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RELIABILITY GROWTH PROJECTION

U.S. ARMY MISSILE RESEARCH AND DEVELOPMENT COMMAND

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PREFACE

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During the development of a system, if good engineering and management practices are followed, the reliability will grow as the program progresses. Advantage is taken of this fact in reliability growth management, wherein expected growth is projected and then compared with actual growth, thus allowing an assessment of reliability progress throughout the development program. This management process is thoroughly

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tito estantacol procedure resolts in an otrostanta factor (2) revisio etaco recipitor esta constructivo da 15 of described in DARCOM-P 702-4 entitled Reliability Growth Management. Reliability growth is a complex process and it is extremely difficult to project for a new program. This report presents two general procedures for projecting such growth. Although it is recognized that the specific procedures used will have to be tailored to the particular situations, it is hoped that a certain amount of uniformity will result.

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1. INTRODUCTION

The purpose of this report is to provide guidelines for the preparation of reliability growth projections. Of course, a purely judgemental approach can be followed, but only as a last resort. It is considered more desirable to channel or control judgement by using actual growth data as a reference. Two general approaches are available - non-parametric and parametric. The non-parametric approach presented in Paragraph 2 was developed for application to missiles, since there is uncertainty concerning the validity of the parametric approach for one-shot items. The procedure involves adjustment of the growth curve of a similar system and program by considering differences in the design and program. Although this method was developed for missiles, it should be equally applicable to ground equipment. The parametric approach recommended in Paragraph 3 is based upon the Duane model. Estimates are made concerning the two parameters or constants in the Duane equation by the use of test data, data from other programs, or assumed typical values. This method which was developed for the time-dependent case has a possible application to missiles as indicated in the procedure. It is also recognized that other parametric models could possibly apply, but at present the Duane model is preferred.

2. NON-PARAMETRIC METHOD

A. Introduction

The usual methods for reliability growth projection are based on the Duane Model (Reference 1). However, this model deals with time to failure type of data and generally is not applicable to success-failure data (oneshot items) unless the number of trials is large. Since missile test programs usually involve relatively small quantities of test items, a different approach to growth projection is necessary. The following procedure, which was developed to meet this need. involves the adjustment of a known reliability growth curve for a similar system in accordance with differences in the hardware design and the development program.

B. Theory

The reliability at any time during the program is assumed to be exponentially related to the failure rate and flight time. Thus for the reference system, the reliability (R_0) at any point in the program is related to the failure rate (λ_0) and the missile flight time (T_0) as follows:

$$R_o = e^{-\lambda_o T_o}$$

The estimation procedure results in an adjustment factor (K) which when multiplied with $\lambda_0 T_0$ yields the λT of

the new system. Therefore, the reliability (R) of the new system at a particular point in the program is estimated as

$$\begin{array}{c} -\lambda T & -K\lambda_{o}T_{o} \\ R = e & = e \end{array}$$

Eliminating the term, $\lambda_0 T_0$, from the above two equations, we obtain the following ratio between the reliability of the new system and the reliability of the reference system at any time during the program:

$$K = R = e^{K + K \ln R}$$

Therefore, for each point (R_0) on the reliability growth curve of the reference system, we can apply the estimated adjustment factor (K) and obtain the corresponding reliability for the growth curve of the new system.

C. Procedure

(1) General. The proposed reliability growth projection procedure consists of three major steps:

- Step 1 Select reference system.
- Step 2 Estimate adjustment factors.
- Step 3 Compute growth curve.

In the first two steps, comparisons must be made of design features, such as type of hardware, design maturity, complexity of hardware, and flight time and program features such as program length, intensity of reliability program, test program, and contractor capability. Step 3 is accomplished by applying the formula of Paragraph B above.

(2) Step 1 - Select Reference System. In the selection of a suitable reference system, it is necessary to choose a system for which reliability growth data is available and whose design and program are reasonably similar to the new system. The closer the reference system compares with the new system, the better the growth estimate will be. Conversely, as the systems diverge, our ability to estimate the numerical effect of differences deteriorates rapidly.

(3) Step 2 - Estimate Adjustment Factors.

(a) Individual Adjustment Factors. Adjustment factors are estimated by comparing the two systems for each of the design and program features indicated in Paragraph (1) above. Individual factors are estimated for each feature or subfeature and represent a judgement of the effects of differences between the new system and the reference system expressed as the multiplier of the reference failure rate. If a lower failure rate is expected, the adjustment factor will be less than one. If a higher failure rate is expected, the factor will be greater than one. No effect will be represented by a factor of one. These

factors may be different for different times in the program.

(b) Features Considered.

1) Types of Hardware - This factor refers to differences such as mechanical vs. electronic vs. optical, etc.

2) Design Maturity - This factor includes any state-of-the-art components which will cause a lag in the growth curve relative to the reference system.

