .GT. 4)NND=4
      NDSP=1
      IF(NND .GT. 2)NDSP=2
      DO 86 K=1,LOADS
      KH=1
      DO 86 KK=1,NND
      DO 86 I=1,NDSP
      KX=NCON (KK)
      EDR(KH,K)=0
      DO 85 J=1,MM
      EDR(KH,K)=EDR(KH,K)+AA(I,J)*DR(KX,K)
   85 KX=KX+1
   86 KH=KH+1
      RETURN
      END
```

```
      SUBROUTINE STRESS(UV,X,Y,MA,MB,MC,SX,SY,SXY,EFST,E,P,ALS,ESR,
     1                  L,ENG,TRIANG,NND)
      DIMENSION UV(12,L),X(1),Y(1),SX(1),SY(1),SXY(1),EX(10),EY(10),
     1EXY(10),A(3,3)       ,EFST(1),ENG(1),ALS(3),ESR(1)
      CALL CRAMER(A,TRIANG,X,Y,MA,MB,MC)
      DO 30 K=1,L
      EX(K)=0.
      EY(K)=0.
      EXY(K)=0.
      KX=0
      DO 20 I=1,3
      IX=I +KX
      EX(K)=EX(K)+A(1,I)*UV(IX,K)
      EY(K)=EY(K)+A(2,I)*UV(IX+1,K)
      EXY(K)=EXY(K)+A(2,I)*UV(IX,K)+A(1,I)*UV(IX+1,K)
   20 KX=KX+1
   30 CONTINUE
      EMU=E/(1.0-P**2)
      G=(0.5*E)/(1.0+P)
      DO 40 K=1,L
      SX(K)=(EX(K)+P*EY(K))*EMU
      SY(K)=(P*EX(K)+EY(K))*EMU
   40 SXY(K)=G*EXY(K)
      DO 90 K=1,L
      AAX = ALS(1)
      AAY = ALS(1)
      AAXY = ALS(3)
      IF (SX(K) .LT. 0.0) AAX = ALS(2)
      IF (SY(K) .LT. 0.0) AAY = ALS(2)
      EFST(K)=SQRT(SX(K)**2+SY(K)**2-SX(K)*SY(K)+3.*(SXY(K)**2))
      ENG(K)=(SX(K)*EX(K)+SY(K)*EY(K)+SXY(K)*EXY(K))*TRIANG
      IF(NND .GT. 4)ENG(K)=(SXY(K)*EXY(K))*TRIANG
      ESR(K) = SQRT((SX(K)/AAX)**2 + (SY(K)/AAY)**2
     1 - ((SX(K)*SY(K))/(AAX*AAY)) + (SXY(K)/AAXY)**2)
      IF (NND .GT. 4) ESR(K) = ABS(SXY(K))/AAXY
 90   CONTINUE
      RETURN
      END
```

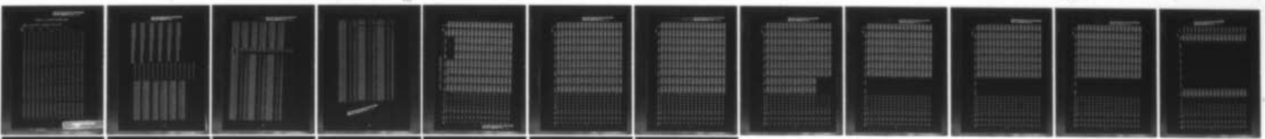
```
      SUBROUTINE QLSTRS(EDR,EDDR,XI,ETA,MAA,MBB,MCC,SX,SY,SXY,EFSTRS,E,  QLSTRS       2
     1PMU,LOADS,SSX,SSY,SSXY,EFFSTR,KTR,EKK,ENG,SFTM,ALS,ESR,NND)        QLSTRS       3
      DIMENSION EDR(12,LOADS),EDDR(12,LOADS),XI(1),ETA(1),MAA(1),MBB(1), QLSTRS       4
     1MCC(1),SX(1),SY(1),SXY(1),EFSTRS(1),SSX(4,LOADS),SSY(4,LOADS),     QLSTRS       5
     2SSXY(4,LOADS),EFFSTR(4,LOADS),KTR(1),EKK(12,12),ENG(1),ENGG(10)    QLSTRS       6
     3,ESR(1),SFTM(1),ALS(1)                                             QLSTRS       7
      DO 115 K=1,LOADS                                                   QLSTRS       8
      SFTM(K) = 0.0                                                      QLSTRS       9
      ENG(K)=0.                                                          QLSTRS      10
      KX=0                                                               QLSTRS      11
      DO 115 I=9,10                                                      QLSTRS      12
      KX=KX+1                                                            QLSTRS      13
      EDR(I,K)=0.                                                        QLSTRS      14
      DO 114 J=1,8                                                       QLSTRS      15
  114 EDR(I,K)=EDR(I,K)+EKK(KX,J)*EDR(J,K)                               QLSTRS      16
      EDR(I,K)=-EDR(I,K)                                                 QLSTRS      17
  115 CONTINUE                                                           QLSTRS      18
      DO 116 K=1,LOADS                                                   QLSTRS      19
      EDDR(5,K)=EDR(9,K)                                                 QLSTRS      20
  116 EDDR(6,K)=EDR(10,K)                                                QLSTRS      21
      KX=1                                                               QLSTRS      22
      KY=3                                                               QLSTRS      23
      QUAD = 0.0                                                         QLSTRS      24
      DO 200 I=1,4                                                       QLSTRS      25
      IF(I .LT. 4)GO TO 117                                              QLSTRS      26
      KX=1                                                               QLSTRS      27
      KY=7                                                               QLSTRS      28
  117 DO 119 J=1,2                                                       QLSTRS      29
      DO 118 K=1,LOADS                                                   QLSTRS      30
      EDDR(J,K)=EDR(KX,K)                                                QLSTRS      31
  118 EDDR(J+2,K)=EDR(KY,K)                                              QLSTRS      32
      KX=KX+1                                                            QLSTRS      33
  119 KY=KY+1                                                            QLSTRS      34
      CALL STRESS(EDDR,XI,ETA,MAA(I),MBB(I),MCC(I),SX,SY,SXY,EFSTRS,     QLSTRS      35
     1           E,PMU,ALS,ESR,LOADS,ENGG,TRIANG,NND)                    QLSTRS      36
      QUAD = QUAD + TRIANG                                               QLSTRS      37
      DO 201 J=1,LOADS                                                   QLSTRS      38
      ENG(J)=ENG(J)+ENGG(J)                                              QLSTRS      39
      SSX(I,J)=SX(J)                                                     QLSTRS      40
      SSY(I,J)=SY(J)                                                     QLSTRS      41
      SSXY(I,J)=SXY(J)                                                   QLSTRS      42
      EFFSTR(I,J)=EFSTRS(J)                                              QLSTRS      43
      IF(NND .GT. 4)EFFSTR(I,J)=ABS(SXY(J))                              QLSTRS      44
      SFTM(J) = SFTM(J) + ESR(J)*TRIANG                                  QLSTRS      45
  201 CONTINUE                                                           QLSTRS      46
  200 CONTINUE                                                           QLSTRS      47
      DO 205 J=1,LOADS                                                   QLSTRS      48
      SFTM(J) = SFTM(J)/QUAD                                             QLSTRS      49
      SFTM(J) = (1.0 - SFTM(J))/SFTM(J)                                  QLSTRS      50
      AMAX=0.                                                            QLSTRS      51
      DO 204 I=1,4                                                       QLSTRS      52
      IF(AMAX .GT. EFFSTR(I,J))GO TO 204                                 QLSTRS      53
      AMAX=EFFSTR(I,J)                                                   QLSTRS      54
      KTR(J)=I                                                           QLSTRS      55
  204 CONTINUE                                                           QLSTRS      56
  205 CONTINUE                                                           QLSTRS      57
      DO 210 J=1,LOADS                                                   QLSTRS      58
      KX=KTR(J)                                                          QLSTRS      59
      AMAX=EFFSTR(KX,J)                                                  QLSTRS      60
      DO 209 I=1,4                                                       QLSTRS      61
  209 EFFSTR(I,J)=EFFSTR(I,J)/AMAX                                       QLSTRS      62
      EFFSTR(KX,J)=AMAX                                                  QLSTRS      63
  210 CONTINUE                                                           QLSTRS      64
      RETURN                                                             QLSTRS      65
      END                                                                QLSTRS      66
```

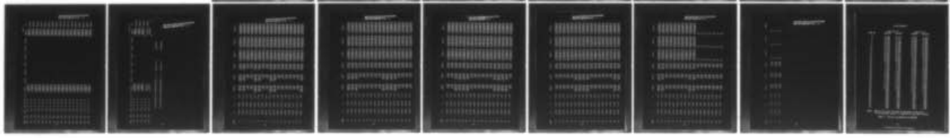
```
SUBROUTINE PRNTOR      74/74    OPT=1      FTN 4.6+446         08/21/78  10.12.39        PAGE      1

      SUBROUTINE PRNTOR(A,B,X,Y,Z,N,M,L,NJ,NP,NN)                           PRNTOR     2
      DIMENSION A(NN,L),B(NN,L),X(1),Y(1),Z(1)                              PRNTOR     3
      NP=NP+1                                                               PRNTOR     4
      LINES=1                                                               PRNTOR     5
      WRITE(6,1)NP                                                          PRNTOR     6
      WRITE(6,2)                                                            PRNTOR     7
      DO 10 I=1,NJ                                                          PRNTOR     8
      IF (LINES+L-54)4,3,3                                                  PRNTOR     9
    3 LINES=1                                                               PRNTOR    10
      WRITE(6, 1) NP                                                        PRNTOR    11
      WRITE(6, 2)                                                           PRNTOR    12
      NP=NP+1                                                               PRNTOR    13
    4 KH=M*I                                                                PRNTOR    14
      KHH=KH-M+1                                                            PRNTOR    15
      IF(M .LT. 3)GO TO 11                                                  PRNTOR    16
      WRITE(6, 9)I,X(I),Y(I),Z(I),( A(J,1),J=KHH,KH),( B(J,1),J=KHH,KH)     PRNTOR    17
      GO TO 12                                                              PRNTOR    18
   11 WRITE(6, 5)I,X(I),Y(I), ( A(J,1),J=KHH,KH),( B(J,1),J=KHH,KH)         PRNTOR    19
   12 IF(L .EQ. 1) GOTO 8                                                   PRNTOR    20
      DO 7   K=2,L                                                          PRNTOR    21
      IF(M .LT. 3)GO TO 13                                                  PRNTOR    22
      WRITE (6,  6) ( A(J,K) ,J=KHH,KH), ( B(J,K), J=KHH,KH)                PRNTOR    23
      GO TO 7                                                               PRNTOR    24
   13 WRITE (6, 15) ( A(J,K) ,J=KHH,KH), ( B(J,K), J=KHH,KH)                PRNTOR    25
    7 CONTINUE                                                              PRNTOR    26
    8 LINES =LINES +L+1                                                     PRNTOR    27
      IF(L     .EQ. 1)LINES=LINES-1                                         PRNTOR    28
   10 CONTINUE                                                              PRNTOR    29
    1 FORMAT(1H1,120X,5HPAGE ,I3/)                                          PRNTOR    30
    2 FORMAT(  1X,5HJOINT,8X,2H-X,8X,2H-Y,8X,2H-Z,8X,7HFORCE-X,             PRNTOR    31
     17X,7HFORCE-Y,7X,7HFORCE-Z,8X,7HDISPL-X,10X,7HDISPL-Y,10X,             PRNTOR    32
     27HDISPL-Z//)                                                          PRNTOR    33
    9 FORMAT(/I5,F14.3,F10.3,F10.3,F12.3,F14.3,F14.3,1PE18.8,               PRNTOR    34
     11PE17.8,1PE17.8)                                                      PRNTOR    35
    5 FORMAT(/I5,F14.3,F10.3,10X,F12.3,F14.3,14X,1PE18.8,1PE17.8)           PRNTOR    36
    6 FORMAT(39X,F12.3,F14.3,F14.3,1PE18.8,1PE17.8,1PE17.8)                 PRNTOR    37
   15 FORMAT(39X,F12.3,F14.3,14X,1PE18.8,1PE17.8)                           PRNTOR    38
      RETURN                                                                PRNTOR    39
      END                                                                   PRNTOR    40
```

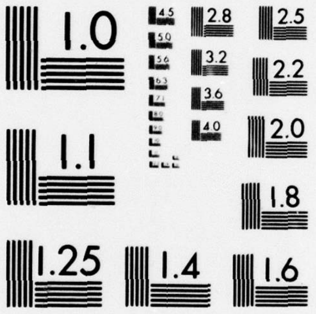
AD-A065 633 AIR FORCE FLIGHT DYNAMICS LAB WRIGHT-PATTERSON AFB OHIO F/G 13/13
'ANALYZE' - ANALYSIS OF AEROSPACE STRUCTURES WITH MEMBRANE ELEM--ETC(U)
DEC 78 V B VENKAYYA, V A TISCHLER
UNCLASSIFIED AFFDL-TR-78-170 NL

2 OF 2
AD
A065633





END
DATE
FILMED
5-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

APPENDIX C: LISTING OF THE SAMPLE DATA

