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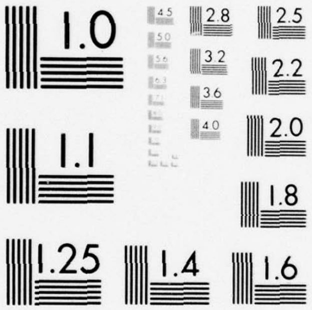
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A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS

by J. Ernest Breeding, Jr.
K.C. Matson
Nourollah Riahi

TECHNICAL REPORT No. JEB-1
Department of Oceanography
Florida State University

MARCH, 1978

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ERRATA SHEET NOVEMBER 1978 N00014-77-C-0329

Breeding, J. Ernest, Jr., K.C. Matson, and Nourallah Riahi, "A Method for Calculating Wave Packet Trajectories and Wave Heights," Report JEB-1, Department of Oceanography, Florida State University, Tallahassee, March 1978. pt. 1

There is a miscopy in Equation (2-48) on page 13. The equation should read

$$\frac{\partial^2 U}{(\partial x')^2} = \frac{\operatorname{csch} I}{2}\left\{-\left[(\sinh I + I)\frac{\partial \upsilon}{\partial x'} + \upsilon\frac{\partial I}{\partial x'}(1 - I\coth I)\right]\coth I\,\frac{\partial I}{\partial x'} + \right.$$
$$+ (\sinh I + I)\frac{\partial^2 \upsilon}{(\partial x')^2} + \frac{\partial \upsilon}{\partial x'}\frac{\partial I}{\partial x'}(2 + \cosh I - I\coth I) + \qquad (2\text{-}48)$$
$$\left. + \upsilon\left[\frac{\partial^2 I}{(\partial x')^2}(1 - I\coth I) + \left(\frac{\partial I}{\partial x'}\right)^2 \operatorname{csch} I\,(I\operatorname{csch} I - \cosh I)\right]\right\}$$

The FORMAT statements at MAIN 122 and MAIN 124 on page 39 should be changed to

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1 FORMAT (6F10.7)

2 FORMAT (12F2.0)
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TECHNICAL REPORT NO. JEB - 1
Department of Oceanography
Florida State University

⑥

A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS

Part 1.

by

⑩

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⑫ 111 p.

⑪ March, 1978

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ABSTRACT

The theory and numerical methods are presented for determining the paths of gravity water wave packets. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \phi$. The symbol ω denotes the angular frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. At each point of the wave packet trajectory the wavelet direction is determined using Snell's law with phase velocity. The wave height is computed along the wave packet paths accounting for the effects of shoaling, refraction, and energy dissipation. The computer program is described and sample printouts and plots are presented.

ACKNOWLEDGMENTS

We express our appreciation to Kimberly Crane Oppenheimer, Pete H. Oppenheimer, S. Shang, and Ellen Walker Breeding for their assistance in preparing this report. The research was supported by the Geography Programs, Earth Sciences Division, Office of Naval Research under Contract No. N00014-77-C-0329.

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CHAPTER I

It has been shown by Breeding (1978) that a wave packet refracts according to Snell's law with the geometric group velocity G where

$$G = (d\omega/dk) \cos \phi \qquad (1\text{-}1)$$

The symbol ω denotes the angular frequency and k is the wave number. The angle ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. The wavelet direction at each point of the wave packet trajectory is determined by Snell's law with phase velocity.

In this work a numerical method is presented for determining the paths of gravity water wave packets. Further, a procedure is developed for computing the wave heights along the paths accounting for the effects of shoaling, refraction, and energy dissipation.

There are a number of papers in which numerical methods are presented for calculating and plotting the trajectories of monochromatic rays. Skovgaard, et al (1975) summarize a number of these methods and present one of their own. The numerical methods for calculating and plotting rays which are presented in this work are based on the Wilson (1966) program. However, extensive modifications of the Wilson program were required in order to compute the path of a wave packet.

In Chapter II, an expression is derived for the ray curvature of a wave packet. The assumption is made that the wave speed contours are locally parallel in the vicinity of each point of a ray. Properties of the packet ray curvature are discussed, and a procedure is described for computing the wavelet directions. A method is described for computing the water depth, phase speed, group speeds, and their spatial derivatives from a square grid of water depth values. To simplify the ray curvature calculations a coordinate system is defined with an axis parallel to the water depth contours at each ray point. The numerical procedure for computing the trajectories of gravity water wave packets is described. Rules for dealing with reflection points, which occur when the ray curvature becomes infinite, are established.

The shoaling, refraction, and friction coefficients used to compute the wave height are presented. The refraction coefficient is evaluated on the assumption that the wave speed contours are locally parallel in the vicinity of a ray point. A method for computing the refraction coefficient near a reflection point is described. A procedure for locating caustics and focal points is discussed. Chapter II ends with a discussion of the wave breaking criterion.

Chapter III contains a description of the computer program and a program listing. A guide to using the program and an explanation of the computer output is given in Chapter IV. The principal notation used in the report is presented following the references.

CHAPTER II THEORY

2.1 Ray Curvature for Wave Packets. The ray curvature κ_v of a ray moving with phase speed v was derived by Munk and Arthur (1952) and Arthur, et al (1952) as

$$\kappa_v = \frac{d\gamma}{ds_v} = \frac{1}{v}\left(\sin\gamma\,\frac{\partial v}{\partial x} - \cos\gamma\,\frac{\partial v}{\partial y}\right) \tag{2-1}$$

where x,y are the Cartesian coordinates, γ is the direction of the ray with respect to the positive x-axis, and s_v is the arc length along the ray.

The ray curvature κ_G for the trajectory of a wave packet is given by

$$\kappa_G = \frac{d\theta}{ds_G} = \frac{1}{G}\left(\sin\theta\,\frac{\partial G}{\partial x} - \cos\theta\,\frac{\partial G}{\partial y}\right) \tag{2-2}$$

where θ is the direction of the ray with respect to the positive x-axis, ds_G is an element of arc length along the ray, and G is the geometric group speed defined by (Breeding, 1978)

$$G = U\cos\phi \tag{2-3}$$

where

$$U = \frac{d\omega}{dk} \tag{2-4}$$

is the collinear group speed, $\omega = 2\pi f$ is the angular frequency, $f = 1/T$ is the frequency, T is the wave period, $k = 2\pi\lambda$ is the wave number, λ is the wavelength, and

$$\phi = \theta - \gamma \tag{2-5}$$

a. Locally parallel wave speed contours

The calculations are simplified by making the assumption that the wave speed contours are locally parallel in the vicinity of each point of the ray trajectory. Further, we chose a x'y'-coordinate system such that the y'-derivatives of v, U, and G are zero. Accordingly, in the primed system the space derivative of Equation (2-3) is

$$\frac{\partial G}{\partial X'} = \frac{\partial U}{\partial X'} \cos \phi' - U \sin \phi' \left(\frac{\partial \theta'}{\partial X'} - \frac{\partial \gamma'}{\partial X'} \right) \tag{2-6}$$

Expressions for the space derivatives of θ' and γ' are derived from Equations (2-1) and (2-2) where

$$\partial X' = \partial s_{v} \cos \gamma' \tag{2-7}$$

$$\partial X' = \partial s_{G} \cos \theta' \tag{2-8}$$

Thus,

$$\frac{\partial \gamma}{\partial X'} = \frac{\tan \gamma'}{v} \frac{\partial v}{\partial X'} \tag{2-9}$$

$$\frac{\partial \theta}{\partial X'} = \frac{\tan \theta'}{G} \frac{\partial G}{\partial X'} \tag{2-10}$$

where it should be noted that $d\gamma = d\gamma'$, and $d\theta = d\theta'$, $\phi = \phi'$, and $\kappa_G = \kappa_G'$. When Equations (2-9) and(2-10) are substituted into (2-6) and the terms rearranged, it is found that

$$\frac{\partial G}{\partial X'} = \frac{\frac{\partial U}{\partial X'} \cos \phi + \frac{U \sin \phi \tan \gamma'}{v} \frac{\partial v}{\partial X'}}{1 + \tan \phi \tan \theta'} \tag{2-11}$$

After Equations (2-2) and (2-11) are combined and the result is simplified, it is found that

$$K_G = \frac{\frac{1}{U} \frac{\partial U}{\partial X'} + \frac{\tan \phi \tan \gamma'}{v} \frac{\partial v}{\partial X'}}{\csc \theta' + \tan \phi \sec \theta'} \tag{2-12}$$

b. Properties of the group ray curvature

The ray curvature of a wave packet defined by Equation (2-12) exhibits some very remarkable properties. Under various conditions the trigonometric terms of the equation can become infinite or have indeterminate forms. The value of κ_G approaches zero as θ' approaches a direction either parallel or perpendicular to the wave speed contours, provided γ' does not have a direction parallel to the contours. This means that given a sufficiently long path, refraction tends to turn the wave packet so that it is directed either parallel or perpendicular to the wave speed contours. In the limit θ' or γ' together approach a direction parallel to the wave speed contours, κ_G approaches a finite nonzero number. As γ', but not θ', approaches a direction parallel to the wave speed contours, κ_G approaches infinity and the wave group undergoes total reflection.

For parallel wave speed contours, the value of θ' at a reflection point can be determined using Snell's law, which is the integrated form of the ray curvature expression. Snell's law for a wave packet can be written

$$\frac{\sin\theta'}{U\cos(\theta'-\gamma')} = C_s \qquad (2\text{-}13)$$

where C_s is a constant. When the wavelet direction is parallel to the wave speed contours (Equation 2-13) becomes

$$\sin\theta' = \pm\, U C_s \sin\theta' \qquad (2\text{-}14)$$

where the sign is positive or negative depending on whether $\sin\gamma' = \pm 1$. Equation (2-14) holds for all values of U and C_s only if θ' is an integral multiple of 180^0. Accordingly, at a reflection point the wavelet direction becomes parallel to the wave speed contours, the wave packet direction becomes perpendicular to the contours, the geometric group velocity goes to zero, and the ray curvature becomes infinite.

c. Wavelet directions

Both the wave packet and wavelet directions must be computed in determining each point of a ray path. Equation (2-1) can be used to calculate γ. However, since the packet and wavelets travel with different velocities, the incremental distances by which they advance are different. The wavelet incremental distance must extend to the wave speed contour reached by the wave packet. This is illustrated in Figure (2-1) where the wave speed contours are assumed to be locally parallel. From the figure, it is seen that

$$ds_U = \frac{\cos\theta'}{\cos\gamma'}\, ds_G \qquad (2\text{-}15)$$

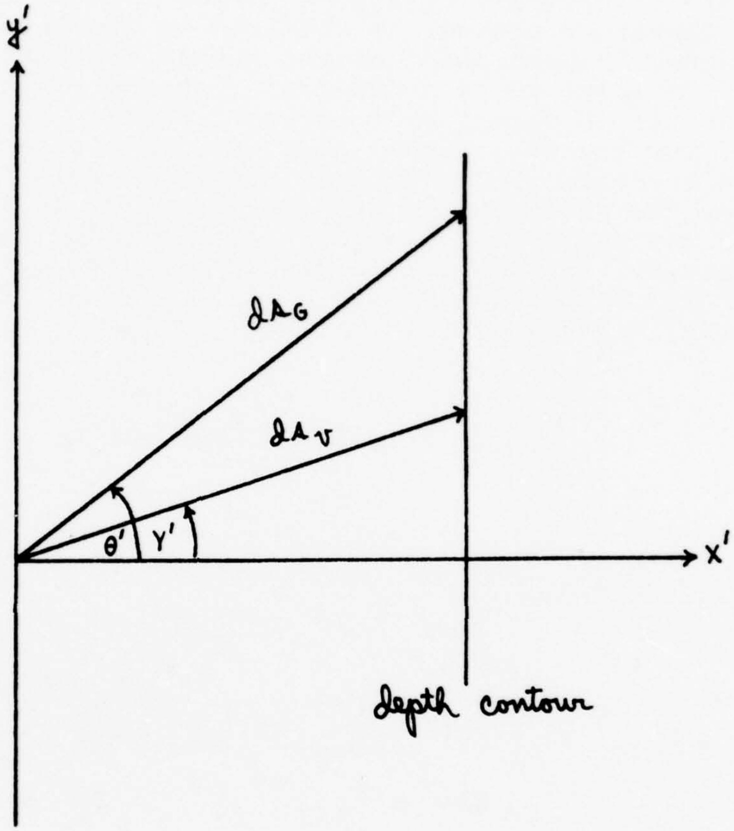

Figure (2-1). RELATIONSHIP BETWEEN THE WAVE PACKET AND WAVELET INCREMENTAL DISTANCES

Equation (2-15) is well behaved except at reflection points where $\cos \gamma' = 0$ and $\kappa_G = \infty$.

The wavelet direction can be calculated using Snell's law with phase velocity. This offers the advantage that the wavelet incremental distance, which must be determined in the ray curvature method, is replaced by the wave speed which is computed at each point of the packet trajectory.

In order to be consistent with the rest of the computations, it is necessary to present Snell's law in a form such that the incident angle is defined with respect to the positive x'-axis. To do this, a number of rules are employed where the subscripts n and (n + 1) refer to consecutive points of a ray and n is a positive integer. The first step, if necessary, is to successively add or subtract 360° from the incident wavelet angle until it is within the range $0 \leq \gamma_n' < 360^\circ$. Then, when Snell's law is given by

$$\gamma_{m+1}^{\;*} = \arcsin\left(\frac{v_{m+1}}{v_m} \sin \gamma_m'\right) \qquad (2\text{-}16)$$

where $-90^\circ \leq \gamma_{n+1}^{\;*} \leq 90^\circ$, the angle γ_{n+1}' is defined by the following scheme.

$$\gamma_{m+1}' = \begin{cases} \gamma_{m+1}^{\;*}, & \gamma_m' \leq 90^\circ \\ 180^\circ - \gamma_{m+1}^{\;*}, & 90^\circ < \gamma_m' \leq 270^\circ \\ 360^\circ + \gamma_{m+1}^{\;*}, & \gamma_m' > 270^\circ \end{cases} \qquad (2\text{-}17)$$

2.2 Spatial Derivatives of G, U, v, and h. In this section relations are presented for connecting the partial derivatives of G , U, v, and h.

a. Determination of h and its partial derivatives

For each ray point the water depth h is interpolated from a quadratic surface equation which is fitted to the water depths at 12 grid points as illustrated in Figure (2-2). The use of a quadratic surface makes it possible to evaluate second derivatives which are required in calculating the wave height. The surface is approximated by the general quadratic equation (Dobson, 1967)

$$h = E_1 + E_2 x + E_3 y + E_4 x^2 + E_5 xy + E_6 y^2 \qquad (2\text{-}18)$$

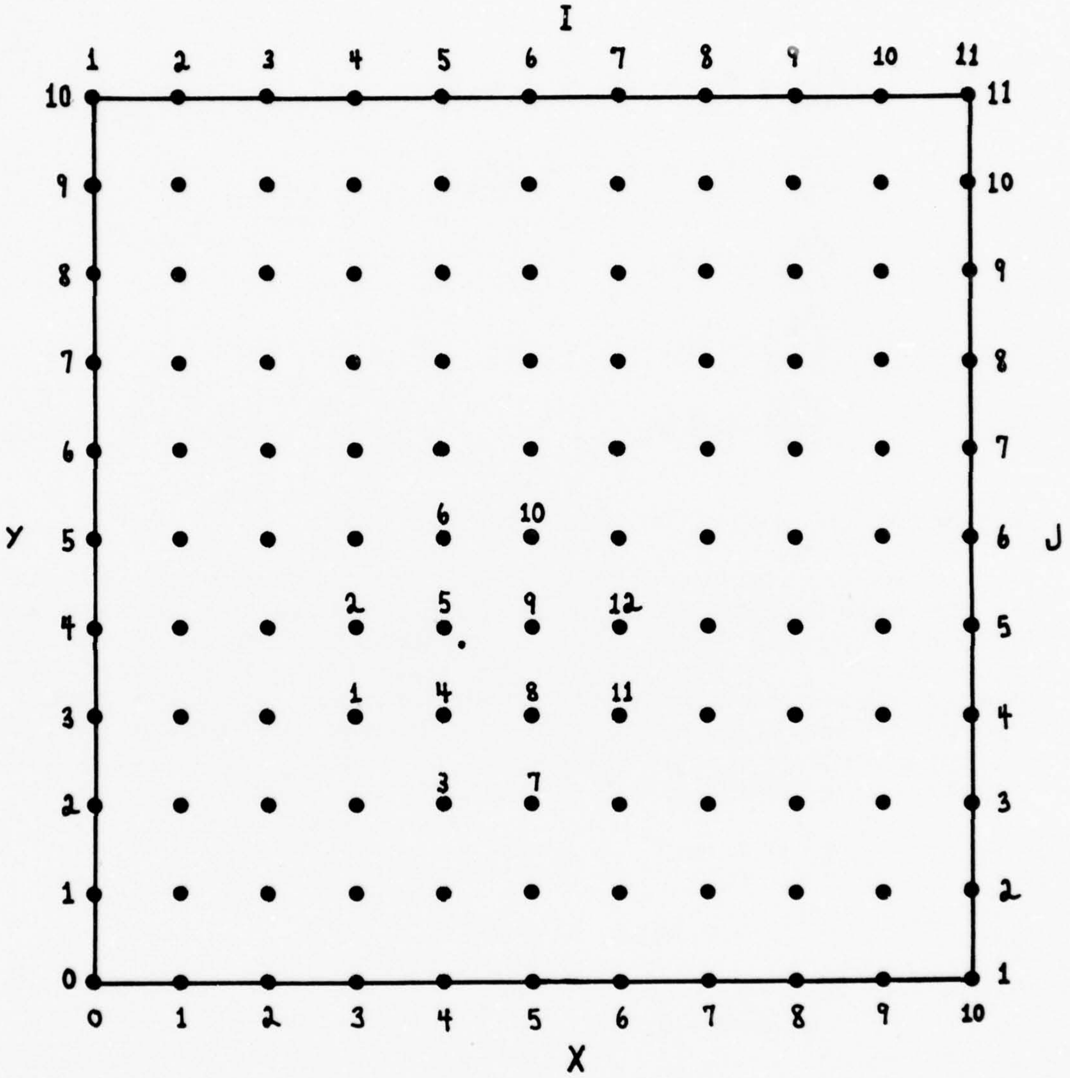

Figure (2-2). SELECTION OF WATER DEPTHS ABOUT RAY POINT (X,Y) = (4.2, 3.8) where the coefficients E are determined by fitting the equation by the method of least squares to the 12 water depth values. The partial derivatives of h are readily determined from Equation (2-18).

$$\frac{\partial h}{\partial x} = E_2 + 2E_4 x + E_5 y \qquad (2\text{-}19)$$

$$\frac{\partial h}{\partial y} = E_3 + E_5 x + 2E_6 y \qquad (2\text{-}20)$$

$$\frac{\partial^2 h}{\partial x^2} = 2E_4 \qquad (2\text{-}21)$$

$$\frac{\partial^2 h}{\partial x \partial y} = E_5 \qquad (2\text{-}22)$$

$$\frac{\partial^2 h}{\partial y^2} = 2E_6 \qquad (2\text{-}23)$$

b. Rotation of axes to make computations

A reference xy-coordinate system aligned with respect to the water depth grid is used to tabulate the particulars of a wave packet trajectory. However, the calculations at each point of the trajectory are made in an x'y'-coordinate system chosen such that the y'-derivatives of h are zero. The relationships between the coordinate systems, a ray, and a depth contour are illustrated in Figure (2-3). Equations relating the coordinate systems are given by

$$x' = x \cos\alpha + y \sin\alpha \qquad (2\text{-}24)$$

$$y' = -x \sin\alpha + y \cos\alpha \qquad (2\text{-}25)$$

$$\theta' = \theta - \alpha \qquad (2\text{-}26)$$

$$\gamma' = \gamma - \alpha \qquad (2\text{-}27)$$

The angle α by which the x'-axis is rotated with respect to the x-axis is given by

$$\tan\alpha = \frac{\partial h}{\partial y} \Big/ \frac{\partial h}{\partial x} \qquad (2\text{-}28)$$

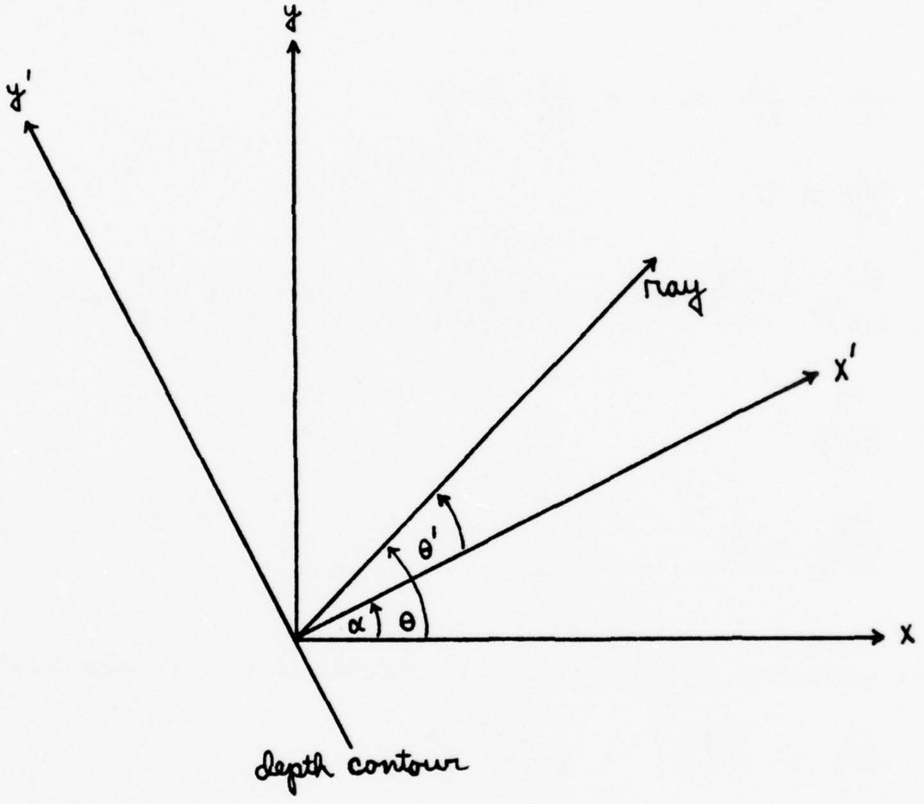

Figure (2-3). RELATIONSHIPS BETWEEN THE COORDINATE SYSTEMS, A RAY, AND A DEPTH CONTOUR

As a consequence, the positive x'-axis is perpendicular to the water depth contour in the direction of increasing h. For the assumption of locally parallel wave speed contours to be good, the variation in α between successive points of a ray must be small.

The partial derivatives of h in the x'y'-coordinate system become

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \cos \alpha + \frac{\partial h}{\partial y} \sin \alpha \qquad (2\text{-}29)$$

$$\frac{\partial h}{\partial y'} = 0 \qquad (2\text{-}30)$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} \cos^2 \alpha + 2 \frac{\partial^2 h}{\partial x \partial y} \sin \alpha \cos \alpha + \frac{\partial^2 h}{\partial y^2} \sin^2 \alpha \qquad (2\text{-}31)$$

$$\frac{\partial^2 h}{(\partial y')^2} = 0 \qquad (2\text{-}32)$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = 0 \qquad (2\text{-}33)$$

Alternative expressions for Equations (2-29) and (2-31) are

$$\frac{\partial h}{\partial x'} = \left[\left(\frac{\partial h}{\partial x} \right)^2 + \left(\frac{\partial h}{\partial y} \right)^2 \right]^{\frac{1}{2}} \qquad (2\text{-}34)$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \qquad (2\text{-}35)$$

c. Derivatives of v

As a convenience in the computations, the spatial derivatives of v are expressed in terms of the spatial derivatives of h. For linear theory (Lamb, 1932) the phase speed of a gravity water wave can be defined

$$v = \frac{1}{a} \tanh \frac{I}{2} \qquad (2\text{-}36)$$

where

$$a = \frac{2\pi}{gT} \qquad (2\text{-}37)$$

$$b = \frac{T}{4\pi} \qquad (2\text{-}38)$$

$$I = \frac{h}{bv} \qquad (2\text{-}39)$$

and g is the acceleration due to gravity. For a given wavelet period, a and b are constants. It can be shown (Wilson, 1966; Dobson, 1967; Breeding, 1972) that

$$\frac{\partial v}{\partial x} = W \frac{\partial h}{\partial x} \qquad (2\text{-}40)$$

$$\frac{\partial v}{\partial y} = W \frac{\partial h}{\partial y} \qquad (2\text{-}41)$$

where

$$W = \frac{v(1 - a^2 v^2)}{[2abv^2 + h(1 - a^2 v^2)]} \qquad (2\text{-}42)$$

Further

$$\frac{dv}{dx'} = W \frac{dh}{dx'} \qquad (2\text{-}43)$$

Differentiation of Equation (2-43) yields after some simplification

$$\frac{d^2 v}{(dx')^2} = W \left[\frac{d^2 h}{(dx')^2} + Y \left(\frac{dh}{dx'} \right)^2 \right] \qquad (2\text{-}44)$$

where

$$Y = - \frac{4abv^2}{[2abv^2 + h(1 - a^2 v^2)]^2} \qquad (2\text{-}45)$$

d. Derivatives of U

For linear wave theory the collinear group speed of a gravity water wave can be defined

$$U = \frac{1}{2}\left(1 + \frac{I}{\sinh I}\right) v \tag{2-46}$$

The spatial derivatives of U can be expressed

$$\frac{\partial U}{\partial X'} = \frac{\operatorname{csch} I}{2} \left[(\sinh I + I)\frac{\partial v}{\partial X'} + v\,\frac{\partial I}{\partial X'}(1 - I\coth I)\right] \tag{2-47}$$

$$\begin{aligned}\frac{\partial^2 U}{(\partial X')^2} = \frac{\operatorname{csch} I}{2} \Bigg\{ & - \left[(\sinh I + I)\frac{\partial v}{\partial X'} + v\,\frac{\partial I}{\partial X'}(1 - I\coth I)\right]\coth I + \\ & + (\sinh I + I)\frac{\partial^2 v}{(\partial X')^2} + \frac{\partial v}{\partial X'}\,\frac{\partial I}{\partial X'}\,(2 + \cosh I - I\coth I) + \\ & + v\left[\frac{\partial^2 I}{(\partial X')^2}(1 - I\coth I) + \left(\frac{\partial I}{\partial X'}\right)^2 \operatorname{csch} I\,(I\operatorname{csch} I - \cosh I)\right]\Bigg\}\end{aligned} \tag{2-48}$$

where

$$\frac{\partial I}{\partial X'} = I\left(\frac{1}{h}\,\frac{\partial h}{\partial X'} - \frac{1}{v}\,\frac{\partial v}{\partial X'}\right) \tag{2-49}$$

$$\begin{aligned}\frac{\partial^2 I}{(\partial X')^2} = \frac{1}{I}\left(\frac{\partial I}{\partial X'}\right)^2 + I\Bigg\{ & \frac{1}{h}\left[\frac{\partial^2 h}{(\partial X')^2} - \frac{1}{h}\left(\frac{\partial h}{\partial X'}\right)^2\right] - \\ & - \frac{1}{v}\left[\frac{\partial^2 v}{(\partial X')^2} - \frac{1}{v}\left(\frac{\partial v}{\partial X'}\right)^2\right]\Bigg\}\end{aligned} \tag{2-50}$$

e. Derivatives of G

Equation (2-11) can be restated as

$$\frac{\partial G}{\partial X'} = \rho\left(\cos\phi\,\frac{\partial U}{\partial X'} + \sigma\,\frac{\partial v}{\partial X'}\right) \tag{2-51}$$

where

$$\rho = (1 + \tan\phi \tan\theta')^{-1} \tag{2-52}$$

$$\sigma = \frac{U}{v} \sin\phi \tan\gamma' \tag{2-53}$$

The second spatial derivative of G is given by

$$\begin{aligned}\frac{\partial^2 G}{(\partial X')^2} = \rho\left(\cos\phi \frac{\partial^2 U}{(\partial X')^2} + \sigma \frac{\partial^2 v}{(\partial X')^2}\right) + \left(\cos\phi \frac{\partial\rho}{\partial X'} -\right. \\ \left. - \rho \sin\phi \frac{\partial\phi'}{\partial X'}\right) \frac{\partial U}{\partial X'} + \left(\rho \frac{\partial\sigma}{\partial X'} + \sigma \frac{\partial\rho}{\partial X'}\right) \frac{\partial v}{\partial X'}\end{aligned} \tag{2-54}$$

where

$$\frac{\partial\rho}{\partial X'} = -\rho^2\left(\tan\theta' \sec^2\phi \frac{\partial\phi'}{\partial X'} + \tan\phi \sec^2\theta' \frac{\partial\theta'}{\partial X'}\right) \tag{2-55}$$

$$\frac{\partial\sigma}{\partial X'} = \sigma\left(\frac{1}{U}\frac{\partial U}{\partial X'} - \frac{1}{v}\frac{\partial v}{\partial X'}\right) + \frac{U}{v}\left(\cos\phi \tan\gamma' \frac{\partial\phi'}{\partial X'} + \sin\phi \sec^2\gamma' \frac{\partial\gamma'}{\partial X'}\right) \tag{2-56}$$

The derivatives of γ' and θ' are defined by Equations (2-9) and (2-10), and

$$\frac{\partial\phi'}{\partial X'} = \frac{\partial\theta'}{\partial X'} - \frac{\partial\gamma'}{\partial X'} \tag{2-57}$$

2.3 Computation of Gravity Water Wave Packet Trajectories. In this section a method is presented for computing the surface trajectories of gravity water wave packets.

a. Determining the path

Successive points of a wave packet trajectory are found by iteration using Equation (2-12) for the ray curvature. The ray curvature is calculated on the assumption that the water depth contours are locally parallel in the vicinity of each point of the trajectory. At each point of the wave packet trajectory the wavelet direction is found using Snell's law with phase velocity as defined by Equations (2-16) and (2-17).

The coordinates of each ray point are defined by

$$x_{n+1} = x_n + D_n \cos \bar{\theta} \tag{2-58}$$

$$y_{n+1} = y_n + D_n \sin \bar{\theta} \tag{2-59}$$

$$\bar{\theta} = \frac{1}{2}(\theta_n + \theta_{n+1}) \tag{2-60}$$

$$\theta_{n+1} = \theta_n + \Delta\theta \tag{2-61}$$

$$\Delta\theta = \frac{1}{2}\left[(K_G)_n + (K_G)_{n+1}\right] D_n \tag{2-62}$$

$$D_M = G(\Delta t)/\mathrm{GRID} \tag{2-63}$$

where D_n is the incremental distance in grid units between the points n and (n + 1) of a ray, GRID is the grid unit distance in units consistent with G, and (Δt) is the time step.

b. Reflection points

Reflection points require special consideration. The waves are assumed to reflect if any of three criteria are satisfied. The first test for reflection is based on Snell's law with phase velocity. Reflection occurs if

$$\left| \frac{v_{n+1}}{v_n} \sin \gamma_n' \right| > 1 \tag{2-64}$$

As a reflection point is approached the ray curvature changes so quickly that calculations of the ray curvature by iteration may cease to converge. If convergence fails reflection is assumed if the following conditions are met.

$$\frac{v_{n+1}}{v_n} > 1 \tag{2-65}$$

$$|\tan \gamma'| > \tan 80°$$

The first condition requires that the phase speed increases between the last two ray points, and the second condition requires that the wavelet direction be consistent with total reflection.

A third criterion is used to specify reflection points in order to maintain accuracy in calculating the wave packet trajectory. Very near a reflection point, due to the rapidly changing ray curvature, the estimates of ray points can become erratic. Therefore, reflection is assumed if the following conditions are met

$$\begin{aligned} \frac{v_{m+1}}{v_m} &> 1 \\ |\tan \gamma'| &> \tan 89.5^\circ \\ |\tan \theta'| &< \tan 75^\circ \end{aligned} \qquad (2\text{-}66)$$

When a reflection point is determined on the basis of either the second or third criteria, it is advisable to examine the printout to determine if the ray particulars are consistent with total reflection. The values of κ_G, γ', θ', and G should exhibit the behavior described in Section (2.1), b.

When a reflection occurs there is an option to either halt the wave packet trajectory or to continue it as a reflection. When the wave packet reflection option is chosen, the reflection angles are determined by the relations

$$\theta_r' = -\theta_i' + 180^\circ \qquad (2\text{-}67)$$

$$\gamma_r' = -\gamma_i' + 180^\circ \qquad (2\text{-}68)$$

The subscript r denotes the reflection angles, and the subscript i signifies the angles at the last ray point reached prior to where the reflection criterion is satisfied.

The ray curvature calculations are more likely to converge and the accuracy of the path is increased if there is a restriction on how much the ray direction can change between successive ray points. Accordingly, if γ' is within 15° of a direction for which the ray curvature is infinite, the time step between ray points is successively halved, as necessary, until $|\Delta\theta|$ is less than 1°. In the event it is necessary to reduce the time step to less than 0.5 seconds the ray is stopped.

