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LASER BEAMS,

by

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ON THE INTERACTION OF TWO HIGH INTENSITY LASER BEAMS*

by

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ABSTRACT

With the rapid development of high energy, coherent, monochromatic laser sources foreseen, the interaction phenomenon between two such sources is of fundamental importance. This work attempts to treat the above interaction by the use of the Lagrangian for a strong electromagnetic field. The quantum mechanical effect of the scattering between plane light waves is simulated within the framework of classical theory under the assumption of an effective non linear interaction between the light waves. The scattered fields are solved for, and expressions are obtained for, the scattered power.

RÉSUMÉ

La mise au point dans un proche avenir de lasers haute énergie à lumière monochromatique et cohérente, rend très importante l'étude des phénomènes d'interaction entre deux faisceaux émis par ces lasers. Cet article essaie d'interpréter cette interaction, en faisant appel au lagrangien d'un champ électromagnétique fort. L'hypothèse d'une interaction efficace non linéaire entre des ondes lumineuses planes permet de simuler les effets quantiques de leur diffusion dans le cadre de l'électromagnétisme classique. Les équations des champs diffusées sont résolues pour obtenir l'expression de la puissance diffusée.

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INTRODUCTION

Heisenberg and Euler [1] and Weisskoff [2] have shown that classical electromagnetism becomes a non linear theory due to the possibility of pair creation. The non linearity results in a correction to the classical Lagrangian $L_0 = 1/2 \ (E^2 - B^2)$, which results in non linear Maxwell equations. Thus, electromagnetic fields interact with one another and the effective scattered fields may be evaluated.

In this work, the Lagrangian in the strong field limit is used to discuss the scattering between two focussed plane waves. With the development of highly powerful sources it might become possible in the near future to generate intense electromagnetic fields and thus provide concrete experimental evidence for the interaction of the waves. Optical lasers show promise of such development. The energies in the visible range are much lower than the electron rest energy mc^2, and the quasi-static approximation can be used throughout. In such an energy limit, the radiation cannot see the structure of the interaction and using some of the results of earlier papers in quantum electrodynamics [3], the interaction Lagrangian can be added to the classical Lagrangian density. It is well known [4] that the ratio of the non linear term L, in the Lagrangian to its classical value L_0 increases logarithmically for fields greater than the critical field. Hence it was concluded that a semi classical approach along the lines of McKenna and Platzman [5] may be reasonably appropriate to investigate the non linear effects in the strong field limit. It is shown that the non linear Lagrangian gives rise to a fluctuating current and that the charge density, unlike the case in the weak field limit [4], vanishes.

The fields F (E, B) are assumed to be slowly varying and satisfy the conditions

$$\frac{h}{mc} \left| \overline{\nabla F} \right| << \left| F \right| \ ; \frac{h}{mc^2} \left| \overline{\frac{\partial F}{\partial F}} \right| << \left| F \right| \tag{1}$$

around the optical range of the spectrum. Slowly varying fields as is known cannot create real electron-positron pairs in practice unless there is pair production by a large number of soft photons which is not considered in the present problem.

The non linear electrodynamic effects can be described with the aid of a field dependent dielectric permittivity and magnetic permeability vacuum which are obtained from the electric displacement D and the magnetic induction H.

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The non linear source terms are introduced into the extended Maxwell equations. The scattered fields are obtained by solution of the reduced wave equations in terms of integrals over the derivatives of the current distributions. The variables are evaluated in the retarded time. Estimates are made for the changes in the dielectric constant and the magnetic permeability. It is also observed that the phase velocity can be greater than the velocity of light in vacuum. The expression for the scattered power is derived as is the ratio of the power in the scattered field to that in the incident field. The system of units in which $\bar{h} = c = 1$ is used unless otherwise specified.

Non Linear Maxwell Equations in the Intense Field Limit

In order to formulate the problem at hand i.e., the interaction of two electromagnetic waves, we must use non linear, in contrast to the generally used linear Maxwell equations. Thus, the Lagrangian density cannot be a quadratic function of the fields and therefore differs from $L_o = 1/2 \ (E^2-B^2)$.

The Lagrangian for any arbitrarily strong electromagnetic field which satisfies the conditions given in (1) is given by Akhiezer and Berestetsky [4].

$$L_1 = \frac{e^2}{8\pi^2} \int \frac{d\eta}{\eta^3} e^{-\eta} \left[i\eta(EB) \left\{ \frac{\cos \eta \ \sqrt{E^{*2} - B^{*2} + 2iE^*B^*+}}{\cos \eta \ \sqrt{E^{*2} - B^{*2} + 2iE^*B^*-}} \right. \right.$$

$$\left. \left. \frac{\cos \eta \sqrt{E^{*2} - B^{*2} - 2iE^*B^*}}{\cos \eta \sqrt{E^{*2} - B^{*2} - 2iE \ B}} \right\} + \frac{m^4}{e^2} + \frac{\eta^2}{3}(B^2-E^2) \right] \tag{2}$$

This Lagrangian was originally derived by Euler and Heisenberg [1], and is a function of the two independent variables E^2-B^2. We differ from the notation of Akhiezer and Berestetsky [4] in that we use B instead of H.

In the case of weak fields, L_1 reduces to

$$L_1 = \frac{2}{45} \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{m^4}\left[(E^2-B^2)^2 + 7(EB)^2\right] + \ - \ - \ - \ - \ - \tag{3}$$

where $\frac{e^2}{4\pi} = \alpha$ is the fine structure constant, in the mass of the electron and E and B are the electric and magnetic fields. This Lagrangian was used by quite a few authors to study the non linear interaction between electromagnetic fields in vacuum. The Maxwell equations that result from such a choice of the Lagrangian are non linear and describe well the interaction of low energy photons. McKenna and Platzman [5] have used these equations to derive the counting rate for the scattering by light in the presence of classical static fields and a semiclassical approach to derive the cross section for the scattering of light by light in the absence of external fields which was already derived quantum mechanically [6].

