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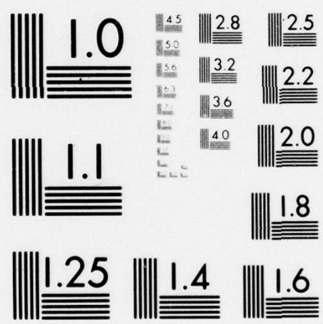

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A COMPARISON OF LYAPUNOV AND HYPERSTABILITY APPROACHES TO ADAPTIVE CONTROL

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A Comparison of Lyapunov and Hyperstability Approaches to Adaptive Control

Kumpati S. Narendra and Lena S. Valavani

1. Introduction: The study of adaptive systems using a stability approach has gained a wide following in recent years [1]. Using this approach adaptive problems are formulated as stability problems of multivariable nonlinear nonautonomous systems. Lyapunov's method and more recently Popov's hyperstability theory have been used as the principal tools in the analysis of such systems.

When Lyapunov's method is employed, the asymptotic stability of a set of error differential equations is studied using a suitable choice of a Lyapunov function candidate. While the theory has been applied effectively to autonomous systems, it has been less decisive in the adaptive context where the equations are nonautonomous. The difficulty lies in the fact that the Lyapunov function candidate $V(x)$ yields a negative semidefinite time derivative $\dot{V}(x,t)$. This, in turn, is not adequate to assure asymptotic stability since LaSalle's theorem [2] for autonomous systems does not carry over to this case.

The hyperstability approach [3,4] which is being increasingly used as an alternative to Lyapunov's method requires the problem to be recast as the stability of a feedback loop with a linear time-invariant operator (corresponding to the controller) in the feedback path. This structure, according to its proponents, provides the designer with greater flexibility in choosing the adaptive laws, since it is merely required to make the feedback block satisfy some passivity conditions for the system to be hyperstable. Further, in some recently published work on the subject, it is also implied that this approach can, in some ways, avoid the shortcomings of Lyapunov's method.

The aim of this brief paper is to examine the conditions which have to be satisfied for the two approaches to be successfully applied to adaptive observers and controllers and also to demonstrate that they are entirely equivalent. Using a typical error model it is shown that hyperstability and asymptotic hyperstability are achieved under exactly the same conditions as stability and asymptotic stability in the sense of Lyapunov. In those cases where the adaptive control problem remains unresolved, the difficulties encountered using Lyapunov's method are shown to have their counterparts in hyperstability theory as well.

2. Lyapunov's Direct Method:

The general statement of the conditions that are sufficient to guarantee the uniform asymptotic stability in the large of the solutions of the differential equation

$$\dot{x} = f(x,t) \qquad f(0,t) \equiv 0 \tag{1}$$

where x and f are n-vectors are very well known and stated in Theorem 1. f is assumed to be sufficiently regular that solutions exist for all $t \geq t_0$.

Definition: A real valued function $\phi(\rho)$ belongs to class $\{K\}$ if it is defined, continuous and strictly increasing for all ρ, $0 \leq \rho \leq \rho_1$ where ρ_1 is arbitrary and $\phi(0) = 0$.

Theorem 1 [Hahn]:

If a function V(x,t) that is defined for all x and t satisfies:

(i) V(x,t) is continuous with respect to x and t for all t.

(ii) V(x,t) is positive definite

i.e. $\alpha \in \{K\}$ exists such that

$$0 \leq \alpha(\|x\|) \leq V(x,t)$$

(iii) V(x,t) is radially unbounded

i.e. α in (ii) is such that $\lim_{\rho\to\infty} \alpha(\rho) = \infty$

(iv) V(x,t) is decrescent

i.e. there exists $\beta \in \{K\}$ such that

$$V(x,t) \leq \beta(\|x\|) \text{ for all } x \in R^n \text{ and } t$$

then a sufficient condition for the uniform asymptotic stability in the large of the solution of equation (1) is that

(v) there exists $\gamma \in \{K\}$ such that the total time derivative of $V(x,t)$ along system trajectories satisfies

$$\dot{V}(x,t) \triangleq \frac{\partial V}{\partial t} + \nabla V^T f(x,t) \leq -\gamma(\|x\|) < 0$$

for all $x \neq 0$ and t.

