

AD A064732

12

AD

TECHNICAL REPORT  
NATICK/TR-79/002

**PRESSURE STABILIZED BEAM  
FINITE ELEMENT**

D D C  
RECEIVED  
FEB 21 1978  
C

Earl C. Steeves

Best Available Copy

Approved for public release;  
distribution unlimited.

December 1978

**UNITED STATES ARMY  
NATICK RESEARCH and DEVELOPMENT COMMAND  
NATICK, MASSACHUSETTS 01760**



**Aero-Mechanical Engineering Laboratory**

70 02 12 033

Approved for public release; distribution unlimited.

Citation of trade names in this report does not constitute an official indorsement or approval of the use of such items.

Destroy this report when no longer needed. Do not return it to the originator.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NATICK/TR-79/002	2. GOVT ACCESSION NO	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PRESSURE STABILIZED BEAM FINITE ELEMENT		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Earl C. Steeves		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Natick Research and Development Command Natick, Mass 01760		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 6.11.01.A 1T161101A91A/07/101
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Natick Research and Development Command ATTN: DRXNM-UE Natick, Ma 01760		12. REPORT DATE December 1978
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 59
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release, distribution unlimited		15. SECURITY CLASS (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
FINITE ELEMENT                      TENTS                                      SHELTERS BEAM                                      FIELD SHELTER                              AIR-SUPPORTED STRUCTURES ARCH                                      NONLINEAR                                      COATED FABRICS PRESSURE STABILIZED              STRUCTURAL ANALYSIS		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
<p>The development of a finite element for a pressure stabilized beam is presented and its adaptation to a computer code for the nonlinear structural analysis of frame-supported tents is described. The finite element is based on theory developed in earlier work and includes transverse shear and differential stiffness effects. Third order polynomials are used to represent the transverse displacements and bending rotations. First order polynomials are used to represent the axial and torsional deformation. A condensation is carried out to reduce the number</p>		

DD FORM 1473  
1 JAN 73

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Abstract (cont'd)

of generalized coordinates to twelve. Two check cases are presented to establish the accuracy of the finite element.

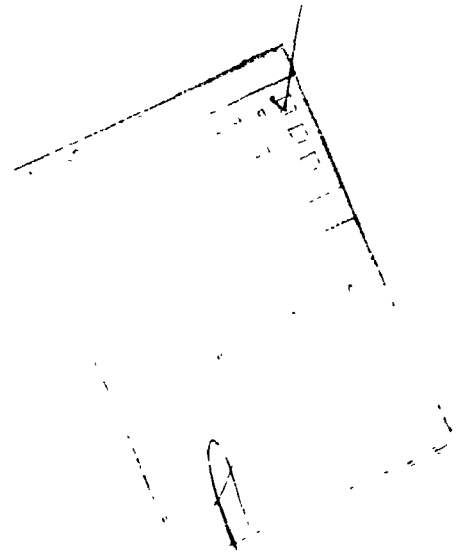
SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

## PREFACE

This work is part of an ongoing investigation of the behavior of pressurized structural elements under load with the ultimate objective of making possible the use of such elements in the support structure of Army tents. The work was funded under the in-House Laboratory Independent Research program as a work unit entitled, "Study of the Stability of Pressure Stabilized Arches and their Structural Assemblies." In the reference citations the organizations "US Army Natick Laboratories" and "US Army Natick Development Center" refer to the organization now called the "US Army Natick Research and Development Command."

## TABLE OF CONTENTS

	Page
LIST OF FIGURES	4
LIST OF TABLES	4
INTRODUCTION	5
DEVELOPMENT OF FINITE ELEMENT	7
CHECK CASES	15
CONCLUDING REMARKS	20
REFERENCES	21
APPENDIX A: Elements of the 8 X 8 Bending Stiffness Matrix	23
APPENDIX B: Input Data and Results for Beam Check Case	27
APPENDIX C: Input Data and Results for Arch Check Case	41



## LIST OF FIGURES

	Page
Figure 1. Tent Concept Using Pressure Stabilized Structural Elements	6
Figure 2. Coordinate System and Sign Convention	8
Figure 3. Finite Element Model for Straight Beam Check Case	16
Figure 4. Finite Element Model for Circular Arch Check Case	17

## LIST OF TABLES

Table 1. Comparison of Finite Element Results with Those from the Exact Solution for the Beam Check Case	18
Table 2. Comparison of Finite Element Results with Those from the Exact Solution for the Arch Check Case	19

# PRESSURE STABILIZED BEAM FINITE ELEMENT

## INTRODUCTION

A systems analysis of Army needs for shelters in the 1985 time frame revealed that the requirements for lightweight tentage of low packaged bulk with minimum setup and disassembly time might be most effectively achieved by a tent with a frame of highly pressurized (compared with present air-supported tents) structural elements supporting a lightweight fabric barrier, as illustrated in Figure 1. Since the structural elements are pressure stabilized, they can be fabricated from lightweight, flexible materials such as cloth, thus reducing the weight and bulk of the transport configuration. The use of bladders or coated fabric for these elements should provide sufficient air retention capability to eliminate the need for the dedicated air supply used with current air-supported tents.

To successfully develop this concept, a structural design capability was needed. To meet this need an investigation of the structural behavior of pressure-stabilized beams and arches was carried out and reported in references 1, 2, and 3. These references give the development of theories with experimental confirmation to predict the structural behavior of pressure stabilized beams and arches as a function of geometry, pressure level and material properties. The theories give adequate predictions of both the deformation and the load-carrying capability. These theories thus serve to characterize the behavior of the structural elements and can be used as a guide to design.

Since these structural elements are to be used in structural frames for tents, a more complete design capability is needed; that is, the capability to predict the deformation and load-carrying capability of frame assemblies made of pressure-stabilized elements and loaded through a fabric barrier. Thus, a structural analysis method capable of treating the interaction of assemblages of frame elements and the interaction of this frame with the fabric barrier is needed. The finite element procedure is well suited to such calculations, and a computer program using that procedure for the analysis of conventional frame tents was available. This program, which is described in references 4 and 5, includes a

1. Earl C. Steeves; A Linear Analysis of the Deformation of Pressure Stabilized Beams; US Army Natick Laboratories; Technical Report 75-47-AMEL, 1975 (AD A006493).
2. Earl C. Steeves; Behavior of Pressure Stabilized Beams Under Load; US Army Natick Development Center; Technical Report 75-82-AMEL; 1975 (AD A010702).
3. Earl C. Steeves; The Structural Behavior of Pressure Stabilized Arches; US Army Research & Development Command; Natick/TR-78/018; 1978.
4. Paul J. Remington, John C. O'Callahan and Richard Madden; Analysis of Stresses and Deflections in Frame Supported Tents; US Army Natick Laboratories; Technical Report 75-31; 1974 (AD A002072).
5. Paul J. Remington, John C. O'Callahan and Richard Madden; Finite Element Analysis of Scale-Model Frame-Supported Tents; US Army Natick Research & Development Command; Technical Report 76-21-AMEL, 1975.



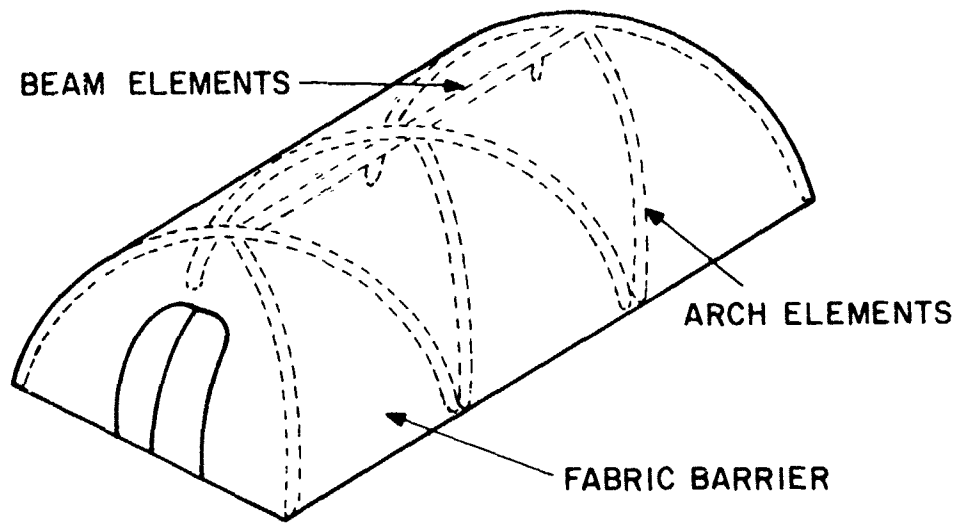


Figure 1. Tent Concept Using Pressure Stabilized Structural Elements

conventional beam element and a nonlinear membrane element for modeling the fabric barrier. Since a pressure stabilized beam element was not included, it was decided to develop such an element and adapt it to this existing computer program. The purpose here is to report this work. This will include the development of the finite element, its adaptation to the computer code and some check cases.

## DEVELOPMENT OF FINITE ELEMENT

We proceed here with the derivation of the finite element with the intended result a 12 X 12 stiffness matrix associated with generalized coordinate set comprised of the three deflections and three rotations of each of the beam end points. The basis for this derivation is the energy principle derived in reference 1. This energy principle which includes bending in one plane and axial deformation is augmented to include bending in two planes, axial deformation, and torsion. The displacement fields are represented by polynomials in the coordinate specifying position along the beam.

Because of the number and order of polynomials used, the number of generalized coordinates exceeds 12, so a condensation is carried out.

