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ANALYTICAL MODELS FOR ACV SEAKEEPING

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A linearized mathematical model is developed from first principles to predict the motion of an air cushion vehicle (ACV) in head seas consisting of sinusoidal waves.

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Nomenclature

a	= wave amplitude
A_b	= bottom area of cell
A_c	= cushion area
A_g	= gap leakage area
A_ℓ	= leakage area
A_o	= equilibrium leakage area
A_p	= craft planform area (= A_c+base area of skirt cells)
A_r	= area of orifice between loop and cell
$A_{33,35,53}$	= equation coefficients
b	= craft beam
$B_{33,35,53}$	= equation coefficients
C_n	= orifice coefficient
$C_{0,1}$	= empirical coefficients from fan map
$C_{33,35,53}$	= equation coefficients
d	= depth of fully extended skirt below c.g.
G	= cushion leakage fraction of gap height
h	= heave displacement
I_y	= moment of inertia
k	= wavenumber
ℓ	= length of craft
m	= mass of craft
M_{ext}	= externally applied moment
P	= cushion pressure (gauge)
P_L	= loop pressure
P_j	= cell or jupe pressure

$Q_{in,out}$	= volume rate of flow in, out of cushion
Δs_i	= length of skirt seal around cushion periphery
T	= average skirt thickness
U_o	= craft speed
U_{sp}	= unit step function
V	= cushion volume
W	= craft weight
Z_{ext}	= externally applied moment
η	= surface elevation
θ	= pitch angle
λ	= wavelength
ρ	= density
ϕ	= angle of the outer face of skirt with respect to z-axis
ω_e	= encounter frequency ($= \frac{2\pi}{\lambda} U_o + \omega$)
ω	= wave frequency

<u>Subscripts</u>

b	refers to bow
i	refers to the i^{th} seal
j	refers to jupe or cell
L	refers to plenum loop
r	refers to relative position with respect to surface
o	refers to equilibrium condition ($a=0$, $h=0$, $P=P_o=\frac{W}{A_p}$)
1	refers to forward subcushion
2	refers to after subcushion

I. Introduction

The problem considered in this report is the theoretical prediction of the motion of an air cushion vehicle (ACV), traveling at constant speed and course, as it encounters head seas consisting of regular, sinusoidal waves. A mathematical model is developed from first principles based upon the dynamic equations of motion and the air flow relations for the air cushions. These equations, in their entirety, are complicated and require numerical evaluation by a digital computer. They have thus formed the fundamental structure for computer simulations of ACV response to waves. In this report, however, these equations will be simplified and linearized so that an analytical solution may be obtained. The solution provides the pitch and heave response of the ACV in terms of the wave parameters and the design particulars of the craft. The purpose of this ideal model is to determine the design parameters which control the various physical processes that affect the seakeeping ability of ACV's.

The approach taken here emphasizes a line of research, in connection with the ACV seakeeping problem, which has not been fully exploited. On the one hand, sophisticated computer simulations are being developed in which the extremely complex, nonlinear differential and algebraic equations are solved numerically (1-6). On the other hand, systematic empirical studies are being done utilizing full scale craft and physical scale models in the towing tank (7-10). However relatively little work has been done using an analytical approach to solve a simplified set of dynamic equations. Though one would expect that a closed form solution to a reduced set of equations would be less accurate than a numerical evaluation of the complete, nonlinear equations, the analytical approach has two important benefits. First of all, physical insight into the problem is gained by close inspection and identification of the terms in the analysis. And, secondly, recognizable dynamic parameters such as damping and restoring force terms can be found explicitly in terms of the craft's design particulars. This, of course, will be of enormous significance in decisions made at the design stage.

Efforts directed towards obtaining analytical solutions for ACV motion problems include studies of ACV heave motion by Hogben (11), Reynolds (12), and Kaplan and Davis (13). Lavis, Bartholomew and Jones (14) used a linear analysis for pitch, heave and roll motion in which coupled damped harmonic oscillator equations were employed. The critical stiffness and damping coefficients, however, were specified empirically. Reynolds, West and Brooks (15) have also developed a linear analysis for the coupled pitch and heave motion of a divided cushion ACV in head seas. In this work, a rather limiting assumption was made that the skirt hem did not come in contact with the wave surface. More recently, Lebel and Swift (16) have obtained a solution to a nonlinear model for heave motion, and Lundblad (17) has developed a linearized pitch and heave model for single cushion, peripheral all-stabilized craft.