```
    1
  ANALYZE  DEMO PROBLEM--INTERMEDIATE COMPLEXITY WING
 158    88    30     2     3    10     1     0     2     1
10.5        .3          .1
60.       60.         35.         60.         60.         35.
10.5        .3          .1          10.5        .3          .1
     3     3     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     2     2     2     2     2     2     2
     2     2     2     2     2     2     2     2     2     2     2     2     2     2
     2     2     2     2     2     2     2     2     2     2     2     2     2     2
     2     2     2     2
     1     2     3     4     5     6     7     8     1     2    11    12    13    14
    15    16    19    20    21    22    23    24    25    26    29    30    31    32
    33    34    35    36    39    40    41    42    43    44    45    46    49    50
    51    52    53    54    55    56    59    60    61    62    63    64    65    66
    69-   70    71    72    73    74    75    76     1     3     5     7     1    11
    13    15    19    21    23    25    29    31    33    35    39    41    43    45
    49    51    53    55    59    61    63    65    69    71    73    75     1    19
    29    39    49    59    69     5    13    23    33    43    53    63    73     9
    17    27    37    47    57    67    77     1     3     5     7     9    11    13
    15    17    19    21    23    25    27    29    31    33    35    37    39    41
    43    45    47    49    51    53    55    57    59    61    63    65    67    69
    71    73    75    77
     3     4     5     6     7     8     9    10    11    12    13    14    15    16
    17    18    21    22    23    24    25    26    27    28    31    32    33    34
    35    36    37    38    41    42    43    44    45    46    47    48    51    52
    53    54    55    56    57    58    61    62    63    64    65    66    67    68
    71    72    73    74    75    76    77    78     2     4     5     8     2    12
    14    16    20    22    24    26    30    32    34    36    40    42    44    46
    50    52    54    56    60    62    64    66    70    72    74    76     2    20
    30    40    50    60    70     6    14    24    34    44    54    64    74    10
    18    28    38    48    58    68    78     2     4     6     8    10    12    14
    16    18    20    22    24    26    28    30    32    34    36    38    40    42
    44    46    48    50    52    54    56    58    60    62    64    66    68    70
    72    74    76    78
    11    12    13    14    15    16    17    18    21    22    23    24    25    26
    27    28    31    32    33    34    35    36    37    38    41    42    43    44
    45    46    47    48    51    52    53    54    55    56    57    58    61    62
    63    64    65    66    67    68    71    72    73    74    75    76    77    78
    81    82    83    84    85    86    87    88     4     6     8    10    12    14
    16    18    22    24    26    28    32    34    36    38    42    44    46    48
    52    54    56    58    62    64    66    68    72    74    76    78    20    30
    40    50    60    70    80    14    24    34    44    54    64    74    84    18
    28    38    48    58    68    78    88     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0
     0     0    11    12    13    14    15    16    19    20    21    22    23    24
    25    26    29    30    31    32    33    34    35    36    39    40    41    42
    43    44    45    46    49    50    51    52    53    54    55    56    59    60
    61    62    63    64    65    66    69    70    71    72    73    74    75    76
    79    80    81    82    83    84    85    86     3     5     7     9    11    13
    15    17    21    23    25    27    31    33    35    37    41    43    45    47
    51    53    55    57    61    63    65    67    71    73    75    77    19    29
    39    49    59    69    79    13    23    33    43    53    63    73    83    17
    27    37    47    57    67    77    87     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
```

```
        0        0        0        0
.0312     .0312     .0312     .0312     .0312     .0312
.0416     .0416     .0312     .0312     .0312     .0312
.0364     .0364     .0364     .0364     .0364     .0364
.0468     .0468     .0468     .0468     .0520     .0520
.0416     .0416     .0624     .0624     .0624     .0624
.0624     .0624     .0468     .0468     .0676     .0676
.0728     .0728     .0780     .0780     .0520     .0520
.0728     .0728     .0936     .0936     .1040     .1040
.0520     .0520     .0780     .0780     .1144     .1144
.1196     .1196     .0468     .0468     .0884     .0884
.1248     .1248     .1404     .1404     .019      .019
.019      .019      .019      .019      .019      .019
.019      .019      .019      .019      .019      .019
.019      .019      .019      .019      .019      .019
.019      .019      .019      .019      .022      .021
.019      .019      .025      .024      .019      .019
.019      .038      .042      .048      .047      .039
.031      .026      .037      .050      .058      .065
.079      .101      .126      .019      .039      .044
.053      .065      .079      .092      .100      .02
.02       .02       .02       .02       .02       .02
.02       .02       .02       .02       .02       .02
.02       .02       .02       .02       .02       .02
.02       .02       .02       .02       .02       .02
.02       .02       .02       .02       .02       .02
.02       .02       .02       .02       .02       .02
.02       .02
    2    2    2    2    2    2    2    2    2    2    2    2    2    2
    2    2    2    2    2    2    2    2    2    2    2    2    2    2
    2    2    2    2    2    2    2    2    2    2    2    2    2    2
    2    2    2    2    2    2    2    2    2    2    2    2    2    2
    2    2    2    2    2    2    2    2    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1    1    1    1    1    1    1    1    1    1    1
    1    1    1    1
   63.5000    90.0000      1.1250    63.5000    90.0000    -1.1250
   70.8330    90.0000      1.3130    70.8330    90.0000    -1.3130
   78.1670    90.0000      1.5000    78.1670    90.0000    -1.5000
   85.5000    90.0000      1.3130    85.5000    90.0000    -1.3130
   92.8330    90.0000      1.1250    92.8330    90.0000    -1.1250
   69.6860    87.4710      1.3490    69.6860    87.4710    -1.3490
   76.0970    84.8510      1.5860    76.0970    84.8510    -1.5860
   82.7460    82.1330      1.4270    82.7460    82.1330    -1.4270
   89.6470    79.3120      1.2590    89.6470    79.3120    -1.2590
   57.2660    77.6690      1.2790    57.2660    77.6690    -1.2790
   63.9920    74.9200      1.5320    63.9920    74.9200    -1.5320
   70.9620    72.0710      1.7990    70.9620    72.0710    -1.7990
   78.1910    69.1160      1.6170    78.1910    69.1160    -1.6170
   85.6920    66.0500      1.4240    85.6920    66.0500    -1.4240
   51.0320    65.3390      1.4330    51.0320    65.3390    -1.4330
   58.2970    62.3690      1.7150    58.2970    62.3690    -1.7150
   65.8260    59.2910      2.0120    65.8260    59.2910    -2.0120
   73.6350    56.1000      1.8070    73.6350    56.1000    -1.8070
   81.7380    52.7870      1.5900    81.7380    52.7870    -1.5900
   44.7990    53.0080      1.5670    44.7990    53.0080    -1.5670
   52.6030    49.8180      1.8980    52.6030    49.8180    -1.8980
   60.6310    46.5120      2.2250    60.6310    46.5120    -2.2250
   69.0790    43.0830      1.9970    69.0790    43.0830    -1.9970
   77.7840    39.5250      1.7560    77.7840    39.5250    -1.7560
   38.5650    40.6730      1.7420    38.5650    40.6730    -1.7420
   46.9080    37.2670      2.0820    46.9080    37.2670    -2.0820
```

```
   55.5550     33.7320      2.4380     55.5550     33.7320     -2.4380
   64.5230     30.0670      2.1870     64.5230     30.0670     -2.1870
   73.8300     26.2620      1.9220     73.8300     26.2620     -1.9220
   32.3310     28.3470      1.8960     32.3310     28.3470     -1.8960
   41.2140     24.7160      2.2650     41.2140     24.7160     -2.2650
   50.4200     20.9530      2.6510     50.4200     20.9530     -2.6510
   59.9670     17.0500      2.3760     59.9670     17.0500     -2.3760
   69.8760     13.0000      2.0580     69.8760     13.0000     -2.0580
   25.1660     14.1730      2.0730     25.1660     14.1730     -2.0730
   35.5830     12.3040      2.4460     35.5830     12.3040     -2.4460
   46.1810     10.4030      2.8270     46.1810     10.4030     -2.8270
   56.9640      8.4690      2.5020     56.9640      8.4690     -2.5020
   67.9380      6.5000      2.1690     67.9380      6.5000     -2.1690
   18.0000      0.0000      2.2500     18.0000      0.0000     -2.2500
   30.0000      0.0000      2.6250     30.0000      0.0000     -2.6250
   42.0000      0.0000      3.0000     42.0000      0.0000     -3.0000
   54.0000      0.0000      2.6250     54.0000      0.0000     -2.6250
   66.0000      0.0000      2.2500     66.0000      0.0000     -2.2500
  235   236   237   238   239   240   241   242   243   244   245   246   247   248
  249   250   251   252   253   254   255   256   257   258   259   260   261   262
  263   264
  142   142
   .290E+02    3    3   .290E+02    3    4  -.280E+04    1    5
  -.596E+04    2    5   .113E+04    3    5   .280E+04    1    6
   .596E+04    2    6   .113E+04    3    6   .309E+02    3    7
   .309E+02    3    8  -.987E+04    1    9  -.978E+04    2    9
   .113E+04    3    9   .987E+04    1   10   .978E+04    2   10
   .113E+04    3   10   .205E+03    1    1  -.738E+04    2    1
   .926E+03    3    1  -.205E+03    1    2   .738E+04    2    2
   .926E+03    3    2   .178E+03    3   11   .178E+03    3   12
   .214E+03    3   13   .214E+03    3   14   .253E+03    3   15
   .253E+03    3   16  -.568E+04    1   17   .232E+04    2   17
   .102E+04    3   17   .568E+04    1   18  -.232E+04    2   18
   .102E+04    3   18   .231E+04    1   19  -.946E+03    2   19
   .723E+03    3   19  -.231E+04    1   20   .946E+03    2   20
   .723E+03    3   20   .314E+03    3   21   .314E+03    3   22
   .326E+03    3   23   .326E+03    3   24   .338E+03    3   25
   .338E+03    3   26  -.407E+04    1   27   .166E+04    2   27
   .902E+03    3   27   .407E+04    1   28  -.166E+04    2   28
   .902E+03    3   28   .174E+04    1   29  -.713E+03    2   29
   .646E+03    3   29  -.174E+04    1   30   .713E+03    2   30
   .646E+03    3   30   .340E+03    3   31   .340E+03    3   32
   .352E+03    3   33   .352E+03    3   34   .365E+03    3   35
   .365E+03    3   36  -.425E+04    1   37   .174E+04    2   37
   .974E+03    3   37   .425E+04    1   38  -.174E+04    2   38
   .974E+03    3   38   .182E+04    1   39  -.743E+03    2   39
   .694E+03    3   39  -.182E+04    1   40   .743E+03    2   40
   .694E+03    3   40   .365E+03    3   41   .365E+03    3   42
   .378E+03    3   43   .378E+03    3   44   .392E+03    3   45
   .392E+03    3   46  -.444E+04    1   47   .182E+04    2   47
   .105E+04    3   47   .444E+04    1   48  -.182E+04    2   48
   .105E+04    3   48   .189E+04    1   49  -.773E+03    2   49
   .742E+03    3   49  -.189E+04    1   50   .773E+03    2   50
   .742E+03    3   50   .390E+03    3   51   .390E+03    3   52
   .404E+03    3   53   .404E+03    3   54   .420E+03    3   55
   .420E+03    3   56  -.464E+04    1   57   .190E+04    2   57
   .112E+04    3   57   .464E+04    1   58  -.190E+04    2   58
   .112E+04    3   58   .229E+04    1   59  -.937E+03    2   59
   .883E+03    3   59  -.229E+04    1   60   .937E+03    2   60
   .883E+03    3   60   .413E+03    3   61   .413E+03    3   62
   .391E+03    3   63   .391E+03    3   64   .368E+03    3   65
   .368E+03    3   66  -.303E+04    1   67   .124E+04    2   67
   .804E+03    3   67   .303E+04    1   68  -.124E+04    2   68
   .804E+03    3   68   .307E+04    1   69  -.520E+03    2   69
   .104E+04    3   69  -.307E+04    1   70   .520E+03    2   70
   .104E+04    3   70   .433E+03    3   71   .433E+03    3   72
```