In carrying out this derivation we use the coordinate system and sign convention given in Figure 2. Shown are the sign conventions associated with the basic theory and that associated with the finite element. The potential energy expression to be used is that given in reference 1 and augmented as indicated above and is expressed in dimensional form as:

$$\begin{aligned}
 \int_0^L \frac{1}{2} \left\{ E \left( \frac{du}{dx} \right)^2 + D \left( \frac{d\phi}{dx} \right)^2 + D \left( \frac{d\psi}{dx} \right)^2 + G \left( \frac{d\theta}{dx} \right)^2 \right. \\
 + C \left( \frac{dv}{dx} - \psi \right)^2 + C \left( \frac{dW}{dx} + \phi \right)^2 \\
 \left. + P \left( \frac{dW}{dx} \right)^2 + P \left( \frac{dv}{dx} \right)^2 \right\} dx \quad (1)
 \end{aligned}$$

This expression is based on the assumptions of uniaxial strain and linear expression for the shear strain in terms of the displacements. Since we seek a finite element with which forces will be applied at the nodes, the distributed forces are not included in (1). The potential energy expression is a quadratic in the displacements and can be written in matrix form as

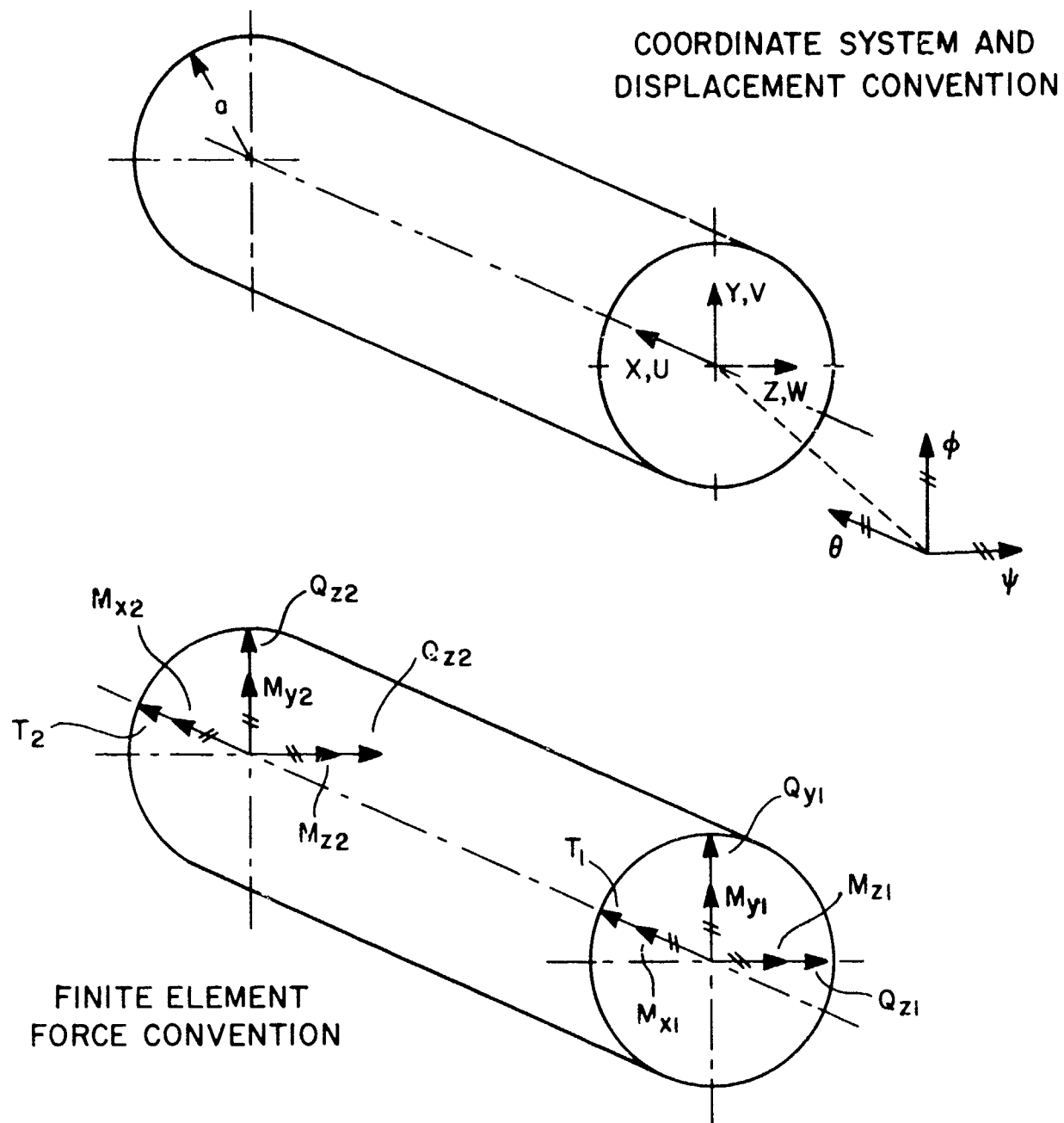


FIGURE 2. COORDINATE SYSTEM AND SIGN CONVENTION

$$\int_0^L \frac{1}{2} \begin{bmatrix} \frac{du}{dx} & \frac{d\phi}{dx} & \frac{dW}{dx} & \phi & \frac{d\theta}{dx} & \frac{d\Psi}{dx} & \frac{dv}{dx} & \Psi \end{bmatrix} \begin{bmatrix} E & & & & & & & \\ & D & & & & & & \\ & & C+P & C & & & & \\ & & C & C & & & & \\ & & & & G & & & \\ & & & & & D & & \\ & & & & & & C+P & -C \\ & & & & & & -C & C \end{bmatrix} \begin{bmatrix} \frac{du}{dx} \\ \frac{d\phi}{dx} \\ \frac{dW}{dx} \\ \phi \\ \frac{d\theta}{dx} \\ \frac{d\Psi}{dx} \\ \frac{dv}{dx} \\ \Psi \end{bmatrix} dx \quad (2)$$

where

$$\begin{aligned} E &= 2\pi a C_{11} & D &= \pi a^3 C_{11} \\ C &= \pi a C_{33} & G &= \pi a^3 C_{33} \\ P &= \pi a^2 p/2 \end{aligned}$$

In these definitions  $C_{11}$ ,  $C_{33}$ , and  $p$  are respectively the elastic modulus, the shear modulus, and the pressure. It can easily be seen from this expression that the axial, torsional, and two bending energies are uncoupled and thus can be treated independently in formulating the finite element stiffness matrix.

The energy resulting from axial deformation is given by the expression  $E(\frac{du}{dx})^2$ , which is the classical expression. With the use of a linear representation for the axial displacement field in terms of the end point axial displacements,  $U_1$  and  $U_2$ , the following energy expression is obtained upon integration:

$$\frac{1}{2} \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} E/L & -E/L \\ -E/L & E/L \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \quad (3)$$

This is the classical stiffness matrix for linear axial behavior.

Similarly, the torsional energy is given by  $G(\frac{d\theta}{dx})^2$  and a representation of  $\theta$  linear in the  $x$  coordinate in terms of the end point rotation  $\theta_1$  and  $\theta_2$ , yields for the energy

$$\frac{1}{2} \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} \begin{bmatrix} G/L & G/L \\ -G/L & G/L \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (4)$$

Examination of (2) reveals that the two bending problems differ only in the algebraic signs of the off diagonal elements. This being so, we can derive the stiffness matrix associated with one of the bending problems and deduce the other from it. We will treat the problem described by  $W$  and  $\phi$ . The formulation of the bending stiffness matrix is not as direct or simple as the above problems or the classical bending problem. This results from the use of two independent parameters,  $W$  and  $\phi$ , to describe the bending problem. The use of four generalized coordinates is generally sufficient to approximately describe the beam bending problem and is thought to be adequate here also. However, dividing these four coordinates between the two independent variables,  $W$  and  $\phi$ , only allows for the use of linear approximation of these variables within the context of polynomial approximation. That is not believed to be an adequate approximation, since for  $W$  it only allows description of rigid body displacements. It was, therefore, decided to use two third order polynomials, one to represent  $W$  and the other to represent  $\phi$ . This gives a stiffness matrix in terms of eight generalized coordinates, so a condensation is performed to reduce the number to four. The bending energy is

$$\frac{1}{2} \int_0^L \begin{bmatrix} \frac{d\phi}{dx} & \frac{dW}{dx} & \phi \end{bmatrix} \begin{bmatrix} D & 0 & 0 \\ 0 & C+P & C \\ 0 & C & C \end{bmatrix} \begin{bmatrix} \frac{d\phi}{dx} \\ \frac{dW}{dx} \\ \phi \end{bmatrix} dx \quad (5)$$

and the polynomial approximations are

$$\begin{bmatrix} \frac{d\phi}{dx} \\ \frac{dW}{dx} \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & h'_1 & 0 & -h'_1 & 0 & h'_3 & 0 & h'_4 \\ h'_1 & 0 & -h'_1 & 0 & h'_3 & 0 & h'_4 & 0 \\ 0 & h_1 & 0 & h_2 & 0 & h_3 & 0 & h_4 \end{bmatrix} \begin{bmatrix} W_1 \\ \phi_1 \\ W_2 \\ \phi_2 \\ W'_1 \\ \phi'_1 \\ W'_2 \\ \phi'_2 \end{bmatrix} \quad (6)$$

