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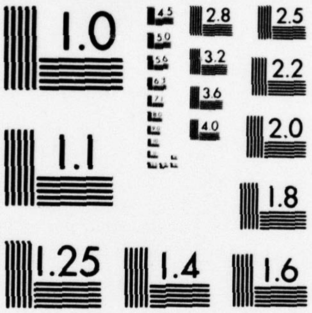
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LEVEL III

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MISCELLANEOUS PAPER H-77-9

A NUMERICAL HINDCAST MODEL FOR WAVE SPECTRA ON WATER BODIES WITH IRREGULAR SHORELINE GEOMETRY

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Report 2

MODEL VERIFICATION WITH OBSERVED WAVE DATA

by

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Hydraulics Laboratory

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P. O. Box 631, Vicksburg, Miss. 39180

December 1978

Report 2 of a Series

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The procedures used to verify a numerical model for wave generation in water bodies of arbitrary shape are described. The verification results for Lakes Erie, Ontario, and Superior are presented. Parameters included in the study include significant wave height and frequency at the spectral peak. Examples of hindcast and measured one-dimensional (frequency) spectra are given. The study indicates a capability of the model to hindcast significant

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20. ABSTRACT (Continued)

wave height such as root-mean-square error of about 1.5 ft and to hindcast the period of peak frequency with a root-mean-square error of 1 sec. These errors include effects from both the wave model and the hindcast model used to estimate the windfield.

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PREFACE

A request for the U. S. Army Engineer Waterways Experiment Station (WES) to conduct an investigation of wave heights on the Great Lakes was made by the U. S. Army Engineer Division, North Central (NCD), in a conference held in Chicago, Illinois, on 22 July 1974. Funds were authorized by NCD on 30 August 1974. The study was conducted during the period from September 1974 to June 1978 in the Coastal Branch, Wave Dynamics Division, Hydraulics Laboratory, WES, under the direction of Mr. H. B. Simmons, Chief of the Hydraulics Laboratory, and Dr. R. W. Whalin, Chief of the Wave Dynamics Division.

Drs. D. T. Resio and C. L. Vincent conducted the study and also prepared the report. Mrs. Rebecca Brooks and Mr. W. D. Corson were especially helpful in performing analytical and programming tasks.

A special acknowledgment is due to Drs. D. Lee Harris and E. F. Thompson (Coastal Engineering Research Center) for their review and constructive comments on the text of this report.

Directors of WES during the conduct of the study and the preparation and publication of this report were COL G. H. Hilt, CE, and COL J. L. Cannon, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) AND METRIC (SI) TO U. S. CUSTOMARY UNITS OF MEASUREMENT

Units of measurement used in this report can be converted as follows:

Multiply	By	To Obtain
U. S. Customary to Metric (SI)		
feet	0.3048	metres
miles (U. S. statute)	1.609344	kilometres
miles per hour	1.609344	kilometres per hour
knots (international)	0.5144444	metres per second
square feet	0.09290304	square metres
square feet per second	0.09290304	square metres per second
feet per second	0.3048	metres per second
Metric (SI) to U. S. Customary		
centimetres	0.3937007	inches
metres	3.280839	feet
square metres per second	10.76391	square feet per second
Celsius degrees or Kelvins	9/5	Fahrenheit degrees*

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* To obtain Fahrenheit (F) temperature readings from Celsius (C) readings, use the following formula: F = 9/5(C) + 32. To obtain Fahrenheit readings from Kelvins (K), use: F = 9/5(K - 273.15) + 32.

A NUMERICAL HINDCAST MODEL FOR WAVE SPECTRA ON WATER BODIES WITH IRREGULAR SHORELINE GEOMETRY

MODEL VERIFICATION WITH OBSERVED WAVE DATA

PART I: INTRODUCTION

1. The wave information used to estimate design wave climates for the Great Lakes was produced with a numerical wave hindcast model based on the calculation of a directional spectrum. The theoretical basis for the model and the numerical techniques used to solve the radiative transfer equation were discussed in the first report of this series.[1] The techniques for estimating a wind field over a lake have likewise been published,[2] and general overviews of the methodology to produce the design wave climate for each lake have been published in a series of technical reports.[3,4,5,6,7] These technical reports also contain a summary of the design wave climate for each lake.

2. In Reference 1 the hindcast model was shown to reproduce the growth of wave height with fetch in accordance with current field evidence. Further, the growth of wave height with time in the wave model was also in the range of other wave models and empirical evidence. The ability of the hindcast model to reproduce adequately both fetch-limited and duration-limited wave growth provides a measure of confidence that the model will produce realistic wave-height estimates, particularly under the extreme meteorological conditions. However, it is desirable to show that the model produces reliable wave estimates for time-varying and spatially inhomogeneous wind fields.

3. Two approaches are available for a verification of a hindcast model. The first constitutes a detailed field study on one or more of the lakes including the collection of wind, air, and sea surface temperature and wave data. Such a verification study was not performed because it is costly and can only be performed for a narrow range of environmental conditions. A second approach was selected involving the assembly of available wave gage data and the use of the model to

hindcast wave estimates for comparison with the gage records. Verification of the model in this manner is less costly and provides an estimate of the error for the entire hindcast methodology. One drawback to this approach is the inability to assign proportions of the error to either wave or wind model. However, it is the total error that is of final interest in evaluating the quality of the hindcast wave climatology. The model can also be tested under a wider range of environmental conditions that would normally be possible in a short field study.

4. Three sets of wave data were used in the verification study (Table 1). One set was collected by the Canadian Department of the Environment[8] and consists of wave spectra collected on Lakes Erie and Ontario. A second set was collected on Lake Superior by a Canadian group headed by Ploeg.[9] A third set of wave data constituted an initially double blind experiment between model hindcasts at the U. S. Army Engineer Waterways Experiment Station (WES) and wave gage records gathered by the U. S. Army Coastal Engineering Research Center (CERC) for Lakes Erie and Michigan. Model verification will cover all of the Great Lakes except Lake Huron, where no deepwater gage data were available.

5. The procedure used to verify the model is as follows. For each set of wave gage records, the wind data at the weather stations used in the actual hindcasts are collected and transformed precisely as done in the hindcast studies. The resulting wind data are used to generate the spatial wind field over the lake and to drive the numerical wave model. The calculated significant wave heights and spectra are then plotted against the gage data and appropriate statistics calculated.

6. The goal of the model development portions of the wave information study was to incorporate, according to the current state of the art, the physics of air-sea interaction. To this end, the model programmed was treated as a universal model in that all coefficients in the wave model are preestablished constants suggested in the literature and were never adjusted on a lake-by-lake basis. The computer runs shown here do not represent attempts at a calibration

or adjustment of the model to each lake. All adjustments of the wind relations in the wind model were made independent of the wave model, prior to the verification runs.

PART II: MODEL DATA REQUIREMENTS

7. The numerical model requires two basic inputs: a bathymetric grid and a time-dependent wind field corresponding to the same mesh as the bathymetric grid. The bathymetric grid is primarily used to describe the geometry of the water body. Since refraction and shoaling are not considered in this model the depths are ignored. The spatial grids used are shown in Figure 1.

Wind Speed Estimates

8. The time-dependent wind field is produced for the model verification using the same procedures for the wave hindcasts. For each prototype hour of calculation, winds from a series of weather stations around a lake are transformed to produce over the lake wind estimates using a transformation method that accounts for velocity and stability dependences in the relation between land and lake winds. These methods are described briefly below.

9. Since long records of historical wind data are available at land sites only, the hindcast study on the Great Lakes was oriented toward the estimation of extremes. An objective method of transforming land winds to lake winds was required. For the hindcast studies, a method of transforming land winds to lake winds based on a two-layer model of the atmosphere was developed. It is described in great detail in Reference 2. In brief summary, from both theoretical and empirical approaches, it is possible to show that the ratio R, of lake-wind speed (U_w) to land-wind speed (U_ℓ),

$$R = U_w/U_\ell$$

can be estimated by a product of two empirical functions A and B

$$R = A(U_\ell) \cdot B(\Delta T)$$

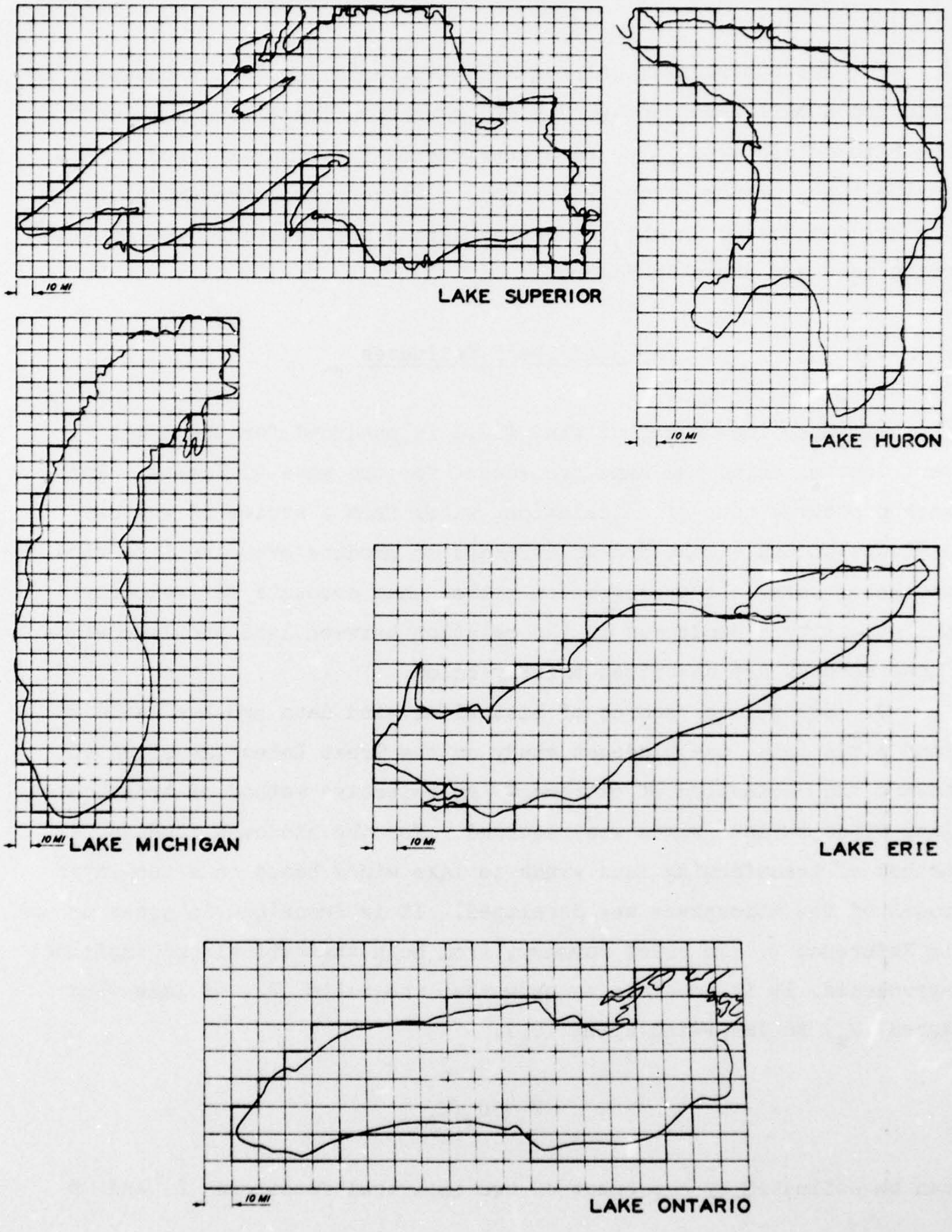


Figure 1. Model grids. The grids used for wave computations have a spacing of 10 miles. Only the mesh intersections overlying a lake are used for calculations

where $A(U_\ell)$ relates R to land-wind speed and $B(\Delta T)$ relates R to the air-lake temperature difference (ΔT). A modification for fetch to account for nearshore development of the boundary layer can also be added.[2] The forms of A and B used were evaluated empirically from over 100,000 observations. An example of the forms of the functions is given in Figure 2. The root-mean-square (RMS) error of estimated wind speed over the lake appears to be less than 5 knots, particularly at high wind speeds (Figure 3). This formulation holds only after full development of a marine boundary layer occurs, normally 10 to 30 miles offshore. The function A is site-dependent. Where different anemometer heights are used at a weather station, the data are converted to a 20-ft level. As a conservative measure, this wind-speed relation was applied everywhere over the lake through a Platzman-type interpolation scheme.[10]

10. In order to illustrate the spatial and temporal variability of the wind field, a set of plots showing the passage of a storm across Lake Superior has been prepared (Figure 4). The wind direction and speed are plotted. This storm illustrates the variability of storm winds over a lake and show how variable fetch lengths can be.

Air-Lake Temperature Differences

11. Prior to 1960, reliable estimates of air-lake temperature are not generally available; and even after 1960, estimates are not always available everyday. Since proper estimation of the atmospheric stability is a prerequisite to adequate wind speed estimation and since the eventual product of the hindcast project is statistical summaries of the wave climate, the mean air-lake temperature difference by month and 10° wind direction class appeared a reasonable approximation. The effect of using statistical estimates of air-lake temperature dependence is discussed in Reference 2 and appears to be minor. The use of statistical estimates of stability is a potential source of error, however, in a verification comparison.