In this report, the pitch and heave response of an ACV to head seas comprised of regular waves is analyzed using a theory which is a special case of the more general, nonlinear mathematical models developed by Carrier and Swift (6). The equations presented are a greatly simplified, linearized version of the general form and are appropriate for instances in which the dynamic variables deviate only slightly from their equilibrium values.

Two craft are considered - a single cushion, peripheral cell-stabilized ACV and a craft having a divided cushion. A description of the support systems used in each configuration is included in reference (6). Immediately following is the analysis for the single cushion craft, while the divided cushion ACV is considered in the last section.

II. Single Cushion, Peripheral Cell-Stabilized ACV

As indicated in Fig. 1, the ACV is in planar motion with its position and orientation given by h and Θ, respectively. For convenience in the analysis, the origin of the x, z system, which is fixed with respect to the craft, will be at the craft center of gravity and over the centroid of the planform area. Whenever possible, advantage will be taken of the smallness of the difference of dependent variables

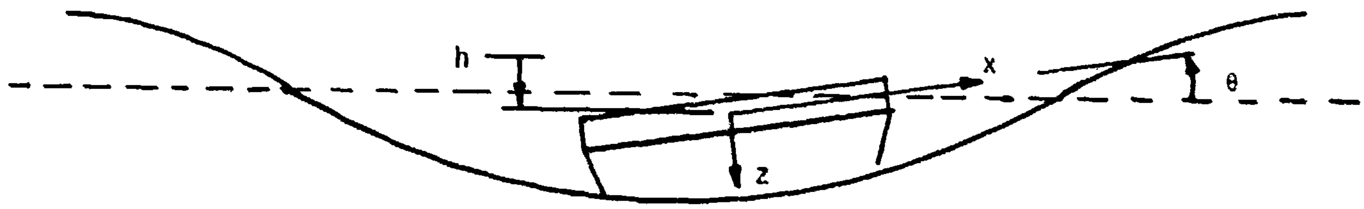

Figure 1 Craft Coordinate System

from their equilibrium values in order to linearize the equations. Products of small quantities will be dropped in favor of linear terms. The craft, traveling at constant speed U_o, encounters head seas consisting of regular, sinusodial waves of amplitude

$$\eta(x,t) = a \sin(kx + \omega_e t) \tag{1}$$

The Froude-Kryloff hypothesis will be employed; that is, the waves are assumed non-compliant.

The equations of motion governing craft response require that inertia forces equal the forces on the cushion and seals. The forces on the cushion or skirt cells are taken as the pressure acting over the externally supported area:

$$m\ddot{h} = \sum_{\substack{i \\ \text{skirt} \\ \text{cells}}} [(P_j)_k - P]A_b - [P - P_o]A_p - (P_j)_{io} \sum_{\substack{i \\ \text{skirt} \\ \text{cells}}} \frac{h-x_i\theta+\eta_i}{\cot\phi} \Delta s_i U_{sp}(h-x_i\theta+\eta_i) \tag{2}$$

$$I_y\ddot{\theta} = \sum_{\substack{i \\ \text{skirt} \\ \text{cells}}} x_i [(P_j)_i - P]A_b + (P_j)_{io} \frac{h-x_i\theta+\eta_i}{\cot\phi} \Delta s_i U_{sp}(h-x_i\theta+\eta_i) - [\eta(x_b) - \eta(x_s) - \ell\theta]bdP_o \tag{3}$$

In the above expressions, it has assumed that the skirt is completely flexible and is without inertia. The hydrodynamic pressure on the seal, when seal contact occurs, is then equal to the cell or jupe pressure. With this approximation, the effect of skirt contact is to increase projected area.

The cushion or cell pressures are determined from a conservation of mass relation. For the main cushion, when compressibility effects are neglected, the rate of cushion volume change must equal the net flow into the cushion:

$$\dot{V} = Q_{in} - Q_{out} \tag{4}$$

The main cushion is fed air directly by a fan supply. The linearized flow-pressure relation represents the "fan map" near the equilibrium operating point and is given by

$$Q_{in} - C_o - C_1 P \tag{5}$$

where

$$C_1 = -\left(\frac{\partial Q_{in}}{\partial P}\right)_o > o$$

The flow out of the main cushion is due to leakage beneath the surrounding skirt. An orifice-coefficient approach will be used; in equation form this is

$$Q_{out} = C_n A_\ell \sqrt{2P/\rho} \tag{7}$$

The leakage area, A_ℓ , includes a part present at equilibrium and additional area due to gap openings between skirt and surface which occur while encountering waves. Not all the physical gap, however, is available to main cushion leakage. The underfed peripheral jet flow from the cellular skirt must also pass beneath the outer skirt along with main cushion leakage flow. Therefore, only a fraction of the physical gap area is allotted to main cushion leakage. The leakage area is then given by