In equation (6)

$$\begin{aligned}
 h_1 &= 1 - 3x^2/L^2 + 2x^3/L^3 & h'_1 &= 6x^2/L^3 - 6x/L^2 \\
 h_2 &= 3x^2/L^2 - 2x^3/L^3 & h'_2 &= 1 - 4x/L + 3x^2/L^2 \\
 h_3 &= x - 2x^2/L + x^3/L^2 & h'_3 &= 3x^2/L^2 - 2x/L \\
 h_4 &= x^3/L^2 - x^2/L
 \end{aligned} \tag{7}$$

The elements of the vector of order eight in equation (6) are the transverse deflection and cross-section rotation and their derivations evaluated at  $x = 0$  and  $x = L$ . The subscript 1 denotes evaluation at  $x = 0$ , and subscript 2 denotes evaluation at  $x = L$ . The prime denotes the derivative of the variable. Substitution of (6) into (5) and integration yields the following energy expression:

$$\frac{1}{2} \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^t & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} \tag{8}$$

where

$$\Delta_1 = \begin{Bmatrix} W_1 \\ \phi_1 \\ W_2 \\ \phi_2 \end{Bmatrix} \quad \Delta_2 = \begin{Bmatrix} W'_1 \\ \phi'_1 \\ W'_2 \\ \phi'_2 \end{Bmatrix} \tag{9}$$

The elements of the matrices  $K_{11}$ ,  $K_{12}$  and  $K_{22}$  are given in Appendix A. We have an eighth-order stiffness matrix which we would like to reduce to fourth order to make it compatible with other finite elements for beam bending but particularly for compatibility with the finite element described in references 4 and 5. To accomplish this the set of generalized coordinates contained in  $\Delta_1$  are chosen to be the set to be used and those in  $\Delta_2$  the set to be removed. It is also assumed that the applied forces do work only through the generalized coordinates  $\Delta_1$  so that the potential energy including the external forces can be written

$$\frac{1}{2} \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^t & K_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} - \begin{bmatrix} \Delta_1^t & \Delta_2^t \end{bmatrix} \begin{Bmatrix} Q \\ O \end{Bmatrix} \tag{10}$$

In this situation the following transformation of coordinates can be used to carry out the desired reduction or condensation:

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{bmatrix} I & \\ & -K_{22}^{-1} K_{12}^t \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} \quad (11)$$

with I denoting the identity matrix. Substitution of (11) into (10) gives for the energy expression

$$\frac{1}{2} \Delta_1^t S \Delta_1 - \Delta_1^t Q \quad (12)$$

Where S is the desired 4 x 4 stiffness matrix expressed in terms of the original submatrices as

$$S = K_{11} - K_{12} K_{22}^{-1} K_{12}^t \quad (13)$$

This completes the development of the stiffness matrices for axial, torsional, and bending deformation and the complete 12 x 12 stiffness matrix associated with the end point displacements and rotations can be assembled. In doing so, attention is paid to the order of the generalized coordinates or end point displacements used in the NONFESA (Nonlinear Finite Element Structural Analysis) program in which this finite element will be used. The desired stiffness matrix appears as follows in the expression for the potential energy:

$$\begin{bmatrix} U_1 & V_1 & W_1 & \theta_1 & \psi_1 & U_2 & V_2 & W_2 & \theta_2 & \psi_2 \end{bmatrix} \begin{bmatrix} E/L & & & & & & & & & -E/L \\ & S_{11} & & & & S_{12} & & & & S_{13} \\ & & S_{11} & & & -S_{12} & & & & S_{13} \\ & & & G/L & & & & & & -S_{14} \\ & & -S_{12} & & S_{22} & & & & & -S_{23} \\ & S_{12} & & & S_{22} & & & & & S_{24} \\ -E/L & & & & & S_{23} & & & & & \\ & S_{13} & & & & S_{23} & & & & S_{33} \\ & & S_{13} & & & -S_{23} & & & & S_{33} \\ & & & G/L & & & & & & -S_{34} \\ & & -S_{14} & & S_{24} & & & & & G/L \\ & S_{14} & & & S_{24} & & & & & -S_{34} \\ & & & & S_{24} & & & & & S_{44} \end{bmatrix} \begin{bmatrix} U_1 \\ V_1 \\ W_1 \\ \theta_1 \\ \psi_1 \\ U_2 \\ V_2 \\ W_2 \\ \theta_2 \\ \psi_2 \end{bmatrix} \quad (14)$$

The elements  $S_{ij}$  in (14) are the elements of the matrix  $S$  given in (13). In assembling this matrix, account has been taken of the algebraic sign difference of the off diagonal elements in the bending problem associated with  $V$  and  $\psi$  that was discussed previously in connection with expression (2). The matrix in (14) is the desired stiffness matrix, and having it we can proceed with its adaptation to the NONFESA.

In carrying out this adaptation, two options were open, the first being to create an entirely new element to be included along with the beam, membrane and truss elements already in the program, and the second being to modify the present beam element by making the pressure stabilized beam element an option on the conventional beam element. Because of the great similarity of this beam element to the conventional one, this second option was chosen. To accomplish this adaptation changes were required in both the program and the input. Program changes were required in the computation of the stiffness matrix and in the computation of the stresses for output. The beam stiffness matrix is computed in the subroutine named NEWBM, and in this subroutine an option was added for calculation of the stiffness matrix according to (14). This option is controlled by the parameter JPRES. For the calculation of the stiffness matrix according to (14) two additional subroutines are needed, ELB to calculate the stiffness matrix  $S$  and GJSOL to solve a system of equations by Gauss-Jordan reduction. In ELB the order eight stiffness matrix of (8) is calculated according to the formulas for the elements given in the appendix. The condensation of the matrix is carried out using subroutine GJSOL, and the order four matrix is returned to subroutine NEWBM for assembly into the  $12 \times 12$  stiffness matrix along with the other elements which are calculated directly into the matrix. The reader may question the need for inclusion of the subroutine GJSOL for Gauss-Jordan reduction when one is already present in NONFESA for solution of the large system of equations generated by the program. The subroutine already included is especially designed for large systems of equations stored on tape in block format so its use on a fourth order system seemed inefficient and overly complex. Thus a very simple 25-line subroutine was added to carry out the condensation. The other programming changes are in subroutine BEAM where the stresses are computed for output from the nodal forces. Because the pressure-stabilized beam is a circular cylindrical membrane, membrane-type stresses are computed as follows:

The uniform stress resulting from axial deformation is computed by dividing the axial nodal forces by the circumference of the cross-section.

The maximum and minimum stresses resulting from bending deformation are computed for each of the two planes of bending. The magnitudes of these parameters are the quantities and their negatives obtained by dividing the nodal moment for each of the planes of bending by the area enclosed by the cross-section.

The average stresses from shear deformation are computed by dividing nodal shear force associated with each of the two planes of bending by the circumference of the cross-section.



These stress calculations are carried out for both ends of the beam.

The only input that changes is that associated with the beam element, with these changes being on the material property card, the element property card and the beam data cards of reference 6. For the pressure stabilized beam the needed input in addition to the control parameter is the beam length, the elastic modulus, Poisson's Ratio, the cross-section radius, and the inflation pressure. The beam length is computed from the beam end point nodal coordinates and this does not constitute a change. The elastic modulus and Poisson's Ratio are entered with the material property card, with the elastic modulus being the first entry following the identification number and Poisson's Ratio the second. The elastic modulus required is of the membrane type having the units of force per unit length. If Poisson's Ratio is known, it can be entered, but typically the parameter that is known is the membrane shear modulus. In the program the shear modulus is computed from the elastic modulus and Poisson's Ratio according to the relation

$$C_{33} = C_{11}/2(1 + \mu) \quad (15)$$

Thus, when the shear modulus and the elastic modulus are known, the Poisson's Ratio to be entered as the second entry on the material property card should be calculated by the expression by solving (15) for  $\mu$  in terms of  $C_{11}$  and  $C_{33}$ . The element property card contains an identification number followed by eight numerical parameters. For the pressure-stabilized beam elements, the first two of the numerical parameters are the cross-section radius and the inflation pressure, in that order. The fifth of these parameters should be entered as a negative number as this is used to control the option on the computation of the stresses from the nodal forces. The remaining five parameters are dummies for the pressure-stabilized beam element, and nonvanishing entries should be made for them. Control of the pressure-stabilized beam option is accomplished by the parameter JPRES on the beam data card (there is one of these cards for every beam). If a given beam is to use the pressure-stabilized beam option the JPRES must be entered as unity in column 68 of the beam data card, otherwise it is to be left blank or entered as zero. If the pressure-stabilized beam element is exercised then the material property and element property numbers entered on the beam data card must correspond to the identification numbers corresponding to material and element property sets for pressure-stabilized beams as described above.

- 6 John C. O'Callahan NONFESA - Nonlinear Finite Element Structural Analysis Code for the Analysis of Stresses and Deflections in Frame Supported Tents, Bolt Beranek and Newman, Inc., Report No. 2803, 1975.

## CHECK CASES

To establish the validity of the pressure stabilized beam finite element two check cases are presented here. A straight beam loaded at its midpoint in two perpendicular planes and a circular arch loaded in its plane are the check cases. The finite element calculations are compared with the exact solutions of the theory given in references 4 and 5.