2.4 Wave Height Calculations. Modification of the wave height due to refraction, shoaling, and energy dissipation is considered. The wave height H increases when adjacent rays converge and it decreases when the rays diverge; this effect is accounted for by the refraction coefficient K_R. The shoaling coefficient K_S accounts for the change in H due to the variation in the propagation speed of the wave packet. The loss in energy due to wave motion at the sea bottom is determined by the friction coefficient K_F.

a. Without energy dissipation

The average rate of energy transmission F can be defined

$$F = E \ell G \tag{2-69}$$

where E is the energy per unit area and ℓ is the perpendicular distance between rays. The energy is assumed to be conserved along the ray. Therefore

$$F_{j+1} = F_j \tag{2-70}$$

where j and (j + 1) denote consecutive ray points. It is further assumed that E is proportional to H^2. Accordingly, it follows from Equations (2-69) and (2-70) that

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} H_j \tag{2-71}$$

where $(K_S)_{j+1}$ and $(K_R)_{j+1}$ are the shoaling and refraction coefficients, respectively, between points j and (j+1).

$$(K_S)_{j+1} = \left(\frac{G_j}{G_{j+1}}\right)^{\frac{1}{2}} \tag{2-72}$$

$$(K_R)_{j+1} = \left(\frac{\ell_j}{\ell_{j+1}}\right)^{\frac{1}{2}} \tag{2-73}$$

If H_0 is the initial wave height, then the wave height at the n-th point is

$$H_m = K_S K_R H_0 \tag{2-74}$$

where

$$K_S = (K_S)_1 (K_S)_2 \cdots (K_S)_m = \left(\frac{G_0}{G_m}\right)^{\frac{1}{2}} \tag{2-75}$$

$$K_R = (K_R)_1 (K_R)_2 \cdots (K_R)_m = \left(\frac{l_0}{l_m}\right)^{\frac{1}{2}} \qquad (2\text{-}76)$$

b. With energy dissipation

To account for energy losses, Equation (2-70) can be restated

$$F_{j+1} = (K_F)^2_{j+1} F_j \qquad (2\text{-}77)$$

where $(K_F)_{j+1}$ is the friction coefficient between the points j and (j+1). As a result, the relationship between the wave heights at consecutive ray points can be expressed by

$$H_{j+1} = (K_S)_{j+1} (K_R)_{j+1} (K_F)_{j+1} \qquad (2\text{-}78)$$

In terms of the initial wave height, the wave height at the n-th point is given by

$$H_m = K_S K_R K_F H_0 \qquad (2\text{-}79)$$

where

$$K_F = (K_F)_1 (K_F)_2 \cdots (K_F)_m = (K_F)_m (K_F)_m \qquad (2\text{-}80)$$

2.5 Refraction Coefficient. In computing K_R it is convenient to define

$$\beta = \frac{l_m}{l_0} \qquad (2\text{-}81)$$

where β is called the ray separation factor. Equation (2-76) for the refraction coefficient becomes

$$K_R = |\beta|^{-\frac{1}{2}} \qquad (2\text{-}82)$$

a. Ray separation equation

In considering the refraction of monochromatic waves, Munk and Arthur (1952) have shown that β can be determined from a second-order differential equation called the ray separation equation. The equation can be stated

$$\frac{\partial^2 \beta}{\partial t^2} + p\frac{\partial \beta}{\partial t} + q\beta = 0 \qquad (2\text{-}83)$$

where t is time. For a wave packet trajectory

$$p = -2\left(\cos\theta \frac{\partial G}{\partial x} + \sin\theta \frac{\partial G}{\partial y}\right) \qquad (2\text{-}84)$$

$$q = G\left(\sin^2\theta \frac{\partial^2 G}{\partial x^2} - 2\sin\theta\cos\theta \frac{\partial^2 G}{\partial x \partial y} + \cos^2\theta \frac{\partial^2 G}{\partial y^2}\right) \qquad (2\text{-}85)$$

The calculations for p and q can be simplified in the same manner as in calculating the wave packet ray curvature by using the x'y'-coordinate system. Then, p and q reduce to

$$p = -2\cos\theta' \frac{\partial G}{\partial x'} \qquad (2\text{-}86)$$

$$q = G\sin^2\theta' \frac{\partial^2 G}{(\partial x')^2} \qquad (2\text{-}87)$$

b. Solution for parallel water depth contours

There is a simple solution to Equations (2-83), (2-86), and (2-87) when the water depth contours are everywhere parallel. Then, with the x'y'-coordinate system defined as in Section 2.2, b, it can be shown that

$$\beta = \frac{\cos\theta_n'}{\cos\theta_0'} \qquad (2\text{-}88)$$

where the subscript 0 denotes the initial value and the subscript n depicts the value at the n-th ray point. The time derivative of Equation (2-88) is

$$\frac{\partial \beta}{\partial t} = -\frac{\sin^2 \theta_n'}{\cos \theta_0'} \frac{\partial G}{\partial x'} \qquad (2\text{-}89)$$

where Equations (2-8) and (2-10) are used.

c. Numerical solutions of the ray separation equation

Several numerical methods can be used to solve the ray separation equation. The assumption is made that the water depth contours are locally parallel in the neighborhood of each ray point; therefore, p and q are defined by Equations (2-86) and (2-87). An easy to use fourth order finite difference solution to Equation (2-83) is the Fox method (Salvadori and Baron, 1961). However, this method has the disadvantage that the time step must be constant between successive ray points. When p and q do not change much between ray points, the general solution of a homogeneous second-order differential equation with constant coefficients (Wylie, 1951) can be used to solve the ray separation equation. This solution has 3 cases depending on the value of $(p^2 - 4q)$. The value of β at each new ray point is found using the values of p and q at the last point. There is usually little difference between the results obtained by this method and the Fox method.

A numerical method which does not require a constant time step and which better accounts for the variation of p and q along a ray is the Runge-Kutta method. This method is stable. It is also self-starting, i.e., values at the previous point are used to find values at the next point (Romanelli, 1960). For these reasons, the Runge-Kutta method was selected for the solution of the ray separation equation.

In order to use the Runge-Kutta method, Equation (2-83) is reduced to a system of first-order equations.

$$\frac{\partial \beta}{\partial t} = u \qquad (2\text{-}90)$$

$$\frac{\partial u}{\partial t} = -(pu + q\beta) \qquad (2\text{-}91)$$

Both fourth and fifth order solutions of β are obtained. The initial conditions are the values of β and dβ/dt at the first ray point. The latter is estimated using Equation (2-89). The solutions require the values of (p_n, q_n) at the n-th ray point and the values (p_{n+1}, q_{n+1}) at the (n+1)-th ray point. Further, the values of (p_1, q_1), (p_2, q_2), (p_3, q_3), (p_4, q_4), and (p_5, q_5) are needed along the ray at points intermediate to the ray points. They are determined, respectively, at time intervals of $(\Delta t)/3$, $(\Delta t)/4$, $0.45573725(\Delta t)$, $2(\Delta t)/3$, and

0.8(Δt) beyond the n-th ray point where Δt is the time step in the calculations.

A fourth order Runge-Kutta method with a minimum truncation error bound is given by Ralston (1962). The solution for the ray separation equation becomes

$$\beta_{m+1} = \beta_m + 0.17476028\,K_1 - 0.55148066\,K_2 + + 1.20553560\,K_3 + 0.17118478\,K_4 \qquad (2\text{-}92)$$

$$\left(\frac{\partial\beta}{\partial t}\right)_{m+1} = \left(\frac{\partial\beta}{\partial t}\right)_m + 0.17476028\,L_1 - 0.55148066\,L_2 + + 1.20553560\,L_3 + 0.17118478\,L_4 \qquad (2\text{-}93)$$

where

$$K_1 = (\Delta t)\left(\frac{\partial\beta}{\partial t}\right)_m \qquad (2\text{-}94)$$

$$L_1 = -(\Delta t)\left[p_m\left(\frac{\partial\beta}{\partial t}\right)_m + q_m\beta_m\right] \qquad (2\text{-}95)$$

$$K_2 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + 0.4L_1\right] \qquad (2\text{-}96)$$

$$L_2 = -(\Delta t)\left[p_2\left(\left(\frac{\partial\beta}{\partial t}\right)_m + 0.4L_1\right) + q_2(\beta_m + 0.4K_1)\right] \qquad (2\text{-}97)$$

$$K_3 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + 0.29697761\,L_1 + 0.15875964\,L_2\right] \qquad (2\text{-}98)$$

$$L_3 = -(\Delta t)\left[p_3\left(\left(\frac{\partial\beta}{\partial t}\right)_m + 0.29697761\,L_1 + 0.15875964\,L_2\right) + + q_3(\beta_m + 0.29697761\,K_1 + 0.15875964\,K_2)\right] \qquad (2\text{-}99)$$

$$K_4 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + 0.21810040\,L_1 - 3.05096516\,L_2 + 3.83286476\,L_3\right] \qquad (2\text{-}100)$$

$$L_4 = -(\Delta t)\left[p_{m+1}\left(\left(\frac{\partial\beta}{\partial t}\right)_m + 0.21810040\,L_1 - 3.05096516\,L_2 + + 3.83286476\,L_3\right) + q_{m+1}(\beta_m + 0.21810040\,K_1 - - 3.05096516\,K_2 + 3.83286476\,K_3)\right] \qquad (2\text{-}101)$$

A disadvantage of the Runge-Kutta method is that there is no simple means for estimating the truncation error (Milne, 1953). One procedure for controlling the error is to compute both fourth and fifth order solutions of β and to adjust the time step so that the two estimates differ by less than an arbitrary amount.

A fifth order Runge-Kutta method is given by Milne (1953). The fifth order solutions for β and dβ/dt are

$$\beta_{m+1}^{(5)} = \beta_m + \frac{1}{192}\left(23K_1 + 125K_6 - 81K_8 + 125K_9\right) \tag{2-102}$$

$$\left(\frac{\partial\beta}{\partial t}\right)_{m+1}^{(5)} = \left(\frac{\partial\beta}{\partial t}\right)_m + \frac{1}{192}\left(23L_1 + 125L_6 - 81L_8 + 125L_9\right) \tag{2-103}$$

where

$$K_5 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{L_1}{3}\right] \tag{2-104}$$

$$L_5 = -(\Delta t)\left[p_1\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{L_1}{3}\right) + q_1\left(\beta_m + \frac{K_1}{3}\right)\right] \tag{2-105}$$

$$K_6 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{6L_5 + 4L_1}{25}\right] \tag{2-106}$$

$$L_6 = -(\Delta t)\left[p_2\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{6L_5 + 4L_1}{25}\right) + q_2\left(\beta_m + \frac{6K_5 + 4K_1}{25}\right)\right] \tag{2-107}$$

$$K_7 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4}\right] \tag{2-108}$$

$$L_7 = -(\Delta t)\left[p_{m+1}\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{15L_6 - 12L_5 + L_1}{4}\right) + \right. \\ \left. + q_{m+1}\left(\beta_m + \frac{15K_6 - 12K_5 + K_1}{4}\right)\right] \tag{2-109}$$

$$K_8 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81}\right] \tag{2-110}$$

$$L_8 = -(\Delta t)\left[p_4\left(\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 - 50L_6 + 90L_5 + 6L_1}{81}\right) + \right. \\ \left. + q_4\left(\beta_m + \frac{8K_7 - 50K_6 + 90K_5 + 6K_1}{81}\right)\right] \tag{2-111}$$

$$K_9 = (\Delta t)\left[\left(\frac{\partial\beta}{\partial t}\right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75}\right] \tag{2-112}$$

$$L_9 = -(\Delta t) \left[p_5 \left(\left(\frac{\partial \beta}{\partial t} \right)_m + \frac{8L_7 + 10L_6 + 36L_5 + 6L_1}{75} \right) + \right.$$

$$\left. + q_5 \left(\beta_m + \frac{8K_7 + 10K_6 + 36K_5 + 6K_1}{75} \right) \right] \qquad (2\text{-}113)$$

The difference between the fourth and fifth order solutions of β and dβ/dt are

$$\varepsilon_\beta = \beta_{m+1} - \beta_{m+1}^{(5)} \qquad (2\text{-}114)$$

$$\varepsilon_{\beta t} = \left(\frac{\partial \beta}{\partial t} \right)_{m+1} - \left(\frac{\partial \beta}{\partial t} \right)_{m+1}^{(5)} \qquad (2\text{-}115)$$

In the calculations both $|\varepsilon_\beta|$ and $|\varepsilon_{\beta t}|$ are monitored. If either is greater than or equal to an arbitrary constant (determined as an input parameter) the time step is halved, the corresponding (n+1)-th ray point is found, and the β and dβ/dt calculations are repeated. This process continues, as necessary, until both $|\varepsilon_\beta|$ and $|\varepsilon_{\beta t}|$ are less than the arbitrary constant. If the time step is reduced to less than 0.5 seconds the ray is stopped.

d. Reflection points

The numerical solutions of the ray separation equation have not produced satisfactory results near reflection points. This is possibly due to the rapid change in p which approaches infinity as the reflection point is approached. The problem occurs when the wavelet direction is within 15° of being parallel to the wave speed contours. Accordingly, for this narrow region, Equations (2-88) and (2-89) for parallel water depth contours are used to evaluate β and dβ/dt. The equation for β is not only well behaved but approaches a constant value at the reflection point.

e. Caustics and focal points

The value of β is monitored along a ray. If the value becomes zero or negative a focal point or caustic is located. In this case the ray is stopped.

2.6 Friction Coefficient. Energy dissipation of the waves due to bottom friction is considered. The friction coefficient is determined using a method based on the theory of Putnam and Johnson (1949) and Bretschneider and Reid (1954). Other energy dissipation methods can be substituted if desired.

In this work the friction factor c_f is defined following Jonsson (1966)

$$\tau = \frac{1}{2} c_f \rho_f u_m^2 \tag{2-116}$$

where τ is the tangential stress per unit area at the bottom, ρ_f is the density of the fluid, and u_m is the maximum velocity of the fluid at the bottom. The definition for τ given by Putnam and Johnson (1949) does not contain the factor ½. When Equation (2-116) is used the friction coefficient becomes

$$K_F = \frac{(K_F)_m}{F (K_F)_m (\Delta s_G)_{n+1} + 1} \tag{2-117}$$

where $(K_f)_m$ is defined by Equation (2-80), $(\Delta s_G)_{n+1}$ is the incremental distance between the ray points n and (n+1), and

$$F = \left(\frac{8\pi^2}{3g}\right)\left(\frac{c_f H_0}{U_0}\right)\left(\frac{K_S}{T \sinh kh}\right)^3 \tag{2-118}$$

2.7 Wave Breaking Criterion. In the program there is an option to determine if the waves break. When this option is chosen, the waves are assumed to break when the following relation is satisfied.

$$\frac{H}{\lambda} > \frac{1}{7} \tanh kh \tag{2-119}$$

CHAPTER III THE COMPUTER PROGRAM

3.1 Description of the Computer Program. The computer program is written in Fortran IV for the Control Data Cyber 70 computer systems and plotters which are compatible with the Calcomp plotting systems. With the exception of the Calcomp subroutines, a description of each program subroutine is presented. The reader is referred to a Calcomp reference manual for descriptions of the Plot, Symbol, and Number subroutines (California Computer Products, Inc., 2411 West La Palma, Anaheim, CA. 92801). With the exception of the plotting subroutines, each one is written so that it is possible to follow the subroutine listing. It will be helpful to make use of the definitions of the symbols presented with the Notation at the end of this work.

Including the 100 by 100 array for the water depth grid the program requires 30976 words (base 10). In order to reduce the size of the computer program card deck and to shorten the program listing, several program statements have frequently been combined on one computer card. This is done by separating the statements by a blank, a dollar sign, and another blank.

There is often a need to prepare forecasts for a number of different water depth grids. Further, it is frequently desirable to make a number of forecasts for the same water depth grid. To make it easier to handle the input data for these situations, the water depth grids (XYGRID) are all stored on one input file. Each grid has its own name, e.g., XYGOM1. The rest of the input parameters (RAYDAT) are stored on another input file, and the data for each run has a name, e.g., RAYDAT1. When using the program an appropriate set of control cards is used to access the input data. If this feature is not desired, the READ statements in MAIN should be changed so that the same input file is used for all the input data.

When checks of the program are made or if there are modifications to the program, it should be noted that English units are used internally in the program for the calculations. In addition, the input and output wave packet and wavelet directions are defined as the directions from which the waves come with respect to true north. Before making calculations these angles are transformed using the following relationships

$$\theta_C = \text{CNVRSA} - \theta_N + 180 \qquad (3\text{-}1)$$

$$\gamma_C = \text{CNVRSA} - \gamma_N + 180 \qquad (3\text{-}2)$$

where the subscript C refers to the calculation coordinate system, the subscript N denotes the true north coordinate system, and CNVRSA is the direction of the positive x-axis of the water depth grid with respect to true north. The angles are in degrees.

3.2 PROGRAM MAIN. The MAIN PROGRAM controls the input, plots, and calculations for all the rays. To begin with, the values of MMAX, LI, and CORI are assigned values, and LII is calculated. Two statements are used to read numbers used in the surface fitting procedure in SURFCE. Then, descriptive information is read which is used for all plots. Next, two read statements are used to obtain input parameters for a specific plot. If MOE ≠ 0 there is a conversion from Metric to English units. The value of CIN is changed from seconds to hours, and the values of AMM, ANN, DY, and SCLI are determined. Next TITLE is called.

If NXCMAT = 0 the water depths are read and stored in CMAT. If NXCMAT ≠ 0 no water depths are read, and the depth grid used in the previous plot is used again. If NCO ≠ 0 sounding depths for the plot are read and NUMCON is called. If NCO = 0 no sounding depths are read; in this case there must be no sounding depth card. SHORE is called if NSH ≠ 0.

Next, the input parameters for a given ray are read. If MOE ≠ 0 there is a conversion from Metric to English units. The values of SDLTAT and WL are defined, and the computational values of the wave packet and wavelet directions are determined. MAXQ is initialized to one and FUD, BRK, REFLCT, RFLBUM, REFRCT, RFRBUM, FLAGR, FLAG3, IFLG, and α are initialized to zero. If COL ≠ 0 the plotter will pause before a ray is plotted. The values of A and AV are changed from degrees to radians. Then RAYN is called. After all rays for a given plot are determined the comment: THIS IS THE END is written on the output.

3.3 Subroutine TITLE. TITLE is called by MAIN to draw labels and straight line borders on a plot. The labels consist of PROJCT, DATE1, DATE2, SCL, CIN, NPLOT, and DIR. If NAX ≠ 0, AXIS2 is called.

3.4 Subroutine AXIS2. AXIS2 is called by TITLE to prepare xy-axes for the plot. The axes are calibrated with tick marks, and the origin and every fifth tick mark are numbered. Finally, each axis is labeled.

3.5 Subroutine NUMCON. If NCO > 0, NUMCON is called by MAIN to locate NCO sounding water depths on a plot. The sounding depths are stored in the array CONTUR. If MOE ≠ 0 the sounding depths are converted from Metric to English values before the calculations are made. The search for the sounding depths begins one grid unit from the end of a column starting with the second y-column. The column is searched separately for each depth. If necessary, the sounding depths are located by linear interpolation.

After all the sounding depths for a given column are found, they are drawn at their respective locations on the plot. In the event MOE ≠ 0 the English values are converted back to Metric values before being drawn on the plot.

This process is repeated for additional y-columns where the next column is determined by adding NNSKIP to the number of the previous y-column. The process stops when the y-column selected is greater than (NN-1). A restriction on the use of this subroutine is that $(\partial h/\partial x)|_{h=0} \geq 0$ for the entire depth grid.

3.6 Subroutine SHORE. If NSH ≠ 0, SHORE is called by MAIN to draw the shoreline on the plot. Beginning with the first y-column in CMAT, each column is searched for the location of the zero water depth. The search in each column begins with the maximum value of x, and if necessary, the point of zero water depth is found by linear interpolation. The shoreline is the line drawn connecting these points. To use this subroutine it is necessary that $(\partial h/\partial x)|_{h=0} \geq 0$.

3.7 Subroutine RAYN. This subroutine is called by MAIN to control the calculation of the wave packet trajectory and the wave particulars along the trajectory. Initially, NDP, NFK, NGO, and FLAG1 are set equal to one. Also, KREST, KCIN, and RCOUNT are put equal to zero. SURFCE is called to calculate ray particulars for the first ray point. FLAG1 and INUM are set equal to zero. The saved values of α, γ, h, v, u, and G are initialized. MOVE is called to calculate the initial value of D, and HEIGHT is called to determine the initial wave height. The travel time is initialized to zero, and the initial wave packet and wavelet directions are converted to degrees and to values measured with respect to true north for later printout. Then the value of NPT is checked to determine how much printout is desired.

If NPT = 0 printout occurs at the first and last ray points. When NPT ≠ 0, PCD is called to calculate PCTDIF and printout occurs for selected ray points depending upon the value of SK. The ray parameters which appear in the output depend upon the value of NPT. The procedure employed to obtain the output when NPT ≠ 0 is presented below. However, there is little difference in the routine used to obtain output when NPT = 0.

Printout occurs for the first ray point or if the number of the ray point is an integral multiple of SK. Then α is changed to degrees, and if MOE ≠ 0 the English values of the ray parameters are changed to Metric values. If the ratio of FUD to LI has no remainder, page and column headings are written. First, the page heading is written. Next, if the ray is at the first point the initial value of $d\beta/dt$ is written. In addition, if MOE = 0 the printout contains: THE OUTPUT IS IN ENGLISH UNITS. H, HGT(FEET). G, U, V(FEET/SECOND). If instead MOE ≠ 0 the printout contains: THE

OUTPUT IS IN METRIC UNITS. H, HGT(METER). G, U, V(METER/SECOND). Then the column headings are written.

FUD is increased by one and the ray particulars are written. One of three formats is used. If RFLBUM ≠ 0 (determined in HEIGHT) the format for a reflection breakup of the time step interval is chosen. Further, RFLBUM is set equal to zero. If RFLBUM = 0 and RFRBUM ≠ 0 (determined in HEIGHT) the format for a refraction breakup is used and RFRBUM is set equal to zero. The remaining format is used if there has not been a breakup of the time step interval.

The value of α is changed back to radians. If MOE ≠ 0 the ray particulars which were changed to Metric values are converted back to English values for use in the calculations. The values of the ray point number, the coordinates of the ray point, and the number of breakup intervals are saved. Then STORE is called.

After returning from STORE, if MIT = 2 (determined in MOVE), if NPT ≠ 0, and if the ratio of FUD to LI has no remainder, then page and column headings are written. FUD is increased by one, and the printed output contains the statement: PACKET CURVATURE AVERAGED. If after returning from STORE, NPT = 0 or MIT = 1 the previous write statements are omitted.

When at the first ray point the value of NDP is checked. If NDP = 2 (determined in SURFCE) the printed output is: RAY REACHED SHORE. The ray is stopped and the program returns to MAIN. If NDP = 1 the number of the ray point is increased by one, and the dimension of the AX and AY arrays is checked (described below). For points beyond the first the value of NGO is checked. If NGO = 1 the ray point number is increased by one, and the size of the AX and AY arrays is checked. If NGO ≠ 1 the ray is stopped, the printed output contains: RAY REACHED GRID BOUNDARY, and the program returns to MAIN.

Before the next ray point is calculated a check is made to determine if there is additional storage space in the AX and AY arrays. If the sum of the number of ray points and the number of tick marks (if any) exceeds MMAX the ray is stopped. The statement in the printed output is: DIMENSION OF OUTPUT-ARRAYS EXCEEDED. The ray particulars for the last point are written if they have not been previously written. Page and column headings appear when appropriate, and the format used is determined as explained above for the output of other ray points. The program returns to MAIN.

If the dimension of the output arrays is not exceeded, the value of G is saved and MOVE is called to find the next ray point. After the return to RAYN the ray is stopped if NDP ≠ 1 or MIT = 3, 4, 5, 6, 7, or 8. The ray particulars for the last ray point are written unless they have already appeared in the output. In addition, one of the following descriptive printouts occurs. If NDP ≠ 1 (determined in SURFCE) the printout is: RAY REACHED SHORE. When MIT = 3 (determined in MOVE) the printout is: PACKET CURVATURE

ITERATION NOT CONVERGING. If MIT = 4 (determined in HEIGHT) the statement is: CAUSTIC OR FOCAL POINT. When MIT = 5 (determined in HEIGHT) the printout contains: WAVE BREAKS. When MIT = 6 (determined in MOVE), which can occur when ROP = 0, there is no descriptive printout. If MIT = 7 (determined in MOVE) the printout is: REFLECTION HANG-UP. Finally, if MIT = 8 (determined in HEIGHT) the printout is: BREAKUP TIME STEP LESS THAN 0.5 SECOND.

After the return from MOVE to RAYN, the ray continues if NDP = 1 and MIT = 1 or 2 (determined in MOVE). The travel time is computed. The values of α, κ_G, h, D, G, v, U, H, K_S, K_F, and K_R are saved in case printout is required at the last ray point. The wave packet and wavelet angles are converted to degrees and are defined with respect to true north. Then, as described above, PCD is called if desired, the ray particulars are written if required, and MOVE is called if appropriate to find the next ray point.

3.8 Subroutine MOVE. MOVE is called by RAYN to determine the path of the wave packet. NUMT and MIT are initialized to one. The values of the geometric group speed and ray curvature are saved for use if the time step interval is divided into smaller intervals. At the second ray point the value of κ_G is saved; at other points the values of the ray curvature and the average ray curvature are saved.

The value of the incremental distance to the next ray point is computed. If at the first ray point the program returns to MOVE. At the second ray point the average ray curvature is set equal to the ray curvature obtained at the first point. For points beyond the second this latter step is ignored.

A check is made to determine if there is a breakup of the time step interval due to the calculation of the ray separation factor. If so, REFRCT was set equal to one in HEIGHT. Further, a check is made to determine if there is or should be a breakup of the time step interval due to a reflection point. If REFLCT ≠ 0 a reflection breakup has occurred; if $|\tan\gamma'| > \tan 75^\circ$ and $|\tan\theta'| < \tan 75^\circ$ a reflection breakup should occur. Accordingly, if REFRCT ≠ 0 or REFLCT ≠ 0 or $|\tan\gamma'| \leq \tan 75^\circ$ or $|\tan\theta'| \geq \tan 75^\circ$ the iteration for the next point begins. Otherwise, REFLCT and RFLBUM are set equal to one before the iteration for the next point begins. The value of RFLBUM determines the format for the printed output of the ray particulars.

A maximum of fifty iterations can occur in locating a new point. On the first iteration the average ray direction to and the position of the next point are estimated using the ray curvature of the present point. Beyond the first iteration the average of the ray curvatures at the present point and the approximated next point is used to obtain a new estimate of the ray point. With each iteration SURFCE is called to calculate the ray curvature and other ray particulars at the estimated position of the next point. After the return to MOVE, the wavelet direction γ' is computed.

If FLAG2 $\neq$ 0 (determined in SURFCE) a reflection occurs due to Snell's law with phase velocity. In this case, REF = 1 and there is a reflection (described below). If FLAG2 = 0, DUD is calculated. If MIT > 2 the program returns to RAYN. If MIT = 2 (described below) a check is made to determine if the ray is too close to a reflection point (described below). If MIT = 1 the value of NDP is examined. If NDP = 2 (determined in SURFCE), $h \leq 0$ so the program returns to RAYN. If NDP $\neq$ 2 the ray curvature at the latest estimate of the new point is averaged with the value at the present point; the average is used in the next approximation of the new point. The average ray curvature is saved on the 48th and 49th iterations.

The check for the convergence of the ray curvature depends upon both the number of the ray point and the number of the iteration. For the first or second ray point and on the first iteration the average ray curvature is saved and the second iteration begins. Beyond the first iteration successive curvature averages are checked, and if they differ by less than 0.00009/D convergence has occurred. The new point is checked to see if it is too close to a reflection point. If convergence has not occurred the iteration continues.

Beyond the second ray point and on the first iteration the ray curvature average for the estimated new point is compared with the ray curvature average of the present point. If the values differ by less than 0.00009/D convergence has occurred. On successive iterations curvature averages are checked to see if they differ by less than 0.00009/D. When convergence occurs the check is made to determine if the new point is too close to a reflection point.

If convergence has not occurred after fifty iterations the ray curvatures on the 48th and 50th iterations are compared to see if they differ by less than 0.00009/D. If so, the ray curvature is assumed to have converged to two values. This would happen if estimates of the new ray point alternate between two grid cells. For this situation MIT = 2, and the average of the ray curvature averages for the 49th and 50th iterations is determined and used to locate the next ray point. The point is checked to see if it is too close to a reflection point.

If after fifty iterations convergence is not achieved a check is made to find if the convergence failed because of a reflection point. This is possible since the ray curvature becomes infinite at a reflection point. Reflection is assumed if DUD > 1, i.e., the phase speed increases between the last two ray points, and if $|\tan \gamma'| > \tan 80^\circ$. Then REF = 2 and the reflection begins (described below). If the conditions for reflection are not met MIT = 3, and the program returns to RAYN.

It is desirable to get close to a reflection point in order to accurately define the ray path. But problems occur if a ray gets too close to a reflection point due to the rapidly increasing ray curvature. This can cause the convergence procedure to fail with a resulting reflection as discussed above. However, very near a reflection point the estimates of ray points become erratic even if the iterations of the ray curvature converge. Accordingly, a reflection is assumed if DUD > 1, $|\tan \gamma'| > \tan 89.5^{\circ}$, and $|\tan \theta'| < \tan 75^{\circ}$. Then REF = 3 and the reflection begins. Otherwise, there is no reflection and a check is made to determine if the new point lies too close to a grid boundary (described below).

A reflection begins with a check of NPT. If NPT ≠ 0 the ratio of FUD to LI is checked. If the ratio has no remainder page and column headings are written. The printout values of θ, γ, α, and the ray number are calculated. If MOE = 0 the ray particulars at the reflection point are written. If MOE ≠ 0 the English values of the ray particulars are converted to metric values before being written. FUD is increased by one and the value of α is converted back to radians. If MOE ≠ 0 the metric values of the ray particulars are converted back to English values. Page and column headings are written if the remainder of the ratio of FUD to LI is zero. Then the type of reflection is written. If REF = 1 the printed output contains: REFLECTION: SNELLS LAW WITH PHASE VELOCITY. When REF = 2 the output is: REFLECTION: PACKET CURVATURE ITERATION NOT CONVERGING. Finally, if REF = 3 the output is: REFLECTION: NEAR REFLECTION POINT. After the write statement FUD is increased by one. The previous write statements are omitted if NPT = 0.

If ROP = 0 the reflection procedure is stopped, MIT = 6, and the program returns to RAYN. If ROP ≠ 0 the ray continues beyond the reflection point. After setting FLAG2 = 0 the reflection angles are calculated and the wavelet direction is saved. RCOUNT is increased by one, and if then RCOUNT ≥ 2 there has been more than one reflection at the same point. In this case MIT = 7 and the program returns to RAYN. If RCOUNT < 2, FLAG1 is set equal to one and SURFCE is called to calculate the ray particulars based on the reflection angles. Then FLAG1 = 0, FLAGR = 1, and the values of the ray separation coefficents are saved. This is followed by iterating to the point after reflection using the procedure discussed above.

With the exception of reflection points and if a ray has not been stopped, each new ray point is checked to determine if the point lies within 1½ grid units of a boundary of the water depth grid. If it does NGO = 2. Otherwise, NGO remains equal to one.

A number of quantities are saved in case a breakup of the time step interval occurs in HEIGHT. The quantitites saved are the coordinates of the previous point, the coordinates of the new point, the previous rotation angle, the present rotation angle, the wave packet direction, the previous wavelet direction, the new wavelet direction, the two previous values of phase speed, and new values of h, v, U, G, and D. Average values of the wave packet and wavelet directions are calculated for use if the values of p and q are determined at intermediate ray points. The values of the wave packet direction, wavelet direction, and ray curvature are updated, and RCOUNT is initialized to zero.

If REFLCT = 1 the values of x and y are updated and HEIGHT is called. If REFLCT ≠ 1 the average values of the packet and wavelet directions are used to determine the values of p and q at the five intermediate points needed for the Runge-Kutta method. In order to keep the wavelet direction from changing on the calls to SURFCE, FLAG1 is set equal to one. After all the values of p and q are computed at the intermediate points SURFCE is called again to reevaluate quantities at the new ray point. Then FLAG1 is set equal to zero, the values of x and y are updated, and HEIGHT is called.

Upon the return to MOVE from HEIGHT, if BRK = 1 there has been a breakup of the time step interval in HEIGHT. Iteration begins for a new point with the new time step interval. If BRK ≠ 1 the printout values of both the wave packet and wavelet directions are placed, if necessary, in the range of 0 to 360°. Then, the program returns to RAYN.

3.9 Subroutine HEIGHT. This subroutine is called to calculate the wave height. For the first ray point HEIGHT is called by RAYN. The values of p and q are initialized, and the tolerance for the fourth and fifth order calculations of β and $d\beta/dt$ is calculated. Further, the initial value of $d\beta/dt$ is computed, and the initial values of the friction, refraction, and shoaling coefficients, and the wave height are determined. The program returns to RAYN.

For ray points beyond the first point this subroutine is called by MOVE. The shoaling and friction coefficients are computed. The values of K_r, p, β, and $d\beta/dt$ are saved in case there is a breakup of the time step interval.

The method used in determining β and $d\beta/dt$ depends upon the values of NFK (determined in SURFCE), REFLCT (determined in MOVE), and $|dh/dx'|$ (determined in SURFCE). If NFK = 1 the values of $d\beta/dt$, the difference between the fourth and fifth order solutions of β, and the difference between the fourth and fifth order solutions of $d\beta/dt$ are equated to zero. That is, β is assumed to be constant since the ray is in deep water. The value of β is used to calculate the refraction coefficient.

If NFK = 2 and REFLCT ≠ 0 the ray is near a reflection point, and β and $d\beta/dt$ are calculated using the analytical solution for parallel wave speed contours. If FLAGR ≠ 0 (determined in MOVE), ROP is put equal to zero; as a result, a ray is not continued beyond a second reflection point should one occur. The refraction coefficient is computed.

If NFK = 2 and REFLCT = 0 the method used to determine

β and $d\beta/dt$ depends upon the value of $|dh/dx'|$. If $|dh/dx'| \leq 0.00001$, then $d\beta/dt$, the difference between the fourth and fifth order solutions of β, and the difference between the fourth and fifth order solutions of $d\beta/dt$ are equated to zero. That is, β is assumed to be constant since the water depth is taken as invariant. If $|dh/dx'| > 0.00001$ the Runge-Kutta method is used to calculate β and $d\beta/dt$. The fourth order and fifth order solutions as well as the difference of these solutions are determined for both β and $d\beta/dt$. However, before making the calculations, if FLAGR ≠ 0 the value of ROP is set equal to zero. The refraction coefficient is calculated using the value of β.

After the refraction coefficient is computed several checks are made to determine if the calculations are sufficiently accurate with the time step which was used. Near a reflection point the ray curvature is large; it is necessary to reduce the time step in order to keep successive changes in the wave packet direction small enough to determine an accurate ray path. When computing the ray separation factor using the Runge-Kutta method it is desirable to be able to reduce the time step, if necessary, to keep the truncation error small. Only one of these checks is made at a time. The check to be made is determined by the value of REFLCT. However, regardless of the value of REFLCT, if IFLG ≠ 0 the value of the breakup time step has previously been determined. In this case the value of INUM is increased by one. The new value of INUM is compared with NUMT to determine if the next ray point should be computed with the breakup time step (described below).

If REFLCT ≠ 0 and IFLG = 0 the change in the absolute value of the ray direction is checked. If the change is less than 1° the time step is not too large and the value of NUMT is checked (explained below). If the change is greater than or equal to 1° there is a breakup of the time step interval (described below).

If REFLCT = 0 and IFLG = 0 the difference between the Runge-Kutta fourth and fifth order solutions of β and the similar difference of the solutions for $d\beta/dt$ are checked. If the absolute values of both differences are less than BZTOL the time step interval is not divided and NUMT is checked. But, if the absolute value of either difference is greater than or equal to BZTOL there is a breakup of the time step interval and REFRCT = 1. Further, RFRBUM = 1 to determine the format of the printed output of the ray particulars.

If the calculations meet the criteria for accuracy using the assigned time step the value of NUMT is checked. If NUMT > 1 the initial time step interval has been broken. In this case IFLG = 1. The value of INUM is increased by one, and the new value of INUM is compared with NUMT to determine if the calculations should continue with the breakup time step (described below). When IFLG = 1 further checks for a breakup of the time step interval are not made at new ray points until the breakup ends and calculations are resumed with the initial time step.