For the case of strong electric and magnetic fields

$$E* = \frac{eE}{m^2} >> 1 \quad \text{and} \quad B* = \frac{eB}{m^2} >> 1, \tag{4}$$

with polarization such that $E*B* = 0$ and the Lagrangian L_1 is given by:

$$L_1 = -\frac{\alpha}{6\pi} \left(\frac{E^2-B^2}{2}\right) \ln\left(\frac{E^2-B^2}{F^2_{crit}}\right) + \text{terms in m, e} - - - - - \tag{5}$$

The terms independent of E and B are neglected. F_{crit} is the critical field strength and is given by $F_{crit} = m^2/e$. L_1 is obtained from the original Euler-Heisenberg Lagrangian [2] for the case when E^2-B^2 is small $\simeq 0$. In the case of either zero electric or magnetic fields, L_1 again reduces to (4).

$$L_1 \simeq \frac{\alpha B^2}{12\pi} \ln\left(\frac{B^2}{F^2_{crit}}\right) \quad \text{for } E = 0$$

$$L_1 \simeq -\frac{\alpha E^2}{12\pi} \ln\left(\frac{E^2}{F^2_{crit}}\right) \quad \text{for } B = 0 \tag{6}$$

For the case of the initial fields being the sum of the fields due to two plane waves, L_1 in equation (5) corresponds to the case when the polarizations of the plane waves ϵ_1, ϵ_2 are parallel to each other.

In the case of perpendicular polarizations, the situation is reversed; $E^2-B^2 = 0$ and E.B is large when the intensities are large. The Lagrangian in this case is obtained from (2) and is given by:

$$L_2 = \frac{\alpha}{2\pi} (E.B) \ln\left(\frac{E.B}{F^2_{crit}}\right) \tag{7}$$

Equations (6) and (7) contrast strongly from (3) for the weak field case. The electric displacement D and the magnetic induction H are determined from the equations:

$$D = \frac{\partial L}{\partial E} \; ; \; H = -\frac{\partial L}{\partial B} \tag{8}$$

where $L = L_0 + L_2$ for perpendicular polarization. Hence, direct differentiation gives

$$D_i = E_i - \frac{\alpha}{6\pi} E_i \left[1 + \ln\left(\frac{(E^2-B^2)}{F^2_{crit}}\right)\right] = E_i - \frac{\alpha}{6\pi} \delta E_i \tag{9a}$$

$$H_i = B_i - \frac{\alpha}{6\pi} B_i \left[1 + Ln\left((E^2-B^2)/F^2_{crit} \right) \right] = B_i - \frac{\alpha}{6\pi} \delta B_i \tag{9b}$$

for the parallel polarization case, and

$$D_i = E_i + \frac{\alpha}{2\pi} B_i \left[1 + Ln\left(E.B/F^2_{crit} \right) \right] = E_i + \frac{\alpha}{2\pi} \delta E_i \tag{10a}$$

$$H_i = B_i + \frac{\alpha}{2\pi} E_i \quad 1 + Ln \quad E.B/F^2_{crit} \quad = B_i + \frac{\alpha}{2\pi} \delta B_i \tag{10b}$$

Equations (9) and (10) differ strongly from each other as well as from the expressions for D and H in the weak field case, which for arbitrary polarization states are:

$$\vec{D} = \vec{E} + \frac{1}{45\pi} \frac{\alpha^2}{m^4} [2(E^2-B^2)\vec{E} + 7 (E.B)\vec{B}] \tag{11a}$$

$$\vec{H} = \vec{B} + \frac{1}{45\pi} \frac{\alpha^2}{m^4} [2(E^2-B^2)\vec{B} - 7 (E.B)\vec{E}] \tag{11b}$$

and $\vec{D}$ and $\vec{H}$ are related to $\vec{E}$ and $\vec{B}$ by the following relationships.

$$D_i = \epsilon_{ik} E_k \tag{12a}$$

$$H_i = B_k/\mu_{ik} \tag{12b}$$

Our notation in (12) differs again from that of Akhiezer and Berestetsky [4] and conforms with that of Adler [7] who has derived expressions for $\vec{D}$ and $\vec{H}$ instead of $\vec{D}$ and $\vec{B}$ as was done by Akhiezer and Berestetsky [4].

A comparison of equations (9) and (12) shows that

$$\epsilon_{ik} = \delta_{ik} \left\{ 1 - \frac{\alpha}{6\pi} - \frac{\alpha}{6\pi} Ln\left[(E^2-B^2)/F^2_{crit} \right] \right\} \tag{13a}$$

$$\mu^{-1}_{ik} = \delta_{ik} \left\{ 1 - \frac{\alpha}{6\pi} - \frac{\alpha}{6\pi} Ln\left[(E^2-B^2)/F^2_{crit} \right] \right\}$$

whereas comparing (12) and (10) gives

$$\epsilon_{ik} = \delta_{ik} \left\{ 1 + \frac{\alpha}{2\pi} \frac{B_i}{E_i} \left[1 + Ln(E.B/F^2_{crit}) \right] \right\} \tag{14a}$$

$$\mu_{ik}^{-1} = \delta_{ik}\left\{1 - \frac{\alpha}{2\pi}\frac{E_i}{B_i}\left[1 + \text{Ln}\left(E.B_{/F^2_{crit}}\right)\right]\right\} \qquad (14b)$$

in the limit of strong field intensities. For two plane waves with polarization vectors parallel to each other, $E^2-B^2=0$ and for

$$1 - \frac{\alpha}{6\pi} = \frac{\alpha}{6\pi}\text{Ln}\left((E^2-B^2)_{/F^2_{crit}}\right)$$

$$\text{i.e. Ln}\left((E^2-B^2)_{/F^2_{crit}}\right) \simeq 6\pi/\alpha \qquad (15a)$$

we have $\epsilon_{i_k} = \mu_{i_k}^{-1} = 0$ in (13) $\qquad (15b)$

For (15b) to hold, the fields have to be almost infinite, which is also true in the case of perpendicular polarizations and is an almost improbable physical situation.