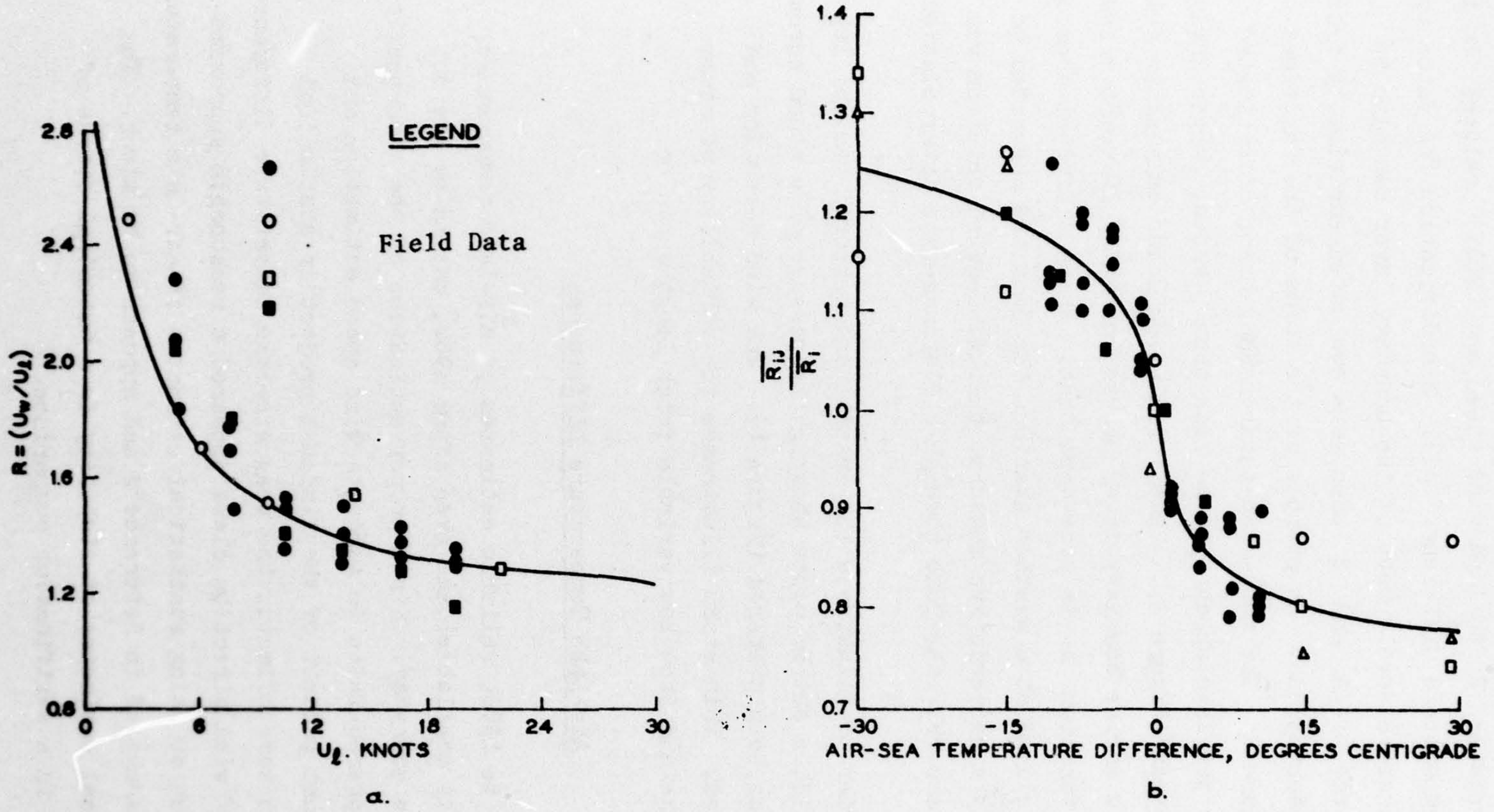


Figure 2. Wind hindcast curves. Estimation of the wind over a lake from wind over land requires a velocity dependent function (A) whose form is given in a. above. The values must be adjusted for atmospheric stability by a function (B) shown in b. above. The field data and theoretical development for functions A and B are discussed in Reference 2

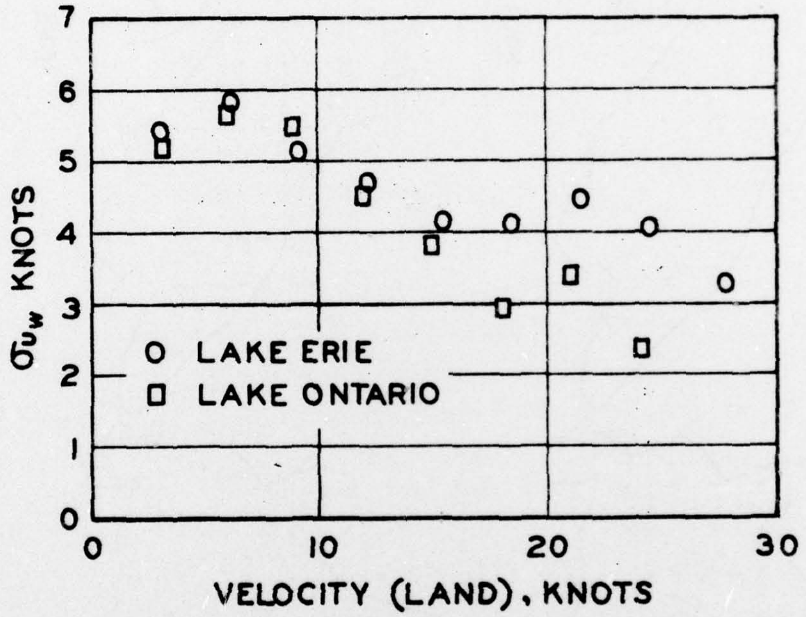
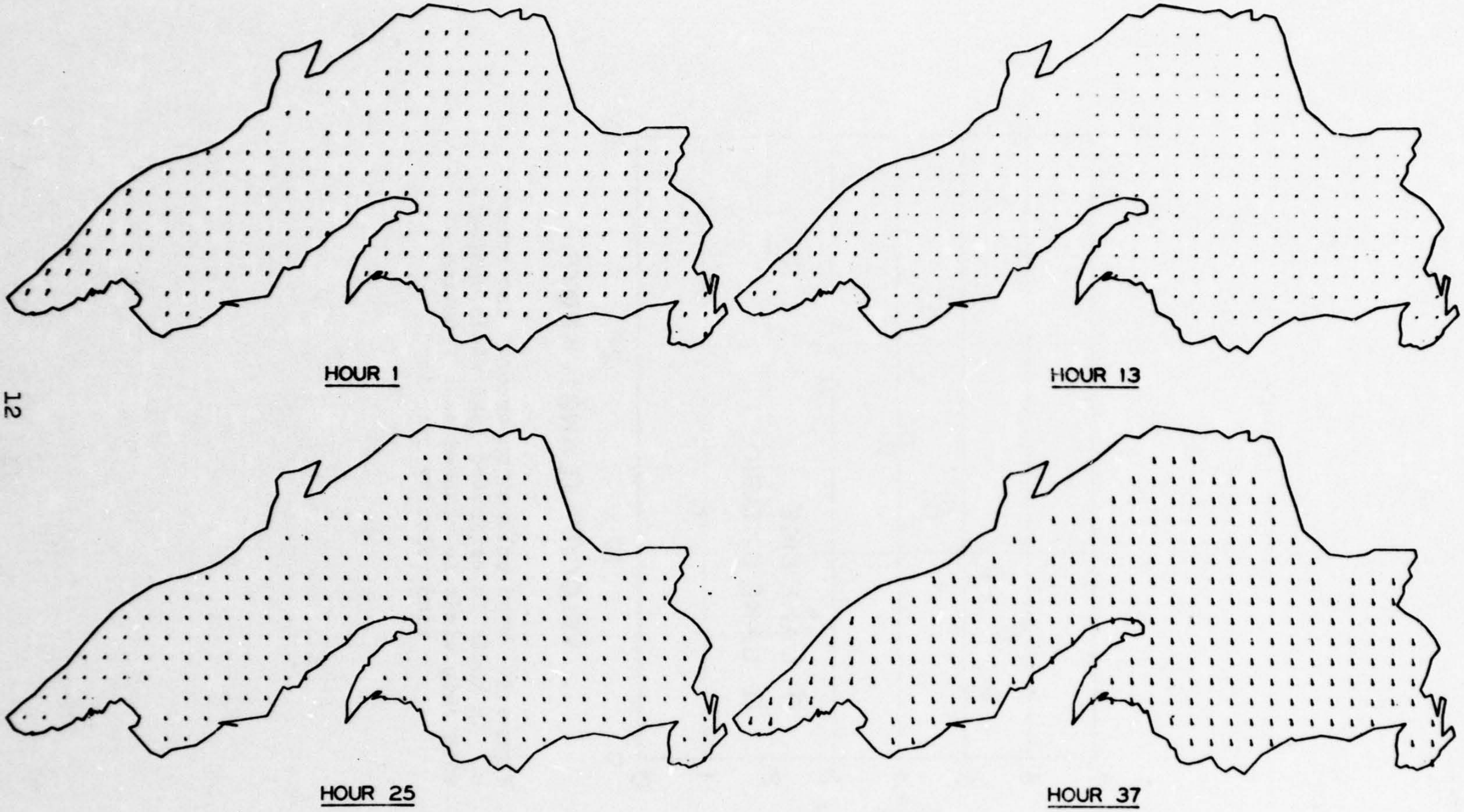


Figure 3. Wind estimation error. The error σ_μ in knots is estimated lake winds compared with ship winds is plotted as a function of wind velocity over land



HOUR 1

HOUR 13

HOUR 25

HOUR 37

Figure 4. Lake Superior wind. The variation in wind speed and direction for differing hours during a typical storm on Lake Superior are plotted

Wave Model Setup

12. The wave model for Lakes Erie, Ontario, Michigan, and Superior was set up with a 10-mile grid. Hourly wind-field information was input, and the radiation transfer equation solved at a time step of 15 min. The frequencies used in the calculations are given in Table 2. The directional increment used was 30°. The frequencies were those used in the hindcast study and were not changed for this study. The model is structured so that in the frequency range above the highest frequency input (in this case 0.23 Hz) the spectral computations are treated in a parametric sense. For other computations, a different frequency set may be more appropriate, in which case a new frequency matrix must be input. The model will accept a variable-spaced frequency interval.

PART III: POTENTIAL PROBLEMS IN COMPARISON OF GAGE AND HINDCAST DATA

13. The comparison of one-dimensional spectrum and significant wave-height data collected at a nondirectional wave gage to similar products estimated through a numerical hindcast model would appear straightforward. If the gage location is in deep water on a wide lake and several miles offshore, the process is fairly simple; however, if the gage is located near the shore, several problems can arise in comparison of wave records resulting from factors other than shallow water.

14. Reference to a schematization of the type grid used in the Great Lakes wave hindcasts (Figure 5) can illustrate the problems. The lake grids used were designed so that calculation points would be placed as close as practical to the U. S. shoreline; these points are labeled A, C, D, E, and F in Figure 5. As in the hindcast study, the spacing and orientation of the grid were chosen to provide a good approximation of fetch distances across the lake and to allow stable integration of the radiative transfer equation.

15. It is evident on the schematization that wave records at the gage site shown in Figure 5 will likely be intermediate to estimates at grid points A and C for conditions when the wind is blowing from directions from NE to NW. If the wind direction is from a southerly direction, however, it is evident that the hindcast values may be far too high because the fetches used in the model (lines A-G, A-I, C-H, and C-A) are several times larger than the actual distance (line J-K). However, since large waves, which are the principal interest in the storm hindcasts, come from the other directions, this misestimation is of little concern. In a comparison of the wave history at the gage with that hindcast, this effect may create substantial error.

16. Another interesting variant of this problem arises when the wind blows parallel to the lake. For grid points like C, D, and E the lake as represented in the model is one grid size wider than in the prototype and as a result there is a fetch distortion. Further, in comparison of the gage data with the estimated data, the form of the

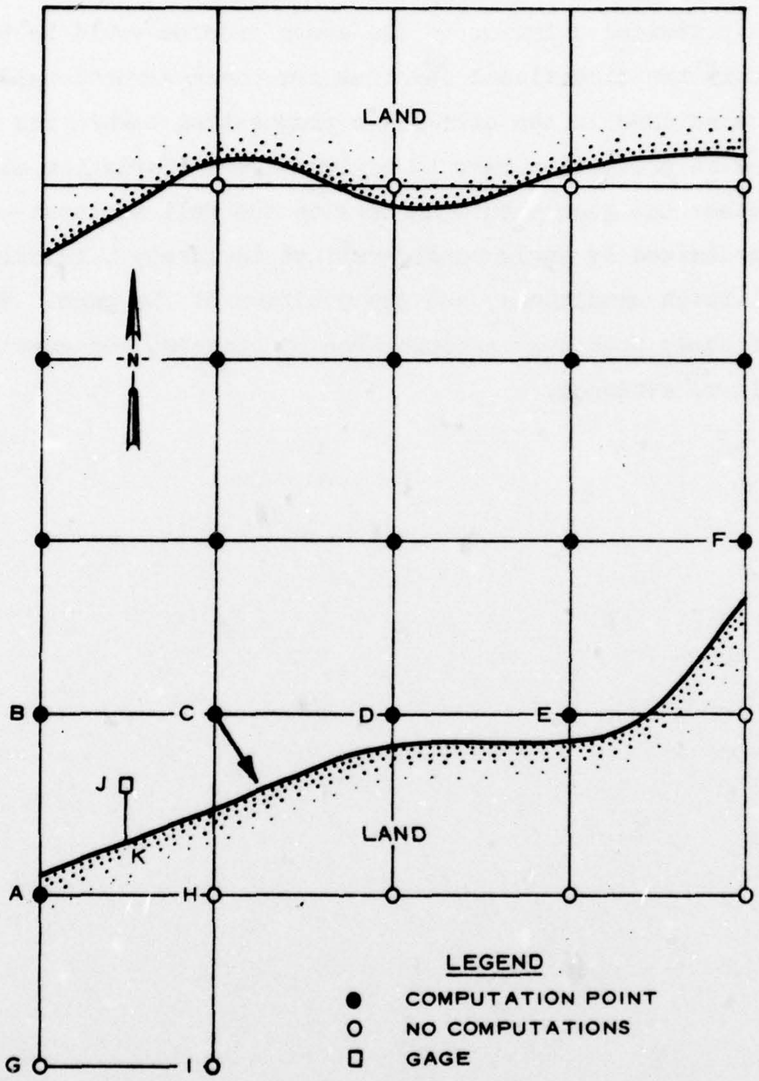


Figure 5. Computation grid scheme. The setup of a grid for a typical lake segment is shown illustrating points in the computation and those not. A gage located at J should have wave values intermediate to grid points A, B, and C. Points A, C, D, E, and F would be used to estimate waves at the shore. Points G, H, and I are on land. In the example, the grid is arranged for a presentation of the north to south fetch across the lake

gradient transverse to the mean wind direction for a wind parallel to a boundary is not known and has not been considered in any field studies.

17. A potential solution to the above problem would be to integrate only the directional spectrum for these directional spectra for these directions in the half-plane propagating toward the gage. This is done in production runs to estimate wave statistics at the shore. Whether the gage should agree with the full spectral estimate or with one limited by angle considerations is likely a function of wind, local fetch conditions, and the position of the gage. The severity of these problems in comparison of hindcast and gage data is difficult to estimate.

PART IV: VERIFICATION ON LAKES ONTARIO AND ERIE: CANADIAN WAVE DATA

18. The source of the wave data used for the initial model verification is the Canadian Department of the Environment. These data consist of a total of over 3000 wave spectra collected at four sites on Lakes Ontario and Erie during the period 1972 and 1973. Four gages are on Lake Ontario: Coburg, Main, Duck, and Toronto; one gage is on Lake Erie: Point Pelee. Spectral densities are provided in 62 frequencies bands of width 7.32×10^{-3} Hz from 0.05 Hz to 0.5 Hz. The quantity and quality of the Canadian data were of great benefit to this study.

19. Since the objective of the wave information study was to produce a design wave climate (signifying large waves), model verification runs were limited only to reproduction of storm conditions. These comparisons encompassed winds for all directions and for all ice-free seasons. In order to reproduce storm conditions, it is necessary to include some start-up time for the model; and as a consequence, many small wave conditions were hindcast. However, the wind model used is not specifically designed for weak wind conditions; and as a result, some error can be expected. For Lake Ontario verification runs, the only wind data used come from Rochester and Buffalo, New York. For Lake Erie verification runs, data from Buffalo, New York, Erie, Pennsylvania, Cleveland and Toledo, Ohio, are available. These stations are all on the U. S. side of the Lakes. Additional stations were not used because of a lack of a long period of record for the study hindcasts. It was desirable to test the entire methodology for error estimates rather than use a finer station network for model verification which would not be representative of the actual hindcast model runs.