$$\begin{aligned} A_\ell &= A_o + G \sum_{\substack{i \\ \text{skirt} \\ \text{cells}}} (-h+x_i\theta-\eta_i)\, \Delta s_i U_{sp}(-h+x_i\theta-\eta_i) \\ &= A_o + A_g,\quad A_o = \frac{C_o - C_1 \frac{W}{A_p}}{C_n\sqrt{\frac{2W}{\rho A_p}}} \end{aligned} \tag{8}$$

where G is the fraction gap area occupied by main cushion leakage. The fraction G will be specified as the ratio of leakage areas, main cushion system over skirt system plus main cushion system, known to exist at equilibrium. In the design considered the jupe pressure of the open-bottomed cells at equilibrium equals the main cushion pressure, so that the gap fraction G may be written as

$$G = \frac{A_0}{A_0 + \sum\limits_{\substack{i \\ \text{skirt} \\ \text{cells}}} (A_i)_0} = \frac{Q_0}{Q_0 + (Q_L)_0} \tag{9}$$

The remaining term in Eq. (4), the rate of volume change, accounts for changes in cushion height and the volume occupied by the waves ("wave pumping"). The rate of cushion volume change is then

$$\dot{V} = A_c(-\dot{h}) - \frac{\lambda a b \omega_e}{\pi} \sin \frac{\pi \ell}{\lambda} \cos \omega_e t \tag{10}$$

Combining Eqs. (4), (5), (7), (8), (10), and linearizing, one obtains

$$A_c \dot{h} + \frac{\lambda a b \omega_e}{\pi} \sin \frac{\pi \ell}{\lambda} \cos \omega_e t = (C_1 + \frac{C_n A_0}{\sqrt{2 \rho P_0}})(P - P_0) + C_n A_g \sqrt{\frac{2P_0}{\rho}} \tag{11}$$

Conservation of mass is also applied to the air flow through each open-bottomed cell. This is a statement that the flow into the jupe from the plenum loop must equal the flow out through the bottom plus the rate of volume change of the cell. In equation form this reads as

$$C_n A_r \sqrt{\frac{2[P_L - (P_j)_i]}{\rho}} = C_n A_i \sqrt{\frac{2(P_j)_i}{\rho}} + T \, \Delta s_i (-\dot{h} + x_i \dot{\theta} - \dot{n}_i), \tag{12}$$

where A_i is the leakage area below the outer skirt available to the escape flow,

$$A_i = (A_i)_0 + (1-G) \Delta s_i (-h + x_i \theta - n_i) U_{sp} (-h + x_i \theta - n_i)$$

and

$$(A_i)_0 = A_r \sqrt{\frac{(P_L)_0 - (P_j)_{io}}{(P_j)_{io}}} \tag{13}$$

Though the loop plenum is fed separately from the main cushion, the exit flow areas for the cushion and the loop-skirt system are proportional. The flow out from the two systems are also nearly proportional since the equilibrium pressure of the open-bottomed cells and the equilibrium cushion pressure are the same. Because of the almost linear relationship between the flow through the loop-skirt system and the main cushion system, the loop pressure will be approximated by

$$P_L = C_p P \tag{14}$$

In this expression, the constant of porportionality, C_p, will equal the ratio of pressures at equilibrium, $C_p - (P_L)_o/P_o$.

The i^{th} cell mass conservation relation, Eq. 12, is linearized by squaring both sides and dropping terms involving the products of the fluctuations of time dependent variables from their equilibrium values. After some manipulation, this may be written in the form

$$\begin{aligned}(P_j)_i - P = &- \frac{2(A_i)_o \Delta s_i}{A_r^2 + (A_i)_o^2}(1-G)(P_j)_o(-h + x_i\theta + \eta_i) \\ &- \frac{(A_i)_o \sqrt{2\rho(P_j)_o}}{C_n[A_r^2 + (A_i)_o^2]} \, T \, \Delta s_i \, (-\dot{h} + x_i\dot{\theta} - \dot{\eta}_i)\end{aligned} \tag{15}$$

At the stern, the seals are clo~ed on the bottom. For these jupes, the cell pressure is always equal to the loop pressure.