In the weak field limit

$$\epsilon_{i_k} = \delta_{i_k} + \frac{1}{45\pi}\frac{\alpha^2}{m^4}\left\{2(E^2-B^2)\delta_{i_k} + 7B_iB_k\right\} \qquad (16a)$$

$$\mu_{ik}^{-1} = \delta_{ik} + \frac{1}{45\pi}\frac{\alpha^2}{m^4}\left\{2(E^2-B^2)\delta_{ik} - 7E_iE_k\right\} \qquad (16b)$$

An extended set of Maxwell's equations occur due to the inclusion of non linear terms and are of the form

$$\vec{\nabla} \times \vec{E} = \frac{\partial\vec{B}}{\partial t} \qquad (17)$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial\vec{D}}{\partial t} \qquad (18)$$

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad (19)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad (20)$$

McKenna and Platzman [5] have shown that no scattering takes place between photons travelling in the same direction. For $\epsilon_1||\epsilon_2$, the extended set of Maxwell's equations can be written as

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$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \tag{21}$$

$$\nabla \times H = \frac{\partial E}{\partial t} + 4\pi \vec{J} \left(-\alpha/_{6\pi} \right) \tag{22}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \left(-\alpha/_{6\pi} \right) \tag{23}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{24}$$

and for $\epsilon_1 \perp^r \epsilon_2$ as:

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \tag{25}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial E}{\partial t} + 4\pi\vec{J} \left(\alpha/_{2\pi} \right) \tag{26}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \left(\alpha/_{2\pi} \right) \tag{27}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{28}$$

where

$$\rho = - \frac{1}{4\pi} \vec{\nabla} \cdot \vec{\delta E} \tag{29a}$$

$$\vec{J} = \frac{1}{4\pi} \left[\frac{\partial(\vec{\delta E})}{\partial \bar{t}} - \vec{\nabla} \times \vec{\delta B} \right] \tag{29b}$$

The non linear terms are therefore included in two source terms ρ and J. Since the non linear terms in the Lagrangian L_1 and L_2 are $\ll L_0$, it is convenient and appropriate to use a power series expansion in $(-\alpha/_{6\pi})$ and $(\alpha/_{2\pi})$ to discuss the effects of scattering.

Therefore the series of expansion in the case of parallel polarizations is,

$$\vec{E} = \vec{E}_0 - \alpha/_{6\pi} \vec{E}_f + ----- \tag{30a}$$

$$\vec{B} = \vec{B}_0 - \alpha/_{6\pi} \vec{B}_f + ----- \tag{30b}$$

and for perpendicular polarization

$$\vec{E} = \vec{E}_o + \alpha/_{2\pi} E_f + \cdots \cdots \tag{31a}$$

$$\vec{B} = \vec{B}_o + \frac{\alpha}{2\pi} \vec{B}_f + \cdots \cdots \tag{31b}$$

$\vec{E}_o$ and $\vec{B}_o$ are solutions of equations (24) and (28) with the source terms $\rho = o$ and $\vec{J} = o$ and these are specified in the next section as the initial fields.

The fields $\vec{E}_f$ and $\vec{B}_f$ are the solutions of the Maxwell's equations (21) to (28) when $\rho = \rho_o = \rho(\vec{E}_o, B_o)$ and $\vec{J} = \vec{J}_o = \vec{J}(E_o, B_o)$ and satisfy the wave equations.

$$\nabla^2 \vec{E}_f - \frac{\partial^2 E_f}{\partial t^2} = 4\pi \left[\nabla \delta_o(\vec{r}, t) + \frac{\partial \vec{J}}{\partial t}(\vec{r}, t) \right] \tag{32a}$$

and $$\nabla^2 \vec{B}_f - \frac{\partial^2 B_f}{\partial t^2} = -4\pi \left[\vec{\nabla} \times \vec{J} \right] \tag{32b}$$

They can be solved by the use of Green's functions and are given by (8).

$$\vec{E}_f \left[\vec{r}, t \right] = \int \left[-\nabla \rho(\vec{r}, t-\underline{R}) - \frac{\partial \vec{J}}{\partial t}(\vec{r}, t-\underline{R}) \right] d^3 r'/R \tag{33a}$$

$$\vec{B}_f \left[r, t \right] = \int \vec{\nabla} \times \vec{J}(r', t-\underline{R}) \, d^3 r' / R \tag{33b}$$

Where $R = \left| r - r' \right|$, $t = t - \underline{R}$ is the retarded time and $d^3 r'$ is the source volume element. In the far field approximation, $\left| r \right| \gg 1$ and

$$R = \left| r - r \right| \approx r \tag{34}$$

THE INITIAL FIELDS

We assume that the initial fields which generate the source terms are a sum of two fields with propagation vectors in opposite directions to each other. In essence we consider the scattering of two electromagnetic fields in the strong field limit. The two impinging waves produce a standing wave pattern which in turn generates the non linear sources.

The initial fields polarize the vacuum and generate a fluctuating current which again produces the scattered fields E_f and B_f. The initial fields are represented as

$$\vec{E}_o = F_1 \epsilon_1 \cos(\omega_1 t - \vec{k}_1 \cdot \vec{r}) + F_2 \vec{\epsilon}_2 \cos(\omega_2 t - \vec{k}_2 \cdot \vec{r}) = \sum_{i=1}^{2} F_i \epsilon_i \cos \phi_i \tag{35a}$$

$$\vec{B}_o = F_1 \vec{\nu}_1 \times \vec{\epsilon}_1 \cos(\omega_1 t - \vec{K}_1 \cdot \vec{r}) + F_2 \vec{\nu}_2 \times \vec{\epsilon}_2 \cos(\omega_2 t - \vec{k}_2 \cdot \vec{r}) = \sum_{i=1}^{2} F_i \nu_i \times \vec{\epsilon}_i \cos \phi_i \tag{35b}$$

where F_1 and F_2 are real and in the centre of mass system of the two laser beams we have

$$\omega_1 = |k_1| = \omega_2 = |k_2|; \ \nu_1 = k_1 \ /|\underline{k_1}| = -\nu_2 = \ k_2/|k_2| \tag{36}$$

The real expressions for the initial fields are due to the fact that they appear non linearly in the expressions for the scattered fields. $\epsilon_1 \ \epsilon_2$ are the unit polarization vectors such that

$$\vec{k}_1 . \vec{\epsilon}_1 = k_2 . \vec{\epsilon}_2 = 0 \tag{37}$$

due to the transversality conditions. For beams of equal intensity $F_1 = F_2$, as is assumed in the present problem. Thus, the initial fields are completely specified. It is also reasonable to assume that the initial fields are plane (5, 9) though at the focal point this is not exactly true.