```
 .370E+03    3   73   .370E+03    3   74   .304E+03    3   75
 .304E+03    3   76  -.137E+04    1   77   .262E+03    2   77
 .446E+03    3   77   .137E+04    1   78   .262E+03    2   78
 .446E+03    3   78
 .295E+02    3    3   .295E+02    3    4  -.242E+04    1    5
-.602E+04    2    5   .979E+03    3    5   .242E+04    1    6
 .602E+04    2    6   .979E+03    3    6   .553E+02    3    7
 .559E+02    3    8  -.402E+04    1    9  -.398E+04    2    9
 .474E+03    3    9   .402E+04    1   10   .398E+04    2   10
 .474E+03    3   10   .351E+03    1    1  -.126E+05    2    1
 .153E+04    3    1  -.351E+03    1    2   .126E+05    2    2
 .153E+04    3    2   .194E+03    3   11   .194E+03    3   12
 .175E+03    3   13   .175E+03    3   14   .157E+03    3   15
 .157E+03    3   16  -.160E+04    1   17   .653E+03    2   17
 .325E+03    3   17   .160E+04    1   18  -.653E+03    2   18
 .325E+03    3   18   .551E+04    1   19  -.225E+04    2   19
 .155E+04    3   19  -.551E+04    1   20   .225E+04    2   20
 .155E+04    3   20   .347E+03    3   21   .347E+03    3   22
 .270E+03    3   23   .270E+03    3   24   .213E+03    3   25
 .213E+03    3   26  -.121E+04    1   27   .496E+03    2   27
 .311E+03    3   27   .121E+04    1   28  -.496E+03    2   28
 .311E+03    3   28   .399E+04    1   29  -.163E+04    2   29
 .131E+04    3   29  -.399E+04    1   30   .163E+04    2   30
 .131E+04    3   30   .375E+03    3   31   .375E+03    3   32
 .291E+03    3   33   .291E+03    3   34   .230E+03    3   35
 .230E+03    3   36  -.127E+04    1   37   .518E+03    2   37
 .336E+03    3   37   .127E+04    1   38  -.518E+03    2   38
 .336E+03    3   38   .416E+04    1   39  -.170E+04    2   39
 .141E+04    3   39  -.416E+04    1   40   .170E+04    2   40
 .141E+04    3   40   .402E+03    3   41   .402E+03    3   42
 .313E+03    3   43   .313E+03    3   44   .247E+03    3   45
 .247E+03    3   46  -.132E+04    1   47   .541E+03    2   47
 .361E+03    3   47   .132E+04    1   48  -.541E+03    2   48
 .361E+03    3   48   .433E+04    1   49  -.177E+04    2   49
 .150E+04    3   49  -.433E+04    1   50   .177E+04    2   50
 .150E+04    3   50   .430E+03    3   51   .430E+03    3   52
 .334E+03    3   53   .334E+03    3   54   .264E+03    3   55
 .264E+03    3   56  -.138E+04    1   57   .565E+03    2   57
 .386E+03    3   57   .138E+04    1   58  -.565E+03    2   58
 .386E+03    3   58   .530E+04    1   59  -.217E+04    2   59
 .182E+04    3   59  -.530E+04    1   60   .217E+04    2   60
 .182E+04    3   60   .458E+03    3   61   .458E+03    3   62
 .326E+03    3   63   .326E+03    3   64   .233E+03    3   65
 .233E+03    3   66  -.922E+03    1   67   .377E+03    2   67
 .287E+03    3   67   .922E+03    1   68  -.377E+03    2   68
 .287E+03    3   68   .716E+04    1   69  -.121E+04    2   69
 .218E+04    3   69  -.716E+04    1   70   .121E+04    2   70
 .218E+04    3   70   .484E+03    3   71   .484E+03    3   72
 .310E+03    3   73   .310E+03    3   74   .194E+03    3   75
 .194E+03    3   76  -.451E+03    1   77   .360E+02    2   77
 .175E+03    3   77   .451E+03    1   78  -.860E+02    2   78
 .175E+03    3   78
```

```
                                          APPENDIX D:  RESULTS OF THE SAMPLE PROBLEM ANALYSIS
                                                                                                                          PAGE    1

MEMB  THICK    AREA TYPE  MA  MB  MC  MD    SIGMA-X    SIGMA-Y   SIGMA-XY    EFSTR-1    EFSTR-2    EFSTR-3    EFSTR-4     ENERGY        MS
   1   .031    9.28   32   1   3  11       .1150E+05 -.1072E+05 -.7223E+04  .2296E+05                                   .6297E+01  .162E+01
                                           .1630E+05 -.1321E+05 -.1825E+05  .4067E+05                                   .1978E+02  .484E+00

   2   .031    9.28   32   2   4  12      -.1150E+05  .1072E+05  .7223E+04  .2296E+05                                   .6297E+01  .162E+01
                                          -.1630E+05  .1321E+05  .1825E+05  .4067E+05                                   .1978E+02  .484E+00

   3   .031   28.51   42   3   5  13  11 -.2799E+05 -.2990E+05  .3225E+05  .8821E+00  .6294E+05  .8299E+00  .7833E+00  .1295E+03  .769E-01
                                        -.1677E+05 -.2533E+05  .1865E+05  .8356E+00  .3926E+05  .7585E+00  .8513E+00  .5026E+02  .760E+00

   4   .031   28.51   42   4   6  14  12  .2799E+05  .2990E+05 -.3225E+05  .8821E+00  .6294E+05  .8299E+00  .7833E+00  .1295E+03  .769E-01
                                         .1677E+05  .2533E+05 -.1865E+05  .8356E+00  .3926E+05  .7585E+00  .8513E+00  .5026E+02  .760E+00

   5   .031   48.79   42   5   7  15  13 -.1431E+05 -.4576E+05  .1269E+05  .6114E+00  .4567E+00  .7033E+00  .4612E+05  .8417E+02  .959E+00
                                        -.8232E+02 -.3900E+05  .1240E+04  .7730E+00  .5601E+00  .6495E+00  .3902E+05  .5953E+02  .113E+01

   6   .031   48.79   42   6   8  16  14  .1431E+05  .4576E+05 -.1269E+05  .6114E+00  .4567E+00  .7033E+00  .4612E+05  .8417E+02  .959E+00
                                         .8232E+02  .3900E+05 -.1240E+04  .7730E+00  .5601E+00  .6495E+00  .3902E+05  .5953E+02  .113E+01

   7   .042   70.24   42   7   9  17  15 -.3013E+05 -.4199E+05  .3984E+05  .9941E+00  .7853E+05  .6949E+00  .6979E+00  .6326E+03 -.100E+00
                                        -.1027E+05 -.2025E+05  .1844E+05  .9579E+00  .3643E+05  .6815E+00  .6397E+00  .1271E+03  .998E+00

   8   .042   70.24   42   8  10  18  16  .3013E+05  .4199E+05 -.3984E+05  .9941E+00  .7853E+05  .6949E+00  .6979E+00  .6326E+03 -.100E+00
                                         .1027E+05  .2025E+05 -.1844E+05  .9579E+00  .3643E+05  .6815E+00  .6397E+00  .1271E+03  .998E+00

   9   .031   96.12   42   1  11  21  19 -.5404E+04 -.5376E+05 -.1147E+05  .9576E+00  .8637E+00  .9235E+00  .5499E+05  .3905E+03  .168E+00
                                        -.1695E+05 -.5036E+05 -.3445E+05  .7437E+05  .7840E+00  .7550E+00  .9578E+00  .6031E+03 -.695E-01

  10   .031   96.12   42   2  12  22  20  .5404E+04  .5376E+05  .1147E+05  .9576E+00  .8637E+00  .9235E+00  .5499E+05  .3905E+03  .168E+00
                                         .1695E+05  .5036E+05  .3445E+05  .7437E+05  .7840E+00  .7550E+00  .9578E+00  .6031E+03 -.695E-01

  11   .031   99.62   42  11  13  23  21 -.6569E+04 -.6259E+05  .1393E+05  .9258E+00  .6427E+05  .8560E+00  .7520E+00  .4990E+03  .596E-01
                                        -.9696E+04 -.4683E+05 -.1269E+04  .8641E+00  .4287E+05  .9388E+00  .7713E+00  .2473E+03  .565E+00

  12   .031   99.62   42  12  14  24  22  .6569E+04  .6259E+05 -.1393E+05  .9258E+00  .6427E+05  .8560E+00  .7520E+00  .4990E+03  .596E-01
                                         .9696E+04  .4683E+05  .1269E+04  .8641E+00  .4287E+05  .9388E+00  .7713E+00  .2473E+03  .565E+00

  13   .036  103.28   42  13  15  25  23 -.2933E+04 -.6248E+05  .1378E+03  .9226E+00  .9765E+00  .6107E+05  .9327E+00  .6557E+03  .248E-01
                                        -.4382E+04 -.4428E+05 -.1371E+05  .9538E+00  .9712E+00  .4847E+05  .9755E+00  .4125E+03  .272E+00

  14   .036  103.28   42  14  16  26  24  .2933E+04  .6248E+05 -.1378E+03  .9226E+00  .9765E+00  .6107E+05  .9327E+00  .6557E+03  .248E-01
                                         .4382E+04  .4428E+05  .1371E+05  .9538E+00  .9712E+00  .4847E+05  .9755E+00  .4125E+03  .272E+00

  15   .036  107.18   42  15  17  27  25  .4674E+02 -.5888E+05  .1810E+05  .8819E+00  .8912E+00  .6673E+05  .9853E+00  .7416E+03 -.422E-01
                                        -.6999E+02 -.4121E+05  .3025E+04  .9163E+00  .9131E+00  .4151E+05  .9963E+00  .3041E+03  .510E+00

  16   .036  107.18   42  16  18  28  26 -.4674E+02  .5888E+05 -.1810E+05  .8819E+00  .8912E+00  .6673E+05  .9853E+00  .7416E+03 -.422E-01
                                         .6999E+02  .4121E+05 -.3025E+04  .9163E+00  .9131E+00  .4151E+05  .9963E+00  .3041E+03  .510E+00

  17   .036  104.16   42  19  21  31  29 -.1214E+04 -.6501E+05 -.7260E+04  .8748E+00  .6562E+05  .9551E+00  .8262E+00  .6621E+03 -.306E-05
                                        -.1743E+04 -.4980E+05 -.2551E+05  .8177E+00  .9962E+00  .6595E+05  .8302E+00  .6464E+03  .746E-03
```

```
                                                                                                                    PAGE    2
MEMB  THICK   AREA  TYPE MA  MB  MC  MD   SIGMA-X     SIGMA-Y    SIGMA-XY   EFSTR-1    EFSTR-2    EFSTR-3    EFSTR-4      ENERGY        MS

  18   .036   104.16  42  20  22  32  30  .1214E+04  .6501E+05  .7260E+04 .8748E+00 .6562E+05 .9551E+00 .8262E+00 .6621E+03 -.306E-05
                                          .1743E+04  .4980E+05  .2551E+05 .8177E+00 .9962E+00 .6595E+05 .8302E+00 .6464E+03  .746E-03

  19   .047   107.95  42  21  23  33  31 -.3887E+04 -.7808E+05  .8556E+04 .9190E+00 .7764E+05 .9303E+00 .8400E+00 .1271E+04 -.162E+00
                                         -.7068E+04 -.6300E+05 -.4625E+04 .9217E+00 .6031E+05 .9904E+00 .9062E+00 .8438E+03  .417E-01

  20   .047   107.95  42  22  24  34  32  .3887E+04  .7808E+05 -.8556E+04 .9190E+00 .7764E+05 .9303E+00 .8400E+00 .1271E+04 -.162E+00
                                          .7068E+04  .6300E+05  .4625E+04 .9217E+00 .6031E+05 .9904E+00 .9062E+00 .8438E+03  .417E-01

  21   .047   111.92  42  23  25  35  33 -.3478E+04 -.7611E+05 -.3260E+04 .9545E+00 .9289E+00 .9762E+00 .7464E+05 .1325E+04 -.167E+00
                                         -.4866E+04 -.6073E+05 -.1670E+05 .9668E+00 .9221E+00 .9593E+00 .6522E+05 .9878E+03 -.416E-01

  22   .047   111.92  42  24  26  36  34  .3478E+04  .7611E+05  .3260E+04 .9545E+00 .9289E+00 .9762E+00 .7464E+05 .1325E+04 -.167E+00
                                          .4866E+04  .6073E+05  .1670E+05 .9668E+00 .9221E+00 .9593E+00 .6522E+05 .9878E+03 -.416E-01

