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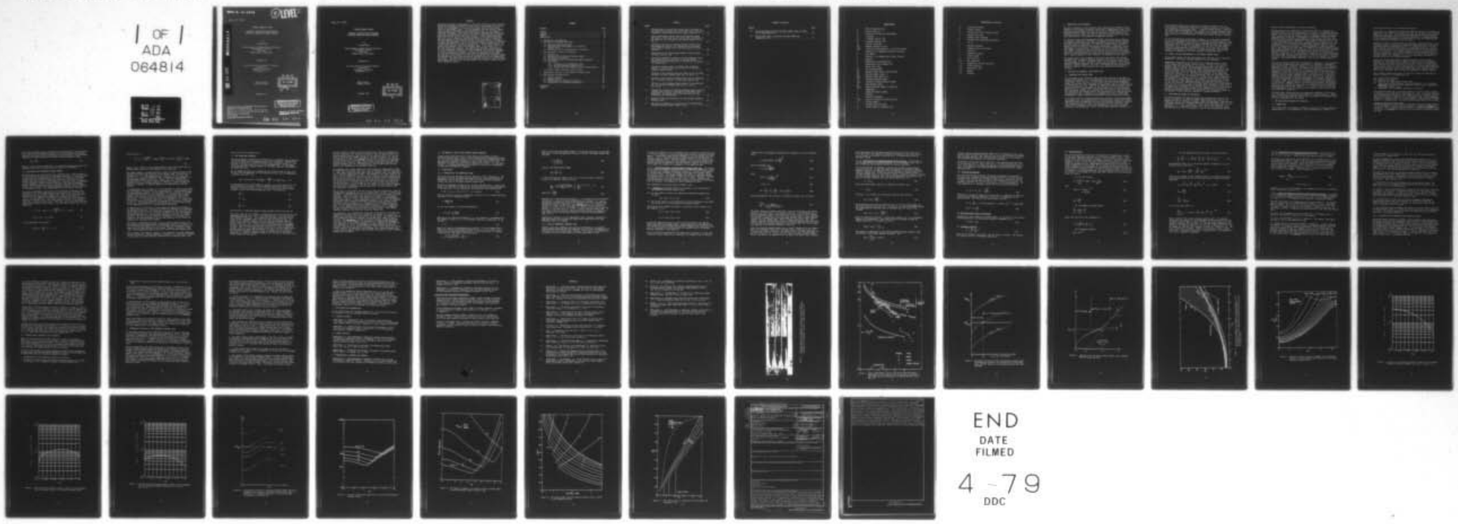

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INTERIM PROGRESS REPORT

STABILITY, TRANSITION AND TURBULENCE IN MIXING LAYERS AND ADJACENT STREAMS

By

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Prepared for

Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, D. C.

Under Contract
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ABSTRACT

This report describes accomplishments in the period 1977-1978 in basic research dealing with phenomena in parallel viscous flows. An experiment has been completed on the development of the mean flow in wakes shed by nozzle arrays typical of gas-dynamic lasers. In this experiment, the wake velocity, temperature and density was mapped in wakes generated at Mach 4 in the Company's Supersonic Wind Tunnel. The results are in general agreement with theoretical predictions developed under an earlier OSR study in which theoretical rules on wake flows were set up for laser cavity design. A second problem addressed dealt with a novel approach to predicting transition in parallel shear flows. The basic physics in this approach consists of the postulate of a universal turbulence Reynolds number, below which self-preserving turbulence is impossible. Although this condition is only one of necessity, formulas derived with it had earlier predicted wake transition remarkably well. In the present period, the postulate was applied to flat plate boundary layers in the range $0 < M < 10$ with and without heat transfer. The computations uncovered a mechanism for the long unexplained "transition reversal" phenomenon and otherwise conformed to earlier transition observations. Finally, the data analysis was completed on an earlier hypersonic boundary layer stability experiment done at AEDC. The data show little change of the stability diagram with heat transfer although the amplification rates were substantially increased when the wall temperature was lowered. The present results were disseminated through two technical reports, three journal articles, three delivered papers and by personal contact with Air Force engineering centers.

CONTENTS

FIGURES

FIGURES (continued)

NOMENCLATURE

b	width of turbulent zone
b_o	initial value of b
C_1	function of the velocity fluctuations
C_2	a constant
C_3	a constant (see Eq. (5))
C_4	a constant (see Eq. (6))
C_f	friction coefficient
$C_D h$	momentum thickness of wake
$F(M)$	a function of Mach number in the friction formula
k	constant in the temperature-viscosity relation
M	Mach number
n	constant in the temperature-viscosity relation
p	pressure
$\bar{Q}$	dimensional heat exchange rate
Q	dimensionless heat exchange rate
R	square root of Re_x
Re'	unit Reynolds number
Re_x	Reynolds number based on wetted length
Re_b	Reynolds number based on b
Re_{bo}	Reynolds number based on b_o
Re_Λ	turbulence Reynolds number
$Re_{\Lambda o}$	threshold turbulence Reynolds number
R_w	wake Reynolds number
$Re_{\theta T}$	momentum Reynolds number at transition
T	temperature
t	temperature defect (wakes)
u	velocity
u'	velocity fluctuation
$u(o)$	velocity at center or on axis of flow
w	velocity defect
x	distance from flow origin
x_o	virtual origin of turbulent flow

NOMENCLATURE (continued)

x_T	transition distance
y	lateral distance
y_o	location of critical transition layer
γ	specific heat ratio
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
Δ	$x_T - x_o$
θ	momentum thickness
Λ	integral scale of turbulence
μ	viscosity
ν	kinematic viscosity
σ	Prandtl number
τ_w	wall friction
$(\ )_e$	edge conditions
$(\ )_o$	stagnation or initial conditions
$(\ )_\infty$	stream conditions
$(\ )_w$	conditions on wall
$(\ )_\ell$	laminar
$(\ )_t$	turbulent

1. INTRODUCTION AND BACKGROUND

The work in progress in this laboratory under AFOSR Contract F44620-75-C-0016 addresses basic problems in parallel shear flows of compressible fluids. These problems are well defined and consist of the issue of hydrodynamic stability of high-speed boundary layers, the issue of transition from the laminar to the turbulent state of a shear flow, and the issue of the prediction of free shear flow characteristics as encountered in engineering devices.

In contrast to many other OSR-supported studies, the present work is done in an industrial organization involved in the engineering of many modern systems for defense and civilian needs. This enhances the relevance of the research performed, makes the applicability of its results highly visible, and frequently allows the testing of the research conclusions by comparison with the performance of actual systems. Furthermore, the Fluid Mechanics Section of this Company is engaged in contract research for military directorates other than AFOSR; these projects are closely related to the subject of the present contract, and provide motivation, direction and testing grounds for the present work. Specific examples of this interaction are mentioned in this report.

The present report covers progress in the period from October 1977 through September 1978. The report addresses separately the subjects covered, namely (a) development of wakes in gasdynamic lasers, (b) hydrodynamic stability of hypersonic boundary layers and (c) a novel theoretical approach to the laminar-turbulent transition program. The status of documentation and publication of results is reported, and interaction with other individuals and agencies is indicated.

2. THE WAKES OF GASDYNAMIC LASER NOZZLE CUSPS

2.1 BACKGROUND AND PREVIOUS WORK

Our involvement in the nozzle-cusp wake problem was motivated by the need to organize the knowledge of the two-dimensional wakes trailing the multiple cusps in gasdynamic laser cavities. For such lasers, the wakes divide homogeneous stream, and interest in the wake arises mainly because of possible attenuation and defocusing of the beam. Knowledge of compressible wakes was already adequate for no heat addition in the volume, if the wake could be characterized in the momentum sense by a drag integral and in the energy sense by a heat integral. What was missing, however, was a consideration of the entire cusp-wake system, an accounting of the sidewall effects, and the question of far wake interaction. The cusp-wake system is important to the designer who needs to compute wake development in terms of plenum, nozzle geometry and heat transfer conditions.

In the period 1975-77, we began systematically to review, calculate and compile into compact form, the characteristics of two-dimensional laminar and turbulent compressible wakes as generated by flows of arbitrary Mach and Reynolds numbers, wall temperature, specific heat ratio and other molecular properties. These results, which appeared in report form (References 1 and 2) and in journal publication form (References 3 and 4), revealed some unexpected phenomena such as the peculiar behavior of a wake which is turbulent starting at the cusp trailing edge ("transitional" wake) and the irregular variation of its fluctuation magnitudes with Reynolds number. To arrive at these results, a combination was used of known flow properties (e.g., laminar and turbulent diffusion rates) and of some new, key assumptions; for example, it was assumed that the virtual wake origin lies at the nozzle throat for the fully laminar and fully turbulent wake.

To test the predictions of References 1 through 4, a triple nozzle arrangement was set up in the Aeronutronic Supersonic Wind Tunnel to produce exit M = 4 and nozzle wall temperatures T_w down to 0.7 T_o. The purpose was to measure the cusp wake properties in this set-up and compare them with the properties (velocity, temperature, etc.), of the above references. By late 1977, the wind tunnel set-up was working smoothly; preliminary results on the mean flow had been obtained and reported in Reference 5. The final experiments, which had been scheduled for completion in the 1977-78 period, are mentioned below.

2.2 NOZZLE CUSP WAKE MEASUREMENTS IN THE PRESENT WORKING PERIOD

In the present period, the wake measurements were completed and are reported in Reference 6. The results are aptly summarized in that reference:

"Experimental measurements are described in the supersonic wakes of cusps formed symmetrically by DeLaval nozzle contours. The nozzle exit air flow, serving as the wake edge condition, was continuous at Mach 4 and nominal Reynolds number of 100,000. The measurements spanned the three cases of the adiabatic laminar wake, the adiabatic laminar wake undergoing turbulent transition downstream of the trailing edge, and the laminar wake produced by a nozzle cusp cooled to 0.7 T_o. The mean flow properties of such wakes were predicted by the author in an earlier theoretical study and the present work aimed at providing a test of the theory. Verification was indeed obtained for key assumptions in the theory such as the location of the virtual origin and the existence of laminar similarity and turbulent self-preservation. As predicted, the nozzle cusp temperature affects the temperature and density, but not the fluid velocity. The non-equilibrium region associated with the trailing edge was found to extend to about 14 boundary layer thicknesses. Chief source of anomalies was the network of trailing-edge source and their reflections which introduce pressure gradients suspected to be responsible for an overall depression of the velocity defect. The disparities are, nevertheless, small and the agreement between theory and experiment is considered adequate."

2.3 DISCUSSION AND UTILITY OF THE RESULTS

The results obtained have largely validated and reinforced the theory of References 1 through 4; by this, the present author implies that he would unhesitatingly use the references to predict trends and magnitudes of wake velocity, temperature, density and turbulence magnitudes in most gasdynamic laser cavities configured like the test of Reference 6. While much of the predictions of References 1, etc., were never in doubt (e.g., the turbulent diffusion rates), some findings of the tests of Reference 6, such as that concerning the virtual origin, are new contributions of practical importance.

Following publication of Reference 6, Peterson (Reference 7) completed and published results of a similar experiment at $M \simeq 6$ and at two different Reynolds numbers. He found that the turbulent theory of Reference 1 agreed with his turbulent results, but that his laminar results lay somewhere between our laminar and transitional predictions. Disagreement by about 50% in the velocity and temperature defects between theory (Reference 1) and the data were also noticed by us in the experiments of Reference 6. Two possible explanations exist. One is that shock-wave interactions in the multiple nozzles are important; in fact, in Reference 6, it is noted that the longitudinal pressure along the wake experiences considerable variations. Peterson (Reference 7) noticed a similar event. The theory, on the other hand, is formulated for constant-pressure wakes, while pressure gradients can be generated by the criss-crossing shocks and wakes. The second possibility is that the laminar wakes studied (in both ours and Peterson's cases) are in fact, transitional. Support for this possibility comes from the finding that the transition theory we advanced (see Section 3) predicts the laminar wake investigated in Reference 6 to be on the threshold of becoming turbulent. Transitional wakes indeed have lower velocity defects, exactly as found.

With the satisfactory outcome of the experiments of Reference 6, this OSR effort has produced a compendium of wake characteristics, supported by experiments, which should become a convenient reference for the discussion of the mean-flow properties of laser cusp wakes. Furthermore, the mean-flow results provided us with the basis of making estimates of the turbulent magnitudes, which are presented in References 2 and 4. The outstanding result of these turbulence predictions is that the rms wideband density fluctuation on the wake plane of symmetry ("axis") reaches a maximum of 17% of the mean density in the cavity regardless of Mach number, Reynolds number, heat transfer, etc. Shortage of time prevented extensive turbulence measurements in the present contract; it may even be argued that the existing background in the turbulence properties of adiabatic wakes (see Reference 1 for example) does not merit further work on this subject. However, there is justification for measuring the turbulence properties for cooled nozzle cusps, since no turbulence data for compressible wakes with heat transfer are available. It is planned to make at least some such measurements in the SWT, during the next contract period using the hardware described in Reference 6.

The double wake system with which the experiment of Reference 6 was carried out in Aeronutronic's Supersonic Wind Tunnel is pictured in Figure 1. Typical results of this experiment are shown in Figure 2 in which the theoretical predictions of Reference 6 are also drawn.

3. THE DISSIPATION CRITERION OF TRANSITION

3.1 INTRODUCTION

After twenty years' involvement in transition research, the present author has become skeptical, if not pessimistic, about the prospects of a "theory" of transition, mainly due to the following observations.

First, there is a persistent and large gap between the physical and mathematical approaches to the transition problem on one hand, and the practice of transition prediction on the other. Fluid dynamicists have been postulating theoretical avenues and performing carefully controlled experiments, in seeming disregard of the engineers and designers who invent their own rules for predicting transition in practical situations - and vice versa.