The source terms are now obtained from (29) since the initial fields are known from (35). We find that $\rho = o$ and J is proportional to the amplitude of the initial field unlike the case of the weak fields as will be shown in the following section.

THE NON LINEAR SOURCE TERMS AND THE SCATTERED FIELDS

Assuming that the propagation vectors of the two incident beams are along the z axis in opposite directions to each other, we can compute the non linear source terms and derive expressions for the scattered fields. From equations (31) and (32) we have for the parallel polarization case

$$\rho = -\frac{1}{4\pi} \nabla.\delta\vec{E} = -\frac{1}{4\pi} \vec{\nabla} .\{\vec{E} [1 + \ln[(\vec{E} \vec{B}^2)/F_{crit}^2]] \}$$

$$= -\frac{F_1}{4\pi} [\nabla. \{\vec{\epsilon}_1.(\cos\phi \pm \cos\phi_2)(1+ \ln 4F_1^2/F_{crit}^2 \cos\phi_1 \cos\phi_2)\} \tag{38}$$

vanishes due to (37) unlike the problem for weak fields.

$$\vec{J} = -2 \ \frac{F_1 \omega_1}{4 \ \pi} [\vec{\epsilon}_1 \ (\cos\phi_1 \tan\phi_2 + \cos\phi_2 \tan\phi_1)] \tag{39}$$

where ϕ_1, ϕ_2 in the C.M system are given by

$$\phi_1 = \omega_1 t - k_1 . r = \omega_1 (t-z) \tag{40}$$

$$\phi_2 = \omega_2 t - k_2 . r = \omega_1 (t+z)$$

$\vec{J}$ is therefore in the opposite direction to that of the incident fields. For the perpendicular polarization case.

$$\rho = -\frac{1}{4\pi} \vec{\nabla}. \{\vec{B} (1 + \ln (B.B/2_{Fcrit}))\} = o \tag{41}$$

due to the transversality conditions and

$$\vec{J} = -\frac{1}{4\pi} \, F_1\omega_1 \left[\vec{\vartheta}_1 \times (\vec{\varepsilon}_1 \tan\phi_2 \cos\phi_1 - \vec{\varepsilon}_2 \tan\phi_1 \cos\phi_2)\right] \tag{42}$$

$\vec{J}$ is in a direction different from that of the electric and magnetic fields in the case of perpendicular polarizations. Equations (39) and (42) show also that the current is fluctuating in space-time and is proportional to the intensity of the field, unlike the problem for weak fields.

Using the expression for $\vec{J}$ and appropriate time and space derivatives, the scattered fields can be obtained from equation (33) by integration over the source volume. The cross sectional area of the beam (A) is assumed to be small λ^2 . We further assume that the interaction of the two beams is over a length $L \approx ?-3\lambda$. Use is made of the approximation that the source can therefore be considered to be almost linear and along the direction of the incident beam as shown in Figure 1. In the calculation of the scattered field the term

$$\frac{\frac{\partial \vec{J}(r',t-R)}{\partial t}}{R} \approx \frac{\frac{\partial \vec{J}(r',t-R)}{\partial t}}{r} \qquad \text{and}$$

$$\int_{v'} \frac{\frac{\partial J(r'-t-R)}{\partial t}}{R} \, d^3 r' = -\frac{1}{4\pi r} \int \left(\frac{\partial^2}{\partial t^2} \delta\vec{E} - \frac{\partial}{\partial t} \vec{\nabla}\times\delta\vec{B} \right) dx'dy'dz'.$$

$$\approx -\frac{1}{4\pi r}\left[\int_{v^1} \frac{\partial^2}{\partial t^2} \delta\vec{E}(r',t-R) \, dx'dy'dz' - \frac{\partial}{\partial t}\int \vec{\nabla}\times\delta\vec{B}(r',t-R) dx'dy'dz' \right] \tag{43}$$

The retarded time (t-R) can be approximated as

$$t_{ret} \approx t - |r-r'| = t - (r^2 + r'^2 - 2r\cdot r')^{\frac{1}{2}}$$

$$\approx t-r \left(1+ \frac{r'^2}{r^2} - \frac{2r\cdot r'}{r^2}\right)^{\frac{1}{2}} \approx t-r\left(1-\frac{2r\cdot r'}{r^2}\right)^{\frac{1}{2}}$$

$$\approx t-r \left(1- \frac{r\cdot r'}{r^2}\right) \approx t- r+\frac{r\cdot r'}{r} \approx t-r+z' \cos\theta' \qquad (r>>r') \tag{44}$$

Where θ' is the angle between the field vector $\vec{r}$ and $\vec{r} \approx \vec{Z}$ since the source is almost along the Z direction. The integrand is now independent of the variable X' and Y', the integration over which give the cross sectional area A of the cylindrical source. Hence

$$-\int_{v} \frac{\frac{\partial \vec{J}}{\partial t}(r',t-R)}{R} \, d^3 r' \approx -\frac{A}{4\pi r}\left[\int_0^L \frac{\partial^2}{\partial t^2} \delta\vec{E} \, dZ' - \frac{\partial}{\partial t}\int_0^L \vec{v}_1 \times \frac{\partial}{\partial Z} \delta B'(Z', t_{ret}) dZ' \right]$$

$$\approx -\frac{A}{4\pi r}\left[\int_0^L \frac{\partial^2}{\partial t^2} \delta\vec{E} \, dZ' - \frac{\partial}{\partial t}\left[\vec{v}_1 \times \delta B'(Z', t_{ret})\right] \Big|_0 \right] \tag{45}$$