Wave-Height Verification

20. The significant wave heights calculated from the Canadian

gages and estimated in the hindcast model have been plotted as a function of a time for the four gage sites (Figure 6, a-h) for a selection of storms. The storms plotted represent a mixture of best and worst agreement at the four sites. Two problems in these comparisons can be related to the sparse wind data available. First is the occurrence of shifts in times of peak waves observed versus predicted. It is expected that although storm wind fields are relatively homogeneous over large areas, regional gradients do exist. The gradients tend to propagate at storm speeds (normally 20 to 50 mph). As a result, one effect of having wind input stations on only one side of the lake, as is the case here, would be for hindcast peaks to be shifted in time with respect to the observed data. Examples of this are given in Figures 6a and 6b; for higher waves this phasing problem is not as evident (Figures 6c and 6f). This is because the higher waves are caused by high winds which tend to be more homogeneous over the lake. For high winds that are local in spatial extent and short in duration, the lakes do not have time to respond fully. A second problem is the presence of small-scale perturbations in the synoptic-scale wind field. Within large storms, there are mesoscale and smaller effects that are not picked up by the wind station grid. As a result, the hindcast can miss locally generated waves (Figure 6c). Likewise, if a pronounced small-scale perturbation is present over one station, the hindcasts should tend to overpredict (Figure 6d). This problem can only be solved with a finer resolution of the wind field. The effect appears most pronounced for low wave heights (6 ft or less).

21. In spite of the two problems noted, the hindcast model appears to satisfactorily simulate wave heights during storms. For the purpose of statistical summary of a large number of hindcasts for design analyses, it is evident that the larger waves are well represented, though perhaps shifted in time of occurrence. This will not affect the statistics or the results. The importance of the smaller scale wind effects tends to be limited to small waves. With simulation of several hundred storms, it is possible that these wind effects will be random and unbiased.

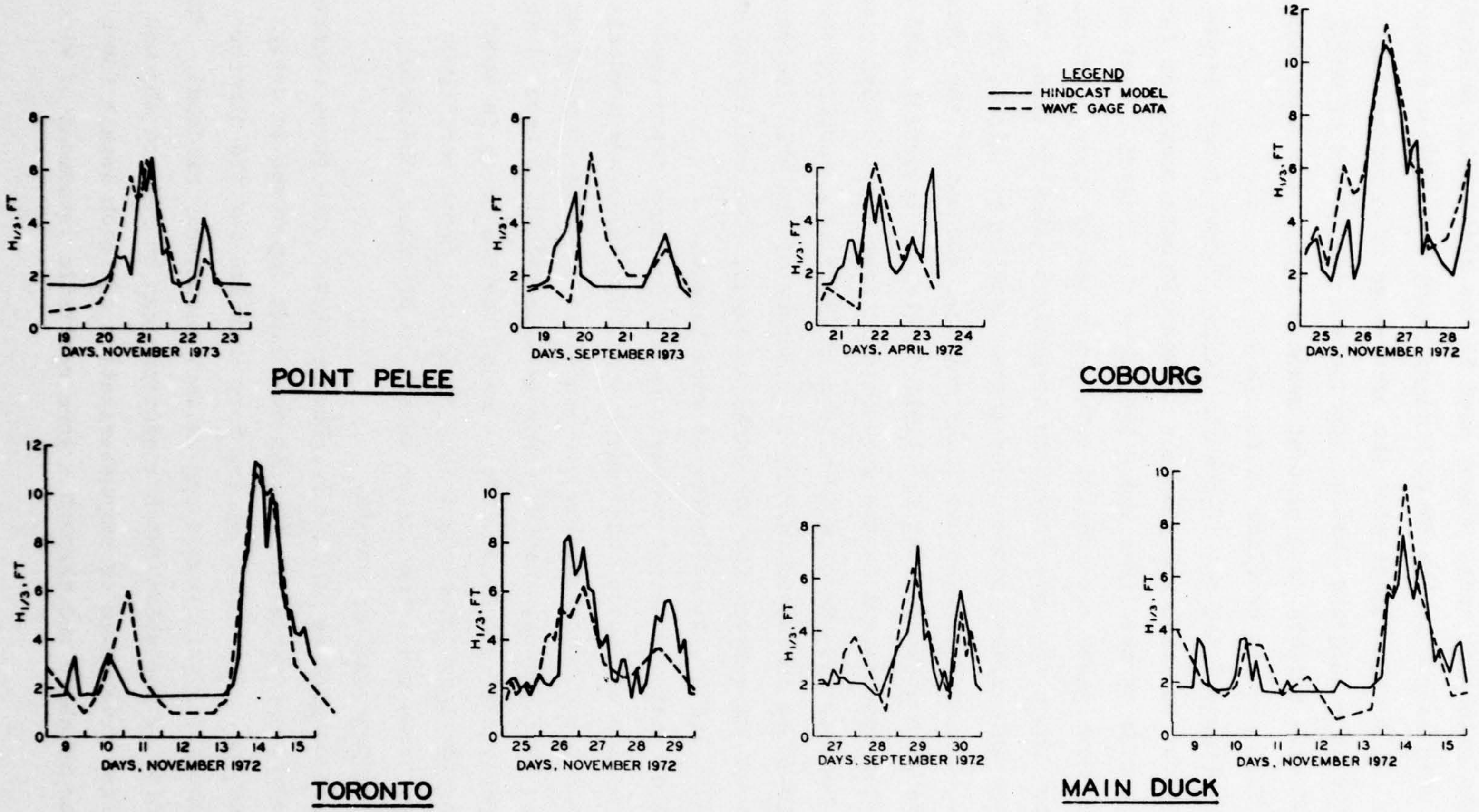


Figure 6. Wave hydrographs for Lakes Erie and Ontario. Predicted and measured significant wave height are plotted for selected dates and sites. Neither the best nor worst examples are given

22. The principal model output used in the statistical analysis of extreme wave height is the largest significant wave height observed in a storm. This requirement plus the occurrence of moderate time shifts in the data suggests that an appropriate comparison of model and observed wave heights is a plot of maximum significant wave height observed. This plot is provided in Figure 7.

23. In Figure 7 the set of results termed "short fetch" represents conditions in which winds have a marked offshore component for the Canadian side of the lakes where the gages are located. Most of the Canadian wave gages are located within 5 miles of shore. Hence, the overwater fetch for wave growth is normally 5 miles or less. Two factors enter into misestimates under these conditions. First, the atmospheric boundary layer is not fully marine. The use of the transformed winds without accounting for limited fetch is incorrect; and as a result, wind speeds used in the wave calculations are too high close to shore. Second, the use of a 10-mile grid leads to inaccurate calculation of fetch as discussed in PART II. Survey of the grid indicated a bias of too long a fetch for the Canadian gages. The results of both effects should lead to misestimates of wave heights.

24. The remaining set of comparisons are for long fetch conditions. These include those cases where the wind fields are predominantly from large storms, and the over-the-lake fetch is 20 miles or greater. In these cases, the RMS error in wave height is about 1 ft. When a comparison of all conditions is made, the RMS error is about 1.5 ft. For the long fetch conditions, which are those conditions in most lake areas where the larger waves are produced, the model predictions appear quite accurate.

25. The comparison of wave heights predicted with those observed indicates excellent agreement for the case most important to design wave hindcasting. The model appears very accurate for the stronger weather systems for which winds can be most accurately hindcast. The overprediction of the short fetch conditions can be reduced through use of a finer grid scale or parameterization of fetch length for grid points close to shore and through a more realistic treatment of winds

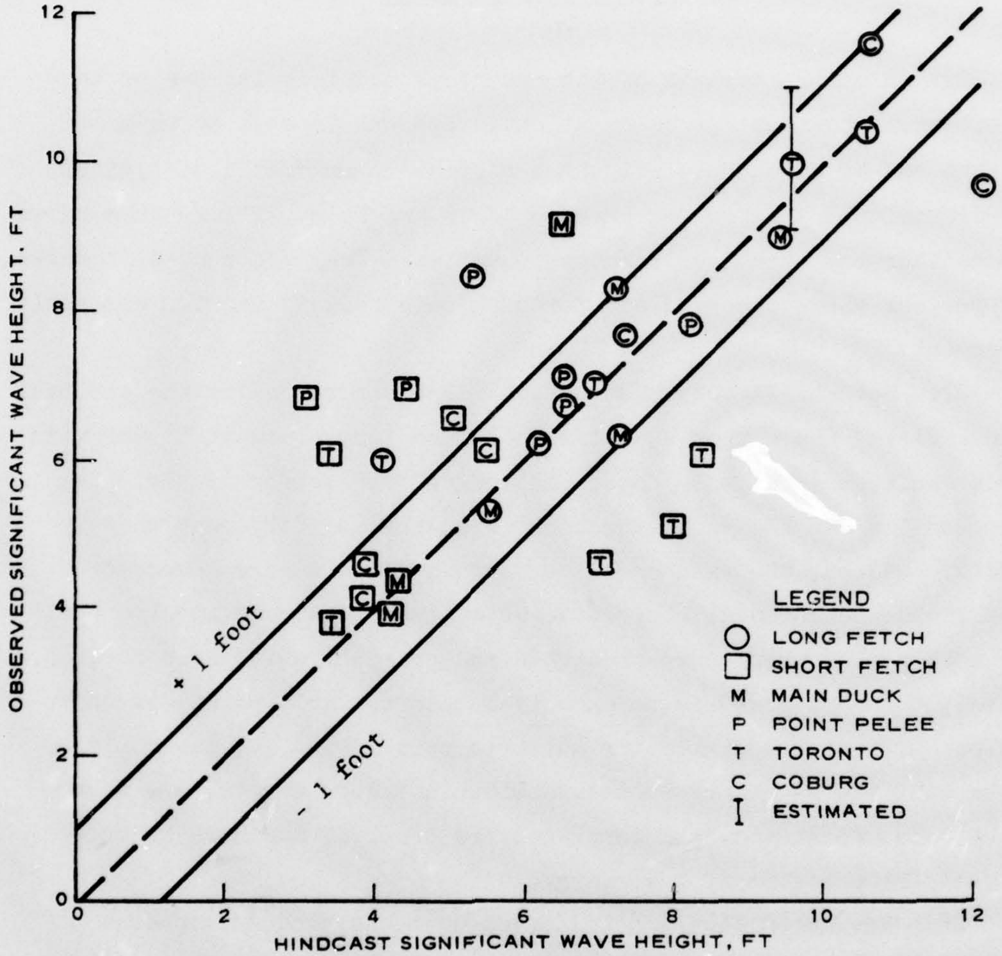


Figure 7. Scattergram of peak significant wave heights. The peak significant wave height hindcast for a storm is plotted against measured peak significant wave height. The values are annotated by site and fetch length. Short fetch comparisons indicate that the location of the gage was closer to shore than the first model grid point for an offshore blowing wind

near shore. Accomplishment of these changes would give a hindcast technique applicable to a range from daily to extreme wave conditions.

Spectral Comparisons

26. A major strength of the numerical model is its use of the directional spectrum for calculations. Spectra as well as wave heights can be an output. Figure 8 presents a selection of typical one-dimensional spectra for Lakes Ontario and Erie. The spectra shown are neither the worst nor the best examples. They are chosen, however, for periods where observed and forecast wave heights are in reasonable agreement.

27. Several factors must be considered in reviewing the spectral data. First, the use of a statistical technique to estimate the wind field has been previously noted to produce time shifts in the data. This can lead to shifts in the peak frequency and different energy levels. Second, the wind model tends to produce a more homogeneous wind field; hence local convective activity may produce spectra that are more peaked than those predicted and possibly shifted in frequence. Third, it is not possible to exactly locate the grid point for calculations with the gage site for multiple sets. Finally, it should be noted that the gages are in moderate but not deep waters, and as a result some refraction and depth-related breaking may have occurred to redistribute some of the energy.

28. Review of Figure 8 indicates that the hindcast spectra reasonably match the observed spectra. The gage spectra are more peaked than those hindcast. The Canadian gage-recorded spectra are presented as raw spectral estimates and according to the Canadian Department of the Environment should be filtered by a 10-point moving average to achieve 95 percent confidence. This would remove some of the peakedness and would tend to increase the energy densities on the forward faces of spectra which would increase the agreement between the measured and hindcast data.

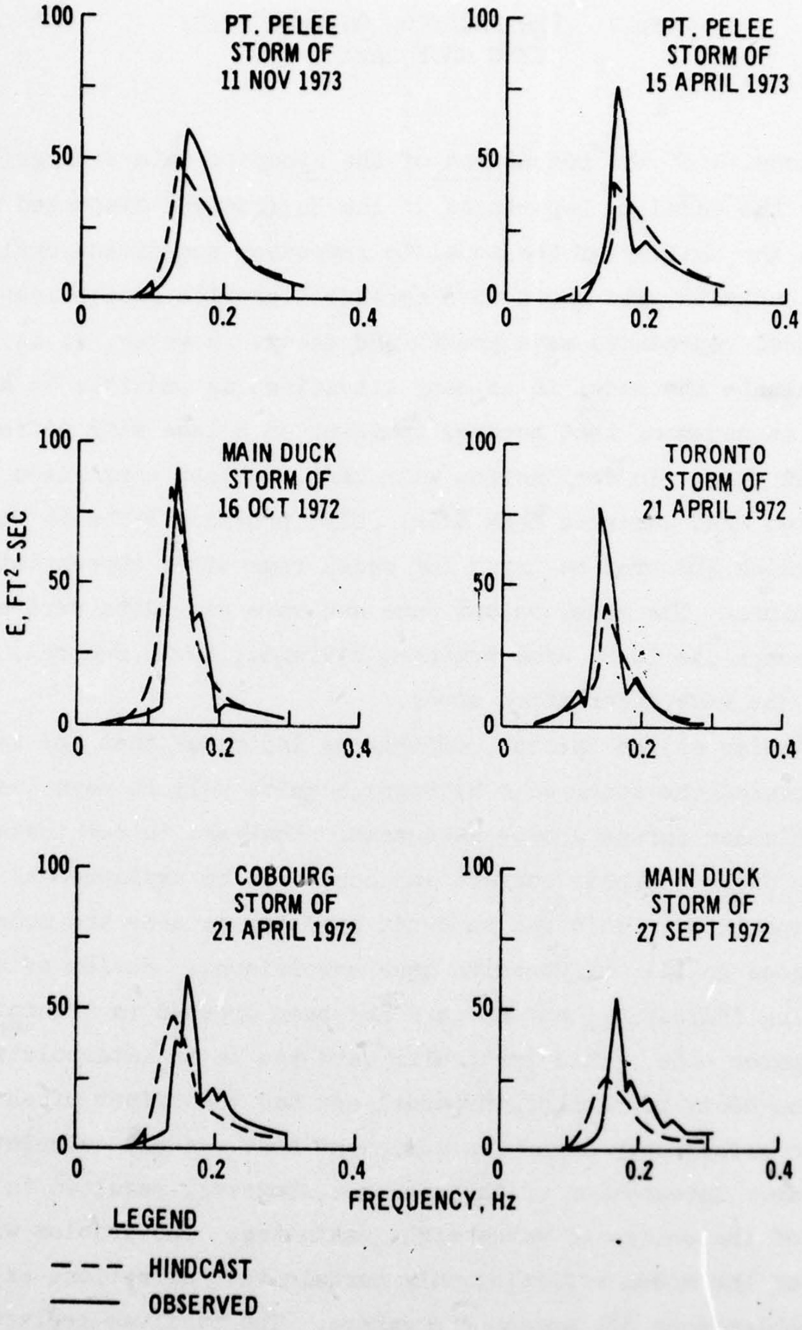


Figure 8. Spectral comparisons. A comparison of hindcast and measured wave spectra for selected times and sites on Lakes Erie and Ontario are given

PART V: VERIFICATION ON LAKE ERIE: CERC GAGE DATA

29. Results of the comparison of the hindcast data with gage data supplied by the Canadian Department of the Environment discussed in PART IV and the ability of the model to reproduce non-dimensional growth with fetch and with time shown in Reference 1 provide good evidence that the wave model reproduces wave growth and decay. However, it is desirable to evaluate the model in as many situations as possible to assure that there is no error that becomes dominant on a lake with different geometry and size. In conjunction with CERC, a blind comparison was made with two CERC gages on Lake Erie. CERC provided WES with storm-wind data which WES used as input for model runs after appropriate transformations. The model output data and wave gage data were exchanged through the U. S. Army Engineer Division, North Central, sponsor of the wave information study.

