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COMPUTER PROGRAMS FOR THE EFFECT OF PLASMA ON A ONE-DIMENSIONAL--ETC(U)

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COMPUTER PROGRAMS FOR THE EFFECT OF PLASMA ON A ONE-DIMENSIONAL SLOT ANTENNA IN CANONICAL GEOMETRIES

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Table of Contents

1. Introduction

The purpose of this report is to document a set of computer programs for plasma effects on one-dimensional slot antennas. This set of programs covers the cases in which the slot is located in an infinite conducting plane, an infinite cylinder, and a sphere. In all three the plasma is stratified parallel to the given surface. Such canonical problems are approximate but useful representations of the true problem of plasma effects on antennas in reentry vehicles and interceptors, and they have the advantage of solvability. In applications the planar and cylindrical models can be used in terms of the tangent to the vehicle at the antenna station and the spherical model is useful when the antenna is located on a spherical aft dome of the vehicle.

The types of plasma effects covered by these models include the antenna radiation pattern, the input impedance and the antenna noise temperature. The models are a set of four computer programs. SLOP is the model of a slot in a ground plane, SLOC is the model of a slot in a cylinder, SLOS is the model of a slot in a sphere, and ABCD is the model of a two-port network which is used to derive input impedances from the results of the former programs. All are written in FORTRAN and stored on the author's user file in the GE, Space Division, Information Systems and Computer Center's L66 computer.

Section 2 gives a general description of the models in terms of the theoretical assumptions and techniques. Section 3 gives the information for the user beginning with input and output. This section also presents a discussion of the detailed theory integrated closely with the FORTRAN algorithms. Generalizations of the models which are possible are discussed in section 4. Such a generalization, covering the important area of two-dimensional aperture antennas, already exists in a GE-RESD computer program which has never been formally documented. The latter program is quite old and has many features which make it undesirable for future revisions. The present models are thus planned for such future revisions.

2. General Description of the Models

These models assume that the antenna can be represented physically as a one-dimensional slot in a perfectly conducting surface which is either a plane, a cylinder, or a sphere. The plasma is represented physically as a medium external to the antenna surface having its properties dependent on the normal distance only. The internal representation of the antenna uses the assumption of a two- port linear network where one port is at the input terminals and the other is the slot.

The assumed physical representation of the problem makes for a convenient mathematical solution. The dominant mode of excitation of a one-dimensional slot in a conducting surface is that having a uniform electric vector across the slot. This uniform field mode is assumed to exist under all conditions. In the regions external to the antenna surface the electromagnetic fields are the superposition of separated solutions of Maxwell's equations. For the planar and the cylindrical geometries, the separation constant can take on any real value, so the external fields are represented by a Fourier integral. For the spherical geometry the separation constant can have only the discrete values associated with the Legendre functions, or spherical harmonics. The physical meanings of these separation constants are such that for a given value the em fields are those for a plane wave in the planar case, a cylindrical wave in the cylindrical case, and a spherical wave the spherical case. The effect of the plasma on each kind of wave is calculated by breaking the plasma profile into a series of uniform parallel layers and matching boundary conditions at each layer interface. The boundary condition at the outer surface of the plasma is such that the wave is outgoing, or radiating, At the antenna surface the superposition of all separated waves must match the assumed field distribution in the slot.

The antenna radiation pattern, including the plasma effect, is calculated by carrying the em field representations described above into the asymptotic far field. For the planar and cylindrical cases, the Fourier integral is evaluated by the method of stationary phase, which gives the result in terms of only the wave which propagates along the line of sight. For the spherical case the far field involves the sum of spherical waves of all orders.

The near-field effect can be represented in terms of the aperture admittance, which is defined as the ratio of the tangential magnetic field to the tangential electric field in the slot. This parameter is calculated by invoking conservation of energy between the field representations in the aperture and in the medium outside the antenna surface. For the planar and cylindrical cases the aperture admittance is the integral of the admittances of the separated waves weighted by the square of the Fourier transform of the aperture electric field. For the spherical case it is the sum of the admittances of the separated waves weighted by the square of the spherical harmonics expansion coefficients for the aperture electric field.

Noise power generated by the plasma and received by the antenna is calculated by invoking Kirchoff's law. This law is used in terms of the statement that the noise power at the antenna input contributed by an element of volume of the plasma is equal to the power absorbed by that element of volume when the power input to the antenna is equal to the power radiated by a black body at the temperature of the given element of volume. The total noise power is the sum over all elements of plasma volume of such noise power contributions. Since the plasma is in the antenna near field, this theory gives a result which is similar in form to that for the aperture admittance. Thus the noise temperature at the aperture is a weighted integral or sum of the noise temperatures for the separated wave solutions, each of which is a sum of the noise temperature contributions of all plasma layers.

The transformation from the aperture surface to the antenna input terminals is done by the program ABCD, which utilizes the two-port black box assumption. It gives the relative value of the aperture electric field, to which the radiation pattern calculation is normalized in the other programs, both with and without plasma over the antenna. It also gives the input impedance and the noise temperature at the input terminals.

3. Detailed Information for the User

Section 3.1 contains all information needed to operate the computer programs. The derivations of the equations are integrated with explanations of the FORTRAN listings in section 3.2.

```
LIST PAR

10        BLOCK DATA
20        COMMON/RAB/COLL(21),EMO(21),NPTS,TO(21),UNCM,YO(21)
30        DATA NPTS,UNCM/21,1./
40        DATA COLL/21*1./
50        DATA EMO/0.,.19,.36,.51,.64,.75,.84,.91,.96,.99,
60       & 1.,.99,.96,.91,.84,.75,.64,.51,.36,.19,0./
70        DATA TO/21*1./
80        DATA YO/0.,.05,.1,.15,.2,.25,.3,.35,.4,.45,.5,
90       & .55,.6,.65,.7,.75,.8,.85,.9,.95,1./
100        END
```

Figure 1   Example of a Block Data Subroutine

3.1 Input and Output

Programs SLOP, SLOC and SLOS have similar input and output requirements. Three means of providing input are used: a block data subroutine, NAMELIST and READ statements. The block data subroutine, which must be added to each program before it will run, has the same requirements for all three. An example is given in fig. 1. This subroutine provides the data on the plasma profile. COLL is the collision frequency in units of 10^6/sec, EMO is the electron density in cm^{-3}, TO is the temperature in $^{\circ}k$ and YO is the distance normal to the wall. NPTS is the number of points in the profile and UNCM is the unit of distance in cm used in the profile variable YO. For example if the data YO were in terms of Y/R_N and R_N were 0.05 inch, UNCM would be equal to 0.127. In the example shown the profile is one cm thick. The collision frequency, electron density, and temperature have maximum values of unity for convenience in this example since, as we will see, NAMELIST input permits scale factors to be used.

The NAMELIST name is INPUT for all four programs. SLOP, SLOC and SLOS take basically the same list of variables. CAYA is k times the aperture width, which is measured along the surface of the sphere for SLOS, where k is 2π divided by the wavelength. CYL is k times the body radius. In the planar case CYL serves the purpose of defining a finite length for the aperture so the output noise temperature will be finite. In this case the aperture length is 2π times CYL, as though the aperture were wrapped around a cylinder of radius CYL. Of course CYL represents the cylinder radius in SLOC and the sphere radius in SLOS. FACC, FACE, FACT and FACY are scale factors for the profile variables collision frequency, electron density, temperature and normal distance, respectively. All corresponding values from the block data subroutine are multiplied by the input values of these scale factors. FMHZ is the frequency in MHz. GA is the normalized aperture conductance, which is used in SLOP and SLOC primarily as a control on whether to calculate the aperture admittance. In SLOS only, MODES is the number of spherical wave modes to be used and THETO is the angular position of the center of the slot. In SLOP and SLOC, only CAYA, CYL and FMHZ are mandatory inputs, all of which must be greater than zero. In SLOS, THETO is also mandatory and greater than zero. The programs assume that the scale factors are unity unless input otherwise. Input nonzero GA in SLOP and SLOC only if the aperture admittance calculation is not desired. In SLOS, a value of MODES (currently 20) which is consistent with certain dimension statements is assumed unless input smaller. Do not input MODES larger than this value.

Fig. 2 Shows a sample run of SLOP. This run incorporates the block data in Fig. 1. which represent a parabolic electron density profile which goes to zero at points one cm apart and has uniform collision frequency and temperature. The first input during execution is via NAMELIST, which sets values for the aperture width, the body radius, the profile scale factors and the frequency. The first line of output gives the result of the first iteration in the calculation of aperture admittance and plasma noise temperature. Reading across, the first number is the number of iterations, the next two are the real and imaginary parts of the aperture admittance in free space, the next pair is the aperture admittance in plasma, and the last is the plasma noise temperature in ^{o}K without accounting for aperture area or conductance. (As noted below the conventions for our theoretical derivations are such that both complex parts of the aperture admittance are normalized by the characteristic admittance of free space and the standard electrical engineering convention is the complex conjugate of ours. Thus, for example, the susceptance in mho is the negative of the second number of the pair divided by 376.7.) Immediately after each line of this kind of output there appears another "=", where the program reads an integer. This integer must have the value unity if further iterations of the admittance and noise temperature calculations are to be done. After the third iteration in this example, this integer is input equal to negative unity, which stops the iterations. (Zero is another option, which will continue the iterations but only with respect to the aperture admittance in free space.) Then the final values of aperture admittance and noise temperature are printed on a line which does not contain the iteration number. The aperture susceptance in free space is corrected for truncation of the iterations and the noise temperature is made to include the effects of the aperture area and conductance on this line of output. This line marks a change to a second mode of operation.

In the second mode of operation the program reads a floating point number at the equal sign. This input denotes the far field line of sight, or incidence, angle to the normal in degrees. Thus in the example the first input is for normal incidence. The next line of output gives the far field gains and losses for a constant value of aperture output power. The first number is the attenuation in dB, where a positive number indicates a loss of signal in the given direction. The second number is the gain in free space in dB and the last number is the gain in plasma in dB. (These gains and losses are defined in greater detail below.) The example illustrates the method of generating the plasma effect on the radiation pattern. If the input incidence angle is 90, or greater, the program reverts to the first mode of input, NAMELIST. In this example the same case is rerun except that the peak electron density is 10^{12} instead of 1.24×10^{12}. Also, input of GA not zero causes the program to skip the aperture admittance and noise temperature calculation. The next time, GA must again be input since the program sets it equal to zero just before reading NAMELIST. Finally, inputting the incidence angle less than zero causes the program to stop executing.

The sample run shown in Fig. 2 illustrates an interesting phenomenon in non-uniform, relatively collisionless plasma. That is, the attenuation at large incidence angles for the electric vector in the plane of incidence is maximum when the peak electron density is near critical density. (Critical density in cm^{-3} is approximately equal to $1.24 \times 10^{-8} f^2$, where f is the frequency in Hz.)

Fig. 3 shows a sample run of SLOC. All input and output formats are identical with those for SLOP. Even the numerical results are qualitatively, although of course not quantitatively, similar.

Fig. 4 shows a sample run of SLOS for the same case, but where the slot is centered at 45° from the pole of the sphere. In the iterated output of aperture admittance and noise temperature the integer denotes the number of modes used. The iteration is continued, up to a maximum of 20 times by inputting unity. It is stopped by inputting zero. There are no other alternatives since it is logically difficult to divorce the calculations with and without plasma in the spherical case. The ouput of the final results shows the total number of modes used because there is an internal criterion on truncation of the number of modes. The operator must watch to see that the number of modes continues to increase when he inputs unity. If the output number should be the same as on the previous output line or if the number 20 is reached, the program has switched from this mode of input/output. Then the input consists of the polar angle of incidence in degrees for output of the far field attenuation and gains without or with plasma. For this sample case the normal incidence direction is 45°, the grazing incidence direction is 135° and 179° is deep within the optical shadow. Input of 180°, which is a singular point for the assumed antenna, switches the program back to the original input/output mode. Input MODES less than 20 to skip the iteration of the output of admittances and noise temperature. Then the program goes immediately to the far field pattern mode of input/output. As shown by this example, MODES does not have to be input again to repeat this modus operandi. To stop execution input 0. when the program reads the incidence angle. Note that there is no option to skip the part of the program which calculates admittances and noise temperature, because this is a logical impossibility in the spherical geometry.

Fig. 5 illustrates a run of the program ABCD. The NAMELIST inputs are the variables P, R, Y, YAO, YO and YY. P and R are arrays of phase (in degrees) and amplitude, respectively, of the (experimentally measured) complex voltage reflection coefficient at the antenna input terminals. For each of these the first element is for the antenna in the free space environment, the second is for a good conductor tightly covering the antenna aperature, and the third is for a thin resistance sheet covering the aperture. The complex array Y is the input admittance corresponding to each of these same three conditions in the same order. Of course Y is redundant with P and R, and it represents an alternative method of input. If the phase and amplitude of the reflection coefficient at the antenna input are input to the calculation then Y need not be input. If it is desired to input Y rather than P and R, P(3) must be input equal to a negative number, or set P=3*-1. for example.

```
 FRUN SLOP;PAR
=$INPUT CAYA=3.,CYL=5.,FACC=1000.,FACE=1.24E12,FACT=5000.,FMHZ=10000.$
 1      9.0280E-01 -1.5816E-02    2.7580E-01  1.1049E-01      5.0516E 01
=1
 2      9.0280E-01 -1.9021E-02    2.7581E-01  1.0787E-01      5.7663E 01
=1
 3      9.0280E-01 -2.0169E-02    2.7581E-01  1.0787E-01      5.7663E 01
=-1
        9.0280E-01 -2.1681E-02    2.7581E-01  1.0787E-01      3.1360E 03
=0.
    5.31        7.35       2.04
=10.
    5.18        7.20       2.08
=20.
    9.29        6.97      -2.32
=30.
   16.07        6.52      -9.55
=40.
   21.93        5.96     -15.97
=50.
   27.01        5.35     -21.66
=60.
   31.78        4.76     -27.02
=70.
   36.84        4.26     -32.58
=80.
   43.75        3.93     -39.82
=85.
   49.97        3.84     -46.13
=87.
   54.45        3.82     -50.63
=89.
   64.01        3.81     -60.19
=90.
=$INPUT FACE=1.E12,GA=.27581$
=89.
   48.54        3.81     -44.73
=90.
=$INPUT FACE=2.5E12,GA=.27581$
=89.
   52.42        3.81     -48.61
=-1.
*
```

Figure 2 Sample Run of Program SLOP

```
PROG SLOC$PAR
=$INPUT DATA=3.,CYL=5.,FACC=1000.,FACE=1.24E12,FACT=5000.,FMHZ=10000.$
 1       9.0708E-01  7.3131E-02       2.7856E-01  1.9553E-01       5.0838E 01
=1
 2       9.0708E-01  6.9804E-02       2.7857E-01  1.9287E-01       5.4198E 01
=1
 3       9.0708E-01  6.8700E-02       2.7857E-01  1.9287E-01       5.4198E 01
=-1
         9.0708E-01  6.7188E-02       2.7857E-01  1.9287E-01       2.9184E 03
=0.
   5.29      5.37      0.08
=10.
   4.89      5.34      0.45
=20.
   8.62      5.20     -3.38
=30.
  15.56      5.17    -10.39
=40.
  21.55      5.15    -16.40
=50.
  26.72      5.31    -21.40
=60.
  31.59      5.84    -25.75
=70.
  36.87      7.05    -29.82
=80.
  44.56      9.90    -34.66
=85.
  52.48     13.27    -39.21
=87.
  58.89     15.99    -42.90
=89.
  74.19     22.42    -51.77
=90.
=$INPUT FACE=1.E12,GA=.27857$
=89.
  58.74     22.42    -36.32
=90.
=$INPUT FACE=2.5E12,GA=.27857$
=89.
  62.64     22.42    -40.22
=-1.