Combining Eqs. (11) and (15) with the equations of motion, Eqs. (2) and (3), one obtains:

$$m\ddot{h} = \sum_{\substack{i \\ \text{skirt cells} \\ \text{on bow and} \\ \text{sides}}} \left(\left[\frac{2(A_i)_o \Delta s_i}{A_r^2 + (A_i)_o^2}(1-G)P_o A_b + \frac{A_p C_n \sqrt{\frac{2P_o}{\rho}}\,\Delta s_i}{C_1 + \frac{C_n A_o}{\sqrt{2\rho P_o}}} G\right](-h + x_i\theta - \eta_i)U_{sp}\right.$$

$$(-h+x_i\theta-\eta_i) - P_o \frac{\Delta s_i}{\cot\phi}(h-x_i\theta+\eta_i)U_{sp}(h-x_i\theta+\eta_i) +$$

$$+ \frac{A_b(A_i)_o \sqrt{2\rho P_o}}{C_n[A_r^2+(A_i)_o^2]} T\,\Delta s_i\,(-\dot{h}+x_i\dot{\theta}-\dot{\eta}_i))$$

$$- (P_L)_o \frac{b}{\cot\phi}(h+\frac{\ell}{2}\theta+\eta_s)U_{sp}(h+\frac{\ell}{2}\theta+\eta_s) +$$

$$+ \frac{A_p C_n \sqrt{\frac{2P_o}{\rho}}\, bG}{(C_1 + \frac{C_n A_o}{\sqrt{2\rho P_o}})}(-h-\frac{\ell}{2}\theta-\eta_s)U_{sp}(-h-\frac{\ell}{2}\theta-\eta_s) -$$

$$- \frac{A_p}{C_1 + \frac{C_n A_o}{\sqrt{2\rho P_o}}}(A_c\dot{h} + \frac{\lambda a b \omega_e}{\pi}\sin\frac{\pi\ell}{\lambda}\cos\omega_e t) \qquad (16)$$

$$I_y\ddot{\theta} = \sum_i x_i \left(- \frac{2A_b(A_i)_o\Delta s_i}{A_r^2+(A_i)_o^2}(1-G)P_o(-h+x_i\theta-\eta_i)U_{sp}(-h+x_i\theta-\eta_i) +\right.$$

skirt cells on bow and sides

$$+ P_o \frac{\Delta s_i}{\cot\phi}(h-x_i\theta+\eta_i)U_{sp}(h-x_i\theta+\eta_i) -$$

$$- \frac{A_b \sqrt{2\rho P_o}}{C_n[A_r^2+(A_i)_o^2]} T\,\Delta s_i(-\dot{h}+x_i\dot{\theta}-\dot{\eta}_i)) -$$

$$- (\ell/2)P_o \frac{b}{\cot\phi}(h+\frac{\ell}{2}\theta+\eta_s)U_{sp}(h+\frac{\ell}{2}\theta+\eta_s) -$$

$$-(\eta_b-\eta_s-\ell\theta)bdP_o \qquad (17)$$

It is observed that these equations now involve stiffness terms which are piecewise linear. That is, there are now forcing terms of the form

$$F(h_r) = k_1 h_r U_{sp}(+h_r)+k_2 h_r U_{sp}(-h_r) \qquad (18)$$

where h_r is the relative skirt height, $h_r = h - x_i\theta + \eta_i$.

In order to completely linearize the equations, some criteria must be set for determining an equivalent stiffness (spring) constant such that

$$F(h_r) \approx K_{eq} h_r \tag{19}$$

The criteria which has given the best results is that the equivalent stiffness be such that the period of free oscillation be the same for the Eq. (19) form as for the Eq. (18) form. This criteria implies that

$$K_{eq} = \frac{4k_1k_2}{k_1 + 2\sqrt{k_1k_2} + k_2} \tag{20}$$

After this approximation is made, the summations may be evaluated by calculating the corresponding integrals along the craft periphery. The equations can then be put in the linearized form

$$(m+A_{33})\ddot{h} + B_{33}\dot{h} + C_{33}h + A_{35}\ddot{\theta} + B_{35}\dot{\theta} + C_{35}\theta = Z_{ext}$$

$$A_{53}\ddot{h} + B_{53}\dot{h} + C_{53}h + (I_y + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + C_{55}\theta = M_{ext} \tag{21}$$

in which the hydrodynamic coefficients and wave forcing terms are given explicitly in Appendix A.