The simplest structure that can be modeled using the pressure stabilized beam finite element is a straight beam, as shown in Figure 3. We have taken a beam 120 cm long having a circular cross section of 3 cm radius and divided it into six elements of equal length. The element numbers in Figure 3 are enclosed in the square symbols and the nodes are numbered one through 8 and denoted by the solid circular symbols. Node eight is a dummy node required to establish the element coordinate system. The global coordinate system is shown in Figure 3, and for this case local coordinate system for each of the elements coincides with the global system. The ends of the beam are simply supported and restrained against rotation about the global x-axis. Concentrated loads are applied at the center of the beam, node number 4, in the Y and Z directions. The beam material properties and the inflation pressure are also tabulated in Figure 3.

It is desirable to be able to model curved members with this finite element, so to establish its validity for this class of structures, a circular arch was modeled as shown in Figure 4. This arch has the form of a semicircle with a radius of 120 cm and the circular cross section has a radius of 4 cm. This arch is modeled with eighteen elements of equal length. The element numbers are included in square symbols and the node points are denoted by the solid circular symbols. Node number twenty is a dummy used to establish the local element coordinate system. The global coordinate system is shown in Figure 4 and the arch material properties and inflation pressure are tabulated there also. The ends of the arch are simply supported, and displacements in the X direction for all nodes are suppressed. A concentrated load of unit magnitude is applied at node number ten in the negative Z direction.

For both the beam and the arch we present the input data and the output results from the NONFESA program and a comparison of the finite element results with the results from the exact solutions of the theory. The computer input and results associated with the NONFESA program are given in Appendices B and C for the beam and arch respectively and the comparison of the finite element results with those from the exact solution are given in Table 1 and 2. The input data in Appendices B and C are presented as illustrative examples and no comments are needed. The output from the NONFESA program also appearing in Appendices B and C can be divided into two parts, the first being a printout of the input data and some computed setup data such as the correspondence between degrees of freedom and equation numbers. It should be noted that the cross-section radius and the inflation pressure are printed under the Beam Geometric Properties heading as the Area X and Area Y, respectively. Also note that the parameter Inertia Y under this same heading is a negative number. As discussed above

ELASTIC MODULUS 2100 N/cm  
 SHEAR MODULUS 96 N/cm  
 PRESSURE 50 N/cm<sup>2</sup>

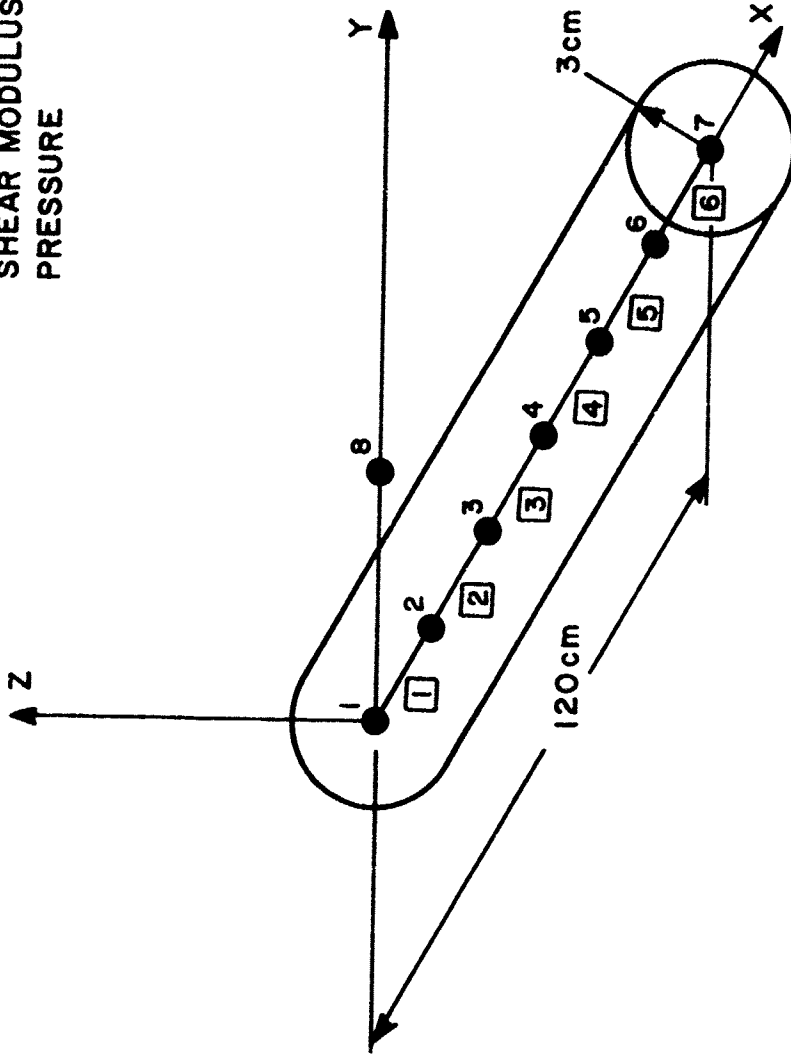


FIGURE 3. FINITE ELEMENT MODEL FOR STRAIGHT BEAM CHECK CASE

ELASTIC MODULUS 2100 N/cm  
 SHEAR MODULUS 96 N/cm  
 PRESSURE 50 N/cm<sup>2</sup>

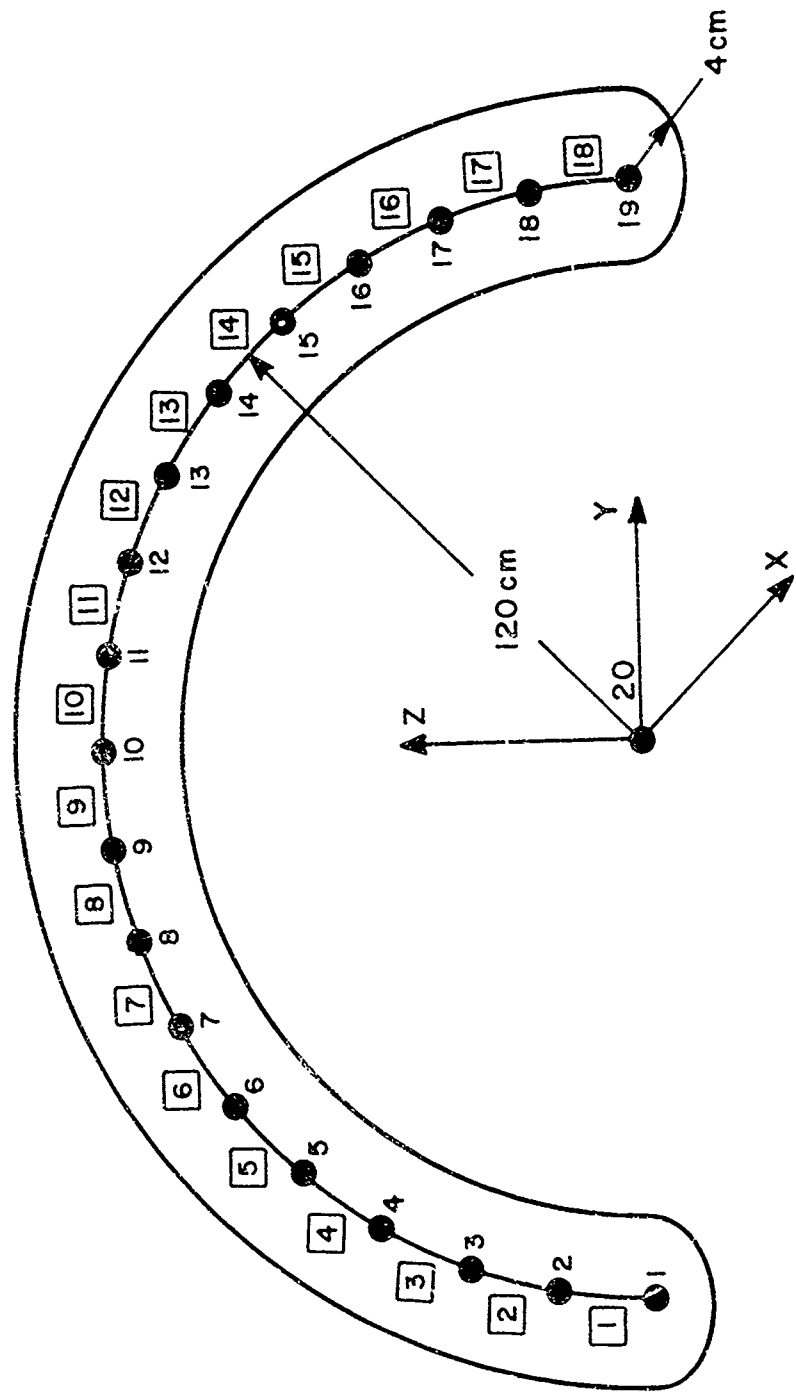


FIGURE 4. FINITE ELEMENT MODEL FOR CIRCULAR ARCH CHECK CASE

**TABLE 1**  
**Comparison of Finite Element Results with Those from the Exact Solution**  
**for the Beam Check Case**

Position (cm)	Node Number	Displacement		Rotation		Bending Stress	
		Finite Element	Exact	Finite Element	Exact	Finite Element	Exact
-60	1	0.	0.	$-3.2 \times 10^{-4}$	$-3.2 \times 10^{-4}$	0.	0
-40	2	0.0071	0.0071	-2.9	-2.9	0.0215	0.0215
-20	3	0.0137	0.0137	-2.1	-2.1	0.0561	0.0561
0	4	0.0189	0.0189	0.	0.	0.125	0.125
20	5	0.0137	0.0137	2.1	2.1	0.0561	0.0561
40	6	0.0071	0.0071	2.9	2.9	0.0251	0.0251
60	7	0.	0.	3.2	3.2	0	0

TABLE 2

Comparison of Finite Element Results with Those  
from the Exact Solution for the Arch Check Case

Angular Position Degrees	Node No.	Normal Displacement cm		Axial Displacement cm		Rotation	
		Finite Element†	Exact	Finite Element	Exact	Finite Element	Exact
-90	1	0.	0.	0.	0.	$1.4 \times 10^{-4}$	$1.4 \times 10^{-4}$
-80	2	- 3.58 $\times 10^{-3}$	- 3.58 $\times 10^{-3}$	-0.52 $\times 10^{-3}$	-0.53 $\times 10^{-3}$	1.3	1.2
-70	3	- 5.82	- 5.81	-1.57	-1.59	0.7	0.7
-60	4	- 6.39	- 6.35	-2.86	-2.91	0.06	0.05
-50	5	- 5.16	- 5.11	-4.11	-4.16	-0.6	-0.6
-40	6	- 2.19	- 2.12	-4.98	-5.03	-1.3	-1.3
-30	7	2.28	2.39	-5.19	-5.24	-1.8	-1.8
-20	8	7.89	8.0	-4.49	-4.54	-2.0	-2.0
-10	9	13.5	14.0	-2.17	-2.78	-1.6	-1.6
0	10	19.2	19.3	0.	0.	0.	0.