Secondly, the pace of the theoretical and microscopic experimental work on the transition problem is inadequate to handle the large number of critical design problems for which transition prediction is an everyday necessity. For example, boundary layer stability analysis and experiment* have been active for over forty years and yet no transition prediction from the stability findings alone is yet possible. It is indeed remarkable that aircraft and missiles have been successfully operating in a spectrum of flight parameters (Mach and Reynolds numbers, heat transfer, etc.), for so long without the benefit of a rational solution to the transition problem.

Third, many current predictions of laminar-turbulent transition quoted by engineers, are invariably based on empirical formulas which are ephemeral in their history and severely limited in their scope. Typical are cases where these formulas derive from curve-fitting of data from indirect measurements of dubious validity taken in uncontrolled experiments. Such approaches are rarely supported by dimensional analysis and feature mysterious numerical constants which require adjustment every time the flow parameters or regimes change.

These thoughts prompted the author to search for a method of predicting the onset of transition to turbulence which is:

(a) based on accepted physics,
(b) mathematically simple,
(c) valid for all common forms of shear flows,
(d) compatible with past knowledge of shear flow behavior, i.e., utilizing established observations of the transition behavior and the character of laminar and turbulent flows,
(e) independent of the microscopic mechanism of transition.

With these provisions one is led almost naturally to seek a minimal condition, or threshold, for the existence of turbulence in a flow. We are thus forced to think in elementary terms and one such elementary path is to consider the minimum possible width of a turbulent zone. Since such a zone contains secondary motions of a continuum fluid, it is clearly unable to reach the small dimensions possible of a laminar fluid; the factor controlling the width of such a zone,

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*Stability experiments, one of which is done under the present program (Section 4), are invaluable as the only rational approach to finding the mechanism of transition. What is stressed here is the need for complementary approach which can predict the occurrence of transition with no sacrifice of physical insight and rigor.

or the size of eddies, must be connected with the molecular or viscous effects. One is thus led to the familiar concept of the viscous dissipation threshold, below which turbulent motion is forbidden by such dissipation. In abbreviated thinking, this recalls the equally familiar turbulence Reynolds number

$$Re_\Lambda = \frac{u'\Lambda}{\nu} \tag{1}$$

where u' is the velocity fluctuation, Λ the correlation length scale and ν the kinematic viscosity. The limit we seek is a limit to this number.

3.2 VARIATION OF THE TURBULENCE REYNOLDS NUMBER

The quantity Re_Λ will be the focus of our discussion and it will be useful if we first consider its magnitude and development generally, as affected by (and as affecting) the common turbulent shear flows. By definition, such flows feature some type of balance between gain and loss of turbulence and this balance allows the scaling of flow properties by similarity laws. Furthermore, we will temporarily consider isobaric flows ($\Delta p = 0$) since the extension to pressure gradients makes the initial discussion cumbersome without adding insight. The scaling and the isobaric assumption simplify the first overview of the behavior of Re_Λ; later embelishments will not change the picture.

We can gain considerable intuition by first developing an overview of the variation of Re_Λ in the common turbulent shear flows such as the wake, the mixing layer, the jet and the boundary layer. Much of this intuition can be obtained by first neglecting the details of transverse variations of u', Λ and ν. For the moment let us therefore utilize only the basic features of turbulent similarity, that u' and Λ bear certain relations to the velocity scale $u(0) - u_e$ and the width scale b; here $u(0)$, u_e and b are the lowest and highest mean velocity in the flow and the flow width, respectively. We can then write

$$u' = C_1 \left[u_e - u(0)\right] = C_1 \left(\frac{u_e - u(0)}{u_e}\right) u_e, \quad C_1 = \text{const} \tag{2}$$

$$\Lambda = \frac{\Lambda}{b}\, b = C_2 b \quad , \; C_2 = \text{const} \tag{3}$$

In the same spirit, we can write

$$\nu = \frac{\nu}{\nu_e}\, \nu_e = \left(\frac{T}{T_e}\right)^n \nu_e \quad , \; n > 0 \tag{4}$$

The net result is

$$Re_\Lambda = C_3 \left(\frac{u_e - u(0)}{u_e} \right) b \, (Re_b) \left(\frac{T}{T_e} \right)^{-n} = C_3 \, wb \, Re' \left(\frac{T}{T_e} \right)^{-n}, \; C_3 = \text{const} \tag{5}$$

where $w \equiv (u_e - u(0))/u_e$ is the velocity defect of the flow, and Re' the unit Reynolds number which is a fixed number for each given flow.

This simple formula is already sufficient to give us a general picture of the turbulence Reynolds number variation in our typical flows. The product wb is dependent on x, i.e., the distance from the origin of the flow (e.g., the exit of a jet, or the wake-shedding body); it depends on the axisymmetric or two-dimensional geometry of the flow. The Re' fixes the characteristic Reynolds number of the problem, and the temperature ratio contains the compressibility and heat transfer information. Thus Re_Λ is found to depend on distance, on geometry, on Reynolds number and on compressibility and heat transfer.

In incompressible flows, defined as $T = T_e$ in Equation (5), some of the classical flow examples we chose have a Re_Λ increasing with x, some decreasing with x and some remaining constant. In the boundary layer Re_Λ increases in the downstream direction because the layer thickness $\delta = b$ increases while w remains constant. The same is true for the free shear (mixing layer) and for the same reasons. The same is also true for the two-dimensional (2-D) jet, but only because b in it increases faster than its w decreases. In a second class of flows of constant Re_Λ belong the 2-D wake and the axi-symmetric (circular), or A/S, jet. The incompressible A/S wake belongs in a class by itself since its Re_Λ decreases with x, i.e., from the body on downstream.

If compressibility is defined as the case where the temperature decreases in the downstream direction, then the factor $(T/T_e)^{-n}$ in Equation (5) accelerates the increase of Re_Λ with x, or causes such an increase in the flows with previously constant Re_Λ. For the 2-D wake, for example, Re_Λ will now increase in the downstream direction, since the wake cooling will decrease the kinematic viscosity. For the 2-D jet, in which Re_Λ increased with x even when $T = T_e$ because of the rapid spreading of the jet, the cooling of the gas during the expansion will accelerate this increase.

The effect of compressibility on Re_Λ is not as clear, at this juncture, for the boundary and free-shear layers; although their increase in Re_Λ with x will not be affected qualitatively, the rate of increase will be affected since in these cases the edge conditions do not change. Separate discussion of these cases is more appropriate when the details of these flows are examined later.

The axisymmetric wake remains a special - and intriguing - case when temperature effects are present. Decaying temperature competes with the decaying wb product; in many cases an increase and then a decrease of Re_Λ is possible here. This case, too, will be discussed later.

3.3 THE TRANSITION CRITERION

Our entire approach is based on the statement that no turbulent flow is possible if Re_Λ lies below a certain magnitude $Re_{\Lambda o}$ which is universal. Conversely, we say that if the flow is turbulent then $Re_\Lambda > Re_{\Lambda o}$ everywhere within it. The question of precisely what $Re_{\Lambda o}$ is will be discussed below. For the moment, let us see what this statement implies for the common turbulent flow examples described just above.

Let us temporarily return to an example with an obvious history of $Re_{\Lambda o}$, such as that of the incompressible 2-D jet idealized so that it can be represented by Equation (5) with $T = T_e$:

$$Re_\Lambda = \text{const bw } Re' = (\text{const})\frac{b}{b_o} w \left(\frac{u_e b_o}{\nu_e} \right) = C_4 \frac{b}{b_o} w \, Re_{bo} \qquad (6)$$

In this expression we already agreed to consider C_4 a scaling ratio which is a pure number for all 2-D jets; thus the Reynolds number Re_{bo}, where b_o is a constant for this particular jet (e.g., the jet exit size or the jet thrust integral) controls the absolute magnitude of Re_Λ. The variations

$$\frac{b}{b_o} \sim x \qquad (7)$$

$$w \sim {}^1/_{\sqrt{x}} \qquad (8)$$

$$\frac{b}{b_o} \sim \sqrt{x} \qquad (9)$$

show how the jet turbulence Reynolds number increases from the jet exit on.

Let us now, with the aid of Figure 3, visualize the Re_Λ variation of such a two-dimensional turbulent jet, first in the case where Re_{bo} is so large that Re_Λ lies everywhere above $Re_{\Lambda o}$. If we compute Re_Λ for such a given jet we will find $Re_\Lambda > Re_{\Lambda o}$ everywhere, and we will thus find no reason why such a jet cannot exist. Consider, however, the jet of the second example shown in Figure 3, where $Re_\Lambda > Re_{\Lambda o}$ beyond a certain distance x_o from the jet origin, but $Re_\Lambda < Re_{\Lambda o}$ for $x < x_o$. If such a jet had been postulated as everywhere turbulent, this postulate would have been invalid for a length x_o beyond the jet exit. That is, if we began checking on the Re_Λ of this jet from infinity forward toward the origin, we would arrive at a point x_o ahead of which we would find the molecular viscosity capable of dissipating the turbulence. Turbulence is, therefore, impossible ahead of x_o; and the jet would have to be laminar between the exit and x_o.

It must be noted, of course, that the condition $Re_\Lambda > Re_{\Lambda o}$ is a necessary one, but not a sufficient one. The fact that the jet will be laminar ahead of x_o is established, but not so that the jet will be turbulent aft of x_o. For that matter, the "high Re_{bo}" example of Figure 3 may also be laminar; all we had said previously is that it is possible for this case to have a fully turbulent jet from the exit on to infinity. The condition of sufficiency is, therefore, lacking and will remain so. Nevertheless, we will now begin referring to the physical boundary x_o as the "transition point," staying aware that the term refers to the threshold in the necessary condition.

The arguments built, above, around the 2-D jet require no change when it comes to other flows of the same class, i.e., flows of monotonic increase of Re_Λ with x. These flows include the free shear layer and boundary layer, the compressible A/S jet and the compressible 2-D wake. The procedure in all these cases is the same: we first assume that the flow is fully turbulent and then, beginning with the far end of the flow, we calculate the Re_Λ proceeding toward the flow origin. We may never find that $Re_\Lambda < Re_{\Lambda o}$, in which case we say that it is possible for the entire flow to be turbulent (or we may risk the statement that the entire flow is turbulent). Or, if we come to a point where Re_Λ just equals $Re_{\Lambda o}$, we can say that the flow ahead of that point is laminar, or even risk the statement that transition to turbulence occurs at that point. We fully intend to take the latter risk in this work.

The situation is even simpler for the class of flows with constant Re_Λ, like the incompressible axisymmetric jet and the incompressible 2-D wake. In these cases, according to everything said so far, the flow is either wholly laminar or wholly turbulent; there is no chance that transition will appear at some distance downstream of the flow origin. The unique case, however, of the axisymmetric wake is more complex. In the incompressible version of this flow, Re_Λ decreases monotonically and two possibilities exist according to our thinking: either this wake is wholly laminar or it is turbulent for some distance behind the body and turns laminar beyond some "relaminarization," or reverse transition, point. In the compressible case (decaying T in Equation (5)), a maximum in the variation $Re_\Lambda(x)$ may even exist and a turbulent "piece" of wake appears somewhere downstream, preceded and followed by laminar flow.

Our association of the threshold $Re_\Lambda = Re_{\Lambda o}$ with transition will look much less premature if we realize that actual transition, as observed to occur in the sample flows discussed above, also follows the same rules. For example, whereas 2-D incompressible wakes are observed to be either wholly laminar or wholly turbulent, the compressible 2-D wake can become turbulent at some downstream location. Even the odd behavior of transition in a compressible axisymmetric wake, predicted in the previous paragraph has been recognized and discussed by Lees (Reference 8). Thus, the experimental evidence strongly supports our postulate, even beyond its stated limitations. What we have here is the ability to make a general and apparently valid statement about laminar-turbulent transition, which is based on physics and which does not require "a priori" extensive mathematical development.

3.4 THE NUMERICAL VALUE OF THE REYNOLDS NUMBER THRESHOLD

To make quantitative predictions of the transition occurrence, we need a numerical value for $Re_{\Lambda o}$. Such a value has been found by reversing the process, i.e., by utilizing transition data, in this instance from two-dimensional wakes. The derivation, reported in Reference 9, gave a value of $Re_{\Lambda o} \approx 15$. This magnitude, consistent with the intuitive expectation of $Re_{\Lambda o} >> 1$, will serve us until it is adjusted (perhaps only slightly) by later data or rationalized by more precise mathematical arguments.

3.5 APPLICATIONS

3.5.1 TRANSITION IN TWO DIMENSIONAL WAKES

The first use of the threshold criterion appeared in 1978 in Reference 9. The application involved two dimensional wakes for which a satisfactory amount of experimental data is available. The results are outlined below, especially because they outline the general method of application.