3) Complexity of Hardware - If failure rate predictions are available for both systems, this factor may be set equal to the ratio of the new to the reference. Otherwise, one will have to examine differing design features and make a judgement of the effect on system failure rate.

4) Flight Time - The adjustment factor for this feature should be the ratio of the new time to the reference time. However, the reference system should be chosen to minimize this difference.

5) Program Length - The reference system should be chosen so that the new program will cover approximately the same time period.

6) Intensity of Reliability Program - Special features such as safety margin test programs and contractor emphasis on reliability should be considered here.

7) Test Program - The achievement of growth from testing is entirely dependent upon the failure analysis and corrective action system. The amount and comprehensiveness of system testing is known to have a direct effect on reliability growth achieved. If the new system involves less system testing in the same time period, the rate of growth will be reduced. Extent of lower level testing can also have a significant effect on growth and should be considered.

8) Contractor Capability -Unfamiliarity of the contractor with Army systems, known slow corrective action system, or other features that tend to reduce growth rate should be considered.

9) Other - Of course, any other features which can affect reliability or reliability growth should be considered.

(c) Overall Adjustment Factor. This factor will be the product of all the individual adjustment factors.

(4) Step 3 - Compute Growth Curve. For selected points in the program, use the adjustment factors for these points determined as described above and reliability values for the reference system to determine the reliability growth curve of the new system from the formula given in Paragraph B.

D. Example

SYSTEM X	REFERENCE System a	ESTIMATED ADJUSTMENT FACTOR	and the second se
One Propulsion System	Two Propulsion Systems	K ₁ = .25	
Requires Complex Seeker	Target Located by Operator	K ₂ = 3.0	
Planning Design Margin Test Program	No Design Margin Program	K ₃ = .75	
Severely Limited Test Program	Good Test Program	K ₄ = 1.5	

The overall adjustment factor is then

 $K = K_1 K_2 K_3 K_4 = (.25)(3.0)(.75)$ (1.5) = .84

If the reliability of System A is R_0 =.70 at time T_0 , the corresponding reliability of System X at time T_0 is then calculated to be

 $K = e^{K \ln R} = e^{.84 \ln .70} = .741$

It is emphasized that the K-factors given above are purely hypothetical and are not to be taken as typical for the differences described.

3. PARAMETRIC METHOD

A. Introduction

In 1962, J. T. Duane (Reference 1) of General Electric proposed the use of the learning curve, $y = Ax^{B}$, to fit reliability growth data. His basic equation was

$$\overline{\lambda} = \frac{f}{T} = KT^{-\alpha}$$

where $\bar{\lambda}$ = cumulative failure rate

f = cumulative number of failures

T =cumulative test hours

 α =growth rate constant

K =constant.

He reported good fits to data for five divergent groups of products with growth rates close to 0.5. In 1970, Selby and Miller (Reference 2) of General Electric developed a reliability planning and management (RPM) procedure using the Duane model adapted to MTBF with a starting point of 10% of the predicted value and a growth rate of 0.5 for a vigorous reliability program.

In 1972, Dr. Larry H. Crow (Reference 3) of AMSAA proposed maximum likelihood curve-fitting procedures for the Duane model and further developed these procedures in 1974 (References 4, 5, 6, 7). In 1974, E. F. Belbot (Reference 8) of AMSAA published a computer program to implement this procedure. This method requires a substantial data base in order to make reasonable projections.

The U. S. Navy has adopted the Selby and Miller approach and has published an extensive coverage in NAVORD OD 44622 dated 31 December 1975 (Reference 9). They have determined an average cumulative MTBF starting point of 10% of the predicted value and recommend a growth rate of 0.5 for a good development program.

B. Theory

The basic Duane equation for cumulative failure rate as given in Paragraph A is

$$\bar{\lambda} = \frac{f}{T} = \kappa T^{-\alpha}$$
(1)

This equation can be converted to cumulative MTBF by inverting the terms of equation (1) as follows:

$$\overline{\text{MTBF}} = \frac{T}{f} = \left(\frac{1}{K}\right) T^{\alpha} = K' T^{\alpha}$$
(2)

The proportionality constant, K', can be determined by solving equation (2) using the initial conditions, $\overline{\text{MTBF}}_{0}$ and T_{0} . Equation then becomes:

$$\overline{\text{MTBF}} = \overline{\text{MTBF}}_{o} (T/T_{o})^{\alpha}$$
(3)

The above equations can be changed to instantaneous (current) failure rates and MTBF's by taking the derivative of equation (1) as follows:

$$\lambda = \frac{df}{dT} = \frac{d(KT^{1-\alpha})}{dT}$$

$$= K(1-\alpha)T^{-\alpha} = \overline{\lambda}(1-\alpha)$$
(4)