The first integral in (45) is evaluated first for the two polarizations. For parallel polarization

$$I_1 = \int_0^L \frac{\partial^2}{\partial t^2}\left\{\vec{E}_0 + E_0 \, \ell n \left[(E_0^2 - B_0^2)\big/_{F_{crit}^2}\right]\right\} dz' \tag{46}$$

where $\vec{E}_0 = E_0(Z', t_{ret})$ and $\vec{B} = \vec{B}_0 (Z', t_{ret})$

for $\theta' \neq 0$

$$\int_0^L \frac{\partial^2}{\partial t^2} \vec{E}_0 \, dz' = - F_1 \omega_1^2 \int_0^L (\vec{\epsilon}_1 \, Cos\phi_1 + \vec{\epsilon}_2 Cos\phi_2) \, dz'$$

$$= - F_1 \omega_1^2 \left[\vec{\epsilon}_1 \frac{Sin\phi_1}{\omega_1(Cos\theta'-1)} \, \Big|_0^L + \frac{\vec{\epsilon}_2 \, Sin\,\phi_2}{\omega_1(Cos\theta'+1)} \, \Big|_0^L \right]$$

$$= - F_1 \omega_1 \left[\vec{\epsilon}_1 \left\{\frac{Sin(\omega_1 t - \omega_1 r + {}_1(Cos\theta - 1)L) - Sin(\omega_1 t - \omega_1 r)}{Cos\theta' - 1}\right\}\right.$$

$$\left. + \vec{\epsilon}_2 \left\{\frac{Sin(\omega t - \omega r + \omega_1(Cos\,\theta' + 1)L) - Sin(\omega t - \omega_1 r)}{Cos\theta' + 1}\right\} \right]$$

$$= - F_1 \omega_1 \left[\vec{\epsilon}_1 \cdot \frac{2Cos(\omega_1 t \cdot \omega_1 r + \omega_1 \frac{(cos\theta \pm 1)L}{2}) \, Sin \, \omega_1 \frac{(Cos\theta'-1)L}{2}}{(Cos\theta'-1)}\right.$$

$$\left. + \vec{\epsilon}_2 \cdot \frac{2Cos(\omega_1 t - \omega_1 r + \omega_1 \frac{(Cos\theta'+1)L}{2})Sin\frac{\omega_1}{2}(Cos\theta'+1)L}{(Cos\theta'+1)} \right] \tag{47}$$

Further,

$$\int_0^L \frac{\partial^2}{\partial t^2}E_0 \, \ell n \left\{(E^2 - B^2) \big/_{F^2_{crit}}\right\} d\,z' = - F_1 \omega_1 \vec{\epsilon} X \left[\frac{1}{(Cos\theta'-1)} Sin Sin\phi_1 \, \ell n\right.$$

$$\left.(4F_1^2 Cos\phi_1 Cos\phi_2/F^2_{crit}) \, \Big|_0^L - \frac{Cos\theta'\pm 1}{Cos\theta'-1} Sin\phi_1 \, \Big|_0^L - \frac{(Cos\theta-1)^2}{Cos\theta'+1} \frac{1}{\omega} \frac{\partial^2}{\partial t^2} \int_0^L\right.$$

$$\frac{Cos2k_1z'}{Cos\phi_1}\,dz' + \frac{2\,Cos\phi_1\,Sin\phi_2}{Cos\phi_2}\Big|_0^L + \frac{Sin\phi_1}{Cos^2\phi_2}\Big|_0^L + \frac{1}{(Cos\phi'+1)}\Big|Sin\phi_2\Big|_0^L$$

$$Sin\phi_2 \ln(4F^2Cos\phi_1Cos\phi_2)/F_{crit}^2\Big|_0^L - \frac{(Cos\theta'-1)Sin\,\phi_2}{Cos\theta'+1}\Big|_0^L - \frac{(Cos\theta'+1)^2}{Cos\theta'-1}$$

$$\frac{1}{\omega}\,\frac{\partial^2}{\partial t^2}\int_0^L \frac{Cos2k_1z'\,dz' + 2Cos\phi_2Sin\phi_1/\,Cos\phi_1}{Cos\phi_2}\Big|_0^L + \frac{Sin\phi_2}{Cos^2\phi_1}\Big|_0^L\Big]$$

$$\tag{48}$$

Thus summing up (47) and (48) will give I_1. Also for perpendicular polarization,

$$-\frac{\partial}{\partial t}\,\vec{v}_1 \times \vec{\delta B}\,(\vec{z}_1,t_{ret})\Big|_0^L = -\vec{\epsilon}_1\,F_1\,\vec{\omega}_1\,X\Big[Sin\phi_1(2+\ln4F_1^2/F_{crit}^2$$

$$Cos\phi_1\,Cos\phi_2)\Big|_0^L + \tan\phi_2Cos\phi_1\Big|_0^L - Sin\phi_2(2+\ln4F_1^2/F_{crit}^2\,Cos\phi_1Cos\phi_2\Big|_0^L$$

$$- \tan\phi_1Cos\phi_2\Big|_0^L = 0 \tag{49}$$

also

$$\frac{\partial^2}{\partial t^2}\int_0^L \vec{\delta E}dz' = -F_1\omega_1\Big[\frac{\vec{v}_1\times\vec{\epsilon}_1}{Cos\theta-1}\,Sin\phi_1(4+\ln(2F_1^2/F_{crit}^2\,Cos\phi_1Co\phi_2)+$$

$$2\,Sec^2\phi_1 + Sec^2\phi_2 - 2\,Sec^2\phi_2\frac{Cos\theta'+1}{Cos\theta'-1})\Big|_0^L + 2\,\tan\phi_2Cos\phi_1\Big|_0^L - \frac{Cos\theta'+1}{Cos\theta'-1}$$

$$Cos\phi_1\tan\phi_2\Big|_0^L + 2\frac{(Cos\theta'+1)}{Cos\theta'-1}\frac{Sin\phi_2Cos\phi_1}{Cos^3\phi_2}\Big|_0^L - \frac{Cos\theta'+1}{Cos\theta'-1}\frac{1}{\omega}\frac{\partial^2}{\partial t^2}\int_0^L$$

$$\frac{Cos\phi_1}{Cos^2\phi_2}\,dz' - \frac{\vec{v}_1\times\vec{\epsilon}_2}{Cos\theta'+1}\Big\{Sin\phi_2\,(4+\ln(2F_1^2\,Cos\phi_1Cos\phi_2/F_{crit}) + Sec^2\phi_1)$$