The point of departure is Equation (1), in which the quantities Λ, ν and u' are sought to be expressed in terms of the transition distance x_T and the parameters of the wake flow. For the compressible two dimensional wake (Reference 9):

$$u'(0) = C_1 \left[u_e - u(0)\right] (t + 1)^{\frac{1}{2}} , \quad C_1 = \text{constant} \tag{10}$$

where (0) refers to the axis (centerplane) values and "e" to the edge conditions, and where t is the "temperature defect"

$$t \equiv \frac{T(0) - T_e}{T_e} \tag{11}$$

For the scale length Λ we obtain from Reference 9

$$\Lambda = C_2 \; C_D h \; \frac{u_e}{u_e - u(0)} \tag{12}$$

where $C_D h$ is the "wake drag thickness," i.e., the integral of its momentum flux. The kinematic viscosity in (1) is furthermore converted to temperature via the relation

$$\mu \sim T^k \tag{13}$$

where k is fixed by the thermodynamics of the gas. It is also argued that at transition, the wake (centerplane) temperature will be close to that of the laminar wake just preceding transition and if the latter occurs at a distance x from the body, then laminar wake theory gives

$$t = \frac{\sqrt{\sigma}(\gamma-1)(1+Q)M_e^{\,2}R_w}{4\sqrt{\pi}} \left(\frac{C_D h}{x} \right)^{\frac{1}{2}} \tag{14}$$

where σ is the molecular Prandtl number, γ the specific heat ratio, M_e the edge Mach number, Q the non-dimensional version of the heat $\bar{Q}$ exchanged between body and flow

$$Q \equiv \frac{\bar{Q}}{\frac{1}{2}\rho_e u_e^3 C_D h} \tag{15}$$

and R_w is the wake Reynolds number

$$Re_w \equiv \frac{u_e}{\nu_e} C_D h \tag{16}$$

If these equations are inserted into (1) at the point x_T where transition occurs, i.e., when $Re_\Lambda = Re_{\Lambda o}$, we obtain

$$\frac{x_T}{C_D h} = \frac{\sigma(\gamma-1)^2 M_e^4 (1+Q)^2 Re_w}{16\pi} \left[\left(\frac{Re_w}{Re_o}\right)^{1/k + 0.5} -1 \right]^{-2} . \tag{17}$$

where $$Re_o \equiv \frac{Re_{\Lambda o}}{C_1 C_2} \tag{18}$$

Equation (17) is the desired end product; it is a formula giving the transition distance x_T in terms of the given parameters of the problem k, σ, γ, M_e, Q, Re_w and $C_D h$; the constants C_1 and C_2 are presumably universal, and have been determined by previous experiments with turbulent wakes (e.g., Reference 10). Equation (17) gives, for the first time, the dependence of the transition distance on the type of gas, for example, x_T should be about twice as large in helium ($\gamma = 1.66$) than in air ($\gamma = 1.4$), all other conditions being equal. It also explains why the observed transition distances increase so rapidly with Mach number and why one can delay transition by heating the wake-shedding body ($Q > 0$).

Comparison of Equation (17) with experimental data, discussed in Reference 9, gave good results and convinced this author that further work on the two-dimensional wake is not needed.

3.5.2 THE TWO DIMENSIONAL BOUNDARY LAYER

Boundary layers were addressed after the work of Reference 9 was completed, since abundant transition data for comparison exist and since the boundary layer transition issue is of great current interest. This study is continuing; the following text describes work done in the spring and summer of 1978.

To apply the criterion (1) to the boundary layer, calculation of the quantities u', Λ and $T(y)$ (where y is the usual height-above-the-wall coordinate) is again needed. In specifying these, existing knowledge is utilized ranging from known data on u' and Λ to the more prosaic textbook formulas for $u(y)$ and $T(y)$ in turbulent boundary layers. Surprising insight, in fact, can be gained by manipulating commonplace notions on those boundary layer transition features frequantly observed in experiments. One such feature, necessary for exploiting the criterion (1) further, is the geometry of the transition zone.

3.5.2.1 The Virtual Origin of the Turbulent Boundary Layer Flow. We can make use of existing knowledge on boundary layers to compose the geometrical picture of transition in a boundary layer shown in Figure 4. The key to this picture is the well-known difference in boundary layer growth rates between laminar and turbulent flows. The laminar boundary layer edge δ is definable despite the diffuse nature of the laminar interface by the simple expedient of picking a percentage of the velocity ratio, e.g. $u(y=\delta) = 0.99u_e$. Similarly, the turbulent boundary layer edge is definable despite the turbulent intermittency, e.g., by picking δ to correspond to a given intermittency factor.

The consequences of the construction of Figure 4 are:

(a) A theoretical transition "point" x_T is defined at the intersection of the laminar and turbulent growth curves.

(b) At this transition "point" the laminar and turbulent thicknesses are the same:

$$\delta_t(x = x_T) = \delta_\ell (x = x_T) \tag{19}$$

(c) The virtual origin x_o of the turbulent flow lies downstream of the origin of the flow (at $x = 0$) and ahead of the transition point.

Since the choice of ordinate in Figure 4 is arbitrary, we can replace it by the integrals δ^* or θ:

$$\delta^*_t (x = x_T) = \delta^*_\ell (x = x_T) \tag{20}$$

$$\theta_t (x = x_T) = \theta_\ell (x = x_T) \tag{21}$$

so that the shape factors such as $(\delta/\theta)_t$ and $(\delta/\theta)_\ell$ would be the same for laminar and turbulent boundary layers. Calculations done in the present contract period for typical laminar and turbulent boundary layers showed that this is so to the extent that we can describe the turbulent case by the present empirical approaches. (See Figure 5).

To act as further reinforcement of the construction of Figure 4, we have calculated the virtual origin of the turbulence in a boundary layer by using the standard form of the momentum thickness (e.g., Reference 11) in the turbulent case,

$$\theta_t = \frac{7}{12} (0.0262)\ f\ (M) \frac{Re_\Delta^{\frac{6}{7}}}{Re'} \tag{22}$$

and in the laminar case,

$$\theta_\ell = 0.68 \frac{(Re_{xT})^{\frac{1}{2}}}{Re'} \tag{23}$$

where

$$f(M) \equiv \left(\frac{2}{2+\frac{\gamma-1}{2}M_e^2} \right)^{\frac{5}{7}} \tag{24}$$

$$\Delta \equiv x_T - x_o \tag{25}$$

$$Re' = \frac{u_e}{\nu_e}\ ,\ Re_\Delta = \frac{u_e}{\nu_e}\Delta\ ,\ Re_{xT} = \frac{u_e}{\nu_e}x_T \tag{26}$$

By equating Equation (22) with (23), according to Equation (21), we obtain

$$\frac{x_T - x_o}{x_T} = \frac{83.8}{[Re_{xT}]^{5/12}[f(M)]^{7/6}} \tag{27}$$

The results plotted in Figure 6 show that at low M_e and high Re_{xT}, the virtual origin x_o lies very close to the transition point x_T. It would appear that in the other extreme of high M_e and low Re_{xT} the virtual origin can lie upstream of the actual flow origin; in the flat plate boundary layer (to which equations (22) and 23) in anyway refer) this would mean that the virtual origin lies upstream of the plat leading edge. There is nothing unreallistic about this concept, since the virtual origin is only an artifice convenient for calculating the flow, and there are many situations in fluid flows where the virtual origin lies upstream of the physical generator of the flow.

Since the transition Reynolds number Re_{xT} is known to depend on M_e (in a manner which is the target of this work), the actual trace of $(x_T-x_o)/x_T$ vs M_e will cut across the curves of Figure 6. The general view is that Re_{xT} increases with M_e quite rapidly once $M_e > 5$, and this will cause this trace to remain below unity in the figure. An example of this is shown by the dashed line in Figure 6 which anticipates the forthcoming results of Re_{xT} vs M_e for the flat plate. It is seen that in the subsonic and supersonic regimes x_o lies rather close to x_T, but shifts to the origin of the flow (i.e., x_o becomes nearly zero) at hypersonic speeds.

3.5.2.2 Calculation of the Reynolds Number Across the Flow. We now return to the original objective of applying Equation (1) to the boundary layer, with a view to deducing the transition location for various conditions.

In the published work of Reference 9, the Re_Λ was assumed constant across a section of the flow. So far, this idea was retained in the previous remarks because we wanted to put emphasis on the principle and to discuss various shear flows in a general way. If we still were to consider Re_Λ constant across a turbulent boundary layer, we could quickly derive a transition criterion utilizing all that has been previously said. For the incompressible 2-D layer on a flat plate, for example, Equation (10) would become (since u(0) = 0):

$$u' = C_1 u_e \tag{28}$$

while (12) would become, using "t" to denote the turbulent case,

$$\Lambda = C_2 \delta_t \tag{29}$$

and since $\nu = \nu_e$, the Re_Λ at any position along the layer would be

$$Re_\Lambda = C_1 C_2 \left(\frac{u_e \delta_t}{\nu_e} \right) \tag{30}$$

The method would then have us assume that very far from the leading edge the layer is turbulent and that Re_Λ in it is given by (1); then as we advance toward the leading edge Re_Λ decreases since δ_t decreases until we arrive at the threshold

$$Re_\Lambda = Re_{\Lambda o} = C_1 C_2 \left(\frac{u_e \delta_{tT}}{\nu_e} \right) \tag{31}$$

where "T" signifies transition. At that point, however, $\delta_{tT} = \delta_{\ell T}$ according to Equation (19); furthermore, a more familiar form is obtained by using the laminar form factor C' to convert $\delta_{\ell T}$ to $\theta_{\ell T}$:

$$Re_{\Lambda o} = C_1 C_2 C' \left(\frac{u_e \theta_\ell}{\nu_e} \right)_T \tag{32}$$

The quantity in parenthesis is the familiar momentum Reynolds number at transition based on the laminar momentum thickness; thus,

$$Re_{\theta T} = \frac{Re_{\Lambda o}}{C_1 C_2 C'} = \text{constant} \tag{33}$$

and the transition momentum Reynolds number for the incompressible flat plate boundary layer has been found. Note that the four constant quantities forming $Re_{\theta T}$, although empirical, have definite physical meaning and can be found in the literature of turbulence, not transition, experiments.

We now come to grips with the fact that Re_Λ is not constant across the flow, but varies with y as u', Λ and ν vary with y. Suitable expressions for u'(y), etc., can be found in the present bibliography on turbulent boundary layers, which is considerable. The following describes the work done so far to pick these functions.

(a) Velocity Fluctuations

The u'(y) variation was obtained by examining experimental data available in the literature. Guidance for the most rational correlation of these data was provided by the original work of Morkovin (Reference 12) according to which u' should scale with the friction velocity and a compressibility factor. The rationale, together with a list of data sources is given in Reference 13. In that reference it is, therefore, pointed out that the correlated form is

$$u' \left(\frac{y}{\delta} \right) = C_1 \left(\frac{y}{\delta} \right) \sqrt{\frac{\tau_w}{\rho(y)}} \tag{34}$$

Inspection of the data revealed that the function C_1 is additionally sensitive to the wall-to-stagnation temperature ratio T_w/T_o but independent of the Reynolds number. The function was curve-fitted as follows:

$$C_1 \left(\frac{y}{\delta}, \frac{T_w}{T_o} \right) = \left[1.814-0.2188516 \frac{y}{\delta} - 2.797874 \left(\frac{y}{\delta} \right)^2 + 1.430676 \left(\frac{y}{\delta} \right)^3 \right]$$

$$\times \left(0.4 + 0.6 \frac{T_w}{T_o} \right) \tag{35}$$

(b) The Longitudinal Scale of Turbulence

Information of scale lengths is extremely meager; on the basis of this author's earlier experiences, the following was adopted, independent of y/δ:

$$\Lambda = 0.2\delta_t = C_2\delta_t \tag{36}$$

(c) Kinematic Viscosity

$$\nu = \nu_e \frac{\rho_e}{\rho} \left(\frac{T}{T_e}\right)^n \tag{37}$$

where we have assumed, quite safely, that the layer is isobaric. The constant n is taken to be 0.75 in subsequent computations.

(d) Other Variables

When re-casting Equation (32) into a form suitable for computations, we need the following additional properties of the turbulent boundary layer: velocity profile form, temperature-velocity relations, skin friction variation, and a connection between skin friction and momentum thickness. Unlike the turbulence properties u' and Λ, these additional properties are supported by a large volume of experimental data. Improvements and additions of such data are constantly appearing in the literature, and the present analysis could be certainly postponed until these properties are determined with great precision for a wide variety of boundary layers. In order to proceed, therefore, we limited our approach to zero-pressure-gradient flat plate boundary layers in $0 < M_e < 10$, $0.2 < T_w/T_o < 1$ in exchange for using established formulas for the needed parameters as follows:

- for the skin friction,

$$C_f = \frac{2\,\tau_w}{\rho_e u_e^2} = 0.0262\ f\,(M_e)\ \frac{1}{Re_x^{1/7}} \tag{38}$$

with

$$f(M_e) \equiv \left(\frac{2}{2+\frac{\gamma-1}{2} M_e^2} \right)^{5/7} \tag{39}$$

$$Re_x \equiv \frac{u_e x}{\nu_e} \tag{40}$$

- for the momentum thickness growth,

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho_e u_e^2} = \frac{C_f}{2}\ . \tag{41}$$

which, with the aid of (38), integrates to

$$\theta = \int_o^x \frac{d\theta}{dx}\,dx = \frac{7}{12}\ C_{fx} \tag{42}$$

- for the velocity profile,

$$\frac{u}{u_e} = (y/\delta)^{1/7} \tag{43}$$

- for the temperature-velocity relation, the Crocco relation

$$\frac{T}{T_e} = \frac{T_w}{T_e} + \left(1 + \frac{\gamma-1}{2} M_e^2 - \frac{T_w}{T_e} \right) \frac{u}{u_e} - \frac{\gamma-1}{2} M_e^2 \left(\frac{u}{u_e} \right)^2 \tag{44}$$

Using Equations (38), (41), (43) and the isobaric assumption $p = p_e$ into Equation (32), we now obtain

$$Re_\Lambda = C_2 C_1 \left(\frac{u_e \delta_t}{\nu_e} \right) \sqrt{\frac{C_f}{2}} \left(\frac{\rho}{\rho_e} \right)^{n+1} \tag{45}$$

Since it is customary to refer Reynolds numbers to the momentum thickness, the following conversions can be next made using the formulas previously presented:

$$C_f = C_3^{\frac{7}{6}} [f(M_e)]^{\frac{7}{6}} \left(\frac{7}{12} \right)^{\frac{1}{6}} Re_\theta^{\frac{-1}{6}} \quad ; \; C_3 = 0.0262 \tag{46}$$

$$Re_\theta \equiv \frac{u_e \theta_t}{\nu_e} \tag{47}$$

$$\frac{u_e \delta_t}{\nu_e} = \frac{\delta_t}{\theta_t} Re_\theta \tag{48}$$

so that the final product is:

$$\frac{Re_\Lambda}{Re_\theta^{11/12}} = 0.01767 \, C_1 \frac{\delta_t}{\theta_t} [f(M_e)]^{\frac{7}{12}} \left(\frac{\rho}{\rho_e} \right)^{n+\frac{1}{2}} \tag{49}$$

Thus, we see that for a given turbulent boundary layer, the transverse variation of Re_Λ depends on the corresponding transverse variations of the fluctuations via C_1 (given in Equation (35)) and of the density ratio obtained by combining (43) with (44). Its edge Mach number dependence enters explicitly via $f(M_e)$ and implicitly via ρ/ρ_e and the shape factor δ/θ. The Re_Λ depends on the ratio T_w/T_o through its effect on C_1 (Equation (35)) and ρ/ρ_e (by the Crocco relation). Finally, the dependence on Re_θ is almost wholly left to the factor dividing Re_Λ in Equation (49); the dependence of δ/θ on Re_θ is apparently negligible.