30. Review of the initial comparisons indicated that the wave model reproduced the storm-wave hydrograph quite well in most instances and in particular during growth sequences. However, in one instance the model decay did not appear correct and could not be explained as a deviation in wind input. This was somewhat baffling because the model decay was quite good in all the Canadian gage comparisons. Review of the model listing indicated that one card had been dropped in transmittal to the computer site. This particular card was in an interpolation block in the decay portion of the model and had the effect of shuffling energy into a frequency-direction component that was not calculated during decay. Integration of the spectrum, however, resulted in the inclusion of the energy in wave-height estimates. The problem was in a portion of the model affecting only certain wave directions explaining why the problem does not appear everywhere. The card was replaced and a selected set of Lake Erie storms rerun. The results did not appreciably change in the peak wave conditions, but the model showed dramatically improved decay. The remainder of the CERC verification discussions will be based upon the rerun storms on Lake Erie.

Wave-Height Verification

31. The wave-height history for the storms rerun is provided in Figure 9. The storms were not chosen on any particular basis and represent most of the storms rerun. For each gage site, the model grid points bracketing the gage were plotted. Results indicate that the model generally reproduced the peak waves of the storm within 2 ft well and reproduced the hydrograph well. The RMS error for each storm at each site and for all storms at a site is summarized in Table 3. The maximum RMS found for any site was 1.6 ft and the minimum was 0.6 ft. The RMS error for all storms and sites was 1.1 ft. The peak-to-peak comparison is 1.2 ft at Cleveland and 0.9 ft at Presque Isle. A scattergram for each site (Figure 10) shows little bias. The statistics are based on the raw data and do not include transformation to an angle-limited form. This inclusion would reduce the error somewhat, but the error, as is, is sufficiently small to suggest that the model functions well.

32. It is desirable to consider the types of deviation in the hindcast from the gage record, making the assumption that the gage data are accurate measurements. The first is underprediction by the wave model best seen in 9-22-75 and 9-24-75 at Presque Isle and Cleveland. These are probably due to too low a wind speed because of the consistency of this occurrence at both gage sites. Given an empirical wind transform, this will occur from time to time. The second type of deviation is when the model wave height is higher than the gage. On 11-27-75 and 11-30-75, the discrepancy is in large part due to a wind blowing parallel to shore. As discussed in PART III this results in a reduction in wave height up to a value of 30 percent but is a function of direction. The deviation of 11-14-75 is an apparent wind misestimate.

Frequency Verification

33. The comparison of spectral shape will show how well the model

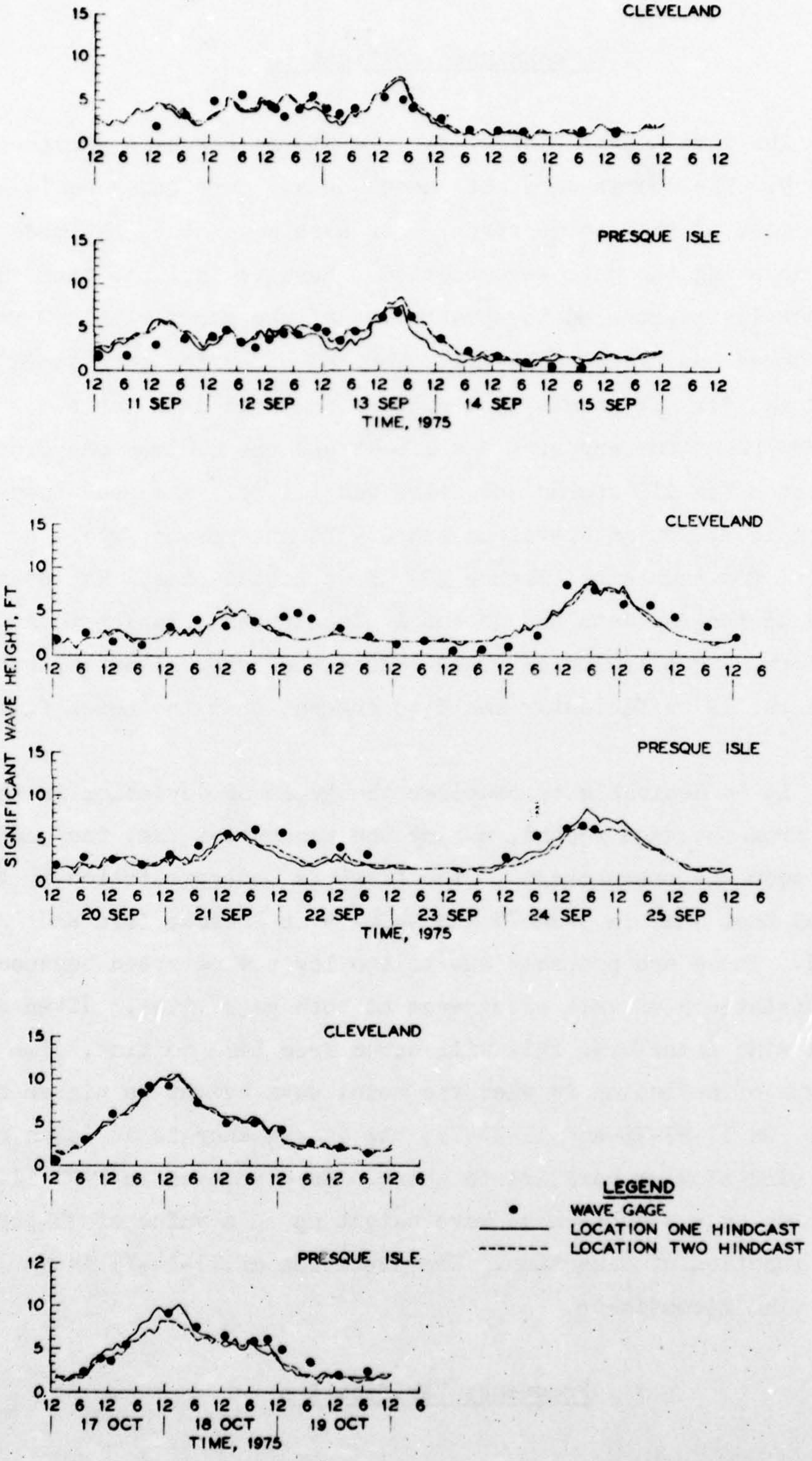


Figure 9. Wave hydrographs for Lake Erie. Measured and hindcast significant wave height are plotted with time. Wave hindcasts are plotted for the two grid points bracketing the gage location for Cleveland

(Sheet 1 of 3)

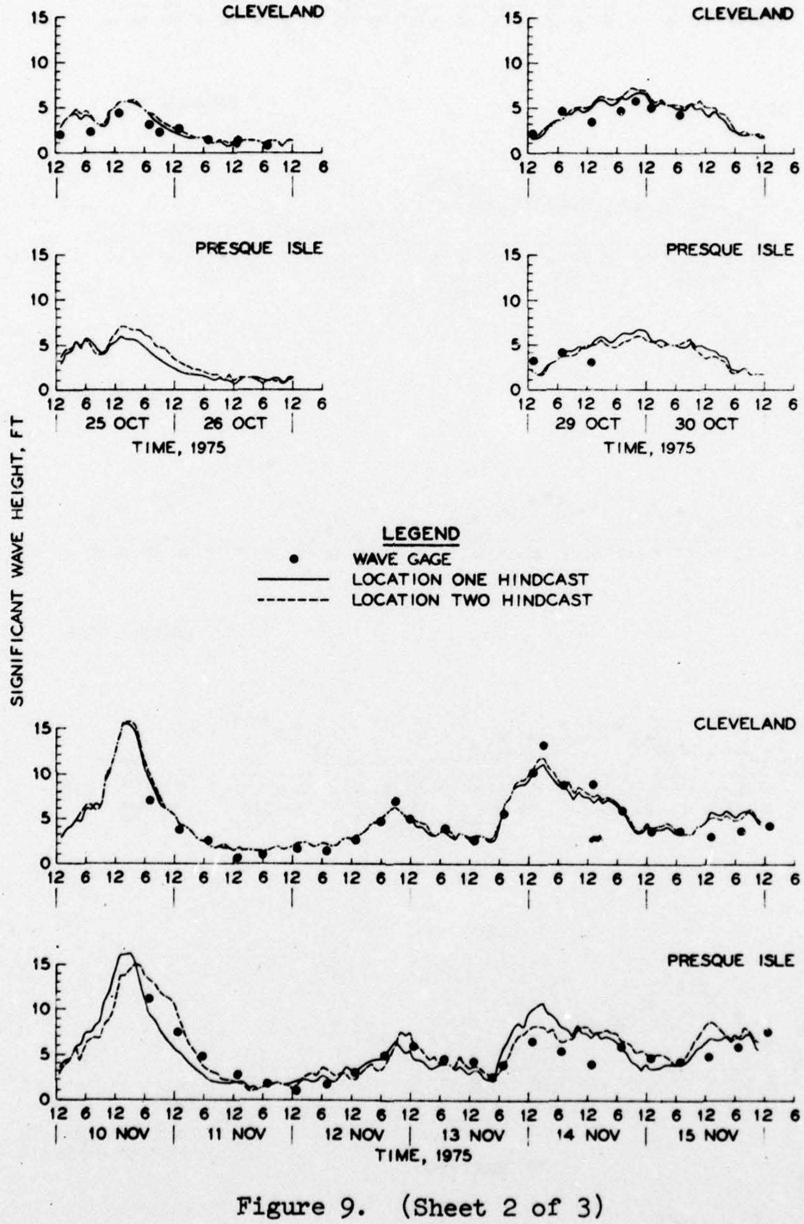


Figure 9. (Sheet 2 of 3)

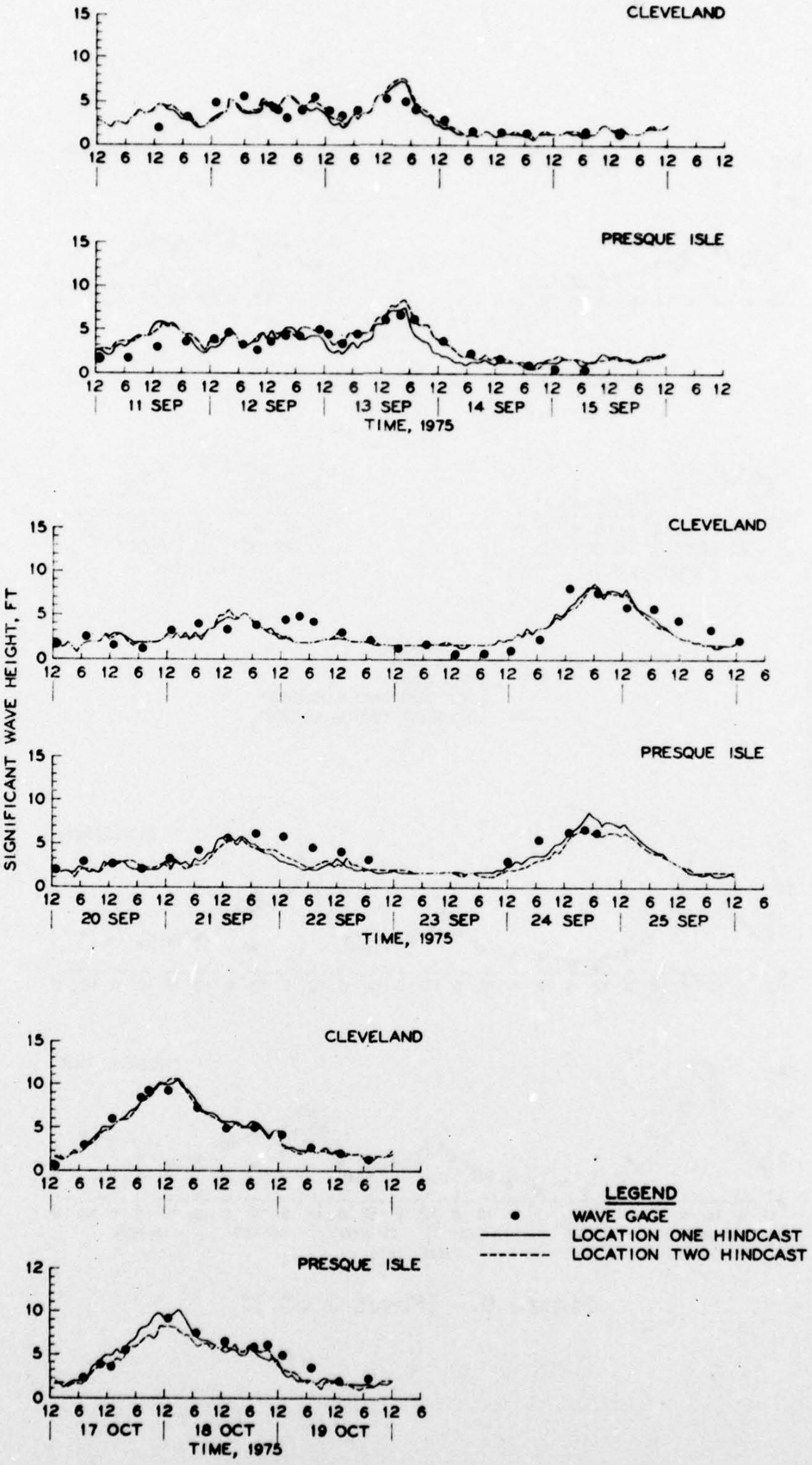


Figure 9. (Sheet 3 of 3)

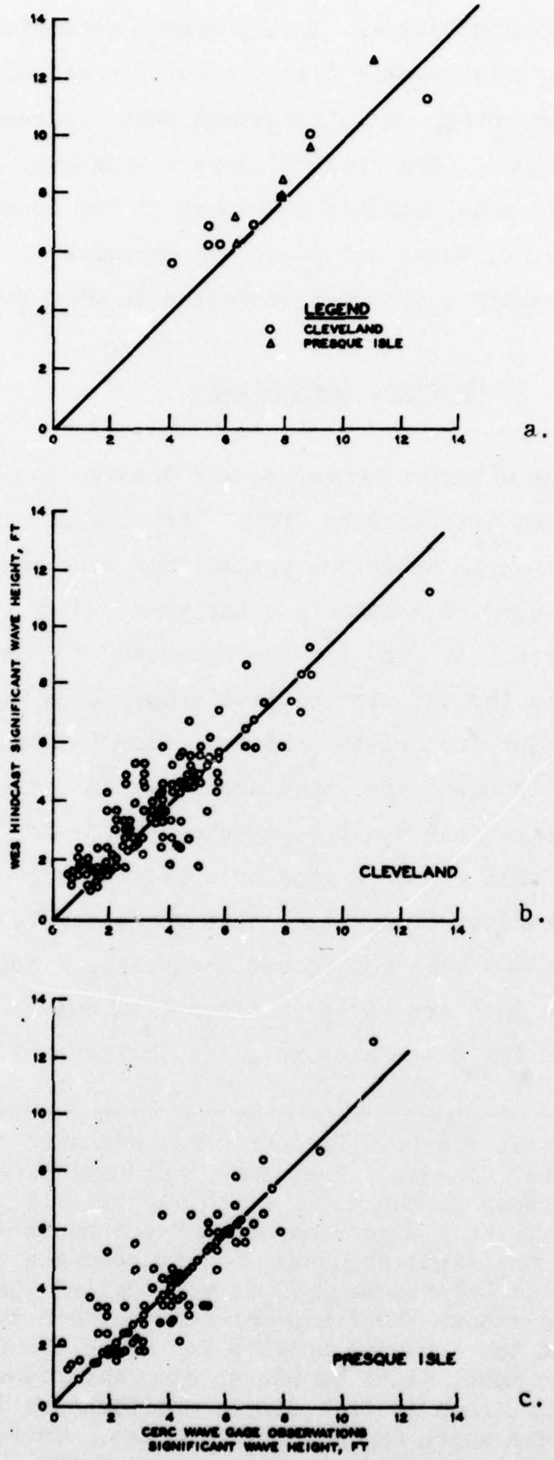


Figure 10. Wave height scattergrams. In (a) the measured peak significant wave height is plotted against the hindcast height for each storm. In (b) and (c) time-paired significant wave heights are plotted separately for the Cleveland and Presque Isle sites

predicts spectral characteristics. A very simple estimate of the model capability to estimate wave period tests is to plot the observed frequency with maximum energy density against that hindcast (Figure 11) for a selection of storms. The results above a frequency of 0.2 Hz are inaccurate because the model assumes that part of the spectrum to be saturated. Below 0.2 Hz, model and prototype agreement is excellent with an RMS error of about 1 sec when converted to wave period.