Since the wave forcing is a harmonic function of time at the encounter frequency, Z_{ext} and M_{ext} can be expressed in the form

$$Z_{ext} = R_e[\overline{Z}_o e^{i\omega_e t}]$$

$$M_{ext} = R_e[\overline{M}_o e^{i\omega_e t}] \tag{22}$$

where $\overline{Z}_o$ and $\overline{M}_o$ are the complex amplitudes of the wave force and moment on the craft. The craft response, $h(t)$ and $\Theta(t)$, are expressed similarly:

$$h(t) = R_e\,[h_o e^{i\omega_e t}]$$

$$\Theta(t) = R_e\,[\Theta_o e^{i\omega_e t}] \tag{23}$$

Substituting Eqs. (22) and (23) into Equation (21) enables the complex amplitudes of the response, h_o and θ_o, to be determined in the frequency domain. The results are:

$$h_o = \{\overline{Z}_o\,[C_{55}-\omega_e^2(I_y+A_{55})+i\omega_e\,B_{55}]-\overline{M}_o[C_{35}-\omega_e^2 A_{35}+i\omega B_{35}]\}\cdot$$

$$\cdot\{[C_{33}-\omega_e^2(m+A_{33}) + i\omega_e\,B_{33}][C_{55}-\omega_e^2(I_y+A_{55}) + i\omega_e\,B_{55}]$$

$$-[C_{53}-\omega_e^2\,A_{53} + i\omega\,B_{53}][C_{53}-\omega_e^2\,A_{35} + i\omega_e\,B_{35}]\}^{-1}$$

$$\theta_o = \{\overline{M}_o[C_{33}-\omega_e^2(m+A_{33}) + i\omega_e\,B_{33}]-\overline{Z}_o[C_{53}-\omega_e^2\,A_{53}+i\omega_e\,B_{53}]\}\cdot$$

$$\cdot\{[C_{33}-\omega_e^2\,(m+A_{33}) + i\omega_e\,B_{33}][C_{55} - \omega_e^2\,(I_y+A_{55}) + i\omega_e\,B_{55}]$$

$$-[C_{53} - \omega_e^2\,A_{53} + i\omega_e\,B_{53}][C_{35} - \omega_e^2\,A_{35} + i\omega_e\,B_{35}]^{-1} \tag{24}$$

Eq. (23) and (24) represent the solution for the craft response variables $\theta(t)$ and $h(t)$. These expressions are written in terms of dynamic parameters which are given explicitly in Appendix A as functions of design particulars.

III. Divided Cushion ACV

The pitch and heave motion of a divided cushion ACV will be analyzed using the same coordinate system as that used in the previous section (i.e., Fig. 1). The vehicle considered here will have its rectangular cushion divided by a transverse stability keel at a point midway between the bow and stern coinciding with the craft center of gravity. (A longitudinal stability keel has no significance in a head seas, pitch and heave analysis). The forward cushion, designated with subscript 1, and the after cushion, designated by subscript 2, are both enclosed externally by a flexible skirt consisting of finger seals. Thus, there is no cell base area and $A_c = A_p$.

The craft, traveling at constant speed U_o, encounters the regular wave head sea given by Eq. (1). Assuming that the skirt system is completely flexible and without inertia, the equations of motion for pitch and heave can be written in the form:

$$m\ddot{h} = -(P_1-P_o)\,Ac/2 - (P_2-P_o)\,Ac/2 - \sum_{\substack{i \\ \text{external} \\ \text{periphery}}} \frac{P_o}{\cot\phi}(h-x_i\theta+\eta_i)U_{sp}(h-x_i\theta+\eta_i)\,\Delta s_i$$

$$I_y\ddot{\theta} = [(P_1-P_o) - (P_2-P_o)]\frac{Ac\ell}{8} + \sum_{\substack{i \\ \text{external} \\ \text{periphery}}} \frac{P_o}{\cot\phi}x_i(h-x_i\theta+\eta_i)U_{sp}(h-x_i\theta+\eta_i) \tag{25}$$

The pressure fluctuations from their equilibrium values are determined by considering the flow relations for each cushion. Since the leakage areas are large, the density is assumed constant, and the rate of cushion volume change equals the net flow into the cushion:

$$\dot{V}_{1,2} = (Q_{in})_{1,2} - (Q_{out})_{1,2} \tag{26}$$

In the craft considered, each cushion is supplied air by a separate fan system. The linearized representation of the fan map is then given by

$$(Q_{in})_{1,2} = C_o - C_1\,P_{1,2} \tag{27}$$

The flow out is expressed as an orifice coefficent modification to Bernoulli's equation:

$$\begin{aligned}(Q_{out})_{1,2} &= C_n(A_\ell)_{1,2}\sqrt{\frac{2\,P_{1,2}}{\rho}} \\ &= C_n(A_\ell)_{1,2}\sqrt{\frac{2}{\rho}}\left(\sqrt{P_o} + \frac{P_{1,2}-P_o}{2\sqrt{P_o}}\right)\end{aligned}$$

where

$$(A_\ell)_{1,2} = \frac{C_o-C_1\,P_o}{C_n\sqrt{\dfrac{2\,W/A_c}{\rho}}} + \sum_{\substack{i \\ \text{external} \\ \text{periphery 1,2}}} \Delta s_i\,(-h_r)_i\,U_{sp}\,(-h_r)_i \tag{28}$$