3.5.2.3 Computations of the Turbulence Reynolds Number. Using Equation (49), the ratio on the l.h.s. was computed for $M_e = 0, 1, 2 \ldots 9, 10$ and $T_w/T_o = 0.2, 0.4, 0.6, 0.8$ and 1. Typical results are shown in Figures 7 through 9.

On the wall, the value of Re_Λ (or $Re_\Lambda/Re_\theta^{11/12}$) is controlled primarily by the wall temperature, while its value at the layer edge is controlled by the magnitude of the fluctuations there. Since the fluctuations (i.e., C_1) decrease, and the density ratio increases, toward the edge Re_Λ usually has a maximum value at some point intermediate to $y = 0$ and to $y = \delta$. Also, while Re_Λ is nearly proportional to Re_θ, it decreases as M_e is increased and also as the wall is cooled. For example, in the adiabatic case

$$\text{Maximum } \frac{Re_\Lambda}{Re_\theta^{11/12}} = 0.33 \text{ at } M_e = 0 \tag{50}$$

$$= 0.09 \text{ at } M_e = 10 \tag{51}$$

Similarly, at $M_\infty = 10$ the maximum Re_Λ value decreases from 0.09 in the adiabatic case to about 0.041 at $T_w/T_o = 0.2$.

3.5.2.4 Transition Predictions from the Computed Reynolds Numbers. In contrast to the hypothetical example of Section 3.5.2.2 where Re_Λ was assumed independent of y, the calculations described above show that Re_Λ can vary considerably in the range $0 < y < \delta$. The meaning of this variation is necessarily complex.

The rules established can be explained with the aid of Figure 10. The boundary layer in question is first assumed turbulent throughout so that its variation $Re_\theta(x)$ is known. Thus, at any x, a curve $Re_\Lambda(y/\delta)$ can be plotted, like those of Figures 7 through 9, except that the ordinate is replaced by just Re_Λ. As done before, this Re_Λ is compared with the threshold $Re_{\Lambda o}$, and three possibilities exist:

(a) $Re_{\Lambda o} > Re_\Lambda$ everywhere (case (a) in Figure 10). The boundary layer is laminar at the position x where this Re_Λ was computed.

(b) $Re_{\Lambda o} < Re_\Lambda$ everywhere (case (b) in Figure 10). The boundary layer can be turbulent at this x; we assume that it is.

(c) The $Re_{\Lambda o}$ and Re_Λ curves cross (case (c) in Figure 10). Strictly speaking, no statement can be made in this case based on the present approach, since the Re_Λ curve in this case was computed (from Equation (49)) assuming that the layer was turbulent for all y/δ. However, we can see that if the layer was turbulent everywhere in the sense that Equations (38), etc., are valid, then Re_Λ would indeed have a physically possible value, which is larger than $Re_{\Lambda o}$. In other words, there is no turbulent boundary layer which could produce such a case as shown by (c) in Figure 10. This case, too, must then represent a "non-turbulent" boundary layer.

The above considerations led us to the only remaining possibility, i.e., that transition (departure from turbulence) occurs as soon as the Re_{Λ_o} line first touches the Re_Λ curve (case (d) in Figure 10). But if this is our definition of transition then, to be consistent, the Re_θ value (of this specific boundary layer) where this happens must be that based on the laminar θ, according to Figure 4. Thus, the value of Re_θ at which this first coincidence of the two curves occurs, is the standard laminar Re_θ of transition.

Following this definition, the transition Reynolds numbers $Re_{\theta T}$ were computed for boundary layers in the chosen range of $0 < M_e < 10$, $0.2 < T_w/T_o < 1$. The results, shown in Figures 11 through 13, display some quantitative and some qualitative features which are remarkable for their similarity to common observations. These features are:

1. For fixed T_w/T_o, $Re_{\theta T}$ first decreases with M_e and then increases. The edge layer ($y/\delta = 1$) is responsible for the decreasing portion and the wall layer ($y \simeq 0$) for the increasing portion.

2. Below $M_e \simeq 4.5$ cooling always delays transition (increases $Re_{\theta T}$, "stabilizes the layer"). Above $M_e \simeq 4.5$ transition reversal occurs. The reversal T_w/T_o decreases as M_e increases; at large M_e (of order 10 for example), cooling moves transition forward (to smaller $Re_{\theta T}$) until a rather low T_w/T_o is reached, below which T_w/T_o has a stabilizing influence again.

3. It follows from the above that the reversal phenomenon ($\partial Re_{\theta T}/\partial M_e$ or $\partial Re_{\theta T}/\partial(T_w/T_o)$ changes sign) occurs whenever the transition-ending role is switched from one y/δ position in the layer to another (e.g., from the wall layer to the edge layer or vice versa).

4. The transition scaling parameters are largely the same as those scaling the turbulent boundary layer, i.e., those factors affecting the friction coefficient C_f, profile ρ/ρ_e and distribution $C_1(y/\delta)$ of the fluctuations in a turbulent layer, etc., also affect transition.

3.5.3 DISCUSSION OF THE BOUNDARY LAYER TRANSITION RESULTS

The results shown in Figures 11 through 13 were obtained by postulating that transition occurs when the straight Re_{Λ_o} lines on Figure 10 first encounter the bottom part of the Re_Λ curve obtained at that position x in the boundary layer. Any further increase in Re_θ will raise the Re_Λ curve above the Re_{Λ_o} curve, and the situation will then resemble case (a). Since the latter case represents the "established" turbulent boundary layer, it is clear that if our transition criterion is valid, it is a criterion for the "end" or completion of transition rather than for its "beginning" or "onset". So we say that every part of the layer, from $y = 0$ to δ, will have to meet the condition $Re_\Lambda \geq Re_{\Lambda_o}$ before the entire boundary layer is considered turbulent.

It is then tempting to consider the case (e) of Figure 10, where Re_θ is just large enough to bring the Re_Λ curve into first contact with the $Re_{\Lambda o}$ limit, as the "beginning" of transition. This suggestion cannot be taken too seriously because of the arguments in Section 3.5.2.4; however, one credible feature of this suggestion is that the zone of the boundary layer at $y = y_o$ (Figure 10) is the place in the layer where the first appearance of the turbulence will occur. That turbulence will, in fact, first appear in specific zones of the boundary layer, rather than uniformly across it, a phenomenon which has intrigued observers for years (Reference 14). Figure 14 plots the location of y_o/δ from the present calculations. This picture shows not only that y_o/δ increases with M_e and T_w/T_o has has been observed, but also that a certain "switching", triggered by heat transfer, occurs at low M_e. The increasing y_o/δ as M_e increases is also in general agreement with the "critical layer" idea of hydrodynamic stability theory, which is also plotted in Figure 14.

In addition to the work necessary to arrive at Figures 11 through 13, several issues related to it were examined. One such issue refers to the shape of the velocity profile and its effect on the density profile. Clearly, Equation (43) i.e., the seventh-power law, is the most elementary one which can be assumed, and it can be improved by replacing it with the Clauser-Coles logarithmic velocity formulation. Also, the Crocco relation is equally crude and can be replaced for example, by the Whitfield-High relation (see Reference 15). Such changes were attempted without any major changes in the results of Figures 11 through 13, so far.

A matter of concern is the precise values of the quantities on the r.h.s. of Equation (49). For example, the variation of u' across the layer, as given by the curve-fit of Equation (35), ignores the existence of the laminar sublayer. At the boundary layer edge, also, the value of the fluctuations u' is of order 0.2; since the edge values are critical in deducing $Re_{\theta T}$, improving the $u'(y = \delta)$ should have some effect on the $Re_{\theta T}$ deduced so far.

3.6 PRESENT ISSUES, OBJECTIVES AND ACTIVITIES

There is little doubt that the simple physical criterion as expressed by Equation (1) has already been very successful in making valid and practically useful predictions of transition onset. This is not only attested to by the wake results (Reference 9) but also by the boundary layer results of Figures 11, 12 and 13. The present and future work to be done on this contract deserves considerable thought because of this very success.

In view of the necessarily restricted contract resources and the requirements for application arising from various engineering centers of the Air Force, the following three areas are considered important to pursue:

1. Strengthening of the underlying concepts of the theory.
2. Extension to boundary layers with other transition-causing factors beside M_e and T_w/T_o, e.g., freestream turbulence, pressure gradient, etc.

3. Application to flows other than boundary layers, e.g., the free shear layer.

We are presently pursuing the first item of this list. The sublayer is being taken into account by modifying the curve-fit of Equation (35) to account for the sublayer existence and thickness. The fact that the fluctuations do not go to zero at $y = \delta$ is also of concern; the fact itself is not in doubt, but it must be rationalized on valid physical grounds. The problem here is analogous to the differences in lateral spreading of heat and momentum in a boundary layer, i.e., a Prandtl number must exist which controls the lateral spread of turbulence as well as of heat.

In the near future, we will attempt to extend Equation (49) to account for pressure gradient, surface roughness and freestream turbulence. It is clear that these will affect $Re_{\theta T}$ by affecting separately the skin friction and fluctuation intensity. Preliminary work has shown that the present concept is perfectly capable of explaining the acceleration of transition by these three agents. The key to success here is the existence or non-existence of turbulent boundary layer data. That is, to find how surface roughness affects transition, it is necessary to know the roughness effect on C_f for <u>turbulent</u> boundary layers.

Also on the schedule is an attempt to apply the present approach to the free shear layer, since this problem is of interest to designers of systems containing heterogeneous mixing. The chief obstacle here is the lack of shear layer turbulence data for use in the theory, as well as of shear layer transition data for comparison with the forthcoming results.

4. HYDRODYNAMIC STABILITY OF THE HYPERSONIC BOUNDARY LAYER

In the present period the final analysis was done and the results published, of the hypersonic boundary layer experiment performed in 1976-77. This work represents a complementary, and more conventional, approach to the boundary layer transition problem than adopted in the "dissipation theory" project described in Section 3 . Whereas the latter is a theoretical approach to transition dealing with a necessary condition for the existence of turbulence, the stability experiment gathered data dealing with the mechanism of transition onset.

Stability experiments under AFOSR sponsorship at Aeronutronic began in 1975, and have already produced nearly half the existing information on hypersonic boundary layer disturbance growth and amplification. The program takes advantage of the author's graduate research in the stability of laminar boundary layers on flat plates at $M_e = 5.8$ (Reference 16). In the 1975-76 period, this investigator analyzed and published (Reference 17) data taken in the laminar and transitional flow over a 5° half-angle cone taken at $M_\infty = 8$ in the AEDC Wind Tunnel B. The two experiments, referenced just above, along with the work of Kendall (Reference 18) hinted at a very complex stability diagram at hypersonic speeds.

The present work was undertaken partly in order to expand the Reynolds number and frequency ranges covered in References 17 and 18, and partly in order to discern the effect of wall cooling on stability. The experiment was done in the spring of 1977 in Wind Tunnel B at AEDC, using a 4° half-angle cone at two temperature ratios, $T_w/T_o = 0.4$ and 0.8. The results were analyzed in the present contract period and presented at the 1978 Heat Transfer and Fluid Mechanics Institute in June 1978 (Reference 19). Briefly, the results were as follows.

1. The data obtained in this experiment are consistent with all other data reported to date at $M_\infty \simeq 7$. A complete picture of disturbance amplification is thus available at this M_∞ in the range $1100 < R < 2400$ (or $1.2 \times 10^6 < Re_x < 5.8 \times 10^6$) and for T_w/T_o down to 0.4. Moreover, the consistency of the results of three independent experiments done with essentially different wind tunnels and models, removes suspicion that the instability was grossly influenced by peculiarities of the flow or the test article.

2. The present results fully vindicate the admonition of Mack and others that boundary layer stability at hypersonic speeds is a complex phenomenon. No fewer than three distinct unstable regions have been found in contrast to the single instability "loop" encountered at subsonic or supersonic speeds. Unstable disturbances exist for $F > 4 \times 10^{-4}$, or considerably higher in frequency than previously thought.

3. The boundaries and amplification rate isotherms have been isolated with considerable precision. The "lower unstable region" has been mapped over the entire range of $1100 < R < 2400$. A lower neutral branch for this region has been tentatively identified but its minimum Reynolds number has not been found. By inference from the work of Mack, this region is the one generated by inclined first mode, and normal second mode instabilities. It is clear that the first mode, which dominates instabilities at lower Mach numbers, is rather unimportant.

4. The middle unstable region, newly mapped in this work, serves to amplify disturbances at considerably higher frequencies than the first region. What is surprising about this region is its rather large amplification rates; these are nearly the same as those found in the lower unstable region, i.e., of order 2 to 5×10^{-3}. Precisely what instability mode is active here is presently unknown.

5. A third unstable region (here called "upper unstable") also exists. Information of this region is still fragmentary but it must involve wavelengths of order 0.2δ or smaller.

6. The passage of each Fourier component through the unstable regions results in the formation of the well-known "laminar waves." Of these, the prominent ones belong to the lower unstable region (and to the second instability mode) while the growth of a weaker "second harmonic" has also been observed, associated with the middle unstable region. Physically, the waves are about twice as long as the boundary layer is thick. The significant feature of these

waves is the new finding that the locus of their maximum amplitude on the stability diagram eventually crosses the upper neutral branch of the lower instability region. By the definition of the neutral branch, this crossing signals the end of the growing period of the laminar waves and the onset of boundary layer transition.