When NUMT $\le$ 1 the time step has its initial value. If $\beta \le 0$ there is a focal point or caustic. Then MIT = 4, BRK = 0, and the program returns to MOVE. When $\beta > 0$ the values of p and q at the present point are set equal to the values at the new point. The wave height is computed. If WBCOP = 0 the program returns to MOVE. If WBCOP $\neq$ 0 a test is made to determine if the wave breaks. Then the program returns to MOVE, and if the wave breaks MIT = 5.

The time step interval is halved for a breakup. It is possible for a time step to be halved many times with successive breakups. Thus, it is necessary to place a lower limit on the value of a time step to prevent an inordinate amount of calculations. The new value of the time step is checked, and if it is less than 0.5 sec the ray is stopped. Then MIT = 8, BRK = 0, and the program returns to MOVE.

If the new time step is greater than 0.5 sec the breakup continues. It is necessary to return to the previous ray point. Accordingly, the saved values of G and D are recovered. To determine a new point in MOVE, BRK = 1. The number of intervals, NUMT, the initial time step is divided into is calculated. If at the second ray point the value of the ray curvature is recovered. At other points both the ray curvature and average ray curvature are restored. Further, the values of θ, γ, α, x, y, β, $d\beta/dt$, p, K_F, and the two previous values of v are recovered. The program returns to MOVE.

There are as many ray points in a breakup as required for the travel time to equal the initial time step. Thus, during a breakup, after each new point is determined there is a check to see if the breakup is complete. If INUM < NUMT the breakup is incomplete and the ray is continued with the breakup time step. The value of D is computed and the values of p and q are updated. If there is a focal point or caustic MIT = 4, BRK = 0, and the program returns to MOVE. Otherwise, BRK = 1 and the program returns to MOVE.

When INUM $\ge$ NUMT the breakup ends and calculations resume with the initial time step. The values of IFLG, INUM, and BRK are set equal to zero, and D is computed. There is a check for a focal point or caustic, p and q are updated, and REFRCT and REFLCT are set equal to zero. The wave height is calculated, if WBCOP $\neq$ 0 there is a check to see if the wave breaks, and the program returns to MOVE, as explained above.

3.10 Subroutine SURFCE. SURFCE is called by RAYN and MOVE to calculate h, α, γ, G, p, q, κ_G, and other ray particulars. At the first ray point twelve values of h from CMAT are selected about the point as shown in Figure (2-2). A quadratic surface is fit to the set of water depths. At successive ray points the quadratic surface is determined only if there is a change in the set of twelve water depths. The water depth and its partial derivatives in the fixed xy-system, $\partial h/\partial x$, $\partial h/\partial y$, $\partial^2 h/\partial x^2$, $\partial^2 h/\partial y^2$, and $\partial^2 h/\partial x \partial y$, are determined at the ray point by interpolating on the quadratic surface.

If $h \leq 0$, NDP = 2 and there is a RETURN. If $h > 0$ the ratio of the water depth to the deep water wavelength is computed. If $h/\lambda_d > 0.64$, which defines deep water, NFK = 1. If $h/\lambda_d \leq 0.64$, NFK = 2. VELCTY is called, and after the return if NFK = 1, W = 0. If NFK = 2, CONDER is called to compute W. The values of $\partial v/\partial x$ and $\partial v/\partial y$ are calculated using W.

At each ray point the water depth contours are assumed to be locally parallel, and a x'y'-coordinate system is chosen such that the y'-derivatives vanish. The value of dh/dx' is computed, and if it exceeds 0.00001 the angle α by which the x'-axis is rotated with respect to the x-axis is computed. If $|dh/dx'| \leq 0.00001$ the water depth is assumed to be constant and α remains constant.

If FLAG1 = 0, γ' is computed, and if necessary it is placed within the range $|\gamma'| \leq 360^\circ$. A check is made to determine if there is total reflection. If there is, FLAG2 = 1 and there is a RETURN. Otherwise, FLAG2 = 0 and the new γ' is computed using Snell's law with phase velocity following a set of rules. Using the values of γ', γ is computed. When FLAG1 $\neq$ 0 these steps for computing the new wavelet direction and the test for total reflection are omitted.

The values of ϕ, G, and dv/dx' are calculated. If NFK = 2, dU/dx' is determined using its unsimplified expression. If NFK $\neq$ 2, the deep water formula is used to calculate dU/dx'. The value of dU/dx' is used in computing dG/dx'.

If NFK $\neq$ 2, the coefficients of the ray separation equation and the ray curvature are set equal to zero. Then there is a RETURN. If NFK = 2, p, $d^2h/(dx')^2$, $d^2v/(dx')^2$, $d^2U/(dx')^2$, $d^2G/(dx')^2$, q, and κ_G are computed. This is followed by a RETURN.

3.11 Subroutine VELCTY. VELCTY is called by SURFCE to calculate v and U. At the first ray point the deep water value of the phase speed and several constants for the ray are computed. If NFK $\neq$ 2 (determined in SURFCE) the deep water value of the phase speed is used, and its value is saved for calculations at the next point. The deep water value of the collinear group speed is calculated, and the program returns to SURFCE.

If NFK = 2 an iterative technique is used to determine v. The iteration continues until the estimate of v differs by less than 0.00005 from an average of the previous estimates of v or until ninety iterations have taken place. The value of v is saved for calculations at the next point, and U is computed using its complete expression. Then the program returns to SURFCE.

3.12 Subroutine CONDER. CONDER is called by SURFCE to compute W. After computing some constants, if NFK = 1 (determined in SURFCE) the program returns to SURFCE. If NFK ≠ 1 the value of W is calculated, then the program returns to SURFCE.

3.13 Subroutine PCD. PCD is called by RAYN. The four water depths in CMAT which are closest to a ray point are compared with the respective depths computed from the twelve point surface fit. The percentage difference of the interpolated water depth to the actual water depth is determined for each grid point, and PCTDIF is the maximum percentage difference of the four values. If the product of the four water depth values in CMAT is zero, PCTDIF = 999.

3.14 Subroutine STORE. STORE is called by RAYN. If CIN ≤ 0 the x and y coordinates of a ray point are stored in the Ax and Ay arrays, respectively. Then, the program returns to RAYN. If CIN > 0 tick marks at travel time intervals equal to CIN are determined along the ray. The travel time is computed using the geometric group speed of the ray. The coordinates of the tick marks are tagged with negative x-values and are stored in sequence with the ray points in the Ax and Ay arrays. The program returns to RAYN.

3.15 Subroutine DRAW. DRAW is called by RAYN to plot each ray. To save plotting time, odd numbered rays begin at their initial points and even numbered rays start at their terminal points. If FAN = 0 a ray is numbered at its initial point, otherwise a ray is numbered at its terminal point. If CIN ≤ 0 a ray has no tick marks. If CIN > 0 tick marks are placed on a ray for those positions where the x coordinate is stored in the Ax array with a negative value. The negative values are changed to positive values.

The coordinates for plotting the tick marks depend upon the positions of the tick mark on the ray and the first ray point that is prior to and located more than a specified distance from the tick mark. The separation requirement is necessary since if the two points are too close together numerical inaccuracies in the calculations prevent the tick mark from being perpendicular to the ray. Every tenth tick mark is larger than the others. The program returns to RAYN.

3.16 Listing of the Computer Program.

```
      PROGRAM MAIN (INPUT, OUTPUT, RAYDAT, XYGRID, PLOTT, TAPE5=INPUT   MAIN
     $,TAPE6=OUTPUT, TAPE1=RAYDAT, TAPE2=XYGRID)                        MAIN
C THIS IS A PROGRAM FOR CALCULATING AND PLOTTING THE PATHS OF SURFACE   MAIN
C GRAVITY WATER WAVE PACKETS AND FOR CALCULATING THE WAVE HEIGHTS       MAIN
C ALONG THESE PATHS CONSIDERING THE EFFECTS OF SHOALING, REFRACTION,    MAIN
C AND ENERGY DISSIPATION.                                               MAIN
C                                                                       MAIN
C THIS PROGRAM WAS COMPLETED NOVEMBER 1977 UNDER A CONTRACT WITH THE    MAIN
C GEOGRAPHY PROGRAMS, EARTH SCIENCES DIVISION, THE OFFICE OF NAVAL      MAIN
C RESEARCH.  THE PROGRAM WAS PREPARED BY                                MAIN
C    J. ERNEST BREEDING, JR., DEPT. OF OCEANOGRAPHY, FLORIDA STATE      MAIN
C      UNIVERSITY, TALLAHASSEE, FL.   32306                             MAIN
C    K. C. MATSON, NAVAL COASTAL SYSTEMS LAB, PANAMA CITY, FL.   32407  MAIN
C    NOUROLLAH RIAHI, DEPT. OF OCEANOGRAPHY, FLORIDA STATE UNIVERSITY   MAIN
C                                                                       MAIN
C THIS PROGRAM IS BASED ON A PROGRAM FOR COMPUTING THE PATHS OF         MAIN
C MONOCHROMATIC RAYS BY                                                 MAIN
C      W. STANLEY WILSON, A METHOD FOR CALCULATING AND PLOTTING SURFACE MAIN
C      WAVE RAYS.  TECHNICAL MEMORANDUM NO. 17, COASTAL ENGINEERING     MAIN
C      RESEARCH CENTER, 57 PP. (1966) (AD 636 771).                     MAIN
C WITH THE EXCEPTION OF THE PLOTTING SUBROUTINES, THE WILSON PROGRAM    MAIN
C WAS EXTENSIVELY MODIFIED IN ORDER TO COMPUTE THE PATH OF A WAVE       MAIN
C PACKET, AND A SUBROUTINE WAS ADDED FOR COMPUTING THE WAVE HEIGHT.     MAIN
C                                                                       MAIN
C THE PROGRAM IS WRITTEN IN FORTRAN IV FOR THE CONTROL DATA CYBER       MAIN
C 70 COMPUTER SYSTEMS AND THE GOULD PLOTTER.                             MAIN
C                                                                       MAIN
C INPUT PARAMETERS.                                                     MAIN
C    A,AV ARE, RESPECTIVELY, THE INITIAL DIRECTIONS FROM WHICH THE WAVE MAIN
C      PACKET AND WAVELETS COME WITH RESPECT TO TRUE NORTH.             MAIN
C    CF IS THE FRICTION FACTOR FOR THE FRICTION COEFFICIENT.            MAIN
C IF CIN IS NOT ZERO IT IS THE TRAVEL TIME IN SECONDS BETWEEN SUCCESSIVEMAIN
C      TICK MARKS ON A RAY.                                             MAIN
C    CMAT IS THE WATER DEPTH GRID.                                      MAIN
C    CNVRSA IS THE DIRECTION OF THE POSITIVE X-AXIS OF THE WATER DEPTH  MAIN
C      GRID WITH RESPECT TO TRUE NORTH.                                 MAIN
C IF COL (IS/IS NOT) ZERO THE PLOTTER (WILL NOT/WILL) PAUSE BEFORE      MAIN
C      A RAY IS PLOTTED.                                                MAIN
C    CONTUR SPECIFIES THE SOUNDING DEPTHS IN FEET OR METERS.            MAIN
C    DATE1, DATE2 DEFINE THE YEAR, MONTH, AND DAY.                      MAIN
C    DCON IS A FACTOR TO CONVERT THE WATER DEPTHS IN CMAT TO FEET       MAIN
C      OR METERS.                                                       MAIN
C    DELTAT IS THE TIME STEP IN SECONDS.                                MAIN
C    DIR IS AN IDENTIFIER.                                              MAIN
C    EM ARE SURFACE FITTING NUMBERS USED WITH CMAT.                     MAIN
C IF FAN (IS/IS NOT) ZERO A RAY IS NUMBERED AT ITS (INITIAL/TERMINAL)   MAIN
C      POINT.                                                           MAIN
C    GRID IS THE NUMBER OF FEET OR METERS PER GRID UNIT FOR A GIVEN RUN.MAIN
C    HGTZ IS THE INITIAL WAVE HEIGHT IN FEET OR METERS.                 MAIN
C    HT IS THE HEIGHT OF THE PLOT IN INCHES OR CENTIMETERS.             MAIN
C    KRTOL DETERMINES THE ACCURACY IN CALCULATING THE REFRACTION        MAIN
C      COEFFICIENT.                                                     MAIN
C    MM,NN ARE THE MAXIMUM X,Y FOR A GIVEN WATER DEPTH GRID.            MAIN
C    NNSKIP IS THE AMOUNT ADDED TO THE Y-COLUMN IN SELECTING THE NEXT   MAIN
C      COLUMN FOR LOCATING SOUNDING VALUES.                             MAIN
C IF MOE (IS/IS NOT) ZERO (ENGLISH/METRIC) UNITS ARE USED.              MAIN
C    MXPLOT IS THE NUMBER OF PLOTS OR COMPUTER RUNS.                    MAIN
```

```
C IF NAX (IS/IS NOT) ZERO THE AXES OF THE PLOT (WILL NOT/WILL) BE
C      CALIBRATED.
C IF NCO IS NOT ZERO IT SPECIFIES THE NUMBER OF SOUNDING VALUES FOR
C      A PLOT.
C     NOR IS THE NUMBER OF RAYS FOR A GIVEN RUN.
C IF NPT IS ZERO PRINTED OUTPUT OCCURS FOR THE INITIAL AND TERMINAL
C      POINTS OF A RAY, AND IF NPT IS NOT ZERO PRINTED OUTPUT OCCURS
C      ADDITIONALLY FOR THOSE POINTS WHICH ARE AN INTEGRAL MULTIPLE
C      OF SK.
C IF NSH (IS/IS NOT) ZERO THE SHORELINE (IS NOT/IS) DRAWN.
C IF NXCMAT IS ZERO A WATER DEPTH GRID IS INPUT FOR THE RUN, AND IF
C      NXCMAT IS NOT ZERO THE DEPTH GRID FOR THE PREVIOUS PLOT IS
C      USED AGAIN.
C     PROJCT IS AN IDENTIFIER.
C IF ROP (IS/IS NOT) ZERO THE RAY (IS NOT/IS) CONTINUED BEYOND A
C      REFLECTION POINT.
C     S ARE SURFACE FITTING NUMBERS USED WITH CMAT.
C     SK.  SEE NPT.
C     TT IS THE WAVELET PERIOD IN SECONDS.
C IF WBCOP (IS/IS NOT) ZERO THE WAVE BREAK TEST (IS NOT/IS) MADE.
C     X,Y ARE THE INITIAL RAY COORDINATES.
C
C OUTPUT OF THE RAY PARTICULARS.
C     ALFA IS THE ANGLE OF THE ROTATED XY-SYSTEM (WHERE THE Y-
C      DERIVATIVES VANISH) RELATIVE TO THE WATER DEPTH GRID XY-SYSTEM.
C     D IS THE DISTANCE IN GRID UNITS BETWEEN SUCCESSIVE RAY POINTS.
C     D(BETA)/DT IS THE INITIAL VALUE OF THE TIME DERIVATIVE OF THE
C      RAY SEPARATION FACTOR.
C     FK IS THE RAY CURVATURE OF THE PACKET IN RADIANS/GRID UNIT.
C     G = U COS (PACK - WAVE) IS THE GEOMETRIC GROUP SPEED IN
C      FEET/SECOND OR METERS/SECOND.
C     H IS THE WATER DEPTH IN FEET OR METERS.
C     HGT IS THE WAVE HEIGHT IN FEET OR METERS.
C     KF IS THE FRICTION COEFFICIENT.
C     KR IS THE REFRACTION COEFFICIENT.
C     KS IS THE SHOALING COEFFICIENT.
C     MAX IS AN INDEX TO NUMBER POINTS ALONG A RAY.
C     NO IS THE NUMBER OF INTERVALS THE INPUT TIME STEP IS DIVIDED INTO.
C     PACK IS THE DIRECTION FROM WHICH THE WAVE PACKET (RAY) COMES.
C     PCTDIF IS THE MAXIMUM OF THE PERCENTAGE DIFFERENCES AT THE 4 GRID
C      POINTS CLOSEST TO THE RAY POINT OF THE SURFACE FIT DERIVED WATER
C      DEPTH RELATIVE TO THE ACTUAL DEPTH.
C     U IS THE COLLINEAR GROUP SPEED IN FEET/SECOND OR METERS/SECOND.
C     V IS THE PHASE SPEED IN FEET/SECOND OR METERS/SECOND.
C     WAVE IS THE DIRECTION FROM WHICH THE WAVELETS (IN A PACKET) COME.
C     X,Y ARE THE COORDINATES OF A RAY POINT.
       DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)
      $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
       REAL KR,KF,KS,KRTOL,KFC
       INTEGER DX1,DX2,ROP,WBCOP,FAN,COL,FUD,BRK,SK,FLAGR,FLAG3
      $,REFLCT,RFLBUM,REFRCT,RFRBUM
       COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1
      $,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX
      $,SVY,SDEP,H,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS
      $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
      $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,PCOUNT,AMM,ANN
      $,REFLCT,RFLBUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,DF,BZ
      $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5
```

```
C THE MAIN PROGRAM IS USED TO READ THE INPUT DATA, TO CONTROL THE
C CALCULATIONS FOR EACH RAY, AND TO PREPARE THE PLOTS.  SUBROUTINES
C TITLE, NUMCON, SHORE, PLOT, AND RAYN ARE CALLED.
      CALL PLOTS (10.,0.,5HPLOTT,120)
      MMAX=2000 $ LI=60 $ LII=(LI-4)/3 $ COFI=12.
      READ (1,1) ((S(DX2,DX1),DX2=1,6),DX1=1,6)
  1   FORMAT (8F10.7)
      READ (1,2) ((EM(DX1,DX2),DX2=1,12),DX1=1,6)
  2   FORMAT (36F2.0)
      READ (1,3) (MXPLOT,PROJCT,DATE1,DATE2,DIR)
  3   FORMAT (I2,1X,3(A6,1X),A6)
      DO 399 NPLOT=1,MXPLOT
      READ (1,4) (NOR,NPT,SK,HT,CIN,NAX,NSH,NCO,NXCMAT)
 4    FORMAT (3(2X,I3),5X,2(F8.3,2X),4(2X,I3))
      READ (1,5) (MM,NN,CNVRSA,GRID,DCON,MOE,NNSKIP)
 5    FORMAT (2(2X,I3),1X,F7.3,2(1X,F9.3),4X,I3,2X,I3)
      IF (MOE .EQ. 0) GO TO 20
C CONVERT TO ENGLISH UNITS FOR CALCULATIONS
      HT=HT/2.54 $ DCON=DCON/0.3048 $ GRID=GRID/0.3048
 20   CIN=CIN/3600. $ AMM=MM-1. $ ANN=NN-1.
      DY=ANN/HT $ SCLI=GRID*DY*COFI
      CALL TITLE (NPLOT,NAX,SCLI,HT)
      IF (NXCMAT .NE. 0) GO TO 3939
      READ (2,11) ((CMAT(J,I),I=1,MM),J=1,NN)
 11   FORMAT (16F5.0)
 3939 IF (NCO .LE. 0) GO TO 493
      READ (1,495) (CONTUR(I),I=1,NCO)
 495  FORMAT (9F8.2)
      CALL NUMCON (MM,NN,NCO,NNSKIP)
 493  IF (NSH .EQ. 0) GO TO 3937
      CALL SHORE (MM,NN)
 3937 DO 15 N=1,NOR
      READ (1,6) (DELTAT,TT,X,Y,A,AV,HGTZ,CF,KRTOL,FOP,WBCOP,FAN,COL)
 6    FORMAT (7(F6.2,2X),2(F6.4,2X),4(I1,1X))
      IF (MOE .EQ. 0) GO TO 22
      HGTZ=HGTZ/0.3048
 22   SOLTAT=DELTAT $ WL=32.2*(TT**2)/6.2831854
      A=CNVRSA-A+180. $ AV=CNVRSA-AV+180. $ MAXG=1 $ FUO=0.
      BRK=0. $ REFLCT=0. $ RFLBUM=0. $ REFRCT=0. $ RFRBUM=0.
      FLAGR=0. $ FLAG3=0. $ IFLG=0 $ ALFA=0.
      IF (COL .EQ. 0) GO TO 4321
      CALL PLOT (3.,0.4,-3)
 4321 A=A*.0174532925 $ AV=AV*.0174532925
      CALL RAYN(X,Y,A,NPLOT,N,MMAX,LI,NPT,LII,AV)
  15  CONTINUE
      CALL PLOT (-3.,-.4,-3)
      CALL PLOT (0.,0.,999)
 399  CONTINUE
      WRITE (6,9999)
 9999 FORMAT (1H1,17H THIS IS THE END.)
      CALL EXIT
      END
```

```
      SUBROUTINE TITLE (NPLOT,NAX,SCLI,HT)
      DIMENSION S(6,6),EM(6,12),C(12),YVH(6),E(6)
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
      REAL KR,KF,KS,KRTOL,KFC
      COMMON S,EM,E,YVH,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1
     $,DATE2,CIN,DIR,ROP,TT,WSCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX
     $,SVY,SDEP,H,DEP,HL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,DF,BZ
     $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5
C IN THIS SUBROUTINE THE PLOT IS LABELED AND STRAIGHT-LINE BORDERS
C ARE DRAWN.  SUBROUTINES SYMBOL, NUMBER, PLOT, AND AXIS2 ARE CALLED.
      CALL PLOT(3.,0.4,3)
      RT=AMM/DY $ XNPLOT=NPLOT
C DRAW LABELS FOR PLOT
      CALL SYMBOL(1.25,0.4,.2,17HPROJ. NO.        ,,90.,17)
      CALL SYMBOL(1.25,2.4,.2,PROJCT,90.,6)
      CALL SYMBOL(1.25,4.0,.2,DATE1,90.,6)
      CALL SYMBOL(1.25,5.2,.2,DATE2,90.,2)
      CALL SYMBOL(1.50,0.4,.2,23HSCL = 1/          , CIN =,90.,23)
      CALL NUMBER(1.50,2.0,.2,SCLI,90.,-1)
      CALL NUMBER(1.50,5.2,.2,CIN*3600.,90.,-1)
      CALL SYMBOL(1.75,0.4,.2,19HPLOT NO.    , DIR. =,90.,19)
      CALL NUMBER(1.75,2.2,.2,XNPLOT,90.,-1)
      CALL SYMBOL(1.75,4.4,.2,DIR,90.,6)
      IF (NAX .NE. 0) GO TO 705
C DRAW STRAIGHT-LINE BORDERS FOR PLOT
      CALL PLOT(3.,0.4,3)
      CALL PLOT(3.,HT+.4,2)
      GO TO 706
 705  CALL AXIS2(3.,0.4,1HY,1,HT,90.,0.,DY)
      CALL AXIS2(3.,.4,1HX,-1,RT,0.,0.,DY)
      CALL PLOT(3.,HT+.4,3)
 706  CALL PLOT(RT+3.,HT+.4,2)
      CALL PLOT(RT+3.,.4,2)
      IF (NAX .NE. 0) GO TO 707
      CALL PLOT(3.,0.4,2)
 707  CALL PLOT(3.,0.4,-3)
      YHT=HT
      RETURN
      END


      SUBROUTINE AXIS2(X,Y,BCD,NC,SIZE,THETA,YMIN,DY)
      DIMENSION BCD(16)
C IN THIS SUBROUTINE THE AXES ARE DRAWN, CALIBRATED, AND LABELED.
C SUBROUTINES PLOT, NUMBER, AND SYMBOL, ARE CALLED.
      BIGN=1.0
      IF (NC .GE. 0) GO TO 2
      BIGN=-1.0
 2    NAC=IABS(NC) $ TH=THETA*.017453294
      N=DY*SIZE+0.5 $ CTH=COS(TH) $ STH=SIN(TH) $ TN=N
      XB=X $ YB=Y $ XA=X-0.1*BIGN*STH $ YA=Y+0.1*BIGN*CTH
```

```
C DRAW AXIS WITH CALIBRATED TICK MARKS                                  AXIS2      11
      CALL PLOT(XA,YA,3)                                                AXIS2      12
      DO 20 I=1,N                                                       AXIS2      13
      CALL PLOT(XB,YB,2)                                                AXIS2      14
      XC=XB+CTH/DY $ YC=YB+STH/DY                                       AXIS2      15
      CALL PLOT(XC,YC,2)                                                AXIS2      16
      XA=XA+CTH/DY $ YA=YA+STH/DY                                       AXIS2      17
      CALL PLOT(XA,YA,2)                                                AXIS2      18
      XB=XC $ YB=YC                                                     AXIS2      19
   20 CONTINUE                                                          AXIS2      20
      ABSV=YMIN+TN $ XA=XB-(.20*BIGN-.05)*STH-.02857*CTH                AXIS2      21
      YA=YB+(.20*BIGN-.05)*CTH-.02857*STH $ N=N+1                       AXIS2      22
C NUMBER THE ORIGIN AND EVERY FIFTH TICK MARK                           AXIS2      23
      DO 30 I=1,N                                                       AXIS2      24
      IF (AMOD(ABSV,5.) .NE. 0.) GO TO 100                              AXIS2      25
      CALL NUMBER(XA,YA,.1,ABSV,THETA,-1)                               AXIS2      26
  100 ABSV=ABSV-1. $ XA=XA-CTH/DY $ YA=YA-STH/DY                        AXIS2      27
   30 CONTINUE                                                          AXIS2      28
C LABEL THE AXIS                                                        AXIS2      29
      TNC=NAC+7 $ XA=X+(SIZE/2.0-.06*TNC)*CTH-(-.07+BIGN*.36)*STH       AXIS2      30
      YA=Y+(SIZE/2.0-.06*TNC)*STH+(-.07+BIGN*.36)*CTH                   AXIS2      31
      CALL SYMBOL(XA,YA,.14,BCD,THETA,NAC)                              AXIS2      32
      RETURN                                                            AXIS2      33
      END                                                               AXIS2      34

      SUBROUTINE NUMCON (MM,NN,NCO,NNSKIP)                              NUMCON      1
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       NUMCON      2
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        NUMCON      3
      REAL KP,KF,KS,KRTOL,KFC                                           NUMCON      4
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,GCON,FAN,DATE1  NUMCON      5
     $,DATE2,CIN,DIR,FOP,TT,W3COP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX   NUMCON      6
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,CUD,KS NUMCON      7
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    NUMCON      8
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,PCOUNT,AMM,ANN      NUMCON      9
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,RFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,OF,BZNUMCON     10
     $,SGZ,BOZ,SBOZ,KRTOL,KP,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5      NUMCON     11
C IN THIS SUBROUTINE SPECIFIED SOUNDING DEPTHS ARE LOCATED AND          NUMCON     12
C DRAWN ON THE PLOT.  SUBROUTINES NUMBER AND PLOT ARE CALLED.           NUMCON     13
      NOD=NN-1 $ MODD=MM-1                                              NUMCON     14
      IF (MOE .EQ. 0) GO TO 2                                           NUMCON     15
C CONVERT TO ENGLISH UNITS FOR CALCULATIONS                             NUMCON     16
      DO 7000 KC=1, NCO                                                 NUMCON     17
      CONTUR(KC)=CONTUR(KC)/0.3048                                      NUMCON     18
 7000 CONTINUE                                                          NUMCON     19
C SELECT Y-COLUMN                                                       NUMCON     20
    2 DO 5000 J=2, NOD, NNSKIP                                          NUMCON     21
      YJ=J-1 $ KKK=1                                                    NUMCON     22
C SELECT SOUNDING DEPTH                                                 NUMCON     23
      DO 8000 KC=1,NCO                                                  NUMCON     24
      KWIT=0 $ NDIF=3 $ I=MM-1                                          NUMCON     25
C SEARCH COLUMN FOR THE GIVEN SOUNDING DEPTH                            NUMCON     26
      DO 1010 II=1,MODD                                                 NUMCON     27
      XI=I-1 $ IL=I+1 $ XL=IL-1                                         NUMCON     28
      IF (KWIT .GT. 0) GO TO 8000                                       NUMCON     29
      IF (CMAT(J,I) .GT. 0) GO TO 20                                    NUMCON     30
      KWIT=1                                                            NUMCON     31
   20 IF (CMAT(J,I)*GCON-CONTUR(KC)) 12,11,13                           NUMCON     32
   11 AX(KKK)=XI $ AY(KKK)=CONTUR(KC) $ KKK=KKK+1 $ NDIF=3              NUMCON     33
      GO TO 1010                                                        NUMCON     34
   12 GO TO (14,77,14),NDIF                                             NUMCON     35
   14 NDIF=1                                                            NUMCON     36
      GO TO 1010                                                        NUMCON     37
   13 GO TO (77,15,15),NDIF                                             NUMCON     38
   15 NDIF=2                                                            NUMCON     39
      GO TO 1010                                                        NUMCON     40
```

```
C LINEARLY INTERPOLATE FOR THE SOUNDING DEPTH                           NUMCON      41
 77    SLPX=(DCON*(CMAT(J,IL)-CMAT(J,I)))/(XL-XI)                        NUMCON      42
       XP=(CONTUR(KC)-DCON*CMAT(J,I))/SLPX+XI                            NUMCON      43
       AX(KKK)=XP $ AY(KKK)=CONTUR(KC) $ KKK=KKK+1                       NUMCON      44
       GO TO (81,82),NDIF                                                NUMCON      45
  81   NDIF=2                                                            NUMCON      46
       GO TO 1010                                                        NUMCON      47
  82   NDIF=1                                                            NUMCON      48
 1010  I=I-1                                                             NUMCON      49
 8000  CONTINUE                                                          NUMCON      50
C DRAW OUT SOUNDING DEPTHS FOR EACH SELECTED Y-COLUMN                    NUMCON      51
       KKK=KKK-1                                                         NUMCON      52
       IF (KKK-1) 5000,668,670                                           NUMCON      53
 670   KKL=KKK-1                                                         NUMCON      54
       DO 997 IA=1,KKL                                                   NUMCON      55
       IAD=IA+1                                                          NUMCON      56
       DO 997 IB=IAD,KKK                                                 NUMCON      57
       IF (AX(IA) .LE. AX(IB)) GO TO 997                                 NUMCON      58
       XMIN=AX(IA) $ AX(IA)=AX(IB) $ AX(IB)=XMIN                         NUMCON      59
       XMIN=AY(IA) $ AY(IA)=AY(IB) $ AY(IB)=XMIN                         NUMCON      60
 997   CONTINUE                                                          NUMCON      61
 668   IF (MOD(J,2) .NE. 0) GO TO 104                                    NUMCON      62
       KONE=KKK $ KADD=-1 $ LAST=1                                       NUMCON      63
       GO TO 105                                                         NUMCON      64
 104   KONE=1 $ KADD=1 $ LAST=KKK                                        NUMCON      65
 105   IF (MOE .EQ. 0) GO TO 4                                           NUMCON      66
C CONVERT SOUNDING DEPTH TO METRIC UNITS BEFORE DRAWING ON PLOT          NUMCON      67
       AY(KONE)=AY(KONE)*0.3048                                          NUMCON      68
 4     CALL NUMBER(AX(KONE)/DY,YJ/DY,0.14,AY(KONE),0.0,-1)               NUMCON      69
       IF (KONE .EQ. LAST) GO TO 5000                                    NUMCON      70
       KONE=KONE+KADD                                                    NUMCON      71
       GO TO 105                                                         NUMCON      72
 5000  CONTINUE                                                          NUMCON      73
       CALL PLOT(0.,0.,-3)                                               NUMCON      74
       RETURN                                                            NUMCON      75
       END                                                               NUMCON      76

       SUBROUTINE SHORE(MM,NN)                                           SHORE        1
       DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       SHORE        2
      $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        SHORE        3
       REAL KF,KF,KS,KRTOL,KFC                                           SHORE        4
       COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1  SHORE        5
      $,DATE2,CIN,CIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX   SHORE        6
      $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS SHORE        7
      $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    SHORE        8
      $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN      SHORE        9
      $,REFLCT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZSHORE       10
      $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5      SHORE       11
C IN THIS SUBROUTINE THE SHORELINE IS DRAWN.  SUBROUTINE PLOT IS USED.  SHORE       12
       PONT(X1,X2,D1,D2)=X1-D1*((X1-X2)/(D1-D2))                         SHORE       13
       IC=3                                                              SHORE       14
C SELECT Y-COLUMN                                                        SHORE       15
       DO 1 J=1,NN                                                       SHORE       16
       YJ=J-1 $ JL=J-1 $ YL=JL-1 $ I=MM                                  SHORE       17
C SEARCH COLUMN FOR ZERO WATER DEPTH STARTING WITH MAXIMUM X             SHORE       18
       DO 2 II=1,MM                                                      SHORE       19
       XI=I-1 $ IL=I+1 $ XL=IL-1                                         SHORE       20
       IF (CMAT(J,I)) 100,200,300                                        SHORE       21
```

```
 100  IF (IC .GT. 2) GO TO 102
C LINEARLY INTERPOLATE FOR ZERO WATER DEPTH
 101  XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))
      CALL PLOT(XP/DY,YJ/DY,IC)
      IC=2
      GO TO 1
 102  IF (J .LE. 1) GO TO 101
      YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
      CALL PLOT (0.0,YP/DY,IC)
      IC=2
      XP=PONT(XI,XL,CMAT(J,I),CMAT(J,IL))
      CALL PLOT(XP/DY,YJ/DY,IC)
      GO TO 1
 200  IF (II .NE. MM) GO TO 201
      CALL PLOT(XI/DY,YJ/DY,IC)
      IF (IC .GT. 2) GO TO 204
      IC=3
      GO TO 1
 204  IC=2
      GO TO 1
 201  IF (IC .LE. 2) GO TO 207
      IF (J .LE. 1) GO TO 207
      YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
      CALL PLOT (0.0,YP/DY,IC)
      IC=2
 207  CALL PLOT(XI/DY,YJ/DY,IC)
      IC=2
      GO TO 1
 300  IF (II .NE. MM) GO TO 2
      IF (IC .GT. 2) GO TO 1
      YP=PONT(YJ,YL,CMAT(J,1),CMAT(JL,1))
      CALL PLOT (0.0,YP/DY,IC)
      IC=3
      GO TO 1
  2   I=I-1
  1   CONTINUE
      CALL PLOT(0.,0.,-3)
      RETURN
      END
```

```
      SUBROUTINE RAYN(X,Y,A,NPLOT,N,MMAX,LI,NPT,LII,AV)
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
      REAL KR,KF,KS,KRTOL,KFC
      INTEGER DX1,DX2,SK,FUJ,RCOUNT,FLAG1,RFLBUM,RFRBUM
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1
     $,DATE2,CIN,DIR,ROP,TT,W3COP,MCE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS
     $.DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
```

```
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN       RAYN
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZRAYN
     $,SEZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5       RAYN
C THIS SUBROUTINE IS USED TO CONTROL THE CALCULATION OF THE WAVE         RAYN
C PACKET PARTICULARS, MOST OF THE PRINTED OUTPUT, AND THE PLOTS OF       RAYN
C THE WAVE PACKET PATHS.  SUBROUTINES SURFCE, MOVE, HEIGHT,              RAYN
C PCD, STORE, AND DRAW ARE CALLED.                                       RAYN
      NDP=1 $ NFK=1 $ NGO=1 $ KREST=0 $ KCIN=0 $ RCOUNT=0. $ FLAG1=1.   RAYN
      CALL SURFCE(X,Y,A,FK,NFK,NDP,AV)                                   RAYN
      FLAG1=0. $ INUM=0 $ SVALFA=ALFA $ SAV=AV $ SDEP=DEP                RAYN
      SAVV=V $ PREV=SAVV $ SAVU=U $ SG=G $ GZERO=G                       RAYN
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI,NPT)                      RAYN
      CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)                           RAYN
      TIMEQ=0. $ ANGLE=A*57.29577951 $ ANGLE=CNVRSA-ANGLE+180.           RAYN
      GAM=AV*57.29577951 $ GAM=CNVRSA-GAM+180.                           RAYN
      IF (NPT .NE. 0) GO TO 150                                          RAYN
      IF (N .LE. 1) GO TO 800                                            RAYN
      IF (MOD(N,LII) .NE. 0) GO TO 803                                   RAYN
C WRITE PAGE AND COLUMN HEADINGS                                         RAYN
 800  WRITE (6,8) (PROJCT,DATE1,DATE2,NPLOT)                             RAYN
 8    FORMAT (1H1,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,//) RAYN
      IF (MOE .NE. 0) GO TO 455                                          RAYN
      WRITE (6,450)                                                      RAYN
 450  FORMAT (1X,52HTHE OUTPUT IS IN ENGLISH UNITS.  H,HGT(FEET).  G,U,VRAYN
     $14H(FEET/SECOND).,//)                                              RAYN
      GO TO 452                                                          RAYN
 455  WRITE (6,451)                                                      RAYN
 451  FORMAT (1X,52HTHE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,VRAYN
     $15H(METER/SECOND).,//)                                             RAYN
 452  WRITE (6,851)                                                      RAYN
851   FORMAT (1X,1HN,2X,6HPERIOD,1X,3HMAX,2X,1HX,6X,1HY,6X,1HH,7X,4HPACKRAYN
     $,3X,4HWAVE,3X,3HHGT,5X,6HDELTAT,1X,2HCF,7X,5HKRTOL,//)             RAYN
  803 ALFA=ALFA*57.29577951                                              RAYN
      IF (MOE .EQ. 0) GO TO 210                                          RAYN
      DEP=DEP*0.3048 $ G=G*0.3048 $ U=U*0.3048                           RAYN
      V=V*0.3048 $ HGT=HGT*0.3048                                        RAYN
C WRITE RAY PARTICULARS FOR THE INITIAL POINT                            RAYN
 210  WRITE (6,853) (N,TT,MAXQ,X,Y,DEP,ANGLE,GAM,HGT,DELTAT,CF,KRTOL)    RAYN
 853  FORMAT (1X,I3,F6.1,1X,I5,2F7.2,F8.2,2F7.2,F8.4,F6.2,2(1X,F8.6))    RAYN
      GO TO 522                                                          RAYN
  3   MAXQ=1+MAXQ                                                        RAYN
      IF (MAXQ+KCIN .LT. MMAX) GO TO 399                                 RAYN
      WRITE (6,401)                                                      RAYN
  401 FORMAT (80X,35HDIMENSION OF OUTPUT-ARRAYS EXCEEDED)                RAYN
      GO TO 15                                                           RAYN
  399 ZCXY=G                                                             RAYN
      CALL MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI,NPT)                      RAYN
      IF (NDP .EQ. 1) GO TO 396                                          RAYN
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```
 402  WRITE (6,403)
 403  FORMAT (80X,17HRAY REACHED SHORE)
      MAXQ=MAXQ-1
      GO TO 15
 396  GO TO (397,397,404,514,515,525,516,528),MIT
 404  WRITE (6,405)
  405 FORMAT (80X,41HPACKET CURVATURE ITERATION NOT CONVERGING)
      MAXQ=MAXQ-1
      GO TO 15
 514  WRITE (6,504)
 504  FORMAT (80X,22HCAUSTIC OR FOCAL POINT)
      MAXQ=MAXQ-1
      GO TO 15
 515  WRITE (6,505)
  505 FORMAT (80X,11HWAVE BREAKS)
      MAXQ=MAXQ-1
      GO TO 15
 525  IF (NPT .NE. 0) GO TO 527
      WRITE (6,526)
 526  FORMAT (80X,10HREFLECTION)
 527  MAXQ=MAXQ-1
      GO TO 15
 516  WRITE (6,517)
 517  FORMAT (80X,18HREFLECTION HANG-UP)
      MAXQ=MAXQ-1
      GO TO 15
 528  WRITE (6,529)
 529  FORMAT (80X,38HBREAKUP TIME STEP LESS THAN 0.5 SECOND)
      MAXQ=MAXQ-1
      GO TO 15
 397  TIMEQ=TIMEQ+(D*GRID/(1800.*(G+ZCXY))) $ PALFA=ALFA*57.29577951
      ANGLE=A*57.29577951 $ ANGLE=CNVRSA-ANGLE+180. $ PFK=FK
      PDEP=DEP $ PD=D $ PG=G $ PV=V $ PU=U $ PHGT=HGT $ PKS=KS
      PKF=KFC $ PKR=KR $ GAM=AV*57.29577951 $ GAM=CNVRSA-GAM+180.
      IF (NPT .EQ. 0) GO TO 161
 160  CALL PCD(C,E,PCTDIF)
      IF (MAXQ .EQ. 1 .OR. MOD(MAXQ,SK) .EQ. 0) GO TO 3041
      GO TO 161
C WRITE RAY PARTICULARS FOR SELECTED RAY POINTS
 3041 ALFA=ALFA*57.29577951
      IF (MOE .EQ. 0) GO TO 200
      DEP=DEP*0.3048 $ G=G*0.3048 $ U=U*0.3048
      V=V*0.3048 $ HGT=HGT*0.3048
 200  IF (MOD(FUD,LI) .NE. 0) GO TO 3043
C WRITE PAGE AND COLUMN HEADINGS
      WRITE (6,7) (PROJCT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,KRTOL)
 7    FORMAT (1H1,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,1H,,
     $1X,7HPERIOD=,F5.1,4HSEC.,1H,,1X,7HRAY NO.,I3,1H,,1X,7HDELTAT=,
     $F6.2,1H,,1X,3HCF=,F8.6,1H,,1X,6HKRTOL=,F8.6,//)
      IF (MAXQ .NE. 1) GO TO 453
      IF (MOE .NE. 0) GO TO 465
      WRITE (6,470) BDZ
 470  FORMAT (1X,52HTHE OUTPUT IS IN ENGLISH UNITS.  H,HGT(FEET).  G,U,V
     $14H(FEET/SECOND).,32X,13HD(BETA)/DT = ,E10.3,//)
      GO TO 453
 465  WRITE (6,471) BDZ
 471  FORMAT (1X,52HTHE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,V
     $15H(METER/SECOND).,31X,13HD(BETA)/DT = ,E10.3,//)
```

```
 453  WRITE (6,150)                                                        RAYN
  150 FORMAT (1X,3HMAX,1X,1HX,6X,1HY,6X,1HH,7X,4HPACK,3X,4HWAVE,3X,1HD,   RAYN
     $9X,2HFK,8X,4HALFA,5X,1HG,5X,1HU,5X,                                 RAYN
     $1HV,5X,3HHGT,5X,2HKS,6X,2HKF,7X,2HNO,1X,2HKR,7X,6HPCTDIF,//)        RAYN
 3043 FUD=FUD+1.                                                          RAYN
      IF (RFLBUM .NE. 0) GO TO 518                                        RAYN
      IF (RFRBUM .NE. 0) GO TO 520                                        RAYN
      WRITE (6,612) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,          RAYN
     $KS,KFC,KR,PCTDIF)                                                   RAYN
 612  FORMAT (1X,I4,2F7.2,F8.2,2F7.2,2E10.2,F8.2,1X,3F6.2,                RAYN
     $3F8.4,4X,F9.4,F7.2)                                                 RAYN
      GO TO 522                                                           RAYN
 518  RFLBUM=RFRBUM=0                                                     RAYN
C USE FORMAT FOR REFLECTION BREAK UP OF TIME STEP INTERVAL                 RAYN
      WRITE (6,613) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,          RAYN
     $KS,KFC,NUMT,KR,PCTDIF)                                              RAYN
 613  FORMAT (1X,I4,2F7.2,F8.2,2F7.2,E10.2,1H*,E9.2,F8.2,1X,3F6.2,        RAYN
     $3F8.4,1X,I3,F9.4,F7.2)                                              RAYN
      GO TO 522                                                           RAYN
 520  RFRBUM=0.                                                           RAYN
C USE FORMAT FOR REFRACTION (BETA) BREAK UP OF TIME STEP INTERVAL          RAYN
      WRITE (6,614) (MAXQ,X,Y,DEP,ANGLE,GAM,D,FK,ALFA,G,U,V,HGT,          RAYN
     $KS,KFC,NUMT,KR,PCTDIF)                                              RAYN
 614  FORMAT (1X,I4,2F7.2,F8.2,2F7.2,2E10.2,F8.2,1X,3F6.2                 RAYN
     $,3F8.4,1X,I3,1H*,F8.4,F7.2)                                         RAYN
 522  ALFA=ALFA*0.01745329                                                RAYN
      IF (MOE .EQ. 0) GO TO 161                                           RAYN
      DEP=DEP/0.3048 $ G=G/0.3048 $ U=U/0.3048                            RAYN
      V=V/0.3048 $ HGT=HGT/0.3048                                         RAYN
 161  KMAX=MAXQ $ PX=X $ PY=Y $ KNUMT=NUMT                                 RAYN
      CALL STORE(X,Y,A,KMAX,TIMEQ,KCIN,KREST)                             RAYN
      GO TO (10,11) MIT                                                   RAYN
 11   IF (NPT .EQ. 0) GO TO 10                                            RAYN
      IF (MOD(FUD,LI) .NE. 0) GO TO 3053                                  RAYN
      WRITE (6,7) (PROJCT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,KRTOL)         RAYN
      WRITE (6,150)                                                       RAYN
 3053 FUD=FUD+1.                                                          RAYN
      WRITE (6,9)                                                         RAYN
 9    FORMAT (80X,25HPACKET CURVATURE AVERAGED)                           RAYN
 10   IF (MAXQ .GT. 1) GO TO 13                                           RAYN
      GO TO (3,402),NDP                                                   RAYN
 13   IF (NGO .EQ. 1) GO TO 3                                             RAYN
      WRITE (6,407)                                                       RAYN
 407  FORMAT (80X,25HRAY REACHED GRID BOUNDARY)                           RAYN
 15   IF (NPT .NE. 0) GO TO 190                                           RAYN
      IF (MOE .EQ. 0) GO TO 212                                           RAYN
      PDEP=PDEP*0.3048 $ PG=PG*0.3048 $ PU=PU*0.3048                      RAYN
      PV=PV*0.3048 $ PHGT=PHGT*0.3048                                     RAYN
C WRITE RAY PARTICULARS FOR THE TERMINAL POINT                            RAYN
 212  WRITE (6,854) (N,TT,KMAX,PX,PY,PDEP,ANGLE,GAM,PHGT)                 RAYN
 854  FORMAT  (1H+,I3,F6.1,1X,I5,2F7.2,F8.2,2F7.2,F8.4,//)                RAYN
 190  IF (MAXQ .LE. 1 .OR. NPT .EQ. 0 .OR.                                RAYN
     $MOD(MAXQ,SK) .EQ. 0) GO TO 1900                                     RAYN
C RAY PARTICULARS HAVE NOT BEEN WRITTEN FOR THE LAST POINT                 RAYN
      IF (MOD(FUD,LI) .NE. 0) GO TO 3031                                  RAYN
      WRITE (6,7) (PROJCT,DATE1,DATE2,NPLOT,TT,N,DELTAT,CF,KRTOL)         RAYN
      WRITE (6,150)                                                       RAYN
 3031 IF (MOE .EQ. 0) GO TO 3030                                          RAYN
```

