



# SIGNAL PROCESSING WITH SAW DEVICES

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### ABSTRACT

The use of surface acoustic wave devices to perform real time Fourier transformation for timelimited signals is well known. In this paper a detailed analysis justifying the implementation most typically employed will be presented, as well as the description of a scheme which extends the above technique by allowing the transformation of a long sequence of contiguous random data. This latter situation, of course, is that normally encountered in a digital communication system.

#### INTRODUCTION

The idea of filtering a signal by Fourier transforming it, multiplying it by the transfer function of the desired filter (in real-time), and inverse transforming the result has been described in various articles (see e.g. [1], [2]). The key component of such a system is the Fourier transformer (or inverse transformer), which is typically implemented with a surface acoustic wave (SAW) tapped delay line. This paper will provide an analytical justification for this type of processing plus describe a technique by which this scheme, nominally applicable to time-limited signals (of the order of tens of microseconds), can be extended to allow filtering and detection of the very long contiguous data streams typical of most digital communication systems.

FOURIER TRANSFORM GENERATION USING SAW DELAY LINES

The use of a linear FM (or chirp) waveform to generate a real-time Fourier transform has been documented in the literature (see [1]-[5]). The following derivation validates this procedure by explicitly showing how one specific implementation, namely a tapped delay line with appropriate tap coefficients, can perform the Fourier transformation to within any desired degree of accurary.

Assume for simplicity that the delay line has a total delay of  $T_1$  seconds. Since a chirp waveform with a large time-bandwidth product has a Fourier transform that is essentially constant over the range of frequencies being chirped and is close to zero elsewhere [6], it is reasonable to consider a chirp pulse as a bandlimited signal. Assuming the chirp waveform is swept from  $w_a$  to  $w_+w_-$ ,  $w_a$  being the carrier frequency, the uniform sampling theorem for bandpass signals [7] states that samples of the waveform taken at the rate  $2(f_a+f_c)$ 

 $\frac{2(f_{a}+f_{c})}{m}$ , where m is the largest integer less than  $\frac{f_{a}+f_{c}}{f_{c}}$ , are sufficient to describe the waveform. Assuming, for simplicity, that  $\frac{f_{a}+f_{c}}{f_{c}}$  is an integer, it follows that if the tap co-

waveform. Assuming, for simplicity, that  $\frac{1}{c}$  is an integer, it follows that if the tap coefficients are chosen such that

 $c_n = h(-\frac{\pi n}{v_c})$ 

(1)

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where h(t) is the chirp waveform, then feeding the output of the delay line into an ideal bandpass filter with impulse response

(2)

$$h_{BP} = P(w_c/2)^{(w-w_a)} + P(w_c/2)^{(w+w_a)}$$

where

 $P_a(x) \stackrel{\Delta}{=} \begin{cases} 0 & \text{elsewhere} \end{cases}$ 

will completely recover h(t). Note that for a chirp signal  $T_1$  seconds long,  $w_c = 2\Delta T_1$ , where  $2\Delta$  is the sweep rate (see below).

For ease of notation, the chirp waveforms will be expressed in complex form. Letting f(t) be a time-limited, approximately bandlimited signal whose Fourier transform is desired, Figure 1 shows a block diagram of how the transform is generated. In Figure 1,  $h_p(t)$  is the impulse response of the tapped delay line and is given by

$$h_{p}(t) = \sum_{n} h(\frac{-\pi n}{v_{c}}) \delta(t + \frac{\pi n}{v_{c}}) .$$
(3)

To perform the Fourier transformation, the slope of the chirp of the filter must be opposite that of the slope of the chirp multiplying f(t), so that h(t) must equal  $e^{j(w_a t + \Delta t^2)}$  and therefore

$$h_{p}(t) = \sum_{n} \exp[j[w_{a}(-\pi n/w_{c}) + \Delta(\pi n/w_{c})^{2}]] \delta(t + (\pi n/w_{c}))$$
(4)