In the above equation, cross-flow leakage between the cushions is assumed neglibible in comparison to leakage to the atmosphere.

The rate of cushion volume change is due to changes in height of the hardstructure base minus the rate of volume change occupied by the wave. In equation form this is

$$\dot{V}_{1,2} = (-\dot{h} \pm \ell/4\ \dot{\theta})(Ac/2) - \frac{\lambda a b \omega_e}{\pi} \sin\frac{\pi\ell}{2\lambda} \cos\left(\frac{\pi\ell}{2\lambda} \pm \omega_e t\right) \tag{29}$$

Combining Eqs. (26)-(29), one obtains the pressure fluctuations from equilibrium as

$$P_{1,2} - P_o = \frac{1}{C_1 + \frac{C_o - C_1 P_o}{2P_o}} \left\{(\dot{h} \mp \ell/4\dot{\theta})\ Ac/2 + \frac{\lambda a b \omega_e}{\pi} \sin\frac{\pi\ell}{2\lambda} \cos\left(\frac{\pi\ell}{2\lambda} \pm \omega_e t\right)\right.$$

$$\left. - C_n\sqrt{\frac{2P_o}{\rho}} \sum_{\substack{i \\ \text{external} \\ \text{periphery 1,2}}} \Delta s_i(-h_r)_i U_{sp}(-h_r)_i\right\} \tag{30}$$

Substituting this result into the equations of motion, Eq. (25), yields,

$$m\ddot{h} = \frac{A_c}{C_1 + \frac{C_o\ C_1 P_o}{2\ P_o}} \left\{ -\frac{\lambda a b \omega_e}{2\pi} \sin\frac{\pi\ell}{\lambda} \cos \omega_e t - \frac{A_c}{2}\dot{h}\right.$$

$$\left. + C_n\sqrt{\frac{P_o}{2\rho}} \sum_{\substack{i \\ \text{external} \\ \text{periphery}}} \Delta s_i(-h_r)_i\ U_{sp}(-h_r)_i\right\} - \frac{P_o}{\cot\phi} \sum_{\substack{i \\ \text{external} \\ \text{periphery}}} \Delta s_i(h_r)_i\ U_{sp}(h_r)_i$$

$$I_y\ddot{\theta} = \frac{A_c\ \ell/8}{C_1 + \frac{C_o - C_1 P_o}{2P_o}} \left[\frac{2\lambda a b \omega_e}{\pi} \sin^2\frac{\pi\ell}{2\lambda} \sin \omega_e t - \frac{A_c\ell}{4}\dot{\theta} - C_n\sqrt{\frac{2P_o}{\rho}} \left[\sum_{\substack{\text{external} \\ \text{periphery } i}} \Delta s_i(-h_r)_i\ U_{sp}(-h_r)_i\right.\right.$$

$$\left.\left. - \sum_{\substack{\text{external} \\ \text{periphery 2}}} \Delta s_i(-h_r)_i\ U_{sp}\ (-h_r)_i\right]\right] + \frac{P_o}{\cot\phi} \sum_{\substack{i \\ \text{external} \\ \text{periphery}}} \Delta s_i\ x_i\ (h_r)_i\ U_{sp}\ (h_r)_i \tag{31}$$

It is seen that the restoring force terms are piecewise linear, the general form of which is given by Eq. (18). For the divided cushion craft, a generating function method was used to determine the equivalent stiffness constant defined by Eq. (19). This criteria requires that an average of the stiffness constants be taken:

$$K_{eq} = \frac{K_1 + K_2}{2} \tag{32}$$

By using this approximation and evaluating the summations by determining the corresponding integrals, Eq. (31) is put in the form of Eq. (21) in which the coefficients are given explicitly in Appendix B. The solutions for the pitch and heave variables, $\theta(t)$ and $h(t)$ are then given by Eqs. (23) and (24).