7. The act of lowering the surface temperature from 0.8 to 0.4 has only a negligible effect on the location and size of the unstable regions and a similarly slight effect on the laminar waves, but the amplification rates nearly double in both the lower and the middle unstable regions. This overall increased rate of amplification could only mean earlier transition to turbulence, exactly as observed. This marks the first occasion where the cooling effects on amplification and transition were recorded simultaneously; their mutually consistent behavior should renew confidence in the relevance of hydrodynamic stability to the boundary layer transition problem.

5. PUBLICATIONS AND PRESENTATIONS

In the period October 1977 through September 1978, the following publications were prepared on the work performed under this contract.

5.1 TECHNICAL REPORTS

- Demetriades, A., "Experimental Test of the Theory of Multiple-Nozzle Cusp Wakes," Aeronutronic Publication U-6395, Ford Aerospace & Communications Corp., Newport Beach, California, December 1977 (also to appear as AFWL TR).

- Demetriades, A., "Laminar Boundary Layer Stability Measurements at Mach 7 Including Wall Temperature Effects," AFOSR TR No. 77-1311 (Aeronutronic Publication No. U-6381), November 1977.

5.2 JOURNAL ARTICLES

- Demetriades, A., "New Experiments on Hypersonic Boundary Layer Stabisity Including Wall Temperature Effects," Proceedings of the 1978 Heat Transfer and Fluid Mechanics Institute (Pullman, Washington, June 26-28, 1978), Stanford University Press, 1978, p. 39.

- Demetriades, A., "Transition to Turbulence in Two-Dimensional Wakes," AIAA J. Vol. 16, No. 6, June 1978, p. 587.

- Demetriades, A., "Turbulent Fluctuations in the Wakes of Gas-Dynamic-Laser Nozzle Cusps," J. of Energy, Vol. 2, 1978.

5.3 PRESENTATIONS AT MEETINGS AND SYMPOSIA

- Demetriades, A., "New Experiments on Hypersonic Boundary Layer Stabisity Including Wall Temperature Effects," presented at the 1978 Heat Transfer and Fluid Mechanics Institute, University of Washington, Pullman, June 26, 1978.

- Demetriades, A., "Basic Research in High Speed Aerodynamics," delivered at seminar, Mechanical Engineering Department, Montana State University, at Bozeman, October 27, 1978.

- Demetriades, A., "Consequences of Imposing a Necessary Condition for the Appearance of Transition," presented at the 31st Annual Meeting, Division of Fluid Dynamics, University of Southern California, Nov 20, 1978.

<u>Interaction with Government Laboratories</u>

- Space and Missile Systems Organization (SAMSO), LAAFS: Exchange of personnel, consultations and technical direction meetings, chiefly on related Contract F04701077-C-0113. The dissipation theory of transition has been presented as a candidate scheme for explaining blunt body boundary layer transition data obtained in the above contract.

- Arnold Engineering Development Center (AEDC), Tullahoma, Tennessee. Exchange of visits relating to boundary layer transition (J. Donaldson, S. Pate, S. Dougherty).

- Air Force Weapons Laboratory (AFWL), Albuquerque, N.M. (P.J. Ortwerth). Exchange of visits, consultations and technical direction regarding laser nozzle cusp wake research and research on wake and shear layer transition.

- Wright Air Development Center, Aeronautical Research Laboratory (WADC/ARL) (W. Hankey, J. Shang). Fluctuation and other data obtained in pressure gradients in Contract F33615-77-C-3016 are useful as inputs to the dissipation theory of transition.

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2. Demetriades, A., "Transition and Turbulence in Gas-Dynamic-Laser Nozzle Wakes," AFWL TR 77-122, Air Force Weapons Laboratory, Kirtland Air Force Base, N.M., November 1977 (see also Aeronutronic Publication No. U-6328).

3. Demetriades, A., "Linearized Analysis of Gasdynamic Laser Wakes with Applications," J. of Energy, Vol. 1, No. 2, March-April 1977, pp 73-74.

4. Demetriades, A., "Turbulent Fluctuations in the Wakes of Gas-Dynamic-Laser Nozzle Cusps," J. of Energy, Vol. 2, 1978.

5. Demetriades, A., "Experiments on the Wakes of Multiple Nozzle Cusps," AFWL TR 77-98, Air Force Weapons Laboratory, Kirtland AFB, N.M., November 1977 (see also Aeronutronic Report No. U-6295).

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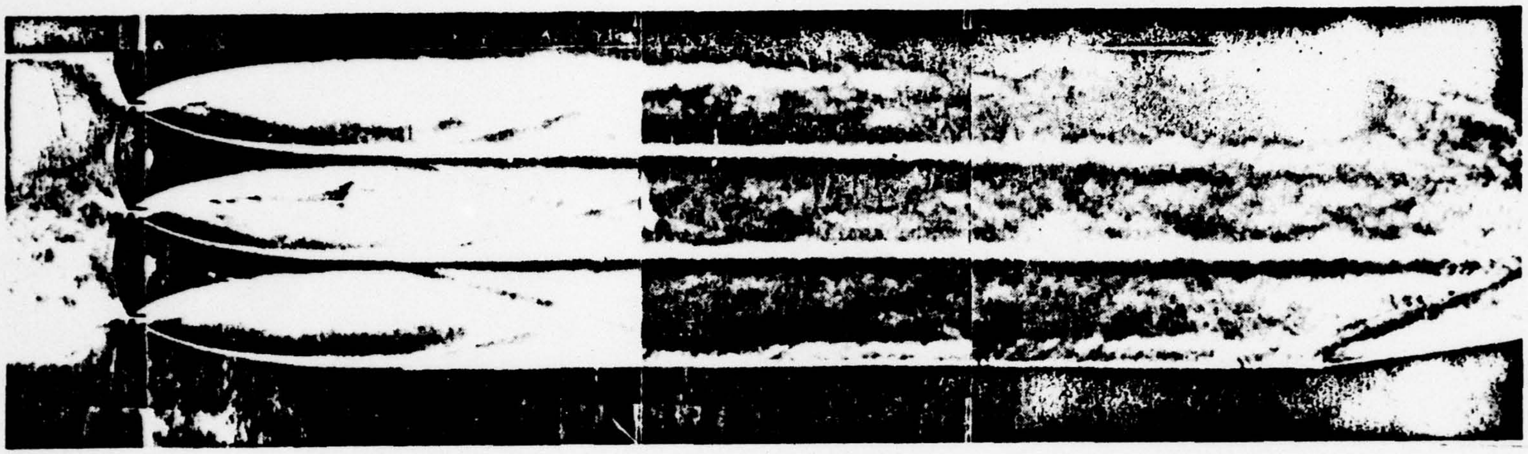

Figure 1. Schlieren photo of the multiple nozzle array as operated in the Supersonic Wind Tunnel for the GDL wake study. Note wakes extending to the diffuser entrance at right (from Ref. 6).

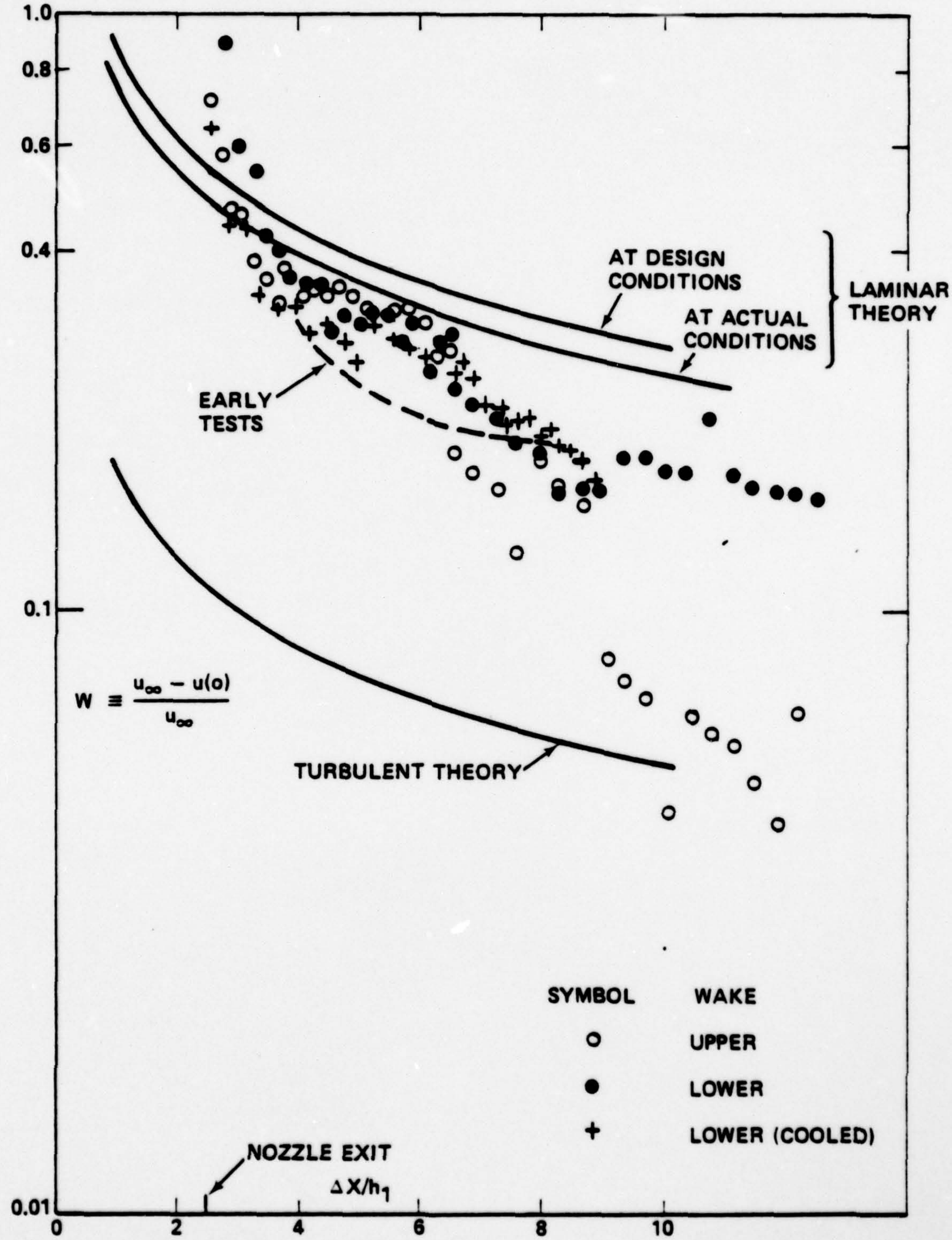

Figure 2. Typical experimental results from the GDL multiple nozzle wake study, in this case of the velocity defect (from Ref. 6). The theory was also developed in the present research (from Ref. 1).

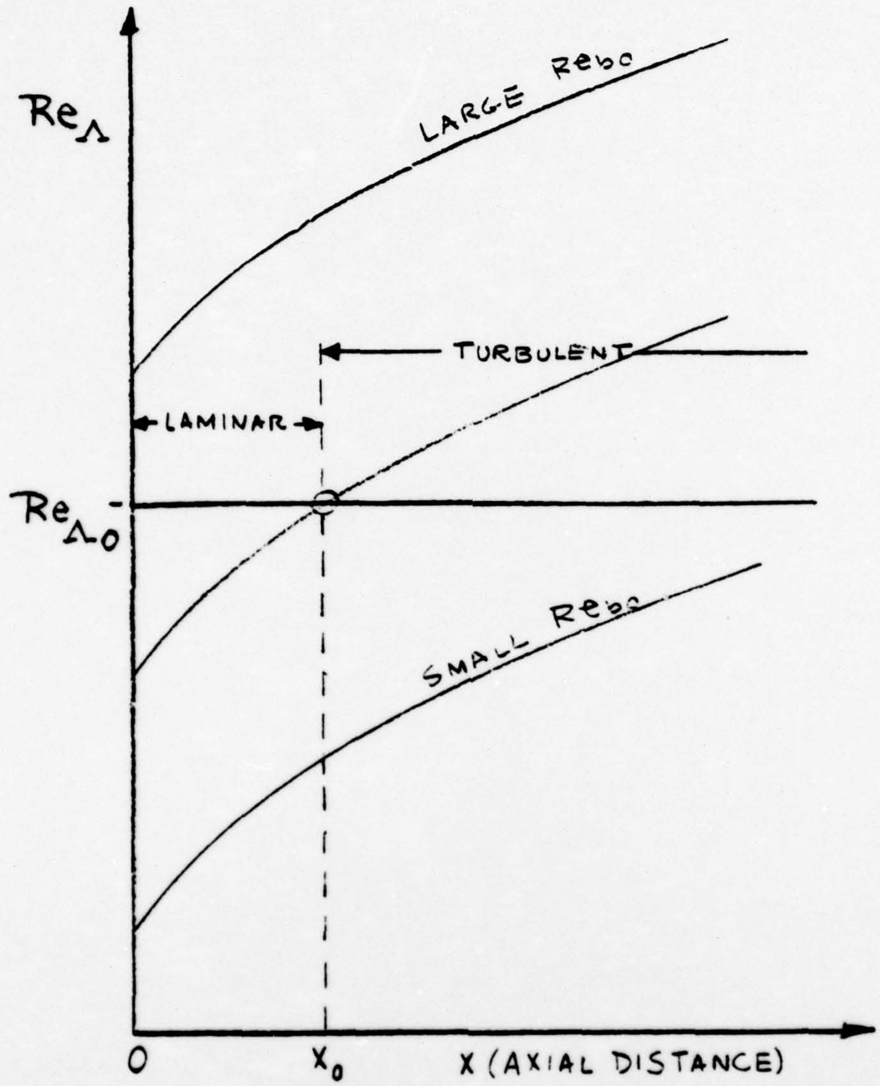

Figure 3. Qualitative variation of turbulence Reynolds number along a two-dimensional turbulent jet, showing how the turbulence-onset criterion applies in the transition-prediction theory developed.