Spectral Comparisons

34. For the gage sites at Cleveland and Presque Isle, over 80 sets of spectral comparisons were made by CERC. The authors did not have access to the spectra until after the comparisons were made. Of these sets, 44 sets were discarded because (a) the wave energy was too low (peak density less than 5 ft^2/Hz) and the frequency of the spectral peak was beyond 0.23 Hz, or (b) the time between comparisons was greater than 3 hr. The justification for deletion in condition (a) above is that the model was programmed to treat the range above 0.23 Hz parametrically, in which case the comparisons would be meaningless.* For condition (b) above, it is evident that either a good or a bad comparison would be difficult to assess because of the time difference involved. Thirty-eight sets of comparisons were considered acceptable. Selections for Cleveland and Presque Isle are given in Figures 12 and 13.

35. The spectra for Cleveland are given in Figure 12. The first

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* It is noted here that the 0.23 Hz constraint was used in the hindcast study to cut computer time in a frequency range relatively unimportant to the original purpose of the model which was to calculate extreme waves on the Great Lakes. Since the tests reported herein were designed to evaluate the hindcast model used to generate the wave information provided in References 3-7, it was decided that it would be inappropriate to change the frequencies over which the spectrum is calculated to treat the low-wave heights for which the CERC gages were functional. If the model is to be run to simulate low-wave height conditions precisely, then it is a simple modification to change the frequency matrix over which the model calculates. Consequently, the 0.23 Hz constraint is not a general limitation of the applicability of the model.

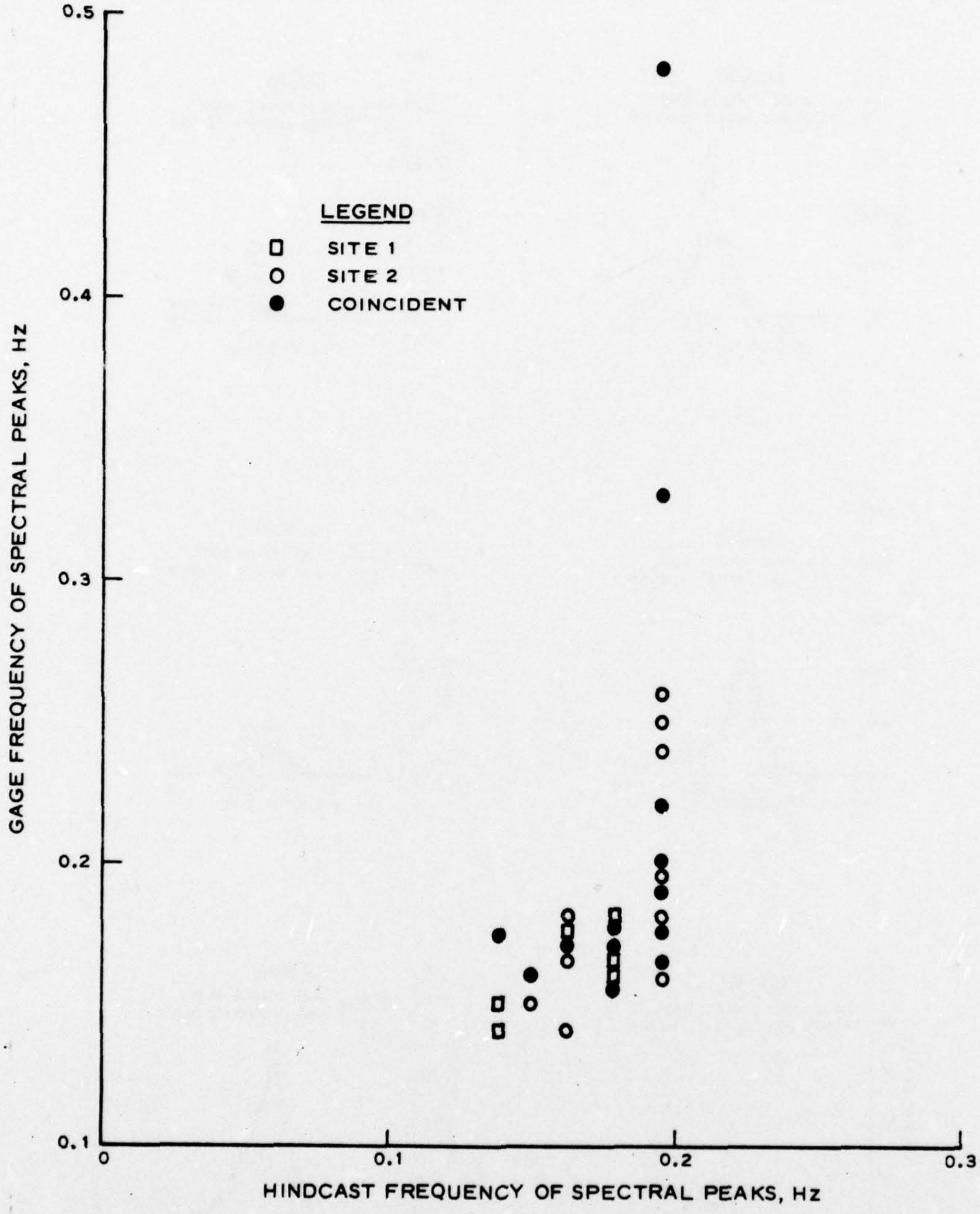


Figure 11. Frequency comparisons. The frequency of the hindcast spectral peak is plotted against that measured for measured frequencies less than 0.23 Hz, the highest frequency included in the model tests

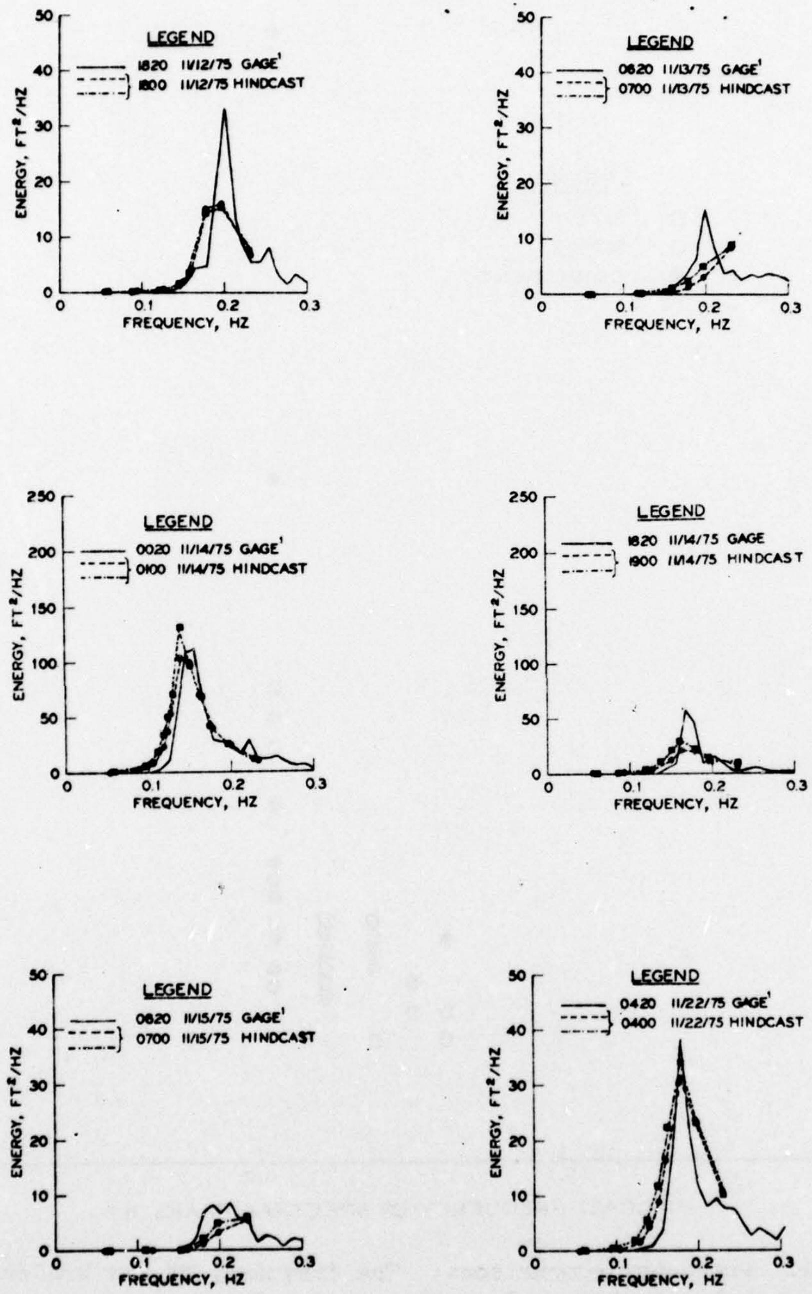


Figure 12. Spectral comparisons - Cleveland. The hindcast spectra are plotted against measured spectra. Examples of good and poor agreement are given. Note that two differing scales are used for the ordinate axis

(Sheet 1 of 2)

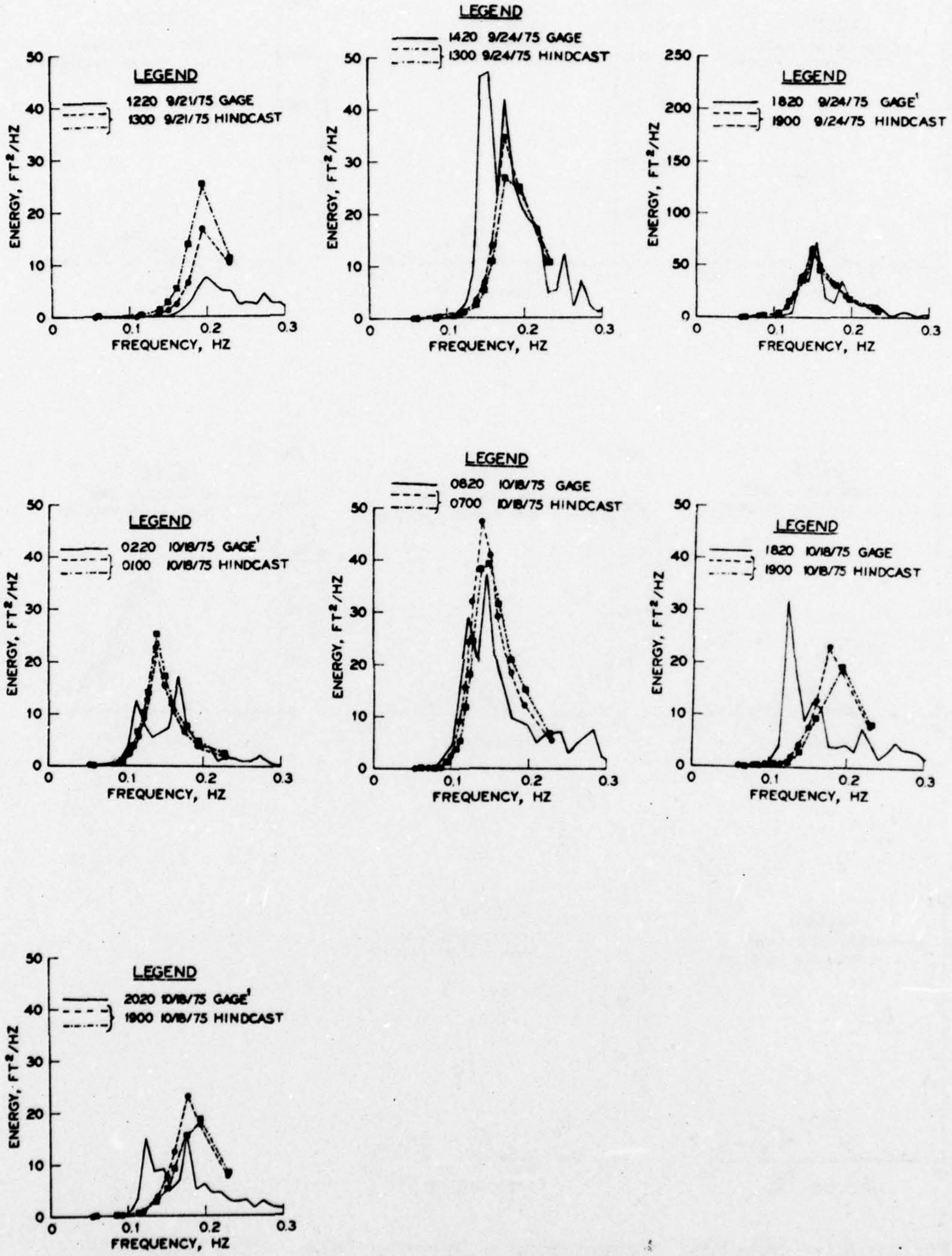


Figure 12. (Sheet 2 of 2)