With the impulse of the filter as given in (4), the output of the filter is given by

$$g(t) = f(t) e^{j[w_{a}t - \Delta t^{-}]} * h_{p}(t)$$

$$= \sum_{n} exp[j[-w_{a}(\pi n/w_{c}) + \Delta(\pi n/w_{c})^{2}][f(t)e^{j(w_{a}t - \Delta t^{2})} * \delta(t + (\pi n/w_{c}))]$$

$$= \frac{\pi}{w_{c}} e^{j[w_{a}t - \Delta t^{2}]} \sum_{n} f(t + \frac{\pi n}{w_{c}}) exp[-j2\Delta t \pi n/w_{c}]$$
(5)

where \* denotes convolution. Since the length of the delay line is T, seconds, it is clear that (5) is only valid for t ([T,T]], since only in that interval of time will the input waveform be fully contained in the delay line. To show that, from (5), one can indeed recover the Fourier transform of f(t), assume f(t) is time-limited to t ([0,T]). Furthermore, assume f(t) is approximately a bandlimited bandpass signal with upper and lower cutoff frequencies given by  $(\Delta T_1/\pi)$  and  $(\Delta T/\pi)$  respectively. The reason for choosing these two frequencies will be obvious shortly. Also, if the signal whose transform is desired is actually a baseband signal, it can easily be shifted into the above frequency range by allowing it to modulate an appropriate carrier.

Since the chirp multiplying f(t) will also be time-limited to [0,T], it will be swept from  $w_a$  to  $w_a-2\Delta T$ , so that the frequency range of the product will extend from w to  $w + 2\Delta T_1$ , precisely the same range that the chirp filter is being swept over. It follows then that f(t) can be expanded as

$$f(t) = \frac{\pi}{v_c} \sum_{n} f(-\frac{\pi n}{v_c}) \frac{\sin v_c(t + \frac{\pi n}{v_c})}{\pi(t + \frac{\pi n}{v_c})}$$
(6)

since (6) merely represents oversampling f(t). Also,

$$F(w) = \frac{\pi}{w_c} \sum_{k} f(-\frac{\pi k}{w_c}) e^{j(\pi k w/w_c)} P_{w_c}(w)$$
(7)

where F(w) is the Fourier transform of f(t). Inserting (6) in (5) yields

$$g(t) = \frac{\pi}{w_{c}} e^{j\left[w_{a}t - \Delta t^{2}\right]} \sum_{n} \sum_{k} \frac{\pi}{w_{c}} f\left(\frac{-\pi k}{w_{c}}\right) \frac{\sin w_{c}\left(t + \frac{\pi}{w_{c}}(n+k)\right)}{\pi\left(t + \frac{\pi}{w_{c}}(n+k)\right)} e^{-j\left(2\Delta t\right)\left(\pi n/w_{c}\right)}$$

$$= e^{j\left(w_{a}t - \Delta t^{2}\right)} \left(\frac{\pi}{w_{c}}\right) \sum_{n} \sum_{k} e^{-j\left(2\Delta t\right)\left(\pi n/w_{c}\right)} f\left(-\frac{\pi k}{w_{c}}\right) \int_{-w_{c}}^{w_{c}} \frac{1}{2\pi} e^{ju\left(t + \left(\pi/w_{c}\right)\left(k+n\right)\right)} du$$

$$= e^{j\left(w_{a}t - \Delta t^{2}\right)} \left(\frac{\pi}{w_{c}}\right) \int_{-w_{c}}^{w_{c}} \sum_{k} f\left(-\frac{\pi k}{w_{c}}\right) e^{j\left(\pi/w_{c}\right)ku} P_{w_{c}}(u) \sum_{n} e^{-j\left(2\Delta t\right)\left(\pi n/w_{c}\right)} e^{j\left(\pi/w_{c}\right)nu} e^{jut/2\pi} du \qquad (8)$$

From (7), the summation over k yields F(u), and the summation over n can be shown to yield (see [8])

$$2 w_{c} \left[ \delta(-2\Delta t + u - 2nw_{c}) \right]$$
(9)

(10)

so that

$$g(t) = e^{j(w_a t - \Delta t^2)} \int_{c}^{v} F(u)e^{jut} \sum_{du} \delta(u - 2\Delta t - 2nw_c) du$$
  
$$= e^{j(w_a t + \Delta t^2)} F(2\Delta t)$$

where the remaining terms in the above sum are zero because f(t) is assumed bandlimited to  $w_{c_1} < w_c$  and the only region in time where (10) represents the output of Figure 1 is  $t \in [T, T_1]$ .