*
```

Figure 3 Sample Run of Program SLOC

```
@RUN SLOS;PAR
=$INPUT CAYA=3.,CYL=5.,FACC=1.E3,FACE=1.24E12,FACT=5.E3,FMHZ=1.E4,THETO=45.$
 1      1.8807E-01 -1.3426E-03      8.5254E-02 -2.4685E-02      2.2022E 03
=1
 2      5.8246E-01 -1.3082E-02      1.8470E-01 -1.8071E-01      6.1510E 03
=1
 3      8.4045E-01 -3.6832E-02      2.1138E-01 -2.7202E-01      7.3787E 03
=1
 4      8.8052E-01 -4.1838E-02      2.1220E-01 -2.7688E-01      7.4191E 03
=1
 5      9.3150E-01 -1.0275E-01      2.1398E-01 -2.9293E-01      7.5093E 03
=1
 6      9.7104E-01 -2.5732E-01      2.1654E-01 -3.2582E-01      7.6412E 03
=1
 7      9.7334E-01 -2.9732E-01      2.1723E-01 -3.3777E-01      7.6769E 03
=1
 8      9.7334E-01 -2.9753E-01      2.1723E-01 -3.3785E-01      7.6771E 03
=0
 8      9.7334E-01 -2.9753E-01      2.1723E-01 -3.3785E-01      3.5341E 04
=45.
   13.25      11.15      -2.10
=135.
   12.55       0.77     -11.79
=179.
   27.81     -16.25     -44.07
=180.
=$INPUT FACE=1.E12,MODES=8$
 8      9.7334E-01 -2.9753E-01      6.1591E-01 -2.4443E-01      3.8193E 03
=45.
    3.76       6.62       2.87
=135.
   -1.00      -3.76      -2.76
=179.
    7.84     -20.78     -28.62
=180.
=$INPUT FACE=2.5E12$
 8      9.7334E-01 -2.9753E-01      1.0104E-01  3.6353E-01      3.1311E 04
=45.
   13.91      14.47       0.57
=135.
   29.67       4.09     -25.58
=179.
   37.77     -12.93     -50.70
=0.

*
```

Figure 4 Sample Run of Program SLOS

```
FRUN ABCD
=$INPUT P=3*0.,R=.1,.9,.3,YA0=.9028,-.0217$
-3.5325E-03  9.1811E-01      7.5330E-13  1.6279E-01
-1.4007E-03  6.5070E 00      2.9870E-13  6.4550E-02
  -0.04
=.276,.168,3136.
0.054   11.6   -0.01   -0.49   137.3
=0.,0.,0.
=$INPUT YA0=.907,.0672$
-1.0959E-02 -9.1742E-01      1.0044E-12 -1.6279E-01
-4.3377E-03 -6.5067E 00      3.9827E-13 -6.4550E-02
  -0.04
=.279,.193,2918.
0.054   10.7   -0.01   -0.50   129.2
=0.,0.,0.
=$INPUT R=3*-1.,Y=6.164,0.,.3965,0.,4.057,0.,YA0=.973,-.298$
 4.8505E-02 -9.0685E-01      2.0081E-12 -1.6277E-01
 1.9232E-02 -6.5033E 00      7.9620E-13 -6.4538E-02
  -0.04
=.217,-.338,35341.
0.043   -4.4   -0.01   -0.60  1246.6
=.616,-.244,3819.
0.074    3.2   -0.02   -0.28   355.0
=.101,.384,31311.
0.067   56.9   -0.02   -0.68   523.3
=0.,0.,-1.

*
```

Figure 5 Sample Run of Program ABCD

YAO is the complex aperture admittance in free space. YO is the characteristic admittance of the input line and YY is the conductance of the thin resistance sheet which corresponds to the third set of data for the complex reflection coefficient or admittance at the input port. The units of Y, YAO, YO and YY must be consistent and the phase convention for P must be consistent with the complex variable convention for YAO. Since a lossless input line and an ideal resistance sheet are assumed, the variables YO and YY are real. By default, as in the example shown, YO corresponds to a 50 ohm line, YY corresponds to a 100 ohm resistance sheet, and the characteristic admittance of free space is the unit of admittance. Except for the values of P and R, which have been assumed arbritrarily, the data for this run have come from the results of the runs from figs 2 through 4 in order. The first eight numbers output are the complex values of the A, B, C and D parameters, which are defined in section 3.2.4. The next output, on a line by itself, is the reflection loss, $1-|R|^2$, in dB for the antenna in free space. Then the program reads input values of the complex aperture admittance in plasma and the plasma noise temperature at the aperture. Of course the units for the aperture admittance must be consistent with those for YAO. The first two numbers in the next line of output are the amplitude and phase of the voltage reflection coefficient at the input when plasma covers the antenna. Next is the corresponding reflection loss in dB. The fourth output is the angle-independent part of the total radiated signal loss in dB. This latter number must be added to the attenuation numbers output by the other programs as a function of angle in order to give the total signal attenuation relative to the signal in free space, which includes reflection, absorption and antenna pattern distortion effects. The last output is the noise temperature at the antenna input. As shown, one normally iterates input values of admittance and noise temperature at the aperture, which allows the operator to repeat the calculations conveniently for a number of different plasma conditions. Inputting zero noise temperature causes the program to go back to new NAMELIST input and inputting negative noise temperature makes it stop executing. The third set of NAMELIST input illustrates input of Y instead of P and R.

3.2 Computational Techniques

This section of the report gives the theoretical derivations as well as the explanation of the FORTRAN. These derivations use a peculiar system of units, which is designed for maximum convenience. The unit of length is the free-space wavelength divided by 2π. The units of the electric field E and the magnetic field H are the same; i.e., the characteristic impedance of free space is unity and all admittances in our equations are numerically equal to their values in mho times 376.7. All fields have the time variation exp $(-i\omega t)$, which is suppressed in all equations. Thus a wave traveling in the positive x direction has the variation exp (ix). Another consequence of this convention for complex quantities is that the standard electrical engineering results are equal to the complex conjugate of these results.

The programming techniques are straightforward. Most of the built-in functions and library routines which are called are of the every day variety. The only exception is FXOPT, which is used only to suppress the error message for exponent underflow. (The L66 computer word does not allow numbers smaller than about 10^{-38}; and when such a number is generated during execution, the computer sets it to zero and normally prints an error message, which FXOPT is used here to suppress.)

LIST 10-500

```
      COMMON/RAB/COLL(21),EMO(21),NPTS,TO(21),UNCM,YO(21)
      COMMON/RAC/T(60),TA(4)
      COMPLEX CA,CB,CC,DIEL(60),Q(4),YA,YAO
      DIMENSION CRIT(2),EM(21),X(60),XA(6),Y(21)
      NAMELIST/INPUT/CAYA,CYL,FACC,FACE,FACT,FACY,FMHZ,GA
      DATA FACC,FACE,FACT,FACY/4*1./
      DATA PI,RAD/3.1415927,.01745329/
      CALL FXOPT(67,1,1,0)
    1 GA=0.
      READ INPUT
      EMC=12405.18*FMHZ*FMHZ/FACE
      COLA=.15915494*FACC/FMHZ
      CAY=2.0958447E-4*UNCM*FMHZ
      CRIT(1)=1.
      CRIT(2)=1.
      NA=1
      DO 10 I=1,NPTS
      Y(I)=FACY*YO(I)
   10 EM(I)=EMO(I)/EMC
      CALL CRITS(EM,Y,NPTS,CRIT,XA,NA)
      A=XA(NA)-XA(1)
      N=0
      MP=2
      DO 60 I=2,NA
      J=I-1
      D=XA(J)
      B=XA(I)-D
      K=50.*B/A
      IF(K.LT.1)K=1
      C=B/K
      E=CAY*C
      DO 50 L=1,K
      N=N+1
      X(N)=E
      F=D+(L-.5)*C
      DO 30 M=MP,NPTS
      IF(Y(M).GT.F)GO TO 35
   30 CONTINUE
   35 MP=M
      MM=MP-1
      AA=(F-Y(MM))/(Y(MP)-Y(MM))
      BB=1.-AA
      COL=COLA*(COLL(MM)*BB+COLL(MP)*AA)
      T(N)=FACT*(TO(MM)*BB+AA*TO(MP))
      IF(AA.GE.1.)GO TO 40
      IF(EM(MM).GT.0..AND.EM(MP).GT.0.)GO TO 38
      G=EM(MM)*BB+EM(MP)*AA
      GO TO 45
   38 G=EM(MM)**BB*EM(MP)**AA
      GO TO 45
```

* **Figure 6 Listing of Program SLOP**

LIST 510-1000

```
 40 G=EM(MP)
 45 G=G/(1.+COL*COL)
    H=G*COL
 50 DIEL(N)=CMPLX(1.-G,H)
 60 CONTINUE
    IF(GA.GT.0.)GO TO 80
    U=0.
    M=1
    ITER=0
    CALL PROP(N,DIEL,X,M,U,Q)
    IF(U.LT.0.)GO TO 80
    AA=.9028
    YAO=CMPLX(AA,0.)
    CC=(Q(1)-Q(3))/(Q(4)-Q(2))
    YA=AA*CC
    A=REAL(CC)
    B=AIMAG(CC)
    TN=AA*(TA(1)+A*TA(2)+B*TA(3)+(A*A+B*B)*TA(4))
    DU=2.*PI/CAYA
    U=1.5*DU
    BA=1.5
    L=1
 61 CALL PROP(N,DIEL,X,M,U,Q)
    IF(U.LT.0.)L=0
 62 BB=1./(PI*(BA-.05/BA))**2
    BC=1.-U*U
    IF(BC)65,69,63
 63 W=SQRT(BC)
    YAO=YAO+BB/W
    IF(L.NE.1)GO TO 69
    CC=(Q(1)-W*Q(3))/(W*Q(4)-Q(2))
    GO TO 68
 65 W=SQRT(-BC)
    YAO=YAO-CMPLX(0.,BB/W)
    IF(L.NE.1)GO TO 69
    CA=CMPLX(0.,W)
    CC=(Q(1)-CA*Q(3))/(CA*Q(4)-Q(2))
 68 YA=YA+BB*CC
    A=REAL(CC)
    B=AIMAG(CC)
    TN=TN+BB*(TA(1)+A*TA(2)+B*TA(3)+(A*A+B*B)*TA(4))
 69 BA=BA+1.
    U=U+DU
    ITER=ITER+1
    PRINT 1000,ITER,YAO,YA,TN
    READ:L
    IF(L)70,62,61
 70 A=.25*CAYA/PI/(PI*(BA-.5))**2
    YAO=YAO+CMPLX(0.,-A)
     GA=REAL(YA)
```

* **Figure 6 (continued)**

```
LIST 1010-1500

      TN=TN*CAYA*CYL/GA
      PRINT 3000,YAC,YA,TN
   80 READ:THET
      IF(THET.LT.0.)GO TO 100
      IF(THET.GE.90.)GO TO 1
      U=SIN(RAD*THET)
      CALL PROP(N,DIEL,X,L,U,Q)
      IF(U.LT.0.)GO TO 80
      W=SQRT(1.-U*U)
      C=.5*CAYA/GA
      D=.5*CAYA*U
      IF(D.GT.0.)C=C*(SIN(D)/D)**2
      D=4.343*ALOG(C)
      CA=Q(4)-Q(2)/W
      CB=Q(1)*Q(4)-Q(2)*Q(3)
      CC=CA/CB
      A=4.343*ALOG(REAL(CC*CONJG(CC)))
      C=D-A
      PRINT 2000,A,L,C
      GO TO 80
  100 STOP
 1000 FORMAT(I3,3X,1P2E12.4,3X,1P2E12.4,1PE15.4)
 2000 FORMAT(3F9.2)
 3000 FORMAT(6X,1P2E12.4,3X,1P2E12.4,1PE15.4)
      END
      SUBROUTINE PROP(N,DIEL,X,M,V,Q)
      COMMON/RAC/T(60),TA(4)
      COMPLEX CA,CB,CC,CD,CE,CF,DIEL(60),Q(4)
      DIMENSION TB(4),TC(4),X(60)
      VS=V*V
      IF(M.LT.0)GO TO 10
      DO 5 I=1,4
      TA(I)=0.
    5 TB(I)=0.
      TB(2)=1.
   10 Q(1)=(1.,0.)
      Q(2)=(0.,0.)
      Q(3)=Q(2)
      Q(4)=Q(1)
      DO 90 I=1,N
      CA=CSQRT(DIEL(I)-CMPLX(VS,0.))
      B=X(I)*AIMAG(CA)
      IF(ABS(B).GT.34.54)GO TO 100
      A=X(I)*REAL(CA)
      C=.5*COS(A)
      D=.5*SIN(A)
      E=EXP(B)
      F=1./E
      G=E+F
      H=F-E
```

*

Figure 6 (continued)

```
LIST 1510-2000

1510        CB=CMPLX(C*G,D*H)
1520        CC=CMPLX(C*H,D*G)
1530        CD=CA*CC
1540        CC=CC/CA
1550        CE=CD/DIEL(I)
1560        CF=CC*DIEL(I)
1570        CA=CB*Q(1)+CE*Q(3)
1580        Q(3)=CB*Q(3)+CF*Q(1)
1590        Q(1)=CA
1600        CA=CB*Q(2)+CE*Q(4)
1610        Q(4)=CB*Q(4)+CF*Q(2)
1620        Q(2)=CA
1630        IF(M.LT.0)GO TO 50
1640        CB=CONJG(Q(3))
1650        CC=CONJG(Q(4))
1660        TC(1)=REAL(Q(1)*CB)
1670        TC(2)=REAL(Q(1)*CC+Q(2)*CB)
1680        TC(3)=AIMAG(Q(1)*CC-Q(2)*CB)
1690        TC(4)=REAL(Q(2)*CC)
1700        DO 20 J=1,4
1710        TA(J)=TA(J)+T(I)*(TB(J)-TC(J))
1720     20 TB(J)=TC(J)
1730     50 A=0.
1740        DO 80 J=1,4
1750     80 A=A+REAL(CONJG(Q(J))*Q(J))
1760        IF(A.GT.1.E12)GO TO 100
1770     90 CONTINUE
1780        RETURN
1790    100 V=-V-1.E-30
1800        RETURN
1810        END
1820        SUBROUTINE CRITS(FI,XI,N,A,XA,NA)
1830        DIMENSION A(2),FI(N),GI(6),XA(6),XI(N)
1840        DATA EPS/1.E-6/
1850        IT=NA
1860        NA=1
1870        DO 1 I=1,4
1880      1 XA(I)=0.
1890        M=N-1
1900        DO 100 I=1,IT
1910        DO 90 J=1,M
1920        K=J+1
1930        FM=FI(J)/A(I)-1.
1940        FP=FI(K)/A(I)-1.
1950        B=FM*FP
1960        IF(B.GT.0.)GO TO 90
1970        IF(B.LT.0.)GO TO 5
1980        IF(ABS(FM).GT.0.)GO TO 90
1990        XA(NA)=XI(J)
2000        IF(NA.EQ.4)GO TO 120
```

Figure 6 (continued)

LIST 2010-2360

```
      NA=NA+1
      GO TO 90
    5 XM=XI(J)
      XP=XI(K)
      IF(FM.LE.-1..OR.FP.LE.-1.)GO TO 10
      B=XM+(XP-XM)*ALOG(A(I)/FI(J))/ALOG(FI(K)/FI(J))
      GO TO 40
   10 B=XM+(XP-XM)*FM/(FM-FP)
   40 XA(NA)=B
      IF(NA.EQ.4)GO TO 120
      NA=NA+1
   90 CONTINUE
  100 CONTINUE
      DO 110 I=2,M
      IF(FI(I-1).LT.FI(I).AND.FI(I).GE.FI(I+1))GO TO 115
  110 CONTINUE
      NA=NA-1
      GO TO 120
  115 XA(NA)=XI(I)
  120 NA=NA+1
      XA(NA)=XI(1)
      NA=NA+1
      XA(NA)=XI(N)
      DO 150 I=1,NA
      B=1.E30
      DO 140 J=1,NA
      IF(XA(J).GE.B)GO TO 140
      K=J
      B=XA(J)
  140 CONTINUE
      GI(I)=B
  150 XA(K)=2.E30
      DO 160 I=1,NA
  160 XA(I)=GI(I)
      RETURN
      END
```

* **Figure 6 (continued)**

3.2.1 SLOP

Fig. 6 is a listing of this program, which consists of a main program and two subroutines. Of course the block data subroutine must be added, as discussed above.

3.2.1.1 Main Program

After reading the NAMELIST type of input, the program generates, in lines 110 through 550, a representation of the plasma profile by a series of uniform layers in terms of an array, DIEL, of the complex dielectric constant and an array, X, of the layer thickness. The equation for the complex dielectric constant K is

$$K = 1 - (N_e/N_c)/(1 + i\nu/\omega) \tag{1}$$

where N_e is the electron density and N_c is the critical electron density. Except for the electron density scale factor, FACE, line 110 gives the equation for the critical density. The array EM represents N_e/N_c at input plasma profile points and the array Y represents the input profile distances in cm. After calling CRITS (see below), there are NA points in this profile, located at the distances XA, between which interpolated uniform layers are desirable. The algorithm from line 210 through 550 performs the interpolation accordingly, with the additional constraints that the total number of layers be approximately fifty and that all their thicknesses be as nearly uniform as possible. The collision frequency is linearly interpolated, as is the temperature array T. The electron density is logarithmically interpolated where possible.