Angular Position Degrees	Node No.	Axial Force, N		Moment N-cm		Stress N/cm	
		Finite Element	Exact	Finite Element	Exact	Finite Element	Exact
-90	1	- 0.525	- 0.500	0.0	0.0	-0.020	-0.019
-80	2	- 0.543	- 0.545	0.895	0.868	-0.039	-0.039
-70	3	- 0.572	- 0.573	1.31	1.27	-0.049	-0.049
-60	4	- 0.583	- 0.585	1.47	1.43	-0.052	-0.052
-50	5	- 0.576	- 0.578	1.46	1.41	-0.052	-0.051
-40	6	- 0.552	- 0.554	1.29	1.24	-0.047	-0.047
-30	7	- 0.511	- 0.513	0.88	0.836	-0.037	-0.037
-20	8	- 0.455	- 0.456	0.046	0.007	-0.019	-0.018
-10	9	- 0.385	- 0.386	-1.66	-1.69	-0.018	0.019
0	10	- 0.347	- 0.304	-5.26	-5.29	-0.091	0.095

under the description of input, this is used for testing whether the beam element is a pressure stabilized element for computation of stresses. The second part of the output contains the computed results for the beam stresses, beam forces and moments, nodal displacements and rotations, and the nodal displacements. As is indicated on the printout, for pressure stabilized beam elements the stress should be interpreted as a membrane stress with units of force per unit length.

The comparison of the finite element results with those given by the exact solution of the beam theory of reference 1 is presented in Table 1. The simple six-element finite element model is in agreement with the exact solution to all decimal places shown for all three parameters, displacement, rotation, and bending stress. The limit on the number of places given in Table 1 is the output of the NONFESA program. The maximum possible percentage difference between the finite element results and the exact solution is then 2.4%. This is good agreement and establishes the validity of the element for straight beams.

The comparison of the finite element results with those given by the exact solution of the arch theory of reference 3 is presented in Table 2. Results are presented for half the arch since the results for the other half can be inferred from those presented by the symmetry of the structure and the loading. The finite element results generally follow the results from the exact solution with differences of less than 5% and in most cases considerably less than 5%. There are, however, three cases where the difference is greater than 5%. Two of these, the axial displacement at node 9 and the axial force at node 10, do not appear to have an easy explanation but the third one, the moment at node 8, appears to be due to the closeness of the node to the point where the moment changes sign. In this case, the exact solution is very close to the value zero, and thus any error in the finite element solution is magnified. In general, this comparison is good and establishes the validity of the finite element for curved structures.

### CONCLUDING REMARKS

The development of a finite element for pressure-stabilized beams is described along with its adaptation to the Nonlinear Finite Element Structural Analysis program. This provides a means for carrying out design analysis of pressure-stabilized frame-supported tents under rather general loading. The validity of the finite element and the correctness of its adaptation to the structural analysis program is established by the comparison of finite element results with exact solution for a straight beam and a circular arch.

## REFERENCES

1. Steeves, Earl C.; A Linear Analysis of the Deformation of Pressure Stabilized Beams; US Army Natick Laboratories, Technical Report 75-47-AMEL; 1975 (AD A006493).
2. Steeves, Earl C.; Behavior of Pressure Stabilized Beams Under Load; US Army Natick Development Center, Technical Report 75-82-AMEL; 1975 (AD A010702).
3. Steeves, Earl C., The Structural Behavior of Pressure Stabilized Arches; US Army Natick Research & Development Command; NATICK/TR-78/018; 1978.
4. Remington, Paul J., John C. O'Callahan and Richard Madden; Analysis of Stresses and Deflections in Frame Supported Tents; US Army Natick Laboratories, Technical Report 75-31; 1974 (AD A002072).
5. Remington, Paul J., John C. O'Callahan and Richard Madden; Finite-Element Analysis of Scale-Model Frame-Supported Tents; US Army Natick Research & Development Command, Technical Report 76-21-AMEL; 1975.
6. O'Callahan, John C.; NONFESA -- Nonlinear Finite Element Structural Analysis Code for the Analysis of Stresses and Deflections in Frame Supported Tents; Bolt Beranek and Newman, Inc., Report No. 2803, 1975.



**APPENDIX A**  
**ELEMENTS OF THE 8 X 8 BENDING**  
**STIFFNESS MATRIX**

## APPENDIX A

### ELEMENTS OF THE 8 x 8 BENDING STIFFNESS MATRIX

The elements of the 8 x 8 bending stiffness matrix are given here in terms of the 4 x 4 submatrices  $K_{11}$ ,  $K_{12}$ , and  $K_{22}$ . In presenting these elements, use is made of the symmetry of the matrices wherever possible

#### Submatrix $K_{11}$

Row index	Column index	Element
1	1	$6(P + C)/5L$
2	1	$C/2$
3	1	$-6(P + C)/5L$
4	1	$C/2$
2	2	$6D/5L + 13CL/35$
3	2	$-C/2$
4	2	$-6D/5L + 9CL/70$
3	3	$6(P + C)/5L$
4	3	$-C/2$
4	4	$6D/5L + 13CL/35$

#### Submatrix $K_{12}$

Row index	Column index	Element
1	1	$(P + C)/10$
2	1	$-CL/10$
3	1	$-(P + C)/10$
4	1	$CL/10$
1	2	$CL/10$

Row index	Column index	Element
2	2	$D/10 + 11CL^2/210$
3	2	$-CL/10$
4	2	$-D/10 + 13CL^2/420$
1	3	$(P + C)/10$
2	3	$CL/10$
3	3	$-(P + C)/10$
4	3	$-CL/10$
1	4	$-CL/10$
2	4	$D/10 - 13CL^2/420$
3	4	$CL/10$
4	4	$-D/10 - 11CL^2/210$

Submatrix  $K_{22}$

Row index	Column index	Element
1	1	$2(P + C)L/15$
2	1	0
3	1	$-(P + C)L/30$
4	1	$-CL^2/60$
2	2	$2DL/15 + CL^3/105$
3	2	$CL^2/60$
4	2	$-DL/30 - CL^3/140$
3	3	$2(P + C)L/15$
3	4	0
4	4	$2DL/15 + CL^3/105$

APPENDIX B  
INPUT DATA AND RESULTS FOR  
BEAM CHECK CASE

INPUT DATA

	PRESSURIZED	BEAM	CHECK	CASE					
1	8	1	1	1	00.0	0.0	0.0	11101000	
2	1	0	0	0	020.0	0.0	0.0		
3	0	0	0	0	040.0	0.0	0.0		
4	0	0	0	0	060.0	0.0	0.0		
5	0	0	0	0	080.0	0.0	0.0		
6	0	0	0	0	0100.0	0.0	0.0		
7	1	1	1	1	0120.0	0.0	0.0		
8	1	1	1	1	10.0	10.0	0.0		
9	1	6	1	0					
10	1	2100.0	9.9375	1					
11	1	4.0	50.0	1.0					
12	1	1	2	8	1.0	-1.0	1.0	1.0	1.0
13	2	3	8	1	0				
14	3	4	8	1	0				
15	4	4	8	1	0				
16	4	5	8	1	0				
17	5	6	8	1	0				
18	6	7	8	1	0				
19	6	7	8	1	0				
20	4	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
21									
22									
23									

## RESULTS

```
*****
*          P R O G R A M   N O N F E S A          *
*          *****                               *
*          *                                     *
*          *  NONLINEAR FINITE ELEMENT STRUCTURAL ANALYSIS *
*          *  PROGRAM FOR DETERMINING DEFLECTIONS AND *
*          *  STRESSES OF TENT STRUCTURES.  THE PROGRAM IS A *
*          *  GENERAL PROGRAM CAPABLE OF ASSEMBLING MANY *
*          *  DIFFERENT ELEMENT TYPES.  THE PRESENT VERSION *
*          *  (MARCH, 1975) CONTAINS ELEMENTS TYPES- *
*          *  1. LINEAR BEAM ELEMENT *
*          *     1.1 LINEAR BEAM *
*          *     1.2 MODIFIED BEAM *
*          *  2. GEOMETRIC NONLINEAR MEMBRANE *
*          *     2.1 LINEAR MATERIAL *
*          *     2.2 NONLINEAR MATERIAL *
*          *     2.3 MATERIAL REFERENCE PLANE *
*          *  3. LINEAR TRUSS ELEMENT *
*          *     3.1 TRUSS (PRE-LOAD) *
*          *     3.2 GUY LINE (TENSILE) *
*          *     3.3 COMPRESSIVE BAR *
*          *                                     *
*          *****                               *
*          *  PROGRAM DEVELOPED AT BOLT BERANEK AND NEWMAN *
*          *  CAMBRIDGE, MASS. BY J.C. OCALLAHAN. *
*          *****
```