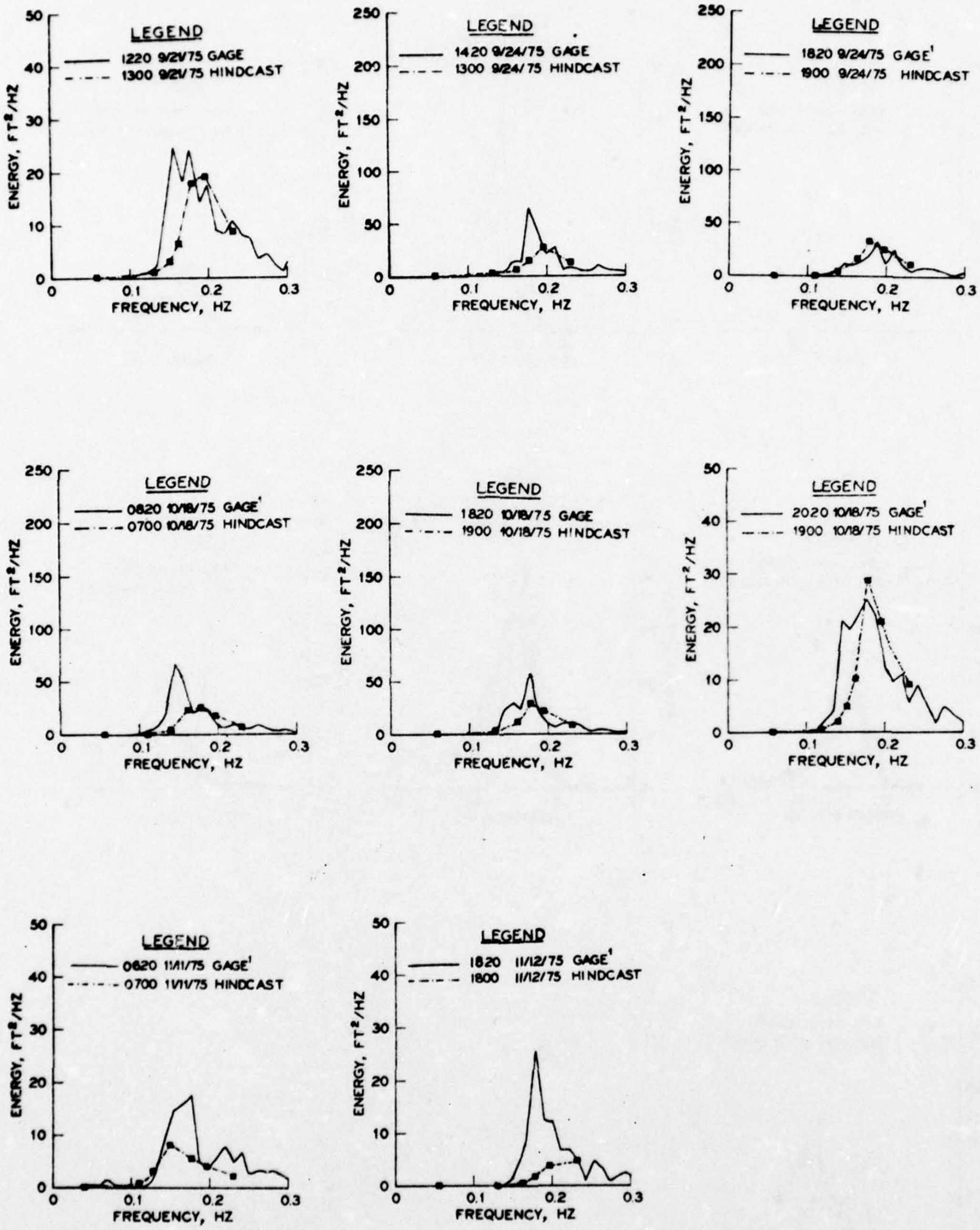


Figure 13. Spectral comparisons - Presque Isle. The hindcast spectra are plotted against measured spectra. Examples of good and poor agreement are given. Note that two differing scales are used for the ordinate axis

(Sheet 1 of 2)

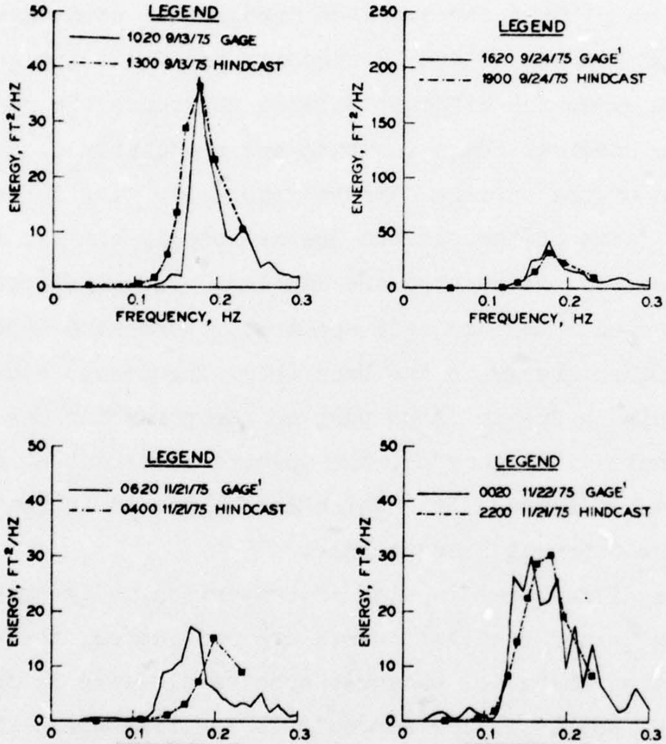


Figure 13. (Sheet 2 of 2)

six are for cases in which the observed and hindcast spectra occur within 1 hr and 20 min of each other and the second set are for cases in which the comparison are from 1 hr and 20 min to 3 hr apart. In each set, two hindcast spectra are shown against one gage spectrum. These spectra are taken at grid points that bracket the gage location. The spectra in Figure 13 are for Presque Isle. The first set (a-d) and second sets follow the same constraints as those in Figure 12. Only one hindcast spectrum is plotted for the gage spectrum in each case.

36. Review of Figures 12 and 13 suggests the following general comments. In most cases the hindcast spectra reproduce the gage spectra well. The spectral peaks for both are reasonably close and the general shapes of the spectra compare favorably. The forward (lower frequency) faces of the spectra are reasonably similar in slope and the peakednesses of the spectra are similar. The gage spectra tend to have more spikes than the hindcast spectra. There also appears a slight bias for higher energy in the back (high frequency) side of the hindcast peak. This is due in large part to the parameter assumption for higher frequencies. In general, the spectra agreement at Cleveland appears better than at Presque Isle which may be due to increased complexity in fetch geometry at Presque Isle.

37. When the differences in time of comparison and geographical location of gage site and hindcast points are considered, the spectral comparisons show that the model produces spectra close to those observed. The variability in results is due not only to the time-space sampling procedures but to the wind input errors inherent to the empirical formulations of the wind field. It should be reiterated that the results obtained were in a blind spectral comparison and that the model was universal, not specifically calibrated for one site or one lake. These spectral comparisons strongly suggest that the combined hindcast model and methodology provided are adequate representation of wave growth on the lakes for engineering purposes.

PART VI: VERIFICATION ON LAKE SUPERIOR: CANADIAN GAGE DATA

38. The verification tests on Lakes Ontario, Erie, and Michigan indicated that the model performed well. However, most of the gage observations are for wave conditions under 13 ft. Since the purpose of the model study was to predict extreme wave conditions, it is desirable to have field verification for waves higher than 13 ft. The combination of a field verification with high waves with a demonstrated ability to predict theoretical growth curves should lend confidence to the extreme wave estimates. Data collected by Ploeg[9] on Lake Superior during 1965 and 1966 were available. The published data consist of wave heights and periods only so no spectral comparisons are made. Storm waves in excess of 18 ft were observed at the gage locations.

39. In the Lake Superior tests, only the storms producing the largest waves at four sites (Figure 14) were run. Five storms were simulated and constituted a total of 25 days of prototype time. Smaller storms were not simulated because the model was assumed to be verified for smaller wave conditions and the additional cost was not believed to be justified.

40. The wave-height histories for the five storms are provided in Figure 15. With the exception of the storm of 19-24 October 1965, the reproduction of the gage histories by the model is quite excellent. No angle integrations or adjustments have been made. The large overprediction on 30 November 1966 at Grand Marais, Michigan, is in part due to an oblique wave approach for waves moving parallel to the shoreline. When the appropriate amount of energy heading toward the gage is integrated, the significant wave height is 10.5 ft.

41. The storm of 19-24 October 1965 presents another problem. The wave direction is directly onshore and the model underpredicts the gage observations. As in the previous verification, the empirical wind estimation technique was used to obtain the lake winds. The peak wind estimates during the storm in the Grand Marais area were in the range of 30 to 50 fps. Ships' observations during the same period indicated wind

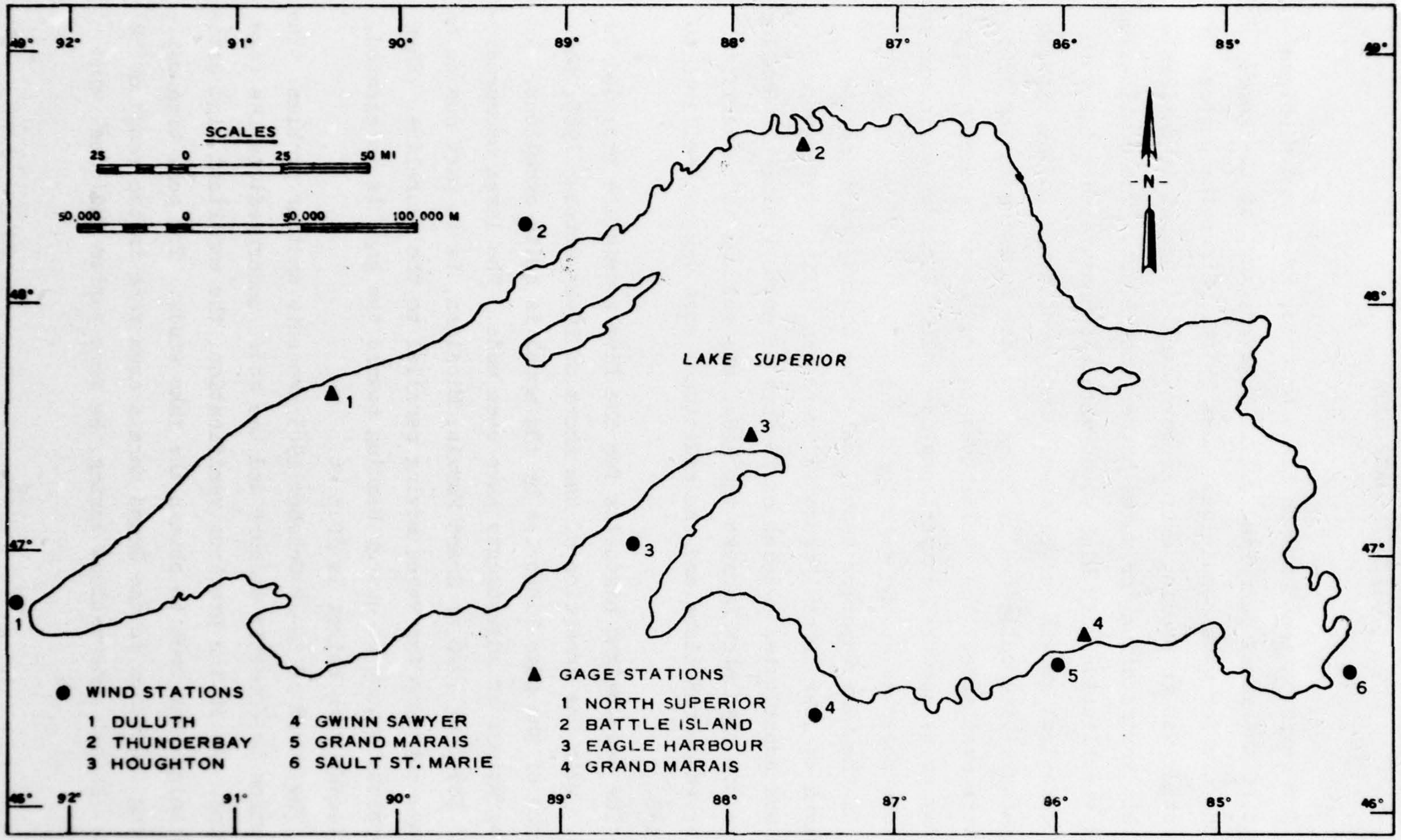


Figure 14. Wind stations and gage locations for Lake Superior hindcasts

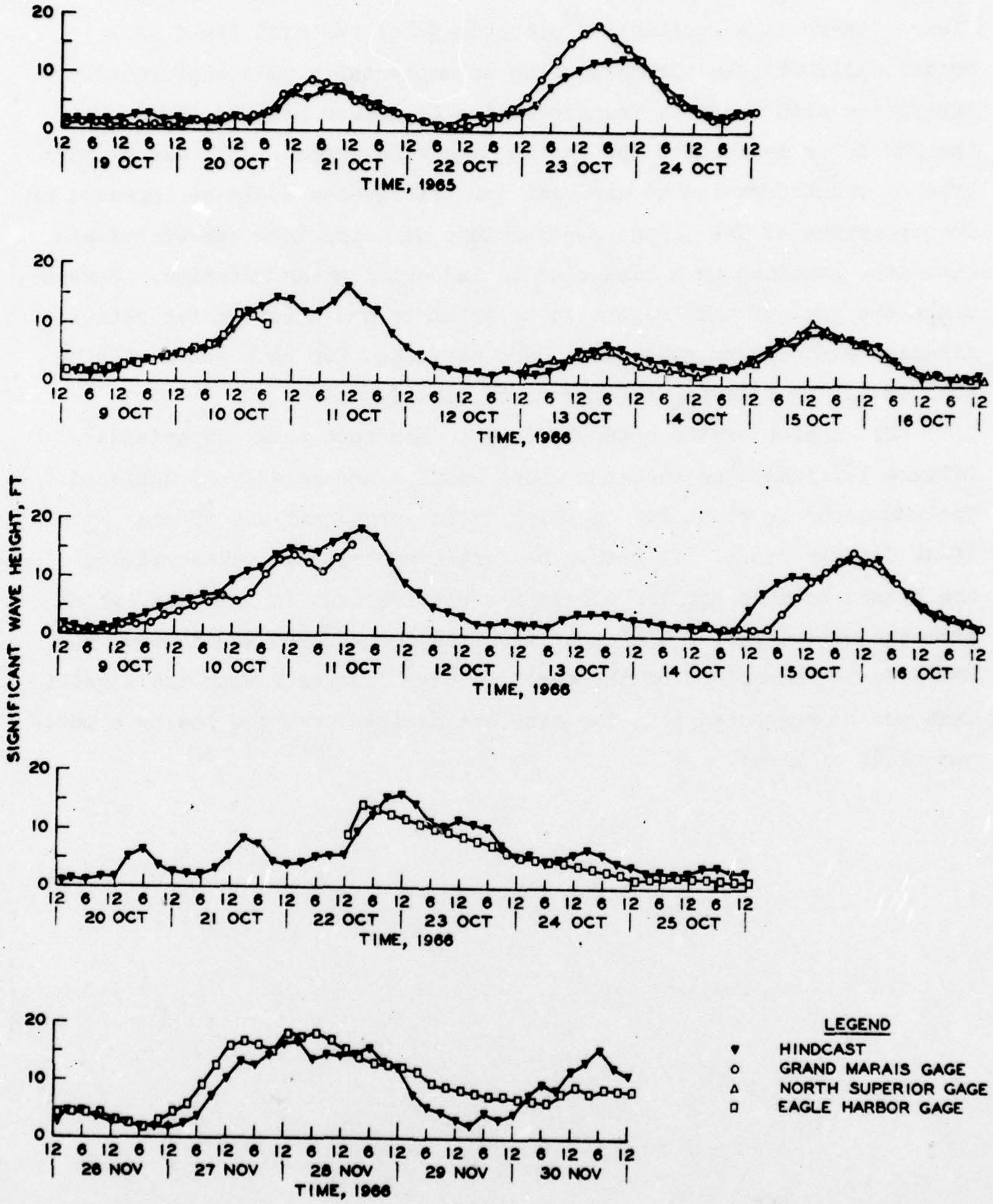


Figure 15. Wave hydrographs for Lake Superior. Predicted and measured significant wave heights are plotted as a function of time and location

speeds in the 50- to 60-fps range with one observation of 79 fps. Clearly there is a substantial misestimate of the wind field as occasionally will be the case using an empirical wind transformation. Experience with the wind transformation technique has suggested that the RMS error is about 8 fps in the high wind speeds. The use of this type of transformation to hindcast specific events could be improved by incorporation of the ships' observations directly into the wind-field estimates obtained by a land-wind to lake-wind transformation. However, since the goal of this report is to estimate the error in the entire hindcast methodology, the unimproved estimates for this storm will be used in all statistical treatments of error.