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      PDEP=PDEP*0.3048 $ PG=PG*0.3048 $ PU=PU*0.3048                    RAYN
      PV=PV*0.3048 $ PHGT=PHGT*0.3048                                   RAYN
 3030 IF (RFLBUM .NE. 0) GO TO 558                                      RAYN
      IF (RFRBUM .NE. 0) GO TO 560                                      RAYN
      WRITE (6,612) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,         RAYN
     $PU,PV,PHGT,PKS,PKF,PKR,PCTDIF)                                    RAYN
      GO TO 1900                                                        RAYN
 558  RFLBUM=RFRBUM=0                                                   RAYN
      WRITE (6,613) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,         RAYN
     $PU,PV,PHGT,PKS,PKF,KNUMT,PKR,PCTDIF)                              RAYN
      GO TO 1900                                                        RAYN
 560  RFRBUM=0.                                                         RAYN
      WRITE (6,614) (KMAX,PX,PY,PDEP,ANGLE,GAM,PD,PFK,PALFA,PG,         RAYN
     $PU,PV,PHGT,PKS,PKF,KNUMT,PKR,PCTDIF)                              RAYN
 1900 CALL DRAW(N,KMAX,KCIN,KREST)                                      RAYN
      RETURN                                                            RAYN
      END                                                               RAYN


      SUBROUTINE MOVE(X,Y,A,FK,NGO,MIT,NFK,NDP,AV,LI,NPT)               MOVE
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       MOVE
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        MOVE
      REAL KP,KF,KS,KPTOL,KFC                                           MOVE
      INTEGER REF,ROP,REFLCT,REFRCT,FLAG1,FLAG2,FLAG3,RCOUNT,FUD,BRK     MOVE
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1  MOVE
     $,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX   MOVE
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS MOVE
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    MOVE
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,FCOUNT,AMM,ANN      MOVE
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,DF,BZMOVE
     $,SBZ,BDZ,SBDZ,KPTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5      MOVE
C IN THIS SUBROUTINE THE PATH OF THE WAVE PACKET IS DETERMINED.         MOVE
C TESTS ARE MADE TO LOCATE A REFLECTION POINT, AND IF DESIRED           MOVE
C THE RAY PATH IS CONTINUED BEYOND THE REFLECTION POINT.                MOVE
C SUBROUTINES SURFCE AND HEIGHT ARE CALLED.                             MOVE
      NUMT=1 $ MIT=1                                                    MOVE
C SAVE VALUES IN CASE OF BREAK UP OF TIME STEP INTERVAL                 MOVE
      SVG=G                                                             MOVE
      IF (MAXQ .NE. 2) GO TO 3033                                       MOVE
      SVFKB=FK                                                          MOVE
      GO TO 202                                                         MOVE
 3033 SVFKB=FKBAR $ SAVFK=FK                                            MOVE
C COMPUTE THE INCREMENTAL DISTANCE TO THE NEXT RAY POINT                MOVE
 202  D=(G*SOLTAT)/GRID                                                 MOVE
 203  IF (MAXQ-2) 38,102,104                                            MOVE
 102  FKBAR=FK                                                          MOVE
C CHECK FOR TIME STEP BREAK UP DUE TO BETA CALCULATION OR REFLECTION    MOVE
 104  IF (REFRCT .NE. 0 .OR. REFLCT .NE. 0                              MOVE
     $.OR. ABS(TAN(SAV-SVALFA)) .LE. 3.7320508                          MOVE
     $.OR. ABS(TAN(A-SVALFA)) .GE. 3.7320508) GO TO 81                  MOVE
      REFLCT=1 $ RFLBUM=1                                               MOVE
C ITERATE TO FIND VALUES FOR THE NEXT POINT                             MOVE
 81   DO 20 IT=1,50                                                     MOVE
 39   DELA=FKBAR*D $ AA=A+DELA $ ABAR=A+0.5*DELA                        MOVE
      DELX=D*COS(ABAR) $ DELY=D*SIN(ABAR) $ XX=X+DELX $ YY=Y+DELY       MOVE
      CALL SURFCE(XX,YY,AA,FKK,NFK,NDP,AAV)                             MOVE
      AVP=AAV-ALFA                                                      MOVE
      IF (FLAG2 .EQ. 0) GO TO 86                                        MOVE
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      REF=1
      GO TO 13
 86   DUD=SAVV/PREV
      GO TO (101,6,38,38,38,38,38,38),MIT
101   IF (NDF .EQ. 2) GO TO 38
      FKBAR=0.5*(FK+FKK)
      IF (IT .NE. 49) GO TO 88
      SVFK=FKBAR
 88   IF (IT-48) 5,37,9
 37   FKKPP=FKBAR
 5    IF (MAXQ .GT. 2) GO TO 9
      IF (IT .LE. 1) GO TO 20
C TEST THE CONVERGENCE OF THE RAY CURVATURE CALCULATIONS
 9    IF (ABS(FKKP-FKBAR) .LE. 0.00009/D) GO TO 6
 20   FKKP=FKBAR
      IF (ABS(FKKPP-FKBAR) .LE. 0.00009/D) GO TO 18
C DETERMINE IF CONVERGENCE FAILED DUE TO A REFLECTION POINT
      IF (DUD .GT. 1.0 .AND.

     $ABS(TAN(AVP)) .GT. 5.6712818) GO TO 91
      MIT=3
      GO TO 38
 91   REF=2
      GO TO 13
 18   FKBAR=.5*(FKBAR+SVFK) $ MIT=2
      GO TO 39
C DETERMINE IF TOO CLOSE TO A REFLECTION POINT
 6    IF (DUD .LE. 1.0 .OR.
     $ABS(TAN(AVP)) .LE. 114.588650 .OR.
     $ABS(TAN(A-SVALFA)) .GE. 3.7320508) GO TO 92
      REF=3
C BEGIN REFLECTION
 13   IF (NPT .EQ. 0) GO TO 14
      IF (MOD(FUD,LI) .NE. 0) GO TO 3043
C WRITE PAGE AND COLUMN HEADINGS
      WRITE (6,96) (PROJCT,DATE1,DATE2,NPLOT,TT,N,SDLTAT,CF,KRTOL)
 96   FORMAT(1H1,11HPROJECT NO.,A6,1H,,2X,2A6,1H,,5X,8HPLOT NO.,I3,1H,,
     $1X,7HPERIOD=,F5.1,4HSEC.,1H,,1X,7HRAY NO.,I3,1H,,1X,7HDELTAT=,
     $F6.2,1H,,1X,3HCF=,F8.6,1H,,1X,6HKRTOL=,F8.6,//)
      WRITE (6,150)
 150  FORMAT (1X,3HMAX,1X,1HX,6X,1HY,6X,1HH,7X,4HPACK,3X,4HWAVE,3X,1HD,
     $9X,2HFK,8X,4HALFA,5X,1HG,5X,1HU,5X,
     $1HV,5X,3HHGT,5X,2HKS,6X,2HKF,7X,2HNC,1X,2HKR,7X,6HPCTDIF,//)
 3043 PACK=A*57.29577951 $ PACK=CNVFSA-PACK+180.
      WAVE=AV*57.29577951 $ WAVE=CNVFSA-WAVE+180. $ KMAX=MAXQ-1
      SVALFA=SVALFA*57.29577951
      IF (MOE .EQ. 0) GO TO 210
      SDEP=SDEP*0.3048 $ SG=SG*0.3048 $ SAVU=SAVU*0.3048
      SAVV=SAVV*0.3048 $ HGT=HGT*0.3048
C WRITE RAY PARTICULARS
 210  WRITE (6,151) (KMAX,X,Y,SDEP,PACK,WAVE,SD,FK,SVALFA,SG,
     $SAVU,SAVV,HGT,KS,KFC,KR)
 151  FORMAT (1X,1H*,I3,2F7.2,F8.2,2F7.2,2E10.2,F8.2,1X,3F6.2,3F8.4
     $,4X,F9.4)
      FUD=FUD+1 $ SVALFA=SVALFA*0.01745329
      IF (MOE .EQ. 0) GO TO 212
      SDEP=SDEP/0.3048 $ SG=SG/0.3048 $ SAVU=SAVU/0.3048
      SAVV=SAVV/0.3048 $ HGT=HGT/0.3048
 212  IF (MOD(FUD,LI) .NE. 0) GO TO 3044
      WRITE (6,96) (PROJCT,DATE1,DATE2,NPLOT,TT,N,SDLTAT,CF,KRTOL)
      WRITE (6,150)
```

```
C WRITE TYPE OF REFLECTION
 3044 GO TO (97,98,99),REF
   97  WRITE (6,152)
  152  FORMAT (1X,43HREFLECTION:  SNELLS LAW WITH PHASE VELOCITY)
       GO TO 300
   98  WRITE (6,153)
  153  FORMAT (1X,44HREFLECTION:  PACKET CURVATURE ITERATION NOT
     $,10HCONVERGING)
       GO TO 300
   99  WRITE (6,154)
  154  FORMAT (1X,34HREFLECTION:  NEAR REFLECTION POINT)
  300  FUD=FUD+1.
  14   IF (ROP .NE. 0) GO TO 301
       MIT=6
       GO TO 38
  301  FLAG2=0.
C COMPUTE REFLECTION ANGLES
       SAV=2.*SVALFA-SAV+3.1415927 $ A=2.*SVALFA-A+3.1415927 $ AV=SAV
       RCOUNT=RCOUNT+1.
C TEST FOR REFLECTION HANG-UP
       IF (RCOUNT .LT. 2) GO TO 305
       MIT=7
       GO TO 38
  305  FLAG1=1.
       CALL SURFCE(X,Y,A,FK,NFK,NDP,SAV)
       FLAG1=0. $ PATI=POT $ QATI=QOT $ FLAGR=1.
       GO TO 102
C DETERMINE IF POINT IS TOO CLOSE TO A GRID BOUNDARY
  92   IF ((XX-1.5)*((AMM-1.5)-XX) .GE. 0.0 .AND.
     $(YY-1.5)*((ANN-1.5)-YY) .GE. 0.0) GO TO 309
       NGO=2
  309  SVX=X $ SVY=Y $ XS=XX $ YS=YY $ SSALFA=SVALFA $ SVALFA=ALFA
       SVA=A $ SVAV=SAV $ SAV=AAV $ RCOUNT=0.
       SPREV=PREV $ PREV=SAVV $ SDEP=DEP $ SAVV=V $ SAVU=U $ SG=G
       AAA=.5*(AA+A) $ A=AA $ AAAV=.5*(AAV+AV) $ AV=AAV $ FK=FKK $ SJ=0
       IF (REFLCT .EQ. 1) GO TO 40
C COMPUTE P AND Q FOR THE INTERMEDIATE POINTS
       XX=X+(1./3.)*DELX*(ABS(COS(AAA)))
       YY=Y+(1./3.)*DELY*(ABS(SIN(AAA))) $ FLAG1=1.
       CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,AAAV)
       P1=POT $ Q1=QOT $ XX=X+.4*DELX*(ABS(COS(AAA)))
       YY=Y+.4*DELY*(ABS(SIN(AAA)))
       CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,AAAV)
       P2=POT $ Q2=QOT $ XX=X+.45573725*DELX*(ABS(COS(AAA)))
       YY=Y+.45573725*DELY*(ABS(SIN(AAA)))
       CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,AAAV)
       P3=POT $ Q3=QOT $ XX=X+(2./3.)*DELX*(ABS(COS(AAA)))
       YY=Y+(2./3.)*DELY*(ABS(SIN(AAA)))
       CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,AAAV)
       P4=POT $ Q4=QOT $ XX=X+.8*DELX*(ABS(COS(AAA)))
       YY=Y+.8*DELY*(ABS(SIN(AAA)))
       CALL SURFCE(XX,YY,AAA,FKK,NFK,NDP,AAAV)
       P5=POT $ Q5=QOT
       CALL SURFCE (XS,YS,AA,FKK,NFK,NDP,AAV)
       FLAG1=0
40     X=XS $ Y=YS
```

```
      CALL HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)
      IF (BRK .EQ. 1) GO TO 203
C PLACE ANGLES IN THE RANGE 0 TO 360 DEGREES FOR THE PRINTED OUTPUT.
      CNVRSA=CNVRSA*.0174532925 $ A=CNVRSA-A+3.1415927
 51   IF (A .GE. 0.0) GO TO 50
      A=A+6.2831853
      GO TO 51
 50   IF (A .LT. 6.2831853) GO TO 52
      A=A-6.2831853
      GO TO 50
 52   A=CNVRSA-A+3.1415927
      AV=CNVRSA-AV+3.1415927
 54   IF (AV .GE. 0.0) GO TO 53
      AV=AV+6.2831853
      GO TO 54
 53   IF (AV .LT. 6.2831853) GO TO 55
      AV=AV-6.2831853
      GO TO 53
 55   AV=CNVRSA-AV+3.1415927 $ CNVRSA=CNVRSA*57.29577951
 38   RETURN
      END

      SUBROUTINE HEIGHT(X,Y,A,FK,NGO,MIT,NFK,NDP,AV)
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
      REAL KP,KF,KS,KRTOL,KFC
     $,L1,L2,L3,L4,L5,L6,L7,L8,L9,K1,K2,K3,K4,K5,K6,K7,K8,K9
      INTEGER DX1,DX2,W3COP,REFLCT,REFRCT,FLAGR,RFLBUM,RFRBUM,BRK
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1
     $,DATE2,CIN,DIR,ROP,TT,W3COP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ
     $,SBZ,BDZ,SBDZ,KRTOL,KR,PCT,P1,P2,P3,P4,P5,QCT,Q1,Q2,Q3,Q4,Q5
C IN THIS SUBROUTINE THE WAVE HEIGHT IS COMPUTED.  THE TIME STEP
C IS SUCCESSIVELY HALVED, IF NECESSARY, TO MAINTAIN THE DESIRED
C ACCURACY IN COMPUTING THE REFRACTION COEFFICIENT, OR THE RAY
C PATH IF NEAR A REFLECTION POINT.
      IF (MAXQ .GT. 1) GO TO 2
      PATI=PCT $ QATI=QCT $ BZTOL=KRTOL**2
C COMPUTE INITIAL D(BETA)/DT
      BDZ=-TAN(A-ALFA)*SIN(A-ALFA)*DGDX/GRID
      KF=BZ=KR=KS=1. $ HGT=HGTZ*KS*KF*KR $ KFC=KF
      GO TO 38
C COMPUTE SHOALING COEFFICIENT
 2    KS=SQRT(ABS(GZERO/G))
C COMPUTE FRICTION COEFFICIENT
      SKH=6.283185308*DEP/(V*TT)
      KF=1./(KFC*.8195*CF*HGTZ*D*GRID/((TT**3)*GZERO)*
     $(2.*KS/(EXP(SKH)-EXP(-SKH)))**3+1.)
      KFC=KFC*KF $ PSAV=PATI $ SBZ=BZ $ SBDZ=BDZ
      IF (NFK .EQ. 1) GO TO 30
      IF (REFLCT .EQ. 0) GO TO 33
C COMPUTE BETA AND D(BETA)/DT ANALYTICALLY NEAR A REFLECTION POINT
      PREA=A-DELA-ALFA $ TPII=COS(PREA)/BZ $ TPI=ACOS(TPII)
      BZ=COS(A-ALFA)/COS(TPI)
      BDZ=-(SIN(A-ALFA)**2)/COS(TPI)*DGDX/GRID
      IF (FLAGR .EQ. 0) GO TO 71
      ROP=0
      GO TO 71
 33   IF (ABS(DHDX/GRID) .GT. 0.00001) GO TO 31
 30   BDZ=EBZ=EBDZ=0.
      GO TO 71
```

```
 31     IF (FLAGR .EQ. 0) GO TO 32                                          HEIGHT     43
                                                                            HEIGHT[1]  44
        ROP = 0                                                             HEIGHT     45
C COMPUTE BETA AND D(BETA)/DT USING RUNGE KUTTA METHOD                      HEIGHT     46
 32     K1=DELTAT*BDZ $ L1=-DELTAT*(PATI*BDZ+QATI*BZ)                       HEIGHT     47
        K2=DELTAT*(BDZ+.4*L1)                                               HEIGHT     48
        L2=-DELTAT*(P2*(BDZ+0.4*L1)+Q2*(BZ+0.4*K1))                         HEIGHT     49
        K3=DELTAT*(BDZ+0.29697761*L1+0.15875964*L2)                         HEIGHT     50
        L3=-DELTAT*(P3*(BDZ+0.29697761*L1+0.15875964*L2)+Q3*(BZ+            HEIGHT     51
       $0.29697761*K1+0.15875964*K2))                                       HEIGHT     52
        K4=DELTAT*(BDZ+0.21810040*L1-3.05096516*L2+3.83286476*L3)           HEIGHT     53
        L4=-DELTAT*(POT*(BDZ+0.21810040*L1-3.05096516*L2+3.83286476*L3)     HEIGHT     54
       $+QOT*(BZ+0.21810040*K1-3.05096516*K2+3.83286476*K3))                HEIGHT     55
        K5=DELTAT*(BDZ+L1/3.)                                               HEIGHT     56
        L5=-DELTAT*(P1*(BDZ+L1/3.)+Q1*(BZ+K1/3.))                           HEIGHT     57

        K6=DELTAT*(BDZ+(6.*L5+4.*L1)/25.)                                   HEIGHT     58
        L6=-DELTAT*(P2*(BDZ+(6.*L5+4.*L1)/25.)+Q2*(BZ+(6.*K5+4.*K1)/25.))  HEIGHT     59
        K7=DELTAT*(BDZ+(15.*L6-12.*L5+L1)/4.)                               HEIGHT     60
        L7=-DELTAT*(POT*(BDZ+(15.*L6-12.*L5+L1)/4.)+QOT*(BZ+(15.*K6         HEIGHT     61
       $-12.*K5+K1)/4.))                                                    HEIGHT     62
        K8=DELTAT*(BDZ+(8.*L7-50.*L6+90.*L5+6.*L1)/81.)                     HEIGHT     63
        L8=-DELTAT*(P4*(BDZ+(8.*L7-50.*L6+90.*L5+6.*L1)/81.)+Q4*(BZ+        HEIGHT     64
       $(8.*K7-50.*K6+90.*K5+6.*K1)/81.))                                   HEIGHT     65
        K9=DELTAT*(BDZ+(8.*L7+10.*L6+36.*L5+6.*L1)/75.)                     HEIGHT     66
        L9=-DELTAT*(P5*(BDZ+(8.*L7+10.*L6+36.*L5+6.*L1)/75.)+Q5*(BZ+        HEIGHT     67
       $(8.*K7+10.*K6+36.*K5+6.*K1)/75.))                                   HEIGHT     68
        BZ5=BZ+(1./192.)*(23.*K1+125.*K6-81.*K8+125.*K9)                    HEIGHT     69
        BDZ5=BDZ+(1./192.)*(23.*L1+125.*L6-81.*L8+125.*L9)                  HEIGHT     70
        BDZ=BDZ+0.17476028*L1-0.55148066*L2+1.20553560*L3+0.17118478*L4     HEIGHT     71
        BZ=BZ+0.17476028*K1-0.55148066*K2+1.20553560*K3+0.17118478*K4       HEIGHT     72
C COMPUTE DIFFERENCE BETWEEN 4TH AND 5TH ORDER SOLUTIONS                    HEIGHT     73
        EBZ=BZ-BZ5 $ EBDZ=BDZ-BDZ5                                          HEIGHT     74
C COMPUTE REFRACTION COEFFICIENT                                            HEIGHT     75
  71    KR=1./(SQRT(ABS(BZ)))                                               HEIGHT     76
        IF (REFLCT .EQ. 0) GO TO 401                                        HEIGHT     77
        IF (IFLG .NE. 0) GO TO 55                                           HEIGHT     78
C NEAR A REFLECTION POINT LIMIT THE CHANGE IN THE PACKET DIRECTION          HEIGHT     79
        IF (ABS(DELA) .LT. 0.017453293) GO TO 22                            HEIGHT     80
        GO TO 58                                                            HEIGHT     81
 401    IF (IFLG .NE. 0) GO TO 55                                           HEIGHT     82
C REQUIRE THAT THE BETA CALCULATION HAS THE DESIRED ACCURACY                HEIGHT     83
        IF (ABS(EBZ) .GE. BZTOL .OR. ABS(EBDZ) .GE. BZTOL) GO TO 21         HEIGHT     84
                                                                            HEIGHT     85
 22     IF (NUMT .LE. 1) GO TO 4                                            HEIGHT     86
        IFLG=1                                                              HEIGHT     87
        GO TO 55                                                            HEIGHT     88
 21     REFRCT=1. $ RFRBUM=1.                                               HEIGHT     89
C BREAK UP TIME STEP INTERVAL AND RESUME CALCULATIONS                       HEIGHT     90
   58 DELTAT=.5*DELTAT                                                      HEIGHT     91
        IF (DELTAT .GE. 0.5) GO TO 81                                       HEIGHT     92
        MIT=8 $ BRK=0.                                                      HEIGHT     93
        GO TO 38                                                            HEIGHT     94
 81     G=SVG $ D=G*DELTAT/GRID $ BRK=1. $ NUMT=2*NUMT                      HEIGHT     95
C RECOVER SAVED VALUES                                                      HEIGHT     96
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[1] This statement has been removed.