  23   .052   116.13  42  25  27  37  35 -.2952E+03 -.7057E+05  .1344E+05 .9426E+00 .9272E+00 .9834E+00 .7417E+05 .1457E+04 -.160E+00
                                         -.1711E+03 -.5466E+05 -.6348E+03 .9340E+00 .8934E+00 .9610E+00 .5459E+05 .7758E+03  .160E+00

  24   .052   116.13  42  26  28  38  36  .2952E+03  .7057E+05 -.1344E+05 .9426E+00 .9272E+00 .9834E+00 .7417E+05 .1457E+04 -.160E+00
                                          .1711E+03  .5466E+05  .6348E+03 .9340E+00 .8934E+00 .9610E+00 .5459E+05 .7758E+03  .160E+00

  25   .042   112.19  42  29  31  41  39 -.8957E+03 -.7312E+05 -.7545E+04 .9307E+00 .7384E+05 .9628E+00 .8933E+00 .1096E+04 -.142E+00
                                         -.3554E+04 -.6398E+05 -.2319E+05 .9448E+00 .7411E+05 .9960E+00 .9430E+00 .1128E+04 -.164E+00

  26   .042   112.19  42  30  32  42  40  .8957E+03  .7312E+05  .7545E+04 .9307E+00 .7384E+05 .9628E+00 .8933E+00 .1096E+04 -.142E+00
                                          .3554E+04  .6398E+05  .2319E+05 .9448E+00 .7411E+05 .9960E+00 .9430E+00 .1128E+04 -.164E+00

  27   .062   116.27  42  31  33  43  41 -.1184E+04 -.8523E+05  .6317E+04 .9283E+00 .8535E+05 .9280E+00 .8518E+00 .2196E+04 -.241E+00
                                         -.3919E+04 -.7137E+05 -.5073E+04 .9270E+00 .7005E+05 .9595E+00 .8850E+00 .1554E+04 -.916E-01

  28   .062   116.27  42  32  34  44  42  .1184E+04  .8523E+05 -.6317E+04 .9283E+00 .8535E+05 .9280E+00 .8518E+00 .2196E+04 -.241E+00
                                          .3919E+04  .7137E+05  .5073E+04 .9270E+00 .7005E+05 .9595E+00 .8850E+00 .1554E+04 -.916E-01

  29   .062   120.54  42  33  35  45  43 -.2180E+04 -.8410E+05 -.6116E+04 .9700E+00 .9637E+00 .9947E+00 .8370E+05 .2448E+04 -.270E+00
                                         -.3938E+04 -.7010E+05 -.1898E+05 .9763E+00 .9541E+00 .9796E+00 .7572E+05 .1955E+04 -.188E+00

  30   .062   120.54  42  34  36  46  44  .2180E+04  .8410E+05  .6116E+04 .9700E+00 .9637E+00 .9947E+00 .8370E+05 .2448E+04 -.270E+90
                                          .3938E+04  .7010E+05  .1898E+05 .9763E+00 .9541E+00 .9796E+00 .7572E+05 .1955E+04 -.188E+00

  31   .062   125.10  42  35  37  47  45 -.1330E+03 -.8168E+05  .1163E+05 .9486E+00 .9356E+00 .9866E+00 .8407E+05 .2452E+04 -.262E+00
                                         -.2666E+03 -.6656E+05 -.2410E+04 .9436E+00 .9048E+00 .9623E+00 .6655E+05 .1506E+04 -.538E-01

  32   .062   125.10  42  36  38  48  46  .1330E+03  .8168E+05 -.1163E+05 .9486E+00 .9356E+00 .9866E+00 .8407E+05 .2452E+04 -.262E+00
                                          .2666E+03  .6656E+05  .2410E+04 .9436E+00 .9048E+00 .9623E+00 .6655E+05 .1506E+04 -.538E-01

  33   .047   120.22  42  39  41  51  49 -.2290E+04 -.8144E+05 -.6196E+04 .9351E+00 .8103E+05 .9466E+00 .8813E+00 .1586E+04 -.213E+00
                                         -.4952E+04 -.7279E+05 -.2167E+05 .9517E+00 .7982E+05 .9827E+00 .9364E+00 .1593E+04 -.221E+00

  34   .047   120.22  42  40  42  52  50  .2290E+04  .8144E+05  .6196E+04 .9351E+00 .8103E+05 .9466E+00 .8813E+00 .1586E+04 -.213E+00
                                          .4952E+04  .7279E+05  .2167E+05 .9517E+00 .7982E+05 .9827E+00 .9364E+00 .1593E+04 -.221E+00
```

```
                                                                                                                    PAGE    3
MEMB  THICK     AREA TYPE  MA  MB  MC  MD    SIGMA-X    SIGMA-Y   SIGMA-XY    EFSTR-1    EFSTR-2    EFSTR-3    EFSTR-4     ENERGY         MS

  35   .068   124.61   42  41  43  53  51 -.2535E+04 -.9870E+05  .3706E+04  .9144E+00  .9767E+05  .9045E+00  .8143E+00  .3242E+04  -.324E+00
                                          -.5892E+04 -.8515E+05 -.6821E+04  .9146E+00  .8321E+05  .9307E+00  .8438E+00  .2461E+04  -.218E+00

  36   .068   124.61   42  42  44  54  52  .2535E+04  .9870E+05 -.3706E+04  .9144E+00  .9767E+05  .9045E+00  .8143E+00  .3242E+04  -.324E+00
                                           .5892E+04  .8515E+05  .6821E+04  .9146E+00  .8321E+05  .9307E+00  .8438E+00  .2461E+04  -.218E+00

  37   .073   129.17   42  43  45  55  53 -.2514E+04 -.9764E+05 -.1000E+05  .9707E+00  .9409E+00  .9710E+00  .9795E+05  .4088E+04  -.369E+00
                                          -.5039E+04 -.8392E+05 -.2218E+05  .9770E+00  .9344E+00  .9588E+00  .9012E+05  .3404E+04  -.310E+00

  38   .073   129.17   42  44  46  56  54  .2514E+04  .9764E+05  .1000E+05  .9707E+00  .9409E+00  .9710E+00  .9795E+05  .4088E+04  -.369E+00
                                           .5039E+04  .8392E+05  .2218E+05  .9770E+00  .9344E+00  .9588E+00  .9012E+05  .3404E+04  -.310E+00

  39   .078   134.06   42  45  47  57  55  .6827E+03 -.9273E+05  .8767E+04  .9358E+00  .8944E+00  .9581E+00  .9430E+05  .3974E+04  -.328E+00
                                          -.8048E+02 -.7785E+05 -.4519E+04  .9302E+00  .8580E+00  .9295E+00  .7821E+05  .2656E+04  -.174E+00

  40   .078   134.06   42  46  48  58  56 -.6827E+03  .9273E+05 -.8767E+04  .9358E+00  .8944E+00  .9581E+00  .9430E+05  .3974E+04  -.328E+00
                                           .8048E+02  .7785E+05  .4519E+04  .9302E+00  .8580E+00  .9295E+00  .7821E+05  .2656E+04  -.174E+00

  41   .052   128.26   42  49  51  61  59 -.4164E+04 -.8117E+05 -.4440E+04  .9479E+00  .7955E+05  .9389E+00  .8863E+00  .1841E+04  -.200E+00
                                          -.7082E+04 -.7521E+05 -.1894E+05  .9589E+00  .7906E+05  .9714E+00  .9316E+00  .1879E+04  -.213E+00

  42   .052   128.26   42  50  52  62  60  .4164E+04  .8117E+05  .4440E+04  .9479E+00  .7955E+05  .9389E+00  .8863E+00  .1841E+04  -.200E+00
                                           .7082E+04  .7521E+05  .1894E+05  .9589E+00  .7906E+05  .9714E+00  .9316E+00  .1879E+04  -.213E+00

  43   .073   132.92   42  51  53  63  61 -.5741E+04 -.1002E+06 -.2650E+03  .9186E+00  .9749E+05  .8897E+00  .8046E+00  .3730E+04  -.318E+00
                                          -.1017E+05 -.8885E+05 -.8870E+04  .9277E+00  .8562E+05  .9199E+00  .8464E+00  .3058E+04  -.241E+00

  44   .073   132.92   42  52  54  64  62  .5741E+04  .1002E+06  .2650E+03  .9186E+00  .9749E+05  .8897E+00  .8046E+00  .3730E+04  -.318E+00
                                           .1017E+05  .8885E+05  .8870E+04  .9277E+00  .8562E+05  .9199E+00  .8464E+00  .3058E+04  -.241E+00

  45   .094   137.80   42  53  55  65  63 -.4183E+04 -.9895E+05 -.1461E+05  .9880E+00  .9451E+00  .9577E+00  .1002E+06  .5896E+04  -.384E+00
                                          -.7067E+04 -.8715E+05 -.2491E+05  .9869E+00  .9307E+00  .9448E+00  .9430E+05  .5106E+04  -.339E+00

  46   .094   137.80   42  54  56  66  64  .4183E+04  .9895E+05  .1461E+05  .9880E+00  .9451E+00  .9577E+00  .1002E+06  .5896E+04  -.384E+00
                                           .7067E+04  .8715E+05  .2491E+05  .9869E+00  .9307E+00  .9448E+00  .9430E+05  .5106E+04  -.339E+00

  47   .104   143.02   42  55  57  67  65  .3288E+03 -.9420E+05  .4538E+04  .9577E+00  .9176E+00  .9596E+00  .9469E+05  .5853E+04  -.339E+00
                                          -.7511E+03 -.8004E+05 -.6725E+04  .9485E+00  .8774E+00  .9299E+00  .8051E+05  .4085E+04  -.206E+00

  48   .104   143.02   42  56  58  68  66 -.3288E+03  .9420E+05 -.4538E+04  .9577E+00  .9176E+00  .9596E+00  .9469E+05  .5853E+04  -.339E+00
                                           .7511E+03  .8004E+05  .6725E+04  .9485E+00  .8774E+00  .9299E+00  .8051E+05  .4085E+04  -.206E+00

  49   .052   146.00   42  59  61  71  69 -.1008E+05 -.7653E+05 -.2280E+04  .9555E+00  .7213E+05  .8764E+00  .8378E+00  .1698E+04  -.897E-01
                                          -.1388E+05 -.7498E+05 -.1692E+05  .9427E+00  .7505E+05  .9165E+00  .8708E+00  .1881E+04  -.139E+00

  50   .052   146.00   42  60  62  72  70  .1008E+05  .7653E+05  .2280E+04  .9555E+00  .7213E+05  .8764E+00  .8378E+00  .1698E+04  -.897E-01
                                           .1388E+05  .7498E+05  .1692E+05  .9427E+00  .7505E+05  .9165E+00  .8708E+00  .1881E+04  -.139E+00

  51   .078   127.77   42  61  63  73  71 -.1190E+05 -.9611E+05 -.3685E+04  .9116E+00  .9098E+05  .8643E+00  .7920E+00  .3337E+04  -.256E+00
                                          -.1700E+05 -.8836E+05 -.1057E+05  .9302E+00  .8324E+05  .9025E+00  .8479E+00  .3039E+04  -.213E+00
```

```
                                                                                                                     PAGE   4
MEMB THICK     AREA TYPE  MA  MB  MC  MD    SIGMA-X    SIGMA-Y   SIGMA-XY   EFSTR-1   EFSTR-2   EFSTR-3   EFSTR-4    ENERGY        MS

  52   .078   127.77  42  62  64  74  72  .1190E+05  .9611E+05  .3685E+04 .9116E+00 .9098E+05 .8643E+00 .7920E+00 .3337E+04 -.256E+00
                                          .1700E+05  .8836E+05  .1057E+05 .9302E+00 .8324E+05 .9025E+00 .8479E+00 .3039E+04 -.213E+00

  53   .114   107.85  42  63  65  75  73 -.6871E+04 -.9669E+05 -.1986E+05 .9958E+05 .9768E+00 .9366E+00 .9623E+00 .5646E+04 -.377E+00
                                         -.1044E+05 -.8613E+05 -.2869E+05 .9538E+05 .9600E+00 .9414E+00 .9814E+00 .5158E+04 -.350E+00

  54   .114   107.85  42  64  66  76  74  .6871E+04  .9669E+05  .1986E+05 .9958E+05 .9768E+00 .9366E+00 .9623E+00 .5646E+04 -.377E+00
                                          .1044E+05  .8613E+05  .2869E+05 .9538E+05 .9600E+00 .9414E+00 .9814E+00 .5158E+04 -.350E+00

  55   .120    86.21  42  65  67  77  75 -.5142E+04 -.1055E+06 -.5026E+04 .9540E+00 .1034E+06 .9344E+00 .9009E+00 .4783E+04 -.385E+00
                                         -.4711E+04 -.8353E+05 -.1274E+05 .8422E+05 .9994E+00 .9671E+00 .9718E+00 .3447E+04 -.275E+00