The aperture admittance and plasma noise temperature are calculated in lines 570 through 1010. To derive the equations for these quantities, let the plane $z = 0$ be the ground plane and let the slot be between the lines $x = \pm a/2$, where a is the width of the slot. Then in this plane let $E_y = 0$, let $E_x = 0$ if $|x| > a/2$, and let $E_x = E_o$ if $|x| < a/2$. Of course E_o is the aperture field strength, the determination of which can be left for later. This assumed distribution of electric field gives rise to a magnetic field vector which is everywhere parallel to the y axis, so $H = H_y$ and we need not denote a vector component as with E. Also, both E and H are independent of y. Maxwell's equations in this situation are:

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = iH \tag{2}$$

$$\frac{\partial H}{\partial z} = iKE_x \tag{3}$$

$$\frac{\partial H}{\partial x} = -iKE_z \tag{4}$$

Accounting for the fact that the gradient of K is parallel to z, these equations combine to give

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} - \frac{1}{K}\frac{\partial K}{\partial z}\frac{\partial H}{\partial z} + KH = 0 \tag{5}$$

The variables separate if the first term is equal to a constant times H. In order to give solutions which are finite for all values of x, let

$$\frac{\partial^2 H}{\partial x^2} = -u^2 H \tag{6}$$

where u is any real number. Since the medium above the groud plane is infinite and since there is not an infinite number of periodically located slots in the ground plane, the allowed values of u are continuous. Therefore the principle of superposition requires that the total solution be an integral over all solutions of (6); i.e.,

$$H(x, z) = \int_{-\infty}^{\infty} h(u, z) \exp(i u x) \, du \tag{7}$$

where $h(u, z)$ is the part of the separated solution which depends on z, and of course also on u. This equation is a Fourier transform and its inverse is

$$h(u,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(x, z) \exp(-i u x) \, dx \tag{8}$$

A precisely similar pair of equations can be written for each component of E, so the vector $E(x, z)$ has a transform in terms of the vector $e(u, z)$.

The aperture admittance Y_a is the ratio of the magnetic field to the tangential electric field in the slot. The fact that this quantity is treated as a constant implies that the magnetic field in the slot is a constant, as is the electric field. This is impossible, so let us use the conservation of energy to estimate Y_a. Thus, using * to denote complex conjugate,

$$a |E_o|^2 Y_a = \int_{-\infty}^{\infty} E_x^* (x, o) \, H(x, o) \, dx \tag{9}$$

where both sides are the integral of $E^* XH$ over the ground plane, the left-hand side using $H = Y_a E_o$ in the slot. Substitution of (7) in (9) and the assumption that the order of integration can be reversed gives, after carrying out the integral on x as the transform of E_x^*,

$$a |E_o|^2 Y_a = 2\pi \int_{-\infty}^{\infty} h(u, o) \, e_x^* (u,o) \, du \tag{10}$$

Now $e_x(u,o)$ is the transform of the assumed tangential electric field distribution in the ground plane.

$$e_x(u,o) = \frac{E_o}{2\pi} \int_{-a/2}^{a/2} \exp(-i u x) \, dx$$

$$= E_o \sin(u a/2) / (\pi u) \tag{11}$$

Therefore

$$Y_a = \frac{2}{\pi a} \int_{-\infty}^{\infty} y(u) \left[\sin(ua/2)/u\right]^2 du \tag{12}$$

where $y(u)$ is the admittance in the plane $z = 0$ for a wave having the separation constant $-u^2$, or having $\exp(i u x)$ variation with x.

A wave having $\exp(i ux)$ variation has $\exp(i w z)$ variation in free space, where $K = 1$, such that, using (5),

$$u^2 + w^2 = 1 \tag{13}$$

For the slot as a radiator, the wave in free space must be outgoing, or moving in the positive z direction, which requires that w be the positive root of (13). Using (3), we also have in free space

$$e_x(u, z) = w \, h(u, z) \tag{14}$$

Therefore, since y (u) is defined as $h(u,o)/e_x(u,o)$,

$$y(u) = 1 \Big/ \sqrt{1 - u^2} \tag{15}$$

Since we require the positive root, we have i times the root of the absolute value when u^2 is greater than unity, which implies that y(u) then lies on the negative imaginary axis. Thus the imaginary part of the free-space aperture admittance, Y_{a_o}, is negative.

In the presence of the plasma, (14) applies only in the space above the plasma. It will be seen, however, that the plasma can be represented in terms of a square matrix Q such that

$$\begin{bmatrix} e_x \\ h \end{bmatrix}_{outer} = Q \begin{bmatrix} e_x \\ h \end{bmatrix}_{inner} \tag{16}$$

where e_x implies e_x (u, z) and similarly for h and where "outer" and "inner" refer to either boundary of the plasma, the inner boundary being at the ground plane where z = o. In view of (14) applying outside the plasma and of the continuity of e_x and h, the left-hand side vector is an arbitrary constant times the vector $\begin{bmatrix} w \\ 1 \end{bmatrix}$. The admittance y(u) is the ratio of the components of the right-hand side vector, so solving (16) we have

$$y(u) = (Q_{11} - wQ_{21}) / (wQ_{22} - Q_{12}) \tag{17}$$

It should be obvious by now that y (u) is an even function of u, so the integral in (12) can be written as twice the integral for only positive values of u. Also, it is convenient to change variables in the integral by letting x = ua/2.

$$Y_a = \frac{2}{\pi} \int_0^\infty y(2x/a)\,(\sin x/x)^2\,dx \tag{18}$$

If a is not too small, a further simplification can be made by ignoring the variation of y within each lobe of the $(\sin x/x)^2$ function.

$$Y_a \simeq \frac{2}{\pi} \Big[y(0) \int_0^\pi (\sin x/x)^2\,dx + \sum_{n=1}^{\infty} y((2n+1)\pi/a) \int_0^\pi \Big[\frac{\sin x}{n\pi + x}\Big]^2 dx \Big] \tag{19}$$

We have numerically evaluated the integrals in this sum and found them to fit the following:

$$Y_a \simeq 0.9028\; y(0) + \sum_{n=1}^{\infty} \frac{y((2n+1)\pi/a)}{\pi^2 [n + 1/2 - 0.05/(n + 1/2)]^2} \tag{20}$$

Turning to the program listing, (20) is found in lines 630, 650, 790, 840, and 880. The series is truncated by the operator according to the discussion in section 3.1. However, the effect of the omitted terms can be estimated analytically for Y_{a_o} if n is sufficiently large at the point of truncation, and this correction factor is given at line 980 of the program. Eq. (17) is found at lines 640, 810, and 870. (Note that the Q matrix is represented in the program by the linear array Q.)

The noise power generated by the plasma and crossing the aperture is the sum of the contributions of all plasma layers. Assuming Kirchoff's law applies, the contribution of a given plasma layer is the total power absorbed by that layer when the power flowing out from the aperture is equal to that which a black body would emit if it had the temperature of that plasma layer and the area of the aperture. The power emitted by a black body at temperature T in the frequency band df is, according to Planck's law,

$$P = \frac{A\, k\, T_o\, df}{2\pi\,(e^{T_o/T} - 1)} \tag{21}$$

where A is the area in our system of nondimensionalized units, k is Boltzmann's constant and T_o is proportional to the frequency. At 10 GHz, To is 0.48 ^{o}K; but we are interested in thousands of ^{o}K temperatures in the plasma. Therefore the exponential function is quite accurately expressed by its linear approximation and

$$P = A\, k\, T\, df/(2\pi) \tag{22}$$

For reasons similar to that which led to this linear function of temperature, the noise power in an electronic circuit is equal to k T_N df, where T_N is the noise temperature. Using this fact to remove k df from the equations, we now have the noise temperature at the aperture equal to the sum over all plasma layers of the noise temperature of the layer as seen at the aperture. Also, this layer noise temperature is the power absorbed by the layer when the aperture output power is $A/(2\pi)$ times the plasma temperature. But the linearity of the plasma absorption implies that the aperture output power may be set equal to a constant if the power absorbed by the layer is scaled by $AT/(2\pi)$ divided by that constant. In our approximation of the aperture field distribution the aperture power is $\frac{1}{2}A|E_o|^2 G_a$ where G_a is the aperture conductance. Thus the aperture noise temperature is

$$T_a = \frac{1}{G_a} \sum_{layers} \left[\frac{\Delta P}{\pi |E_o|^2}\right] T \tag{23}$$

where ΔP is the power absorbed in the layer when the aperture radiates and has electric field E_o.

The power absorbed in a plasma layer is equal to the drop from the inner to the outer boundary in the power flow P. Using the Fourier transform representations for E and H,

$$P = \frac{1}{2} b \,\mathrm{Re}\left\{ \iiint_{-\infty}^{\infty} e_x(u,z)\, h^*(v,z) \exp\left[i(u-v)x\right] du\, dv\, dx \right\} \tag{24}$$

where b is the length of the slot. (even though the model is stated in terms of a semi-infinite slot having a uniform field, the noise temperature is proportional to its length.) Assuming that the x integration may be performed first, its result vanishes unless u = v. Basic mathematics requires that the resulting function weights the integral over all v as follows:

$$\int_{-\infty}^{\infty} f(v) \int_{-\infty}^{\infty} \exp[i(u-v)x]\,dx\,dv = 2\pi f(u) \tag{25}$$

Therefore

$$P = \pi b \int_{-\infty}^{\infty} Re\left[e_x(u,z)\,h^*(u,z)\right] du \tag{26}$$

Now use (16) to give e_x and h, except that Q represents propagation from the inner plasma boundary to the one of current interest rather than the outermost one. Also, at the inner boundary use h (u, o) = e_o y, where e_o represents e (u,o), which is given by (11), and y represents y (u), which is given by (17).

$$\begin{aligned} Re[e_x(u,z)h^*(u,z)] = |e_o|^2 [\, & Re(Q_{11}Q_{21}^*) + Re(Q_{11}Q_{22}^* \\ & + Q_{12}Q_{21}^*)\,Re(y) + Im(Q_{11}Q_{22}^* - Q_{12}Q_{21}^*)\,Im(y) + Re(Q_{12}Q_{22}^*)|y|^2 \,] \end{aligned} \tag{27}$$

The result of this equation is the following form:

$$\begin{aligned} T_a = \frac{2b}{2\pi G_a} \Big\{ \frac{2}{\pi} \int_0^{\infty} (\sin x/x)^2 \, [\, & T_1 + T_2\,Re(y) \\ & + T_3\,Im(y) + T_4 |y|^2 \,]\,dx \Big\} \end{aligned} \tag{28}$$

where y is y (2 x/a). The part of this expression enclosed by { } is similar to (18), so it is actually a by-product of the aperture admittance calculation. We use (20) to accumulate the integral numerically. In the program the array TA represents $T_1, \ldots, T_4$ and the integral is accumulated at lines 680 and 910. The factor outside the brackets is included at line 1010 after the iterations are concluded. Of course a is CAYA. However the aperture length b is taken as 2π times the body radius, CYL.

Far field effects are calculated between lines 1030 and 1200. The theory for this calculation is based on the Fourier integral. The form of the fields outside the plasma in terms of their transforms e and h may be gotten by letting K=1 and substituting the Fourier integrals in (3) and (4). Also, in (3) we recall that the transforms of the fields vary as exp (iwz). Thus in free space, $e_z = -ue_x/w$ and we have

$$E_x(x,z) = \int_{-\infty}^{\infty} e_x(u,d)\exp[i(ux + w(z-d))]\,du \tag{29}$$

$$E_z(x,z) = -\int_{-\infty}^{\infty} (u/w)\,e_x(u,d)\exp[i(ux + w(z-d))]\,du \tag{30}$$

where z = d is the boundary of the outer edge of the plasma and where w is the positive root of (13). We require the limits of the expressions when the distance z approaches infinity. In these limits the integrals have the form $\int_{-\infty}^{\infty} f(u)\,e^{i\phi}\,du$, where the phase ϕ tends to infinity.

Not only does the phase tend to infinity but also its rate of variation with u tends to infinity for most values of u. The contributions to the integral vanish in all regimes of u where the phase variations are infinite, and the only contribution is from the neighborhood where ϕ does not vary. Thus integrals of this type are evaluated by the method of "stationary phase". Now

$$\phi = ux + \sqrt{1-u^2}\,(z-d) \tag{31}$$

$$\frac{d\phi}{du} = x - u(z-d)/\sqrt{1-u^2} \tag{32}$$

Therefore the phase is stationary when $u = u_o$, where

$$u_o = x/r = \sin\theta \tag{33}$$

where r is the radial distance from (o,d) to (x, z) and θ is the incidence angle, or the angle between the line of sight to (x, z) and the normal. The Taylor series expansion of ϕ about u_o is

$$\phi = r - (u-u_o)^2 r/(2\cos^2\theta) + \cdots \tag{34}$$

Use only these terms in the integral and replace f (u) by f (u_o), giving

$$\int_{-\infty}^{\infty} f(u)e^{i\phi}du = 2f(u_o)e^{ir}\int_0^{\infty}\exp[-iu^2r/(2\cos^2\theta)]du$$

$$= f(u_o)e^{ir}\cos\theta\sqrt{2\pi/(ir)} \tag{35}$$

Therefore*

$$E_x(x,z) \sim e_x(\sin\theta,d)\,e^{ir}\cos\theta\sqrt{2\pi/(ir)} \tag{36}$$

$$E_z(x,z) \sim -E_x(x,z)\tan\theta \tag{37}$$

Thus the far field is transverse and is given by

$$E(r,\theta) \sim e_x(\sin\theta,d)\,e^{ir}\sqrt{2\pi/ir} \tag{38}$$

Now (16) gives e_x at z = d in terms of the Q matrix and the values of e_x and h at z = o; then (17) gives h in terms of the Q matrix and e_x since y (u) is h/e_x at z = o; and finally e_x (sin θ, o) is given by (11). Therefore

$$E(r,\theta) \sim \sqrt{\frac{2}{i\pi r}}\; e^{ir}\left[\frac{Q_{11}Q_{22} - Q_{12}Q_{11}}{Q_{22} - Q_{12}/\cos\theta}\right]\frac{E_o\sin(\frac{1}{2}a\sin\theta)}{\sin\theta} \tag{39}$$

where the Q matrix is evaluated for u = sin θ. This is the final equation for the far field of the slot in the presence of the layered plasma and in terms of the assumed field E_o in the slot.

*

Throughout this report the symbol $\sim$ means "asymptotically equals", usually in the far field limit.

Of principal interest in the problem of plasma effects is the far field at a given angle θ relative to what its value would be in the absence of the plasma. Thus the total signal attenuation is determined by the ratio of the results of (39) with/without plasma. It will be seen that E_o depends on whether or not plasma is present and that the Q matrix is the identity in the absence of plasma. Therefore

$$\frac{E(r,\theta)}{E^o(r,\theta)} = \left[\frac{Q_{11}Q_{22} - Q_{12}Q_{21}}{Q_{22} - Q_{12}/\cos\theta}\right]\frac{E_o}{E_o^o} \tag{40}$$

where superscript zero denotes values in the absence of plasma. In this equation only the factor involving the Q matrix is a function of the angle θ, so this factor represents the far field pattern distortion function, which is calculated in terms of dB loss in lines 1140 through 1170 of the program. Of possible additional interest is the absolute power density in the far field. But since the power at the aperture relative to the primary input power is a function of the circuit behavior of the antenna, which is determined elsewhere, consider the power gain relative to the aperture power. This "aperture gain" function is the ratio of $\pi r|E(r,\theta)|^2/[G_a|E_o|^2 a]$.

$$G = \frac{a}{2G_a}\left[\frac{\sin(\frac{1}{2}a\sin\theta)}{\frac{1}{2}a\sin\theta}\right]^2\left|\frac{Q_{11}Q_{22} - Q_{12}Q_{21}}{Q_{22} - Q_{12}/\cos\theta}\right|^2 \tag{41}$$

The quantity is calculated in dB at lines 1100 through 1180 of the program. The part of it which does not depend on the Q matrix is also output at line 1190. Note that this latter quantity only approximately represents the "aperture gain" in free space, it being necessary to correct it by the ratio of the aperture conductance with/without plasma in dB. However, it is convenient to output it in this form and a simple calculation based on the printout of aperture admittance gives the correction factor.