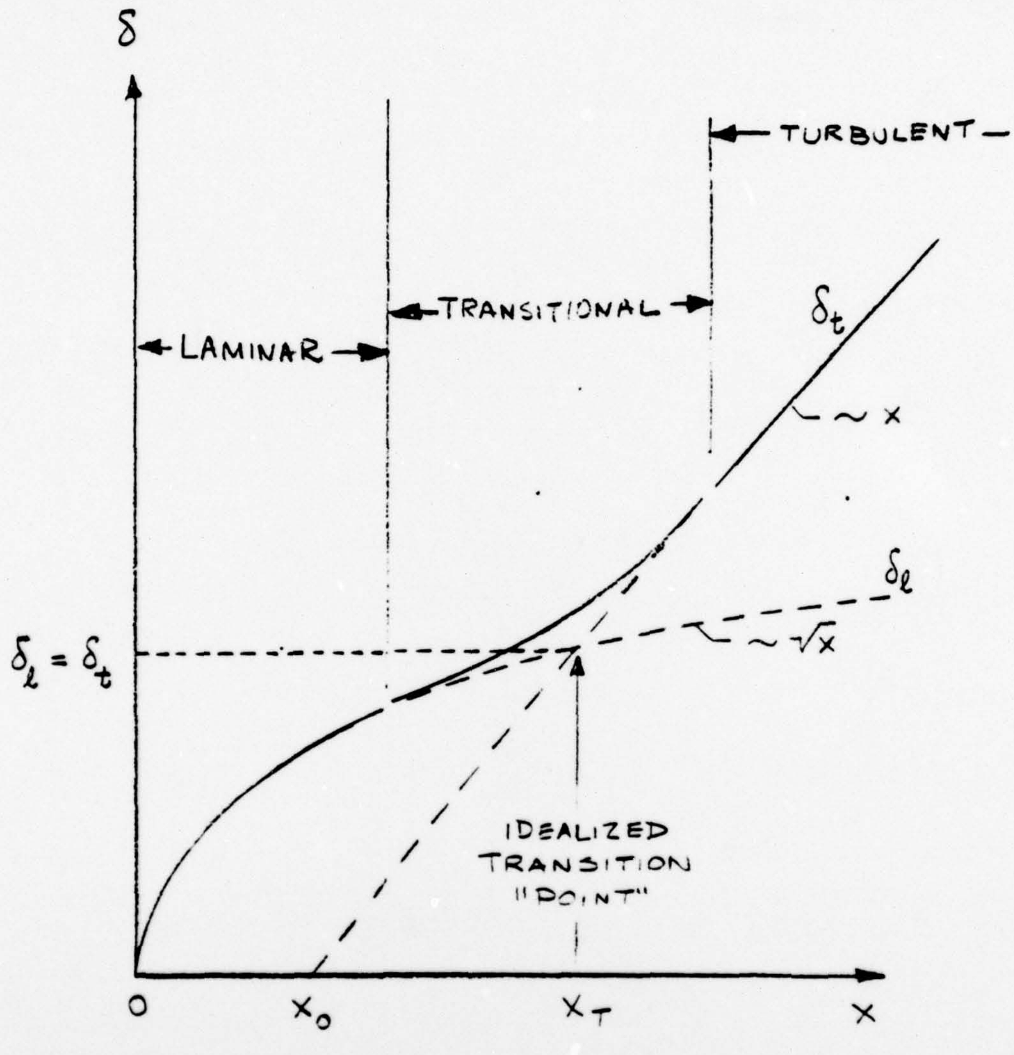

Figure 4. Construction of the time-averaged boundary layer thickness variation along the flow.

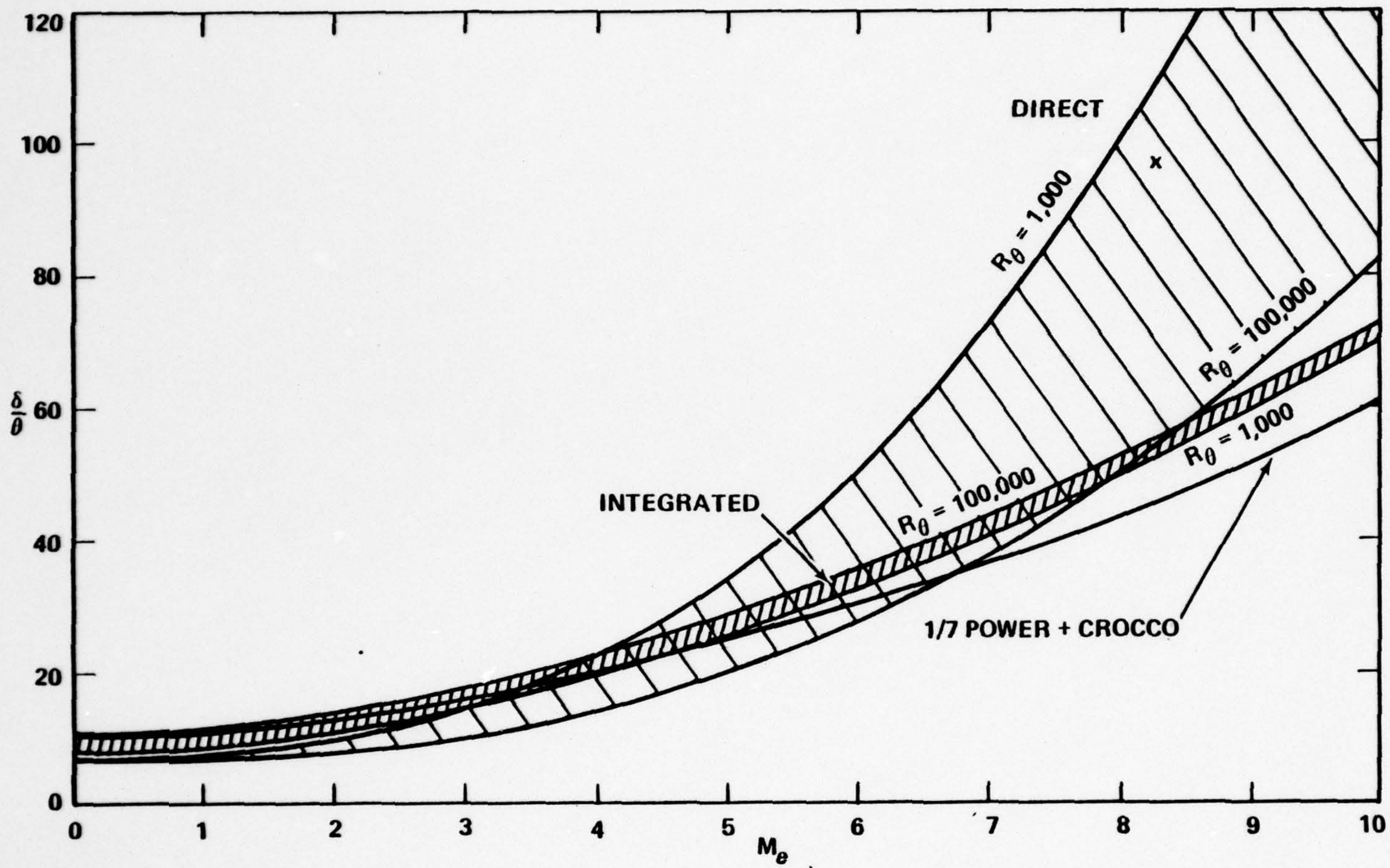

Figure 5. Form factor computed for adiabatic turbulent boundary layers. The integrated results are closer to the experimental evidence and are also identical with theoretical calculations for laminar flow.

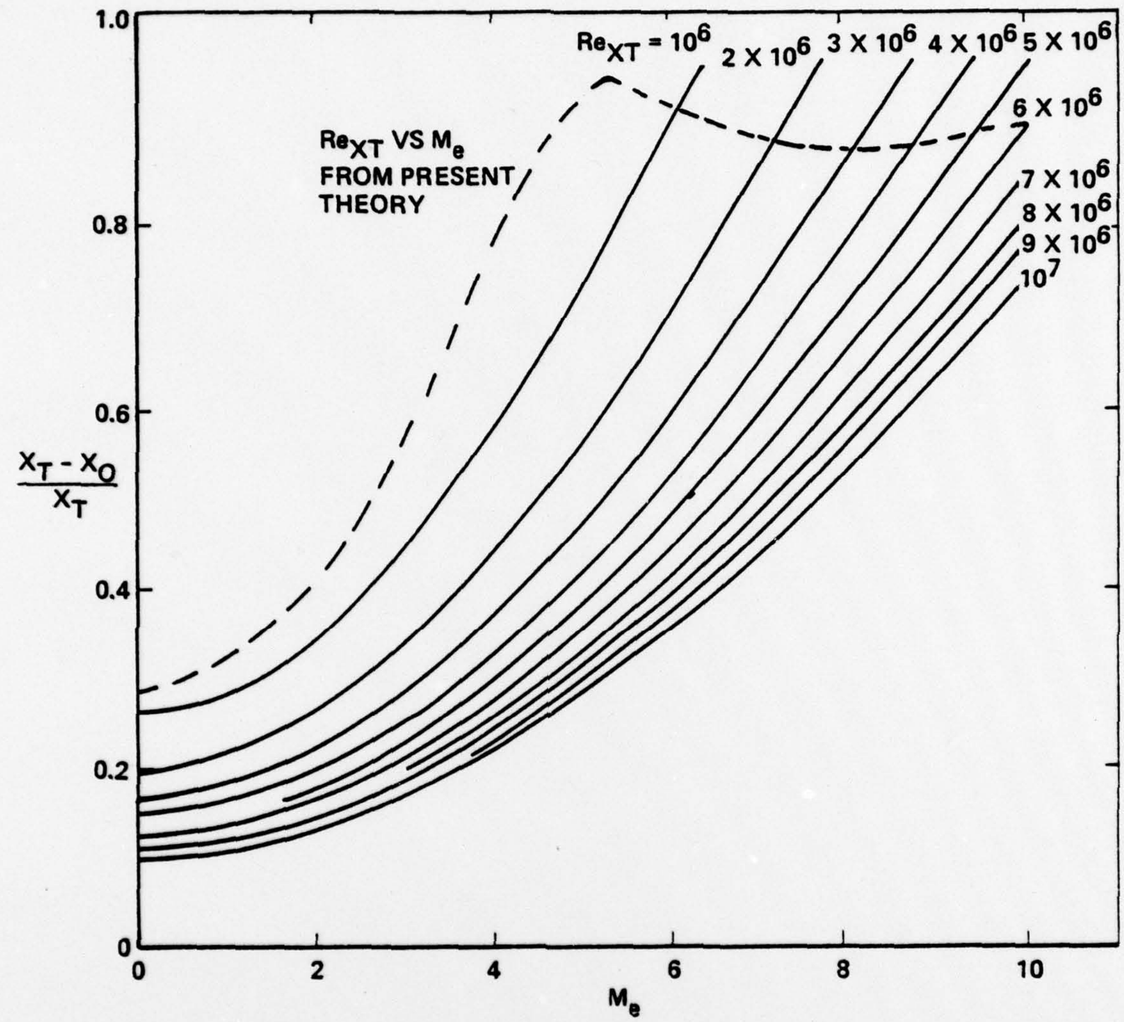

Figure 6. Location of virtual origin of boundary layer turbulence relative to the transition point location, for flat-plate adiabatic boundary layers.

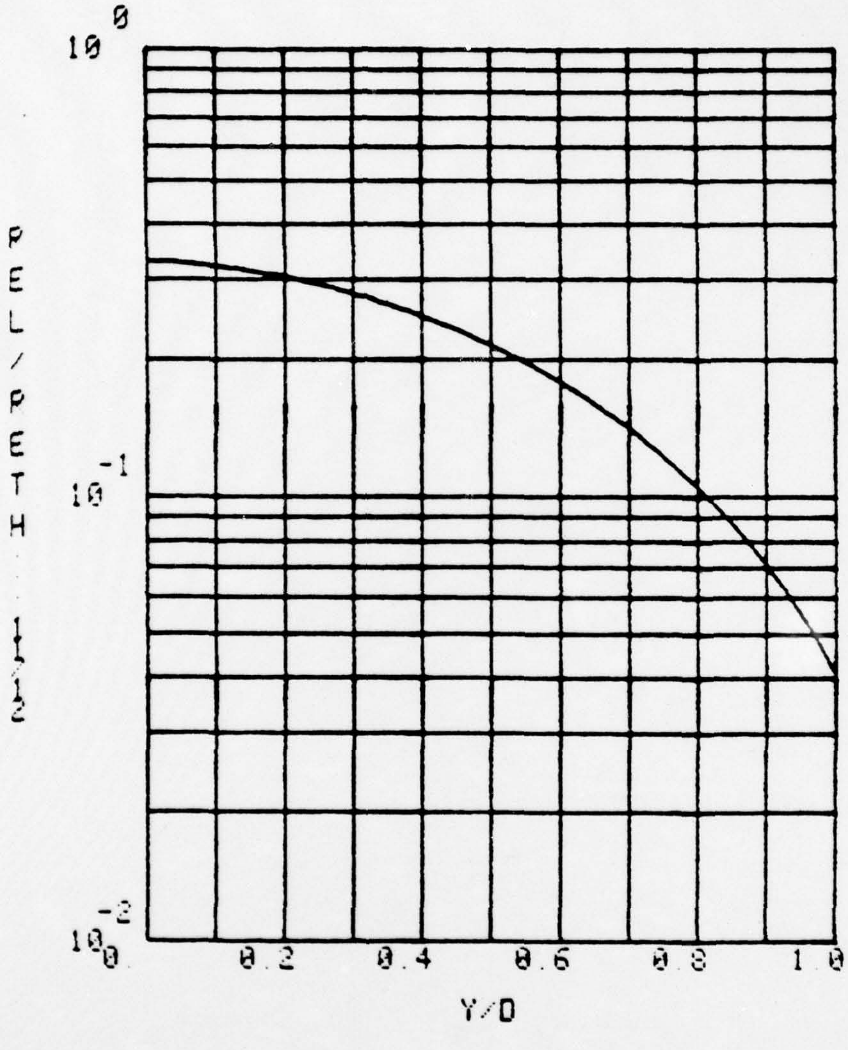

Figure 7. Variation of the turbulence Reynolds number for the low speed adiabatic flat-plate boundary layer ($M_e = 0$, $T_w/T_o = 1$).