PRESSURIZED BEAM CHECK CASE

NUMBER OF NODAL POINTS = 8  
NUMBER OF ELEMENT TYPES = 1  
MAXIMUM NUMBER OF CYCLES = 0  
NUMBER OF LOAD STEPS = 0  
CONVERGENCE TOLERANCE = .000  
STARTING MEMBRANE TENSION = .000  
DERIVATIVE SPACING = .100+00

PRINT CONTROL

(0--OFF, 1--ON, EXCEPT FOR DISPLACEMENTS)

1. DELTA DISPLACEMENTS = 1  
2. ELEMENT STRESSES = 1  
3. BEAM FORCES = 1  
4. DISPLACEMENTS(1--OFF) = 0  
5. NODAL POSITIONS = 1  
6. DEBUG-MEMBRANE  
ELEMENT STIFFNESS = 0  
7. DEBUG-MEMBRANE  
GEOMETRIC PROP = 0  
8. DEBUG-MEMBRANE  
MATERIAL COEFS = 0

NODAL POINT INPUT DATA

NODE NUMBER	X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z
1	1	1	1	1	1	1	.000	.000	.000
2	0	0	0	0	0	0	20.000	.000	.000
3	0	0	0	0	0	0	40.000	.000	.000
4	0	0	0	0	0	0	60.000	.000	.000
5	0	0	0	0	0	0	80.000	.000	.000
6	0	0	0	0	0	0	100.000	.000	.000
7	1	1	1	1	1	1	120.000	.000	.000
8	1	1	1	1	1	1	.000	10.000	.000

GENERATED NODAL DATA

NODE NUMBER	X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z
1	1	1	1	1	1	1	.000	.000	.000
2	0	0	0	0	0	0	20.000	.000	.000
3	0	0	0	0	0	0	40.000	.000	.000
4	0	0	0	0	0	0	60.000	.000	.000
5	0	0	0	0	0	0	80.000	.000	.000
6	0	0	0	0	0	0	100.000	.000	.000
7	1	1	1	1	1	1	120.000	.000	.000
8	1	1	1	1	1	1	.000	10.000	.000

EQUATION NUMBERS

EQUATION NUMBER	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	1	2
2	3	4	5	6	7	8
3	9	10	11	12	13	14
4	15	16	17	18	19	20
5	21	22	23	24	25	26
6	27	28	29	30	31	32
7	0	0	0	0	33	34
8	0	0	0	0	0	0



T H R E E   D I M E N S I O N A L   B E A M   E L E M E N T S

NUMBER OF BEAMS = 6  
NUMBER OF GEOMETRIC PROPERTY SETS = 1  
NUMBER OF FIXED END FORCE SETS = 0  
NUMBER OF MATERIALS = 1

MATERIAL 1      YOUNG S  
                  MODULUS    : 21000+04

                  POISSON S  
                  RATIO        : 9.93750

BEAM GEOMETRIC PROPERTIES

ELEMENT TYPE	AREA X	AREA Y	AREA Z	INERTIA X	INERTIA Y	INERTIA Z	FIBER DISTANCE DY	FIBER DISTANCE DZ
1	.4000+01	.5000+02	.1000+01	.1000+01	-.1000+01	.1000+01	.1000+01	.1000+01

BEAM ELEMENT PARAMETERS

BEAM NO	NODES		MATL		GEOM		ELEM LOADS	SPEC CODE	END CODES	
	I	J	K	NO	NO	NO			I	J
1	1	2	8	1	1	1	0	0	0	0
2	2	3	8	1	1	1	0	0	0	0
3	3	4	8	1	1	1	0	0	0	0
4	4	5	8	1	1	1	0	0	0	0
5	5	6	8	1	1	1	0	0	0	0
6	6	7	8	1	1	1	0	0	0	0

TOTAL NUMBLR OF EQUATIONS = 34  
 BANDWIDTH = 12  
 NUMBER OF EQUATIONS IN A BLOCK = 34  
 NUMBER OF BLOCKS = 1

.....MODAL POINT LOADS

NODE NO.	RX	RY	RZ	MX	MY	MZ
4	.000	1.000	1.000	.000	.000	.000

..... B E A M   S T R E S S E S .....  
 BEAM NO.    NODE I    AXIAL STRESS    2 BENDING STRESSES (+3 DIR)    3 BENDING STRESSES (+2 DIR)    (-2 DIR)    AVERAGE SHEAR STRESSES (+2 DIR)    (+3 DIR)

FOR PRESSURE STABILIZED BEAM ELEMENTS STRESSES ARE  
 MEMBRANE TYPE WITH UNITS FORCE/UNIT LENGTH

BEAM NO.	NODE I	NODE J	AXIAL STRESS	2 BENDING STRESSES (+3 DIR)	3 BENDING STRESSES (+2 DIR)	(-2 DIR)	AVERAGE SHEAR STRESSES (+2 DIR)	(+3 DIR)
1	1	2	.0000	-.0000	.0000	-.0000	-.0199	-.0199
	2	1	.0000	.0215	-.0215	.0215	.0199	.0199
2	2	3	.0000	.0215	-.0215	.0215	-.0199	-.0199
	3	2	.0000	-.0561	.0561	-.0561	.0199	.0199
3	3	4	.0000	.0561	-.0561	.0561	-.0199	-.0199
	4	3	.0000	-.1250	.1250	-.1250	.0199	.0199
4	4	5	.0000	.1250	-.1250	.1250	-.0199	-.0199
	5	4	.0000	-.0561	.0561	-.0561	.0199	.0199
5	5	6	.0000	.0561	-.0561	.0561	-.0199	-.0199
	6	5	.0000	-.0215	.0215	-.0215	.0199	.0199
6	6	7	.0000	.0215	-.0215	.0215	-.0199	-.0199
	7	6	.0000	-.0000	.0000	-.0000	.0199	.0199

..... B E A M F O R C E S A N D M O M E N T S .....

BEAM (NODE I) AXIAL	SHEAR	SHEAR	TORSION	BENDING	BENDING
NO. (NODE J) R1	R2	R3	M1	M2	M3
1 .000 .000	-.500+00 .500+00	-.500+00 .500+00	.000 .000	.596--07 .108+01	-.596--07 -.108+01
2 .000 .000	-.500+00 .500+00	-.500+00 .500+00	.000 .000	-.108+01 .232+01	.108+01 -.232+01
3 .000 .000	-.500+00 .500+00	-.500+00 .500+00	.000 .000	-.282+01 .628+01	.282+01 -.628+01
4 .000 .000	.500+00 -.500+00	.500+00 -.500+00	.000 .000	-.628+01 .282+01	.628+01 -.282+01
5 .000 .000	.500+00 -.500+00	.500+00 -.500+00	.000 .000	-.282+01 .108+01	.282+01 -.108+01
6 .000 .000	.500+00 -.500+00	.500+00 -.500+00	.000 .000	-.108+01 -.238-06	.108+01 .238-06

.....TOTAL NODAL.....  
DISPLACEMENTS AND ROTATIONS.....

NODE	X	Y	Z	XX	YY	ZZ
1	.000	.000	.000	.00	-.32-03	.32-03
2	.000	.710-02	.710-02	.00	-.29-03	.29-03
3	.000	.137-01	.137-01	.00	-.21-03	.21-03
4	.000	.189-01	.189-01	.00	-.27-11	.27-11
5	.000	.137-01	.137-01	.00	.21-03	-.21-03
6	.000	.710-02	.710-02	.00	.29-03	-.29-03
7	.000	.000	.000	.00	.32-03	-.32-03
8	.000	.000	.000	.00	.00	.00



N O D A L P O S I T I O N S

NODE	X	Y	Z
1	.000	.000	.000
2	20.000	.007	.007
3	40.000	.014	.014
4	60.000	.019	.019
5	80.000	.014	.014
6	100.000	.007	.007
7	120.000	.000	.000
8	.000	10.000	.000