  56   .120    86.21  42  66  68  78  76  .5142E+04  .1055E+06  .5026E+04 .9540E+00 .1034E+06 .9344E+00 .9009E+00 .4783E+04 -.385E+00
                                          .4711E+04  .8353E+05  .1274E+05 .8422E+05 .9994E+00 .9671E+00 .9718E+00 .3447E+04 -.275E+00

  57   .047   154.48  42  69  71  81  79 -.8991E+04 -.5774E+05  .1264E+05 .8564E+00 .5809E+05 .7597E+00 .6361E+00 .8280E+03  .284E+00
                                         -.1319E+05 -.6258E+05  .1173E+04 .9624E+00 .5717E+05 .9229E+00 .8978E+00 .1119E+04  .112E+00

  58   .047   154.48  42  70  72  82  80  .6991E+04  .5774E+05 -.1264E+05 .8564E+00 .5809E+05 .7597E+00 .6361E+00 .8280E+03  .284E+00
                                          .1319E+05  .6258E+05 -.1173E+04 .9624E+00 .5717E+05 .9229E+00 .8978E+00 .1119E+04  .112E+00

  59   .088   133.04  42  71  73  83  81 -.1496E+05 -.9825E+05  .1254E+05 .8348E+00 .9423E+05 .7451E+00 .6164E+00 .3488E+04 -.194E+00
                                         -.1792E+05 -.9922E+05  .5423E+04 .8238E+00 .9207E+05 .7552E+00 .6233E+00 .3432E+04 -.177E+00

  60   .088   133.04  42  72  74  84  82  .1496E+05  .9825E+05 -.1254E+05 .8348E+00 .9423E+05 .7451E+00 .6164E+00 .3488E+04 -.194E+00
                                          .1792E+05  .9922E+05 -.5423E+04 .8238E+00 .9207E+05 .7552E+00 .6233E+00 .3432E+04 -.177E+00

  61   .125   111.00  42  73  75  85  83 -.1676E+05 -.1030E+06  .4519E+04 .9402E+00 .8096E+00 .8914E+00 .9606E+05 .5562E+04 -.317E+00
                                         -.1743E+05 -.1037E+06 -.3232E+04 .9211E+00 .7478E+00 .8545E+00 .9638E+05 .5317E+04 -.298E+00

  62   .125   111.00  42  74  76  86  84  .1676E+05  .1030E+06 -.4519E+04 .9402E+00 .8096E+00 .8914E+00 .9606E+05 .5562E+04 -.317E+00
                                          .1743E+05  .1037E+06  .3232E+04 .9211E+00 .7478E+00 .8545E+00 .9638E+05 .5317E+04 -.298E+00

  63   .140    88.43  42  75  77  87  85 -.1974E+05 -.1330E+06 -.9329E+03 .8307E+00 .1243E+06 .7206E+00 .6100E+00 .6185E+04 -.378E+00
                                         -.1822E+05 -.1085E+06 -.1128E+05 .8707E+00 .1025E+06 .7400E+00 .6621E+00 .4523E+04 -.274E+00

  64   .140    88.43  42  76  78  88  86  .1974E+05  .1330E+06  .9329E+03 .8307E+00 .1243E+06 .7206E+00 .6100E+00 .6185E+04 -.378E+00
                                          .1822E+05  .1085E+06  .1128E+05 .8707E+00 .1025E+06 .7400E+00 .6621E+00 .4523E+04 -.274E+00

  65   .019    17.88  51   1   2   4   3                       -.9111E+04 .7004E+00 .1000E+01 .7433E+00 .9111E+04 .2660E+01  .346E+01
                                                                .1206E+05 .1206E+05 .6665E+00 .9522E+00 .6665E+00 .4266E+01  .254E+01

  66   .019    20.63  51   3   4   6   5                       -.6863E+04 .6863E+04 .4466E+00 .9310E+00 .4466E+00 .1290E+01  .624E+01
                                                                .9370E+04 .7972E+00 .1000E+01 .8225E+00 .9370E+04 .3530E+01  .313E+01

  67   .019    20.63  51   5   6   8   7                       -.1524E+05 .9795E+00 .1000E+01 .9766E+00 .1524E+05 .1103E+02  .132E+01
                                                               -.2907E+04 .9249E+00 .3980E+00 .2907E+04 .3980E+00 .2217E+00  .167E+02

  68   .019    17.88  51   7   8  10   9                       -.1918E+05 .6865E+00 .1918E+05 .6342E+00 .1000E+01 .1113E+02  .120E+01
                                                                .1267E+04 .7208E+00 .9500E+00 .1267E+04 .9500E+00 .5547E-01  .297E+02
```

```
                                                                                                              PAGE    5
MEMB  THICK   AREA  TYPE MA  MB  MC  MD   SIGMA-X    SIGMA-Y    SIGMA-XY  EFSTR-1   EFSTR-2   EFSTR-3   EFSTR-4    ENERGY     MS

 69   .019    16.53  51   1   2  12  11                         .1604E+05 .1604E+05 .9315E+00 .9886E+00 .9315E+00 .9283E+01  .127E+01
                                                                .1542E+05 .9933E+00 .1000E+01 .9944E+00 .1542E+05 .9190E+01  .128E+01

 70   .019    20.33  51  11  12  14  13                         .1711E+05 .1711E+05 .9722E+00 .9958E+00 .9722E+00 .1358E+02  .108E+01
                                                                .1741E+05 .9738E+00 .1000E+01 .9777E+00 .1741E+05 .1415E+02  .103E+01

 71   .019    21.64  51  13  14  16  15                         .6507E+04 .9694E+00 .1000E+01 .9660E+00 .6507E+04 .2087E+01  .447E+01
                                                                .6298E+03 .8215E+00 .1000E+01 .8016E+00 .6298E+03 .1676E-01  .603E+02

 72   .019    20.02  51  15  16  18  17                         .1861E+05 .9780E+00 .1000E+01 .9750E+00 .1861E+05 .1594E+02  .903E+00
                                                                .7627E+04 .9238E+00 .1000E+01 .9137E+00 .7627E+04 .2528E+01  .378E+01

 73   .019    20.42  51  19  20  22  21                         .5618E+04 .5618E+04 .5699E+00 .9290E+00 .5699E+00 .9479E+00  .714E+01
                                                                .2101E+04 .2101E+04 .5648E+00 .9281E+00 .5648E+00 .1319E+00  .208E+02

 74   .019    25.08  51  21  22  24  23                         .1324E+05 .1324E+05 .8481E+00 .9775E+00 .8481E+00 .8768E+01  .188E+01
                                                                .1142E+05 .1142E+05 .9108E+00 .9868E+00 .9108E+00 .6979E+01  .222E+01

 75   .019    26.68  51  23  24  26  25                        -.1243E+05 .9867E+00 .8690E+00 .1243E+05 .8690E+00 .8448E+01  .202E+01
                                                               -.8234E+04 .9877E+00 .8784E+00 .8234E+04 .8784E+00 .3741E+01  .354E+01

 76   .019    24.64  51  25  26  28  27                        -.3760E+04 .9210E+00 .3378E+00 .3760E+04 .3378E+00 .4234E+00  .134E+02
                                                               -.2709E+04 .9186E+00 .3182E+00 .2709E+04 .3182E+00 .2165E+00  .193E+02

 77   .019    24.71  51  29  30  32  31                         .9061E+04 .9061E+04 .6170E+00 .9370E+00 .6170E+00 .3136E+01  .388E+01
                                                                .1785E+05 .1785E+05 .8510E+00 .9755E+00 .8510E+00 .1572E+02  .113E+01

 78   .019    30.31  51  31  32  34  33                         .1558E+05 .1558E+05 .8066E+00 .9715E+00 .8066E+00 .1403E+02  .151E+01
                                                                .2239E+05 .2239E+05 .8983E+00 .9850E+00 .8983E+00 .3202E+02  .654E+00

 79   .019    32.22  51  33  34  36  35                        -.7198E+04 .9724E+00 .7288E+00 .7198E+04 .7288E+00 .2950E+01  .467E+01
                                                               -.4738E+04 .9697E+00 .7030E+00 .4738E+04 .7030E+00 .1244E+01  .776E+01

 80   .019    29.74  51  35  36  38  37                         .5312E+04 .5420E+00 .1000E+01 .4795E+00 .5312E+04 .1248E+01  .771E+01
                                                                .3777E+04 .4824E+00 .1000E+01 .4118E+00 .3777E+04 .6005E+00  .118E+02

 81   .019    29.38  51  39  40  42  41                         .1107E+05 .1107E+05 .6622E+00 .9446E+00 .6622E+00 .5839E+01  .288E+01
                                                                .1803E+05 .1803E+05 .8320E+00 .9725E+00 .8320E+00 .1869E+02  .114E+01

 82   .019    36.03  51  41  42  44  43                         .1677E+05 .1677E+05 .8029E+00 .9710E+00 .8029E+00 .1923E+02  .134E+01
                                                                .2222E+05 .2222E+05 .8851E+00 .9831E+00 .8851E+00 .3694E+02  .679E+00

 83   .019    38.26  51  43  44  46  45                        -.2998E+04 .9208E+00 .2275E+00 .2998E+04 .2275E+00 .3929E+00  .187E+02
                                                               -.3271E+04 .9463E+00 .4761E+00 .3271E+04 .4761E+00 .5642E+00  .138E+02

 84   .019    35.29  51  45  46  48  47                         .1080E+05 .7385E+00 .1080E+05 .7026E+00 .1000E+01 .7365E+01  .276E+01
                                                                .5858E+04 .6141E+00 .1000E+01 .5612E+00 .5858E+04 .1920E+01  .652E+01

 85   .019    34.47  51  49  50  52  51                         .2118E+05 .2118E+05 .8250E+00 .9714E+00 .8250E+00 .3000E+02  .827E+00
                                                                .2915E+05 .2915E+05 .8932E+00 .9826E+00 .8932E+00 .6129E+02  .275E+00
```

```
                                                                                                                    PAGE    6
MEMB  THICK    AREA  TYPE MA  MB  MC  MD    SIGMA-X     SIGMA-Y     SIGMA-XY  EFSTR-1   EFSTR-2   EFSTR-3   EFSTR-4   ENERGY      MS
  86  .019    42.22  51  51  52  54  53                             .2382E+05 .2382E+05 .8621E+00 .9799E+00 .8621E+00 .4852E+02  .587E+00
                                                                    .2996E+05 .2996E+05 .9167E+00 .9878E+00 .9167E+00 .8146E+02  .223E+00

  87  .019    44.81  51  53  54  56  55                             .5239E+04 .5939E+00 .5239E+04 .5473E+00 .1000E+01 .1920E+01  .750E+01
                                                                    .1077E+04 .4567E+00 .1000E+01 .6239E+00 .1077E+04 .7915E-01  .413E+02

  88  .019    41.32  51  55  56  58  57                             .1824E+05 .8404E+00 .1000E+01 .8184E+00 .1824E+05 .2730E+02  .110E+01
                                                                    .9185E+04 .7438E+00 .1000E+01 .7085E+00 .9185E+04 .6270E+01  .341E+01

  89  .022    39.93  51  59  60  62  61                             .3399E+05 .3399E+05 .9016E+00 .9840E+00 .9016E+00 .1128E+03  .880E-01
                                                                    .4273E+05 .4273E+05 .9334E+00 .9891E+00 .9334E+00 .1846E+03 -.150E+00

  90  .021    48.89  51  61  62  64  63                             .3281E+05 .3281E+05 .9212E+00 .9885E+00 .9212E+00 .1256E+03  .114E+00
                                                                    .3978E+05 .3978E+05 .9537E+00 .9933E+00 .9537E+00 .1914E+03 -.977E-01

  91  .019    51.85  51  63  64  66  65                             .2047E+05 .9171E+00 .1000E+01 .9075E+00 .2047E+05 .4682E+02  .788E+00
                                                                    .9836E+04 .8774E+00 .1000E+01 .8632E+00 .9836E+04 .1037E+02  .280E+01

  92  .019    47.79  51  65  66  68  67                             .3553E+05 .9244E+00 .1000E+01 .9140E+00 .3553E+05 .1310E+03  .263E-01
                                                                    .1906E+05 .8884E+00 .1000E+01 .8730E+00 .1906E+05 .3627E+02  .952E+00

  93  .025    47.83  51  69  70  72  71                             .3507E+05 .3507E+05 .9731E+00 .9959E+00 .9731E+00 .1769E+03  .127E-01
                                                                    .4035E+05 .4035E+05 .9603E+00 .9939E+00 .9603E+00 .2309E+03 -.114E+00