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      IF (MAXQ .NE. 2) GO TO 59                                         HEIGHT      97
      FK=SVFKB                                                          HEIGHT      98
      GO TO 61                                                          HEIGHT      99
 59   FKBAR=SVFKB $ FK=SAVFK                                            HEIGHT     100
 61   A=SVA $ SAV=SVAV $ SAVV=PREV $ PREV=SPREV $ SVALFA=SSALFA          HEIGHT     101
      X=SVX $ Y=SVY $ BZ=SBZ $ BDZ=SBDZ $ PATI=PSAV $ KFC=KFC/KF        HEIGHT     102
      GO TO 38                                                          HEIGHT     103
  55  INUM=INUM+1.                                                      HEIGHT     104
      IF (INUM .LT. NUMT) GO TO 64                                      HEIGHT     105
C RESUME CALCULATIONS WITH ORIGINAL TIME STEP                            HEIGHT     106
      IFLG=0 $ INUM=0 $ DELTAT=SDLTAT $ BRK=0                           HEIGHT     107
      D=G*DELTAT/GRID                                                   HEIGHT     108
      GO TO 4                                                           HEIGHT     109
 64   D=G*DELTAT/GRID $ PATI=POT $ QATI=QOT                             HEIGHT     110
C TEST FOR FOCAL POINT OR CAUSTIC                                        HEIGHT     111
      IF (BZ .GT. 0.) GO TO 67                                          HEIGHT     112
 68   MIT=4 $ BRK=0.                                                    HEIGHT     113
      GO TO 38                                                          HEIGHT     114
  67  BRK=1.                                                            HEIGHT     115
      GO TO 38                                                          HEIGHT     116
 4    IF (BZ .LE. 0.) GO TO 68                                          HEIGHT     117
      PATI=POT $ QATI=QOT $ REFRCT=0 $ REFLCT=0                         HEIGHT     118
C COMPUTE WAVE HEIGHT                                                    HEIGHT     119
      HGT=HGTZ*KS*KFC*KR                                                HEIGHT     120
      IF (WBCOP .EQ. 0) GO TO 38                                        HEIGHT     121
C TEST FOR WAVE BREAK                                                    HEIGHT     122
      IF (HGT/(V*TT) .LE. (1./7.)*TANH(SKH)) GO TO 38                   HEIGHT     123
      MIT=5                                                             HEIGHT     124
  38  RETURN                                                            HEIGHT     125
      END                                                               HEIGHT     126


      SUBROUTINE SURFCE(X,Y,A,FK,NFK,NDP,AV)                            SURFACE      1
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       SURFACE      2
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        SURFACE      3
      REAL KF,KF,KS,KRTOL,KFC                                           SURFACE      4
      INTEGER FLAG1,FLAG2                                               SURFACE      5
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1  SURFACE      6
     $,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX   SURFACE      7
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS SURFACE      8
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    SURFACE      9
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN      SURFACE     10
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,DF,BZSURFACE     11
     $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5      SURFACE     12
C IN THIS SUBROUTINE THE WATER DEPTH, ROTATION ANGLE, WAVELET           SURFACE     13
C DIRECTION, GEOMETRIC GROUP VELOCITY, COEFFICIENTS OF THE RAY          SURFACE     14
C SEPARATION EQUATION, AND THE PACKET RAY CURVATURE ARE COMPUTED.       SURFACE     15
C SUBROUTINES VELCTY AND CONDER ARE CALLED.                             SURFACE     16
      I=X $ J=Y $ FI=I $ FJ=J $ XL=X+1.-FI $ YL=Y+1.-FJ                 SURFACE     17
      IF (MAXQ .LE. 1) GO TO 1                                          SURFACE     18
      IF (ZI .NE. FI) GO TO 1                                           SURFACE     19
      IF (ZJ .EQ. FJ) GO TO 3                                           SURFACE     20
 1    ZI=FI $ ZJ=FJ                                                     SURFACE     21
```

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C SELECT 12 WATER DEPTHS ABOUT RAY POINT                               SURFACE
      C(1)=CMAT(J+1,I) $ C(2)=CMAT(J+2,I) $ C(3)=CMAT(J,I+1)          SURFACE
      C(4)=CMAT(J+1,I+1) $ C(5)=CMAT(J+2,I+1) $ C(6)=CMAT(J+3,I+1)    SURFACE
      C(7)=CMAT(J,I+2) $ C(8)=CMAT(J+1,I+2) $ C(9)=CMAT(J+2,I+2)      SURFACE
      C(10)=CMAT(J+3,I+2) $ C(11)=CMAT(J+1,I+3) $ C(12)=CMAT(J+2,I+3) SURFACE
C FIT QUADRATIC SURFACE TO 12 WATER DEPTHS                             SURFACE
      DO 318 II=1,6                                                   SURFACE
      YVW(II)=0.                                                      SURFACE
      DO 318 L=1,12                                                   SURFACE
      YVW(II)=YVW(II)+C(L)*EM(II,L)                                   SURFACE
 318  CONTINUE                                                        SURFACE
      DO 319 II=1,6                                                   SURFACE
      E(II)=0.                                                        SURFACE
      DO 319 JJ=1,6                                                   SURFACE
      E(II)=E(II)+S(JJ,II)*YVW(JJ)                                    SURFACE
 319  CONTINUE                                                        SURFACE
C COMPUTE INTERPOLATED WATER DEPTH                                     SURFACE
  3   DEP=(E(1)+E(2)*XL+E(3)*YL+E(4)*XL**2+E(5)*XL*YL+E(6)*YL**2)*DCON SURFACE
C COMPUTE PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED XY-SYSTEM        SURFACE
      HX=(E(2)+2.*E(4)*XL+E(5)*YL)*DCON                               SURFACE
      HY=(E(3)+E(5)*XL+E(6)*2.*YL)*DCON                               SURFACE
      HXX=2.*E(4)*DCON $ HYY=2.*E(6)*DCON $ HXY=E(5)*DCON             SURFACE
      IF (DEP .GT. 0.) GO TO 324                                      SURFACE
      NDP=2                                                           SURFACE
      GO TO 403                                                       SURFACE
 324  IF (DEP/WL .GT. .64) GO TO 322                                  SURFACE
      NFK=2                                                           SURFACE
      GO TO 323                                                       SURFACE
 322  NFK=1                                                           SURFACE
 323  CALL VELCTY(V,TT,MAXQ,DEP,NFK,U)                                SURFACE
      IF (NFK .EQ. 2) GO TO 402                                       SURFACE
      W=0.                                                            SURFACE
      GO TO 10                                                        SURFACE
  402 DN=1.                                                           SURFACE
      CALL CONDER(DN,TT,V,MAXQ,NFK)                                   SURFACE
      W=DN                                                            SURFACE

 10   VX=W*HX $ VY=W*HY $ DHDX=SQRT((HX**2)+(HY**2))                  SURFACE
      IF (ABS(DHDX/GRID) .GT. 0.00001) GO TO 8                        SURFACE
      GO TO 9                                                         SURFACE
C COMPUTE ROTATION ANGLE                                               SURFACE
 8    ALFA=ATAN2(HY,HX)                                               SURFACE
 9    IF (FLAG1 .NE. 0) GO TO 12                                      SURFACE
C COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELLS LAW      SURFACE
C WITH V AND TEST FOR TOTAL REFLECTION DUE TO THE WAVELETS             SURFACE
      GP=SAV-ALFA                                                     SURFACE
 14   IF (ABS(GP) .LE. 6.2831853) GO TO 13                            SURFACE
      IF (GP) 16,13,17                                                SURFACE
 16   GP=GP+6.2831853                                                 SURFACE
      GO TO 14                                                        SURFACE
 17   GP=GP-6.2831853                                                 SURFACE
      GO TO 14                                                        SURFACE
 13   ARG1=V*SIN(GP)/SAVV                                             SURFACE
      IF (ABS(ARG1) .LE. 1.) GO TO 18                                 SURFACE
      FLAG2=1.                                                        SURFACE
      GO TO 403                                                       SURFACE
 18   FLAG2=0. $ GPT=ASIN(ARG1)                                       SURFACE
```

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      IF (ABS(GP) .LE. 4.7123889) GO TO 20                              SURFACE  78
      AVP=6.2831853+GPT                                                 SURFACE  79
      GO TO 22                                                          SURFACE  80
   20 IF (ABS(GP) .LE. 1.5707963) GO TO 23                              SURFACE  81
      AVP=3.1415927-GPT                                                 SURFACE  82
      GO TO 22                                                          SURFACE  83
   23 AVP=GPT                                                           SURFACE  84
   22 AV=AVP+ALFA                                                       SURFACE  85
   12 PHI=A-AV $ G=U*COS(PHI)                                           SURFACE  86
      DVDX=W*DHDX $ BAR3=12.5663706/TT $ BAR4=BAR3*DEP/V                SURFACE  87
      IF (NFK .EQ. 2) GO TO 25                                          SURFACE  88
      DUDX=DVDX/2.                                                      SURFACE  89
      GO TO 27                                                          SURFACE  90
   25 DUDX=(1./(EXP(BAR4)-EXP(-BAR4)))*(BAR3*DHDX-BAR4*(BAR3*DHDX-BAR4  SURFACE  91
     $*DVDX)*((EXP(BAR4)+EXP(-BAR4))/(EXP(BAR4)-EXP(-BAR4)))+DVDX*(EXP( SURFACE  92
     $BAR4)-EXP(-BAR4))/2.)                                             SURFACE  93
   27 RHO=1./(1.+TAN(PHI)*TAN(A-ALFA)) $ SIGMA=U*SIN(PHI)*TAN(AV-ALFA)/VSURFACE  94
      DGDX=RHO*(DUDX*COS(PHI)+SIGMA*DVDX)                               SURFACE  95
      IF (NFK .EQ. 2) GO TO 28                                          SURFACE  96
      POT=0. $ QOT=0. $ FK=0.                                           SURFACE  97
      GO TO 403                                                         SURFACE  98
C COMPUTE P IN ROTATED XY-SYSTEM                                         SURFACE  99
   28 POT=(-2.*COS(A-ALFA)*DGDX)/GRID $ DAVDX=TAN(AV-ALFA)*DVDX/V       SURFACE 100
      DADX=TAN(A-ALFA)*DGDX/G $ DPHIDX=DADX-DAVDX                       SURFACE 101
      DRHODX=-(RHO**2)*(TAN(A-ALFA)*DPHIDX/(COS(PHI)**2)+               SURFACE 102
     $TAN(PHI)*DADX/(COS(A-ALFA)**2))                                   SURFACE 103
      DSIGDX=SIGMA*(DUDX/U-DVDX/V)+U*(COS(PHI)*TAN(AV-ALFA)*DPHIDX+     SURFACE 104
     $SIN(PHI)*DAVDX/(COS(AV-ALFA)**2))/V                               SURFACE 105
      DHDXX=(COS(ALFA)**2)*HXX+2.*SIN(ALFA)*COS(ALFA)*HXY+(SIN(ALFA)**  SURFACE 106
     $2)*HYY                                                            SURFACE 107
      SMA=6.2831854/(32.2*TT) $ SMAB=1./64.4                            SURFACE 108
      DVDXX=W*(DHDXX+(DHDX**2)*(-4.*SMAB*(V**2)/((2.*SMAB*(V**2)+DEP*   SURFACE 109
     $(1.-(SMA*V)**2))**2)))                                            SURFACE 110
      DIDX=BAR4*(DHDX/DEP-DVDX/V)                                       SURFACE 111
      DIDXX=(DIDX**2)/BAR4+BAR4*((DHDXX-(DHDX**2)/DEP)/DEP-(DVDXX       SURFACE 112
     $-(DVDX**2)/V)/V)                                                  SURFACE 113
   30 DUDXX=(1./(EXP(BAR4)-EXP(-BAR4)))*(-(((EXP(BAR4)-EXP(-BAR4))*.5+  SURFACE 114
     $BAR4)*DVDX+V*DIDX*(1.-BAR4/TANH(BAR4)))*DIDX/TANH(BAR4)           SURFACE 115
     $+((EXP(BAR4)                                                      SURFACE 116
     $-EXP(-BAR4))*.5+BAR4)*DVDXX+DVDX*DIDX*(2.+(EXP(BAR4)+EXP(-BAR4))* SURFACE 117
     $.5-BAR4/TANH(BAR4))+V*(DIDXX*(1.-BAR4/TANH(BAR4))+(DIDX**2)*2/(   SURFACE 118
     $EXP(BAR4)-EXP(-BAR4))*(BAR4/((EXP(BAR4)-EXP(-BAR4))*.5)-(EXP(BAR4 SURFACE 119
     $)+EXP(-BAR4))*.5)))                                               SURFACE 120
   32 DGDXX=RHO*(COS(PHI)*DUDXX+SIGMA*DVDXX)+(COS(PHI)*DRHODX-RHO*SIN   SURFACE 121
     $(PHI)*DPHIDX)*DUDX+(RHO*DSIGDX+SIGMA*DRHODX)*DVDX                 SURFACE 122
C COMPUTE Q IN ROTATED XY-SYSTEM                                         SURFACE 123
      QOT=(G*(SIN(A-ALFA)**2)*DGDXX)/(GRID**2)                          SURFACE 124
C COMPUTE PACKET RAY CURVATURE IN ROTATED XY-SYSTEM                      SURFACE 125
      FK=SIN(A-ALFA)*DGDX/G                                             SURFACE 126
  403 RETURN                                                            SURFACE 127
      END                                                               SURFACE 128
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      SUBROUTINE VELCTY(V,TT,MAXQ,DEP,NFK,U)                            VELCTY
C IN THIS SUBROUTINE THE PHASE VELOCITY AND COLLINEAR GROUP             VELCTY
C VELOCITY ARE COMPUTED.                                                VELCTY
      IF (MAXQ .GT. 1) GO TO 102                                        VELCTY
      BAR=6.2831854/TT $ CXXO=TT*32.2/6.2831854 $ CCC=CXXO              VELCTY
      GO TO 103                                                         VELCTY
  102 CCC=XCXY                                                          VELCTY
  103 IF (NFK .EQ. 2) GO TO 105                                         VELCTY
      V=CXXO                                                            VELCTY
      GO TO 106                                                         VELCTY
  105 DO 1000 M=1,90                                                    VELCTY
      V=CXXO*TANH(BAR*DEP/CCC)                                          VELCTY
      IF (ABS(V-CCC) .LT. 0.00005) GO TO 106                            VELCTY
 1000 CCC=(V+CCC)/2.                                                    VELCTY
  106 XCXY=V $ BAR2=2.*BAR*DEP/V                                        VELCTY
      IF (NFK .EQ. 2) GO TO 3036                                        VELCTY
      U=.5*V                                                            VELCTY
      GO TO 107                                                         VELCTY
 3036 U=.5*V*(1.+2.*BAR2/(EXP(BAR2)-EXP(-BAR2)))                        VELCTY
  107 RETURN                                                            VELCTY
      END                                                               VELCTY

      SUBROUTINE CONDER(DN,TT,V,MAXQ,NFK)                               CONDER
C IN THIS SUBROUTINE W=DN IS COMPUTED.                                  CONDER
      C1=TT/12.5663708 $ C2=6.2831854/(32.2*TT)                         CONDER
      IF (NFK .EQ. 1) GO TO 105                                         CONDER
      C3=C2*V $ A1=C3/(1+C3) $ A2=C3/(1-C3) $ A3=ALOG(1.+C3)            CONDER
      A4=ALOG(1.-C3) $ DN=(DN/C1)*(1./(A1+A2+A3+(-A4)))                 CONDER
  105 RETURN                                                            CONDER
      END                                                               CONDER

      SUBROUTINE PCD(C,E,PCTDIF)                                        PCD
      DIMENSION E(6),C(12)                                              PCD
C IN THIS SUBROUTINE THE DIFFERENCE BETWEEN THE WATER DEPTH AND THE     PCD
C DEPTH COMPUTED FROM THE 12-POINT SURFACE FIT IS DETERMINED FOR        PCD
C THE 4 GRID POINTS CLOSEST TO THE RAY POINT AND THE MAXIMUM            PCD
C PERCENTAGE DIFFERENCE OF THE 4 IS DETERMINED.                         PCD
      IF (C(4)*C(5)*C(8)*C(9) .NE. 0.) GO TO 901                        PCD
      PCTDIF=999.                                                       PCD
      GO TO 902                                                         PCD
  901 P1=ABS((C(4)-(E(1)+E(2)+E(3)+E(4)+E(5)+E(6)))/C(4))               PCD
      P2=ABS((C(5)-(E(1)+E(2)+2.*E(3)+E(4)+E(5)*2.+E(6)*4.))/C(5))      PCD
      P3=ABS((C(8)-(E(1)+E(2)*2.+E(3)+E(4)*4.+E(5)*2.+E(6)))/C(8))      PCD
      P4=ABS((C(9)-(E(1)+E(2)*2.+E(3)*2.+E(4)*4.+E(5)*4.+E(6)*4.))/C(9))PCD
      PCTDIF=100.*AMAX1(P1,P2,P3,P4)                                    PCD
  902 RETURN                                                            PCD
      END                                                               PCD

      SUBROUTINE STORE(X,Y,A,KMAX,TIMEQ,KCIN,KREST)                     STORE
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       STORE
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        STORE
      REAL KP,KF,KS,KPTOL,KFC                                           STORE
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,CCON,FAN,DATE1  STORE
     $,DATE2,CIN,DIR,POP,TT,WBCOP,MOE,DY,DELTAT,SOLTAT,D,HGT,HGTZ,SVX   STORE
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS STORE
     $,DGDX,SVA,TFI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    STORE
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN      STORE
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     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZSTORE
     $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5       STORE
C IN THIS SUBROUTINE THE COORDINATES OF EACH RAY POINT ARE STORED.      STORE
C IF DESIRED, THE LOCATION OF TICK MARKS AT EQUAL TIME INTERVALS        STORE
C ALONG THE RAY ARE COMPUTED AND STORED.                                STORE
      IF (CIN .LE. 0) GO TO 403                                         STORE
      IF (KMAX .GT. 1) GO TO 401                                        STORE
      AT=0.                                                             STORE
C STORE COORDINATES OF POINT                                            STORE
  403 KQ=KMAX+KCIN $ AX(KQ)=X $ AY(KQ)=Y                                STORE
      IF (CIN .LE. 0.) GO TO 205                                        STORE
  402 ZA=A $ ZCXY=G                                                     STORE
      GO TO 205                                                         STORE
  401 ET=TIMEQ-AT                                                       STORE
      IF (CIN-ET) 405,404,403                                           STORE
C RAY POINT AND TICK MARK COINCIDE, STORE WITH NEGATIVE X               STORE
  404 KQ=KMAX+KCIN $ AX(KQ)=-X $ AY(KQ)=Y $ KREST=KREST+1 $ AT=AT+CIN   STORE
      GO TO 402                                                         STORE
C COMPUTE LOCATION OF TICK MARK AND STORE WITH NEGATIVE X               STORE
  405 DSC=(ET-CIN)*(G+ZCXY)*3600./(GRID*2.)                             STORE
      AA=(A+ZA)/2. $ XM=DSC*COS(AA) $ YM=DSC*SIN(AA)                    STORE
      KQ=KMAX+KCIN $ AX(KQ)=-X+XM $ AY(KQ)=Y-YM                         STORE
      KREST=KREST+1 $ KCIN=KCIN+1 $ AT=AT+CIN                           STORE
      GO TO 401                                                         STORE
  205 RETURN                                                            STORE
      END                                                               STORE


      SUBROUTINE DRAW (N,KMAX,KCIN,KREST)                               DRAW
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)                       DRAW
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)                        DRAW
      INTEGER FAN                                                       DRAW
      REAL KR,KF,KS,KRTOL,KFC                                           DRAW
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1  DRAW
     $,DATE2,CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX   DRAW
     $,SVY,SDEP,W,DEP,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS DRAW
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX    DRAW
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FUD,NUMT,INUM,IFLG,RCOUNT,AMM,ANN      DRAW
     $,REFLCT,RFLBUM,REFRCT,RFRBUM,BFK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZDRAW
     $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5       DRAW
C IN THIS SUBROUTINE THE RAYS ARE DRAWN AND NUMBERED.  IF DESIRED,      DRAW
C TICK MARKS ARE DRAWN ON THE RAY AT EQUAL TIME INTERVALS.              DRAW
C SUBROUTINES NUMBER AND PLOT ARE CALLED.                               DRAW
      XN=N $ KMAX=KMAX+KCIN                                             DRAW
      IF (AX(KMAX) .GE. 0.) GO TO 601                                   DRAW
      AX(KMAX)=-AX(KMAX) $ KREST=KREST-1                                DRAW
  601 IF (MOD(N,2) .NE. 0) GO TO 104                                    DRAW
C BEGIN EVEN-NUMBERED RAY WITH THE TERMINAL POINT                       DRAW
      KTWO=KMAX-1 $ KADD=-1 $ LAST=1 $ MC=KREST+1                       DRAW
      IF (FAN .EQ. 0) GO TO 201                                         DRAW
      CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.1,XN,0.0,-1)               DRAW
  201 CALL PLOT (AX(KMAX)/DY,AY(KMAX)/DY,3)                             DRAW
      IF (KMAX .LE. 1) GO TO 106                                        DRAW
      GO TO 105                                                         DRAW
C BEGIN ODD-NUMBERED RAY WITH THE INITIAL POINT                         DRAW
```

```
 104  KTWO=2 $ KADD=1 $ LAST=KMAX $ MC=0.
      IF (FAN .NE. 0) GO TO 111
C NUMBER RAY AT THE INITIAL POINT
      CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)
 111  CALL PLOT (AX(1)/DY,AY(1)/DY,3)
      IF (KMAX .LE. 1) GO TO 106
 105  IF (CIN .LE. 0.) GO TO 300
      IF (AX(KTWO) .LT. 0.) GO TO 302
C DRAW SEGMENT OF RAY
 300  CALL PLOT (AX(KTWO)/DY,AY(KTWO)/DY,2)
      GO TO 303
 302  AX(KTWO)=-AX(KTWO) $ WI=.05 $ MC=MC+KADD
      IF (MOD(MC,10) .NE. 0) GO TO 500
      WI=.10
 500  XPN=AX(KTWO)/DY $ YPN=AY(KTWO)/DY $ KQ=KTWO-KADD
 430  XPL=AX(KQ)/DY $ YPL=AY(KQ)/DY
      IF (ABS(XPN-XPL) .LT. 0.0005 .AND.
     $ABS(YPN-YPL) .LT. 0.0005) GO TO 410
      GO TO 420
C POINTS TOO CLOSE TOGETHER
 410  KQ=KQ-KADD
      GO TO 430
 420  DSC=SQRT((XPN-XPL)**2+(YPN-YPL)**2)

      CALL PLOT(XPN,YPN,2)
      XB=WI*(YPN-YPL)/DSC $ YB=-WI*(XPN-XPL)/DSC
C DRAW TICK MARK ON RAY
      CALL PLOT (XPN+XB,YPN+YB,2)
      CALL PLOT (XPN-XB,YPN-YB,2)
      CALL PLOT (XPN,YPN,2)

 303  IF (KTWO .EQ. LAST) GO TO 106
      KTWO=KTWO+KADD
      GO TO 105
 106  IF (KADD .GE. 0) GO TO 108
      IF (FAN .NE. 0) GO TO 205
      CALL NUMBER (AX(1)/DY,AY(1)/DY,0.1,XN,0.0,-1)
      GO TO 205
 108  IF (FAN .EQ. 0) GO TO 205
C NUMBER RAY AT THE TERMINAL POINT
      CALL NUMBER (AX(KMAX)/DY,AY(KMAX)/DY,0.1,XN,0.0,-1)
 205  RETURN
      END
```

CHAPTER IV USING THE COMPUTER PROGRAM

4.1 Preparation of the Water Depth Grid. Once a coastal area is selected for making wave forecasts a water depth grid must be prepared. Details with numerous illustrations for preparing water depth grids are given by Wilson (1966). It is necessary to obtain charts of the region of interest containing sufficiently detailed bathymetric information.

A water depth grid is rectangular in shape. The value of x varies between 0 and AMM while y varies from 0 to ANN. The values of AMM and ANN are defined by

$$AMM = MM - 1 \qquad (4\text{-}1)$$

$$ANN = NN - 1 \qquad (4\text{-}2)$$

where MM is the number of water depth values in a y-column and NN is the number of columns. The value of MM must be an integral multiple of 16. If another number is preferred the format statement in the computer program used to input the water depth values must be changed. The maximum values of MM and NN depend upon the storage capacity of the computer. In the computer program presented in this report the values of MM and NN are assumed not to exceed 100. If the grid requirements exceed the storage capacity of the computer the coastal region of interest can be divided into several overlapping grids.

The xy-coordinate system is right-handed with the x-axis extending seaward. The direction of the x-axis with respect to true north is defined as CNVRSA. The use of CNVSRA makes it possible to define the input and output ray directions with respect to true north.

The distance between water depths in the x- or y-directions is a grid interval or grid unit and is denoted by GRID. This distance must be small enough for the water depth grid to describe adequately the bottom topography. If it is desirable for rays to start in deep water the grid must extend at least several grid units seaward of the deep water depth of the largest wave period of interest. In this report deep water is defined as any depth greater than 0.64 λ_d where λ_d is the deep water wavelength. This definition of deep water is chosen since the collinear group speed is nearly invariant for greater water depths.

To determine the location of the water depths to be read from a chart lines can be drawn on tracing paper parallel to the x- and y-axes of the grid and separated a distance equal to a grid unit. The tracing paper is placed on the chart and water depths are estimated for the points defined by the intersection of the grid lines. The water depths can be recorded in any system of units.

One of the program options is to have the shoreline drawn on a plot. In order for the location of the shoreline to be computed it is necessary to determine negative values of water depths for at least two grid points landward of the shoreline. The negative values are determined by drawing the reflection of water depth contours on land with respect to the shoreline. Zero water depths can be used to fill out a column for grid points more than two grid units landward of the shore.

4.2 Preparing a Computer Run. The way in which data is prepared for a computer run is illustrated on the coding form in Figure (4-1). Eight types of computer cards are used. The columns available for each parameter are outlined by rectangles. The positions of decimal points for real numbers are indicated. If there is no decimal point the number is an integer and is placed in the rectangle as far to the right as possible. The input parameters must appear on each card as shown, and the card types must be in the order indicated.

Six computer cards are required to input the data for both S and EM. These numbers are used in the surface fitting routine and they are the same for all computer runs.

For the third type of computer card, MXPLOT is the number of runs for a given operation of the computer program. The PROJCT is a 6-character label of any combination of letters and numbers. The label can be used, for example, to indicate a project number. An alternative use is to identify which water depth grid is used for the run. It appears in both the printed output and on the plot. DATE1 and DATE2 are used to date the run. DATE1 can be used for the year and the month in the form ZZ/YY/. DATE2 can be used for the day in the form XX. The DIR is another 6-character label of any combination of letters and numbers. This label appears only on the plot. One possible label is WAVPAK, which can denote that wave packet trajectories are presented. If the rays have a common initial direction, DIR can be used to indicate that direction.

The number of rays for a given run, NOR, is input on the fourth type of computer card. The values of NPT and SK determine the amount of printed output. If NPT is not zero there is printed output for the first ray point, those points which are an integral multiple of SK, and the last point. If NPT is zero printed output occurs only for the initial and terminal ray points. The value of HT is the length of the y-axis of the plot in inches or centimeters. If CIN is not zero tick marks are placed on the rays at equal intervals of travel time given by the value of CIN in

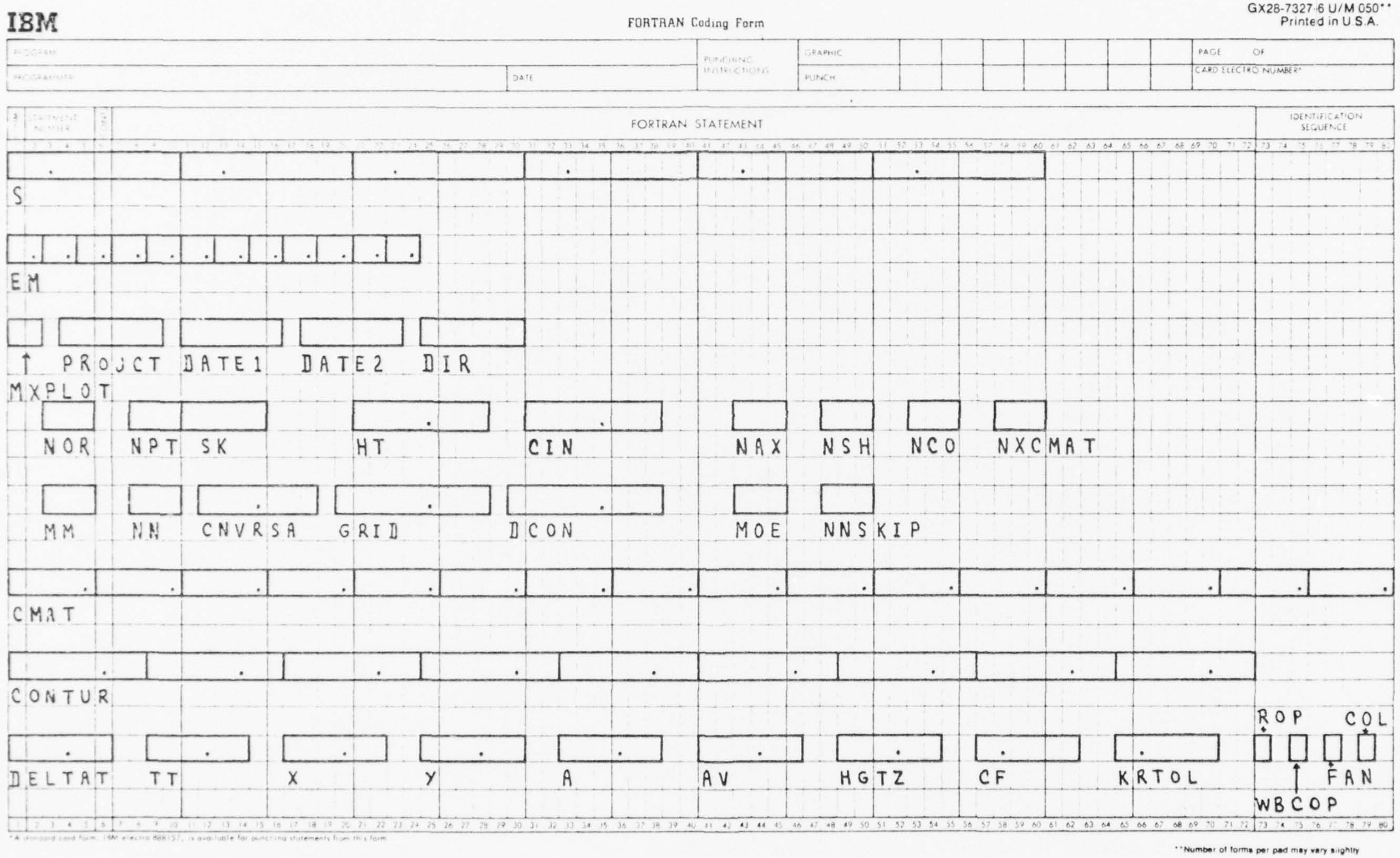


Figure (4-1). FORMAT OF THE INPUT PARAMETERS seconds. If no tick marks are desired CIN is zero.

The x- and y-axes of the plot will be calibrated and labeled if NAX is not zero. If NAX is zero the plot borders are drawn but the axes are not calibrated. The shoreline is drawn on the plot if NSH is not zero. If the shoreline is not desired NSH must be zero. The number of sounding water depths for a plot is NCO. There cannot be more than 9 sounding depths, and they are input on the CONTUR computer card. If NCO is zero there are no sounding depths; in this case the CONTUR card must be removed from the input. If a water depth grid is to be read in the input for the computer run the value of NXCMAT is zero. If NXCMAT is not zero the depth grid for the previous run is used again. This situation can arise if MXPLOT is greater than one.

The fifth type of computer card contains the input dimensions for the water depth grid. A description of the quantities MM, NN, CNVRSA, and GRID are described in the previous section. The angle CNVRSA is given in degrees and GRID is given in feet or meters. The value of DCON is chosen so that the product of DCON and a water depth in CMAT yields a value with units of feet or meters. If English units are to be used in the input and output MOE is zero. If MOE is not zero Metric units are used. The value of NNSKIP is the amount by which y is incremented in selecting columns for locating sounding water depths. For example, if NNSKIP is 15 and NN is 64, sounding water depths are located for the 2, 17, 32, 47, and 62 y-columns.

The sixth type of computer card is used to input the water depth grid (CMAT). The units of CMAT determine the value of DCON. There are 16 water depths on each card. The water depths are entered column by column starting with the first column. There are NN columns. In each column the water depths are entered starting with the land values, if any, and proceeding seaward. There are MM values per column. The format for entering the water depths does not include numbers beyond the decimal points. Near shore it may be desirable to record water depths to the nearest tenth of a foot or meter. On some computer systems it is possible to enter data routinely in this form with the indicated format for CMAT being overridden. If such a capability is not available it may be desirable to alter the format statement for CMAT in MAIN.

If NCO is not zero the CONTUR computer card is used to input the sounding water depths in feet or meters. The number of sounding depths must agree with NCO which should not exceed 9. If NCO is zero the CONTUR card must be removed.

The eighth type of computer card is used to input the particulars for each ray. There must be as many ray cards as declared in the input for NOR. The initial time step interval between ray points in seconds is DELTAT. The wave period in seconds is TT, and X, Y are the initial ray coordinates.[1] The initial wave packet and wavelet directions are A and AV, respectively. The directions are in degrees and are the directions from which the waves come with respect to true north. The initial wave height in feet or meters is HGTZ.

The friction factor is CF. The value of KRTOL determines the accuracy of the calculations of the refraction coefficient with the exception of near reflection points. As a general rule, if accuracy is required to the second decimal point KRTOL is 0.01. If accuracy is desired to the third decimal point KRTOL is 0.001.

To continue a ray beyond a reflection point ROP is set unequal to zero. If ROP is zero a ray is stopped at a reflection point. A test is made to determine if a wave breaks if WBCOP is not zero. If WBCOP is zero there is no test to determine if a wave breaks. If the ray is to be numbered at its terminal point FAN is set unequal to zero. A group of rays should be numbered at their terminal points if they have a common origin. If FAN is zero the ray is numbered at its initial point.

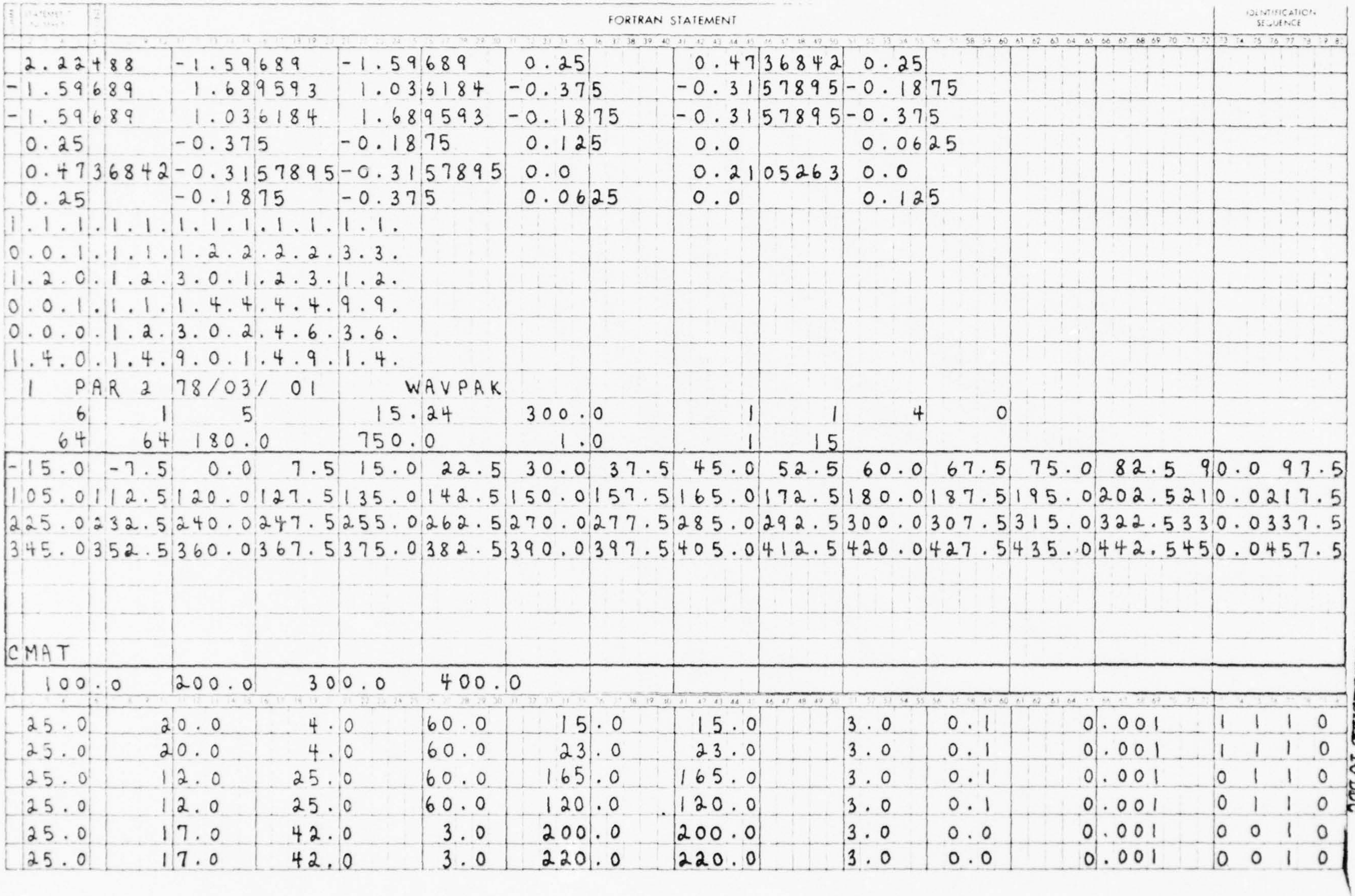
A sample of input data for a computer run with 6 rays is shown in Figure (4-2). Since the water depth contours are parallel, only one of the 64 columns of water depth values is shown in the rectangle labeled CMAT. The computer output for this run is presented in Section (4.6). Therefore, if desired, these input data can be used to check the computer program.

4.3 The Printed Output when NPT ≠ 0. The most detailed computer printout is obtained when NPT ≠ 0. The first thing that occurs in the printout is the page heading. This contains the PROJCT, date, plot number, ray period, ray number, input time step, friction factor, and KRTOL. If at the first ray point this is followed by a statement denoting whether English or Metric units are used in the output. Further, the initial value of the time derivative of the ray separation factor is given. The column headings appear next in the output. Beyond the first point of a ray the page and column headings occur after every 60 lines of additional printout.

The column headings identify the ray particulars which appear in the output. They contain the ray point number MAX, the ray coordinates X, Y, and the water depth H in meters or feet. The wave packet and wavelet directions are denoted, respectively, by PACK and WAVE. These are the directions in degrees from which the waves come with respect to true north. The distance in grid units between ray points is given by D, and FK is the packet ray curvature in radians per grid unit. The angle in degrees by which the x'y'-coordinate system is rotated with respect to the positive X-axis for computations is given by ALFA. The geometric group speed, collinear group speed, and phase speed are denoted, respectively, by G, U,

---

[1] The initial ray points should be at least two grid units from a grid boundary.