**APPENDIX C**

**INPUT DATA AND RESULTS FOR  
ARCH CHECK CASE**

### INPUT DATA

ARCH	CHECK	CASE	FOR P.	B.	FINITE	ELEMENT						
1												
2	20	1	0	0	0	0	.0000	.0000	.000001101000	0	0	0
3		1	1	1	0	0	.0000	-120.0000	.0000	.0000	0	0
4		2	1	0	0	0	.0000	-113.1772	20.8380	0	0	0
5		3	1	0	0	0	.0000	-112.7628	41.0424	0	0	0
6		4	1	0	0	0	.0000	-103.9224	60.0000	0	0	0
7		5	1	0	0	0	.0000	-91.9248	77.1336	0	0	0
8		6	1	0	0	0	.0000	-77.1336	91.9248	0	0	0
9		7	1	0	0	0	.0000	-60.0000	103.9224	0	0	0
10		8	1	0	0	0	.0000	-41.0424	112.7628	0	0	0
11		9	1	0	0	0	.0000	-20.8380	118.1772	0	0	0
12		10	1	0	0	0	.0000	.0000	120.0000	0	0	0
13		11	1	0	0	0	.0000	20.8380	118.1772	0	0	0
14		12	1	0	0	0	.0000	41.0424	112.7628	0	0	0
15		13	1	0	0	0	.0000	60.0000	103.9224	0	0	0
16		14	1	0	0	0	.0000	77.1336	91.9248	0	0	0
17		15	1	0	0	0	.0000	91.9248	77.1336	0	0	0
18		16	1	0	0	0	.0000	103.9224	60.0000	0	0	0
19		17	1	0	0	0	.0000	112.7628	41.0424	0	0	0
20		18	1	0	0	0	.0000	118.1772	20.8380	0	0	0
21		19	1	1	0	0	.0000	120.0000	.0000	0	0	0
22		20	1	1	1	1	.0000	.0000	.0000	.0000	0	0
23		1	18	1	1	0	1					
24		1	2100.0000	9.9375								
25		1	4.0000	50.0000	1.000	1.000	1.0000	-1.00	1.00001.0001.000	0	1	0
26		1	1	2	20	1	0	0	0	0	1	0
27		2	2	3	20	1	0	0	0	0	1	0
28		3	3	4	20	1	0	0	0	0	1	0
29		4	4	5	20	1	0	0	0	0	1	0
30		5	5	6	20	1	0	0	0	0	1	0
31		6	6	7	20	1	0	0	0	0	1	0
32		7	7	8	20	1	0	0	0	0	1	0
33		8	8	9	20	1	0	0	0	0	1	0
34		9	9	10	20	1	0	0	0	0	1	0
35		10	10	11	20	1	0	0	0	0	1	0
36		11	11	12	20	1	0	0	0	0	1	0
37		12	12	13	20	1	0	0	0	0	1	0
38		13	13	14	20	1	0	0	0	0	1	0
39		14	14	15	20	1	0	0	0	0	1	0
40		15	15	16	20	1	0	0	0	0	1	0
41		16	16	17	20	1	0	0	0	0	1	0
42		17	17	18	20	1	0	0	0	0	1	0
43		18	18	19	20	1	0	0	0	0	1	0
44												
45												
46												
47												

## RESULTS

```
*****
* P R O G R A M   N O N F E S A
* *****
* NONLINEAR FINITE ELEMENT STRUCTURAL ANALYSIS
* PROGRAM FOR DETERMINING DEFLECTIONS AND
* STRESSES OF TENT STRUCTURES. THE PROGRAM IS A
* GENERAL PROGRAM CAPABLE OF ASSEMBLING MANY
* DIFFERENT ELEMENT TYPES. THE PRESENT VERSION
* (MARCH, 1975) CONTAINS ELEMENTS TYPES-
* 1. LINEAR BEAM ELEMENT
*   1.1 LINEAR BEAM
*   1.2 MODIFIED BEAM
* 2. GEOMETRIC NONLINEAR MEMBRANE
*   2.1 LINEAR MATERIAL
*   2.2 NONLINEAR MATERIAL
*   2.3 MATERIAL REFERENCE PLANE
* 3. LINEAR TRUSS ELEMENT
*   3.1 TRUSS (PRE-LOAD)
*   3.2 GUY LINE (TENSILE)
*   3.3 COMPRESSIVE BAR
* *****
* PROGRAM DEVELOPED AT BCLT BERANEK AND NEWMAN
* CAMBRIDGE, MASS. BY J.C. OCALLAHAN.
* *****
```

ARCH CHECK CASE FOR P. B. FINITE ELEMENT

NUMBER OF NODAL POINTS = 20  
NUMBER OF ELEMENT TYPES = 1  
MAXIMUM NUMBER OF CYCLES = 0  
NUMBER OF LOAD STEPS = 0  
CONVERGENCE TOLERANCE = .000  
STARTING MEMBRANE TENSION = .000  
DERIVATIVE SPACING = .100+00

PRINT CONTROL  
(0--OFF, 1--ON, EXCEPT FOR DISPLACEMENTS)

1. DELTA DISPLACEMENTS = 0  
2. ELEMENT STRESSES = 1  
3. BEAM FORCES = 1  
4. DISPLACEMENTS(1--OFF) = 0  
5. NODAL POSITIONS = 1  
6. DEBUG-MEMBRANE = 0  
ELEMENT STIFFNESS  
7. DEBUG-MEMBRANE = 0  
GEOMETRIC PROP  
8. DEBUG-MEMBRANE = 0  
MATERIAL COEFS

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES			NODAL POINT COORDINATES					
	X	Y	Z	XX	YY	ZZ	X	Y	Z
1	1	1	1	0	0	0	.000	-120.000	.000
2	1	0	0	0	0	0	.000	-118.177	20.838
3	1	0	0	0	0	0	.000	-112.763	41.042
4	1	0	0	0	0	0	.000	-103.922	60.000
5	1	0	0	0	0	0	.000	-91.925	77.134
6	1	0	0	0	0	0	.000	-77.134	91.925
7	1	0	0	0	0	0	.000	-60.000	103.922
8	1	0	0	0	0	0	.000	-41.042	112.763
9	1	0	0	0	0	0	.000	-20.838	118.177
10	1	0	0	0	0	0	.000	.000	120.000
11	1	0	0	0	0	0	.000	20.838	118.177
12	1	0	0	0	0	0	.000	41.042	112.763
13	1	0	0	0	0	0	.000	60.000	103.922
14	1	0	0	0	0	0	.000	77.134	91.925
15	1	0	0	0	0	0	.000	91.925	77.134
16	1	0	0	0	0	0	.000	103.922	60.000
17	1	0	0	0	0	0	.000	112.763	41.042
18	1	0	0	0	0	0	.000	118.177	20.838
19	1	1	1	0	0	0	.000	120.000	.000
20	1	1	1	1	1	1	.000	.000	.000

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES			NODAL POINT COORDINATES					
	X	Y	Z	XX	YY	ZZ	X	Y	Z
1	1	1	1	0	0	0	.000	-120.000	.000
2	1	0	0	0	0	0	.000	-118.177	20.838
3	1	0	0	0	0	0	.000	-112.763	41.042
4	1	0	0	0	0	0	.000	-103.922	60.000
5	1	0	0	0	0	0	.000	-91.925	77.134
6	1	0	0	0	0	0	.000	-77.134	91.925
7	1	0	0	0	0	0	.000	-60.000	103.922
8	1	0	0	0	0	0	.000	-41.042	112.763
9	1	0	0	0	0	0	.000	-20.838	118.177
10	1	0	0	0	0	0	.000	.000	120.000
11	1	0	0	0	0	0	.000	20.838	118.177
12	1	0	0	0	0	0	.000	41.042	112.763
13	1	0	0	0	0	0	.000	60.000	103.922
14	1	0	0	0	0	0	.000	77.134	91.925
15	1	0	0	0	0	0	.000	91.925	77.134
16	1	0	0	0	0	0	.000	103.922	60.000
17	1	0	0	0	0	0	.000	112.763	41.042
18	1	0	0	0	0	0	.000	118.177	20.838
19	1	1	1	0	0	0	.000	120.000	.000
20	1	1	1	1	1	1	.000	.000	.000

EQUATION NUMBER	X	Y	Z	XX	YY	ZZ
1	0	0	0	1	2	3
2	0	4	5	6	7	8
3	0	9	10	11	12	13

4	0	14	15	16	17	18
5	0	19	20	21	22	23
6	0	24	25	26	27	28
7	0	29	30	31	32	33
8	0	34	35	36	37	38
9	0	39	40	41	42	43
10	0	44	45	46	47	48
11	0	49	50	51	52	53
12	0	54	55	56	57	58
13	0	59	60	61	62	63
14	0	64	65	66	67	68
15	0	69	70	71	72	73
16	0	74	75	76	77	78
17	0	79	80	81	82	83
18	0	84	85	86	87	88
19	0	0	0	89	90	91
20	0	0	0	0	0	0

T H R E E   D I M E N S I O N A L   B E A M   E L E M E N T S

NUMBER OF BEAMS = 18  
NUMBER OF GEOMETRIC PROPERTY SETS = 1  
NUMBER OF FIXED END FORCE SETS = 0  
NUMBER OF MATERIALS = 1



MATERIAL	YOUNG S MODULUS	POISSON S RATIO
1	.21000+04	9.93750

BEAM: GEOMETRIC PROPERTIES

ELEMENT TYPE	AREA X	AREA Y	AREA Z	INERTIA X	INERTIA Y	INERTIA Z	FIBER DISTANCE CY DZ
1	.4000+01	.5000+02	.1000+01	.1000+01	-.1000+01	.1000+01	.1000+01 .1000+01

BEAM ELEMENT PARAMETERS

BEAM NO	NODES		MATL GEOM		ELEM LOADS	SPEC CODE	END CODES	
	I	J	K	NO			I	J
1	1	2	20	1	0	0	0	0
2	2	3	20	1	0	0	0	0
3	3	4	20	1	0	0	0	0
4	4	5	20	1	0	0	0	0
5	5	6	20	1	0	0	0	0
6	6	7	20	1	0	0	0	0
7	7	8	20	1	0	0	0	0
8	8	9	20	1	0	0	0	0
9	9	10	20	1	0	0	0	0
10	10	11	20	1	0	0	0	0
11	11	12	20	1	0	0	0	0
12	12	13	20	1	0	0	0	0
13	13	14	20	1	0	0	0	0
14	14	15	20	1	0	0	0	0
15	15	16	20	1	0	0	0	0
16	16	17	20	1	0	0	0	0
17	17	18	20	1	0	0	0	0
18	18	19	20	1	0	0	0	0