3.2.1.2 Subroutine PROP

The purpose of this subroutine is to accumulate the values of the propagation matrix Q and the four temperature factors appearing in (28). In the calling sequence, N is the number of plasma layers, DIEL is the array of their complex dielectric constants, X is the array of their thicknesses, M is an input index which bypasses the temperature factors if M is negative, V corresponds to the input value of u or $\sin\theta$, and Q is a linear complex array constituting Q_{11}, Q_{12}, Q_{21}, and Q_{22} in that order. Error indication is made by returning V equal to the negative of the value which was input.

The equations from which the propagation matrix is derived are given above. In particular, substitution of the Fourier integrals in (3) gives

$$\frac{dh}{dz} = i\,Ke_x \tag{42}$$

Also, consider a given plasma layer in which the complex dielectric constant K is constant. Then the Fourier integrals substituted in (5) give

$$\frac{d^2 h}{dz^2} + \gamma^2 h = 0 \tag{43}$$

where

$$\gamma^2 = K - u^2 \tag{44}$$

Of course the solution of (43) is

$$h = a_1 e^{i\gamma z} + a_2 e^{-i\gamma z} \tag{45}$$

where a_1 and a_2 are arbitrary constants. Now (42) gives

$$e_x = (\gamma/K)(a_1 e^{i\gamma z} - a_2 e^{-i\gamma z}) \tag{46}$$

Of course the origin of z is arbitrary in all of these equations so let us place it at the inner boundary of the layer in question. Then, letting z = o in (45) and (46), solving for a_1 and a_2, and substituting back in (45) and (46), we get the matrix equation

$$\begin{bmatrix} e_x \\ h \end{bmatrix}_{outer} = P \begin{bmatrix} e_x \\ h \end{bmatrix}_{inner} \tag{47}$$

where

$$P = \begin{bmatrix} \cos(\gamma z) & i(\gamma/K)\sin(\gamma z) \\ i(K/\gamma)\sin(\gamma z) & \cos(\gamma z) \end{bmatrix} \tag{48}$$

Of course z in this equation is the thickness of the layer. The meaning of "outer" and "inner" in (47) refers to the individual plasma layer while (16), which defines Q, refers to the entire plasma. Now both e_x and h are required to be continuous at all boundaries between layers, so the Q matrix for the first two layers would be the product $P_2 P_1$ where P_j denotes the P matrix for the j^{th} layer from the innermost plasma boundary. In general the Q matrix is the cumulative product of the P matrices for all layers, where the addition of a layer in outward order gives Q as P times the old value of Q.

In terms of the program listing, the value of Q is initialized as the identity in lines 1360 - 1390. The effect of all plasma layers is accumulated in the loop from line 1400 to line 1770. CA is γ at line 1410. The distinct elements of P are calculated as CB, CE and CF in lines 1420 - 1560. The cumulative matrix multiplication PQ is done at lines 1570 - 1620. Note that the error guards at lines 1430 and 1730 - 1760 are required by the limited exponent ($10^{\pm 38}$) of the L66 computer word.

Comparison of (26), (27) and (28) shows that T_1 through T_4 are each equal to the layer temperature times the drop across the layer in the value of the corresponding combination of the elements of the cumulative Q matrix as given in (27). The arrays TB and TC denote these four combinations of elements of Q, TB being for the inner edge of the layer and TC for the outer. As 1340 and 1350 indicate all combinations are initially zero except for the second, which is initially unity. Current values of TC are calculated at lines 1640 - 1690. As noted above, the array TA is the four T's in (28), and their values are accumulated at line 1710.

3.2.1.3 Subroutine CRITS

In long experience with calculations of em wave propagation in plasma, we have found that spurious effects can be induced by approximating a smooth plasma distribution in terms of a series of uniform layers. Such spurious effects are greatest when one or more of the layers is at critical electron density. They tend to be minimum and of no effect on the results when the critical density is midway between the densities of adjacent layers. In other words if the critical density point of the smooth profile lies at a boundary point between adjacent layers of the layered approximation, the latter tends to be a good approximation. This subroutine is designed to produce this effect, together with the main program.

In the calling sequence the array FI is the input profile of relative electron density, XI is the input profile distances, N is the number of points in the input profile, and the array A represents NA (up to two) values of relative electron density which are to be considered critical. (Although not done, for convenience, in this application, it is possible to consider the point where γ vanishes in addition to where K does as a critical point.) During execution NA is changed to indicate the number of locations used in the array XA. The meaning of the array XA is a set of points in the coordinates of the input profile at which layer boundaries must be placed. Layers of constant thickness may be used between all values of XA.

The subroutine is designed to work under the assumption that the input plasma profile has only one relative maximum. If it has more than one, subsequent crossings of the critical points will be ignored. The logic of the routine can be seen by studying the listing.

3.2.2 SLOC

Fig. 7 is a listing of this program, which consists of a main program and five subroutines. Subroutine CRITS is identical to the same subroutine in SLOP, so it is not described in this section.

3.2.2.1 Main Program

The first 58 lines of this program are similar to the first 55 lines of SLOP, where the arrays of complex dielectric constants and thicknesses of the

```
LIST 10-500

      COMMON/RAB/COLL(21),EMO(21),NPTS,TO(21),UNCM,YO(21)
      COMMON/RAC/T(60),TA(4)
      COMPLEX CA,CB,CC,DIEL(60),G(4),YA,YAO
      DIMENSION CRIT(2),EM(21),X(60),XA(6),Y(21),YIP(60)
      NAMELIST/INPUT/CAYA,CYL,FACC,FACE,FACT,FACY,FMHZ,GA
      DATA FACC,FACE,FACT,FACY/4*1./
      DATA PI,RAD/3.1415927,.01745329/
      CALL FXOPT(67,1,1,0)
    1 GA=0.
      READ INPUT
      EMC=12405.18*FMHZ*FMHZ/FACE
      COLA=.15915494*FACC/FMHZ
      CAY=2.0958447E-4*UNCM*FMHZ
      CRIT(1)=1.
      CRIT(2)=1.
      NA=1
      DO 10 I=1,NPTS
      Y(I)=FACY*YO(I)
   10 EM(I)=EMO(I)/EMC
      CALL CRITS(EM,Y,NPTS,CRIT,XA,NA)
      A=XA(NA)-XA(1)
      N=0
      MP=2
      CAYR=CYL
      DO 60 I=2,NA
      J=I-1
      D=XA(J)
      B=XA(I)-D
      K=50.*B/A
      IF(K.LT.1)K=1
      C=B/K
      E=CAY*C
      DO 50 L=1,K
      N=N+1
      X(N)=E
      YIP(N)=E/CAYR
      CAYR=CAYR+E
      F=D+(L-.5)*C
      DO 30 M=MP,NPTS
      IF(Y(M).GT.F)GO TO 35
   30 CONTINUE
   35 MP=M
      MM=MP-1
      AA=(F-Y(MM))/(Y(MP)-Y(MM))
      BB=1.-AA
      COL=COLA*(COLL(MM)*BB+COLL(MP)*AA)
      T(N)=FACT*(TO(MM)*BB+AA*TO(MP))
      IF(AA.GE.1.)GO TO 40
      IF(EM(MM).GT.0..AND.EM(MP).GT.0.)GO TO 38
      G=EM(MM)*BB+EM(MP)*AA
```

*

Figure 7. Listing of Program SLOC

LIST 510-1000

```
      GO TO 45
   38 G=EM(MM)**BB*EM(MP)**AA
      GO TO 45
   40 G=EM(MP)
   45 G=G/(1.+COL*COL)
      H=G*COL
   50 DIEL(N)=CMPLX(1.-G,H)
   60 CONTINUE
      IF(GA.GT.0.)GO TO 80
      U=0.
      ITER=0
      CALL WFP(N,DIEL,X,YIP,CYL,U,Q)
      IF(U.LT.0.)GO TO 80
      CALL HANK(CYL,A,B,C,D)
      AA=.9028
      CA=CMPLX(A,B)
      CB=CMPLX(-D,C)
      YAO=AA*CB/CA
      CALL HANK(CAYR,A,B,C,D)
      CA=CMPLX(A,B)
      CB=CMPLX(-D,C)
      CC=(Q(1)*CB+Q(3)*CA)/(Q(4)*CA+Q(2)*CB)
      YA=AA*CC
      A=REAL(CC)
      B=AIMAG(CC)
      TN=AA*(TA(1)+B*TA(2)-A*TA(3)+(A*A+B*B)*TA(4))
      DU=2.*PI/CAYA
      U=1.5*DU
      BA=1.5
      L=1
   61 CALL WFP(N,DIEL,X,YIP,CYL,U,Q)
      IF(U.LT.0.)L=0
   62 BB=1./(PI*(BA-.05/BA))**2
      BC=1.-U*U
      IF(BC)65,69,63
   63 W=SQRT(BC)
      BC=W*CYL
      CALL HANK(BC,A,B,C,D)
      CA=CMPLX(W*A,W*B)
      CB=CMPLX(-D,C)
      YAO=YAO+BB*CB/CA
      IF(L.NE.1)GO TO 69
      BC=W*CAYR
      CALL HANK(BC,A,B,C,D)
      CA=CMPLX(W*A,W*B)
      CB=CMPLX(-D,C)
      CC=(Q(1)*CB+Q(3)*CA)/(Q(4)*CA+Q(2)*CB)
      GO TO 66
   65 W=SQRT(-BC)
       BC=W*CYL
```

* **Figure 7 (continued)**

```
LIST 1010-1500

1010       CALL HANKI(BC,A,B)
1020       YAO=YAO-CMPLX(0.,BB*B/A/W)
1030       IF(L.NE.1)GO TO 69
1040       BC=W*CAYR
1050       CALL HANKI(BC,A,B)
1060       CA=CMPLX(0.,B)
1070       CC=(0.,0.)
1080       CB=W*Q(4)*A-Q(2)*CA
1090       IF(CABS(CB).GT.0.)CC=(W*Q(3)*A-Q(1)*CA)/CB
1100    68 YA=YA+BB*CC
1110       A=REAL(CC)
1120       B=AIMAG(CC)
1130       TN=TN+BB*(TA(1)+B*TA(2)-A*TA(3)+(A*A+B*B)*TA(4))
1140    69 BA=BA+1.
1150       U=U+DU
1160       ITER=ITER+1
1170       PRINT 1000,ITER,YAO,YA,TN
1180       READ:L
1190       IF(L)70,62,61
1200    70 A=.25*CAYA/PI/(PI*(BA-.5))**2
1210       YAO=YAO+CMPLX(0.,-A)
1220       GA=REAL(YA)
1230       TN=TN*CAYA*CYL/GA
1240       PRINT 3000,YAO,YA,TN
1250    80 READ:THET
1260       IF(THET.LT.0.)GO TO 100
1270       IF(THET.GE.90.)GO TO 1
1280       U=SIN(RAD*THET)
1290       CALL WFP(N,DIEL,X,YIP,CYL,U,Q)
1300       IF(U.LT.0.)GO TO 80
1310       W=SQRT(1.-U*U)
1320       AA=W*CAYR
1330       CALL HANK(AA,A,B,C,D)
1340       CA=CMPLX(A,B)
1350       CB=CMPLX(-D/W,C/W)
1360       AA=W*CYL
1370       CALL HANK(AA,A,B,C,D)
1380       BB=A*A+B*B
1390       CC=CMPLX(A,B)*(Q(1)*Q(4)-Q(2)*Q(3))/(Q(4)*CA+Q(2)*CB)
1400       A=-4.343*ALOG(REAL(CC*CONJG(CC)))
1410       C=2.*CAYA/(CYL*GA*(PI*W)**2*BB)
1420       D=.5*CAYA*U
1430       IF(D.GT.0.)C=C*(SIN(D)/D)**2
1440       D=4.343*ALOG(C)
1450       C=D-A
1460       PRINT 2000,A,B,C
1470       GO TO 80
1480   100 STOP
1490  1000 FORMAT(I3,3X,1P2E12.4,3X,1P2E12.4,1PE15.4)
1500  2000 FORMAT(3F9.2)
```

Figure 7 (continued)

```
LIST 1510-2000

1510 3000 FORMAT(6X,1P2E12.4,3X,1P2E12.4,1PE15.4)
1520      END
1530      SUBROUTINE WFP(LAYERS,DIEL,DK,YIP,CAYR,U,Q)
1540      COMMON/RAC/T(60),TA(4)
1550      COMPLEX CA,CB,CC,DIEL(LAYERS),EYE,ONE,P,Q(4),QQ,R,S,ZERO
1560      DIMENSION DK(LAYERS),TB(4),TC(4),YIP(LAYERS)
1570      DATA EYE,ONE,ZERO/(0.,1.),(1.,0.),(0.,0.)/
1580      DATA EPSO,STP,TEST/1.E-12,88.,1.E12/
1590      US=U*U
1600      EN=0.
1610      DO 5 I=1,4
1620      TA(I)=0.
1630    5 TB(I)=0.
1640      TB(3)=1.
1650      Q(1)=ONE
1660      Q(2)=ZERO
1670      Q(3)=ZERO
1680      Q(4)=ONE
1690      RKM=CAYR
1700      J=1
1710   10 CA=CSQRT(DIEL(J)-US)
1720      RKP=RKM+DK(J)
1730      IF(RKP*CABS(CA).GT.STP)GO TO 30
1740      DEL=YIP(J)
1750      A=RKM*REAL(CA)
1760      B=RKM*AIMAG(CA)
1770      CALL PROP(EN,A,B,DEL,P,QQ,R,S)
1780      CB=-EYE*CA*QQ/DIEL(J)
1790      CC=EYE*DIEL(J)*R/CA
1800      CA=CC*Q(1)+S*Q(3)
1810      Q(1)=P*Q(1)+CB*Q(3)
1820      Q(3)=CA
1830      CA=CC*Q(2)+S*Q(4)
1840      Q(2)=P*Q(2)+CB*Q(4)
1850      Q(4)=CA
1860      CB=CONJG(Q(3))
1870      CC=CONJG(CA)
1880      TC(1)=REAL(Q(1)*CB)
1890      TC(2)=AIMAG(Q(2)*CB-Q(1)*CC)
1900      TC(3)=REAL(Q(2)*CB+Q(1)*CC)
1910      TC(4)=REAL(Q(2)*CC)
1920      A=RKP/CAYR
1930      DO 15 I=1,4
1940      TC(I)=A*TC(I)
1950   15 TA(I)=TA(I)+T(J)*(TC(I)-TB(I))
1960      IF(J.EQ.LAYERS)RETURN
1970      J=J+1
1980      RKM=RKP
1990      AI=0.
2000      DO 20 I=1,4
```

* **Figure 7 (continued)**

LIST 2010-2500

```
      TB(I)=TC(I)
      A=REAL(Q(I))
      IF(A.GT.TEST)GO TO 30
      A1=A1+A*A
      A=AIMAG(Q(I))
      IF(A.GT.TEST)GO TO 30
      A1=A1+A*A
   20 CONTINUE
      A=1.-.0625*EPS0*A1
      IF(A.GT.0.)GO TO 10
   30 G=-G-1.E-30
      RETURN
      END
      SUBROUTINE HANK(X,FJ0,FY0,FJ,FY)
      DATA TP/.63661977/
      A=X/3.
      IF(A.GT.1.)GO TO 10
      A=.1*A*A
      B=TP*ALOG(.5*X)
      FJ0=(((((210.*A-394.44)*A+444.479)*A-316.3866)*A+126.56208)*A
     & -22.499997)*A+1.
      FY0=B*FJ0+.36746691-(((((248.46*A-427.916)*A+426.1214)*A
     & -253.00117)*A+74.350384)*A-6.0559366)*A
      FJ=X*(.5+(((((11.09*A-31.761)*A+44.3319)*A-39.54289)*A
     & +21.093573)*A-5.6249985)*A)
      FY=B*FJ+((((((2787.3*A-4009.76)*A+3123.951)*A-1316.4827)*A
     & +216.82709)*A+2.212091)*A-.6366198)/X
      RETURN
   10 A=.3/X
      B=SQRT(X)
      F=((((((144.76*A-72.805)*A+13.7237)*A-.09512)*A-.55274)*A
     & -7.7E-6)*A+.79788456)/B
      T=X+(((((135.58*A-29.333)*A-5.4125)*A+2.62573)*A-.003954)*A
     & -.4166397)*A-.78539816
      FJ0=F*COS(T)
      FY0=F*SIN(T)
      F=(.79788456-(((((200.33*A-113.653)*A+24.9511)*A-.17105)*A
     & -1.659667)*A-1.56E-5)*A)/B
      T=X-2.35619449-(((((291.66*A-79.824)*A-7.4348)*A+6.37879)*A
     & -.00565)*A-1.2499612)*A
      FJ=F*COS(T)
      FY=F*SIN(T)
      RETURN
      END
      SUBROUTINE HANKI(X,FK0,FK)
      A=.5*X
      IF(A.GT.1.)GO TO 10
      B=ALOG(A)
      A=.1*A*A
      T=A/3.515625
```