```
 2.22488  -1.59689  -1.59689   0.25      0.4736842 0.25
-1.59689   1.689593  1.036184 -0.375    -0.3157895-0.1875
-1.59689   1.036184  1.689593 -0.1875   -0.3157895-0.375
 0.25     -0.375    -0.1875    0.125     0.0       0.0625
 0.4736842-0.3157895-0.3157895 0.0       0.2105263 0.0
 0.25     -0.1875   -0.375     0.0625    0.0       0.125
1.1.1.1.1.1.1.1.1.1.1.1.1.1.
0.0.1.1.1.1.2.2.2.2.3.3.
1.2.0.1.2.3.0.1.2.3.1.2.
0.0.1.1.1.1.4.4.4.4.9.9.
0.0.0.1.2.3.0.2.4.6.3.6.
1.4.0.1.4.9.0.1.4.9.1.4.
 1  PAR 2 78/03/ 01     WAVPAK
    6    1    5       15.24    300.0        1    1    4    0
   64   64 180.0     750.0       1.0        1   15
-15.0 -7.5  0.0  7.5 15.0 22.5 30.0 37.5 45.0 52.5 60.0 67.5 75.0 82.5 90.0 97.5
105.0112.5120.0127.5135.0142.5150.0157.5165.0172.5180.0187.5195.0202.5210.0217.5
225.0232.5240.0247.5255.0262.5270.0277.5285.0292.5300.0307.5315.0322.5330.0337.5
345.0352.5360.0367.5375.0382.5390.0397.5405.0412.5420.0427.5435.0442.5450.0457.5
CMAT
  100.0   200.0   300.0   400.0
 25.0    20.0     4.0    60.0    15.0    15.0     3.0  0.1      0.001   1 1 1 0
 25.0    20.0     4.0    60.0    23.0    23.0     3.0  0.1      0.001   1 1 1 0
 25.0    12.0    25.0    60.0   165.0   165.0     3.0  0.1      0.001   0 1 1 0
 25.0    12.0    25.0    60.0   120.0   120.0     3.0  0.1      0.001   0 1 1 0
 25.0    17.0    42.0     3.0   200.0   200.0     3.0  0.0      0.001   0 0 1 0
 25.0    17.0    42.0     3.0   220.0   220.0     3.0  0.0      0.001   0 0 1 0
```

Figure (4-2). SAMPLE INPUT DATA FOR A COMPUTER RUN and V. The units are in meters/second or feet/second. The wave height is HGT in meters or feet. The shoaling coefficient is identified by KS, and KF represents the friction coefficient. The number of breakups, if any, of the initial time step is NO. The refraction coefficient is defined by KR. An estimate of how well the computed water bottom surface fits the actual water depths is given by PCTDIF (see the NOTATION). The smaller the value the better the fit. The units of H, HGT, G, U, and V are identified in the output.

The travel time along the wave packet trajectory does not appear in the output. However, it can be determined by subtracting one from MAX and multiplying the result by the input time step.

The ray particulars are printed out for the first ray point, those points which are an integral multiple of SK, and the last point. Printout occurs for a reflection point should one occur. Note that the number of ray points, NO, which occur when there is a breakup of the input time step interval is not counted in MAX. There is no printout for ray points which occur within a breakup.

If there is printed output and a ray is near a reflection point (Sections (2.3), b, (2.5), d, (3.8), and 3.9)) an asterisk appears in the printout to the left of the value of FK. If there is printed output when there is a breakup of the input time step interval due to the calculations of the refraction coefficient (Sections (2.5), c and (3.9)) an asterisk appears in the printout to the left of the value of KR. If there is no printout (SK > 1) the asterisk appears with the ray particulars for the next ray point where printout occurs. Only the asterisk with FK appears in the next printout should both conditions for an asterisk be satisfied at preceding ray points where there is no printed output. If the ray is at a reflection point (Sections (2.3), b and (3.8)) an asterisk occurs in the printout to the left of the value of MAX.

A number of descriptive printouts appear with the ray particulars when certain types of calculations occur or when a ray terminates. If the ray curvature of the wave packet is averaged in computing a ray point (Section (3.8)) the following printout appears.

(1) PACKET CURVATURE AVERAGED

If SK = 1 this descriptive printout follows the ray pariculars. If SK > 1 the descriptive printout occurs even if there is no printed output of the ray particulars. In this case, the curvature is averaged for a ray point between the ray points preceding and following the descriptive printout.

When there is a reflection one of three descriptive printouts occurs (Section (2.3), b).

(2) REFLECTION: SNELLS LAW WITH PHASE VELOCITY

(3) REFLECTION: PACKET CURVATURE ITERATION NOT CONVERGING

(4) REFLECTION: NEAR REFLECTION POINT

The ray point where one of these three descriptive printouts occurs is the last ray point if ROP = 0.

When a ray terminates one of the following descriptive printouts can appear in the output.

(5) DIMENSION OF OUTPUT-ARRAYS EXCEEDED

(6) RAY REACHED SHORE

(7) RAY REACHED GRID BOUNDARY

(8) PACKET CURVATURE ITERATION NOT CONVERGING

(9) CAUSTIC OR FOCAL POINT

(10) WAVE BREAKS

(11) REFLECTION HANG-UP

(12) BREAKUP TIME STEP LESS THAN 0.5 SECOND

Printout (5) occurs if the sum of the number of ray points and tick marks is equal to or greater than the array dimension MMAX. Printout (6) is obtained if the water depth becomes zero or negative. Printout (7) results if the ray point is within 1.5 grid units of a grid boundary. The conditions for a reflection point are not satisfied if Printout (8) occurs. Printout (9) is produced if the ray separation factor becomes zero or negative. The condition for Printout (10) is given in Section (2.7). Printout (11) is obtained if there are successive reflections at the same ray point. Printout (12) can occur if the calculation time step becomes too small in either calculating the ray path near a reflection point (Section (2.3), b) or in calculating the ray separation factor (Section (2.5), c).

4.4 The Printed Output when NPT = 0. When NPT = 0 there is printed output at only the first and last ray points. The page heading contains PROJCT, the date, and plot number. A statement signifies whether English or Metric units are employed. The column headings define the ray number N, the wave period, MAX, X, Y, H, PACK, WAVE, HGT, the input time step, the friction factor, and KRTOL. All of these ray particulars appear in the printed output at the first ray point. At the last ray point the input time step, the friction factor, and KRTOL are not repeated. In their place is a descriptive statement which explains why the ray terminated.

When NPT = 0, Printout (1) does not appear. There is no descriptive printout for a reflection unless the reflection occurs at the last ray point (ROP = 0). Then the following descriptive printout occurs.

(13) REFLECTION

The remaining descriptive printouts are the same as discussed in Section (4.3).

4.5 The Plots. Each plot contains a label consisting of PROJCT, the date, the scale factor, the time in seconds between tick marks on a ray, if any, the plot number, and DIR. If NAX ≠ 0 the axes of the plot are calibrated and labeled. If NSH ≠ 0 the shoreline is drawn. If NCO ≠ 0 sounding water depth values are labeled. Each ray is numbered. If FAN = 0 the number appears at the initial ray point, and if FAN ≠ 0 the ray is numbered at its terminal point.

4.6 Examples of Computer Output. Figures (4-3) through (4-8) show the printed output for the 6 rays of the sample input data presented in Figure (4-2). The plot of the rays is shown in Figure (4-9). The examples illustrate rays beginning both at an intermediate water depth and in deep water. Three different wave periods are considered. The second ray undergoes a reflection (Section (2.1), b and Section (2.5), d). For the last two rays the friction factor is assumed to be zero. Tick marks and sounding water depths are shown on the plot.

Figure (4-10) shows a portion of the Gulf of Mexico off the southwestern Florida coast. A water depth grid was prepared for this region with GRID = 14886.2 feet (4.537 km) and CNVRSA = 180°. A ray plot for this region is shown in Figure (4-11). Figure (4-12) contains the printed output for this plot when NPT = 0. The first two rays start at an intermediate water depth, whereas the remaining rays begin in deep water. Figure (4-13) displays printed output for the first portion of ray number 1. Since the water depth contours are not parallel there is a variation in ALFA.

Ray number 2 has a reflection. Figure (4-14) shows a listing of the ray particulars near the reflection point. The wave packet and wavelet angles in the xy- and x'y'-coordinate systems are defined by Equations (3-1), (3-2), (2-26), and (2-27). At the reflection point the angles in the xy-coordinate system are $\theta_C = 274.86^\circ$ and $\gamma_C = 1.94^\circ$. In the x'y'-coordinate system $\theta' = 2.41^\circ$ and $\gamma' = 89.49^\circ$.

Ray number 12 illustrates the importance of examining the ray particulars in the printout. Figure (4-15) shows the printed output for this ray. A message in the output states there is a reflection. However, a reflection is not likely since H, θ', γ', FK, and G do not exhibit the behavior characteristic of a reflection. At the ray point where the reflection is indicated $\theta' = 115.78^{\circ}$ and $\gamma' = 117.52^{\circ}$. This false reflection is the result of a large change in ALFA between successive ray points. When this occurs the water depth grid is not sufficiently detailed to adequately represent the changing water depth contours.

```
PROJECT NO. PAR 2,  78/03/01     ,      PLOT NO.  1, PERIOD= 20.0SEC., RAY NO.  1, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,V(METER/SECOND).                                  D(BETA)/DT = -2.201E-04

MAX X        Y        H         PACK   WAVE    D          FK          ALFA     G      U      V      HGT      KS       KF         NO KR      PCTDIF

  1    4.00  60.00    15.00   15.00  15.00  3.75E-01 -5.48E-02     -.00   11.25  11.25  11.83  3.0000  1.0000  1.0000            1.0000    .01
  5    5.61  59.50    27.04   19.07  19.89  4.73E-01 -3.26E-02     -.00   14.19  14.19  15.55  2.5065   .8901   .9283     2*     1.0111    .01
 10    8.00  58.58    45.02   22.57  25.16  5.56E-01 -1.68E-02     -.00   16.67  16.69  19.43  2.2695   .8213   .9004     2*     1.0230    .01
 15   10.64  57.43    64.81   24.49  29.45  5.94E-01 -7.25E-03     -.01   18.00  18.07  22.47  2.1755   .7904   .8904            1.0303    .01
 20   13.40  56.14    85.49   25.20  32.86  6.15E-01 -1.33E-03     -.01   18.48  18.65  24.81  2.1413   .7800   .8858            1.0331    .01
 25   16.19  54.83   106.40   25.09  35.55  6.15E-01  2.19E-03     -.01   18.41  18.72  26.57  2.1384   .7815   .8834            1.0325    .01
 30   18.95  53.55   127.13   24.53  37.58  6.04E-01  4.04E-03     -.01   18.02  18.50  27.87  2.1529   .7899   .8821            1.0299    .01
 35   21.66  52.34   147.45   23.77  39.10  5.87E-01  4.74E-03     -.01   17.50  18.14  28.83  2.1763   .8017   .8814            1.0267    .01
 40   24.30  51.20   167.27   22.99  40.22  5.69E-01  4.74E-03     -.02   16.95  17.75  29.51  2.2037   .8145   .8809            1.0235    .01
 45   26.88  50.12   186.59   22.25  41.04  5.51E-01  4.34E-03     -.02   16.44  17.36  30.01  2.2304   .8271   .8807            1.0206    .01
 50   29.39  49.11   205.44   21.61  41.63  5.36E-01  3.77E-03     -.02   15.99  17.02  30.36  2.2555   .8385   .8805            1.0182    .01
 55   31.85  48.15   223.87   21.08  42.05  5.23E-01  3.16E-03     -.02   15.62  16.73  30.62  2.2776   .8485   .8805            1.0163    .01
 60   34.26  47.23   241.98   20.64  42.36  5.12E-01  2.58E-03     -.02   15.32  16.49  30.80  2.2962   .8568   .8804            1.0147    .01
 65   36.64  46.34   259.80   20.30  42.57  5.04E-01  2.07E-03     -.03   15.08  16.29  30.92  2.3115   .8636   .8804            1.0135    .01
 70   38.99  45.48   277.39   20.03  42.73  4.97E-01  1.63E-03     -.03   14.89  16.14  31.01  2.3238   .8691   .8804            1.0125    .01
 75   41.31  44.64   294.82   19.81  42.84  4.92E-01  1.27E-03     -.03   14.74  16.02  31.08  2.3336   .8734   .8803            1.0117    .01
 80   43.62  43.81   312.11   19.65  42.92  4.88E-01  9.86E-04     -.03   14.63  15.92  31.13  2.3412   .8768   .8803            1.0110    .01
 85   45.91  43.00   329.29   19.52  42.97  4.85E-01  7.57E-04     -.03   14.54  15.85  31.16  2.3470   .8794   .8803            1.0105    .01
 90   48.19  42.19   346.40   19.43  43.02  4.83E-01  5.78E-04     -.03   14.47  15.79  31.18  2.3514   .8814   .8803            1.0101    .01
 95   50.46  41.39   363.44   19.36  43.04  4.81E-01  4.39E-04     -.04   14.42  15.75  31.20  2.3547   .8830   .8803            1.0098    .01
100   52.73  40.60   380.43   19.30  43.06  4.80E-01  3.32E-04     -.04   14.39  15.72  31.21  2.3571   .8841   .8803            1.0095    .01
105   54.99  39.81   397.38   19.26  43.08  4.79E-01  2.50E-04     -.04   14.36  15.69  31.22  2.3589   .8850   .8803            1.0092    .01
110   57.24  39.02   414.27   19.25  43.11  4.76E-01  0.            -.04   14.29  15.62  31.24  2.3648   .8872   .8803            1.0092    .01
115   59.48  38.24   431.12   19.25  43.11  4.76E-01  0.            -.04   14.29  15.62  31.24  2.3648   .8872   .8803            1.0092    .01
120   61.73  37.45   447.97   19.25  43.11  4.76E-01  0.            -.04   14.29  15.62  31.24  2.3648   .8872   .8803            1.0092    .01
                                                                                  RAY REACHED GRID BOUNDARY
```

Figure (4-3). LISTING FOR RAY NUMBER 1 OF SAMPLE INPUT DATA

```
PROJECT NO. PAR 2,  78/03/01     ,     PLOT NO.  1, PERIOD= 20.0SEC., RAY NO.  2, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  C,U,V(METER/SECOND).                          D(BETA)/DT = -5.264E-04

MAX X       Y        H          BACK    WAVE    D         FK          ALFA     C      U      V     HGT      KS       KF        NO KR        PCTDIF

   1    4.70   60.00     15.30    23.0    23.00   3.75E-01  -3.27E-02   -.00   11.25  11.25  11.83   3.0000  1.0000   1.0000          1.0000     .01
   5    5.51   59.05     26.30    34.22   30.48   4.08E-01  -5.00E-02   -.00   14.05  14.05  15.35   2.5464   .8947    .9276    2*    1.0272     .01
  10    7.05   57.60     42.35    34.68   33.72   5.45E-01  -2.71E-02   -.00   16.77  16.41  18.93   2.3517   .8288    .8934    2*    1.0583     .01
  15    9.93   56.22     59.45    37.74   45.95   5.87E-01  -1.16E-02   -.01   17.61  17.79  21.74   2.2787   .7994    .8811    2*    1.0789     .01
  20   12.26   54.38     76.35    36.66   52.22   5.99E-01   4.99E-04   -.01   17.97  16.43  23.33   2.2529   .7910    .8750    2*    1.0849     .01
  25   14.53   52.87     91.45    37.06   57.04   5.85E-01   1.13E-02   -.01   17.58  18.73  25.65   2.2519   .7996    .8717    2*    1.0767     .01
  30   16.93   50.93    111.74    35.06   62.07   5.66E-01   2.06E-02   -.01   16.53  18.09  26.34   2.2772   .8249    .8597    1*    1.0581     .01
  35   19.12   49.07    128.03    31.28   67.30   5.26E-01   2.90E-02   -.01   14.94  18.48  27.94   2.3397   .8676    .8684          1.0350     .01
  40   21.18   48.02    143.86    26.89   71.30   4.47E-01   3.58E-02   -.01   13.01  18.21  28.68   2.4513   .9298    .8676          1.0129     .01
  45   23.05   47.37    157.70    22.41   74.74   3.79E-01   4.11E-02   -.02   10.96  17.94  29.21   2.6206  1.0128    .8670           .9948     .01
  50   24.52   46.78    169.60    18.19   77.71   3.12E-01*  4.59E-02   -.02    8.58  17.70  29.58   2.8552  1.1192    .8665    1      .9814     .01
  55   25.93   46.40    179.47    14.07   80.30   2.51E-01*  5.16E-02   -.02    7.14  17.55  29.84   3.1696  1.2552    .8661    1      .9719     .01
  60   26.98   46.16    187.34    10.95   82.61   1.93E-01*  6.15E-02   -.02    5.46  17.35  30.03   3.5984  1.4351    .8659    1      .9654     .01
  65   27.72   46.03    193.33     7.84   84.70   1.41E-01*  7.66E-02   -.02    3.92  17.24  30.15   4.2265  1.6939    .8654    1      .9611     .01
  70   28.37   45.96    197.44     4.94   86.65   9.13E-02*  1.13E-01   -.02    2.47  17.10  30.23   5.3031  2.1323    .8651    1      .9583     .01
  75   28.64   45.94    199.52     2.11   88.58   4.48E-02*  2.50E-01   -.02    1.06  17.12  30.27   8.0901  3.2622    .8645    1      .9569     .01
*  77  28.70   45.94    200.28      .86   89.41   2.52E-02   5.06E-01   -.02     .47  17.11  30.28  12.6433  5.0984    .8641           .9566
REFLECTION:    SNELLS LAW WITH PHASE VELOCITY
  80   28.64   45.94    199.50   177.92   91.90   2.52E-02*  2.40E-01   -.02    1.07  17.12  30.27   8.0451  3.2442    .8678    1      .9559     .01
  85   28.39   45.97    197.89   175.58   93.11   6.09E-02*  1.19E-01   -.02    2.29  17.15  30.23   5.5480  2.2356    .8635    1      .9580     .01
  90   27.90   45.78    194.40   173.34   94.65   1.09E-01*  8.11E-02   -.02    3.53  17.22  30.17   4.4398  1.7657    .8632    1      .9601     .01
  95   27.26   45.70    189.43   170.29   96.75   1.54E-01*  6.37E-02   -.02    4.91  17.31  30.07   3.7763  1.5140    .8629    1      .9635     .01
 100   26.36   45.60    182.72   167.29   99.03   2.03E-01*  5.36E-02   -.02    6.41  17.44  29.92   3.3189  1.3241    .8626    1      .9686     .01
 105   25.22   45.31    174.15   163.86  101.10   2.58E-01*  4.74E-02   -.02    8.07  17.61  29.71   2.9794  1.1801    .8622    1      .9760     .01
 110   23.83   44.85    163.75   160.08  103.61   3.17E-01*  4.28E-02   -.02    9.89  17.82  29.41   2.7193  1.0661    .8618    1      .9866     .01
 115   22.18   44.39    151.35   155.95  106.87   3.81E-01*  3.87E-02   -.01   11.83  18.07  28.98   2.5213   .9748    .8613    1     1.0012     .01
 120   20.29   43.25    137.03   151.66  110.41   4.47E-01   3.38E-02   -.01   13.76  18.33  28.38   2.3790   .9032    .8606          1.0202     .01
 125   18.21   42.37    121.55   147.59  114.40   5.09E-01   2.57E-02   -.01   15.56  18.59  27.55   2.2845   .8501    .8596          1.0422     .01
 130   15.95   40.52    104.86   144.28  119.11   5.57E-01   1.71E-02   -.01   16.95  18.73  26.40   2.2292   .8145    .8580          1.0632     .01
 135   13.67   38.79     87.51   142.27  124.35   5.92E-01   7.23E-03   -.01   17.77  18.68  25.03   2.2007   .7955    .8556    2*    1.0778     .01
 140   11.32   36.95     69.91   141.95  130.25   5.96E-01  -3.87E-03   -.01   17.89  18.27  23.11   2.1881   .7927    .8513    2*    1.0808     .01
 145    8.90   35.10     52.37   143.66  136.95   5.74E-01  -1.71E-02   -.01   17.21  17.32  21.67   2.1875   .8084    .8434    2*    1.0694     .01
 150    6.71   33.60     35.34   147.68  144.66   5.18E-01  -3.53E-02   -.00   15.53  15.55  17.52   2.2054   .8511    .8265    2*    1.0451     .01
 155    4.07   32.43     19.75   154.31  153.61   4.21E-01  -6.56E-02   -.00   12.59  12.59  13.46   2.2473   .9450    .7825    2*    1.0130     .01
 160    2.99   31.78      7.40   163.59  163.67   2.74E-01  -1.32E-01   -.00    8.21   8.21   8.42   2.1760  1.1704    .6308    4*     .9825  999.00
 165    2.00   31.59       .60   175.25  175.41   3.06E-02  -5.15E-01   -.00    2.42   2.42   2.42    .4978  2.1559    .0798    4*     .9644  999.00
                                                                                  RAY REACHED SHORE
```

Figure (4-4). LISTING FOR RAY NUMBER 2 OF SAMPLE INPUT DATA

```
PROJECT NO. PAR 2,  78/03/01     ,      PLOT NO.   1, PERIOD= 12.0SEC., RAY NO.   3, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.   H,HGT(METER).   G,U,V(METER/SECOND).                              D(BETA)/DT = -0.

MAX X       Y        H          PACK    WAVE   D              FK            ALFA        G       U       V      HGT        KS         KF          NO KR          PCTDIF

  1   25.00    60.00    172.49 165.00 165.00   3.12E-01   0.                -.02      9.37    9.37  18.74   3.0000   1.0000   1.0000          1.0000      .01
  5   23.79    59.68    163.44 165.00 165.00   3.12E-01   0.                -.02      9.37    9.37  18.74   3.0000   1.0000   1.0000          1.0000      .01
 10   22.28    59.27    152.13 165.00 165.00   3.12E-01   0.                -.01      9.37    9.37  18.74   3.0000   1.0000   1.0000          1.0000      .01
 15   20.77    58.87    140.80 164.99 165.01   3.14E-01   4.80E-04          -.01      9.42    9.42  18.73   2.9922    .9974   1.0000          1.0000      .01
 20   19.26    58.46    129.41 164.94 165.02   3.15E-01   8.01E-04          -.01      9.46    9.46  18.72   2.9868    .9956   1.0000          1.0000      .01
 25   17.73    58.05    117.97 164.85 165.04   3.17E-01   1.31E-03          -.01      9.51    9.51  18.69   2.9781    .9925   1.0000          1.0002      .01
 30   16.20    57.63    106.46 164.71 165.08   3.19E-01   2.09E-03          -.01      9.61    9.61  18.65   2.9642    .9877    .9999          1.0004      .01
 35   14.64    57.20     94.82 164.47 165.15   3.24E-01   3.21E-03          -.01      9.76    9.76  18.57   2.9427    .9802    .9998          1.0009      .01
 40   13.06    56.76     82.98 164.11 165.27   3.31E-01   4.64E-03          -.01      9.98    9.98  18.41   2.9110    .9691    .9995          1.0017      .01
 45   11.45    56.29     70.85 163.59 165.50   3.41E-01   6.11E-03          -.01     10.30   10.30  18.13   2.8672    .9540    .9988          1.0030      .01
 50    9.78    55.79     58.34 162.95 165.93   3.54E-01   6.75E-03          -.01     10.70   10.71  17.61   2.8126    .9361    .9970          1.0046      .01
 55    8.05    55.25     45.39 162.32 166.68   3.67E-01   4.69E-03          -.00     11.08   11.11  16.69   2.7546    .9198    .9921          1.0062      .01
 60    6.28    54.68     32.12 162.19 167.97   3.73E-01  -3.62E-03          -.00     11.16   11.22  15.10   2.7069    .9165    .9782          1.0065      .01
 65    4.54    54.14     19.08 163.52 170.09   3.45E-01  -2.51E-02          -.00     10.35   10.42  12.46   2.6733    .9514    .9340    2*    1.0028      .01
 70    3.03    53.74      7.74 167.71 173.34   2.59E-01  -7.98E-02          -.00      7.78    7.81   8.40   2.4881   1.0978    .7603    2*     .9936      .01
 75    2.11    53.59       .82 175.62 177.77   9.34E-02  -3.41E-01          -.00      2.80    2.80   2.83    .7026   1.8290    .1302    4*     .9838  999.00
                                                                                               PACKET CURVATURE AVERAGED
                                                                                               RAY REACHED SHORE
 76    2.03    53.59       .24 177.06 178.79   5.08E-02  -8.03E-01          -.00      1.52    1.52   1.53    .1902   2.4810    .0260    4*     .9828  999.00
```

Figure (4-5). LISTING FOR RAY NUMBER 3 OF SAMPLE INPUT DATA

```
PROJECT NO. PAR 2,  78/03/01     ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO.  4, DELTAT= 25.00, CF= .100000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,V(METER/SECOND).                                  D(BETA)/DT = -0.

MAX X       Y       H        PACK   WAVE   D          FK          ALFA      G       U       V     HGT      KS       KF       NO KR      PCTDIF

   1   25.00   60.00  172.49 120.00 120.00  3.12E-01  0.              -.02     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
   5   24.38   58.92  167.81 120.00 120.00  3.12E-01  0.              -.02     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
  10   23.59   57.57  161.95 120.00 120.00  3.12E-01  0.              -.02     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
  15   22.81   56.21  156.09 120.00 120.00  3.12E-01  0.              -.02     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
  20   22.03   54.86  150.23 120.00 120.00  3.12E-01  0.              -.01     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
  25   21.25   53.51  144.38 120.00 120.00  3.12E-01  0.              -.01     9.37    9.37 18.74  3.0000  1.0000  1.0000           1.0000    .01
  30   20.47   52.15  139.50 119.86 120.09  3.14E-01  1.77E-03       -.01     9.43    9.43 18.73  2.9919   .9971  1.0000    1*     1.0002    .01
  35   19.69   50.78  132.64 119.70 120.12  3.15E-01  2.29E-03       -.01     9.44    9.44 18.72  2.9918   .9962  1.0000           1.0011    .01
  40   18.91   49.41  126.81 119.47 120.16  3.15E-01  2.93E-03       -.01     9.47    9.47 18.71  2.9932   .9950  1.0000           1.0027    .01
  45   18.13   48.04  121.01 119.13 120.23  3.16E-01  3.72E-03       -.01     9.49    9.50 18.70  2.9968   .9976  1.0000           1.0054    .01
  50   17.37   46.65  115.24 118.61 120.31  3.17E-01  4.65E-03       -.01     9.53    9.53 18.69  3.0029   .9918   .9999           1.0094    .01
  55   16.61   45.25  109.54 118.35 120.42  3.19E-01  5.73E-03       -.01     9.57    9.58 18.66  3.0124   .9895   .9999           1.0149    .01
  60   15.85   43.84  103.89 117.78 120.57  3.20E-01  6.92E-03       -.01     9.62    9.63 18.64  3.0260   .9869   .9998           1.0223    .01
  65   15.11   42.42   98.34 117.10 120.77  3.22E-01  8.17E-03       -.01     9.68    9.70 18.60  3.0446   .9838   .9997           1.0319    .01
  70   14.38   40.97   92.98 116.29 121.03  3.25E-01  9.38E-03       -.01     9.75    9.79 18.55  3.0688   .9802   .9996           1.0440    .01
  75   13.67   39.50   87.55 115.38 121.35  3.27E-01  1.04E-02       -.01     9.83    9.88 18.48  3.0994   .9764   .9994           1.0588    .01
  80   12.98   38.01   82.36 114.36 121.77  3.30E-01  1.12E-02       -.01     9.91   10.00 18.40  3.1364   .9724   .9990           1.0763    .01
  85   12.31   36.49   77.34 113.29 122.27  3.33E-01  1.15E-02       -.01    10.00   10.12 18.30  3.1793   .9683   .9985           1.0961    .01
  90   11.67   34.95   72.50 112.18 122.88  3.35E-01  1.13E-02       -.01    10.08   10.25 18.18  3.2264   .9644   .9977           1.1177    .01
  95   11.05   33.39   67.84 111.11 123.59  3.38E-01  1.06E-02       -.01    10.15   10.40 18.03  3.2749   .9609   .9966           1.1339    .01
 100   10.45   31.80   63.36 110.13 124.42  3.40E-01  9.36E-03       -.01    10.22   10.54 17.85  3.3203   .9578   .9950           1.1613    .01
 105    9.87   30.19   59.04 109.30 125.37  3.42E-01  7.63E-03       -.01    10.27   10.69 17.65  3.3574   .9553   .9928           1.1800    .01
 110    9.32   28.57   54.87 108.65 126.45  3.43E-01  5.49E-03       -.01    10.31   10.83 17.41  3.3803   .9535   .9897           1.1940    .01
 115    8.77   26.94   50.79 108.23 127.66  3.44E-01  2.92E-03       -.01    10.33   10.96 17.13  3.3826   .9523   .9856           1.2013    .01
 120    8.24   25.30   46.77 108.09 129.03  3.45E-01 -1.17E-04       -.00    10.34   11.07 16.81  3.3582   .9519   .9799           1.2001    .01
 125    7.70   23.66   42.74 108.28 130.60  3.45E-01 -3.80E-03       -.00    10.33   11.17 16.43  3.3016   .9524   .9722           1.1886    .01

 130    7.15   22.03   38.64 108.87 132.40  3.44E-01 -8.47E-03       -.00    10.30   11.23 15.98  3.2976   .9541   .9617           1.1653    .01
 135    6.58   20.42   34.39 110.00 134.53  3.41E-01 -1.48E-02       -.00    10.22   11.24 15.43  3.0718   .9574   .9471           1.1292    .01
 140    5.98   18.83   29.86 111.79 137.10  3.37E-01 -2.39E-02       -.00    10.10   11.17 14.73  2.8918   .9635   .9269    2*     1.0793    .01
 145    5.33   17.30   24.95 114.71 140.31  3.29E-01 -3.85E-02       -.00     9.88   10.95 13.82  2.6619   .9741   .8974    2*     1.0150    .01
 150    4.60   15.86   19.47 119.36 144.52  3.16E-01 -6.46E-02       -.00     9.48   10.47 12.56  2.3774   .9945   .8510    2*      .9353    .01
 155    3.77   14.57   13.27 127.03 150.36  2.89E-01 -1.19E-01       -.00     8.68    9.45 10.70  2.0279  1.0392   .7700    4*      .8448    .01
 160    2.87   13.60    6.52 140.46 159.00  2.31E-01 -2.68E-01       -.00     6.92    7.30  7.76  1.5556  1.1640   .5964    8*      .7470 999.00
 165    2.12   13.17     .92 164.30 172.06  9.79E-02 -1.06E+00       -.00     2.94    2.97  2.99   .5232  1.7859   .1459   16*      .6692 999.00
                                                                                          RAY REACHED SHORE
 166    2.04   13.15     .33 170.51 175.22  5.97E-02 -1.83E+00       -.00     1.79    1.80  1.80   .2180  2.2870   .0481   16*      .6613 999.00
```

Figure (4-6). LISTING FOR RAY NUMBER 4 OF SAMPLE INPUT DATA

```
PROJECT NO. PAR 2, 78/03/01    ,     PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  5, DELTAT= 25.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,V(METER/SECOND).                          D(BETA)/DT = -0.