$$-2\frac{(Cos\theta'-1)}{Cos\theta'+1}\,Sec^2\phi_1)\Big|_0^L + 2\,\tan\phi_1Cos\phi_2\Big|_0^L - \frac{Cos\theta'-1}{Cos\theta'+1}\,Cos\phi_2\tan\phi_1\Big|_0^L +$$

$$2\ \frac{Cos\Theta'_{-1}}{Cos\Theta'+_1}\ \frac{Sin^{\phi_1}}{Cos\phi_1}\ Cos\phi_2 \Bigg|_0^L\ -(\frac{Cos\Theta-_1}{Cos_\Theta\pm_1})\cdot\ \frac{1}{w}\ \frac{\partial^2}{\partial t^2}\ \int_0^L \frac{Cos\phi_2}{Cos\phi_1}\ dz' \Bigg\} \Bigg]$$

$$(50)$$

and finally

$$-\ \frac{\partial}{\partial t}\int \nabla\times\ \delta B dz'\ =\ -F_1\omega_1\Bigg[\vec{v}_1\ \times\vec{\epsilon}_1\ \Bigg\{2\ Sin\phi_1\ (1+\tfrac{1}{2}ln\ (E.B/F^2_{crit})+tan\phi_2$$

$$Cos\phi_1\ \Bigg\}\Bigg|_0 \Bigg[+\vec{v}_1\ \times\vec{\epsilon}_2\ \Bigg\{2\ Sin\phi_2(1+\tfrac{1}{2}ln\frac{E.B}{F^2_{crit}})\ +tan\phi_1\ Cos\phi_2\Bigg\}\Bigg|_0^L \Bigg]\qquad (51)$$

Thus, using equations (47) to (51), E can be evaluated for parallel
polarizations for Θ' and is given by

$$E_f\ =\frac{A}{4\pi r}\ F_1\omega_1\vec{\epsilon}_1\ \Bigg[\frac{1}{(Cos\Theta'-1)}\Bigg\{Sin\phi_1(3+ln(4F^2_1\ Cos\phi_1 Cos\phi_2/F^2_{crit})+$$

$$Sec^2\phi_2\ -\ \frac{Cos\Theta+1}{Cos\Theta'-1})\Bigg\}\ \Bigg|_0^L\ -\ \frac{Cos\Theta-_1}{Cos\Theta+_1}\ \frac{1}{\omega}\ \frac{\partial^2}{\partial t^2}\int_0^L\frac{Cos^2 k\ z'}{Cos\phi_1}\ dz'+\frac{1}{(Cos\Theta+1)}$$

$$\Bigg\{Sin\phi_2\ (3+ln\ (4F^2_1 Cos\phi_1 Cos\phi_2/F^2_{crit})+\ Sec^2\phi_1-\ \frac{(Cos\Theta'-1)}{Cos\Theta'+1}\Bigg\}\Bigg|_0^L\ -$$

$$\frac{Cos\Theta'+1}{Cos\Theta'-1}\ \frac{1}{\omega}\ \frac{\partial^2}{\partial t^2}\int_0^L\ \frac{Cos2\kappa_1 z'}{Cos\phi_2}\ dz'\Bigg]$$

$$(52)$$

These integrals are more conveniently evaluated in the calculations for the
emission power due to the scattered fields which involve an integration
over time. For $\theta'=0$, i.e. along the axis of propagation,

$$E_f(r_1 t)\ =\frac{A}{4\pi r}\ F_1\omega^2_1\ L\vec{\epsilon}_1 Cos(\omega_1 t-\omega_1 r)\{\ 3+ln\Big(8F^2_1\ /\ F^2_{crit}Cos(\omega_1 t-\omega_1 r)\Big)\ -$$

$$Sec^2(\omega_1 t-\omega_1 r)\ \}\qquad (53)$$

Equation (53) shows that due to scattering, the incident wave, $F_1\vec{\epsilon}_1$ cos

$(\omega_1 t\ -\omega_1 r)$ is modulated by a kernel function K (r, t) given by

$$K(r,t)\ =\frac{A}{4\pi r}\ L\omega^2_1\ \Big[3+ln\Big(8\ F^2_1\ Cos(\omega_1 t\ -\omega_1 r)/\ F^2_{crit}\Big)-Sec^2(\omega_1 t-\omega_1 r)\Big]$$

$$(54)$$

The forward scattering can be interpreted as causing a change in the dielectric constant of vacuum. Similarly, the electric field for perpendicular polarization can also be evaluated and is given by

$$E_f = \frac{A}{4\pi r} F_1 L \omega_1^2 \left[\frac{\vec{v}_1 \times \vec{\epsilon}_1}{2\delta_o} \left\{ (Sin(\omega_1 t - \omega_1 r - 2\delta o) - Sin(\omega_1 t - \omega_1 r)) \times (5 + ln 2F_1^2/F_{crit}^2 - 2/\delta_o \right. \right.$$

$$+ 6\delta o) + Sin(\omega_1 t - \omega_1 r + 2\delta o) \; (ln \; Cos^2(\omega_1 t - \omega_1 r + 2\delta o) + \frac{3}{Cos^2(\omega_1 t - \omega_1 r + 2\delta o)}$$

$$+ 2\delta_o \; ln(2F_1 Cos^2 \frac{(\omega_1 t - \omega_1 r + 2\delta o)}{F_{crit}^2} - Sin(\omega_1 t - \omega_1 r) \; (ln \; Cos^2(\omega_1 t - \omega_1 r) \frac{3}{Cos^2(\omega_1 t - \omega_1 r)}$$

$$+ 2\delta_1 ln \left[\frac{2F_1 \; Cos^2(\omega_1 t - \omega_1 r)}{F_{crit}^2} \right] \right\} \Big|_0^L - \frac{\delta o + 1}{\delta \bar{o} \omega} \frac{\partial^2}{\partial t^2} \int_0^L \frac{Cos\phi_1}{Cos\phi_2} \; dz \; - \frac{\vec{v}_1 \times \vec{\epsilon}_1}{2\delta_o + \omega_1^1} \left\{ Sin(\omega_1 t - \right.$$