* **Figure 7 (continued)**

```
LIST 2510-3000

2510       C=(((((4581.3*T+3607.68)*T+2659.732)*T+1206.7492)*T
2520      & +308.99424)*T+35.156229)*T+1.
2530       FK0=(((((7.4*A+10.75)*A+26.2698)*A+34.8859)*A+23.069756)*A
2540      & +4.227842)*A-.57721566-B*C
2550       C=(((((324.11*T+301.532)*T+265.8733)*T+150.84934)*T
2560      & +51.498869)*T+8.7890594)*T+.5
2570       FK=X*B*C+(1.-(((((46.86*A+110.404)*A+191.9402)*A+181.56897)*A
2580      & +67.278579)*A-1.5443144)*A)/X
2590       RETURN
2600    10 A=.2/X
2610       B=EXP(-X)/SQRT(X)
2620       FK0=B*((((((532.08*A-251.54)*A+58.7872)*A-10.62446)*A
2630      & +2.189568)*A-.7832358)*A+1.25331414)
2640       FK=B*(1.25331414-(((((682.45*A-325.614)*A+78.0353)*A
2650      & -15.04268)*A+3.65562)*A-2.3498619)*A)
2660       RETURN
2670       END
2680       SUBROUTINE PROP(EN,XZR,XZI,Y,C,D,CP,DP)
2690       COMPLEX C,CP,D,DP
2700       DIMENSION AJ(2,2,5),P(2,2),Q(2,2),U(2,2),W(2,2)
2710       A=XZR*XZR
2720       B=XZI*XZI
2730       E=A+B
2740       F=SQRT(E)
2750       G=F+.01
2760       LIM=6.+.1*F+.2*EN/SQRT(G)+(41.+.4*F+.04*E+(4.6-.7*ALOG(G))*EN)*Y
2770       ENS=EN*EN
2780       XSA=A-B
2790       XSB=2.*XZR*XZI
2800       YS=Y*Y
2810       A=E*Y
2820       AJ(1,1,3)=XZR/A
2830       AJ(1,2,3)=-XZI/A
2840       AJ(2,1,4)=1.
2850       AJ(2,2,4)=0.
2860       DO 3 K=1,2
2870       DO 2 J=1,2
2880       AJ(J,K,1)=0.
2890       AJ(J,K,2)=0.
2900       U(K,J)=0.
2910     2 W(K,J)=0.
2920       AJ(2,K,3)=0.
2930     3 AJ(1,K,4)=0.
2940       L=5
2950       LA=4
2960       LB=3
2970       LC=2
2980       LD=1
2990       A=1.
3000       DO 10 J=1,LIM
```

* **Figure 7 (continued)**

```
LIST 3010-3500

3010       G=YS/A
3020       A=A+1.
3030       G=G/A
3040       E=(3./A-2.)*Y
3050       F=(ENS-(A-2.)**2)*G
3060       DO 5 I=1,2
3070       DO 4 K=1,2
3080       P(K,I)=E*AJ(K,I,LA)+F*AJ(K,I,LB)
3090     4 Q(K,I)=(AJ(K,I,LB)+2.*Y*AJ(K,I,LC)+YS*AJ(K,I,LD))*G
3100     5 CONTINUE
3110       DO 7 K=1,2
3120       AJ(K,1,L)=P(K,1)-XSA*Q(K,1)+XSB*Q(K,2)
3130       AJ(K,2,L)=P(K,2)-XSA*Q(K,2)-XSB*Q(K,1)
3140       DO 6 I=1,2
3150       P(K,I)=U(K,I)
3160       Q(K,I)=W(K,I)
3170       U(K,I)=U(K,I)+AJ(K,I,L)
3180     6 W(K,I)=W(K,I)+A*AJ(K,I,L)
3190     7 CONTINUE
3200       E=0.
3210       DO 8 K=1,2
3220       DO 8 I=1,2
3230       E=E+(U(K,I)-P(K,I))**2+(W(K,I)-Q(K,I))**2
3240     8 CONTINUE
3250       IF(E.LE.0.)GO TO 11
3260       LD=LC
3270       LC=LB
3280       LB=LA
3290       LA=L
3300       L=1+MOD(LA,5)
3310    10 CONTINUE
3320    11 A=Y*XZR
3330       B=Y*XZI
3340       C=CMPLX(1.+A*U(1,1)-B*U(1,2),A*U(1,2)+B*U(1,1))
3350       E=1.+U(2,1)
3360       D=CMPLX(A*E-B*U(2,2),A*U(2,2)+B*E)
3370       CP=CMPLX(W(1,1),W(1,2))
3380       DP=CMPLX(1.+W(2,1),W(2,2))
3390       RETURN
3400       END
3410       SUBROUTINE CRITS(FI,XI,N,A,XA,NA)
3420       DIMENSION A(2),FI(N),GI(6),XA(6),XI(N)
3430       DATA EPS/1.E-6/
3440       IT=NA
3450       NA=1
3460       DO 1 I=1,4
3470     1 XA(I)=0.
3480       M=N-1
3490       DO 100 I=1,IT
3500       DO 90 J=1,M
```

* **Figure 7 (continued)**

```
LIST 3510-3950

3510       K=J+1
3520       FM=FI(J)/A(I)-1.
3530       FP=FI(K)/A(I)-1.
3540       B=FM*FP
3550       IF(B.GT.0.)GO TO 90
3560       IF(B.LT.0.)GO TO 5
3570       IF(ABS(FM).GT.0.)GO TO 90
3580       XA(NA)=XI(J)
3590       IF(NA.EQ.4)GO TO 120
3600       NA=NA+1
3610       GO TO 90
3620     5 XM=XI(J)
3630       XP=XI(K)
3640       IF(FM.LE.-1..OR.FP.LE.-1.)GO TO 10
3650       B=XM+(XP-XM)*ALOG(A(I)/FI(J))/ALOG(FI(K)/FI(J))
3660       GO TO 40
3670    10 B=XM+(XP-XM)*FM/(FM-FP)
3680    40 XA(NA)=B
3690       IF(NA.EQ.4)GO TO 120
3700       NA=NA+1
3710    90 CONTINUE
3720   100 CONTINUE
3730       DO 110 I=2,M
3740       IF(FI(I-1).LT.FI(I).AND.FI(I).GE.FI(I+1))GO TO 115
3750   110 CONTINUE
3760       NA=NA-1
3770       GO TO 120
3780   115 XA(NA)=XI(I)
3790   120 NA=NA+1
3800       XA(NA)=XI(1)
3810       NA=NA+1
3820       XA(NA)=XI(N)
3830       DO 150 I=1,NA
3840       B=1.E30
3850       DO 140 J=1,NA
3860       IF(XA(J).GE.B)GO TO 140
3870       K=J
3880       B=XA(J)
3890   140 CONTINUE
3900       GI(I)=B
3910   150 XA(K)=2.E30
3920       DO 160 I=1,NA
3930   160 XA(I)=GI(I)
3940       RETURN
3950       END
```

* **Figure 7 (continued)**

plasma layers are defined. The only exceptions are the lines 240, 360 and 370. The array YIP is the ratio of the thickness of the layer to the cylindrical radius at its inner edge and CAYR becomes the cylindrical radius of the outer edge of the plasma.

The aperture admittance and plasma noise temperature are calculated in lines 600 through 1230. The derivations are based on the assumption of a cylinder of radius R having a gap of width a about its circumference. The plasma is composed of a series of co-axial layers, each of uniform but differing density. In cylindrical coordinates let $E_\phi = 0$, $E_z = 0$ if $|z| > a/2$, and $E_z = E_o$ if $|z| < a/2$, on the cylinder of radius R. Thus throughout all space the magnetic field $H = H_\phi$, $E_\phi = 0$ and all fields are independent of ϕ. In the cylindrical coordinate system (ρ, ϕ, z) Maxwell's equations are

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = iH \quad (49)$$

$$\frac{\partial H}{\partial z} = iKE_\rho \quad (50)$$

$$\frac{\partial}{\partial \rho}(\rho H) = -iK\rho E_z \quad (51)$$

Where K is a function of ρ only. These equations have the same solutions by separation of variables and superposition as in the planar case except that exp (i u z) separates instead of the same function of x. Typical Fourier transforms are

$$E_z(\rho, z) = \int_{-\infty}^{\infty} e_z(\rho, u) \exp(iuz)\, du \quad (52)$$

$$e_z(\rho, u) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_z(\rho, z) \exp(-iuz)\, dz \quad (53)$$

and similary for E_ρ and H.

Let us preserve the convention implied by the derivation used in the planar case, that the aperture admittance is positive for power flowing outward through the aperture. Then Y_a is defined as the negative of H/E_o in the aperture and the analog of (9) is

$$a|E_o|^2 Y_a = -\int_{-\infty}^{\infty} E_z^*(R, z)\, H(R, z)\, dz \quad (54)$$

Substituting the transform for H, reversing the order of integration, and using the inverse transform of E_z we get

$$a|E_o|^2 Y_a = -2\pi \int_{-\infty}^{\infty} h(R, u)\, e_z^*(R, u)\, du \quad (55)$$

Now e_z (R,u) is identical to e_x (u, o) as given by (11). Therefore (12) and, as a result, (18), (19) and finally (20) apply for the cylindrical as well as the planar case. Of course, however, the admittance function y(u) is the negative of the magnetic field divided by the tangential electric field at radius R for a cylindrical wave which varies as exp (i u z).

Substitution of the Fourier transforms gives Maxwell's equation as follows, on elimination of the ρ component,

$$\gamma^2 h = iK \frac{de_z}{d\rho} \tag{56}$$

$$\frac{d}{d\rho}(\rho h) = -iK\rho\, e_z \tag{57}$$

where γ is given by (44). Elimination of h gives

$$\frac{d}{d\rho}\left(\frac{K\rho}{\gamma^2}\frac{de_z}{d\rho}\right) + K\rho\, e_z = 0 \tag{58}$$

In free space, where K = 1 and γ = w, this is Bessel's equation of zero order in the variable ρw. The solution corresponding to outgoing radiation in our convention is $H_o\,(w\rho)$, which is the Hankel function of the first kind. Also, (56) states that h is i/w times the derivative of this function. Therefore for free space,

$$y\,(u) = \frac{i\; H_1\;(wR)}{w\; H_o\;(wR)} \tag{59}$$

where we recall that $w = \sqrt{1 - u^2}$. Even when plasma is present the external variations of e_z and h are the same, so let R_p represent the cylindrical radius of the outer edge of the plasma and use the matrix Q to represent the plasma effect on the tangential fields. Therefore let

$$A\begin{bmatrix} iw\; H_o(wR_p) \\ H_1(wR_p)\end{bmatrix} = Q\begin{bmatrix} e_z \\ h\end{bmatrix}_{\rho=R} \tag{60}$$

where A is an arbitrary scalar. Now, since y (u) = - h/e_z at ρ = R,

$$y(u) = \frac{i\,Q_{11}\,H_1(wR_p) + w\,Q_{21}\,H_o(wR_p)}{w\,Q_{22}\,H_o(wR_p) + i\,Q_{12}\,H_1(wR_p)} \tag{61}$$

Turning to the interpretation of the computer program listing, note that in calls of subroutine HANK the variables A, B, C and D are the real and imaginary parts first for H_o and second for H_1. Therefore (59) shows up in lines 680 and 910, where CA and CB are w H_o and i H_1, respectively. Similarly (61) is seen in lines 720 and 970. The logic also includes a branch covering imaginary values of w, which uses the identities

$$i\,H_o(ix) = (2/\pi)\,K_o(x) \tag{62}$$

$$H_1(ix) = -(2/\pi)\,K_1(x) \tag{63}$$

where K_o and K_1 are the modified Bessel functions. Noting that w equals i times a real quantity, called W in this branch of the program, and that calls of HANKI give K_o as A and K_1 as B, (59) shows up in line 1020. Also, (61) is represented by line 1090.

The derivation of the plasma noise temperature is the same as before up through (23). The power flowing out across a given layer boundary at radius ρ is

$$P = -\pi \rho \, Re\left\{ \iiint_{-\infty}^{\infty} e_z(\rho,u)\, h^*(\rho,v) \exp[i(u-v)z]\, du\, dv\, dz \right\}$$

$$= -2\pi^2 \rho \int_{-\infty}^{\infty} Re\left[e_z(\rho,u)\, h^*(\rho,u) \right] du \tag{64}$$

Now the fact that e_z and h are transformed by the matrix Q from their values at ρ = R, where h = - y e_z and e_z = e_o, gives

$$Re[e_z(\rho,u)\, h^*(\rho,u)] = |e_o|^2 \big[Re(Q_{11} Q_{21}^*)$$
$$- Re(Q_{12}Q_{21}^* + Q_{11}Q_{22}^*)\, Re(y)$$
$$+ Im(Q_{12}Q_{21}^* - Q_{11}Q_{22}^*)\, Im(y) + Re(Q_{12}Q_{22}^*)|y|^2 \big] \tag{65}$$

Therefore

$$T_a = \frac{aR}{G_a}\left\{ \frac{2}{\pi} \int_0^{\infty} (\sin x/x)^2 \big[T_1 + T_2\, Im(y) \right.$$
$$\left. - T_3\, Re(y) + T_4 |y|^2 \big] dx \right\} \tag{66}$$

As before the array TA represents T_1, ..., T_4 in the program. The integral is accumulated at lines 760 and 1130; and the multiplying factor is included at line 1230, where a is CAYA and R is CYL.

The far field effects are calculated between lines 1250 and 1470. In the far field e_z is the solution of Bessel's equation for n = o and for argument $w\rho$, so

$$E_z(\rho, z) = i \int_{-\infty}^{\infty} w\, A\, H_o(w\rho) \exp(iuz)\, du$$

$$\sim \int_{-\infty}^{\infty} \sqrt{\frac{2iw}{\pi}}\; A \exp[i(w\rho + uz)]\, du \tag{67}$$

where A is a function of u determined by the boundary conditions and where the asymptotic form of the Hankel function has been used. Now the phase is stationary in this integral when

$$u = u_o = z/r = \sin\theta \tag{68}$$

where, as before, θ is the angle between the line of sight and the normal. (The fact that now r is taken from the cylinder axis instead of from the edge of the plasma makes negligible difference in the limit of infinite r.) The phase expands as in (34) about this point of stationary value and therefore (35) still applies. The result is

$$E_z(r,\theta) \sim 2A \cos\theta \; e^{ir}/r \tag{69}$$

where A is evaluated for u = sin θ. Now (49) and (50) give, at this point of stationary phase

$$E_\rho(r,\theta) \sim -\tan\theta \;\; E_z(r,\theta) \tag{70}$$

Therefore the far field is transverse and is given by

$$E(r,\theta) \sim 2A \; e^{ir}/r \tag{71}$$

The coefficient A is determined by (60) from elimination of h at ρ = R. Thus

$$E(r,\theta) \sim \frac{2E_0 \, e^{ir} \sin(ua/2)\,(Q_{11}Q_{22} - Q_{12}Q_{21})}{\pi u r \; (iwQ_{22}H_0 - Q_{12}H_1)} \tag{72}$$

where, of course, u = sin θ , w = cos θ , and H_o and H_1 are the Hankel functions of argument wR_p. In the absence of plasma Q is the identity matrix and R_p = R, so

$$\frac{E(r,\theta)}{E^\circ(r,\theta)} = \left[\frac{H_0(wR)(Q_{11}Q_{22} - Q_{12}Q_{21})}{Q_{22}H_0 + iQ_{12}H_1/w}\right]\frac{E_0}{E_0^\circ} \tag{73}$$

The part of this equation within [ ] is at line 1390 in the program and line 1400 gives its value in dB down. The "aperture gain" is $4\pi r^2$ times the square of the far field divided by the aperture power.

$$G = \frac{2a}{RG_2(\pi w)^2}\left[\frac{\sin(ua/2)}{ua/2}\right]^2 \left|\frac{Q_{11}Q_{22} - Q_{12}Q_{21}}{Q_{22}H_0 + iQ_{12}H_1/w}\right|^2 \tag{74}$$

The value of this quantity in dB is the FORTRAN variable C at line 1450. Except for the lack of normalization by the free-space conductance, as noted in SLOP, the FORTRAN variable D is the free-space value.