MAX X       Y       H       PACK   WAVE    D          FK          ALFA      G      U      V      HGT      KS       KF        NO KR       PCTDIF

   1   42.00    3.00  299.99 200.00 200.00  4.43E-01   0.           -.03  13.28  13.28  26.55  3.0000  1.0000  1.0000          1.0000     .01
   5   40.34    3.61  287.51 200.00 199.99  4.43E-01  -2.82E-04    -.03  13.34  13.34  26.54  2.9930   .9977  1.0000          1.0000     .01
  10   38.25    4.37  271.83 200.04 199.98  4.45E-01  -4.02E-04    -.03  13.37  13.37  26.53  2.9900   .9966  1.0000          1.0000     .01
  15   36.15    5.13  256.13 200.10 199.97  4.47E-01  -5.69E-04    -.02  13.41  13.41  26.51  2.9859   .9952  1.0000          1.0001     .01
  20   34.05    5.90  240.37 200.18 199.95  4.48E-01  -7.99E-04    -.02  13.46  13.46  26.49  2.9802   .9931  1.0000          1.0003     .01
  25   31.94    6.68  224.54 200.30 199.92  4.51E-01  -1.11E-03    -.02  13.54  13.54  26.45  2.9724   .9903  1.0000          1.0005     .01
  30   29.82    7.47  208.63 200.46 199.88  4.54E-01  -1.51E-03    -.02  13.65  13.65  26.40  2.9619   .9863  1.0000          1.0010     .01
  35   27.68    8.27  192.60 200.68 199.81  4.59E-01  -2.02E-03    -.02  13.79  13.80  26.32  2.9480   .9811  1.0000          1.0016     .01
  40   25.52    9.09  176.39 200.98 199.72  4.65E-01  -2.63E-03    -.02  13.99  13.99  26.19  2.9299   .9742  1.0000          1.0025     .01
  45   23.33    9.94  159.97 201.37 199.56  4.73E-01  -3.30E-03    -.02  14.24  14.25  26.00  2.9072   .9655  1.0000          1.0037     .01
  50   21.10   10.82  143.28 201.86 199.34  4.83E-01  -3.93E-03    -.01  14.56  14.57  25.71  2.8801   .9550  1.0000          1.0053     .01
  55   18.83   11.75  126.25 202.43 198.99  4.95E-01  -4.32E-03    -.01  14.93  14.96  25.27  2.8498   .9431  1.0000          1.0073     .01
  60   16.52   12.72  108.87 203.06 198.48  5.08E-01  -4.15E-03    -.01  15.32  15.37  24.61  2.8193   .9310  1.0000          1.0094     .01
  65   14.15   13.74   91.12 203.60 197.73  5.20E-01  -2.90E-03    -.01  15.66  15.74  23.65  2.7940   .9209  1.0000          1.0113     .01
  70   11.75   14.80   73.12 203.83 196.66  5.27E-01   1.53E-04    -.01  15.80  15.93  22.25  2.7831   .9166  1.0000          1.0121     .01
  75    9.35   15.85   55.10 203.40 195.14  5.21E-01   6.13E-03    -.01  15.54  15.70  20.27  2.8019   .9244  1.0000          1.0103     .01
  80    7.01   16.83   37.57 201.80 193.04  4.94E-01   1.71E-02    -.00  14.53  14.70  17.52  2.8804   .9559  1.0000          1.0044     .01
  85    4.07   17.62   21.53 198.41 190.25  4.12E-01   3.76E-02    -.00  12.36  12.49  13.81  3.0897  1.0365  1.0000    2*     .9937     .01
  90    3.14   18.11    8.54 192.65 186.64  2.86E-01   8.37E-02    -.00   8.58   8.63   8.98  3.6583  1.2441  1.0000    2*     .9801     .01
  95    2.13   18.28    1.01 184.55 182.32  1.04E-01   2.91E-01    -.00   3.12   3.12   3.14  6.0038  2.0633  1.0000    4*     .9699  999.00
                                                                               RAY REACHED SHORE
  96    2.05   18.29     .35 182.67 181.36  6.13E-02   5.01E-01    -.00   1.84   1.84   1.84  7.8091  2.6865, 1.0000    4*     .9689  999.00
```

Figure (4-7). LISTING FOR RAY NUMBER 5 OF SAMPLE INPUT DATA

```
PROJECT NO. PAR 2,  78/03/01    ,      PLOT NO.  1, PERIOD= 17.0SEC., RAY NO.  6, DELTAT= 25.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS.  H,HGT(METER).  G,U,V(METER/SECOND).                        D(BETA)/DT = -0.

MAX X       Y        H         PACK   WAVE   D           FK             ALFA      G      U      V      HGT       KS        KF          NO KR         PCTDIF

   1   42.00     3.00   299.99 220.00 220.00   4.43E-01   0.              -.03   13.28  13.28  26.55  3.0000   1.0000   1.0000               1.0000     .01
   5   40.64     4.14   289.82 220.00 220.00   4.43E-01   0.              -.03   13.28  13.28  26.55  3.0000   1.0000   1.0000               1.0000     .01
  10   38.94     5.57   277.06 220.07 219.96   4.45E-01  -6.71E-04        -.03   13.36  13.36  26.53  2.9913    .9970   1.0000               1.0001     .01
  15   37.24     7.00   264.28 220.16 219.94   4.46E-01  -8.92E-04        -.03   13.38  13.38  26.52  2.9889    .9960   1.0000               1.0003     .01
  20   35.53     8.44   251.49 220.29 219.91   4.47E-01  -1.18E-03        -.02   13.42  13.42  26.51  2.9862    .9947   1.0000               1.0008     .01
  25   33.83     9.89   238.69 220.45 219.83   4.49E-01  -1.54E-03        -.02   13.47  13.47  26.49  2.9832    .9929   1.0000               1.0015     .01
  30   32.12    11.36   225.88 220.67 219.83   4.51E-01  -2.00E-03        -.02   13.53  13.53  26.46  2.9798    .9906   1.0000               1.0027     .01
  35   30.41    12.83   213.05 220.96 219.75   4.53E-01  -2.56E-03        -.02   13.61  13.61  26.42  2.9760    .9876   1.0000               1.0044     .01
  40   28.69    14.33   200.20 221.33 219.65   4.56E-01  -3.23E-03        -.02   13.71  13.72  26.36  2.9718    .9839   1.0000               1.0068     .01
  45   26.98    15.85   187.32 221.79 219.51   4.61E-01  -4.00E-03        -.02   13.84  13.85  26.28  2.9671    .9793   1.0000               1.0099     .01
  50   25.26    17.41   174.42 222.37 219.31   4.66E-01  -4.83E-03        -.02   14.00  14.02  26.17  2.9623    .9738   1.0000               1.0140     .01
  55   23.53    19.00   161.50 223.06 219.04   4.72E-01  -5.65E-03        -.02   14.19  14.22  26.02  2.9576    .9673   1.0000               1.0192     .01
  60   21.81    20.63   148.55 223.07 218.67   4.79E-01  -6.35E-03        -.01   14.41  14.47  25.81  2.9535    .9600   1.0000               1.0255     .01
  65   20.08    22.32   135.58 224.78 218.17   4.86E-01  -6.77E-03        -.01   14.64  14.74  25.53  2.9505    .9522   1.0000               1.0328     .01
  70   18.35    24.07   122.61 225.73 217.50   4.95E-01  -6.70E-03        -.01   14.89  15.04  25.15  2.9489    .9444   1.0000               1.0408     .01
  75   16.62    25.87   109.65 226.65 216.63   5.02E-01  -5.89E-03        -.01   15.12  15.35  24.64  2.9488    .9372   1.0000               1.0488     .01
  80   14.89    27.72    96.71 227.39 215.50   5.09E-01  -4.12E-03        -.01   15.30  15.63  23.99  2.9493    .9316   1.0000               1.0553     .01
  85   13.17    29.61    83.78 227.78 214.06   5.13E-01  -1.09E-03        -.01   15.40  15.85  23.14  2.9490    .9286   1.0000               1.0585     .01
  90   11.45    31.51    70.84 227.62 212.24   5.13E-01   3.57E-03        -.01   15.36  15.93  22.04  2.9463    .9298   1.0000               1.0562     .01
  95    9.72    33.38    57.87 226.63 209.95   5.06E-01   1.06E-02        -.01   15.11  15.77  20.62  2.9408    .9373   1.0000               1.0458     .01
 100    7.96    35.16    44.83 224.41 207.05   4.90E-01   2.14E-02        -.00   14.55  15.24  18.79  2.9378    .9553   1.0000               1.0251     .01
 105    6.24    36.75    31.82 220.41 203.33   4.49E-01   3.96E-02        -.00   13.48  14.10  16.36  2.9564    .9925   1.0000     2*         .9929     .01
 110    4.56    38.04    19.23 213.81 198.52   3.86E-01   7.42E-02        -.00   11.57  12.00  13.12  3.0555   1.0712   1.0000     4*         .9508     .01
 115    3.09    38.87     8.17 203.51 192.28   2.77E-01   1.56E-01        -.00    8.30   8.46   8.78  3.4367   1.2651   1.0000     4*         .9055     .01
 120    2.14    39.17     1.05 188.79 184.45   1.06E-01   5.33E-01        -.00    3.18   3.19   3.21  5.3480   2.0427   1.0000     8*         .8727  999.00
                                                                                             RAY REACHED SHORE
 121    2.05    39.18      .40 185.40 182.73   6.54E-02   8.86E-01        -.00    1.96   1.96   1.97  6.7868   2.6016   1.0000     8*         .8696  999.00

THIS IS THE END.
```

Figure (4-8). LISTING FOR RAY NUMBER 6 OF SAMPLE INPUT DATA

PROJ. NO. PAR 2, 78/03/01
SCL = 1/310039 , CIN = 300
PLOT NO. 1 , DIR. = WAVPAK

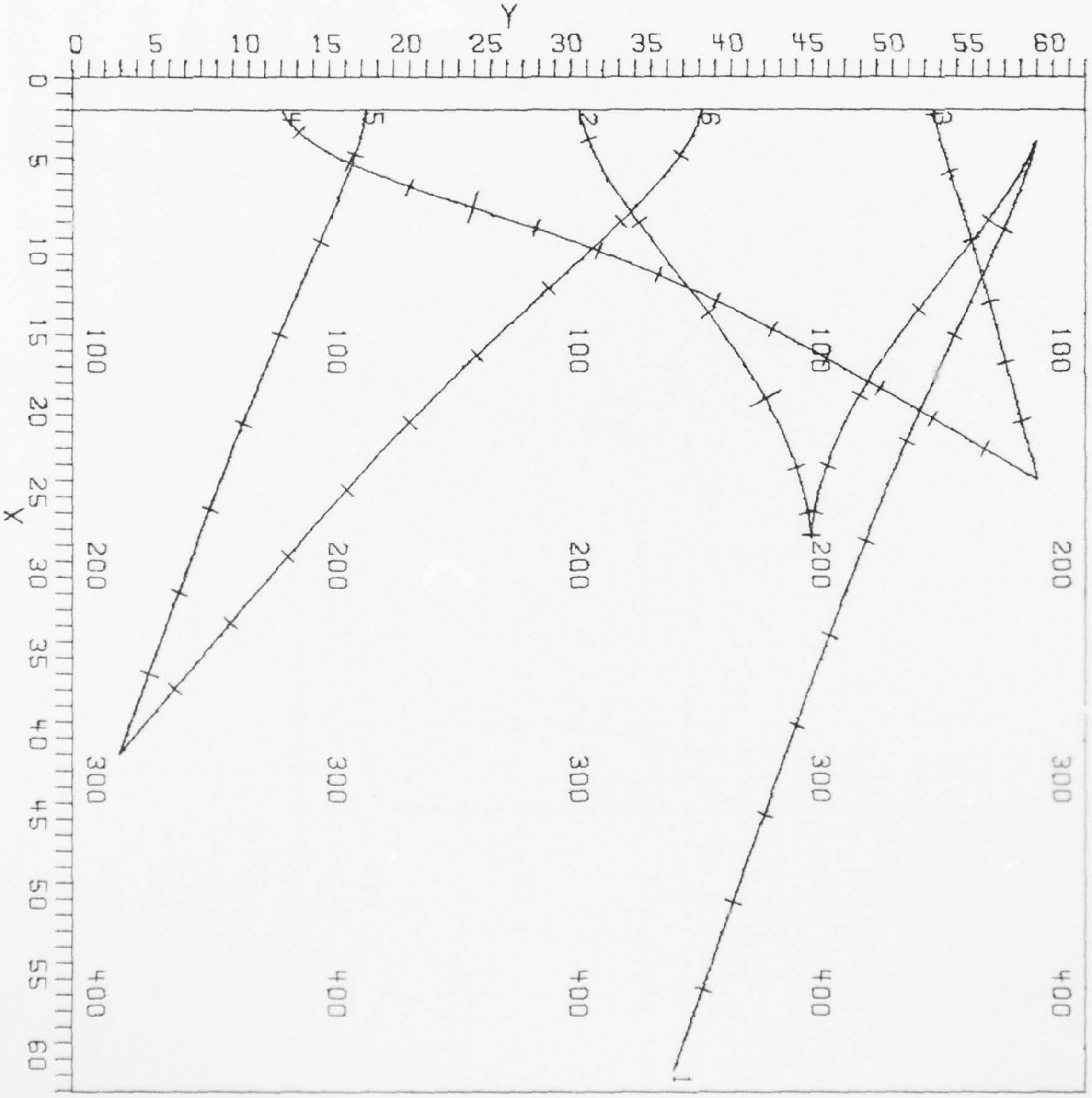


Figure (4-9). PLOT FOR SAMPLE INPUT DATA

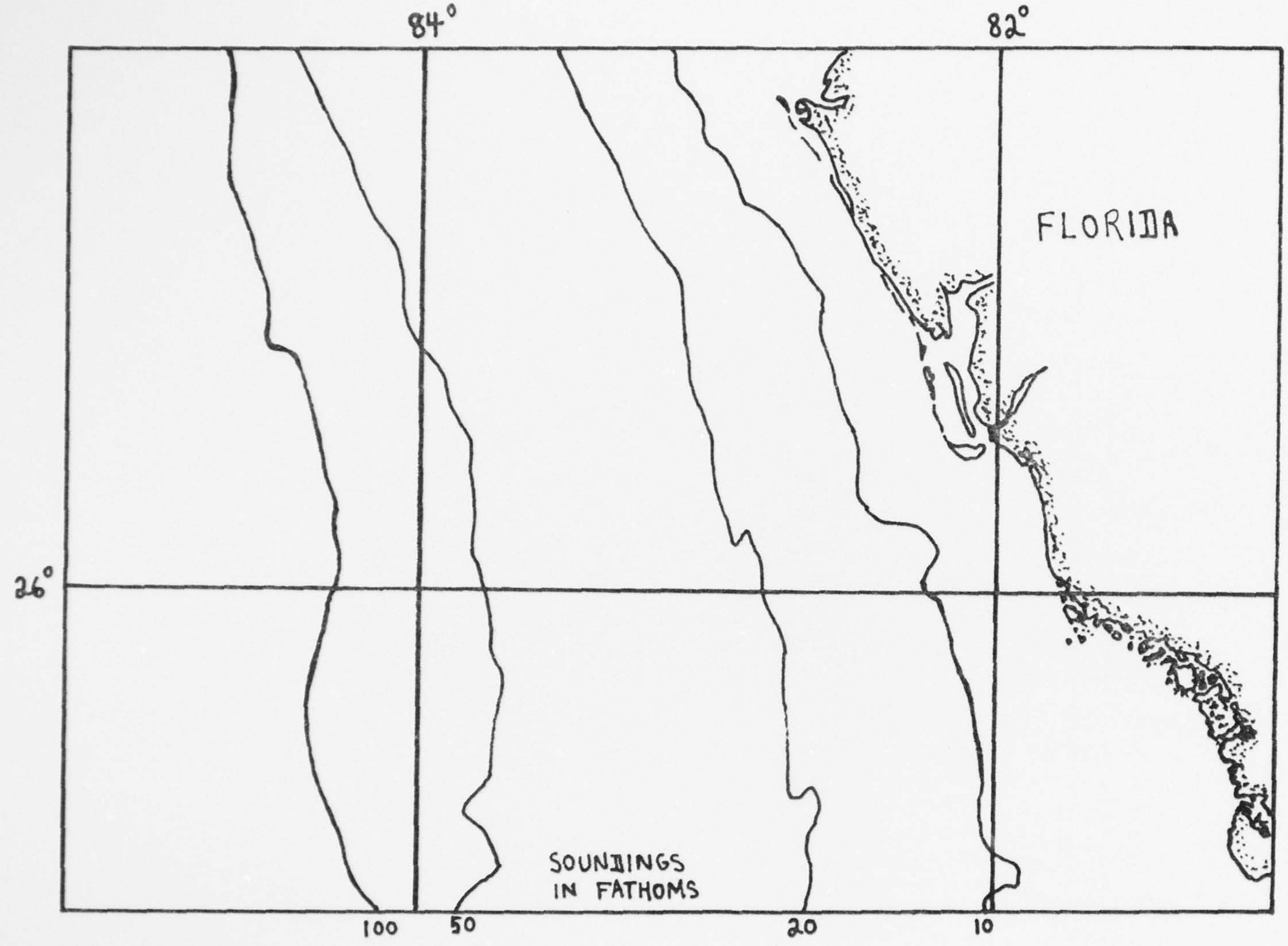


Figure (4-10). GULF OF MEXICO OFF THE SOUTHWESTERN FLORIDA COAST

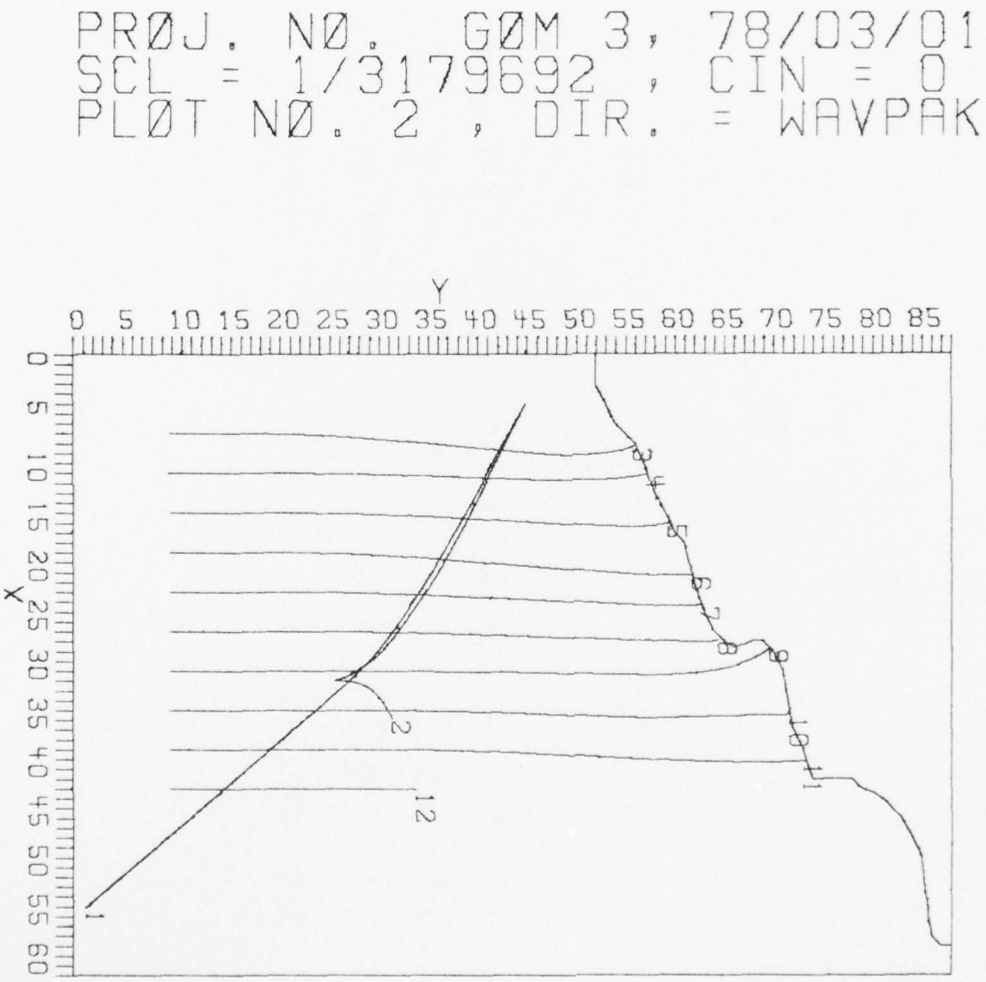


Figure (4-11). PLOT OF RAYS OFF THE SOUTHWESTERN FLORIDA COAST

```
PROJECT NO. GOM 3, 78/03/01    ,    PLOT NO.  2

THE OUTPUT IS IN ENGLISH UNITS.  H,HGT(FEET).  G,U,V(FEET/SECOND).

N  PERIOD MAX  X        Y       H         PACK   WAVE    HGT      DELTAT CF        KRTOL

 1  12.0        1    5.00   46.00    74.74   30.00  30.00   5.0000 50.00   .005000  .001000
 1  12.0      736   56.12    1.4410918.00    49.23  10.06   5.0781                       RAY REACHED GRID BOUNDARY

 2  12.0        1    5.00   46.00    74.74   28.00  28.00   5.0000 50.00   .005000  .001000
 2  12.0      572   36.78   32.50   234.50  339.42 340.50   3.4110                       BREAKUP TIME STEP LESS THAN 0.5 SECOND

 3  12.0        1    8.00   10.00   741.40  270.00 270.00   5.0000 50.00   .005000  .001000
 3  12.0      427    9.32   57.00     9.70  247.29 243.51   4.6359                       RAY REACHED SHORE

 4  12.0        1   12.00   10.00   779.94  270.00 270.00   5.0000 50.00   .005000  .001000
 4  12.0      434   12.14   58.21     5.87  254.32 249.39   4.9579                       WAVE BREAKS

 5  12.0        1   16.00   10.00   804.32  270.00 270.00   5.0000 50.00   .005000  .001000
 5  12.0      455   17.07   60.45     7.25  253.75 249.48   5.3563                       WAVE BREAKS

 6  12.0        1   20.00   10.00   877.93  270.00 270.00   5.0000 50.00   .005000  .001000
 6  12.0      477   22.32   62.88     6.43  268.84 266.49   5.3723                       WAVE BREAKS

 7  12.0        1   24.00   10.00  1012.37  270.00 270.00   5.0000 50.00   .005000  .001000
 7  12.0      487   25.31   63.73     6.44  259.49 256.76   5.2451                       WAVE BREAKS

 8  12.0        1   28.00   10.00  1188.92  270.00 270.00   5.0000 50.00   .005000  .001000
 8  12.0      517   28.86   65.53     5.27  244.79 241.54   4.5792                       WAVE BREAKS

 9  12.0        1   32.00   10.00  1620.58  270.00 270.00   5.0000 50.00   .005000  .001000
 9  12.0      573   29.72   70.73     1.83  227.40 226.14   1.5455                       WAVE BREAKS

10  12.0        1   36.00   10.00  2375.57  270.00 270.00   5.0000 50.00   .005000  .001000
10  12.0      576   36.47   72.78     5.44  266.90 264.53   4.5235                       WAVE BREAKS

11  12.0        1   40.00   10.00  3096.22  270.00 270.00   5.0000 50.00   .005000  .001000
11  12.0      585   41.15   74.18     6.05  257.92 252.36   4.9206                       WAVE BREAKS

12  12.0        1   44.00   10.00  3720.83  270.00 270.00   5.0000 50.00   .005000  .001000
12  12.0      239   44.06   34.91   232.41  271.22 269.48   4.7737                       REFLECTION

THIS IS THE END.
```

Figure (4-12). LISTING OF RAYS WHEN NPT = 0

```
PROJECT NO. GOM 3,  78/03/01    ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO.  1, DELTAT= 50.00, CF= .005000, KRTOL= .001000

THE OUTPUT IS IN ENGLISH UNITS.  H,HGT(FEET).  G,U,V(FEET/SECOND).                              D(BETA)/DT = -4.694E-05

MAX X      Y       H         PACK   WAVE   D           FK          ALFA      G      U      V      HGT       KS       KF        NO KR         PCTDIF

   1    5.00   46.00     74.74   30.00   30.00   1.19E-01   2.02E-02   -74.29   35.42 35.42 43.83   5.0000   1.0000   1.0000          1.0000   11.91
   5    5.42   45.76     83.58   29.57   27.85   1.21E-01   1.31E-02   -72.61   36.02 36.03 45.70   4.9695    .9917    .9982    1*    1.0040   13.95
  10    5.95   45.46     88.43   29.20   26.55   1.22E-01   8.38E-03   -77.57   36.25 36.29 46.64   4.9604    .9886    .9962          1.0073   13.95
  15    6.48   45.17     93.56   28.98   25.60   1.22E-01   4.26E-03   -67.33   36.43 36.50 47.58   4.9481    .9860    .9946          1.0091    9.39
  20    7.02   44.87     94.50   28.86   25.24   1.22E-01   4.27E-03   -67.26   36.46 36.53 47.75   4.9434    .9857    .9930    2*    1.0101    4.82
  25    7.55   44.58     98.87   28.73   24.51   1.23E-01   2.65E-03   -62.58   36.56 36.66 48.49   4.9325    .9843    .9916          1.0108    4.82
  30    8.09   44.28    105.25   28.66   23.83   1.23E-01   7.73E-04   -43.26   36.67 36.80 49.51   4.9211    .9829    .9903          1.0112    3.30
  35    8.63   43.99    107.22   28.64   23.73   1.23E-01   4.24E-04   -43.85   36.69 36.82 49.81   4.9154    .9826    .9892          1.0114    3.49
  40    9.17   43.69    110.18   28.63   23.52   1.23E-01   2.35E-04   -51.03   36.71 36.86 50.25   4.9100    .9823    .9882          1.0116    2.17
  45    9.71   43.39    113.05   28.62   23.31   1.23E-01   5.23E-05   -46.62   36.72 36.88 50.66   4.9057    .9822    .9872          1.0118    2.17
  50   10.26   43.10    116.74   28.62   23.09   1.23E-01  -1.60E-04   -44.83   36.72 36.89 51.17   4.9024    .9822    .9864          1.0120    1.82
  55   10.80   42.80    120.78   28.63   22.82   1.23E-01  -5.71E-04   -49.60   36.69 36.88 51.70   4.9009    .9825    .9856          1.0122    2.17
  60   11.34   42.51    125.47   28.66   22.47   1.23E-01  -1.06E-03   -49.94   36.64 36.85 52.26   4.9016    .9832    .9849          1.0123    1.95
  65   11.88   42.21    129.85   28.70   22.21   1.23E-01  -8.80E-04   -44.66   36.57 36.81 52.80   4.9033    .9841    .9842          1.0124    1.95
  70   12.42   41.92    132.73   28.72   22.10   1.23E-01  -4.72E-04   -40.10   36.53 36.77 53.12   4.9041    .9848    .9837          1.0125    2.32
  75   12.95   41.62    135.49   28.73   22.02   1.23E-01  -1.26E-04   -32.24   36.48 36.73 53.42   4.9052    .9854    .9831          1.0126    2.32
  80   13.49   41.33    139.26   28.74   21.94   1.22E-01  -4.09E-04   -37.85   36.40 36.66 53.82   4.9082    .9864    .9826          1.0127    2.22
  85   14.03   41.04    140.42   28.76   21.92   1.22E-01  -1.23E-03   -59.19   36.38 36.64 53.94   4.9081    .9868    .9822          1.0128    1.36
  90   14.56   40.74    141.40   28.80   21.84   1.22E-01  -1.65E-03   -56.42   36.35 36.62 54.04   4.9080    .9872    .9817          1.0129    1.92
  95   15.09   40.45    144.35   28.87   21.61   1.22E-01  -3.01E-03   -62.25   36.27 36.56 54.32   4.9113    .9883    .9813          1.0128    1.97
 100   15.63   40.15    147.77   28.98   21.34   1.22E-01  -3.16E-03   -58.24   36.15 36.48 54.65   4.9159    .9898    .9809          1.0126    1.97
 105   16.16   39.86    152.37   29.08   21.01   1.21E-01  -3.33E-03   -55.80   35.99 36.35 55.11   4.9239    .9921    .9805          1.0124    1.26
 110   16.69   39.57    156.57   29.20   20.77   1.21E-01  -3.14E-03   -59.25   35.86 36.25 55.43   4.9301    .9939    .9802          1.0121    1.28
 115   17.21   39.27    159.97   29.30   20.54   1.20E-01  -2.82E-03   -57.97   35.73 36.15 55.70   4.9365    .9957    .9799          1.0119     .94
 120   17.73   38.98    161.25   29.41   20.46   1.20E-01  -4.36E-03   -69.90   35.67 36.12 55.81   4.9365    .9965    .9796          1.0118    1.49
 125   18.25   38.68    164.90   29.59   20.12   1.19E-01  -5.14E-03   -68.34   35.52 36.01 56.09   4.9473    .9987    .9794          1.0117    1.61
 130   18.77   38.39    168.08   29.76   19.87   1.19E-01  -5.10E-03   -65.65   35.37 35.91 56.32   4.9549   1.0007    .9791          1.0114    1.61
 135   19.28   38.09    172.22   29.92   19.60   1.18E-01  -4.25E-03   -60.15   35.20 35.78 56.61   4.9650   1.0032    .9789          1.0112    1.95
 140   19.80   37.80    175.16   30.07   19.43   1.18E-01  -4.92E-03   -59.09   35.07 35.68 56.81   4.9720   1.0050    .9787          1.0110    1.68
 145   20.30   37.50    178.30   30.24   19.23   1.17E-01  -6.13E-03   -69.24   34.92 35.58 57.02   4.9805   1.0071    .9785          1.0108    2.34
 150   20.81   37.21    181.91   30.44   18.98   1.17E-01  -6.37E-03   -65.46   34.75 35.46 57.24   4.9910   1.0096    .9783          1.0107    2.34
 155   21.31   36.91    185.47   30.67   18.71   1.16E-01  -6.77E-03   -76.08   34.57 35.34 57.45   5.0023   1.0122    .9781    1*    1.0105    2.15
 160   21.81   36.61    187.57   30.98   18.45   1.16E-01  -1.26E-02   -88.51   34.43 35.27 57.57   5.0112   1.0143    .9780          1.0104    2.15
 165   22.30   36.32    188.24   31.40   18.35   1.15E-01  -1.23E-02   -85.68   34.34 35.25 57.61   5.0152   1.0157    .9778          1.0099    2.81
 170   22.79   36.01    190.57   31.80   18.05   1.15E-01  -1.20E-02   -84.75   34.16 35.17 57.74   5.0227   1.0183    .9777          1.0090    2.81
 175   23.28   35.71    193.73   32.44   17.45   1.14E-01* -1.10E-02   -81.79   33.87 35.06 57.90   5.0085   1.0227    .9775    1     1.0020     .54
 180   23.75   35.40    196.08   32.75   17.23   1.13E-01  -7.31E-03   -77.38   33.71 34.98 58.03   5.0002   1.0251    .9774           .9961     .54
 185   24.23   35.10    198.20   32.92   17.10   1.13E-01  -3.42E-03   -59.07   33.59 34.91 58.13   4.9906   1.0269    .9773    1*     .9945    1.42
 190   24.70   34.79    202.72   33.12   16.95   1.12E-01  -7.26E-03   -69.70   33.45 34.83 58.25   4.9826   1.0291    .9772           .9910    3.54
 195   25.17   34.48    206.59   33.33   16.68   1.12E-01  -4.94E-03   -59.18   33.16 34.63 58.52   4.9839   1.0333    .9771           .9873    1.06
 200   25.63   34.18    210.26   33.48   16.52   1.11E-01  -5.10E-03   -64.23   33.01 34.51 58.68   4.9763   1.0359    .9770           .9836    1.06
 205   26.10   33.87    207.42   33.71   16.83   1.11E-01  -1.04E-02   -81.23   33.11 34.60 58.56   4.9524   1.0343    .9769    1*     .9803    1.70
 210   26.56   33.56    211.65   34.04   16.60   1.11E-01  -1.10E-02   -72.82   32.94 34.49 58.79   4.9440   1.0376    .9768           .9756    1.70
 215   27.01   33.25    217.24   34.40   16.24   1.10E-01  -1.23E-02   -72.65   32.57 34.28 58.96   4.9400   1.0428    .9767           .9700    1.97
 220   27.45   32.94    223.06   34.83   15.92   1.09E-01  -1.59E-02   -74.20   32.26 34.09 59.17   4.9320   1.0479    .9766           .9638    2.03
 225   27.90   32.63    228.93   35.38   15.57   1.07E-01  -2.04E-02   -78.97   31.90 33.91 59.37   4.9247   1.0537    .9766           .9571    2.03
 230   28.33   32.32    236.10   36.08   15.13   1.06E-01  -2.77E-02   -82.53   31.47 33.70 59.58   4.9208   1.0609    .9765           .9499    1.30
 235   28.76   32.01    242.25   37.10   14.64   1.04E-01  -4.40E-02   -87.97   30.97 33.52 59.76   4.9174   1.0694    .9765    2*     .9418    1.30
 240   29.17   31.69    250.02   38.14   14.18   1.02E-01  -3.72E-02   -83.14   30.45 33.30 59.96   4.9125   1.0785    .9764    2*     .9330    1.46
 245   29.56   31.38    259.15   39.11   13.69   1.00E-01  -3.66E-02   -81.27   29.86 33.06 60.16   4.9137   1.0891    .9764    2*     .9241    1.46
 250   29.94   31.06    269.85   40.14   13.23   9.82E-02  -3.53E-02   -79.84   29.25 32.80 60.37   4.9185   1.1005    .9764    2*     .9155    1.46
 255   30.31   30.74    283.41   41.01   12.77   9.62E-02  -3.01E-02   -79.39   28.64 32.51 60.59   4.9277   1.1122    .9764    2*     .9076    1.06
 260   30.67   30.43    294.49   41.83   12.43   9.45E-02  -2.95E-02   -80.61   28.13 32.29 60.74   4.9338   1.1221    .9763    2*     .9007    1.06
 265   31.02   30.11    304.63   42.63   12.14   9.32E-02  -3.58E-02   -84.60   27.67 32.11 60.85   4.9421   1.1314    .9763    1*     .8948     .86
 270   31.36   29.80    314.82   43.46   11.36   9.17E-02  -2.90E-02   -83.30   27.21 31.95 60.95   4.9550   1.1409    .9763    1*     .8897     .52
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Figure (4-13). LISTING OF RAY MOVING TOWARD DEEP WATER

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PROJECT NO. GOM 3,  78/03/01    ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO.  2, DELTAT= 50.00, CF= .005000, KRTOL= .001000

MAX X      Y      H          PACK   WAVE    D          FK           ALFA      G      U     V      HGT       KS        KF       NO KR       PCTDIF

