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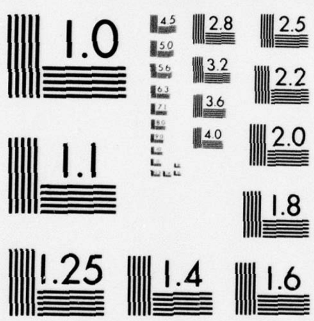

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A METHOD FOR CALCULATING WAVE PACKET TRAJECTORIES AND WAVE HEIGHTS: PART II

by J. Ernest Breeding, Jr.

TECHNICAL REPORT NO. JEB-4

Department of Oceanography
Florida State University

November, 1978

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ABSTRACT

In Part I a method for calculating wave packet trajectories and wave heights is based on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method is extended in order to predict the modification to surface gravity water waves in shoaling water when the water depth contours are not parallel. The calculations are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. Example printouts and plots are presented to illustrate the wave prediction method. It is discovered that when waves initially approach sinuous water depth contours symmetrically with respect to the beach there can be more energy in the bays than at the headlands.

ACKNOWLEDGMENTS

I thank Shelley Horton for her assistance in preparing the computer program and figures in this report. The research was supported by the Geography Programs, Earth Sciences Division, Office of Naval Research under contract No. N00014-77-C-0329.

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CHAPTER I INTRODUCTION

This is a supplement to the report by Breeding, J. Ernest, Jr., K.C. Matson, and Nourollah Riahi, "A Method for Calculating Wave Packet Trajectories and Wave Heights," Department of Oceanography, Florida State University, Tallahassee, Report No. JEB-1 (1978) hereafter referred to as Part I. The program for predicting the modification to waves in shoaling water described in Part I is developed on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method results in accurate wave predictions for many examples of bottom topography. However, the wave height predictions are not accurate when the water depth contours are highly non-parallel in the region of a ray point.

The objective in this report is to extend Part I in order to make accurate wave predictions when the water depth contours are not parallel. The theory is developed in Chapter II. The calculations are greatly simplified by making them in a x'y'-coordinate system which is chosen so that at each ray point the positive x'-axis is in the direction of the gradient of the water depth. As a result, the first partial derivatives of the water depth and wave velocities with respect to y' vanish. Further, there is a simplification in the second partial derivatives involving y'. In Chapter III modifications to the computer program are described. To illustrate the wave prediction program two examples of nonparallel water depth contours are presented in Chapter IV. An interesting result is that when waves initially approach sinuous water depth contours symmetrically with respect to the beach (headon) it is possible for there to be more energy in the bays than at the headlands. This is in contrast to the result for monochromatic trajectories where under similar conditions there is always more energy predicted at the headlands than in the bays.

CHAPTER II THEORY FOR NONPARALLEL WATER DEPTH CONTOURS

2.1 Spatial Derivatives of the Water Depth. At each point of a wave packet trajectory the calculations are made in a x'y'-coordinate system where the x'-axis is taken in the direction of the gradient of the water depth. The particulars of a wave packet trajectory are tabulated in a xy-coordinate system which retains a fixed orientation with respect to the water depth grid. The relationships between these coordinate systems and a specific ray point for a set of nonparallel water depth contours are shown in Figure (2-1). Equations relating these coordinate systems are given by

$$x' = x \cos\alpha + y \sin\alpha \tag{2-1}$$

$$y' = -x \sin\alpha + y \cos\alpha \tag{2-2}$$

$$\tan\alpha = \frac{\partial h}{\partial y} \Big/ \frac{\partial h}{\partial x} \tag{2-3}$$

where α is the angle by which the x'-axis is rotated with respect to the x-axis and h is the water depth.