Positive definiteness of V and negative semidefiniteness of $\dot{V}$ result in stability. The fact that V is decrescent assures uniform stability. Condition (iii) on radial unboundedness yields stability in the large. Uniform asymptotic stability is guaranteed by the additional requirement that $\dot{V}(x,t)$ is negative definite.

The above theorem has been applied extensively in systems theory. Quite often it is found that $\dot{V}(x,t)$ is only negative semidefinite (as in the adaptive control problem discussed in this paper) and in such cases only uniform stability rather than uniform asymptotic stability can be concluded.

For autonomous systems defined by

$$\dot{x} = f(x) \tag{2}$$

a result due to LaSalle [2] assures asymptotic stability even when $\dot{V}(x)$ is semi-definite provided $\dot{V} \equiv 0$ cannot occur along any system trajectory other than $x(t;x_0,t_0) \equiv 0$. LaSalle's theorem also carries over to periodic systems where $f(x,t + T) = f(x,t)$ for some $T > 0$. In this case if $V(x,t) = V(x,t + T)$ satisfies the conditions (i)-(iv) and $\dot{V}(x,t) \leq 0$ and $\dot{V} \not\equiv 0$ on any trajectory, the system is asymptotically stable in the large.

From the above comments it is seen that the asymptotic stability of non-autonomous systems cannot be directly concluded from the negative semidefiniteness of the function $\dot{V}(x,t)$ and that additional properties of $f(x,t)$ must be used. Recent works in this area have addressed themselves to this problem [5],[6],[7].

The following lemma of Barbălat [8] is found to be important for the proof of asymptotic stability using Lyapunov's method as well as asymptotic hyperstability using Popov's theory.

Lemma: If g is a real function of the real variable t defined and uniformly continuous for $t > 0$ and if the limit of the integral

$$\int_0^t g(\tau)d\tau \tag{3}$$

as t tends to infinity exists and is a finite number, then

$$\lim_{t\to\infty} g(t) = 0.$$

If $\dot{V}(x,t)$ is identified with $g(t)$ in the above lemma, then if $\dot{V}(x,t)$ is uniformly continuous, every solution of the differential equation (1) would be such that $\lim_{t\to\infty} \dot{V}(x(t),t) = 0$.

An alternative form of Barbălat's lemma is also of interest.

Lemma: If g is a real function of the real variable t defined and uniformly continuous for $t \geq 0$ and if for every $\delta > 0$ there exists $\theta \in (0,\delta)$ such that

$$\lim_{t\to\infty} \int_t^{t+\theta} g(\tau)d\tau = 0$$

then

$$\lim_{t\to\infty} g(t) = 0. \tag{4}$$

The questions that arise in stability problems associated with adaptive control are best illustrated by applying the above theorems and lemmas to a typical error model.

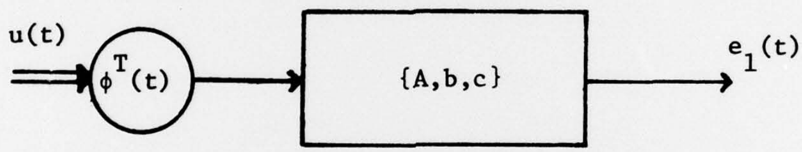
Error Model in Adaptive Control:

The following error model occurs very frequently in model reference adaptive control. We use Lyapunov's method to analyze its stability properties in this section and compare it with the hyperstability approach in Section 3.

The error model is described by the differential equations

$$\begin{aligned} \dot{e} &= Ae + b\phi^T(t)u(t) \\ e_1 &= c^T e \end{aligned} \tag{5}$$

where the transfer function $c^T(sI-A)^{-1}b$ is strictly positive real (Figure 1).



Strictly Positive Real

Transfer Function

Figure 1

$u(t)$ and $\phi(t)$ are m-vectors, $e(t)$ is an n-vector, $e_1(t)$ a scalar and A,b,c are matrices and vectors of appropriate dimensions. In the adaptive control problem the vector $e(t)$ represents the state error between plant and model and the elements of the vector $\phi(t)$ the parameter errors. Although $\phi(t)$ is unknown, $\dot{\phi}(t)$ can be adjusted using available signals to make $e(t)$ tend to zero as $t \to \infty$. If

$$\dot{\phi} = -\Gamma e_1(t)u(t) \qquad \Gamma = \Gamma^T > 0 \tag{6}$$

is used as the adaptive law, the stability of the equilibrium state of equations

(5) and (6) has to be analyzed. The (n+m) equations may also be written as

$$\begin{bmatrix} \dot{e}(t) \\ \hline \dot{\phi}(t) \end{bmatrix} = \left[\begin{array}{c|c} A & bu^T(t) \\ \hline -\Gamma u(t)c^T & 0 \end{array}\right] \begin{bmatrix} e(t) \\ \hline \phi(t) \end{bmatrix} \tag{7}$$