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APPENDIX A

The coefficients and wave forcing terms in Eq. (21) for the single cushion, peripheral cell-stabilized craft are as follows:

$$A_{33} = A_{35} = A_{53} = A_{55} = 0$$

$$B_{33} = \frac{(A_i)_0 T A_b \sqrt{2\rho P_0}\,(2\ell+b)}{C_n[(A_i)_0^2 + A_r^2]} + \frac{A_c A_p}{C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}}$$

$$B_{35} = B_{53} = \frac{-(A_i)_0 T A_b \sqrt{2\rho P_0}\, b\ell}{2C_n[(A_i)_0^2 + A_r^2]}$$

$$B_{55} = \frac{(A_i)_0 T A_b \sqrt{2\rho P_0}\,[2\ell^3 + 3\ell^2 b]}{12\, C_n[(A_i)_0^2 + A_r^2]}$$

$$C_{33} = \frac{4P_0}{\cot\phi}\left[\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2} + \frac{C_n G A_p \sqrt{2\rho P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]}\right]\left[\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2}\right.$$

$$+ \frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]} + \frac{P_0}{\cot\phi} + \left[\frac{4P_0}{\cot\phi}\left[\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2}\right.\right.$$

$$\left.\left.\left. + \frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]}\right]\right]^{1/2}\right]^{-1} [2\ell+b] + \frac{4bC_n G A_p (P_L)_0 \sqrt{2P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]\cot\phi}$$

$$\left[\frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]} + \frac{(P_L)_0}{\cot\phi} + \left[\frac{4C_n G A_p (P_L)_0 \sqrt{2P_0}}{\sqrt{\rho}\,[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]\cot\phi}\right]^{1/2}\right]^{-1}$$

$$C_{35} = \frac{b\ell}{2} [[\frac{4C_nGA_p(P_L)_0\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]\cot\phi}][\frac{C_nGA_p\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]} + \frac{(P_L)_0}{\cot\phi}$$

$$+ [\frac{4C_nGA_p(P_L)_0\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]\cot\phi}]^{1/2}]^{-1} - \frac{4P_0}{\cot\phi}\ [\frac{2(1-G)(A_i)_0P_0A_b}{(A_i)_0^2+A_r^2}$$

$$+ \frac{C_nGA_p\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]}]\ [\frac{2(1-G)(A_i)_0P_0A_b}{(A_i)_0^2+A_r^2} + \frac{C_nGA_p\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]} + \frac{P_0}{\cot\phi}$$

$$+ [\frac{4P_0}{\cot\phi}\ [\frac{2(1-G)(A_i)_0P_0A_b}{(A_i)_0^2+A_r^2} + \frac{C_nGA_p\ \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_nA_0}{\sqrt{2\rho P_0}}]}]]^{1/2}]^{-1}$$

$$C_{53} = -\ \frac{b\ell}{2}\ [\frac{8P_0A_b(A_i)_0P_0(1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}][\frac{P_0}{\cot\phi} + \frac{2A_b(A_i)_0P_0(1-G)}{(A_i)_0^2+A_r^2}$$

$$+ [\frac{8P_0A_b(A_i)_0P_0(1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}]^{1/2}]^{-1}$$

$$C_{55} = -b\ell dP_0 + \frac{[2\ell^3+3\ell^2b]}{12}\ [\frac{8P_0A_b(A_i)_0P_0(1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}][\frac{P_0}{\cot\phi} + \frac{2A_b(A_i)_0P_0(1-G)}{(A_i)_0^2+A_r^2}$$

$$+ [\frac{8P_0A_b(A_i)_0P_0(1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}]^{1/2}]^{-1}$$

$$Z_{ext} = [- \frac{(A_i)_0 T A_b \sqrt{2\rho P_0}}{C_n[(A_i)_0^2 + A_r^2]}][ba\omega_e \cos(\frac{k\ell}{2} + \omega_e t) + \frac{2\lambda a\omega_e}{\pi} \sin \frac{\pi\ell}{\lambda} \cos\omega_e t]$$

$$- \frac{4P_0}{\cot\phi} [\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2} + \frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]}][\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2}$$

$$+ \frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]} + \frac{P_0}{\cot\phi} + [\frac{4P_0}{\cot\phi} [\frac{2(1-G)(A_i)_0 P_0 A_b}{(A_i)_0^2 + A_r^2}$$

$$+ \frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]}]^{1/2}]^{-1} [ba \sin(\frac{k\ell}{2} + \omega_e t) + \frac{2\lambda a}{\pi} \sin\omega_e t]$$