3.2.2.2 Subroutine WFP

The purpose of this subroutine is to accumulate the values of the propagation matrix Q and the four temperature factors appearing in (66). In the calling sequence, LAYERS is the number of plasma layers, DIEL is the array of their complex dielectric constants, DK is the array of their thicknesses, YIP is the array of their relative thicknesses in terms of the thickness divided by the inner radius, CAYR is the radius of the innermost boundary, U is the input value of u or sin θ , and Q is the linear complex array representing Q_{11}, Q_{12}, Q_{21} and Q_{22}. Error indication is made by returning U equal to the negative of its input value.

In a given plasma layer, having a constant value of K, (58) becomes Bessel's equation of zero order,

$$e_z'' + e_z'/(\gamma\rho) + e_z = 0 \tag{75}$$

Where ' denotes differentiation with respect to $\gamma\rho$. Also (56) may be written as

$$h = (iK/\gamma)\, e_z' \tag{76}$$

Hopefully not confusing the reader, let us denote a pair of linearly independent solutions of (75) by u and v. Then in the given layer

$$e_z = a_1 u + a_2 v \tag{77}$$

$$h = (iK/\gamma)(a_1 u' + a_2 v') \tag{78}$$

Now let subscript o denote the value of a quantity at the inner boundary of the layer. Then the undetermined coefficients a_1 and a_2 can be solved for in terms of e_{zo}, h_o, u_o, v_o, u_o' and v_o'. Now, in order to simplify the calculations and since we use nonstandard functions (see subroutine PROP) for u and v, let $v_o = u_o' = o$. Then the vector (e_z, h) is the propagation matrix P times (e_{zo}, h_o), where

$$P = \begin{bmatrix} u/u_o & -i\gamma v/(K v_o') \\ iK u'/(\gamma u_o) & v'/v_o' \end{bmatrix} \tag{79}$$

Subroutine PROP outputs u/u_o, v/v_o' , u'/u_o, and v'/v_o' as P,QQ, R, and S at line 1770. CA represents γ at line 1710. Lines 1800 through 1850 are the accumulation of the Q matrix from multiplication by P.

Lines 1860 through 1950 and line 2010 involve the accumulation of the four temperature factors in (66). Inspection of (65) in comparison with that equation reveals the reason for the form of each term in lines 1880 - 1910. These factors are corrected by the ratio ρ/R in lines 1920 and 1940 because (64) contains ρ while (66) has R. Finally, the difference is taken in terms of "outer" minus "inner" (TC - TB in line 1950) to give the power lost in the layer, because of the negative sign in (64).

3.2.2.3 Subroutine HANK

This routine calculates the real and imaginary parts of the Hankel functions of order zero and unity and for real argument. In the calling sequence, X is the argument, FJO is the real part of H_o, FYO is the imaginary part of H_o, FJ is the real part of H_1, and FY is the imaginary part of H_1. The method uses the polynomial approximations on pages 369-370 of Abramowitz and Stegun (Ref. 1). These polynomial equations are written in the form of continued products in order to obviate the calculation of powers.

3.2.2.4 Subroutine HANKI

This routine calculates the modified Bessel function of the second kind of orders zero and unity. In the calling sequence, X is the argument, FKO is K_o, and FK is K_1. The method is the polynomial approximations on pages 378-379 of Ref. 1. Again the continued products form is used for economical computation of power series.

3.2.2.5 Subroutine PROP

The purpose of this routine is to calculate the elements of the matrix P as in (79). It is more general than needed by subroutine WFP since the variable EN in the calling sequence refers to the order of Bessel's equation. The variables XZR and XZI are the real and imaginary parts of the value of $\gamma\rho$ for the inner boundary of the layer, and Y is the relative layer thickness. The outputs are C, D, CP, and DP, which represents u/u_o, v/v_o', u'/u_o, and v'/v_o' respectively.

The method uses series expansions in the relative layer thickness Y, where Y is the thickness divided by the radius at the inner boundary. The fact that u/u_o and v'/v_o', being on the diagonal of P, must approach unity as Y approaches zero implies that the leading term in series expansions must be unity. Similarly the lowest order term of the other two quantities is at least first order. Therefore let

$$u/u_o = 1 + XY\sum_{j=1}^{\infty} c_j \tag{80}$$

$$v/v_o' = XY\left(1 + \sum_{j=1}^{\infty} d_j\right) \tag{81}$$

$$u'/u_o = \sum_{j=1}^{\infty} (j+1)c_j \tag{82}$$

$$v'/v_o' = 1 + \sum_{j=1}^{\infty} (j+1)d_j \tag{83}$$

In these equations X is the value of $\gamma\rho$ at the inner boundary of the layer and the c_j and d_j are each equal to a complex constant, which depends on j, times the j^{th} power of Y. The c_j and d_j in (82) and (83) are identical with those in (80) and (81) since the second two quantities are the derivatives of the first two with respect to XY. Now u/u_o and v/v_o' must satisfy

$$z^2\frac{d^2u}{dz^2} + z\frac{du}{dz} + (z^2 - n^2)u = 0 \tag{84}$$

where $z = \gamma\rho$ in the layer. In particular

$$z = X(1+Y) \tag{85}$$

where, of course, X is constant and Y is considered variable. Substitution of (80) and (81) in (84) and separation of terms of like powers of Y, keeping in mind the implicit proportionality between c_j or d_j and Y^j, gives recurrance relations for the c_j and d_j. The same relation holds for both, so let a_j denote either c_j or d_j

$$j(j+1)a_j + (2j-1)jYa_{j-1} + [(j-1)^2 - n^2]Y^2 a_{j-2}$$
$$+ X^2Y^2(a_{j-2} + 2Ya_{j-3} + Y^2a_{j-4}) = 0 \tag{86}$$

This equation applies liberally for either term as long as

$$a_j = o \quad , \quad j < -1 \tag{87}$$

$$c_o = d_{-1} = o \tag{88}$$

$$c_{-1} = 1/(XY) \tag{89}$$

$$d_o = 1 \tag{90}$$

It is not difficult to see these equations in the listing of the computer program. However, see ref. 2 for a more detailed account of the theory, programming, and checkout of this routine, which is identical in that report with the present routine. Reference 2 also contains additional details concerning subroutine WFP.

3.2.3 SLOS

Fig. 8 is a listing of this program, which consists of a main program and five subroutines. Subroutine CRITS is identical to the same subroutine in SLOP, so it is not described in this section.

3.2.3.1 Main Program

With two exceptions the first 60 lines of this program are similar to the first 58 lines of SLOC. One exception is that the aperture conductance GA is not initially set to zero since the logic is not a function of this quantity as an input. The other exception is that the array DIEL, which is defined at line 590, is the square root of the complex dielectric constant of the layer.

The aperture admittance and plasma noise temperature are calculated in lines 610 through 1120. The derivations are based on the assumption of a sphere of radius R having a gap of width a at a given latitude. The plasma is composed of a series of concentric spherical layers, each of uniform but differing density. In spherical coordinates (r, θ, ϕ) let $E_\phi = 0$, $E_\theta = 0$ if $|\theta - \theta_o| > a/(2R)$, and $E_\theta = E_o$ if $|\theta - \theta_o| < a/(2R)$ on the sphere of radius R, where θ_o is the location of the center of the aperture. Thus throughout all space $H = H_\phi$, $E_\phi = 0$ and all fields are independent of ϕ. Maxwell's equations are

$$\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial \theta} = irH \tag{91}$$

$$\frac{\partial}{\partial \theta}(H \sin\theta) = -ikrE_r \sin\theta \tag{92}$$

$$\frac{\partial}{\partial r}(rH) = ikrE_\theta \tag{93}$$

```
LIST 10-500
      COMMON/RAB/COLL(21),EMO(21),NPTS,TO(21),UNCM,YO(21)
      COMMON/RAC/T(60),TA(4)
      COMPLEX CA,CB,CC,DIEL(60),HN(21),HNO(21),Q(4),YA,YAO
      DIMENSION CRIT(2),EM(21),EON(20),PNM(21),X(60),XA(6),Y(21),YIP(60)
      NAMELIST/INPUT/CAYA,CYL,FACC,FACE,FACT,FACY,FMHZ,MODES,THETO
      DATA MODES/20/
      DATA FACC,FACE,FACT,FACY/4*1./
      DATA PI,RAD/3.1415927,.01745329/
      CALL FXOPT(67,1,1,0)
    1 READ INPUT
      MOP=MODES+1
      EMC=12405.18*FMHZ*FMHZ/FACE
      COLA=.15915494*FACC/FMHZ
      CAY=2.0958447E-4*UNCM*FMHZ
      CRIT(1)=1.
      CRIT(2)=1.
      NA=1
      DO 10 I=1,NPTS
      Y(I)=FACY*YO(I)
   10 EM(I)=EMO(I)/EMC
      CALL CRITS(EM,Y,NPTS,CRIT,XA,NA)
      A=XA(NA)-XA(1)
      N=0
      MP=2
      CAYR=CYL
      DO 60 I=2,NA
      J=I-1
      D=XA(J)
      B=XA(I)-D
      K=50.*B/A
      IF(K.LT.1)K=1
      C=B/K
      E=CAY*C
      DO 50 L=1,K
      N=N+1
      X(N)=E
      YIP(N)=E/CAYR
      CAYR=CAYR+E
      F=D+(L-.5)*C
      DO 30 M=MP,NPTS
      IF(Y(M).GT.F)GO TO 35
   30 CONTINUE
   35 MP=M
      MM=MP-1
      AA=(F-Y(MM))/(Y(MP)-Y(MM))
      BB=1.-AA
      COL=COLA*(COLL(MM)*BB+COLL(MP)*AA)
      T(N)=FACT*(TO(MM)*BB+AA*TO(MP))
      IF(AA.GE.1.)GO TO 40
      IF(EM(MM).GT.0..AND.EM(MP).GT.0.)GO TO 38
```

* **Figure 8 Listing of Program SLOS**

```
LIST 510-1000

510          G=EM(MM)*BB+EM(MP)*AA
520          GO TO 45
530      38  G=EM(MM)**BB*EM(MP)**AA
540          GO TO 45
550      40  G=EM(MP)
560      45  G=G/(1.+COL*COL)
570          H=G*COL
580          CA=CMPLX(1.-G,H)
590      50  DIEL(N)=CSQRT(CA)
600      60  CONTINUE
610          A=RAD*THETO
620          B=.5*CAYA/CYL
630          BB=COS(A+B)
640          XX=COS(A-B)-BB
650          DX=.1*XX
660          A=.5*DX+BB
670          DO 61 M=1,MODES
680      61  EON(M)=0.
690          DO 70 L=1,10
700          B=SQRT(1.-A*A)
710          CALL LEG(A,B,MOP,PNM)
720          DO 65 M=1,MODES
730      65  EON(M)=EON(M)+PNM(M+1)
740      70  A=A+DX
750          DX=.5*DX
760          A=1.
770          DO 71 M=1,MODES
780          B=A+1.
790          EON(M)=DX*(A+B)*EON(M)/A/B
800      71  A=B
810          CALL HANK(MOP,CYL,HNO)
820          CALL HANK(MOP,CAYR,HN)
830          YA=(0.,0.)
840          YAO=YA
850          TN=0.
860          DO 78 K=1,MODES
870          M=K
880          CALL WFP(N,DIEL,X,YIP,CYL,M,Q)
890          IF(M)80,1,76
900      76  MP=M+1
910          A=2.*M*MP*EON(M)**2/(M+MP)
920          B=A/XX
930          CC=HNO(M)/HNO(MP)-M/CYL
940          YAO=YAO+CMPLX(0.,B)/CC
950          CB=CMPLX(AIMAG(HN(M)),-REAL(HN(M)))/HN(MP)+CMPLX(0.,M/CAYR)
960          CA=(Q(4)*CAYR-C(2)*CB)/(Q(1)*CB-Q(3)*CAYR)
970          YA=YA+B*CA/CYL
980          TN=TN+A*(TA(1)+TA(2)*REAL(CA)+TA(3)*AIMAG(CA)+TA(4)*
990         & REAL(CA*CONJG(CA)))
1000          CA=EON(M)/HN(MP)
```

* **Figure 8 (continued)**

LIST 1010-1500

```
      HN(M)=(Q(1)*Q(4)-Q(2)*Q(3))*CA/(CB*Q(1)-CAYR*Q(3))
      HNO(M)=EON(M)/CC/HNO(MP)
      IF(MODES.NE.20)GO TO 78
      PRINT 1000,M,YAO,YA,TN
      IF(M.EQ.MODES)GO TO 90
      READ:L
      IF(L.EQ.0)GO TO 90
   78 CONTINUE
      GO TO 90
   80 M=-M
   90 GA=REAL(YA)
      TN=TN/GA
      PRINT 1000,M,YAO,YA,TN
   95 READ:ALPH
      IF(ALPH.LE.0.)GO TO 200
      IF(ALPH.GE.180.)GO TO 1
      A=COS(RAD*ALPH)
      B=SQRT(1.-A*A)
      CALL LEG(A,B,M,PNM)
      CA=(0.,0.)
      CB=CA
      DO 100 N=1,M
      A=PNM(N+1)
      CA=CMPLX(-AIMAG(CA),REAL(CA))+A*HNO(N)
      CB=CMPLX(-AIMAG(CB),REAL(CB))+A*HN(N)
  100 CONTINUE
      CC=CA/CB
      A=4.343*ALOG(REAL(CC*CONJG(CC)))
      B=4.343*ALOG(2.*REAL(CA*CONJG(CA))/CYL/CYL/GA/XX)
      C=B-A
      PRINT 2000,A,B,C
      GO TO 95
  200 STOP
 1000 FORMAT(I3,3X,1P2E12.4,3X,1P2E12.4,1PE15.4)
 2000 FORMAT(3F9.2)
      END
      SUBROUTINE WFP(LAYERS,DIEL,DK,YIP,CAYR,N,Q)
      COMMON/RAC/T(60),TA(4)
      COMPLEX CA,CB,CC,DIEL(LAYERS),EYE,ONE,P,Q(4),QQ,R,S,ZERO
      DIMENSION DK(LAYERS),TB(4),TC(4),YIP(LAYERS)
      DATA EYE,ONE,ZERO/(0.,1.),(1.,0.),(0.,0.)/
      DATA EPS0,STP,TEST/1.E-12,88.,1.E12/
      EN=N
      DO 5 I=1,4
      TA(I)=0.
    5 TB(I)=0.
      TB(2)=CAYR
      Q(1)=ONE
      Q(2)=ZERO
      Q(3)=ZERO
```

* **Figure 8 (continued)**

```
LIST 1510-2000

1510        Q(4)=ONE
1520        RKM=CAYR
1530        J=1
1540    10  CA=DIEL(J)
1550        RKP=RKM+DK(J)
1560        IF(RKP*CABS(CA).GT.STP)GO TO 30
1570        DEL=YIP(J)
1580        A=RKM*REAL(CA)
1590        B=RKM*AIMAG(CA)
1600        CALL PROP(EN,A,B,DEL,P,QQ,R,S)
1610        CB=EYE*QQ*CA*RKM
1620        CC=-EYE*R/CA/RKP
1630        S=S*RKM/RKP
1640        CA=CC*Q(1)+S*Q(3)
1650        Q(1)=P*Q(1)+CB*Q(3)
1660        Q(3)=CA
1670        CA=CC*Q(2)+S*Q(4)
1680        Q(2)=P*Q(2)+CB*Q(4)
1690        Q(4)=CA
1700        CB=CONJG(Q(1))
1710        CC=CONJG(Q(2))
1720        A=RKP
1730        TC(1)=A*REAL(Q(4)*CC)
1740        TC(2)=A*REAL(Q(3)*CC+Q(4)*CB)
1750        TC(3)=A*AIMAG(Q(4)*CB-Q(3)*CC)
1760        TC(4)=A*REAL(CB*Q(3))
1770        DO 15 I=1,4
1780    15  TA(I)=TA(I)+T(J)*(TB(I)-TC(I))
1790        IF(J.EQ.LAYERS)RETURN
1800        J=J+1
1810        RKM=RKP
1820        AI=0.
1830        DO 20 I=1,4
1840        TB(I)=TC(I)
1850        A=REAL(Q(I))
1860        IF(A.GT.TEST)GO TO 30
1870        AI=AI+A*A
1880        A=AIMAG(Q(I))
1890        IF(A.GT.TEST)GO TO 30
1900        AI=AI+A*A
1910    20  CONTINUE
1920        A=1.-.0625*EPSO*AI
1930        IF(A.GT.0.)GO TO 10
1940    30  N=1-N
1950        RETURN
1960        END
1970        SUBROUTINE HANK(M,X,HN)
1980        COMPLEX HN(21)
1990        A=SIN(X)/X
2000        B=COS(X)/X
```