Choosing a Lyapunov function candidate

$$V(e,\phi) = \frac{1}{2}[e^T Pe + \phi^T \Gamma^{-1} \phi] \tag{8}$$

the time derivative $\dot{V}(e,\phi)$ can be written as

$$\begin{aligned} \dot{V}(e,\phi) = \frac{1}{2} e^T(t)[A^T P+PA]e(t) + e^T(t)Pb\phi^T(t)u(t) \\ + \phi^T(t)\Gamma^{-1}\dot{\phi}(t) \end{aligned} \tag{9}$$

By the Kalman-Yacubovich Lemma [9], if $c^T(sI-A)^{-1}b$ is strictly positive real, it can be shown that

$$\dot{V}(e,\phi) = -\frac{1}{2} e^T(t)[qq^T+\varepsilon L]e(t) \tag{10}$$

where q is a vector, $L = L^T > 0$ and ε a positive constant. Hence V is a non-increasing function of time which is bounded below and hence converges to a finite value V_∞.

We first consider the case when the input u(t) is uniformly bounded. In such a case we have

$$\lim_{t\to\infty} \int_0^t \dot{V}(t)dt = V_\infty - V(0) \tag{11}$$

which is a finite number and $\dot{V}(t)$ uniformly continuous since $\dot{e}(t)$ and hence $\ddot{V}(t)$ is bounded. Hence by lemma 1

$$\lim_{t\to\infty} \dot{V}(t) = \lim_{t\to\infty} \frac{1}{2} e^T(t)(qq^T+\varepsilon L)e(t) = 0$$

or

$$\lim_{t\to\infty} e(t) = 0. \tag{12}$$

By equation (6), since u(t) is bounded, it follows that

$$\lim_{t\to\infty} \dot{\phi}(t) = 0 \tag{13}$$

Hence $\phi(t)$ is a bounded vector whose time-derivative approaches zero as $t \to \infty$.

It has also been shown by Morgan and Narendra [6] that if u(t) in equation (7) is "sufficiently rich"

$$\lim_{t\to\infty} \phi(t) = 0.$$

The results stated so far may now be summarized as follows:

1) If $V(e,\phi)$ is positive definite and $\dot{V}(e,\phi)$ is negative semidefinite, $e(t),\phi(t)$ are bounded if $e(0),\phi(0)$ are bounded.

2) If the input u(t) is bounded, $\ddot{V}(e,\phi)$ is bounded and hence

$$\lim_{t\to\infty} e(t) = 0 \qquad \lim_{t\to\infty} \dot{\phi}(t) = 0$$

3) If u(t) is bounded and "sufficiently rich"

$$\lim_{t\to\infty} \phi(t) = 0.$$

<u>Unbounded Input Vector u(t)</u>:

The problem becomes considerably more complicated when the input vector u(t) is bounded for all $t \in [0,\infty)$ but is not uniformly bounded. It is now no longer possible to conclude from the above analysis that e(t) and $\dot{\phi}(t)$ behave as described in condition (2). However, it was recently shown [14] that even when the inputs are unbounded the system (7) can be uniformly asymptotically stable provided the inputs are "uniformly exciting". Some of the principal difficulties in the resolution of the adaptive control problem are related to these questions that arise, when the input is unbounded, and are indicated in Section 4.