Instantaneous MTBF is then obtained from the reciprocal of the instantaneous failure rate assuming the system follows the exponential distribution at any point in time:

$$MTBF = 1/\lambda = 1/(\overline{\lambda}(1-\alpha)) = \frac{1}{MTBF}/(1-\alpha)$$
(5)

This equation can be put in a more convenient form by combining with equation (3) as follows:

$$MTBF = \frac{\overline{MTBF}_{o}}{(1-\alpha)} (T/T_{o})^{\alpha}$$

(6)

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If the initial MTBF estimated is instantaneous rather than cumulative, Equation 6 can be written as follows using relation (5):

 $MTBF = MTBF_{o} (T/T_{o})^{\alpha}$ (7)

C. Procedure

(1) General. The Duane model is a two-parameter function and therefore it is necessary to provide two estimates to determine the growth curve. These are the initial MTBF and the growth rate. This model may then be used to estimate the test time required to achieve a specified reliability goal or, as is usually the case, to project the reliability growth for a given test program. To establish the growth curve on a linear calendar time basis for management purposes, it is then necessary to correlate the Duane curve, which is on a test time basis, with the test schedule.

(2) Estimation of Initial MTBF. Three methods are available for determining the initial MTBF of a system:

(a) Use Test Data. If sufficient test data is available or can be generated, the initial cumulative MTBF is obtained by dividing the test time by the number of failures:

 $\overline{\text{MTBF}}_{\text{O}} = T_{\text{O}}/f_{\text{O}}$

NAVORD OD 44622 recommends that at least five failures should be generated.

(b) Use Historical Data. If test data is not available, the next best approach is to utilize information concerning the initial MTBF of similar systems. The initial MTBF of a similar system should be corrected for difference in complexity by multiplying by the ratio of the predicted MTBF of the new system to the predicted MTBF of the similar system. If predictions are not available, the ratio should be estimated by the use of engineering judgement.

(c) Assume Average Value. In the absence of test data or information from a similar system, the only recourse remaining is to assume the initial instantaneous MTBF to be approximately 20% of the predicted MTBF (Handbook prediction). This value represents the average of a number of differing devices (see Reference 9). The cumulative test time (T_0) corresponding to this estimate of initial MTBF (MTBF₀) should be assumed equal to one predicted MTBF. The effect of this parameter on the growth curve is shown in Figure 1.

(3) Estimation of Growth Rate. The reliability growth rate is determined by the effectiveness of testing and data feedback and by the



quality of engineering and management support. For a vigorous program, such as is generally the case in the defense industry, a growth rate of $\alpha = 0.5$ is typical and may be assumed. If one has information contrary to the above, such as lower growth rates of similar programs or knowledge of lack of contractor ability or emphasis, then a lower growth rate should be estimated. The effect of this parameter on the growth curve is shown in Figure 2.

(4) Computation of Growth Curve. One obtains MTBF as a function of test time by substituting the initial MTBF, initial test time, and reliability growth rate estimates into Equation (6) or Equation (7), as appropriate, and solving the equation for various test times. The value of cumulative test time (and therefore instantaneous MTBF) is then determined for selected points in calendar time by referring to the test schedule.

(5) Application to Non-timedependent data. If the data obtained is success - failure data of a series of trials of approximately the same time span, and if the time per trial is small relative to the MTBF, the above procedure and equations can be applied by substituting the cumulative number of trials (N) for the cumulative test time (T). The instantaneous failure rates or MCBF's obtained can be converted to reliability (probability of success) values by applying the exponential formula,

$$R = e^{-\lambda} = e^{-1/MCBF}$$

If the underlying process is not strictly time related, then the proper estimator of instantaneous reliability is the binomial estimator,

$$R = \frac{ds}{dN} = 1 - \frac{df}{dN} = 1 - \lambda = 1 - \frac{1}{MCBF}$$

where s is the cumulative number of successes. This estimator is the one employed in the AMSAA model for missile applications. The above estimators are approximately equal above a reliability of .90.

D. Example

Consider a system with a predicted MTBF of 100 hours. In the absence of information about the initial conditions and the growth rate, we assume

$$MTBF_{o} = 100 \times 0.20 = 20 \text{ Hours}$$

 $T_{o} = 100 \text{ HOURS}$
 $\alpha = 0.5$

These values are then substituted into Equation (7) as follows:

$$MTBF = MTBF_{0}(T/T_{0})^{\alpha}$$
$$= 20 (T/100)^{0.5}$$
$$= 2T^{0.5}$$



Points in the growth curve can then be computed as shown in the following table, which assumes a constant test rate of 100 hours/month:

MONTHS CALENDAR TIME	HOURS TEST TIME	HOURS INSTAN- TANEOUS MTBF
ban public	100	20.0
5	500	44.8
10	1000	63.2
15 .	1500	77.4
20	2000	89.4
25	2500	100.0

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