* **Figure 8 (continued)**

```
LIST 2010-2500

2010          HN(1)=CMPLX(A,-B)
2020          HN(2)=CMPLX(A/X-B,-B/X-A)
2030          IF(M.LT.3)RETURN
2040          K=7+SQRT(3.*X*(X+8.))+.2*ALOG(X)
2050          J=MAX0(K,M)
2060          A=2./X
2070        1 B=(J+1.5)*A
2080          U=1.E-20
2090          V=B*U
2100          K=J
2110          DO 10 I=1,J
2120          T=U
2130          U=V
2140          B=B-A
2150          V=B*U-T
2160          IF(ABS(V).GT.1.E35)GO TO 21
2170          IF(K.LE.M.AND.K.GT.2)HN(K)=CMPLX(V,0.)
2180       10 K=K-1
2190          IF(ABS(U).GT.ABS(V))GO TO 11
2200          F=REAL(HN(1))/V
2210          GO TO 12
2220       11 F=REAL(HN(2))/U
2230       12 B=3./X
2240          U=AIMAG(HN(1))
2250          V=AIMAG(HN(2))
2260          DO 20 I=3,M
2270          T=B*V-U
2280          U=V
2290          V=T
2300          T=F*REAL(HN(I))
2310          HN(I)=CMPLX(T,V)
2320       20 B=B+A
2330          RETURN
2340       21 J=I-1
2350          GO TO 1
2360          END
2370          SUBROUTINE LEG(X,Y,M,P)
2380          DIMENSION P(21)
2390          P(1)=0.
2400          P(2)=Y
2410          IF(M.LT.3)RETURN
2420          EM=1.
2430          DO 10 I=3,M
2440          EN=EM+1.
2450          A=EN+EM
2460          P(I)=(A*X*P(I-1)-EN*P(I-2))/EM
2470       10 EM=EN
2480          RETURN
2490          END
2500          SUBROUTINE PROP(EN,XZR,XZI,Y,C,D,CP,DP)
```

Figure 8 (continued)

```
LIST 2510-3000
      COMPLEX C,CP,D,DP
      DIMENSION AJ(2,2,5),P(2,2),Q(2,2),U(2,2),W(2,2)
      A=XZR*XZR
      B=XZI*XZI
      E=A+B
      F=SQRT(E)
      G=F+.01
      LIM=6.+.1*F+.2*EN/SQRT(G)+(41.+.4*F+.04*E+(4.6-.7*ALOG(G))*EN)*Y
      ENS=EN*EN+EN
      XSA=A-B
      XSB=2.*XZR*XZI
      YS=Y*Y
      A=E*Y
      AJ(1,1,3)=XZR/A
      AJ(1,2,3)=-XZI/A
      AJ(2,1,4)=1.
      AJ(2,2,4)=0.
      DO 3 K=1,2
      DO 2 J=1,2
      AJ(J,K,1)=0.
      AJ(J,K,2)=0.
      U(K,J)=0.
    2 W(K,J)=0.
      AJ(2,K,3)=0.
    3 AJ(1,K,4)=0.
      L=5
      LA=4
      LB=3
      LC=2
      LD=1
      A=1.
      DO 10 J=1,LIM
      G=YS/A
      A=A+1.
      G=G/A
      E=(4./A-2.)*Y
      F=(ENS-(A-2.)*(A-3.))*G
      DO 5 I=1,2
      DO 4 K=1,2
      P(K,I)=E*AJ(K,I,LA)+F*AJ(K,I,LB)
    4 Q(K,I)=(AJ(K,I,LB)+2.*Y*AJ(K,I,LC)+YS*AJ(K,I,LD))*G
    5 CONTINUE
      DO 7 K=1,2
      AJ(K,1,L)=P(K,1)-XSA*Q(K,1)+XSB*Q(K,2)
      AJ(K,2,L)=P(K,2)-XSA*Q(K,2)-XSB*Q(K,1)
      DO 6 I=1,2
      P(K,I)=U(K,I)
      Q(K,I)=W(K,I)
      U(K,I)=U(K,I)+AJ(K,I,L)
    6 W(K,I)=W(K,I)+A*AJ(K,I,L)
```

* **Figure 8 (continued)**

LIST 3010-3500

```
      7 CONTINUE
        E=0.
        DO 8 K=1,2
        DO 8 I=1,2
        E=E+(U(K,I)-P(K,I))**2+(W(K,I)-Q(K,I))**2
      8 CONTINUE
        IF(E.LE.0.)GO TO 11
        LD=LC
        LC=LB
        LB=LA
        LA=L
        L=1+MOD(LA,5)
     10 CONTINUE
     11 A=Y*XZR
        B=Y*XZI
        C=CMPLX(1.+A*U(1,1)-B*U(1,2),A*U(1,2)+B*U(1,1))
        E=1.+U(2,1)
        D=CMPLX(A*E-B*U(2,2),A*U(2,2)+B*E)
        CP=CMPLX(W(1,1),W(1,2))
        DP=CMPLX(1.+W(2,1),W(2,2))
        RETURN
        END
        SUBROUTINE CRITS(FI,XI,N,A,XA,NA)
        DIMENSION A(2),FI(N),GI(6),XA(6),XI(N)
        DATA EPS/1.E-6/
        IT=NA
        NA=1
        DO 1 I=1,4
      1 XA(I)=0.
        M=N-1
        DO 100 I=1,IT
        DO 90 J=1,M
        K=J+1
        FM=FI(J)/A(I)-1.
        FP=FI(K)/A(I)-1.
        B=FM*FP
        IF(B.GT.0.)GO TO 90
        IF(B.LT.0.)GO TO 5
        IF(ABS(FM).GT.0.)GO TO 90
        XA(NA)=XI(J)
        IF(NA.EQ.4)GO TO 120
        NA=NA+1
        GO TO 90
      5 XM=XI(J)
        XP=XI(K)
        IF(FM.LE.-1..OR.FP.LE.-1.)GO TO 10
        B=XM+(XP-XM)*ALOG(A(I)/FI(J))/ALOG(FI(K)/FI(J))
        GO TO 40
     10 B=XM+(XP-XM)*FM/(FM-FP)
     40 XA(NA)=B
```

* **Figure 8 (continued)**

```
LIST 3510-3770

3510       IF(NA.EQ.4)GO TO 120
3520       NA=NA+1
3530   90  CONTINUE
3540  100  CONTINUE
3550       DO 110 I=2,M
3560       IF(FI(I-1).LT.FI(I).AND.FI(I).GE.FI(I+1))GO TO 115
3570  110  CONTINUE
3580       NA=NA-1
3590       GO TO 120
3600  115  XA(NA)=XI(I)
3610  120  NA=NA+1
3620       XA(NA)=XI(1)
3630       NA=NA+1
3640       XA(NA)=XI(N)
3650       DO 150 I=1,NA
3660       B=1.E30
3670       DO 140 J=1,NA
3680       IF(XA(J).GE.B)GO TO 140
3690       K=J
3700       B=XA(J)
3710  140  CONTINUE
3720       GI(I)=B
3730  150  XA(K)=2.E30
3740       DO 160 I=1,NA
3750  160  XA(I)=GI(I)
3760       RETURN
3770       END

*
```

Figure 8 (continued)

where K is a function of r only. Elimination of the components of E from these equations gives

$$\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(H\sin\theta)\right) + Kr\frac{\partial}{\partial r}\left(\frac{1}{K}\frac{\partial}{\partial r}(rH)\right) + Kr^2 H = 0 \tag{94}$$

Separation of variables can be accomplished by letting

$$\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(H\sin\theta)\right) = -n(n+1)H \tag{95}$$

which is Legendre's equation of order unity and degree n. Thus let

$$H(r,\theta) = \sum_{n=1}^{\infty} h(r,n)\, P_n'(\cos\theta) \tag{96}$$

Now (93) implies that E_θ has this same dependence on θ.

$$E_\theta(r,\theta) = \sum_{n=1}^{\infty} e_\theta(r,n)\, P_n'(\cos\theta) \tag{97}$$

Also, (92) implies that E_r depends on P_n (cos θ).

$$E_r(r,\theta) = \sum_{n=1}^{\infty} e_r(r,n)\, P_n(\cos\theta) \tag{98}$$

To calculate the aperture admittance start with

$$A|E_0|^2 Y_a = 2\pi R^2 \int_0^{\pi} E_\theta^*(R,\theta)\, H(R,\theta)\sin\theta\, d\theta \tag{99}$$

where A is the area of the slot.

$$\begin{aligned} A &= 2\pi R^2 \int_{\theta_0 - \frac{a}{2R}}^{\theta_0 + \frac{a}{2R}} \sin\theta\, d\theta \\ &= 2\pi R^2\left[\cos\left(\theta_0 - \frac{a}{2R}\right) - \cos\left(\theta_0 + \frac{a}{2R}\right)\right] \\ &= 2\pi R^2 (x_2 - x_1) \end{aligned} \tag{100}$$

Substitute (96) and (97) in (99) and evaluate the integral using the orthogonality of the Legendre functions with respect to the degree n. Then, using y (n) = h (R, n)/ e_θ (R,n),

$$Y_a = \frac{2\sum_{n=1}^{\infty} \frac{n(n+1)}{(2n+1)} \left|\frac{e_0}{E_0}\right|^2 y(n)}{x_2 - x_1} \tag{101}$$

Where $e_0 = e_\theta$ (R, n). Orthogonality gives

$$e_0 = \frac{(2n+1)E_0}{2n(n+1)} \int_{x_1}^{x_2} P_n'(x)\, dx \tag{102}$$

where x_1 and x_2 are implied by (100).

Substitution of the series solutions in (93) and (94) gives

$$i\,K\,r\,e_\theta = \frac{d}{dr}(r\,h) \tag{103}$$

$$K\,r\,\frac{d}{dr}\left[\frac{1}{K}\,\frac{d}{dr}(r\,h)\right] + \left[Kr^2 - n\,(n+1)\right] h = o \tag{104}$$

If K is constant this last is the equation of the spherical Bessel functions of order n and argument Kr. In free space the solution corresponding to outgoing waves is the spherical Bessel function of the third kind, $h_n^{(1)}(r)$. (From here on we drop $^{(1)}$ from the notation since our time convention makes it clear that we need only those functions which tend to e^{ir}/r at large r.) Thus y (n) in free space is given by

$$y(n) = \frac{i}{h_{n-1}(R)/h_n(R) - n/R} \tag{105}$$

In the presence of plasma let

$$\begin{bmatrix} r\,h \\ e_\theta \end{bmatrix} = Q \begin{bmatrix} r\,h \\ e_\theta \end{bmatrix}_{r=R} \tag{106}$$

Replacing R by R_p, the radius of the outer edge of the plasma, (105) gives the ratio h/e_θ at that radius. Then (106) gives

$$y(n) = -\frac{Q_{22}R_p - Z\,Q_{12}}{R(Q_{11}Z - Q_{21}R_p)} \tag{107}$$

where

$$Z = in/R_p - i\,h_{n-1}(R_p)/h_n(R_p) \tag{108}$$

Now in terms of the FORTRAN, XX at line 640 is $(x_2 - x_1)$ and EON(M) becomes e_o/E_o for M = n after line 800. At line 920, B is the coefficient of y (n) in the sum of (101). CC at line 930 is the denominator of (105) and line 940 completes the accumulative value of Y_{ao}. CB at line 950 is Z as given by (108), and CA at line 960 is Ry (n) as given by (107). Y_a is accumulated at line 970.

The power flowing through a sphere of radius r is

$$\begin{aligned} P &= \pi r^2 \int_{-1}^{1} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Re\left[e_\theta(r,m)\,h^*(r,n)\right] P_m^1(x)\,P_n^1(x)\,dx \\ &= 2\pi \sum_{n=1}^{\infty} \frac{n(n+1)}{(2n+1)}\,Re\left[r^2 e_\theta(r,n)\,h^*(r,n)\right] \end{aligned} \tag{109}$$

Now use (106) to get

$$\text{Re}[r^2 e_\theta(r,n) h^*(r,n)] = r|e_o|^2 [\text{Re}(Q_{22}Q_{12}^*)$$
$$+ \text{Re}(Q_{21}Q_{12}^* + Q_{22}Q_{11}^*)\,\text{Re}(Ry)$$
$$+ \text{Im}(Q_{22}Q_{11}^* - Q_{21}Q_{12}^*)\,\text{Im}(Ry)$$
$$+ \text{Re}(Q_{21}Q_{11}^*)\,|Ry|^2] \qquad (110)$$

where Ry is Ry (n) as given by (107). Now, therefore, (23) gives

$$T_a = \frac{2}{G_2}\sum_{n=1}^{\infty}\frac{n(n+1)}{(2n+1)}\left|\frac{e_o}{E_o}\right|^2 [T_1 + T_2\,\text{Re}(Ry)$$
$$+ T_3\,\text{Im}(Ry) + T_4|Ry|^2] \qquad (111)$$

This equation appears in lines 980 and 990, except that G_a is included at line 1120.

The asymptotic form of h_n (r) is $- ie^{ir}/(i^n r)$. Therefore in the far field, H varies as e^{ir}/r. Now (93) implies that $E_\theta = H$ in the far field and (92) implies that $r^2 |E_r|^2$ tends to zero as r approaches infinity. Thus the far field is transverse and can be related directly to either E_θ or H.

$$E_\theta(r,\theta) \sim \frac{e^{ir}}{ir}\sum_{n=1}^{\infty}\frac{h(R_p,n)}{i^n h_n(R_p)}\,P_n'(\cos\theta) \qquad (112)$$

The quantity Z, which is given by (108), is the free-space impedance (ratio of $e_\theta(R_p, n)$ to h (R_p, n)) at radius R_p, so e_θ = Zh at the outer edge of the plasma. Using (106) we then get

$$h(R_p,n) = \frac{e_o(Q_{11}Q_{22} - Q_{12}Q_{21})}{ZQ_{11} - R_pQ_{21}} \qquad (113)$$

The "aperture gain" is $2\pi r^2$ times $|E_\theta(r,\theta)|^2$ divided by the total aperture power, $\frac{1}{2}A\,|E_o|^2 G_a$.

$$G = \frac{2\,|r\,E_\theta(r,\theta)/E_o|^2}{R^2(x_2 - x_1)G_a} \qquad (114)$$

In the program the quantity $[h(R_p, n)/(E_o h_n(R_p))]$ is stored versus n in the array HN at line 1010. The value of this quantity in free space, which is gotten by replacing R_p by R, by letting Q be the identity matrix, and by replacing Z by the reciprocal of y (n) as given by (105), is stored in HNO at line 1020. The sum in (112) for each case is accumulated by neglecting to keep account of the net phase, as at lines 1240, and 1250; i.e., the effect of i^{-n} is gotten by adding the new term without it to i times the previously accumulated sum. This is simply a FORTRAN short cut.

3.2.3.2 Subroutine WFP

The purpose of this subroutine is to accumulate the values of the propagation matrix Q and the four temperature factors appearing in (111). In the calling sequence, LAYERS is the number of plasma layers, DIEL is the array of the square root margin of their complex dielectric constants, DK is the array of their thickness, YIP is the array of their relative thicknesses, CAYR is the radius of the innermost boundary, N is the value of the order n of spherical wave, and Q is the linear complex array representing Q_{11}, Q_{21} and Q_{22}. Error indication is made by returning N = 1 - n. In other words N < 1 indicates error since n > 0.