Finally, (10) can be rewritten as  

$$e^{j(w_a t + \Delta t^2)} F(2\Delta t) = e^{j(w_a t + \Delta t^2)} F(w),$$
 (11)

where  $w \stackrel{\Delta}{=} 2\Delta t$ , so that a coherent demodulation to remove the chirp carrier yields (to within a constant; the desired transform F(w).

In deriving (11), two approximations were made, one that the signal f(t) is bandlimited and the other that the number of taps in the delay line was infinite. Clearly, both assumptions will be violated in practice, but to the extent that one has a reasonably bandlimited signal (with respect to the range of frequencies over which one is sweeping with the chirp) and one can implement a delay line long enough to encompass most of the energy of the signal, one will be able to come very close to generating F(w) for some appropriate range of values of w. Specifically, the range over which  $F(2\Delta t)$  yields a true estimate of the Fourier transform of f(t)is  $w \in [2\Delta T, 2\Delta T_1]$  so that, as pointed out in [1], values of F(w) about and including w=0 cannot be obtained (assuming the carrier frequencies of both chirps are the same). Consequently, if one wants the transform of a lowpass function f(t), one should input  $f(t) \cos w_1$  into the system of Figure 1, where  $w_1 = \Delta(T_1-T)$ , rather than f(t) itself.

## PROCESSING CONTIGUOUS DATA

It can be seen from the previous section that the technique for Fourier transformation works satisfactorily as long as the input waveform is time-limited to a small enough value (i.e. tens of microseconds). Since the input to most digital communication systems can be viewed for all practical purposes as an infinitely long sequence of a contiguous time pulses, some means of altering the procedure described above must be available.

One such scheme is to create two parallel processing branches and periodically switch segments of the input waveform into each branch. This is illustrated in Figure 2. This system is essentially the same system described and analyzed in [1] and [2], except for the addition of the extra parallel branch of processing. Given that this witching is necessary, it is of interest to do it in such a manner that will minimize any distortion. Towards that end, assume each pulse is of length T and assume the length of the delay line  $T_1$  is set to 2T. Referring to Figure 2, and denoting by  $P_1, i=1,2,...$ , the ith pulse of the input signal and by F(j) and  $F_1^{-1}(j)$ , j=1,2, the jth forward and inverse transformers respectively, then P. is fully contained in F(1) for the first time at t=T, and is fully contained in F(1) for the last time at t= $T_1=2T$ . At this point the input to  $F_1(1)$  due to  $P_1$  is fully contained in  $F^{-1}(j)$ , the output of  $F^{-1}(1)$  due to  $P_1$  is a signal which is valid for T seconds only. In order that there be no distortion in inverse transforming the input to  $F^{-1}(1)$ , it is necessary that no other pulse cause an input to  $F^{-1}(1)$  in the interval t $\in [T_1, T_1+T]$ . For  $T_1=2T$ , this reduces to t $\in [2T, 3T]$ . Since the next input to F(1) is  $P_3$  ( $P_2$ ,  $P_4$ ,  $P_5$ ,... all go to F(2)), the next pulse into F(1) does not enter F(1) until 2T and thus does not enter  $F^{-1}(1)$  until  $T_1 + T = 3T$ . In other words, for the system shown in Figure 2 with  $T_1=2T$ , adjacent pulses do not interfere with one another in either the forward or inverse transformers. Furthermore, since the valid output of the inverse transformer for any pulse is precisely T seconds (i.e. the length of the input pulse), there is no intersymbol interference due to H(w). That is, while H(++) certainly distorts the input pulses and in general disperses them in time over a duration greater than T seconds, the timing of the system can be adjusted through appropriate switching to only allow into the final filter the appropriate T-second output of  $F^{-1}(1)$  or  $F^{-1}(2)$ .

#### CONCLUSION

An analysis has been presented illustrating how a SAW delay line can be used to perform a realtime Fourier transformation of a time-limited input signal, and a procedure was described showing how that technique could be extended to allow the system to handle the long contiguous random data stream typical of the inputs to most digital communication systems.

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Figure 1



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