  94  .024    56.78  51  71  72  74  73                             .3339E+05 .3339E+05 .9609E+00 .9947E+00 .9609E+00 .1803E+03  .707E-01
                                                                    .3765E+05 .3765E+05 .9704E+00 .9960E+00 .9704E+00 .2317E+03 -.555E-01

  95  .019    58.38  51  73  74  76  75                             .2013E+05 .9299E+00 .1000E+01 .9208E+00 .2013E+05 .5165E+02  .806E+00
                                                                    .1044E+05 .8627E+00 .1000E+01 .8448E+00 .1044E+05 .1295E+02  .262E+01

  96  .019    52.08  51  75  76  78  77                             .3446E+05 .9363E+00 .3446E+05 .9265E+00 .1000E+01 .1359E+03  .514E-01
                                                                    .1935E+05 .8909E+00 .1935E+05 .8742E+00 .1000E+01 .4083E+02  .921E+00

  97  .019    33.22  51   1   2  20  19                             .2823E+05 .5030E+00 .1000E+01 .5628E+00 .2823E+05 .4006E+02  .616E+00
                                                                    .2950E+05 .5008E+00 .1000E+01 .5609E+00 .2950E+05 .4368E+02  .548E+00

  98  .038    37.47  51  19  20  30  29                             .2322E+05 .5048E+00 .1000E+01 .5580E+00 .2322E+05 .6105E+02  .967E+00
                                                                    .3619E+05 .7861E+00 .1000E+01 .8091E+00 .3619E+05 .1890E+03  .756E-01

  99  .042    41.73  51  29  30  40  39                             .2351E+05 .5100E+00 .1000E+01 .5575E+00 .2351E+05 .7717E+02  .940E+00
                                                                    .3734E+05 .7469E+00 .1000E+01 .7714E+00 .3734E+05 .2386E+03  .654E-01

 100  .046    45.99  51  39  40  50  49                             .2147E+05 .5071E+00 .1000E+01 .5510E+00 .2147E+05 .8077E+02  .113E+01
                                                                    .3563E+05 .7444E+00 .1000E+01 .7671E+00 .3563E+05 .2727E+03  .119E+00

 101  .047    50.27  51  49  50  60  59                             .1719E+05 .4849E+00 .1000E+01 .5268E+00 .1719E+05 .5432E+02  .170E+01
                                                                    .3306E+05 .7613E+00 .1000E+01 .7807E+00 .3306E+05 .2550E+03  .195E+00

 102  .039    63.04  51  59  60  70  69                             .7158E+04 .2163E+00 .1000E+01 .1125E+00 .7158E+04 .8023E+01  .742E+01
                                                                    .2773E+05 .7138E+00 .1000E+01 .7382E+00 .2773E+05 .1789E+03  .462E+00
```

```
                                                                                                      PAGE    7
MEMB  THICK    AREA  TYPE MA  MB  MC  MD    SIGMA-X    SIGMA-Y    SIGMA-XY   EFSTR-1    EFSTR-2    EFSTR-3    EFSTR-4    ENERGY       MS

 103   .031   68.66  51  69  70  80  79                          -.1611E+05 .1611E+05 .7141E+00 .9775E+00 .7141E+00 .5087E+02  .155E+01
                                                                  .2108E+05 .7273E+00 .1000E+01 .7488E+00 .2108E+05 .9049E+02  .910E+00

 104   .028   17.13  51   5   6  14  13                           .3274E+05 .9408E+00 .1008E+01 .9441E+00 .3274E+05 .6008E+02  .101E+00
                                                                  .2893E+05 .9529E+00 .2893E+05 .9555E+00 .1000E+01 .4746E+02  .238E+00

 105   .037   46.62  51  13  14  24  23                           .3590E+05 .6549E+00 .1000E+01 .6957E+00 .3590E+05 .2007E+03  .163E+00
                                                                  .3404E+05 .7371E+00 .1000E+01 .7682E+00 .3404E+05 .1940E+03  .173E+00

 106   .050   52.49  51  23  24  34  33                           .3651E+05 .6604E+00 .1000E+01 .6964E+00 .3651E+05 .3165E+03  .142E+00
                                                                  .3336E+05 .7059E+00 .1000E+01 .7370E+00 .3336E+05 .2752E+03  .218E+00

 107   .058   58.35  51  33  34  44  43                           .3644E+05 .6974E+00 .1000E+01 .7264E+00 .3644E+05 .4196E+03  .122E+00
                                                                  .3574E+05 .7450E+00 .1000E+01 .7694E+00 .3574E+05 .4214E+03  .114E+00

 108   .065   64.23  51  43  44  54  53                           .3587E+05 .7076E+00 .1000E+01 .7331E+00 .3587E+05 .5054E+03  .134E+00
                                                                  .3734E+05 .7611E+00 .1000E+01 .7820E+00 .3734E+05 .5750E+03  .580E-01

 109   .079   70.09  51  53  54  64  63                           .3292E+05 .7315E+00 .1000E+01 .7530E+00 .3292E+05 .5763E+03  .220E+00
                                                                  .3682E+05 .7913E+00 .1000E+01 .8081E+00 .3682E+05 .7623E+03  .561E-01

 110   .101   62.28  51  63  64  74  73                           .2801E+05 .8269E+00 .1000E+01 .8377E+00 .2801E+05 .5173E+03  .364E+00
                                                                  .3458E+05 .8742E+00 .1000E+01 .8821E+00 .3458E+05 .8248E+03  .778E-01

 111   .126   65.33  51  73  74  84  83                           .2707E+05 .8415E+00 .2707E+05 .8507E+00 .1000E+01 .6409E+03  .401E+00
                                                                  .3443E+05 .8804E+00 .3443E+05 .8873E+00 .1000E+01 .1076E+04  .790E-01

 112   .019   26.59  51   9  10  18  17                           .2408E+05 .4719E+00 .1000E+01 .5281E+00 .2408E+05 .2270E+02  .936[illegible]+00
                                                                  .1754E+05 .6494E+00 .1000E+01 .6867E+00 .1754E+05 .1394E+02  .13[illegible]+01

 113   .038   37.13  51  17  18  28  27                           .3838E+05 .6822E+00 .1000E+01 .7191E+00 .3838E+05 .1920E+03  .718E-01
                                                                  .1921E+05 .5042E+00 .1921E+05 .5616E+00 .1000E+01 .4148E+02  .137E+01

 114   .044   41.71  51  27  28  38  37                           .3838E+05 .6823E+00 .1000E+01 .7155E+00 .3838E+05 .2494E+03  .730E-01
                                                                  .1798E+05 .4506E+00 .1000E+01 .5079E+00 .1798E+05 .4524E+02  .163E+01

 115   .053   46.30  51  37  38  48  47                           .3824E+05 .6923E+00 .1000E+01 .7214E+00 .3824E+05 .3335E+03  .720E-01
                                                                  .1792E+05 .4469E+00 .1000E+01 .4992E+00 .1792E+05 .5978E+02  .165E+01

 116   .065   50.90  51  47  48  58  57                           .3736E+05 .7126E+00 .1000E+01 .7374E+00 .3736E+05 .4366E+03  .857E-01
                                                                  .1729E+05 .4703E+00 .1000E+01 .5160E+00 .1729E+05 .7618E+02  .171E+01

 117   .079   55.49  51  57  58  68  67                           .3705E+05 .7423E+00 .1000E+01 .7628E+00 .3705E+05 .5839E+03  .777E-01
                                                                  .1691E+05 .5213E+00 .1000E+01 .5594E+00 .1691E+05 .1004E+03  .169E+01

 118   .092   28.87  51  67  68  78  77                           .3721E+05 .9343E+00 .1000E+01 .9367E+00 .3721E+05 .4269E+03 -.280E-01
                                                                  .1626E+05 .8773E+00 .1000E+01 .8819E+00 .1626E+05 .7716E+02  .129E+01

 119   .100   29.97  51  77  78  88  87                           .4091E+05 .9334E+00 .4091E+05 .9358E+00 .1000E+01 .5817E+03 -.115E+00
                                                                  .1869E+05 .8847E+00 .1869E+05 .8888E+00 .1000E+01 .1158E+03  .985E+00
```

```
                                                                                                            PAGE   8
MEMB  THICK   AREA  TYPE MA  MB  MC  MD   SIGMA-X      SIGMA-Y    SIGMA-XY   EFSTR-1   EFSTR-2   EFSTR-3   EFSTR-4    ENERGY       MS
 120   .020   2.25   21   1   2           -.2653E-05                                                                .1508E-19
                                          -.2653E-05                                                                .1508E-19  .226E+11

 121   .020   2.63   21   3   4           -.4091E-05                                                                .4186E-19
                                          -.3637E-05                                                                .3307E-19  .165E+11

 122   .020   3.00   21   5   6           -.1592E-05                                                                .7238E-20
                                          -.1990E-05                                                                .1131E-19  .302E+11

 123   .020   2.63   21   7   8           -.5000E-05                                                                .6253E-19
                                          -.6364E-05                                                                .1013E-18  .943E+10

 124   .020   2.25   21   9  10           -.7428E-05                                                                .1182E-18
                                          -.6897E-05                                                                .1019E-18  .870E+10

 125   .020   2.70   21  11  12           -.4203E-05                                                                .4540E-19
                                          -.3097F-05                                                                .2465E-19  .194E+11

 126   .020   3.17   21  13  14           -.3387E-05                                                                .3465E-19
                                          -.2446E-05                                                                .1808E-19  .245E+11

 127   .020   2.85   21  15  16           -.3555E-05                                                                .3436E-19
                                          -.2300E-05                                                                .1438E-19  .261E+11

 128   .020   2.52   21  17  18           -.5452E-05                                                                .7128E-19
                                          -.2370E-05                                                                .1347E-19  .253E+11

 129   .020   2.56   21  19  20           -.3267E-05                                                                .2600E-19
                                          -.3500E-05                                                                .2984E-19  .171E+11

 130   .020   3.06   21  21  22           -.1169E-05                                                                .3986E-20
                                          -.9740E-06                                                                .2768E-20  .616E+11

 131   .020   3.60   21  23  24           -.1327E-05                                                                .6035E-20
                                          -.1161E-05                                                                .4620E-20  .517E+11

 132   .020   3.23   21  25  26           -.2215E-05                                                                .1511E-19
                                          -.2030E-05                                                                .1269E-19  .296E+11

 133   .020   2.85   21  27  28           -.1677E-05                                                                .7624E-20
                                          -.2305E-05                                                                .1441E-19  .260E+11

 134   .020   2.87   21  29  30           -.1354E-05                                                                .5001E-20
                                          -.1874E-05                                                                .9589E-20  .320E+11

 135   .020   3.43   21  31  32           -.1131E-05                                                                .4179E-20
                                          -.1479E-05                                                                .7147E-20  .406E+11

 136   .020   4.02   21  33  34           -.6675E-06                                                                .1707E-20
                                          -.7416E-06                                                                .2108E-20  .809E+11
```

```
                                                                                                                         PAGE    9
MEMB  THICK   AREA  TYPE MA   MB   MC   MD    SIGMA-X      SIGMA-Y     SIGMA-XY    EFSTR-1    EFSTR-2    EFSTR-3    EFSTR-4     ENERGY       MS

 137   .020    3.61  21   35   36              -.1321E-05                                                                      .6008E-20
                                               -.9909E-06                                                                      .3380E-20  .606E+11

 138   .020    3.18  21   37   38              -.1032E-05                                                                      .3227E-20
                                               -.1220E-05                                                                      .4508E-20  .492E+11

 139   .020    3.17  21   39   40              -.8462E-06                                                                      .2165E-20
                                               -.1128E-05                                                                      .3848E-20  .532E+11

 140   .020    3.80  21   41   42              -.8648E-06                                                                      .2704E-20
                                               -.7075E-06                                                                      .1810E-20  .848E+11

 141   .020    4.45  21   43   44              -.2682E-06                                                                      .3050E-21
                                               -.3353E-06                                                                      .4765E-21  .179E+12

 142   .020    3.99  21   45   46              -.5978E-06                                                                      .1359E-20
                                               -.5978E-06                                                                      .1359E-20  .100E+12

 143   .020    3.51  21   47   48              -.2549E-06                                                                      .2174E-21
                                               -.2124E-06                                                                      .1509E-21  .282E+12

 144   .020    3.48  21   49   50              -.4711E-06                                                                      .7364E-21
                                               -.2570E-06                                                                      .2191E-21  .233E+12

 145   .020    4.16  21   51   52              -.2508E-06                                                                      .2495E-21
                                               -.1433E-06                                                                      .8148E-22  .419E+12

 146   .020    4.88  21   53   54              -.1530E-06                                                                      .1087E-21
                                               -.1530E-06                                                                      .1087E-21  .392E+12