Let prime denote differentiation with respect to a variable $z = \sqrt{K}\, r$. Then in a uniform medium (103) can be written as

$$e_\theta = \frac{-i}{z} (r h)' \tag{115}$$

Also (104) is

$$z^2 \, (rh)'' + \left[z^2 - n\,(n+1)\right] (rh) = 0 \tag{116}$$

Let u and v denote two linearly independent solutions of (116). Then in the layer in question

$$rh = a_1\, u + a_2\, v \tag{117}$$

$$e_\theta = -\frac{i}{z} (a_1\, u' + a_2\, v') \tag{118}$$

As in section 3.2.2.2 let $v_o = u_o' = 0$. Then the vector (rh, e_θ) equals P times the vector $(r_o\, h_o, e_{\theta_o})$, where

$$P = \begin{bmatrix} u/u_o & i z_o\, v/v_o' \\ -(i/z)(u'/u_o) & (z_o/z)(v'/v_o') \end{bmatrix} \tag{119}$$

This matrix is generated in lines 1540 - 1630 and the accumulation of Q by multiplication by P is done in lines 1640 - 1690. The four temperature factors in (111) are accumulated in lines 1700 through 1780 and 1840. Each term in Q , including r, which is called A at line 1720, can be seen at lines 1730 - 1760.

3.2.3.3 Subroutine HANK

This routine calculates the spherical Hankel functions of various orders. In the calling sequence, M is the number of orders to be calculated, X is the argument, and HN is the complex array of Hankel functions. The function of order n is stored in the $(n+1)^{th}$ location of HN. The method uses downward recurrence starting with arbitrary values at a sufficiently large order to get the real parts. Then the imaginary parts are generated by upward recurrence starting with the closed-form values for orders zero and unity. The real parts are normalized so as to agree with their two lowest order closed-form values. This routine is similar in theory and development to subroutine HANK in ref. 2, except that that routine is for ordinary Hankel functions.

3.2.3.4 Subroutine LEG

This routine calculates the associated Legendre polynomials of order unity and various degrees in the calling sequence, X is the argument, cos θ; Y is sin θ; M is the number of degrees to be calculated; and the array P is the output Legendre functions. As in HANK, the function of degree n is stored in the (n + 1)th location of the array P.

The method uses upward recurrence starting with $P_o^1(x) = o$ (line 2390) and $P_1^1(x) = \sin\theta$ (line 2400). The recurrence relation is

$$n P_{n+1}'(x) = (2n+1) x P_n'(x) - (n+1) P_{n-1}'(x) \tag{120}$$

which is line 2460.

3.2.3.5 Subroutine PROP

This routine is identical in function and calling sequence to the routine of the same name in SLOC, which is described in 3.2.2.5. The listing differs from the former one only at lines 2590, 2860 and 2870. These differences occur when (80) and (81) are substituted in (116) instead of in (84). Then, instead of (86), the recurrence relation is

$$j(j+1)a_j + 2j(j-1)Ya_{j-1} + [(j-1)(j-2) - n(n+1)]Y^2 a_{j-2} + X^2Y^2(a_{j-2} + 2Ya_{j-3} + Y^2 a_{j-4}) = 0 \tag{121}$$

Therefore n^2 is replaced by $(n^2 + n)$, as in line 2590. The variable E at line 2860 represents the negative of the coefficient of a_{j-1} divided by the coefficient of a_j. The variable F at line 2870 is the same kind of ratio with respect to a_{j-2}.

```
LIST ABCD

      COMPLEX A,B,C,CC,D,RR,Y(3),YA,YAO
      DIMENSION P(3),R(3)
      NAMELIST/INPUT/P,R,Y,YAO,YO,YY
      DATA YO,YY/7.534,3.767/
    1 READ INPUT
      IF(R(3).LT.0.)GO TO 11
      DO 10 I=1,3
      E=R(I)
      G=P(I)/57.296
      F=E*COS(G)
      G=E*SIN(G)
   10 Y(I)=CMPLX(YO-YO*F,-YO*G)/CMPLX(1.+F,G)
      GO TO 12
   11 R(I)=CABS((YO-Y(I))/(YO+Y(I)))
   12 A=YY*(Y(3)-Y(2))/(Y(1)-Y(3))-YAO
      C=A*Y(1)+(Y(1)-Y(2))*YAO
      B=1./CSQRT(A*Y(2)-C)
      D=B*Y(2)
      A=A*B
      C=C*B
      PRINT 100,A,B,C,D
      F=4.343*ALOG(1.-R(1)**2)
      PRINT 200,F
   13 READ:YA,TN
      IF(TN)20,1,15
   15 CC=YO*(A+B*YA)+C+D*YA
      RR=(YO*(A+B*YA)-C-D*YA)/CC
      F=CABS(RR)
      G=57.296*ATAN2(AIMAG(RR),REAL(RR))
      H=4.343*ALOG(1.-REAL(RR*CONJG(RR)))
      RR=CC/(YO*(A+B*YAO)+C+D*YAO)
      Q=4.343*ALOG(REAL(RR*CONJG(RR)))
      S=4.*REAL(YA)*YO/REAL(CC*CONJG(CC))
      T=S*TN
      PRINT 300,F,G,H,Q,T
      GO TO 13
   20 STOP
  100 FORMAT(1P2E12.4,3X,1P2E12.4/1P2E12.4,3X,1P2E12.4)
  200 FORMAT(F8.2)
  300 FORMAT(F6.3,F7.1,2F8.2,F9.1)
      END

*
```

Figure 9 Listing of Program ABCD

3.2.4 ABCD

Fig. 9 is a listing of this program, the objective of which is to transform the results of calculations for conditions at the aperture to conditions at the antenna input. The method assumes that the antenna can be represented as a linear two-port network and that sufficient experimental data exist to define the network parameters which relate voltages and currents at the input to electric and magnetic fields at the aperture.

See, for example, chapter 1 of reference 3 for a more detailed discussion of two-port networks. Such networks are describable in terms of the following:

$$Y = \frac{C + D Y_a}{A + B Y_a} \tag{122}$$

$$AD - BC = 1 \tag{123}$$

Where Y is the admittance at the input; Y_a is the aperture admittance; and A, B, C and D are complex constants which do not change when external effects change the values of Y and Y_a. Rather than attempt to calculate A, B, C and D from some physical model of the antenna, let us rely on experimental data to infer their values, assuming that our previous calculations of Y_a are correct. First we have the calculated aperture admittance in free space, Y_{a_0}, for which let the measured input admittance be Y_1. This gives us one equation in the four unknowns using (122). A second equation is already given by (123). A third is obtained if Y_2 is the input admittance which is measured when the aperture is covered tightly by a sheet of metal foil. Such shorting of the aperture implies $Y_a = \infty$ since the electric field must vanish. Then (122) gives $Y_2 = D/B$. One more equation is needed since there are four unknowns. To get this last equation, consider the effect of placing a thin resistance sheet over the aperture. Regardless of the assumed geometry of the aperture, its admittance is always calculated from a linear combination of wave admittances. The boundary conditions for a thin resistance sheet are that the tangential electric field is the same on either side and the tangential magnetic field has a discontinuity equal to the current in the sheet. This boundary condition implies that the wave admittance is equal to its value in the absence of the thin sheet plus the conductance of the sheet. In other words the sheet is in parallel with the aperture. Then, to a good approximation, the aperture admittance in the presence of the sheet is $Y_{a_0} + Y_s$, where Y_s represents the conductance of the sheet.

Now the three simultaneous equations, which supplement (123), are

$$Y_1 A + Y_1 Y_{a_0} B - C - Y_{a_0} D = 0 \tag{124}$$

$$Y_2 B - D = 0 \tag{125}$$

$$Y_3 A + Y_3 (Y_{a_0} + Y_s) B - C - (Y_{a_0} + Y_s) D = 0 \tag{126}$$

Substitute D from (125) in (124) and (126) and divide each equation by B, giving two simultaneous equations in A/B and C/B, the solution of which is

$$A/B = (Y_3 - Y_2)Y_5/(Y_1 - Y_3) - Y_{a_0} \qquad (127)$$

$$C/B = (A/B)Y_1 + (Y_1 - Y_2)Y_{a_0} \qquad (128)$$

Now divide both sides of (123) by B^2, using Y_2 for D/B, to give

$$B = 1/\sqrt{(A/B)Y_2 - C/B} \qquad (129)$$

This value of B then gives A and C from their ratios to B, which are already known, and $D = Y_2B$. In the program the names A and C are temporarily used for A/B and C/B at lines 150 and 160, which represent (127) and (128). Line 170 is (129) and the final values of A, C and D are derived at lines 180 - 200.

If a calibrated signal is input on a transmission line of characteristic admittance Y_o and the complex voltage reflection coefficient R is measured, the total voltage at the input is (1 + R) times the input voltage. Also, the total current at the input is Y_o (1 - R) times the input voltage and therefore the admittance Y at the input is

$$Y = Y_o\left(\frac{1 - R}{1 + R}\right) \qquad (130)$$

This forms an alternative way of prescribing input conditions for the above calculations of A, B, C and D parameters. Thus the complex voltage reflection coefficients for each of the three conditions can be used to specify Y_1, Y_2 and Y_3, given Y_o. This is done using (130) in lines 70 - 120 of the program. Of course inversion of (130) gives

$$R = \frac{Y_o - Y}{Y_o + Y} \qquad (131)$$

which is done at line 140 if the Y values were input. The dB loss by reflection in free space is calculated at line 220.

Now if, owing to the presence of plasma, the aperture admittance takes on some new value Y_a, then (122) gives the new value of input admittance Y and (131) gives the reflection coefficient at the input. This gives

$$R = \frac{Y_o(A + BY_a) - C - DY_a}{Y_o(A + BY_a) + C + DY_a} \qquad (132)$$

This equation is found at lines 260-270, and lines 280-300 calculate the amplitude, phase and dB loss corresponding to R.

Plasma effects on radiation patterns were calculated in SLOP, SLOC and SLOS with respect to the total electric field at the antenna aperture, E_o, which is itself affected by plasma. The matrix equation corresponding to (122) is

$$\begin{bmatrix} V \\ I \end{bmatrix} = F \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_o \\ H_o \end{bmatrix} \tag{133}$$

where V and I are the total voltage and current at the input port, E_o and H_o are the electric and magnetic fields at the aperture, and F is a scaler constant which converts the aperture fields to equivalent voltages and currents. Invert this equation, keeping (123) in mind, let $V = V_o(1+R)$ and $I = Y_oV_o(1-R)$, and use (132) for R. Then

$$E_o = \frac{2Y_oV_o/F}{Y_o(A+BY_a)+C+DY_a} \tag{134}$$

where V_o is the input signal voltage. Using this equation with/without plasma,

$$\frac{E_o^o}{E_o} = \frac{Y_o(A+BY_a)+C+DY_a}{Y_o(A+BY_{a_o})+C+DY_{a_o}} \tag{135}$$

This quantity is calculated at line 310 and in terms of dB loss at line 320.

Finally the noise power at the aperture needs to be transformed to the input. We calculate this effect by continuing the physical model of equating the emission to the absorption. That is, if the power input to the antenna were set equal to the noise temperature which is seen at the aperture, as calculated above, then the power at the aperture would equal the noise temperature which is seen at the antenna input port. In symbols, let the aperture noise temperature $T_a = \frac{1}{2} Y_o |V_o|^2$ while letting the noise temperature at the input port $T_n = \frac{1}{2} G_a |F E_o|^2$.

$$T_n = \frac{4G_aY_aT_a}{|Y_o(A+BY_a)+C+DY_a|^2} \tag{136}$$

This equation is applied at lines 330-340, where TN denotes T_a and T denotes T_n.

4. Conclusion

The models presented in this report are the lowest-order examples of more general theory for plasma-covered aperature antennas in canonical geometries. The possible generalizations are of various types. Let us discuss some of these here.

The infinite integrals for the aperture admittance and noise temperature induced by the plasma in the planar and cylindrical geometries are a subject for generalization in terms of numerical techniques. The techniques used here are best adapted to the situation in which the aperture is large. In particular the integral in (18) was evaluated by neglecting the variation of y (2x/a) within each lobe of the sin x/x function; i.e., Y (u) is assumed to vary little when u varies by $2\pi/a$, which can be true only when a is large. Obviously, this numerical technique is easily improved upon by evaluating the integral over each lobe of the transform of the aperture field distribution by the trapezoidal rule, for example. With little revision of the logic, one could even let the operator truncate these integrations as is now done with the sum over the lobes of sin x/x. Thus, in a given iteration of the trapezoidal approximation, we would add the effect of including all points midway between those already included, via a do-loop.

The assumed uniform field in the aperture could be changed to some other, fairly arbitrary, distribution. The only restriction which suggests itself is that the electric field parallel to the edge of the aperture must vanish at the edge. The general techniques used in this report would still apply. The near field effects (aperture admittance and plasma noise temperature) are gotten by invoking conservation of energy at the aperture between the assumed field distribution and the separation-of-variables-solutions of Maxwell's equations just outside the aperture. And the far field is gotten in relation to the assumed aperture electric field, ignoring matching to the magnetic field. An interesting example of an aperture field distribution is the case in which E_θ varies as cos ϕ in the slot on a sphere. Then the field does not vanish at $\theta = 0$ or π, as it does when the aperture field is uniform; but in general, the field depends on both θ and ϕ.

The present models are for one-dimensional apertures (infinitely long for planar geometry and circumferential for cylindrical and spherical geometries). It is possible to revise them to treat two-dimensional apertures. The simplest example would be a rectangular aperture in which the electric field is parallel to the short dimension and has a cosine distribution of amplitude along the long dimension. In the calculations of aperture admittance and noise temperature, the single integral or sum would be replaced by a double integral or sum. Far field effects would be calculated in only slightly different ways from the present models. For example in the planar case, the present model gives the relative effect of the plasma on the far field for directions which are in the plane of the short dimension of the aperture. All that needs to be added is the effect of plasma on the far field in directions for which the magnetic field lies in the plane of propagation. Circular apertures in a ground plane can be analyzed also. The relative far field is the same as for a rectangular aperture in terms of the polarization of the antenna relative to the plane of the line of sight. The near field effects for a circular aperture require reformulation in terms of the transforms of the aperture field distributions.

It is possible to generalize these models to include more than one antenna aperture. The theory is quite similar to the above, except that the field distribution at the body surface is that for the array instead of for a single aperture.

The transform of this field distribution is slightly more complicated, and the admittance is a symmetric matrix rather than a scalar. If the same plasma profile covers all elements of the array, the admittance matrix comes from a straightforward calculation and far field effects for each aperture are no different than they would be for a single aperture. Additional pattern distortion may be induced by the plasma effect on the admittance matrix, and such effects could be calculated using the concept of an n-port network, for example. An array has the capability to produce a narrow beam, which will be distorted by the plasma even if the plasma is uniform over all elements of the array. This effect comes about because the far field loss induced by the plasma is a strong function of angle of incidence. Thus the parts of the beam nearer the normal are less attenuated by plasma than are those farther from the normal, which tends to pull the beam toward the normal. This effect can be calculated either by modeling the beam shape and applying the attenuation by plasma at a number of closely spaced angles in the beam or by analytically relating the beam pointing error to the rate of change of attenuation with incidence angle.

The case in which the plasma varies from one aperture to another can not be calculated directly by these canonical models since their essential feature is the assumption that the plasma varies only in the normal direction. However, one can estimate the effects of such transverse gradients by treating each aperture as a separate canonical problem. The far field effects can be derived by considering the effect of differing plasmas on the coherent addition of all elements in forming the beam. Such a calculation can be done by analytical approximation in terms of the rate of variation of the effects of plasma on amplitude and phase versus distance across the array. The near field effects can be approximated by letting the mutual admittance between two given elements be equal to the average value obtained when both are covered by either plasma. All of these effects can be calculated for the case when the plasma has random fluctuations among array elements as well as when the variations are smooth.

Some of these model improvements already exist in another, much older, computer program which has never been documented formally. However, the logic and numerical techniques in that program are cumbersome, so we intend to incorporate such improvements, when needed, in the present programs.

5. References

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This report documents a set of computer programs for plasma effects on one-dimensional slot antennas in planar, cylindrical and spherical surfaces. The plasma in each case is assumed to have electron density variation only in the normal direction. The programs calculate radiation pattern, input impedance and noise temperature as generated by the plasma. Various generalizations, which can be calculated by minor changes to the programs, are described.

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