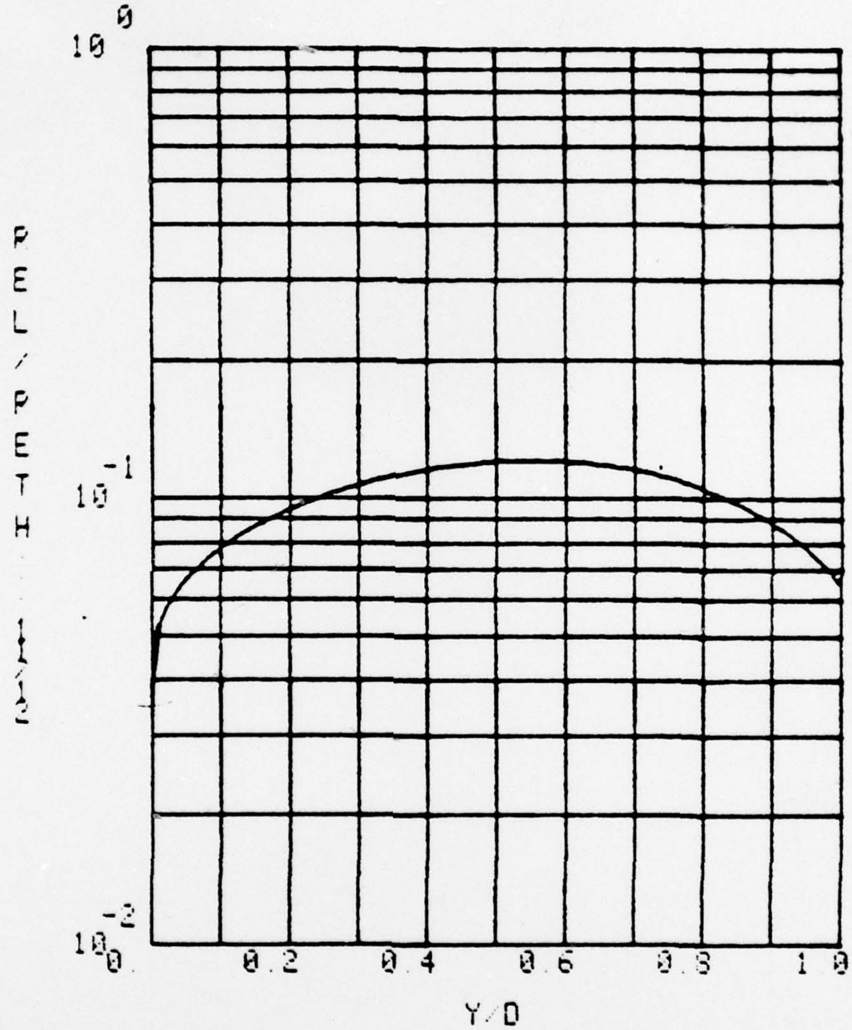

Figure 8. Variation of the turbulence Reynolds number for the hypersonic (M_e = 6) adiabatic (T_w/T_o = 1) flat-plate boundary layer.

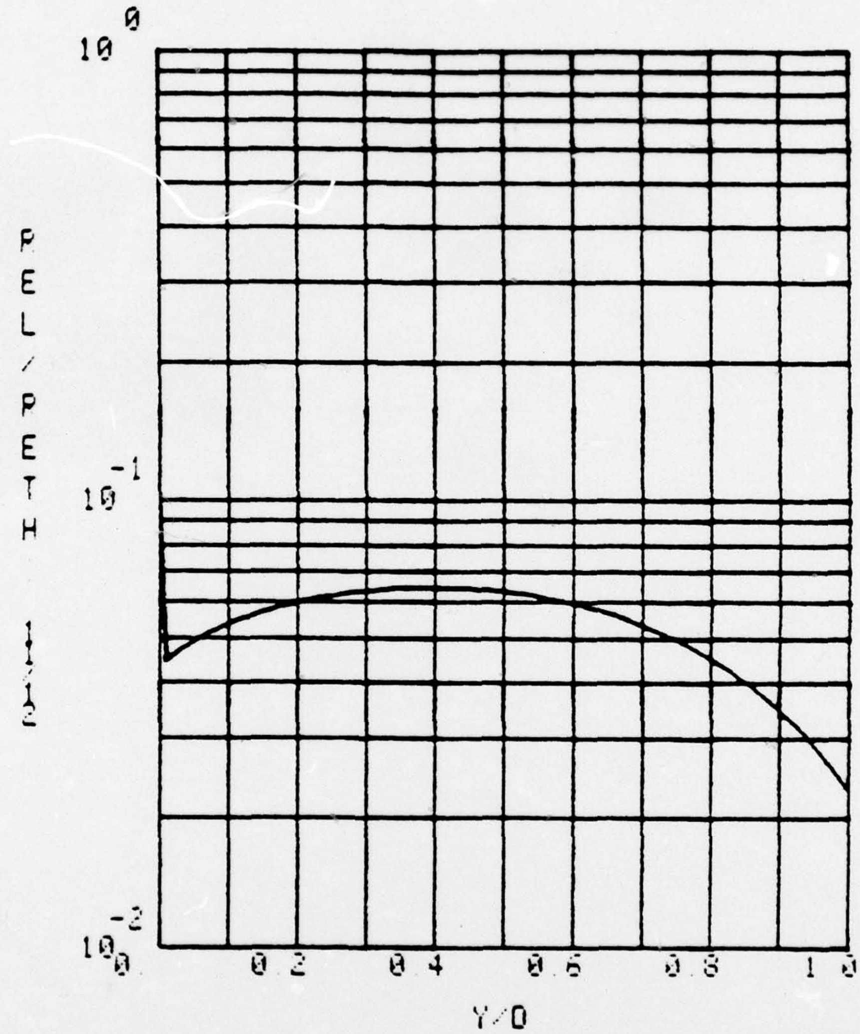

Figure 9. Variation of the turbulence Reynolds number for the hypersonic (M_e = 6) flat-plate boundary layer which is cooled to T_w = 0.2 T_o.

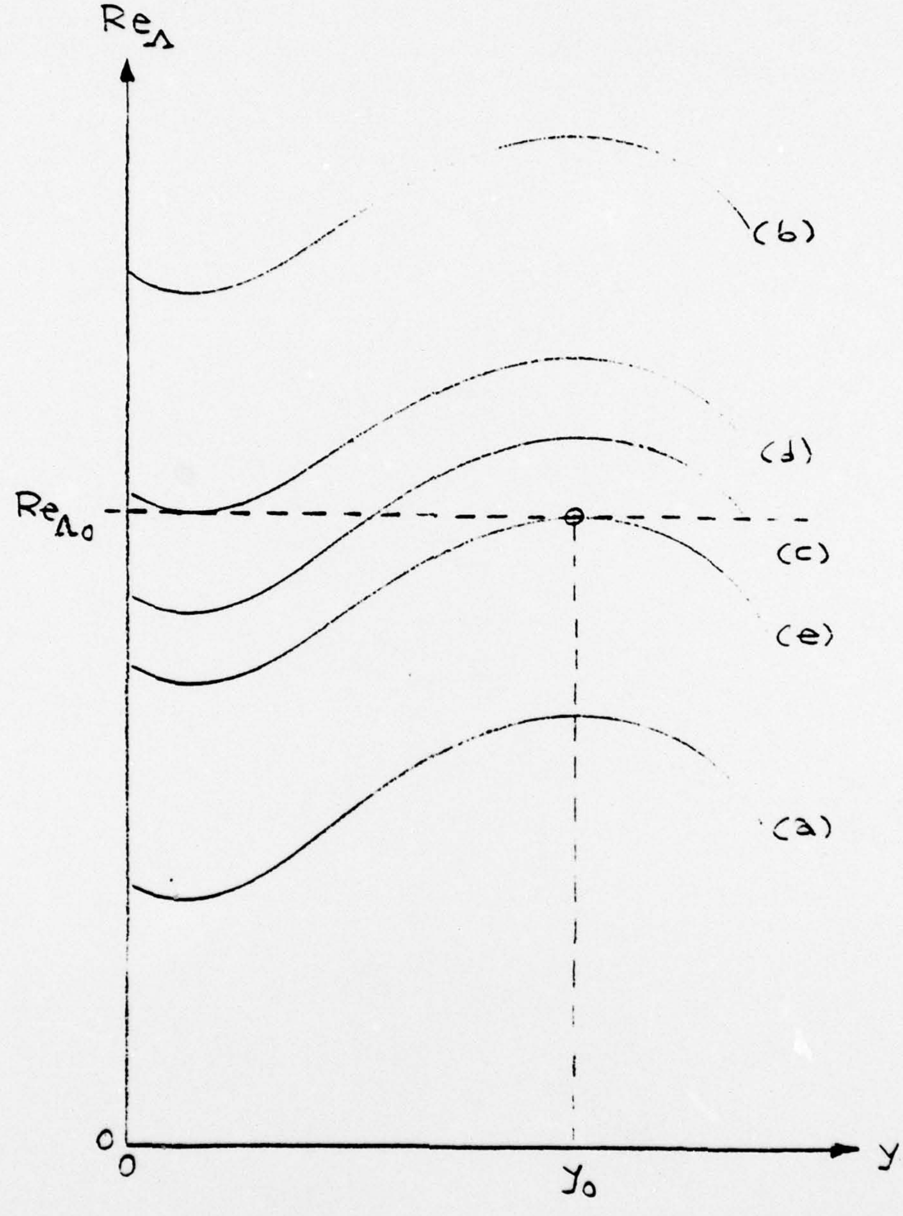

Figure 10. Illustrative variation of turbulence Reynolds number across the boundary layer, serving to show how the transitional case is determined. The various cases shown result from the same M_e and T_w/T_o but different Re_θ.

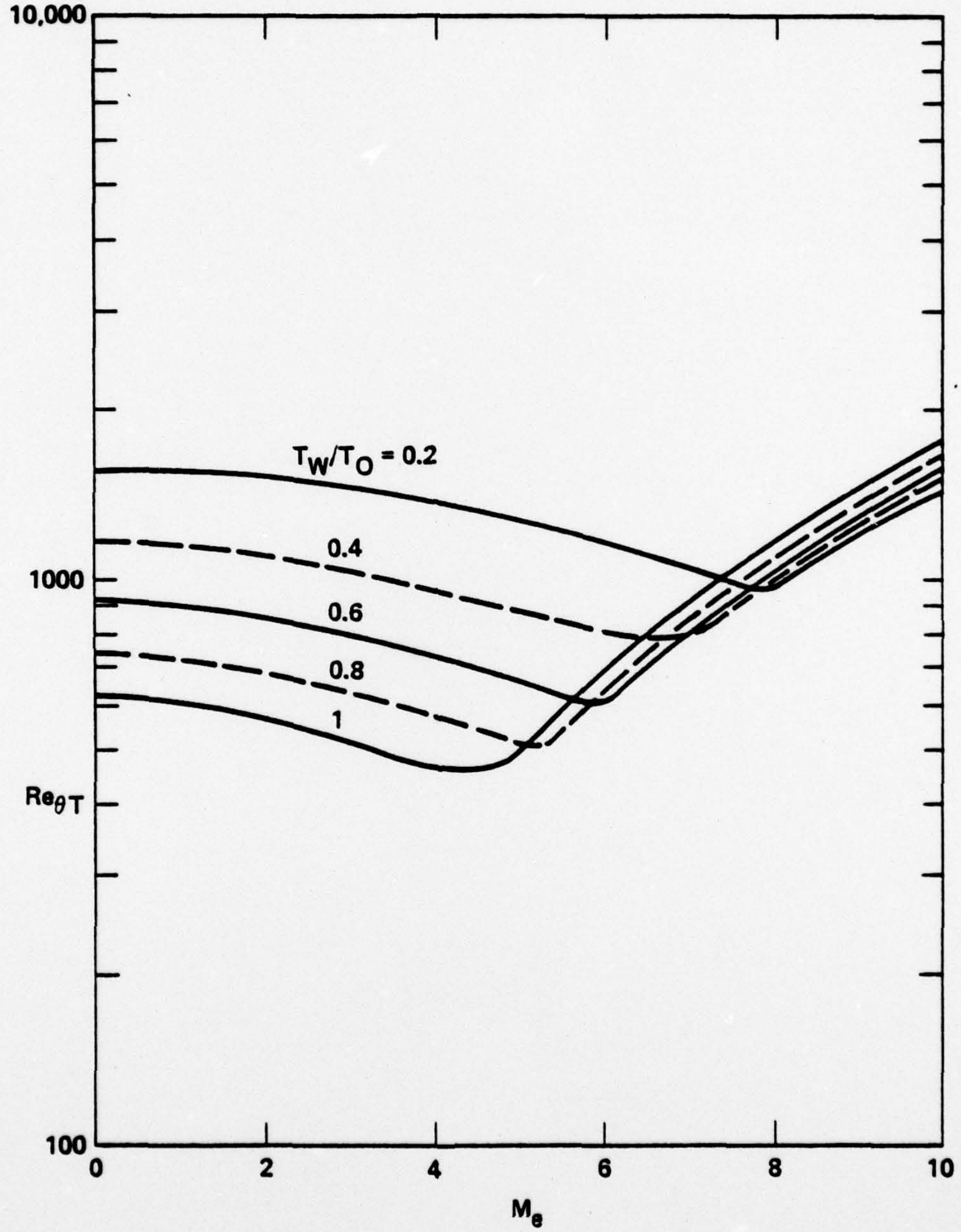

Figure 11. Results of transition predictions for zero pressure gradient boundary layers.