3. Hyperstability Theory:

In 1963 Popov introduced the concept of Hyperstability [10] as a natural extension of absolute stability. Consider a feedback system with two blocks B_1 and B_2 as shown in Figure 2. Let B_2 have input y(t) and output v(t) and let

$$\int_0^t y(\tau)v(\tau)d\tau \geq 0 \quad \text{for all } t \geq 0 \tag{14}$$

The block B_1 is said to be hyperstable if every system of the type shown in Figure 2

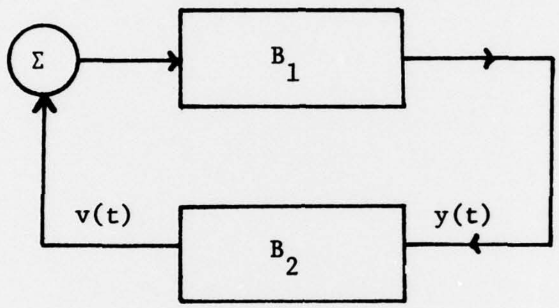


Figure 2

is stable with any block B_2 satisfying relation (14). The most significant results are obtained when B_1 is a linear time-invariant operator and in this case Popov established the equivalence of hyperstability and positive realness of the transfer function B_1. For an extensive treatment, Popov's book [11] on the subject is the primary source. For a lucid presentation of key results of Popov's theory, the reader is referred to the paper by Anderson [12]. In this section we shall merely state the principal theorems on hyperstability and clarify the key points using the same example that was discussed in Section 2.

Consider a completely controllable and completely observable system B_1 with m-inputs and m-outputs described by

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t) \\ y(t) &= Cx(t) + Dw(t) \end{aligned}\right\} \text{System } B_1 \tag{15}$$

where w(t) and y(t) are (mxl) vectors, x(t) is an (nxl) state vector and A,B,C,D are constant matrices of appropriate dimensions.

Hyperstability of B_1 is then defined by the property which requires that the state x(t) be bounded for a certain class of inputs v(t). This class is defined by those v(•) which satisfy for all T

$$\int_0^T w^T(t)y(t)dt \leq \delta[\|x(0)\|] \sup_{0 \leq t \leq T} \|x(t)\| \tag{16}$$

where δ is a positive constant. For the purposes of our discussion it is adequate to limit ourselves to the class of inputs which satisfy

$$\int_0^T w^T(t)y(t)dt \leq \ell^2 \tag{17}$$

where ℓ is an arbitrary constant independent of T.

In the adaptive control error model used in this paper we are concerned only with single input - single output systems and hence w(t) and y(t) are assumed to be scalars in the discussions.

<u>Definition</u>: The system B_1 is hyperstable with respect to any w(t) which satisfies (17) if there exists a positive constant k such that

$$\|x(t)\| \leq k[\|x(0)\| + \ell] \quad \text{for all } t \tag{18}$$

<u>Definition</u>: The system B_1 is said to be asymptotically hyperstable with respect to any w(t) satisfying (17) which is also bounded if the inequality (18) holds together with

$$\lim_{t\to\infty} x(t) = 0 \tag{19}$$

The main theorems of Popov may now be stated as follows:

<u>Theorem 2 (Hyperstability)</u>: A necessary and sufficient condition for the system B_1 to be hyperstable is that the transfer matrix

$$Z(s) = D + C(sI-A)^{-1}B \tag{20}$$

be positive real.

Sufficiency follows from the fact that when the system is positive real

$$\int_0^T w^T(\tau)y(\tau)d\tau \geq x^T(T)Px(T) - x^T(0)Px(0) \tag{21}$$

for some $P = P^T > 0$ and all T. This together with (17) implies (18) and hence hyperstability.

If the system is not positive real, it can be shown [12] that an input $u(t)$ defined for $t \geq 0$ exists satisfying condition (17) such that $x(t)$ is unbounded. This assures necessity in Theorem 2.

Theorem 3: Asymptotic Hyperstability:

A necessary and sufficient condition for the system B_1 to be asymptotically hyperstable is that $Z(s)$ in (20) is strictly positive real.

If B_1 is strictly positive real, it can be shown that a positive definite function $\rho(x)$ exists such that

$$\int_0^T \rho(x)dx \leq \int_0^T w^T(t)y(t)dt. \tag{22}$$

When $w(t)$ is bounded, the function $\rho(x(t))$ can also be shown to be uniformly continuous. Hence, from inequalities (22) and (17) and Barbălat's lemma it follows that $\lim_{t\to\infty} \rho(x(t)) = 0$ or $\lim_{t\to\infty} x(t) = 0$.

Error Model: We now apply the above theorems to the error model discussed in Section 2. The system (5) with the adaptive law (6) can be represented by the feedback loop shown in Figure 3.