$$\omega_1 r + 2\delta o) - Sin \; (\omega_1 t - \omega_1 r)) \; (5 + ln F_1^2 / F_{crit}^2 - 2/\delta o + 6\delta o) + Sin(\omega_1 t - \omega_1 r + 2\delta o)$$

$$(ln Cos^2(\omega_1 t - \omega_1 r + 2\delta o) + \frac{3}{Cos^2(\omega_1 t - \omega_1 r + 2\delta o)} + 2\delta o \; ln \left[\frac{2 F_1^2}{F_{crit}^2} Cos^2(\omega_1 t - \omega_1 r + 2\delta o) \right]$$

$$- Sin(\omega_1 t - \omega_1 r)(ln Cos^2(\omega_1 t - \omega_1 r) + \frac{3}{Cos^2(\omega_1 t - \omega_1 r)} + 2\delta o \; ln \left[\frac{2F_1^2}{F_{crit}^2} Cos^2(\omega_1 t - \omega_1 r) \right] \right\} \Big|_0^L$$

$$- \frac{1}{\omega}(1 + \frac{2}{2\delta o + \omega_1^1}) \; \frac{\partial^2}{\partial t^2} \int_0^L \frac{Cos\phi_2}{Cos^2\phi_1} \; dz^1 \right] \tag{55}$$

Where $\delta o = (Cos\theta - 1)\omega_1 L/2$

For $\theta^1 = o$

$$E_f = \frac{A}{4\pi r} F_1 \omega_1^2 \left[\frac{\vec{v} \times \vec{\epsilon}_1}{1} \left\{ L \; Cos(\omega_1 t - \omega_1 r) \; (3 + ln \; \left[\frac{2F_1^2}{F_{crit}^2} \frac{Cos(\omega_1 t - \omega_1 r)}{2} \right] \frac{1}{Cos^2(\omega t - \omega_1 r)}) \right\} \right.$$

$$\left. - \frac{\vec{\epsilon}_1}{\omega_1} Sin \; (\omega_1 t - \omega_1 r) \left\{ ln \left[2F_1^2 Cos \; (\omega_1 t - \omega_1 r)/F_{crit}^2 \right] + 2 \right\} \right] \tag{56}$$

McKenna and Platzman (5), have shown that in the weak field limit, for the initial field as the sum of two plane waves, no scattering other than forward scattering would have been described unlike the case for strong fields. The scattered fields contain terms proportional to the interaction length. Also from equations (13) and (14) we have for the phase velocity V_p,

$$V_p = \sqrt{\frac{1}{\mu\varepsilon}} \geq 1$$

and from equation (13), for $F_1 \simeq 10^2 F_{crit}$, $\delta\varepsilon \approx \delta\mu^{-1} \approx 10^{-3}$ in accord with the results of Eber and Tsai (10). B_f can now be calculated from (33) for the different polarizations.

For parallel polarization and $\theta' \neq 0$

$$\vec{B}_f(r,t) = -\frac{A\omega_1}{2\pi r} F_1 \nu \hat{x} \vec{\varepsilon}_1 \cos(\omega_1 t - \omega_1 r + \delta_0) \sin\delta_0 \tag{57}$$

whereas for $\theta' = 0$

$$\vec{B}_f(r,t) = 0 \tag{58}$$

showing that along the axis of propagation the magnetic field vanishes.

For perpendicular polarization, and $\theta' \neq 0$

$$\vec{B}_f(r,t) = -\frac{A\omega_1}{\pi r} F_1 (\vec{\varepsilon}_2 - \vec{\varepsilon}_1) \cos(\omega_1 t - \omega_1 r + \delta_0) \sin\delta_0 \tag{59}$$

and for $\theta' = 0$

$$\vec{B}_f(r,t) = 0 \tag{60}$$

showing that (58) is also true for perpendicular polarization.

The time averaged power dP radiated into the solid angle $d\Omega$ by the scattered field can be evaluated. We give the expression for the radial component of the time averaged Poynting vector and multiply by $r^2 d\Omega$ to get the average energy per unit time radiated into $d\Omega$. For parallel polarization,

$$dP = \frac{\vec{\nu}}{4\pi} \int_0^T \frac{(\vec{E}_f \times \vec{B}_f)}{T} dt \ r^2 d\Omega$$

$$= \frac{\alpha A^2}{12\pi^3} F_1^2 \omega_1^2 d^2 \ \Omega - \frac{1}{2\pi\omega L} \left[\frac{\sin\delta_0 \cos\delta_0}{\delta_0} \frac{\delta_0 + 2}{\delta_0 + 3} - \frac{\sin^2\delta_0}{2\delta_0^2} \left(3 + \frac{1}{\delta_0} + \ln \frac{4F_1^2}{F_{crit}^2}\right) - \frac{2\sin^2\delta_0 \cos\delta_0}{\delta_0} \right] \tag{61}$$

The average incident power in beams 1 and 2 $= \frac{F_1^2 A}{4\pi}$ and hence the ratio of the emission power to the incident energy flux can be estimated.

$$R'' = \frac{P'' \text{ scattered}}{P'' \text{ inc}} = \frac{\alpha\omega_1^2 A}{6\pi^2 C^2} \left[\frac{2}{\omega L} \int_0^{\omega L} \left\{ -\frac{\sin 2\delta_0}{2\delta_0} \frac{\delta_0 + 2}{\delta_0 + 3} + \frac{\sin\delta_0}{2\delta_0^2} \left(3 + \frac{1}{\delta_0} + \ln 4F_1^2/F_{crit}^2\right) \right. \right.$$

$$\left. \left. + \frac{\sin 2\delta_0 \sin\delta_0}{\delta_0} \right\} d\delta_0 \right] \tag{62}$$

The expression in the square brackets after integration is just a constant since ωL is fixed and can be expressed in terms of the sine and cosine integrals.