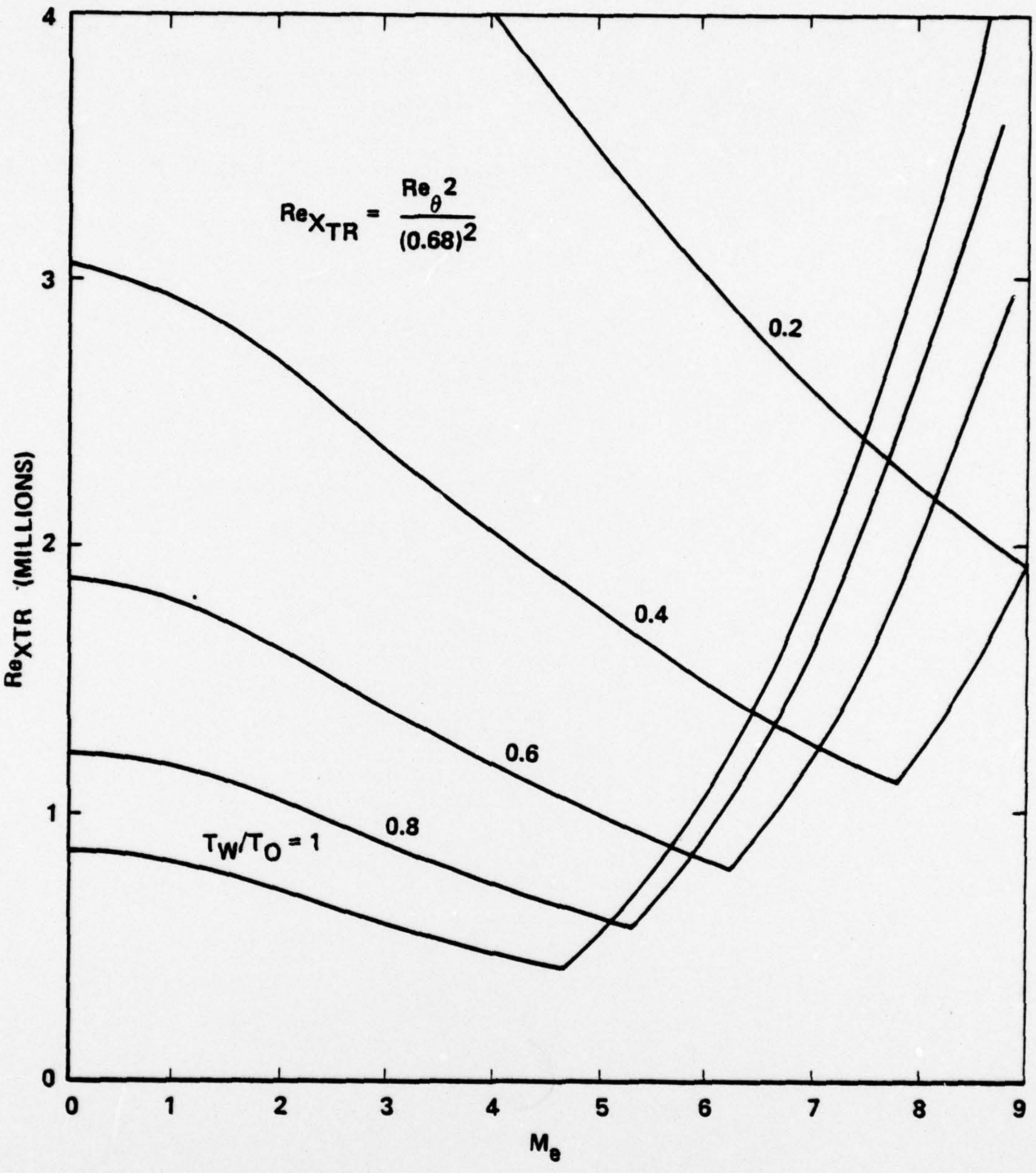

Figure 12. The results of Figure 11 as recast in terms of wetted length transition Reynolds number instead of Re_θ.

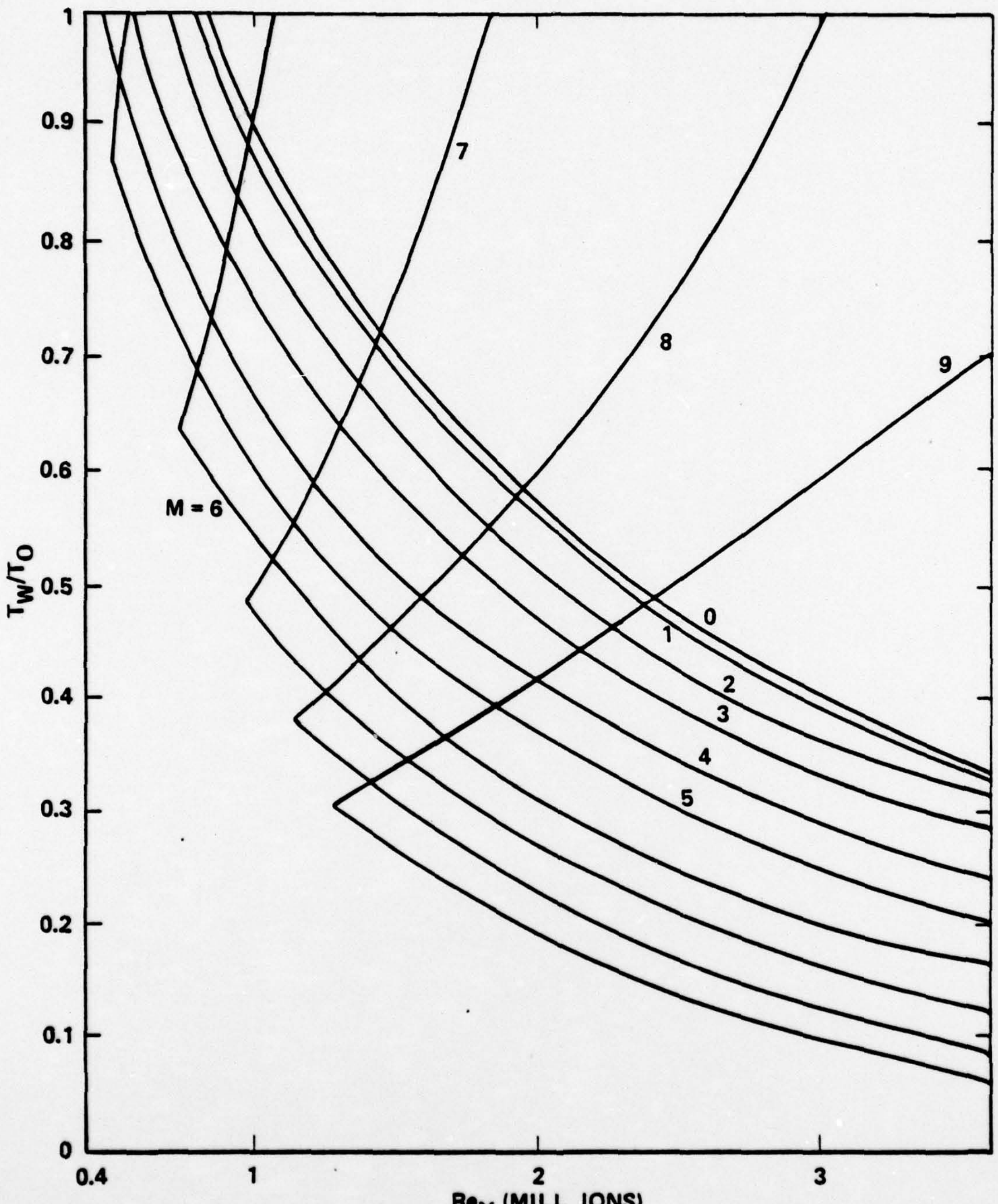

Figure 13. The wetted length transition Reynolds number recast in terms of the temperature ratio.

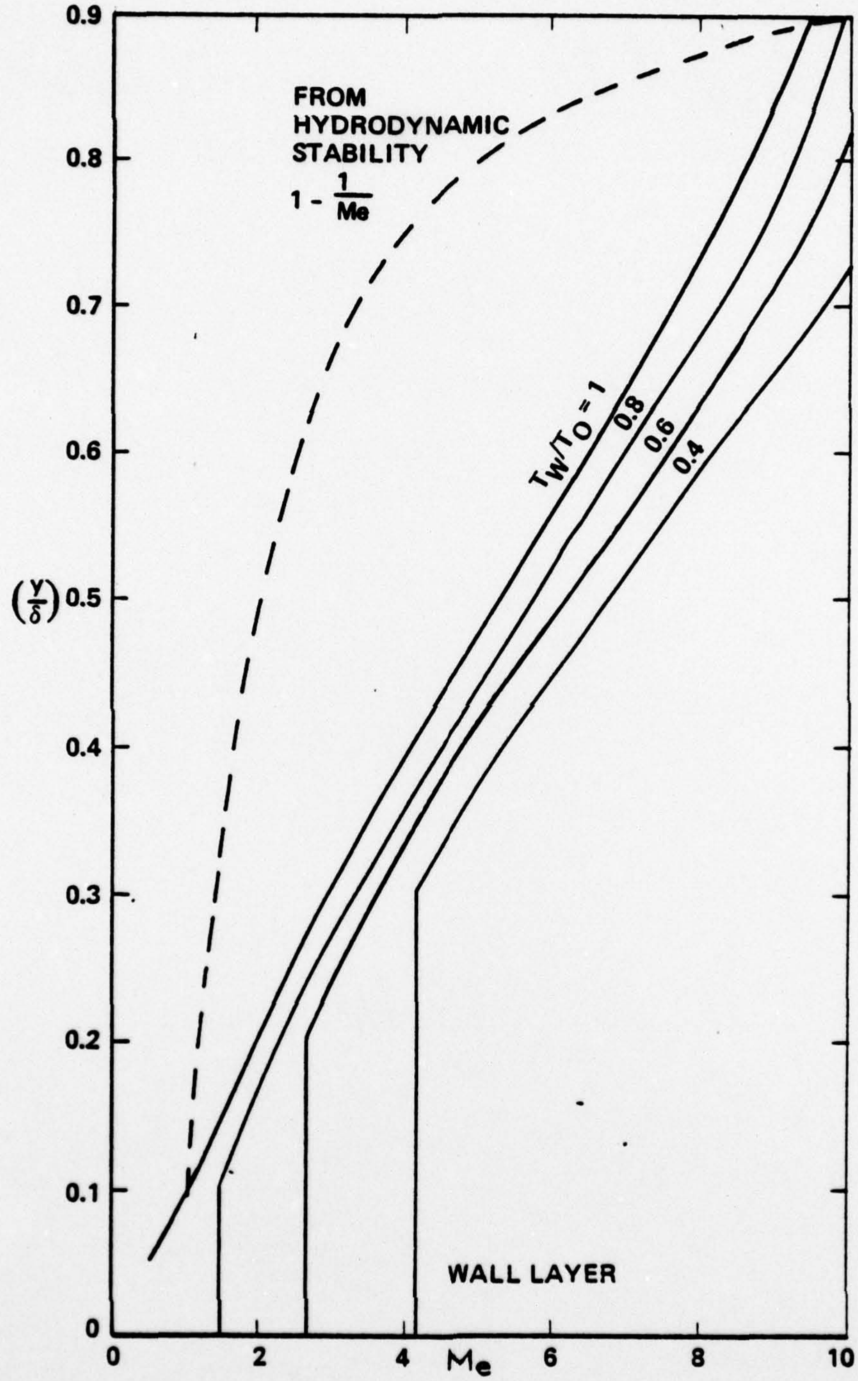

Figure 14. The critical layer y_o variation with Mach number and temperature ratio.

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1. REPORT NUMBER: AFOSR-TR- 79-0074	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): STABILITY, TRANSITION AND TURBULENCE IN MIXING LAYERS AND ADJACENT STREAMS	5. TYPE OF REPORT & PERIOD COVERED: INTERIM Progress rept. 1 Oct 77 - 30 Sep 78
	6. PERFORMING ORG. REPORT NUMBER: Report No U-6488
7. AUTHOR(s): A DEMETRIADES	8. CONTRACT OR GRANT NUMBER(s): F44620-75-C-0016
9. PERFORMING ORGANIZATION NAME AND ADDRESS: FORD AEROSPACE & COMMUNICATION CORPORATION, AERONUTRONIC DIVISION, FORD ROAD, NEWPORT BEACH, CA 92663	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 2307A2, 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS: AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA, BLDG 410, BOLLING AIR FORCE BASE, D C 20332	12. REPORT DATE: Nov 78
	13. NUMBER OF PAGES: 46
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

STABILITY
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TRANSITION REYNOLDS NUMBER

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes accomplishments in the period 1977-1978 in basic research dealing with phenomena in parallel viscous flows. An experiment has been completed on the development of the mean flow in wakes shed by nozzle arrays typical of gas-dynamic lasers. In this experiment, the wake velocity, temperature and density was mapped in wakes generated at Mach 4 in the Company's Supersonic Wind Tunnel. The results are in general agreement with theoretical predictions developed under an earlier OSR study in which theoretical rules on wake flows were set up for laser cavity design. A second problem addressed dealt with a

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) novel approach to predicting transition in parallel shear flows. The basic physics in this approach consists of the postulate of a universal turbulence Reynolds number, below which self-preserving turbulence is impossible. Although this condition is only one of necessity, formulas derived with it had earlier predicted wake transition remarkably well. In the present period, the postulate was applied to flate plate boundary layers in the range $0 < M < 10$ with and without heat transfer. The computations uncovered a mechanism for the long unexplained "transition reversal" phenomenon and otherwise conformed to earlier transition observations. Finally, the data analysis was completed on an earlier hypersonic boundary layer stability experiment done at AEDC. The data show little change of the stability diagram with heat transfer although the amplification rates were substantially increased when the wall temperature was lowered. The present results were disseminated through two technical reports, three journal articles, three delivered papers and by personal contact with Air Force engineering centers.