42. A plot of the observed peak to hindcast peak comparisons (Figure 16) indicates that the model results are relatively unbiased. The RMS error is about 2.5 ft which is in large part due to the 19-24 October storm. It should be further noted that these results are biased because smaller storms are not included in the test, which from the experience of the other verification studies would lower the RMS error. Comparison of the spectral peak frequency with the significant period suggested that the hindcast periods were too low by a maximum value of 2 sec.

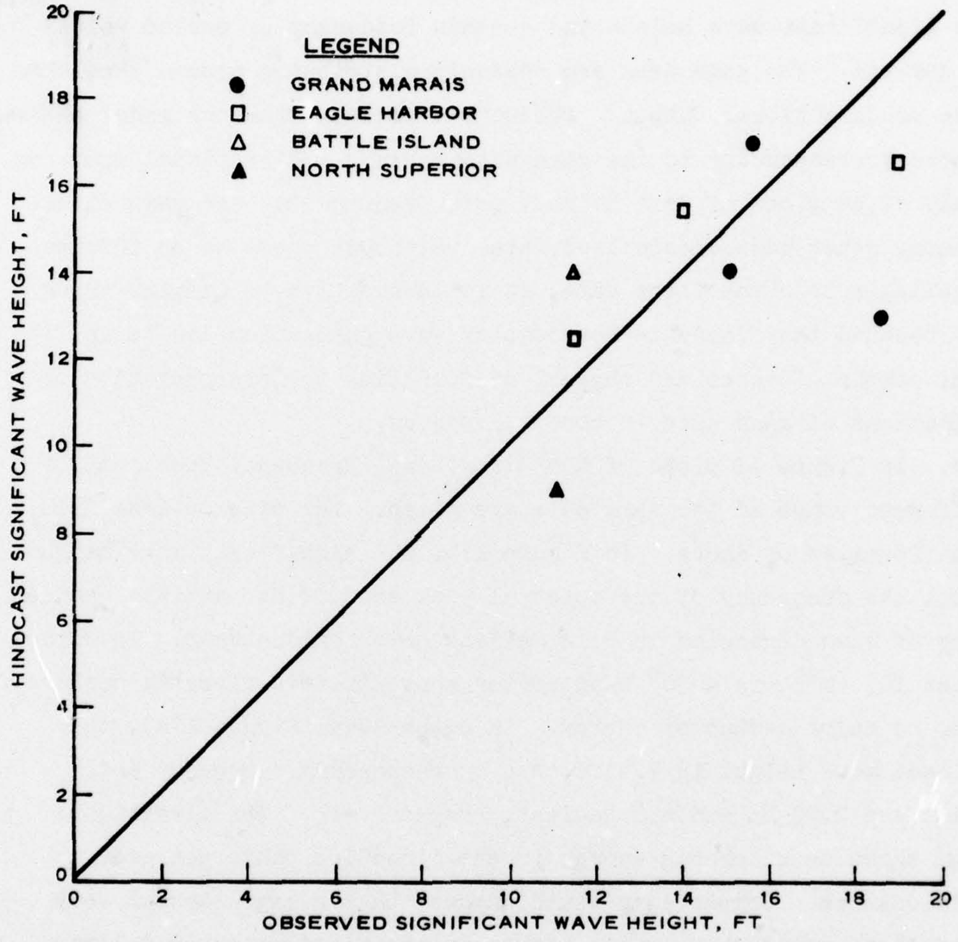


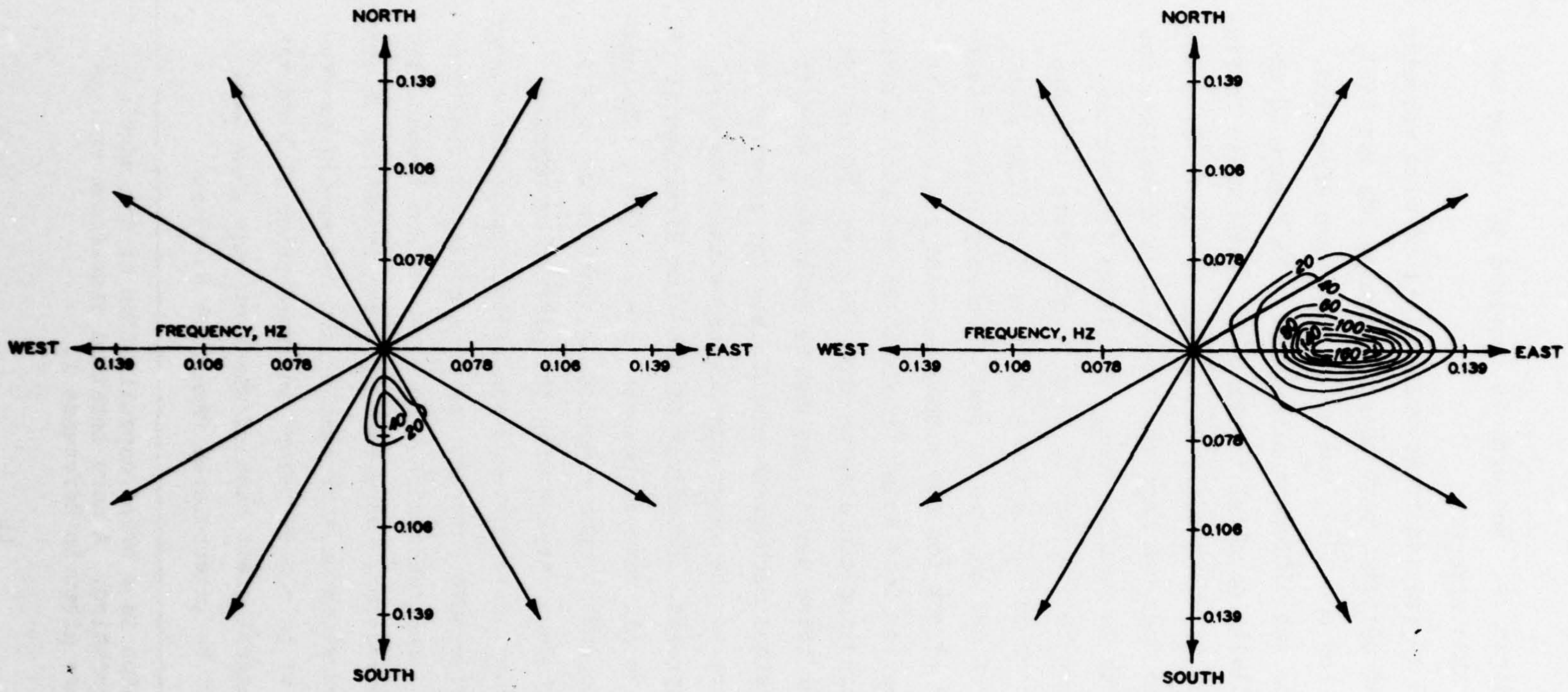
Figure 16. Scattergram of peak significant wave heights. Hindcast peak significant wave heights are plotted against those measured

PART VII: OTHER MODEL OUTPUT

43. The gage data by which the model has been compared are quite primitive in that only the one-dimensional frequency spectrum at most is available for comparison; integrated parameters of this spectrum such as significant wave height and certain frequency or period values can be derived. The gage data are sparsely distributed around the lakes and have no directional output. Products available from the model other than those corresponding to the gage data include a directional spectrum and field value plots of wave height, peak frequencies, and wave directions among other more specialized data. Although there is no information available to check these data, it is instructive to examine these outputs because they indicate how complex wave generation can be in limited, complex fetches and suggest difficulties in interpretation or extrapolations of gage data in coastal regions.

44. In Figure 17 plots of the directional frequency spectrum for two different times at the same site are given. The site on Lake Erie is about 10 miles offshore. In Figure 17b, the significant wave height is 12 ft, the frequency of the spectral peak as 0.12 Hz, and the central tendency of wave direction is 0.12 radians north of due east. In direction band 10, which is a 30° band center approximately directly onshore, there is no major amount of energy. In comparison (Figure 17a), the significant wave height is 7.4 ft, the corresponding frequency and direction are 0.20 Hz and 4.8 radians, respectively. The directional spectrum shows considerable energy in the direction bands centered directly onshore. Comparisons would suggest that a gage located very close to shore would show little energy in the first case and a large amount of energy in the second, in direct contrast to the observation 10 miles offshore.

45. The application of wave gage data taken far offshore to sites very close to shore would appear tenuous without some directional filtering. It seems clear that as a gage is placed closer to the shore some filtering of the alongshore moving waves must occur. It is not



a. 11 OCT '75

WAVE DIRECTION................SOUTH
SIGNIFICANT WAVE HEIGHT....7.4 FEET
CONTOUR INTERVAL.........20 CM^2 SEC

b. 16 OCT '75

WAVE DIRECTION..................EAST
SIGNIFICANT WAVE HEIGHT....12 FEET
CONTOUR INTERVAL..........20 CM^{2}SEC

Figure 17. Directional spectra. In (a) a directional spectrum with a mean angle of propagation toward shore is shown. In (b) a directional spectrum with a mean angle of propagation parallel to shore is plotted. Even though the spectrum in (b) has considerably more energy, practically none is propagating toward shore

clear how steep the gradient is. Research is required to define the magnitude and extent of this effect.

46. Figures 18 and 19 provide field value plots of wind velocity (Figures 18a and 19a), significant wave height (Figures 18b and 19b), peak frequency (Figures 18c and 19c), and an average wave direction (Figures 18d and 19d) for two different situations. In Figure 18 the wind direction is essentially parallel to the U. S. shore while in Figure 19 the wind direction is essentially orthogonal to the U. S. shoreline. Although Lake Erie does not have too irregular a shoreline, the effect of the irregularity is readily seen. In Figure 18a, in the northeast part of the lake to the east of Long Point, there is a decrease in wave height which is due to the protuberance of Long Point into the lake. The effect can be seen in peak frequency plots (Figure 18c) as well. A similar effect for the orthogonal wind field can be seen to the south and west of Long Point (Figures 19b and 19c). Variations in average wave direction can also be seen (Figures 18d and 19d).

47. The reasons for these deviations can be understood when it is realized that the physical mechanisms responsible for transfer of energy from the atmosphere to the water surface and within the wave field have directional spreads. Briefly,* if the wind direction is θ, wind energy is imparted to all wave components $\alpha = \theta \pm 90^\circ$. The magnitude of the input is dependent upon a nonlinear function of $\alpha - \theta$ with the functional dependence varying with the particular physical mechanism. To further complicate matters, the redistribution of energy within the spectrum by wave-wave interactions is a nonlinear function of $\alpha - \phi$ where ϕ is a mean wave direction with ϕ not necessarily equal to θ, the wind direction. As a result, wave generation in areas of irregular fetch geometry can be very complex and not easily generalized. The implication of this goes beyond interpretation of hindcast comparison and should suggest great caution whenever wave gage data (unless directional) must be interpolated from site to site.

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* The following discussion is a brief capsulization of the angular dependence in wind generation. A more detailed discussion and appropriate references are given in Reference 1.

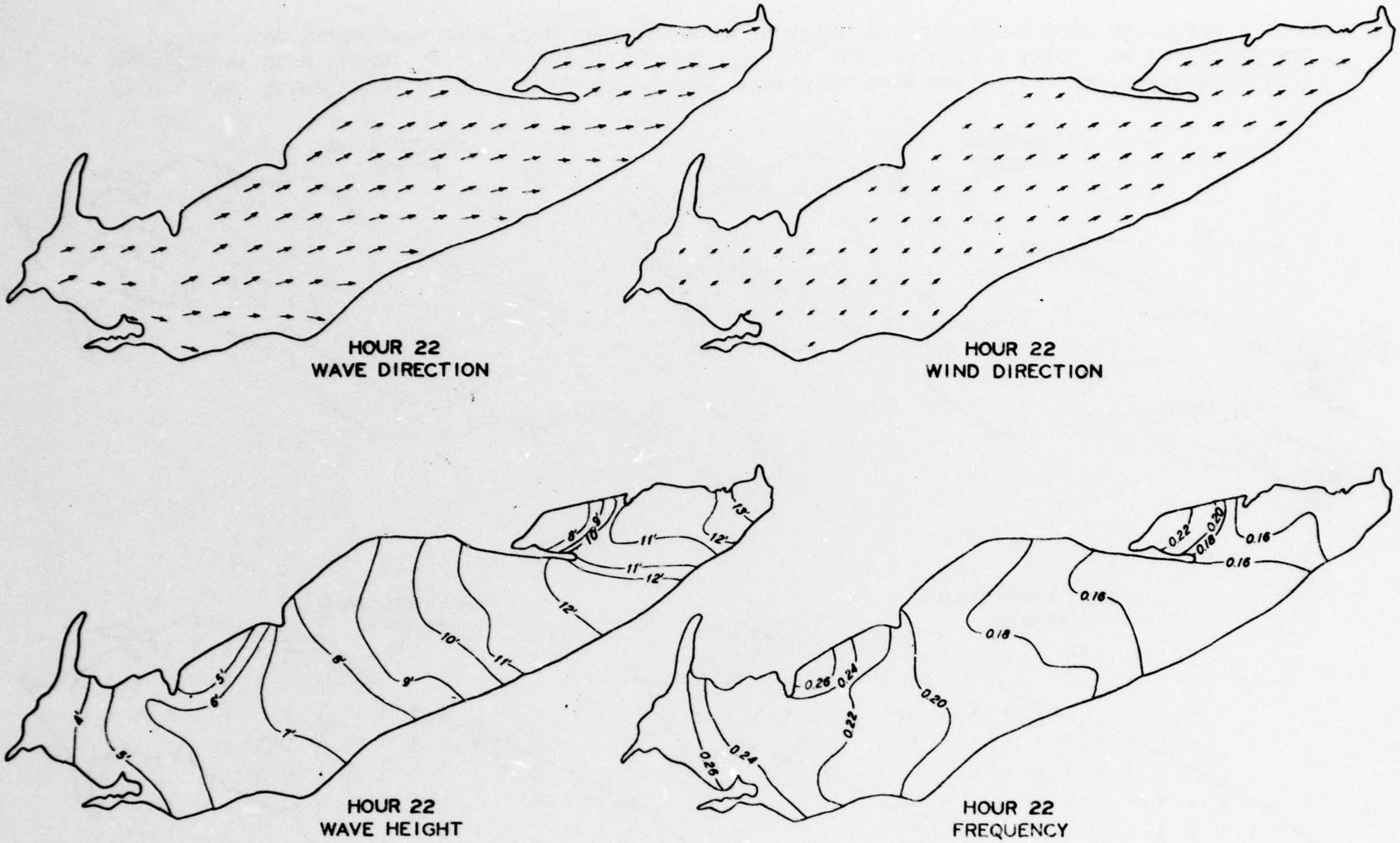

Figure 18. Other model outputs - alongshore winds. The mean wave angle (a), wind direction (b), significant height (c), and peak frequency (d) are plotted over a lake. It is evident that the wave parameters vary significantly with location for a uniform wind direction