 147   .020    4.37  21   55   56              -.2729E-06                                                                      .3103E-21
                                               -.2900E-06                                                                      .3503E-21  .207E+12

 148   .020    3.84  21   57   58              -.2717E-06                                                                      .2703E-21
                                               -.2523E-06                                                                      .2331E-21  .238E+12

 149   .020    3.79  21   59   60              -.1377E-06                                                                      .6850E-22
                                               -.1180E-06                                                                      .5033E-22  .508E+12

 150   .020    4.53  21   61   62              -.1071E-06                                                                      .4944E-22
                                               -.1153E-06                                                                      .5734E-22  .520E+12

 151   .020    5.30  21   63   64               .7036E-08                                                                      .2500E-24
                                                .2111E-07                                                                      .2250E-23  .284E+13

 152   .020    4.75  21   65   66              -.4710E-07                                                                      .1004E-22
                                               -.5495E-07                                                                      .1367E-22  .109E+13

 153   .020    4.18  21   67   68              -.2680E-07                                                                      .2856E-23
                                               -.1340E-07                                                                      .7141E-24  .448E+13
```

```
                                                                                                       PAGE  10
MEMB  THICK    AREA  TYPE MA  MB  MC  MD    SIGMA-X     SIGMA-Y    SIGMA-XY   EFSTR-1   EFSTR-2   EFSTR-3   EFSTR-4    ENERGY       MS
 154   .020    4.15  21  69  70            -.2249E-08                                                                .1998E-25
                                           -.2249E-07                                                                .1998E-23  .267E+13
 155   .020    4.89  21  71  72            -.1906E-08                                                                .1693E-25
                                           -.1144E-07                                                                .6095E-24  .525E+13
 156   .020    5.65  21  73  74             .4289E-07                                                                .9903E-23
                                            .3959E-07                                                                .8438E-23  .152E+13
 157   .020    5.00  21  75  76            0.                                                                       0.
                                           -.5591E-08                                                                .1490E-24  .107E+14
 158   .020    4.34  21  77  78             .2150E-08                                                                .1909E-25
                                           0.                                                                       0.                R

                THE TOTAL ENERGY FOR LOADING CONDITION  1 IS     .1632E+06  (U)                   .1632E+06(W)

                THE TOTAL ENERGY FOR LOADING CONDITION  2 IS     .1340E+06  (U)                   .1340E+06(W)
```

```
JOINT      -X       -Y        -Z       FORCE-X      FORCE-Y      FORCE-Z       DISPL-X          DISPL-Y          DISPL-Z

  1      63.500   90.000    1.125      205.000    -7380.000      926.000   -1.53156595E-01  -3.37310884E-01   1.50853503E+01
                                        351.000   -12600.000     1530.000   -5.12626745E-02  -3.47255404E-01   1.44830153E+01

  2      63.500   90.000   -1.125     -205.000     7380.000      926.000    1.53156595E-01   3.37310884E-01   1.50853503E+01
                                       -351.000    12600.000     1530.000    5.12626745E-02   3.47255404E-01   1.44830153E+01

  3      70.833   90.000    1.313        0.000        0.000       29.000   -1.68003833E-01  -3.64600013E-01   1.60655917E+01
                                          0.000        0.000       29.500   -4.39865366E-02  -3.51717978E-01   1.47515352E+01

  4      70.833   90.000   -1.313        0.000        0.000       29.000    1.68003833E-01   3.64600013E-01   1.60655917E+01
                                          0.000        0.000       29.500    4.39865366E-02   3.51717978E-01   1.47515352E+01

  5      78.167   90.000    1.500    -2800.000    -6960.000     1130.000   -2.11916646E-01  -4.68729580E-01   1.70648982E+01
                                      -2420.000    -6020.000      979.000   -6.02508514E-02  -4.36737902E-01   1.50078963E+01

  6      78.167   90.000   -1.500     2800.000     6960.000     1130.000    2.11916646E-01   4.68729580E-01   1.70648982E+01
                                       2420.000     6020.000      979.000    6.02508514E-02   4.36737902E-01   1.50078963E+01

  7      85.500   90.000    1.313        0.000        0.000       90.900   -1.87143175E-01  -3.97589127E-01   1.81325540E+01
                                          0.000        0.000       55.900   -4.46781197E-02  -3.61211376E-01   1.52850121E+01

  8      85.500   90.000   -1.313        0.000        0.000       90.900    1.87143175E-01   3.97589127E-01   1.81325540E+01
                                          0.000        0.000       55.900    4.46781197E-02   3.61211376E-01   1.52850121E+01

  9      92.833   90.000    1.125    -9870.000    -9780.000     1130.000   -1.84017545E-01  -3.99077236E-01   1.92778711E+01
                                      -4020.000    -3980.000      474.000   -4.66155072E-02  -3.38626999E-01   1.55597198E+01

 10      92.833   90.000   -1.125     9870.000     9780.000     1130.000    1.84017545E-01   3.99077236E-01   1.92778711E+01
                                       4020.000     3980.000      474.000    4.66155072E-02   3.38626999E-01   1.55597198E+01

 11      69.686   87.471    1.349        0.000        0.000      178.000   -1.68393505E-01  -3.77052511E-01   1.51335589E+01
                                          0.000        0.000      194.000   -4.12649435E-02  -3.66040446E-01   1.39597807E+01

 12      69.686   87.471   -1.349        0.000        0.000      178.000    1.68393505E-01   3.77052511E-01   1.51335589E+01
                                          0.000        0.000      194.000    4.12649435E-02   3.66040446E-01   1.39597807E+01

 13      76.097   84.851    1.586        0.000        0.000      214.000   -2.07968475E-01  -4.73436669E-01   1.51675480E+01
                                          0.000        0.000      175.000   -5.65356821E-02  -4.44512470E-01   1.34203495E+01

 14      76.097   84.851   -1.586        0.000        0.000      214.000    2.07968475E-01   4.73436669E-01   1.51675480E+01
                                          0.000        0.000      175.000    5.65356821E-02   4.44512470E-01   1.34203495E+01

 15      82.746   82.133    1.427        0.000        0.000      253.000   -1.84877630E-01  -4.24220570E-01   1.52133131E+01
                                          0.000        0.000      157.000   -4.44832377E-02  -3.86436603E-01   1.28926685E+01

 16      82.746   82.133   -1.427        0.000        0.000      253.000    1.84877630E-01   4.24220570E-01   1.52133131E+01
                                          0.000        0.000      157.000    4.44832377E-02   3.86436603E-01   1.28926685E+01

 17      89.647   79.312    1.259    -5680.000     2320.000     1020.000   -1.66548864E-01  -3.88465419E-01   1.52287247E+01
                                      -1600.000      653.000      325.000   -3.91972815E-02  -3.49018684E-01   1.23217341E+01
```

```
JOINT      -X       -Y       -Z      FORCE-X      FORCE-Y      FORCE-Z        DISPL-X          DISPL-Y          DISPL-Z

  18     89.647   79.312   -1.259    5680.000    -2320.000     1020.000   1.66548864E-01   3.88455419E-01   1.52287247E+01
                                     1600.000     -653.000      325.000   3.91972815E-02   3.49018684E-01   1.23217341E+01

  19     57.266   77.669    1.279    2310.000     -946.000      723.000  -1.50103626E-01  -3.22284307E-01   1.08415205E+01
                                     5510.000    -2250.000     1550.000  -5.17496373E-02  -3.21113405E-01   1.07102222E+01

  20     57.266   77.669   -1.279   -2310.000      946.000      723.000   1.50103626E-01   3.22284307E-01   1.08415205E+01
                                    -5510.000     2250.000     1550.000   5.17496373E-02   3.21113405E-01   1.07102222E+01

  21     63.992   74.920    1.532       0.000        0.000      314.000  -1.66550822E-01  -3.78524461E-01   1.09061029E+01
                                        0.000        0.000      347.000  -5.19120222E-02  -3.69407025E-01   1.02800835E+01

  22     63.992   74.920   -1.532       0.000        0.000      314.000   1.66550822E-01   3.78524461E-01   1.09061029E+01
                                        0.000        0.000      347.000   5.19120222E-02   3.69407025E-01   1.02800835E+01

  23     70.962   72.071    1.799       0.000        0.000      326.000  -1.92739326E-01  -4.65077469E-01   1.09137101E+01
                                        0.000        0.000      270.000  -5.85946987E-02  -4.41946078E-01   9.79710603E+00

  24     70.962   72.071   -1.799       0.000        0.000      326.000   1.92739326E-01   4.65077469E-01   1.09137101E+01
                                        0.000        0.000      270.000   5.85946987E-02   4.41946078E-01   9.79710603E+00

  25     78.191   69.116    1.617       0.000        0.000      338.000  -1.58157580E-01  -4.11940504E-01   1.09200094E+01
                                        0.000        0.000      213.000  -3.81557674E-02  -3.80672377E-01   9.30515091E+00

  26     78.191   69.116   -1.617       0.000        0.000      338.000   1.58157580E-01   4.11940504E-01   1.09200094E+01
                                        0.000        0.000      213.000   3.81557674E-02   3.80672377E-01   9.30515091E+00

  27     85.692   66.050    1.424   -4070.000     1660.000      902.000  -1.34810839E-01  -3.86352653E-01   1.08426840E+01
                                    -1210.000      496.000      311.000  -2.74908182E-02  -3.45888242E-01   8.73800722E+00

  28     85.692   66.050   -1.424    4070.000    -1660.000      902.000   1.34810839E-01   3.86352653E-01   1.08426840E+01
                                     1210.000     -496.000      311.000   2.74908182E-02   3.45888242E-01   8.73800722E+00

  29     51.032   65.339    1.433    1740.000     -713.000      646.000  -1.44133543E-01  -2.96809892E-01   7.28959532E+00
                                     3990.000    -1630.000     1310.000  -6.58601383E-02  -3.04333605E-01   7.48498407E+00

  30     51.032   65.339   -1.433   -1740.000      713.000      646.000   1.44133543E-01   2.96809892E-01   7.28959532E+00
                                    -3990.000     1630.000     1310.000   6.58601383E-02   3.04333605E-01   7.48498407E+00

  31     58.297   62.369    1.715       0.000        0.000      340.000  -1.51869900E-01  -3.44981522E-01   7.35327780E+00
                                        0.000        0.000      375.000  -5.51758216E-02  -3.37208706E-01   7.12717600E+00

  32     58.297   62.369   -1.715       0.000        0.000      340.000   1.51869900E-01   3.44981522E-01   7.35327780E+00
                                        0.000        0.000      375.000   5.51758216E-02   3.37208706E-01   7.12717600E+00

  33     65.826   59.291    2.012       0.000        0.000      352.000  -1.67953885E-01  -4.26140523E-01   7.33441024E+00
                                        0.000        0.000      291.000  -5.57287692E-02  -4.08652457E-01   6.69261135E+00

  34     65.826   59.291   -2.012       0.000        0.000      352.000   1.67953885E-01   4.26140523E-01   7.33441024E+00
                                        0.000        0.000      291.000   5.57287692E-02   4.08652457E-01   6.69261135E+00
```

```
JOINT      -X       -Y       -Z      FORCE-X     FORCE-Y     FORCE-Z      DISPL-X          DISPL-Y          DISPL-Z

  35     73.635   56.100    1.807      0.000       0.000     365.000   -1.31598297E-01  -3.76577355E-01   7.28909697E+00
                                       0.000       0.000     230.000   -3.10292813E-02  -3.49981104E-01   6.24447656E+00

  36     73.635   56.100   -1.807      0.000       0.000     365.000    1.31598297E-01   3.76577355E-01   7.28909697E+00
                                       0.000       0.000     230.000    3.10292813E-02   3.49981104E-01   6.24447656E+00

  37     81.738   52.787    1.590  -4250.000    1740.000     974.000   -1.09022988E-01  -3.57295406E-01   7.13862489E+00
                                   -1270.000     518.000     336.000   -1.82188447E-02  -3.19885301E-01   5.70271391E+00

  38     81.738   52.787   -1.590   4250.000   -1740.000     974.000    1.09022988E-01   3.57295406E-01   7.13862489E+00
                                    1270.000    -518.000     336.000    1.82188447E-02   3.19885301E-01   5.70271391E+00

  39     44.799   53.008    1.587   1820.000    -743.000     694.000   -1.27880637E-01  -2.49846167E-01   4.43438691E+00
                                    4160.000   -1700.000    1410.000   -6.63969885E-02  -2.62488420E-01   4.78522101E+00

  40     44.799   53.008   -1.587  -1820.000     743.000     694.000    1.27880637E-01   2.49846167E-01   4.43438691E+00
                                   -4160.000    1700.000    1410.000    6.63969885E-02   2.62488420E-01   4.78522101E+00