 300   32.00   28.85   345.20   58.49   1.85   5.88E-02*-5.29E-02   -83.34   17.40 31.56 61.16   5.3060   1.4268   .9757   1    .7623   .46
 305   32.15   28.60   353.13   59.41   1.67   5.69E-02*-5.92E-02   -84.17   16.81 31.48 61.21   5.3786   1.4517   .9757   1    .7595   .46
 310   32.29   28.36   360.75   60.41   1.41   5.48E-02*-6.85E-02   -84.97   16.17 31.41 61.24   5.4609   1.4739   .9757   1    .7564   .46
 315   32.42   28.13   368.03   61.57   1.14   5.25E-02*-8.31E-02   -85.75   15.47 31.35 61.27   5.5561   1.5131   .9757   1    .7529   .46
 320   32.54   27.90   375.68   63.36    .78   4.93E-02*-1.66E-01   -87.31   14.41 31.29 61.30   5.7186   1.5680   .9757   1    .7476   .40
 325   32.64   27.69   382.07   65.73    .44   4.48E-02*-1.89E-01   -87.48   13.06 31.25 61.32   5.9537   1.6467   .9757   1    .7411   .40
 330   32.72   27.50   387.89   68.14    .11   4.02E-02*-2.13E-01   -87.59   11.68 31.21 61.33   6.2477   1.7418   .9757   1    .7353   .40
 335   32.79   27.33   393.37   70.54 359.80   3.55E-02*-2.36E-01   -87.64   10.28 31.18 61.35   6.6114   1.8560   .9757   1    .7302   .40
 340   32.84   27.17   397.65   72.34 359.51   3.09E-02*-2.55E-01   -87.63    8.93 31.15 61.36   7.0532   1.9914   .9757   1    .7260   .40
 345   32.88   27.03   401.63   74.36 359.23   2.66E-02*-2.79E-01   -87.60    7.67 31.13 61.37   7.5755   2.1485   .9757   1    .7227   .40
 350   32.91   26.92   403.30   77.91 358.99   2.15E-02*-4.97E-01   -88.01    6.03 31.12 61.37   8.4990   2.4232   .9757   1    .7189   .79
 355   32.92   26.83   405.98   80.44 358.90   1.61E-02*-4.75E-01   -87.86    4.52 31.11 61.38   9.7845   2.7992   .9757   1    .7165   .79
 360   32.93   26.76   408.02   82.27 358.55   1.21E-02*-4.62E-01   -87.75    3.40 31.10 61.38  11.2569   3.2263   .9757   1    .7152   .79
 365   32.94   26.71   409.57   83.62 358.34   9.10E-03*-4.61E-01   -87.65    2.56 31.09 61.38  12.9692   3.7207   .9757   1    .7145   .79
 370   32.94   26.67   410.74   84.64 358.16   6.80E-03*-4.78E-01   -87.58    1.91 31.08 61.39  15.0206   4.3115   .9757   1    .7141   .79
*373   32.95   26.66   411.30   85.14 358.06   5.65E-03 -4.99E-01   -87.55    1.58 31.08 61.39  16.4924   4.7349   .9757        .7140
REFLECTION:   NEAR REFLECTION POINT
 375   32.95   26.67   410.96  270.24 355.98   5.63E-03*-4.09E-01   -87.57    1.77 31.08 61.39  15.6004   4.4784   .9757   1    .7141   .79
 380   32.95   26.70   409.97  270.86 355.84   7.06E-03*-2.70E-01   -87.64    2.18 31.09 61.38  14.0548   4.0336   .9757   1    .7143   .79
 385   32.95   26.74   408.79  271.36 355.71   8.26E-03*-1.92E-01   -87.73    2.52 31.09 61.38  13.0586   3.7467   .9757   1    .7144   .79
 390   32.95   26.78   407.44  271.80 355.59   9.29E-03*-1.44E-01   -87.83    2.82 31.10 61.38  12.3469   3.5416   .9757   1    .7146   .79
 395   32.95   26.83   405.95  272.16 356.47   1.02E-02*-1.11E-01   -87.94    3.08 31.11 61.38  11.8172   3.3869   .9757   1    .7148   .79
 400   32.95   26.89   404.35  272.47 356.36   1.10E-02*-8.37E-02   -88.07    3.31 31.11 61.37  11.4039   3.2697   .9757   1    .7149   .79
 405   32.95   26.94   402.65  272.74 356.26   1.17E-02*-7.20E-02   -88.20    3.52 31.12 61.37  11.0734   3.1744   .9757   1    .7151   .79
 410   32.96   27.00   402.65  272.98 355.27   1.23E-02*-9.51E-02   -88.01    3.64 31.12 61.37  10.8831   3.1193   .9757   1    .7152   .40
 415   32.96   27.07   400.84  273.30 356.16   1.29E-02*-7.75E-02   -88.16    3.87 31.13 61.37  10.5552   3.0246   .9757   1    .7154   .40
 420   32.97   27.13   398.92  273.58 356.05   1.36E-02*-6.44E-02   -88.31    4.06 31.14 61.36  10.2856   2.9467   .9757   1    .7155   .40
 425   32.97   27.20   396.99  273.82 355.94   1.42E-02*-5.45E-02   -88.47    4.27 31.15 61.36  10.0576   2.8806   .9757   1    .7157   .40
 430   32.97   27.27   394.77  274.03 355.84   1.48E-02*-4.67E-02   -88.64    4.44 31.17 61.35   9.8626   2.8245   .9757   1    .7158   .40
 435   32.98   27.35   392.56  274.23 355.74   1.53E-02*-4.06E-02   -88.82    4.60 31.18 61.35   9.6917   2.7751   .9757   1    .7159   .40
 440   32.99   27.43   390.26  274.40 355.64   1.59E-02*-3.56E-02   -89.00    4.75 31.19 61.34   9.5398   2.7312   .9757   1    .7160   .40
 445   32.99   27.51   387.89  274.56 355.55   1.63E-02*-3.15E-02   -89.19    4.89 31.21 61.33   9.4039   2.6920   .9757   1    .7161   .40
 450   33.00   27.59   385.43  274.70 355.45   1.68E-02*-2.82E-02   -89.38    5.02 31.22 61.33   9.2805   2.6564   .9757   1    .7162   .40
 455   33.01   27.68   380.64  274.88 355.24   1.75E-02*-4.02E-02   -88.85    5.23 31.26 61.31   9.0923   2.6021   .9756   1    .7163   .42
 460   33.01   27.77   378.04  275.09 355.12   1.81E-02*-4.70E-02   -88.64    5.42 31.27 61.30   8.9368   2.5571   .9756   1    .7164   .42
 465   33.02   27.86   375.37  275.36 354.98   1.88E-02*-5.53E-02   -88.41    5.64 31.29 61.30   8.7636   2.5068   .9756   1    .7166   .42
 470   33.03   27.95   372.59  275.68 354.83   1.96E-02*-6.55E-02   -88.16    5.90 31.31 61.29   8.5707   2.4508   .9756   1    .7169   .42
 475   33.04   28.05   370.63  276.06 354.72   2.05E-02*-6.61E-02   -88.20    6.16 31.33 61.28   8.3876   2.3974   .9756   1    .7172   .74
 480   33.05   28.16   367.55  276.50 354.54   2.16E-02*-8.07E-02   -87.91    6.50 31.35 61.27   8.1726   2.3346   .9756   1    .7176   .74
 485   33.07   28.27   364.29  277.06 354.33   2.29E-02*-9.93E-02   -87.60    6.92 31.38 61.26   7.9269   2.2627   .9756   1    .7182   .74
 490   33.08   28.39   360.62  277.83 354.09   2.47E-02*-1.24E-01   -87.26    7.46 31.41 61.24   7.6424   2.1789   .9756   1    .7190   .74
 495   33.10   28.52   357.06  278.86 353.80   2.69E-02*-1.56E-01   -86.69    8.17 31.44 61.22   7.3160   2.0821   .9756   1    .7203   .74
 500   33.12   28.66   352.95  280.31 353.45   2.99E-02*-1.99E-01   -86.47    9.13 31.48 61.20   6.9417   1.9698   .9756   1    .7224   .74
 505   33.16   28.82   348.36  282.42 353.03   3.41E-02*-2.59E-01   -85.98   10.47 31.53 61.18   6.5147   1.8394   .9756   1    .7261   .74
 510   33.20   29.00   343.11  285.60 352.50   4.02E-02*-3.45E-01   -85.39   12.43 31.59 61.15   6.0364   1.6884   .9756   1    .7329   .74
 515   33.27   29.21   338.42  289.29 352.06   4.72E-02*-2.62E-01   -85.73   14.47 31.64 61.13   5.6641   1.5644   .9756   1    .7422   .61
 520   33.36   29.45   331.55  292.91 351.47   5.42E-02*-2.33E-01   -85.83   16.55 31.72 61.08   5.3771   1.4631   .9755   1    .7534   .61
 525   33.48   29.71   323.81  296.63 350.83   6.11E-02*-2.13E-01   -85.94   18.61 31.82 61.02   5.1615   1.3795   .9756   1    .7670   .61
 530   33.64   30.00   315.12  300.59 350.09   6.97E-02*-3.39E-01   -84.51   20.75 31.95 60.95   4.9989   1.3066   .9756   2    .7844   .55
 535   33.85   30.31   305.75  306.71 349.15   7.96E-02*-2.50E-01   -84.65   23.69 32.09 60.86   4.8855   1.2229   .9756   2    .8190   .55
 540   34.11   30.63   296.31  312.15 348.24   8.76E-02*-2.13E-01   -84.82   26.07 32.26 60.76   4.8901   1.1656   .9756   2    .8553   .50
 545   34.43   30.95   286.38  317.29 347.26   9.32E-02*-1.79E-01   -84.73   28.11 32.45 60.63   4.9571   1.1226   .9755   1    .9053   .50
 550   34.80   31.27   275.56  320.88 346.34   9.83E-02*-1.18E-01   -86.38   29.50 32.67 60.47   5.0422   1.0958   .9755   1    .9434  1.14
 555   35.20   31.57   267.23  324.48 345.53   1.02E-01*-1.17E-01   -85.44   30.67 32.86 60.32   5.1926   1.0747   .9755   1    .9906   .57
 560   35.63   31.87   257.80  327.35 344.68   1.06E-01*-9.04E-02   -88.01   31.59 33.10 60.13   5.3338   1.0588   .9755   1   1.0328   .57
 565   36.09   32.15   240.37  329.70 342.84   1.09E-01*-9.02E-02   -83.21   32.69 33.57 59.71   5.4479   1.0409   .9754   1   1.0731  2.86
 570   36.57   32.41   235.28  333.80 341.43   1.12E-01*-2.14E-01   -74.55   33.42 33.72 59.56   6.0392   1.0296   .9754   8   1.2028  2.86
                                                                                BREAKUP TIME STEP LESS THAN 0.5 SECOND
 572   36.78   32.50   234.50  339.39 340.51   1.13E-01 -1.03E+00   -70.67   33.74 33.74 59.54   3.4704   1.0247   .9754  64*    .6945  2.86
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Figure (4-14). LISTING OF RAY PARTICULARS NEAR A REFLECTION POINT

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PROJECT NO. GOM 3,  78/03/01     ,     PLOT NO.  1, PERIOD= 12.0SEC., RAY NO. 12, DELTAT= 50.00, CF= .005000, KRTOL= .001000

THE OUTPUT IS IN ENGLISH UNITS.  H,HGT(FEET).  G,U,V(FEET/SECOND).                                  D(BETA)/DT = -0.

MAX X        Y       H         PACK   WAVE    D            FK             ALFA      G      U      V      HGT        KS         KF              NO KR       PCTDIF

   1  44.00   10.00 3720.80 270.00 270.00  1.03E-01  0.                 -92.34   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.76
   5  44.00   10.41 3362.36 270.00 270.00  1.03E-01  0.                 -91.42   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.76
  10  44.00   10.93 2962.20 270.00 270.00  1.03E-01  0.                 -89.98   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.76
  15  44.00   11.45 2645.70 270.00 270.00  1.03E-01  0.                 -87.69   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .29
  20  44.00   11.96 2282.37 270.00 270.00  1.03E-01  0.                 -85.39   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .29

  25  44.00   12.48 1907.93 270.00 270.00  1.03E-01  0.                 -82.21   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    2.68
  30  44.00   13.00 1604.53 270.00 270.00  1.03E-01  0.                 -79.00   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    2.68
  35  44.00   13.51 1346.31 270.00 270.00  1.03E-01  0.                 -77.58   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    3.47
  40  44.00   14.03 1110.63 270.00 270.00  1.03E-01  0.                 -76.19   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    5.92
  45  44.00   14.54  945.12 270.00 270.00  1.03E-01  0.                 -74.63   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    5.92
  50  44.00   15.06  818.98 270.00 270.00  1.03E-01  0.                 -73.19   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    6.49
  55  44.00   15.58  732.32 270.00 270.00  1.03E-01  0.                 -68.45   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    6.49
  60  44.00   16.09  779.91 270.00 270.00  1.03E-01  0.                 -50.91   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .16
  65  44.00   16.61  718.25 270.00 270.00  1.03E-01  0.                 -67.79   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .16
  70  44.00   17.13  693.85 270.00 270.00  1.03E-01  0.                 -88.18   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.15
  75  44.00   17.64  672.75 270.00 270.00  1.03E-01  0.                -107.89   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.15
  80  44.00   18.16  652.37 270.00 270.00  1.03E-01  0.                -113.88   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.15
  85  44.00   18.68  629.12 270.00 270.00  1.03E-01  0.                -115.65   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.15
  90  44.00   19.19  610.54 270.00 270.00  1.03E-01  0.                -113.24   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .26
  95  44.00   19.71  588.96 270.00 270.00  1.03E-01  0.                -108.00   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .26
 100  44.00   20.22  561.81 270.00 270.00  1.03E-01  0.                -106.21   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.12
 105  44.00   20.74  544.95 270.00 270.00  1.03E-01  0.                -104.33   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.12
 110  44.00   21.26  533.94 270.00 270.00  1.03E-01  0.                -100.87   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.39
 115  44.00   21.77  524.98 270.00 270.00  1.03E-01  0.                 -97.69   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000    1.39
 120  44.00   22.29  524.26 270.00 270.00  1.03E-01  0.                 -91.97   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .61
 125  44.00   22.31  519.07 270.00 270.00  1.03E-01  0.                 -84.15   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .61
 130  44.00   23.32  513.25 270.00 270.00  1.03E-01  0.                 -85.16   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .69
 135  44.00   23.84  505.22 270.00 270.00  1.03E-01  0.                 -82.52   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .69
 140  44.00   24.36  496.74 270.00 270.00  1.03E-01  0.                 -82.77   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .64
 145  44.00   24.87  485.77 270.00 270.00  1.03E-01  0.                 -82.78   30.75 30.75 61.50   5.0000   1.0000   1.0000          1.0000     .64
 150  44.00   25.39  471.15 270.00 269.99  1.03E-01 -2.96E-04           -80.10   30.89 30.89 61.46   4.9886    .9977   1.0000          1.0000     .16
 155  44.00   25.91  458.31 270.01 269.99  1.04E-01 -3.81E-04           -79.00   30.92 30.92 61.45   4.9863    .9973   1.0000          1.0000     .16
 160  44.00   26.43  446.62 270.02 269.99  1.04E-01 -5.53E-04           -76.63   30.95 30.95 61.44   4.9838    .9968   1.0000          1.0000     .37
 165  44.00   26.95  433.10 270.04 269.99  1.04E-01 -6.17E-04           -78.39   30.99 30.99 61.42   4.9804    .9961   1.0000          1.0000     .37
 170  44.00   27.47  419.48 270.06 269.98  1.04E-01 -5.83E-04           -81.08   31.04 31.04 61.40   4.9764    .9952   1.0000          1.0000     .60
 175  44.00   27.99  404.71 270.07 269.98  1.04E-01 -6.28E-04           -82.51   31.11 31.11 61.37   4.9711    .9941   1.0000          1.0001     .60
 180  44.00   28.51  387.59 270.09 269.98  1.05E-01 -5.59E-04           -84.51   31.21 31.21 61.33   4.9636    .9926   1.0000          1.0001     .66
 185  44.00   29.04  374.19 270.11 269.97  1.05E-01 -9.34E-04           -82.18   31.30 31.30 61.29   4.9563    .9911   1.0000          1.0001     .87
 190  44.00   29.56  359.44 270.14 269.96  1.05E-01 -1.26E-03           -81.19   31.42 31.42 61.23   4.9469    .9892   1.0000          1.0002     .87
 195  44.01   30.09  344.17 270.18 269.95  1.06E-01 -1.41E-03           -81.27   31.57 31.57 61.16   4.9353    .9869   1.0000          1.0002     .76
 200  44.01   30.62  330.16 270.23 269.94  1.06E-01 -1.86E-03           -80.20   31.74 31.74 61.07   4.9228    .9843   1.0000          1.0003     .76
 205  44.01   31.16  313.33 270.29 269.92  1.07E-01 -2.09E-03           -81.06   31.97 31.97 60.94   4.9051    .9807   1.0000          1.0004     .43
 210  44.01   31.70  299.92 270.36 269.89  1.06E-01 -2.94E-03           -77.04   32.19 32.19 60.80   4.8887    .9773   1.0000          1.0004     .43
 215  44.02   32.24  293.44 270.47 269.87  1.08E-01 -4.20E-03           -73.07   32.31 32.31 60.72   4.8802    .9756   1.0000          1.0005    2.58
 220  44.02   32.78  279.23 270.61 269.82  1.09E-01 -4.95E-03           -75.92   32.59 32.59 60.52   4.8598    .9713    .9999          1.0007    2.58
 225  44.03   33.33  259.33 270.79 269.72  1.11E-01 -5.72E-03           -73.26   33.05 33.06 60.16   4.8270    .9645    .9999          1.0010    4.78
 230  44.04   33.89  247.40 270.96 269.65  1.12E-01 -4.84E-03           -73.49   33.36 33.37 59.89   4.8055    .9630    .9999          1.0012    4.78
 235  44.05   34.45  235.14 271.13 269.54  1.13E-01 -4.22E-03           -66.60   33.71 33.72 59.56   4.7808    .9551    .9998          1.0013    3.36
*239  44.06   34.91  232.41 271.22 269.48  1.13E-01 -2.22E-03           -26.99   33.79 33.81 59.48   4.7737    .9539    .9998          1.0010
REFLECTION:   SNELLS LAW WITH PHASE VELOCITY
 239  44.06   34.91  232.41 271.22 269.48  1.13E-01 -2.22E-03           -26.99   33.79 33.81 59.48   4.7737    .9539    .9998          1.0010    3.36
```

Figure (4-15). LISTING FOR RAY WITH A FALSE REFLECTION

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NOTATION

The principal symbols are defined. The expressions in parentheses are alternate representations of the symbol being defined.

A	(ANGLE, PACK, SVA) The program symbol for θ. In the input and output A is the direction in degrees from which the wave packet comes with respect to true north. Internally in the program A is the direction in radians with which the wave packet moves with respect to the positive x-axis.
AA	The wave packet direction at the new ray point.
AAA, AAAV	The average of the values at the new and previous ray points, respectively, of the wave packet and wavelet directions.
ABAR	The average of the wave packet directions at the present and new points.
ALFA	(SVALFA, PALFA) The program symbol for α.
AMM, ANN	The maximum values of x and y, respectively, for a water depth grid.
AV	(AAV, GAM, WAVE, SAV, SVAV) The program symbol for γ. The directions of AV are defined following the same conventions used in the definitions of A.
AVP	(GP) The program symbol for γ'.
AX, AY	The arrays used to store the locations of ray points and tick marks.
a	A constant for a given wave period.
BDZ	(SBDZ) The program symbol for $d\beta/dt$.
BDZ5	The fifth order Runge-Kutta solution of $d\beta/dt$.
BRK	Is zero except during a breakup of the time step interval when the value is one. After returning to MOVE from HEIGHT, the value of BRK determines where the program resumes.

BZ	(SBZ) The program symbol for β.
BZTOL	The limiting value for $\lvert EBZ \rvert$ and $\lvert EBDZ \rvert$ in the Runge-Kutta calculations of β and $d\beta/dt$. If $\lvert EBZ \rvert$ or $\lvert EBDZ \rvert$ exceeds or is equal to BZTOL the time step interval is halved.
BZ5	The fifth order Runge-Kutta solution of β.
b	A constant for a given wave period.
C	An array of 12 water depths from CMAT used to fit a quadratic surface in the vicinity of the ray point.
CF	The program symbol for c_f.
CIN	If CIN is not zero, its value is the travel time between tick marks along a ray. In the input CIN is in seconds, but for the calculations CIN is converted to hours. If CIN is zero there are no tick marks on a ray.
CMAT	The water depth grid in a two dimensional array.
CNVRSA	The direction of the positive x-axis of the water depth grid with respect to true north. The use of this conversion angle permits the ray directions to be defined with respect to true north in the input and output.
COL	If COL is not zero the plotter will pause before the ray is plotted. If COL is zero the plotter does not pause.
CONTUR	An array containing the sounding water depths in feet or meters. There can be as many as 9 values.
CORI	A conversion factor used in calculating SCLI.
C_s	The constant of Snell's law.
c_f	The friction factor.
D	(PD) The incremental distance in grid units between ray points.
DADX	The program symbol for $d\theta/dx'$.
DATE1, DATE2	The year, month, and day.

DAVDX	The program symbol for $d\gamma/dx'$.
D(BETA)/DT	The initial value of $d\beta/dt$.
DCON	A conversion factor to convert the water depths in CMAT to feet or meters.
DELA	The change in the wave packet direction from the present to the new ray point.
DELTAT	(SDLTAT) The time step interval in seconds between ray points.
DELX, DELY	The change in the values from the present to the new ray points of the x and y coordinates, respectively.
DEP	(SDEP, PDEP) The program symbol for h.
DGDX	(SADGDX) The program symbol for dG/dx'.
DGDXX	The program symbol for $d^2G/(dx')^2$.
DHDX	The program symbol for dh/dx'.
DHDXX	The program symbol for $d^2h/(dx')^2$.
DIR	A label of 6 letters and numbers used to identify a plot.
DN	An expression used to compute W.
DPHIDX	The program symbol for $d\phi/dx'$.
DRHODX	The program symbol for $d\rho/dx'$.
DSIGDX	The program symbol for $d\sigma/dx'$.
DUD	The ratio of the phase speed at the present ray point to the value at the previous ray point.
DVDX	The program symbol for dv/dx'.
DVDXX	The program symbol for $d^2v/(dx')^2$.
DUDX	The program symbol for dU/dx'.
DUDXX	The program symbol for $d^2U/(dx')^2$.
DY	The number of grid units per inch or centimeter for a particular plot.

E	An array of 6 coefficients of the quadratic surface equation which is fitted to the 12 water depths in the array C.
EBDZ	The program symbol for $\varepsilon_{\beta t}$.
EBZ	The program symbol for ε_{β}.
EM	A two dimensional array of numbers used in computing the array E.
$\mathcal{E}$	The wave energy per unit area.
F	An expression used in the derivation of the friction coefficient.
FAN	If FAN is zero the rays are numbered at their initial points. When FAN is not zero the rays are numbered at their terminal points.
FK	(SVFKB, SAVFK, PFK) The program symbol for κ_G. It is measured in radians/grid unit.
FKBAR	(SVFKB) The average of the ray curvature at the present and new ray points.
FKK	The value of κ_G in radians/grid unit at the new ray point.
FLAGR	Is set equal to zero in MAIN at the beginning of a ray. The value is changed to one in MOVE if there is a reflection and the ray is continued. If FLAGR is one, in HEIGHT the value of ROP is set equal to zero.
FLAG1	If FLAG1 is zero the wavelet direction is computed and the test for total reflection is made in SURFCE. If FLAG1 is not zero these calculations are not made.
FLAG2	If there is total reflection due to the wavelets FLAG2 is set equal to one. Otherwise, FLAG2 is set equal to zero. The reflection test is made in SURFCE.
FLAG3	In MAIN, FLAG3 is set equal to zero at the beginning of a ray. FLAG3 is not used in the program, and it is available for checks or modifications to the program.

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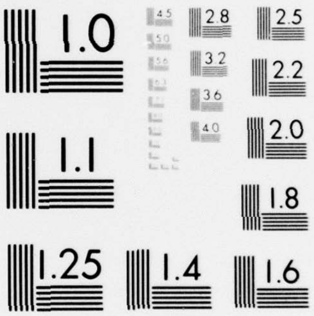

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FUD	An index used to determine when to write page and column headings depending upon the number of lines of printout.
f	The frequency of the wave (1/T).
F	The average rate of energy transmission of the waves.
G	(ZCXY, SG, SVG, PG) The geometric group speed.
GRID	The number of feet or meters per grid unit for a particular water depth grid.
GZERO	The value of the geometric group speed at the first ray point.
g	The acceleration due to gravity.
H	In the theory H is the wave height. In the printed output H represents the water depth.
HGT	(PHGT) The program symbol for the wave height.
HGTZ	The initial value of HGT.
HT	The length of the y-axis in inches or centimeters for a given plot.
HX	The program symbol for $\partial h/\partial x$.
HXX	The program symbol for $\partial^2 h/\partial x^2$.
HXY	The program symbol for $\partial^2 h/\partial x \partial y$.
HY	The program symbol for $\partial h/\partial y$.
HYY	The program symbol for $\partial^2 h/\partial y^2$.
h	The water depth.
I	A variable depending on the ratio of water depth to phase speed.
IFLG	When IFLG is zero a check is made in HEIGHT to determine if there should be a breakup of the time step interval in order to maintain the desired accuracy in the calculations of either β or the ray path. If there is a division of the time step interval, IFLG is set equal to one once the time step interval is sufficiently reduced. When IFLG equals

	one further checks for a breakup of the time step interval are not made at new ray points until the breakup ends and calculations are resumed with the initial time step.
INUM	An index to count the ray points within the broken interval when there is a division of the initial time step interval.
j	An index to number ray points.
K	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$.
KCIN	The number of tick marks along a ray which do not coincide with ray points.
KF	In the program calculations KF is an expression used to evaluate K_F. In the printed output KF is a label for the values of K_F.
KFC	(PKF) The program symbol for K_F.
KMAX	The same as MAX except in DRAW where it is the sum of MAX and KCIN.
KR	(PKR) The program symbol for K_R.
KREST	The number of tick marks along a ray.
KRTOL	Determines the accuracy in the Runge-Kutta calculations of the refraction coefficient. BZTOL depends upon KRTOL.
KS	(PKS) The program symbol for K_S.
K_F	The friction coefficient.
K_R	The refraction coefficient.
K_S	The shoaling coefficient.
k	The wave number $2\pi/\lambda$.
L	Expressions used in the Runge-Kutta calculations of β and $d\beta/dt$.
LI	When NPT is not zero LI is used to determine the number of lines of printout between page and column headings.
LII	When NPT is zero LII is used to determine the number of lines of printout between page and column headings.

ℓ	The perpendicular distance between rays.
MAX	(MAXQ, KMAX) An index to number points along a ray at time intervals equal to the initial time step.
MIT	If MIT is 1 the wave packet curvature approximations in MOVE converge to one value. If MIT is 2 the curvature approximations converge to two values. If the curvature approximations do not converge and there is no reflection Mit is 3. If MIT is 4 a caustic or focal point is computed in HEIGHT. MIT is 5 if it is determined in HEIGHT that the wave breaks. When there is a reflection but the ray is not continued MIT is 6. If MIT is 7 more than one reflection from the same point is determined in MOVE. If MIT is 8 the breakup time step determined in HEIGHT is less than 0.5 seconds.
MM	The dimension of x for a particular water depth grid.
MMAX	The dimension of the AX and AY arrays.
MOE	If MOE is zero the input and output are in English units. If MOE is not zero the input and output are in Metric units.
MXPLOT	The number of runs and the number of plots for a given operation of the computer program.
N	The ray number.
NAX	If NAX is zero the plot has borders but the x- and y-axes are not calibrated. If NAX is not zero the x- and y-axes are calibrated and labeled.
NCO	The number of sounding water depths for a plot. The values are stored in the CONTUR array. The number of sounding depths cannot exceed 9. If NCO is zero there are no sounding depths for the plot.
NDP	The water depth is determined in SURFCE. If the value is greater than 0, NDP is 1 (initialized in RAYN). If the water depth equals or is less than 0, NDP is 2.

NFK	The value of NFK is determined in SURFCE. If the ratio of the water depth to the deep water wavelength is greater than 0.64, NFK is 1. Otherwise, NFK is 2.
NGO	The value of NGO is determined in MOVE. If a ray point lies within one and one half grid units of a grid boundary NGO is 2. Otherwise, NGO is 1. (Initialized in RAYN.)
NN	The dimension of y for a particular water depth grid.
NNSKIP	The amount by which y is incremented in selecting columns for locating sounding water depths.
NO	(NUMT, KNUMT) The number of divisions when there is a breakup of the initial time step interval.
NOR	The number of rays for a given run.
NPLOT	The plot number.
NPT	If NPT is zero printed output occurs only for the initial and terminal ray points. If NPT is not zero printed output occurs for the first ray point, those points which are an integral multiple of SK, and the last point.
NSH	If NSH is zero the shoreline is not drawn on a plot. If NSH is not zero the shoreline is drawn.
NXCMAT	If NXCMAT is zero a water depth grid is read in the input for the run. If NXCMAT is not zero the depth grid for the previous run is used again.
n	An index to number ray points.
PATI	(PSAV) A program symbol for p. The value of p at the point prior to the new ray point.
PCTDIF	An estimate of how well the quadratic surface fits the 12 water depths used to derive it. At each of the 4 water depths closest to the ray point, the percentage difference between the water depth derived from the surface fit and the actual depth is computed. PCTDIF is the maximum of these differences.

PHI	The program symbol for ϕ.
POT	(SPOT) A program symbol for p. The value of p at the new ray point.
PREV	(SPREV) The value of v at the previous ray point.
PROJCT	A label of 6 letters and numbers used to identify a computer run.
P(i)	Program symbols for p. The values of p at points intermediate to the new and previous ray points where i = 1, 2, ..., 5.
p	A coefficient of the ray separation equation.
QATI	A program symbol for q. The value of q at the point prior to the new ray point.
QOT	(SQOT) A program symbol for q. The value of q at the new ray point.
Q(i)	Program symbols for q. The values of q at points intermediate to the new and previous ray points where i = 1, 2, ..., 5.
q	A coefficient of the ray separation equation.
RCOUNT	An index to count the number of reflections at a ray point.
REF	The value of REF is determined in MOVE and it denotes the kind of reflection. When there is reflection due to Snell's law with phase velocity REF is 1. When reflection occurs because the packet curvature iteration is not converging REF is [illegible] If there is reflection because the ray point is too near a reflection point REF is 3.
REFLCT	In MAIN, REFLCT is set equal to zero at the beginning of a ray. In MOVE, REFLCT is set equal to one for those ray points where the conditions for being close to a reflection point are met.
REFRCT	In MAIN, REFRCT is set equal to zero at the beginning of a ray. In HEIGHT, REFRCT is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$.

RFLBUM	In MAIN, RFLBUM is set equal to zero at the beginning of a ray. In MOVE, RFLBUM is set equal to one for those ray points where the conditions for being close to a reflection point are met. The value of RFLBUM is used to determine which format statement to use in the output of the ray particulars.
RFRBUM	In MAIN, RFRBUM is set equal to zero at the beginning of a ray. In HEIGHT, RFRBUM is set equal to one for those ray points where there is a breakup of the time step interval due to insufficient accuracy in the Runge-Kutta calculations of β and $d\beta/dt$. The value of RFRBUM is used to determine which format statement to use in the output of the ray particulars.
RHO	The program symbol for ρ.
ROP	The initial value of ROP is determined in the input data. If ROP is zero a ray is not continued beyond a reflection point. If ROP is not zero a ray is continued beyond a reflection point. After a reflection ROP is set equal to zero so that a ray is not continued beyond a second reflection point if one should exist.
RT	The length of the x-axis in inches or centimeters for a given plot.
S	A two dimensional array of numbers used in computing the array E.
SCL	The scale of the plot.
SCLI	The reciprocal of SCL.
SIGMA	The program symbol for σ.
SK	See NPT.
SSALFA	The program symbol for the value of α at the previous ray point.
s_G	The arc length of a wave packet trajectory (ray).
s_V	The arc length of a monochromatic ray.
T	The wave period.

TIME	(TIMEQ) The travel time along a ray.
TPI	The initial value of the wave packet direction used in the analytical solutions of β and $d\beta/dt$.
TT	The program symbol for T.
t	Time.
U	(SAVU, PU) The collinear group speed $d\omega/dk$.
u_m	The maximum velocity of the fluid at the bottom.
V	(SAVV, PV) The program symbol for v.
VX	The program symbol for $\partial v/\partial x$.
VY	The program symbol for $\partial v/\partial y$.
v	The phase speed of a monochromatic wave.
W	An expression used to relate the first spatial derivatives of v and h.
WBCOP	If WBCOP is zero no test is made to determine if the wave breaks. If WBCOP is not zero a test is made in HEIGHT to determine if the wave breaks.
WL	The program symbol for the deep water value of λ.
X	(SVX, PX) The program symbol for x.
XX	(XS) The program symbol for x at the new ray point.
x	A Cartesian coordinate of the water depth grid.
x'	A Cartesian coordinate in a system chosen such that $\partial h/\partial y' = 0$.
Y	(SVY, PY) The program symbol for y. In the theory an expression used to relate the second spatial derivatives of v to the spatial derivatives of h.
YVW	A one dimensional array used in computing the array E.
YY	(YS) The program symbol for y at the new ray point.

y	A Cartesian coordinate of the water depth grid.
y'	A Cartesian coordinate in a system chosen such that $\partial h/\partial y' = 0$.
α	The angle the x'-axis is rotated with respect to the x-axis such that $\partial h/\partial y' = 0$.
β	The ray separation factor.
γ	The wavelet direction defined with respect to the positive x-axis.
γ'	The wavelet direction defined with respect to the positive x'-axis.
γ^*	A quantity used in calculating the wavelet direction using Snell's law with phase velocity.
Δt	The time step interval between ray points.
ε_β	The difference between the fourth and fifth order Runge-Kutta solutions of β.
$\varepsilon_{\beta t}$	The difference between the fourth and fifth order Runge-Kutta solutions of $d\beta/dt$.
θ	The wave packet (ray) direction defined with respect to the positive x-axis.
θ'	The wave packet (ray) direction defined with respect to the positive x'-axis.
κ_G	The ray curvature of the wave packet.
κ_V	The ray curvature of a monochromatic wave.
κ_G'	The same as κ_G.
λ	The wavelength
π	3.1415927
ρ	An expression used in the spatial derivatives of G.
ρ_f	The density of the fluid.
σ	An expression used in the spatial derivatives of G.

τ	The tangential stress per unit area at the bottom.
ϕ	The angle $(\theta-\gamma)$.
ϕ'	The same as ϕ.
ω	The radian frequencey $(2\pi f)$ of the wave.

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REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER | 2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Method for Calculating Wave Packet Trajectories and Wave Heights	5. TYPE OF REPORT & PERIOD COVERED Technical
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. Ernest Breeding, Jr., K.C. Matson, and Nourollah Riahi	8. CONTRACT OR GRANT NUMBER(s) N00014-77-C-0329
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Oceanography The Florida State University Tallahassee, FL 32306	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR388-138
11. CONTROLLING OFFICE NAME AND ADDRESS Geography Programs Department of the Navy Office of Naval Research, Arlington, VA	12. REPORT DATE March, 1978
	13. NUMBER OF PAGES 107
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report)
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

-wave packets
-wave refraction
-packet ray curvature
-total reflection
-wave height
-shoaling coefficient
-refraction coefficient
-friction coefficient
-geometric group velocity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The theory and numerical methods are presented for determining the paths of gravity water wave packets. A ray curvature expression is used to determine the wave packet trajectories where the speed of the packet is given by $G = (d\omega/dk) \cos \phi$. The symbol ω denotes the angular frequency, k is the wave number, and ϕ is the difference between the direction of the wave packet and the direction of the wavelets within the packet. At each point of the

phi

const

omega

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BLOCK 20

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wave packet trajectory the wavelet direction is determined using Snell's law with phase velocity. The wave height is computed along the wave packet paths accounting for the effects of shoaling, refraction, and energy dissipation. The computer program is described and sample printouts and plots are presented.