The partial derivatives of h in the x'y'-coordinate system with respect to the partial derivatives of h in the xy-coordinate system are given by

$$\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \cos\alpha + \frac{\partial h}{\partial y} \sin\alpha \tag{2-4}$$

$$\frac{\partial h}{\partial y'} = -\frac{\partial h}{\partial x} \sin\alpha + \frac{\partial h}{\partial y} \cos\alpha = 0 \tag{2-5}$$

$$\frac{\partial^2 h}{(\partial x')^2} = \frac{\partial^2 h}{\partial x^2} \cos^2\alpha + 2\frac{\partial^2 h}{\partial x \partial y} \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial y^2} \sin^2\alpha \tag{2-6}$$

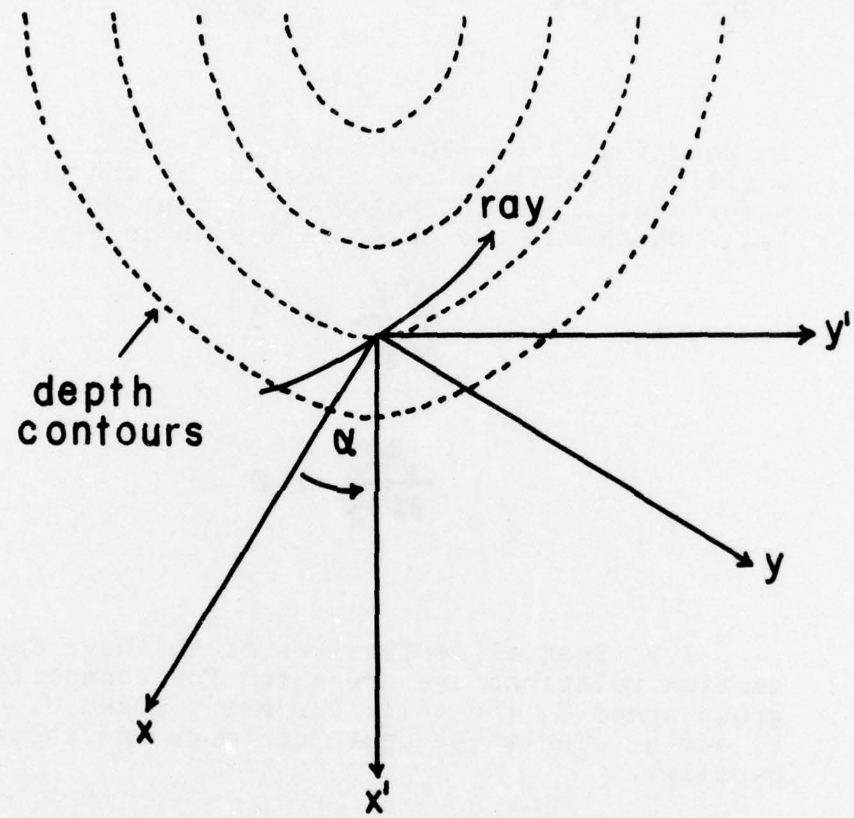

Figure (2-1). Relationships between the coordinate systems and the water depth contours.

$$\frac{\partial^2 h}{(\partial y')^2} = \frac{\partial^2 h}{\partial x^2} \sin^2\alpha - 2\frac{\partial^2 h}{\partial x \partial y} \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial y^2} \cos^2\alpha \tag{2-7}$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = \left(\frac{\partial^2 h}{\partial y^2} - \frac{\partial^2 h}{\partial x^2}\right) \sin\alpha \cos\alpha + \frac{\partial^2 h}{\partial x \partial y}(\cos^2\alpha - \sin^2\alpha) \tag{2-8}$$

In Equation (2-5), $\partial h/\partial y' = 0$ as a result of choosing the positive x'-axis in the direction of the gradient of the water depth. If the water depth contours are locally parallel, which was the case dealt with in Part I, then

$$\frac{\partial^2 h}{(\partial y')^2} = 0 \tag{2-9}$$

$$\frac{\partial^2 h}{\partial x' \partial y'} = 0 \tag{2-10}$$

2.2 Spatial Derivatives of the Wave Speeds. In this section relations are presented for connecting the geometric group speed G, the collinear group speed U, the phase speed v, and h. The water depth contours are assumed to be nonparallel.

a. Derivatives of v

The phase speed of a surface gravity water wave is defined (Part I)

$$v = \frac{1}{a} \tanh \frac{1}{2} \tag{2-11}$$

in which

$$a = \frac{2\pi}{gT} \tag{2-12}$$

$$b = \frac{T}{4\pi} \tag{2-13}$$

$$I = \frac{h}{bv} \tag{2-14}$$

where T is the wave period and g is the acceleration due to gravity.