  41     52.603   49.818    1.898      0.000       0.000     365.000   -1.30665075E-01  -2.92457379E-01   4.49810288E+00
                                       0.000       0.000     402.000   -5.32217253E-02  -2.88831709E-01   4.51417272E+00

  42     52.603   49.818   -1.898      0.000       0.000     365.000    1.30665075E-01   2.92457379E-01   4.49810288E+00
                                       0.000       0.000     402.000    5.32217253E-02   2.88831709E-01   4.51417272E+00

  43     60.691   46.512    2.225      0.000       0.000     378.000   -1.39343038E-01  -3.65573242E-01   4.46771253E+00
                                       0.000       0.000     313.000   -5.06267100E-02  -3.53593407E-01   4.15762025E+00

  44     60.691   46.512   -2.225      0.000       0.000     378.000    1.39343038E-01   3.65573242E-01   4.46771253E+00
                                       0.000       0.000     313.000    5.06267100E-02   3.53593407E-01   4.15762025E+00

  45     69.079   43.083    1.997      0.000       0.000     392.000   -9.89174689E-02  -3.17475451E-01   4.39030667E+00
                                       0.000       0.000     247.000   -2.03986747E-02  -2.95573830E-01   3.77680570E+00

  46     69.079   43.083   -1.997      0.000       0.000     392.000    9.89174689E-02   3.17475451E-01   4.39030667E+00
                                       0.000       0.000     247.000    2.03986747E-02   2.95573830E-01   3.77680570E+00

  47     77.784   39.525    1.756  -4440.000    1820.000    1050.000   -7.54646233E-02  -3.05230452E-01   4.18278166E+00
                                   -1320.000     541.000     361.000   -4.84215299E-03  -2.71275070E-01   3.28711508E+00

  48     77.784   39.525   -1.756   4440.000   -1820.000    1050.000    7.54646233E-02   3.05230452E-01   4.18278166E+00
                                    1320.000    -541.000     361.000    4.84215299E-03   2.71275070E-01   3.28711508E+00

  49     38.565   40.678    1.742   1890.000    -773.000     742.000   -1.05739666E-01  -1.86543915E-01   2.32438568E+00
                                    4330.000   -1770.000    1500.000   -6.20971554E-02  -2.03050212E-01   2.71329020E+00

  50     38.565   40.678   -1.742  -1890.000     773.000     742.000    1.05739666E-01   1.86543915E-01   2.32438568E+00
                                   -4330.000    1770.000    1500.000    6.20971554E-02   2.03050212E-01   2.71329020E+00

  51     46.908   37.267    2.082      0.000       0.000     390.000   -1.03846760E-01  -2.20394079E-01   2.37590002E+00
                                       0.000       0.000     430.000   -4.74240236E-02  -2.21283761E-01   2.51245255E+00
```

```
 JOINT        -X        -Y        -Z      FORCE-X      FORCE-Y      FORCE-Z        DISPL-X           DISPL-Y           DISPL-Z

   52      46.908    37.267    -2.082       0.000        0.000      390.000    1.03846760E-01    2.20394079E-01    2.37590002E+00
                                            0.000        0.000      430.000    4.74240236E-02    2.21283761E-01    2.51245255E+00
   53      55.555    33.732     2.438       0.000        0.000      404.000   -1.03947059E-01   -2.76969324E-01    2.33343889E+00
                                            0.000        0.000      334.000   -4.12753198E-02   -2.71403093E-01    2.23062811E+00
   54      55.555    33.732    -2.438       0.000        0.000      404.000    1.03947059E-01    2.76969324E-01    2.33343889E+00
                                            0.000        0.000      334.000    4.12763198E-02    2.71403093E-01    2.23062811E+00
   55      64.523    30.067     2.187       0.000        0.000      420.000   -6.26198328E-02   -2.33032555E-01    2.24239658E+00
                                            0.000        0.000      264.000   -8.20874278E-03   -2.16845636E-01    1.93907498E+00
   56      64.523    30.067    -2.187       0.000        0.000      420.000    6.26198328E-02    2.33032555E-01    2.24239658E+00
                                            0.000        0.000      264.000    8.20874278E-03    2.16845636E-01    1.93907498E+00
   57      73.830    26.262     1.922   -4640.000     1900.000     1120.000   -4.16609901E-02   -2.33004248E-01    2.00549282E+00
                                        -1380.000      565.000      386.000    7.39064873E-03   -2.04101821E-01    1.53137282E+00
   58      73.830    26.262    -1.922    4640.000    -1900.000     1120.000    4.16609901E-02    2.33004248E-01    2.00549282E+00
                                         1380.000     -565.000      386.000   -7.39064873E-03    2.04101821E-01    1.53137282E+00
   59      32.331    28.347     1.896    2290.000     -937.000      883.000   -7.97542288E-02   -1.15152948E-01    9.39622230E-01
                                         5300.000    -2170.000     1820.000   -5.29909849E-02   -1.34105539E-01    1.27310587E+00
   60      32.331    28.347    -1.896   -2290.000      937.000      883.000    7.97542288E-02    1.15152948E-01    9.39622230E-01
                                        -5300.000     2170.000     1820.000    5.29909849E-02    1.34105539E-01    1.27310587E+00
   61      41.214    24.716     2.265       0.000        0.000      413.000   -7.49426919E-02   -1.39588501E-01    9.71136133E-01
                                            0.000        0.000      458.000   -3.82899685E-02   -1.43064119E-01    1.12826391E+00
   62      41.214    24.716    -2.265       0.000        0.000      413.000    7.49426919E-02    1.39588501E-01    9.71136133E-01
                                            0.000        0.000      458.000    3.82899686E-02    1.43064119E-01    1.12826391E+00
   63      50.420    20.953     2.651       0.000        0.000      391.000   -7.05172863E-02   -1.74586532E-01    9.25682390E-01
                                            0.000        0.000      326.000   -3.24885445E-02   -1.75226816E-01    9.21641638E-01
   64      50.420    20.953    -2.651       0.000        0.000      391.000    7.05172863E-02    1.74586532E-01    9.25682390E-01
                                            0.000        0.000      326.000    3.24885445E-02    1.75226816E-01    9.21641638E-01
   65      59.967    17.050     2.376       0.000        0.000      368.000   -3.28161189E-02   -1.33328204E-01    8.32872079E-01
                                            0.000        0.000      233.000   -9.31470120E-04   -1.24469418E-01    7.28927885E-01
   66      59.967    17.050    -2.376       0.000        0.000      368.000    3.28161189E-02    1.33328204E-01    8.32872079E-01
                                            0.000        0.000      233.000    9.31470120E-04    1.24469418E-01    7.28927885E-01
   67      69.876    13.000     2.088   -3030.000     1240.000      804.000   -1.42984873E-02   -1.42639484E-01    5.96688240E-01
                                         -922.000      377.000      287.000    1.38097400E-02   -1.21961995E-01    4.29246873E-01
   68      69.876    13.000    -2.088    3030.000    -1240.000      804.000    1.42984873E-02    1.42639484E-01    5.96688240E-01
                                          922.000     -377.000      287.000   -1.38097400E-02    1.21961995E-01    4.29246873E-01
```

```
 JOINT       -X       -Y        -Z      FORCE-X     FORCE-Y     FORCE-Z      DISPL-X          DISPL-Y          DISPL-Z


   69      25.166   14.173    2.073    3070.000    -520.000    1040.000  -4.22288799E-02  -4.09255801E-02   1.50235366E-01
                                       7160.000   -1210.000    2180.000  -3.46186534E-02  -6.02115756E-02   3.26853118E-01

   70      25.166   14.173   -2.073   -3070.000     520.000    1040.000   4.22288799E-02   4.09255801E-02   1.50235366E-01
                                      -7160.000    1210.000    2180.000   3.46186534E-02   6.02115756E-02   3.26853118E-01

   71      35.583   12.304    2.446       0.000       0.000     433.000  -4.53298717E-02  -5.96720986E-02   2.19892172E-01
                                          0.000       0.000     484.000  -2.79621842E-02  -6.11086852E-02   3.17463874E-01

   72      35.583   12.304   -2.446       0.000       0.000     433.000   4.53298717E-02   5.96720986E-02   2.19892172E-01
                                          0.000       0.000     484.000   2.79621842E-02   6.11086852E-02   3.17463874E-01

   73      46.181   10.403    2.827       0.000       0.000     370.000  -4.38219757E-02  -8.83518278E-02   2.58556125E-01
                                          0.000       0.000     310.000  -2.57691521E-02  -9.13801375E-02   2.71681902E-01

   74      46.181   10.403   -2.827       0.000       0.000     370.000   4.38219757E-02   8.83518278E-02   2.58556125E-01
                                          0.000       0.000     310.000   2.57691521E-02   9.13801375E-02   2.71681902E-01

   75      56.964    8.469    2.502       0.000       0.000     304.000  -1.98183986E-02  -5.92354093E-02   2.81201334E-01
                                          0.000       0.000     194.000  -2.48140918E-03  -5.57468272E-02   2.49187955E-01

   76      56.964    8.469   -2.502       0.000       0.000     304.000   1.98183986E-02   5.92354093E-02   2.81201334E-01
                                          0.000       0.000     194.000   2.48140918E-03   5.57468272E-02   2.49187955E-01

   77      67.938    6.500    2.169   -1370.000     262.000     446.000  -4.24715599E-03  -8.06102137E-02   1.86891812E-01
                                       -451.000      86.000     175.000   1.17652403E-02  -6.82306147E-02   1.24812534E-01

   78      67.938    6.500   -2.169    1370.000    -262.000     446.000   4.24715599E-03   8.06102137E-02   1.86891812E-01
                                        451.000     -86.000     175.000  -1.17652403E-02   6.82306147E-02   1.24812534E-01

   79      18.000    0.000    2.250       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   80      18.000    0.000   -2.250       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   81      30.000    0.000    2.625       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   82      30.000    0.000   -2.625       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   83      42.000    0.000    3.000       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   84      42.000    0.000   -3.000       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.

   85      54.000    0.000    2.625       0.000       0.000       0.000  0.               0.               0.
                                          0.000       0.000       0.000  0.               0.               0.
```

```
JOINT      -X       -Y       -Z       FORCE-X     FORCE-Y     FORCE-Z    DISPL-X      DISPL-Y      DISPL-Z

 86      54.000    0.000   -2.625      0.000       0.000       0.000   0.           0.           0.
                                       0.000       0.000       0.000   0.           0.           0.

 87      66.000    0.000    2.250      0.000       0.000       0.000   0.           0.           0.
                                       0.000       0.000       0.000   0.           0.           0.

 88      66.000    0.000   -2.250      0.000       0.000       0.000   0.           0.           0.
                                       0.000       0.000       0.000   0.           0.           0.
```

Z-DISPLACEMENTS

	LOAD CASE #1		LOAD CASE #2	
NODE NO	ANALYZE	NASTRAN	ANALYZE	NASTRAN
1*	15.085	15.170	14.483	14.595
3	16.066	16.151	14.752	14.849
5	17.065	17.144	15.008	15.086
7	18.133	18.198	15.285	15.344
9	19.278	19.328	15.560	15.599
11	15.134	15.218	13.960	14.059
13	15.168	15.246	13.420	13.500
15	15.213	15.286	12.893	12.956
17	15.229	15.297	12.322	12.368
19	10.842	10.913	10.710	10.810
21	10.906	10.977	10.280	10.368
23	10.914	10.979	9.797	9.866
25	10.920	10.982	9.305	9.359
27	10.843	10.902	8.738	8.780
29	7.290	7.354	7.485	7.578
31	7.353	7.414	7.127	7.205
33	7.334	7.386	6.693	6.749
35	7.289	7.338	6.244	6.289
37	7.139	7.188	5.703	5.738
39	4.434	4.492	4.785	4.870
41	4.498	4.547	4.514	4.580
43	4.468	4.504	4.158	4.199
45	4.390	4.425	3.777	3.810
47	4.183	4.222	3.287	3.315
49	2.324	2.378	2.713	2.792
51	2.376	2.414	2.512	2.566
53	2.333	2.353	2.231	2.256
55	2.242	2.263	1.939	1.960
57	2.005	2.034	1.531	1.551
59	.940	.987	1.273	1.343
61	.971	.998	1.128	1.169
63	.926	.930	.922	.932
65	.833	.839	.729	.737
67	.597	.611	.429	.439
69	.150	.175	.327	.368
71	.220	.236	.317	.342
73	.259	.259	.272	.276
75	.281	.283	.249	.252
77	.187	.195	.125	.130

*Note: Results are given for nodes on the upper surface only. The displacement pattern is identical on the lower surface.

TABLE 2: Results From ANALYZE and NASTRAN

*U.S.Government Printing Office: 1979 — 657-002/441