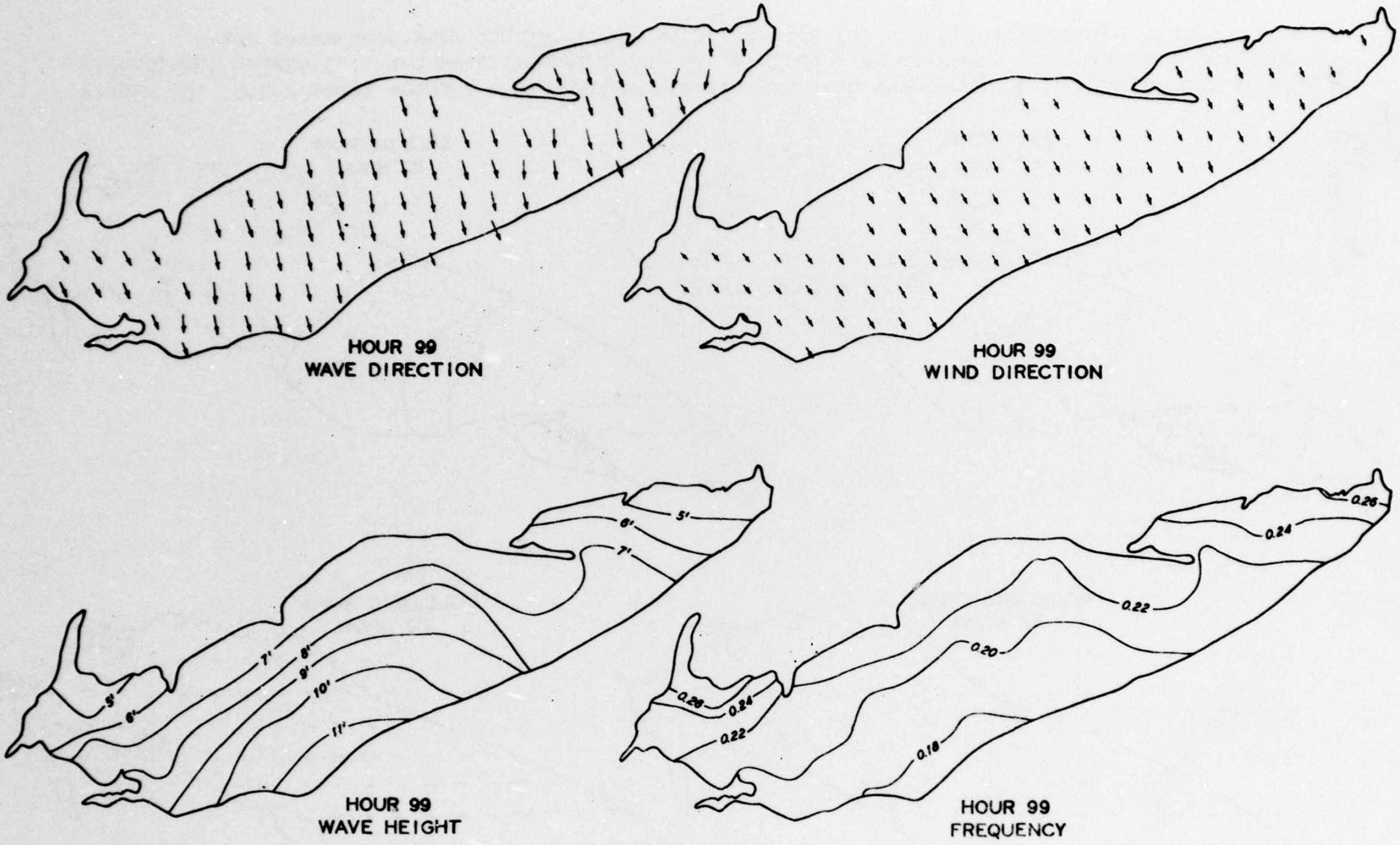

Figure 19. Other model outputs - onshore winds. The mean wave angle (a), wind direction (b), significant wave height (c), and peak frequency (d) are plotted over a lake. As in Figure 18, the wave parameters vary significantly with location for a uniform wind direction

PART VIII: MODEL COMPUTATION ASPECTS

48. The numerical model was originally programmed and tested on a CDC 7600 at the Los Alamos Scientific Laboratory in New Mexico on their unique CROSS operating system. The model was later adjusted to run on two different CDC 7600's with a SCOPE operating system. The model initially was developed for Lakes Erie and Ontario. Modifications were required to economize storage for the larger lakes, particularly after changeover to the SCOPE operating system.

49. A series of model statistics for each lake have been compiled (Table 4). These benchmarks are all related to a CDC 7600 with SCOPE 2 operating system. It should be noted that for Lake Superior the large core memory requirement of 272,000 words is close to the capacity of the CDC 7600 when all buffers and other system requirements are included. Thus the computational grid points are limited to approximately 275 points. The 60-bit accuracy of the CDC 7600 is not really necessary, however, as most values can be contained within 30 bits. So through simple word-splitting techniques the grid size can reach 550 points with only modest program changes and slight run time increase. Beyond 550 points, a more sophisticated revision of the model would be needed.

50. The run time of the model is directly linked to the number of calculation points. Run time varies from 48 sec for Lake Ontario to 206 sec for Lake Superior for 24 hr of prototype hindcasts. For forecasts using the model, the run time for all lakes for 24 hr is 624 sec (10 min); for 48 hr, 1248 sec (21 min); and for 72 hr, 1872 sec (31 min) on a CDC 7600. On a slower computer, such as a CDC 6600, it would be approximately five times longer.

51. The above statistics are based on a 10-mile grid mesh and a 15-min time step. For wave simulations on other bodies of water the grid and time scales can be varied. The only constraint on these two scales is that the grid and time step should be so chosen that waves of the lowest frequency used in the model run should not travel more than the distance between grid points in the selected time step:

$$\Delta x > 2.56 \, \Delta t / f_L$$

where

Δx = the grid mesh

Δt = time step

f_L = the lowest frequency used on the lake

52. Another implicit constraint to model use is the appropriate selection of frequency and angle increments. These two increments must be selected on the basis of the size of the water body and the accuracy of wind direction data. It is possible to choose a different set of angles and frequencies (with some modest modifications to the model required) but size of the storage limits the number to approximately

$$NF \times NA * NGP = 275{,}000$$

where

NF = the number of frequencies

NA = the number of angle classes

NGP = the number of computation points

PART IX: SUMMARY

53. The principal reason for development of this numerical hindcast model was to provide estimates of the largest wave heights for storms of record which then would be statistically analyzed to provide a wave climatology for design (i.e., extreme wave) conditions on the Great Lakes. Since the interest is in large wave heights, a number of simplifications in simulating storm winds proved applicable.[2] Simplifications in the numerical hindcast model were made as well. These principally involve arrangement of the computing grid and choice of frequency intervals for spectral calculations. Thus the model as now formulated should be considered applicable for storm simulations. Meteorologic conditions where winds are weak or organized in small, local convective cells are not properly treated in the wind model. Wave growths that have peak frequencies remaining greater than 0.23 Hz are simulated not correctly in the model as now formulated, but these waves are small and are not of interest in a design wave climate. If wave estimates with peak frequencies less than 0.23 are required, only the input frequency array needs to be changed.

54. When the majority of meteorological conditions producing large waves at a coastal site are considered, it is evident that the principal conditions consist of fast wind speeds over the longest fetches at a site that gives wave-approach directions approximately normal to shore. Figure 20 provides a plot of largest significant wave hindcast to largest significant wave height observed for storm conditions analyzed in this report that meet the criterion of a long fetch with a wave approach nearly orthogonal to shore. It is evident from this figure that the wave model provides unbiased estimates of the design waves that are consistently within an RMS error of 1.5 to 2.0 ft. On Lakes Ontario, Erie, and Michigan the RMS error has been shown to be 1.0 to 1.5 ft. On Lake Superior the error is 2.0 to 2.5 ft, but this is believed to be due to poorer wind estimates on Lake Superior.

55. No data are available on Lake Huron to provide a verification. However, given the extensive verifications achieved on the diverse

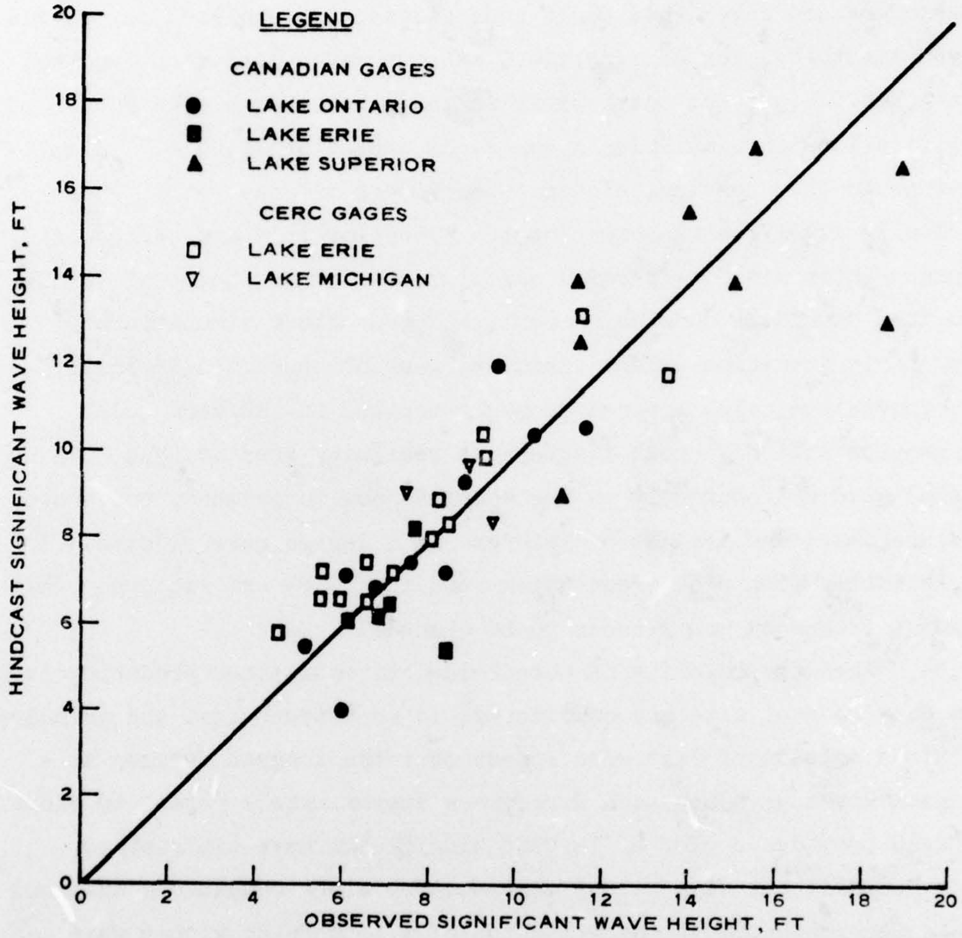

Figure 20. Wave height scattergram. Hindcast peak significant wave height is plotted against that measured for all storms for which the long fetch (20 miles) condition applies. The RMS is about 1.5 ft

geometries of the other lakes, and adequate spacing of wind stations around Lake Huron, there is no reason to expect that the model error will be outside the range of the 1.5 to 2.0 ft established.

56. Wave-height history comparisons, presented herein, also show that the model does reasonably well on the lower wave-height estimates during lower wind speed conditions. For oblique angle approach of wave to shore, integration of the directional spectrum appears to provide adequate estimates of the wave conditions. However, it would be desirable to further check such situations through comparison of the model with an array of wave gages that would traverse the first 10 miles from the shore.

57. Comparison of the model spectral estimates indicates that the model performs especially well in predicting spectral shape. The peak frequency and period parameters appear reasonable on all lakes when comparisons are made, although the values for Lake Superior appear somewhat low.

58. The plots of directional spectral wave height, peak frequency, and direction fields appear reasonable although insufficient data are available to check them in detail. These data show that the spatial gradients in the directional characteristics of the spectrum are fairly large for complex fetch geometries. Most importantly, the data suggest that the use of nondirectional gage data in engineering planning and design problems can be very misleading, particularly if the gage location is far offshore from the site of interest. The complex gradients in wave characteristics also suggest that interpolation of gage data from sites upcoast or downcoast requires significant caution.

59. It should be stressed that the wave estimates in the verification studies on the four lakes were made with what is termed a universal model. All coefficient and arbitrary values required in the wave model were set according to published values and prior to any of the verification runs and were never changed. Thus, the wave model is not adjusted for each lake individually and in essence remains constant. The wind coefficients are adjusted for each site, but the adjustments are made on the basis of comparisons with other wind data not wave data.

The ability of the wave model to produce verifications as excellent as seen here without lake-by-lake adjustment of coefficients strongly suggests that the physics of air-sea interaction on the scale of the Great Lakes are reasonably reproduced and provide more confidence in the application of the hindcast design information for engineering and planning purposes. The verification study presented herein also represents one of the most extensive performed for any numerical wave model, used either in research or for operational purposes.

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8. Marine Environmental Data Service, "A Summary of Available Wave Data Products," Fisheries and Marine Service, Environment Canada, Ottawa, Ontario, 1977.
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Table 1

Wave Data Used for Model Verification

	PART IV	PART V	PART VI
Source	Canadian Department of the Environment Ottawa, Ontario	Coastal Engineering Research Center, Ft. Belvoir, VA	National Research Council of Canada[9]
Lake	Erie, Ontario	Erie, Michigan	Superior
Dates	Ice-free seasons 1972, 1973	Ice-free seasons 1975	Ice-free seasons 1965, 1966

Table 2

Frequencies Used in the Wave Model, in Hertz (Hz)

0.056	0.095	0.139
0.061	0.100	0.150
0.067	0.106	0.161
0.073	0.111	0.178
0.078	0.117	0.195
0.084	0.123	0.230
0.089	0.128	

Table 3

RMS Error in Wave Height, Lake Erie

Storm	Cleveland	Presque Isle
9-11-75	0.36	0.24
9-20-75	0.41	0.34
10-17-75	0.18	0.22
10-25-75	0.38	--
10-29-75	0.34	--
11-10-75	0.30	0.32
11-20-75	0.31	0.49
11-26-75	0.33	0.40
All storms	0.33	0.33
Peak-to-peak	0.36	0.28
All sites, all storms	0.33	
All sites, all storms (peak to peak)	0.32	

Table 4

Model Statistics for CDC 7600

	Lake				
Model Statistic	Ontario	Erie	Michigan	Huron	Superior
Spatial mesh (miles)	10	10	10.	10	10
Time step	15	15	15	15	15
Grid size	10x21	13x25	33x15	23x17	18x37
Computation points	60	86	216	200	273
Small core memory* (60-bit words)	62,000	62,000	62,000	62,000	62,000
Large core memory** (60-bit words)	67,600	92,560	217,000	202,000	272,100
Computer time† in seconds per prototype hour of simulation	2.0	3.0	6.5	6.2	8.6

* For the smaller lakes some reduction is possible.

** Does not include buffer and other machine-required space.

† Based on a SCOPE 2.1 operating system.

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Resio, Donald T

A numerical hindcast model for wave spectra on water bodies with irregular shoreline geometry; Report 2: Model verification with observed wave data / by Donald T. Resio, C. Linwood Vincent. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1978.

53, [2] p. : ill. ; 27 cm. (Miscellaneous paper - U. S. Army Engineer Waterways Experiment Station ; H-77-9, Report 2)

Prepared for U. S. Army Engineer Division, North Central, Chicago, Illinois.

References: p. 53.

1. Great Lakes. 2. Mathematical models. 3. Water wave generation. 4. Water wave hindcasting. 5. Water wave spectra. 6. Water waves. I. Vincent, Charles Linwood, joint author. II. United States. Army. Corps of Engineers. North Central Division. III. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Miscellaneous paper ; H-77-9, Report 2.

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