The first partial derivatives of v in the x'y'-coordinate system are given by

$$\frac{\partial v}{\partial x'} = W \frac{\partial h}{\partial x'} \tag{2-15}$$

$$\frac{\partial v}{\partial y'} = W \frac{\partial h}{\partial y'} = 0 \tag{2-16}$$

where

$$W = \frac{v(1 - a^2 v^2)}{[2abv^2 + h(1 - a^2 v^2)]} \tag{2-17}$$

The second partial derivatives of v are defined by

$$\frac{\partial^2 v}{(\partial x')^2} = W \left[\frac{\partial^2 h}{(\partial x')^2} + \gamma \left(\frac{\partial h}{\partial x'} \right)^2 \right] \tag{2-18}$$

$$\frac{\partial^2 v}{(\partial y')^2} = W \frac{\partial^2 h}{(\partial y')^2} \tag{2-19}$$

$$\frac{\partial^2 v}{\partial x' \partial y'} = W \frac{\partial^2 h}{\partial x' \partial y'} \tag{2-20}$$

where

$$Y = -\frac{4ab v^2}{[2ab v^2 + h(1-a^2v^2)]^2} \qquad (2\text{-}21)$$

b. Derivatives of U

The collinear group speed of a surface gravity water wave is defined (Part I)

$$U = \frac{1}{2}\left(1 + \frac{I}{\sinh I}\right) v \qquad (2\text{-}22)$$

The first partial derivatives of U in the x'y'-coordinate system are given by

$$\frac{\partial U}{\partial x'} = \frac{\operatorname{csch} I}{2}\left[(\sinh I + I)\frac{\partial v}{\partial x'} + v\frac{\partial I}{\partial x'}(1 - I\coth I)\right] \qquad (2\text{-}23)$$

$$\frac{\partial U}{\partial y'} = \frac{\operatorname{csch} I}{2}\left[(\sinh I + I)\frac{\partial v}{\partial y'} + v\frac{\partial I}{\partial y'}(1 - I\coth I)\right] = 0 \qquad (2\text{-}24)$$

where

$$\frac{\partial I}{\partial x'} = I\left(\frac{1}{h}\frac{\partial h}{\partial x'} - \frac{1}{v}\frac{\partial v}{\partial x'}\right) \qquad (2\text{-}25)$$

$$\frac{\partial I}{\partial y'} = I\left(\frac{1}{h}\frac{\partial h}{\partial y'} - \frac{1}{v}\frac{\partial v}{\partial y'}\right) = 0 \qquad (2\text{-}26)$$

The second partial derivatives of U are given by

$$\frac{\partial^2 U}{(\partial x')^2} = \frac{\operatorname{csch} I}{2}\left[(\sinh I + I)\frac{\partial^2 v}{(\partial x')^2} + v\frac{\partial^2 I}{(\partial x')^2}(1 - I\coth I)\right] + $$
$$+ \frac{\partial I}{\partial x'}\left\{-\frac{\partial U}{\partial x'}\coth I + \frac{\operatorname{csch} I}{2}\left[\frac{\partial v}{\partial x'}(2 + \cosh I - I\coth I) + \right.\right.$$
$$\left.\left. + v\frac{\partial I}{\partial x'}\operatorname{csch} I\,(I\operatorname{csch} I - \cosh I)\right]\right\} \quad (2\text{-}27)$$