Since
$$\int_0^T v(t)e_1(t)dt = -\int_0^T \phi^T(t)\dot{\phi}(t)dt$$
$$= \frac{\|\phi(0)\|^2 - \|\phi(T)\|^2}{2} \leq \frac{\|\phi(0)\|^2}{2} \tag{23}$$

by Theorem 2 the state vector e(t) of B_1 is bounded. Similarly, by considering

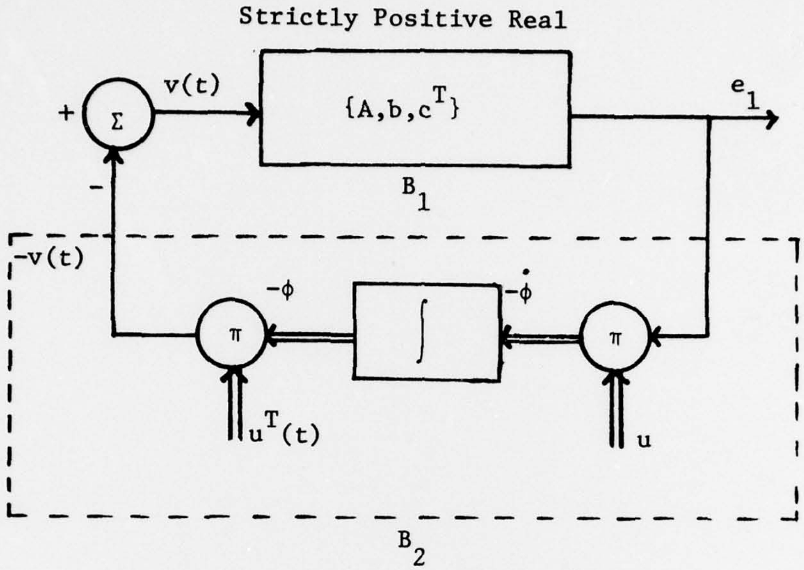


Figure 3

the integrator as the system with a positive real transfer function, the state vector $\phi(t)$ can be shown to be bounded. This corresponds to condition (1) in Section 2.

If u(t) is bounded, the input v(t) to B_1 is bounded. Since B_1 has a strictly positive real transfer function, by Theorem 3, e(t) and $e_1(t)$ tend to zero as $t \to \infty$. This corresponds to condition (2) in Section 2.

The convergence of $\phi(t)$ to zero when u(t) is "sufficiently rich" has to be proven along lines indicated in [5] and is common to the two approaches discussed in this paper.

When u(t) is unbounded, Barbălat's lemma can no longer be directly applied and the same comments made at the end of Section 2 are also valid here.

4. The Adaptive Control Problem:

The adaptive control of a single input - single output linear time-invariant plant was discussed by the authors in a recent paper [13]. It was shown that the stability approach could be applied if the relative degree (n-m) (where the unknown plant transfer function has n poles and m zeros) and the sign of the plant gain are known and the plant zeros lie entirely in the left half plane. Two cases were considered in [13]. In case (i), if $m \geq n-2$, using the error model of Sections 2 and 3 the plant output was shown to approach a desired model output asymptotically. In case (ii), where $m \leq n-3$, additional signals have to be fed back into the model and only a conjecture was made regarding the stability of the adaptive system. Without going into details, our aim in this section is to relate the above two cases to the error model considered in Sections 2 and 3. For ease of discussion we consider a simplified model of the plant but the results carry over to the general problem in [13].

A typical adaptive control problem is shown in Figure 4. The plant has p outputs $y_1(t), y_2(t), \ldots, y_p(t)$ and p adjustable parameters $\theta_1(t), \theta_2(t), \ldots, \theta_p(t)$ which are denoted by the elements of an output vector $y(t)$ and a parameter vector $\theta(t)$ respectively. The output $y_m(t)$ of a stable reference model is the desired output and it is required to adjust $\theta(t)$ so that $y_1(t)$ asymptotically approaches $y_m(t)$ or

$$\lim_{t\to\infty} |y_m(t) - y_1(t)| \triangleq \lim_{t\to\infty} |e_1(t)| = 0$$

It is known that a constant parameter vector θ^* exists such that when $\theta(t) \equiv \theta^*$, the transfer function from the reference input to $y_1(t)$ matches exactly the model transfer function. The aim of the adaptive controller is to adjust $\theta(t)$ in such a manner that $\lim_{t\to\infty} \theta(t) = \theta^*$.