Thus

$$R'' = \frac{\omega \omega_1^2 A^2}{6\pi^2 c^2}\left[\frac{3}{3\omega L}\left\{\int_0^{\omega L}\frac{Sint}{t}\,dt_+ \left(3+\ln\frac{4F_1^2}{F^2_{crit}}\right)\int_0^{\omega L}\frac{Sint}{t}\,dt+\frac{3}{2}+Ci(2\omega L)\right.\right.$$

$$\left.-Ci(o)+2Ci(oL)-2Ci(3\omega L)\right] \tag{63}$$

where $Si(x) = -\frac{\pi}{2}+\int_0^x\frac{Sint}{t}\,dt$ is the sine integral and $Ci(x) = C+\ln x+$

$\int_0^x\frac{Cos\ t-1}{t}\,dt$ is the cosine integral and C is the Euler's constant

≈ 0.577.

For $\lambda = 3000\ A^o$, $L = 3\lambda$, $r = 1m$ and $A\approx 10^{-8}cm^2$ (the source dealt with is not an ideal source for which $A \approx \frac{4}{\pi}\lambda^2\approx\pi \times 10^{-9}cm^2$).

$$R'' \approx 10^{-5} \tag{64}$$

For perpendicular polarization,

$$dP = \frac{\alpha^2 F_1^2\omega_1^3 AL}{16_\pi 3}\,d\Omega\left[Sin^2\delta_o\ \left(5+6\delta_o-\frac{2}{\delta_o}+\ln_2 F_1^2/F^2_{crit}\right)+Sin\delta_o(1-Cos2\delta_o)\right.$$

$$\left.(6+\ln_2 F_1^2/F^2_{crit})\right] \tag{65}$$

The quantity in the square brackets when integrated over θ' or equivalently over δ_o gives,

$$\frac{2}{\omega L}\left\{-\frac{C}{2}\,(5+\ln^2 F_1^2/F^2_{crit}) - 3\omega L+2\left[Si(\omega L)-Si(o)\right]\right\} \tag{66}$$

Where C and Si (X) have been previously defined. Again the ratio of the scattered to the incident power can be estimated and for the same parameters as in the parallel polarization case,

$$R^\perp = \frac{P^\perp_{scat}}{P^\perp_{inc}} \approx 10^{-5} \tag{67}$$

CONCLUSIONS

The scattering of light waves in the optical range in the ultra strong limit of the Euler Heisenberg Lagrangian has been investigated together with the differences from the weak field problem. The radiated fields were derived and shown to be phase shifted for both electric and magnetic fields. The consideration of the Euler-Heisenberg Lagrangian in its complete form without any approximation might lead to the generation of higher harmonics and is an interesting problem requiring further work.

The emission power is estimated for both polarization cases and compared with the incident flux. The ratios are certainly non negligible and far greater than that for the weak field case. The direct creation of e^- - e^+ pairs in intense magnetic fields is also of great astrophysical significance (11). It is interesting to note that the phenomenon of the charged vacuum in strong electrostatic fields has been recently well understood (12). A typical and important feature of the quantized theory is vacuum polarization. The contribution of vacuum polarization has an analogy to the inducing Coulomb potential $V(r)$. Thus the physics of very strong electrostatic fields permits the study of phenomena not encountered in ordinary atomic physics. The incorporation of the electromagnetic non linearities of matter into Maxwell's equations has therefore shown promise in the solution of a number of boundary value problems which are important for the understanding of optical systems as well as instrumentation. In the case flux densities of $\approx 10^{22}$ W/cm^2 are reached as predicted by the Russians in 1982 (13) it would be possible to verify some of the predicted results of this work.

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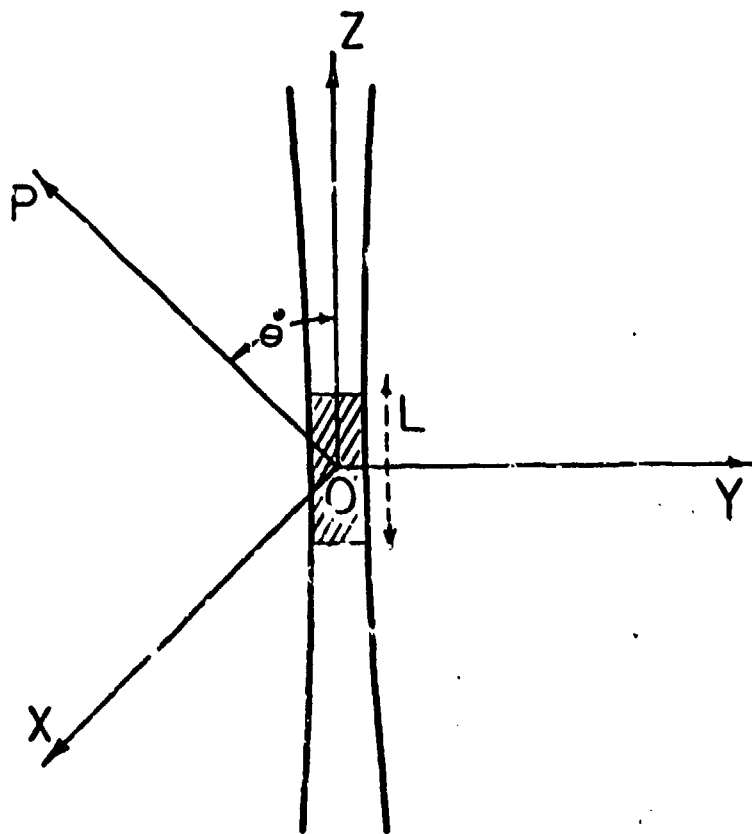13.   J. Moss., New Scientist, 75 P24, 7 July, 1977

Figure 1   Non Linear Scattering of Light in the Limit of Ultra-Strong Fields

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With the rapid development of high energy, coherent, monochromatic laser sources foreseen, the interaction phenomenon between two such sources is of fundamental importance. This work attempts to treat the above interaction by use of the Lagrangian for a strong electromagnetic field. The quantum mechanical effect of the scattering between plane light waves is simulated within the framework of classical theory under the assumption of an effective non linear interaction between the light waves. The scattered fields are solved for, and expressions are obtained for, the scattered power.

KEY WORDS

Laser interaction
High intensity fields
Scattering
Non linear waves interaction

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