$$\frac{\partial^2 U}{(\partial y')^2} = \frac{\operatorname{csch} I}{2}\left[(\sinh I + I)\frac{\partial^2 v}{(\partial y')^2} + v\frac{\partial^2 I}{(\partial y')^2}(1 - I\coth I)\right] \quad (2\text{-}28)$$

$$\frac{\partial^2 U}{\partial x'\partial y'} = \frac{\operatorname{csch} I}{2}\left[(\sinh I + I)\frac{\partial^2 v}{\partial x'\partial y'} + v\frac{\partial^2 I}{\partial x'\partial y'}(1 - I\coth I)\right] \quad (2\text{-}29)$$

where

$$\frac{\partial^2 I}{(\partial x')^2} = \frac{1}{I}\left(\frac{\partial I}{\partial x'}\right)^2 + I\left\{\frac{1}{h}\left[\frac{\partial^2 h}{(\partial x')^2} - \frac{1}{h}\left(\frac{\partial h}{\partial x'}\right)^2\right] - \right. \quad (2\text{-}30)$$
$$\left. - \frac{1}{v}\left[\frac{\partial^2 v}{(\partial x')^2} - \frac{1}{v}\left(\frac{\partial v}{\partial x'}\right)^2\right]\right\}$$

$$\frac{\partial^2 I}{(\partial y')^2} = I\left(\frac{1}{h}\frac{\partial^2 h}{(\partial y')^2} - \frac{1}{v}\frac{\partial^2 v}{(\partial y')^2}\right) \quad (2\text{-}31)$$

$$\frac{\partial^2 I}{\partial x'\partial y'} = I\left(\frac{1}{h}\frac{\partial^2 h}{\partial x'\partial y'} - \frac{1}{v}\frac{\partial^2 v}{\partial x'\partial y'}\right) \quad (2\text{-}32)$$

c. Derivatives of G

The geometric group speed is defined by

$$G = U\cos\phi \quad (2\text{-}33)$$

where

$$\phi = \theta' - \gamma' \tag{2-34}$$

in which θ' is the direction of the wave packet, γ' is the direction of the wavelets, and both directions are measured with respect to the positive x'-axis. The first spatial derivatives of G are

$$\frac{\partial G}{\partial x'} = \frac{\partial U}{\partial x'} \cos\phi - U \sin\phi \frac{\partial \phi}{\partial x'} \tag{2-35}$$

$$\frac{\partial G}{\partial y'} = \frac{\partial U}{\partial y'} \cos\phi - U \sin\phi \frac{\partial \phi}{\partial y'} = 0 \tag{2-36}$$

The ray curvature for a wave packet is given by (Part I)

$$\frac{d\theta'}{ds_G} = \cos\theta' \frac{\partial \theta'}{\partial x'} + \sin\theta' \frac{\partial \theta'}{\partial y'} = \frac{1}{G}\left(\sin\theta' \frac{\partial G}{\partial x'} - \cos\theta' \frac{\partial G}{\partial y'}\right) \tag{2-37}$$

where ds_G is an element of arc length along the packet trajectory. Since the first derivatives in y' vanish it is found that

$$\frac{\partial \theta'}{\partial x'} = \frac{\tan\theta'}{G} \frac{\partial G}{\partial x'} \tag{2-38}$$

$$\frac{\partial \theta'}{\partial y'} = \frac{-\cot\theta'}{G} \frac{\partial G}{\partial y'} = 0 \tag{2-39}$$

In like manner, when considering the ray curvature of the wavelets it is found that

$$\frac{\partial \gamma'}{\partial x'} = \frac{\tan\gamma'}{v} \frac{\partial v}{\partial x'} \tag{2-40}$$

$$\frac{\partial \gamma'}{\partial y'} = -\frac{\cot \gamma'}{v}\frac{\partial v}{\partial y'} = 0 \qquad (2\text{-}41)$$