$$- \frac{\lambda ab\omega_e A_p \sin \frac{\pi\ell}{\lambda} \cos\omega_e t}{\pi[C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]} - [ba \sin(- \frac{k\ell}{2} + \omega_e t)]$$

$$\cdot [\frac{4C_n G A_p (P_L)_0 \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]\cot\phi}][\frac{C_n G A_p \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]} + \frac{(P_L)_0}{\cot\phi}$$

$$+ [\frac{4C_n G A_p (P_L)_0 \sqrt{2P_0}}{\sqrt{\rho}\ [C_1 + \frac{C_n A_0}{\sqrt{2\rho P_0}}]\cot\phi}]^{1/2}]^{-1}$$

$$M_{ext} = \left[\frac{(A_i)_0 TA_b \sqrt{2\rho P_0}}{C_n[(A_i)_0^2+A_r^2]}\right]\left[\frac{b\ell a\omega_e}{\lambda} \cos\left(\frac{k\ell}{2} + \omega_e t\right) - \left(\frac{4a\omega_e}{k^2} \sin \frac{k\ell}{2}\right.\right.$$

$$\left.\left. + \frac{2a\ell\omega_e}{k} \cos \frac{k\ell}{2}\right) \sin\omega_e t\right] - bdP_0 a \sin\left(- \frac{k\ell}{2} + \omega_e t\right) - bdP_0 a \sin \cdot$$

$$\cdot\left(\frac{k\ell}{2} + \omega_e t\right) + \left[\frac{b\ell a}{2} \sin\left(\frac{k\ell}{2} + \omega_e t\right) + \left(\frac{4a}{k^2} \sin \frac{k\ell}{2} - \frac{2a\ell}{k} \cos \frac{k\ell}{2}\right) \cos\omega_e t\right]\cdot$$

$$\left[\frac{8P_0 A_b (A_i)_0 P_0 (1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}\right]\left[\frac{P_0}{\cot\phi} + \frac{2A_b (A_i)_0 P_0 (1-G)}{(A_i)_0^2+A_r^2} + \left[\frac{8P_0 A_b (A_i)_0 P_0 (1-G)}{[(A_i)_0^2+A_r^2]\cot\phi}\right]^{1/2}\right]^{-1}$$

APPENDIX B

The coefficients and wave forcing terms in Eq. (21) for the divided cushion ACV are as follows:

$$A_{33} = A_{35} = A_{53} = A_{55} = 0$$

$$B_{33} = \frac{A_c^2 P_0}{(C_0 + C_1 P_0)}$$

$$B_{35} = B_{53} = 0$$

$$B_{55} = \frac{\ell^2 A_c^2 P_0}{16(C_0 + C_1 P_0)}$$

$$C_{33} = (b + \ell)\ [\frac{C_n \sqrt{2P_0/\rho}\ A_c P_0}{C_0 + C_1 P_0} + \frac{P_0}{\cot\phi}]$$

$$C_{35} = C_{53} = 0$$

$$Z = a(b \cos\frac{\pi\ell}{\lambda} + \frac{\lambda}{\pi}\sin\frac{\pi\ell}{\lambda})\ [\frac{C_n \sqrt{2P_0/\rho}\ A_c P_0}{C_0 + C_1 P_0} + \frac{P_0}{\cot\phi}]\sin\omega_e t$$

$$+ \frac{ab\ \lambda\ \omega_e\ A_c P_0}{\pi(C_0 + C_1 P_0)}\sin\frac{\pi\ell}{\lambda}\cos\omega_e t$$

$$M = [\frac{ab\lambda\ell\omega_e\ A_c P_0}{2\pi(C_0 + C_1 P_0)}\sin^2(\frac{\pi\ell}{2\lambda})]\sin\omega_e t$$

$$+ \left\{ \frac{a\ell\lambda}{2\pi}\ [(\frac{C_n \sqrt{2P_0/\rho}\ A_c P_0}{2(C_0 + C_1 P_0)} + \frac{P_0}{\cot\phi})\cos\frac{\pi\ell}{\lambda} - \frac{C_n \sqrt{2P_0/\rho}\ A_c P_0}{C_0 + C_1 P_0}] \right.$$

$$\left. - [\frac{ab\ell}{2}(\frac{C_n \sqrt{2P_0/\rho}\ A_c P_0}{2(C_0 + C_1 P_0)} + \frac{P_0}{\cot\phi}) + \frac{a\lambda^2}{2\pi^2}\ \frac{P_0}{\cot\phi}]\sin\frac{\pi\ell}{\lambda} \right\}\cos\omega_e t$$