TOTAL NUMBER OF EQUATIONS = 91  
 BANDWIDTH = 10  
 NUMBER OF EQUATIONS IN A BLOCK = 91  
 NUMBER OF BLOCKS = 1

.....NODAL POINT LOADS

NODE NO.	RX	RY	APPLIED LOADS	MY	MZ
			RZ		
10	.000	.000	-1.000	.000	.000
			MX		
			.000		

..... B E A M   S T R E S S E S .....  
 BEAM NO.    NODE I    NODE J    AXIAL STRESS    2 BENDING STRESSES (+3 DIR)    3 BENDING STRESSES (+2 DIR)    AVERAGE SHEAR STRESSES (+2 DIR)    (+3 DIR)

FOR PRESSURE STABILIZED BEAM ELEMENTS STRESSES ARE MEMBRANE TYPE WITH UNITS FORCE/UNIT LENGTH

BEAM NO.	NODE I	NODE J	AXIAL STRESS	2 BENDING STRESSES (+3 DIR)	3 BENDING STRESSES (+2 DIR)	AVERAGE SHEAR STRESSES (+2 DIR)	(+3 DIR)
1	1	2	-.0209	.0000	.0000	-.0000	.0000
	2	3	-.0209	.0000	-.0178	.0178	-.0103
2	2	3	-.0224	.0000	-.0178	.0178	.0066
	3	4	-.0224	.0000	-.0261	.0261	-.0066
3	3	4	-.0232	.0000	-.0261	.0261	.0026
	4	5	-.0232	.0000	-.0293	.0293	-.0026
4	4	5	-.0233	.0000	-.0293	.0293	-.0015
	5	6	-.0233	.0000	-.0291	.0291	.0015
5	5	6	-.0226	.0000	-.0291	.0291	-.0055
	6	7	-.0226	.0000	-.0256	.0256	.0055
6	6	7	-.0213	.0000	-.0256	.0256	-.0093
	7	8	-.0213	.0000	-.0175	.0175	.0093
7	7	8	-.0194	.0000	-.0175	.0175	-.0129
	8	9	-.0194	.0000	-.0009	.0009	.0129
8	8	9	-.0169	.0000	-.0009	.0009	-.0161
	9	10	-.0169	.0000	.0331	-.0331	.0161
9	9	10	-.0138	.0000	.0331	.0331	-.0188
	10	11	-.0138	.0000	.1046	-.1046	.0188
10	10	11	-.0138	.0000	.1046	.1046	-.0188
	11	12	-.0169	.0000	.0331	-.0331	.0161
	12	13	-.0169	.0000	-.0009	.0009	-.0161
12	12	13	-.0194	.0000	-.0009	.0009	.0129
	13	14	-.0194	.0000	-.0175	.0175	-.0129
13	13	14	-.0213	.0000	-.0175	.0175	.0093
	14	15	-.0213	.0000	-.0256	.0256	-.0093
14	14	15	-.0226	.0000	-.0256	.0256	.0055
	15	16	-.0226	.0000	-.0291	.0291	-.0055
15	15	16	-.0233	.0000	-.0291	.0291	.0015
	16		-.0233	.0000	-.0293	.0293	-.0015
16	16		-.0232	.0000	-.0293	.0293	.0026
			-.0232	.0000	-.0261	.0261	-.0026

17	17	-0.24	.0000	-.0261	.0261	-.0066	.0000
	18	-.0224	.0000	-.0178	.0178	.0066	.0000
18	18	-.0209	.0000	-.0178	.0178	-.0103	.0000
	19	-.0209	.0000	.0000	-.0000	.0103	.0000

.....B E A M F O R C E S A N D M O M E N T S .....

BEAM (NODE I) AXIAL NO. (NODE J) R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1 .525+00 -.525+00	.260+00 -.260+00	.000 .000	.000 .000	.000 .000	.596-C7 .895+00
2 .562+00 -.562+00	.165+00 -.165+00	.000 .000	.000 .000	.000 .000	-.895+00 .131+01
3 .582+00 -.582+00	.648-C1 -.648-C1	.000 .000	.000 .000	.000 .000	-.131+01 .147+01
4 .584+00 -.584+00	-.372-01 .372-01	.000 .000	.000 .000	.000 .000	-.147+01 .146+01
5 .569+00 -.569+00	-.138+00 .138+00	.000 .000	.000 .000	.000 .000	-.146+01 .129+01
6 .536+00 -.536+00	-.235+00 .235+00	.000 .000	.000 .000	.000 .000	-.129+01 .880+00
7 .487+00 -.487+00	-.324+00 .324+00	.000 .000	.000 .000	.000 .000	-.880+00 .465-01
8 .424+00 -.424+00	-.404+00 .404+00	.000 .000	.000 .000	.000 .000	-.465-01 -.166+01
9 .347+00 -.347+00	-.472+00 .472+00	.000 .000	.000 .000	.000 .000	.166+01 -.526+01
10 .347+00 -.347+00	.472+00 -.472+00	.000 .000	.000 .000	.000 .000	.526+01 -.166+01
11 .424+00 -.424+00	.404+00 -.404+00	.000 .000	.000 .000	.000 .000	.166+01 .465-01
12 .487+00 -.487+00	.324+00 -.324+00	.000 .000	.000 .000	.000 .000	-.465-01 .880+00
13 .536+00 -.536+00	.235+00 -.235+00	.000 .000	.000 .000	.000 .000	-.880+00 .129+01

14	.569+00	.138+00	.000	.000	.000	-.129+01
	-.569+00	-.138+00	.000	.000	.000	.146+01
15	.584+00	.372-01	.000	.000	.000	-.146+01
	-.584+00	-.372-01	.000	.000	.000	.147+01
16	.582+00	-.648-01	.000	.000	.000	-.147+01
	-.582+00	.648-01	.000	.000	.000	.131+01
17	.562+00	-.165+00	.000	.000	.000	-.131+01
	-.562+00	.165+00	.000	.000	.000	.895+00
18	.525+00	-.260+00	.000	.000	.000	-.895+00
	-.525+00	.260+00	.000	.000	.000	-.209-06



*****DISPLACEMENTS AND ROTATIONS*****						
NODE	X	Y	Z	XX	YY	ZZ
1	.000	.000	.000	.14-03	.00	.00
2	-.362-02	.108-03	.108-03	.12-03	.00	.00
3	-.601-02	.518-03	.518-03	.71-04	.00	.00
4	-.697-02	.711-03	.711-03	.57-05	.00	.00
5	-.660-02	.170-03	.170-03	-.63-04	.00	.00
6	-.523-02	-.152-02	-.152-02	-.13-03	.00	.00
7	-.335-02	-.457-02	-.457-02	-.18-03	.00	.00
8	-.152-02	-.895-02	-.895-02	-.20-03	.00	.00
9	-.298-03	-.142-01	-.142-01	-.16-03	.00	.00
10	.175-07	-.192-01	-.192-01	.11-09	.00	.00
11	.298-03	-.142-01	-.142-01	.16-03	.00	.00
12	.152-02	-.895-02	-.895-02	.20-03	.00	.00
13	.335-02	-.457-02	-.457-02	.18-03	.00	.00
14	.523-02	-.152-02	-.152-02	.13-03	.00	.00
15	.660-02	.170-03	.170-03	.63-04	.00	.00
16	.697-02	.711-03	.711-03	-.57-05	.00	.00
17	.601-02	.518-03	.518-03	-.71-04	.00	.00
18	.362-02	.108-03	.108-03	-.12-03	.00	.00
19	.000	.000	.000	-.14-03	.00	.00
20	.000	.000	.000	.00	.00	.00

N O D A L P O S I T I O N S

NODE	X	Y	Z
1	.000	-120.000	.000
2	.000	-118.181	20.838
3	.000	-112.769	41.043
4	.000	-103.929	60.001
5	.000	-91.931	77.134
6	.000	-77.139	91.923
7	.000	-60.003	103.918
8	.000	-41.044	112.754
9	.000	-20.838	118.163
10	.000	.000	119.981
11	.000	20.838	118.163
12	.000	41.044	112.754
13	.000	60.003	103.918
14	.000	77.139	91.923
15	.000	91.931	77.134
16	.000	103.929	60.001
17	.000	112.769	41.043
18	.000	118.181	20.838
19	.000	120.000	.000
20	.000	.000	.000

## SYMBOLS

$a$	Beam cross-section
$C$	shear stiffness
$C_{11}$	elastic modulus
$C_{33}$	shear modulus
$D$	bending stiffness
$E$	axial stiffness
$G$	torsional stiffness
$h_i$	shape functions used in displacement approximations
$K_{ij}$	submatrices of the $8 \times 8$ bending stiffness matrix
$L$	beam length
$P$	pressure parameter
$p$	pressure
$Q$	vector of generalized forces
$S$	$4 \times 4$ bending stiffness matrix
$S_{ij}$	elements of the matrix $S$
$u$	axial displacement
$U_1, U_2$	nodal axial displacements
$v, w$	bending displacements
$V_1, V_2, W_1, W_2$	nodal bending displacements
$x$	axial coordinate
$\Delta_1, \Delta_2$	vector defined by equation (9)

$\theta, \phi, \psi$  cross-section rotations  
 $\theta_1, \theta_2$  nodal torsional rotations  
 $\phi_1, \phi_2, \psi_1, \psi_2$  nodal bending rotations  
 $( )'$  denotes differentiation with respect to  $x$