Case (i): Let the model transfer function be positive real. In this simple case, if $\theta^* - \theta(t) = \phi(t)$, the error between model and plant states, $e(t)$, can be described by the error model of Figures 1 and 3 with $u(t)$ replaced by $y(t)$.

It follows from Sections 2 and 3 that $e(t)$ is bounded.

Since the model states are bounded, we conclude that $y(t)$ is bounded and this, in turn, assures that $\lim_{t\to\infty} e_1(t) = 0$.

Case (ii): When the model transfer function is not positive real, differentiators have to be used after $\theta(t)$ in Figure 4 to use the same approach. If the controller is to be differentiator free, it was suggested in [13] and [17] that auxiliary signals should be fed instead into the input of the model.

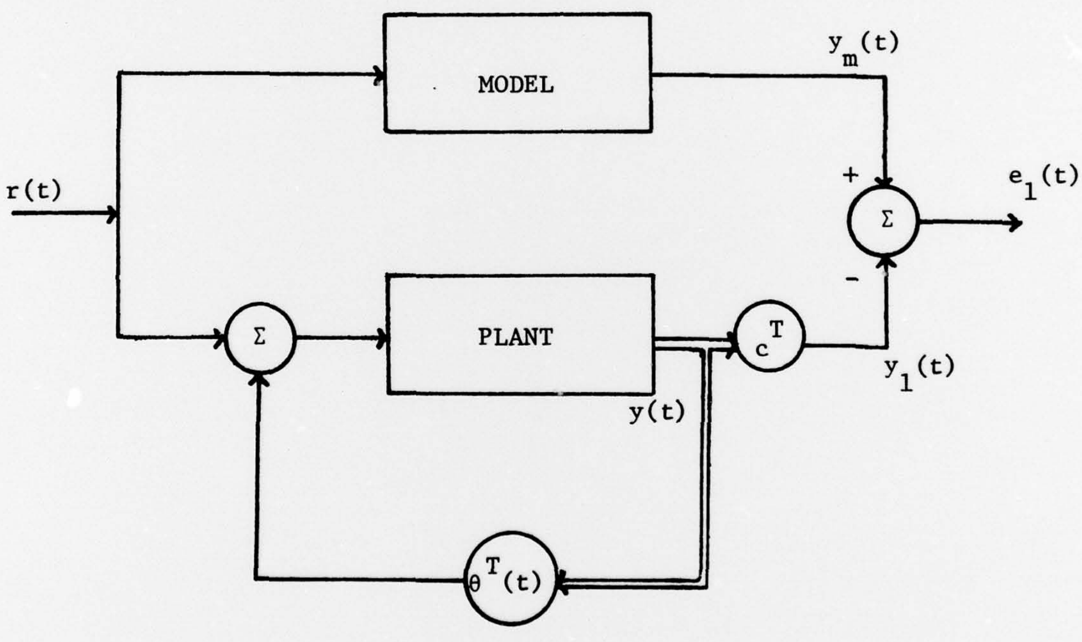


Figure 4

Once again the same error model as before is obtained and, while we can conclude stability (or hyperstability) and, hence, boundedness of $e(t)$, we can no longer conclude that the state $x_m(t)$ of the model - and hence the outputs $y(t)$ of the plant - are bounded. This, in turn, implies that the error model of Sections 2 and 3 may not be asymptotically stable (or asymptotically hyperstable). In other words, the model and plant outputs may both grow without bound even while the error between them is bounded. The conjecture made in [13] implies that such a situation cannot arise.

As mentioned earlier, it has recently been shown in [14] that, even when the inputs are unbounded, the system is uniformly asymptotically stable, if they are "uniformly exciting". Work is currently in progress to utilize these results to demonstrate that the boundedness of the error also implies the boundedness of the plant output in the adaptive control problem and will be reported in [15] and [16].

Conclusion: Lyapunov's direct method and Popov's hyperstability theory yield identical results when applied to the adaptive control problem. The comparison is made in terms of an error model when the input to the model is uniformly bounded. In such a case, stability and asymptotic stability are achieved under exactly the same conditions as hyperstability and asymptotic hyperstability. When the inputs to the error model are unbounded, the problem is not completely resolved using either method.

Recent work on error models with unbounded inputs has yielded conditions for uniform asymptotic stability. These will play a significant role in the complete resolution of the adaptive control problem.

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