Therefore

$$\frac{\partial \phi}{\partial x'} = \frac{\tan \theta'}{G}\frac{\partial G}{\partial x'} - \frac{\tan \gamma'}{v}\frac{\partial v}{\partial x'} \qquad (2\text{-}42)$$

$$\frac{\partial \phi}{\partial y'} = -\frac{\cot \theta'}{G}\frac{\partial G}{\partial y'} + \frac{\cot \gamma'}{v}\frac{\partial v}{\partial y'} = 0 \qquad (2\text{-}43)$$

When Equation (2-42) is substituted into (2-35) and the result is simplified it is found that

$$\frac{\partial G}{\partial x'} = \rho\left(\frac{\partial U}{\partial x'}\cos\phi + \sigma\frac{\partial v}{\partial x'}\right) \qquad (2\text{-}44)$$

where

$$\rho = (1 + \tan\phi \tan\theta')^{-1} \qquad (2\text{-}45)$$

$$\sigma = \frac{U}{v}\sin\phi \tan\gamma' \qquad (2\text{-}46)$$

The second partial derivative of G with respect to x' is given by

$$\frac{\partial^2 G}{(\partial x')^2} = \rho\left(\frac{\partial^2 U}{(\partial x')^2}\cos\phi + \sigma\frac{\partial^2 v}{(\partial x')^2}\right) + \left(\cos\phi\frac{\partial \rho}{\partial x'} - \right.$$

$$\left. - \rho\sin\phi\frac{\partial \phi}{\partial x'}\right)\frac{\partial U}{\partial x'} + \left(\rho\frac{\partial \sigma}{\partial x'} + \sigma\frac{\partial \rho}{\partial x'}\right)\frac{\partial v}{\partial x'} \qquad (2\text{-}47)$$

where

$$\frac{\partial \rho}{\partial x'} = -\rho^2\left(\tan\theta' \sec^2\phi\frac{\partial \phi}{\partial x'} + \tan\phi \sec^2\theta'\frac{\partial \theta'}{\partial x'}\right) \qquad (2\text{-}48)$$

$$\frac{\partial \sigma}{\partial x'} = \sigma \left(\frac{1}{U} \frac{\partial U}{\partial x'} - \frac{1}{v} \frac{\partial v}{\partial x'} \right) + \frac{U}{v} \left(\cos\phi \tan\gamma' \frac{\partial \phi}{\partial x'} + \right.$$

$$\left. + \sin\phi \sec^2\gamma' \frac{\partial \gamma'}{\partial x'} \right) \qquad (2\text{-}49)$$

The remaining second order spatial derivatives of G are obtained by differentiating Equation (2-36). Since the first derivatives in y' vanish it is found that

$$\frac{\partial^2 G}{(\partial y')^2} = \frac{\partial^2 U}{(\partial y')^2} \cos\phi - U \sin\phi \frac{\partial^2 \phi}{(\partial y')^2} \qquad (2\text{-}50)$$

$$\frac{\partial^2 G}{\partial x' \partial y'} = \frac{\partial^2 U}{\partial x' \partial y'} \cos\phi - U \sin\phi \frac{\partial^2 \phi}{\partial x' \partial y'} \qquad (2\text{-}51)$$

The second partial derivative of ϕ with respect to y' is found by differentiating Equation (2-43). The result is

$$\frac{\partial^2 \phi}{(\partial y')^2} = - \frac{\cot\theta'}{G} \frac{\partial^2 G}{(\partial y')^2} + \frac{\cot\gamma'}{v} \frac{\partial^2 v}{(\partial y')^2} \qquad (2\text{-}52)$$

When Equation (2-52) is substituted into (2-50) and the terms rearranged it is found that

$$\frac{\partial^2 G}{(\partial y')^2} = \zeta \left(\frac{\partial^2 U}{(\partial y')^2} \cos\phi - \xi \frac{\partial^2 v}{(\partial y')^2} \right) \qquad (2\text{-}53)$$

where

$$\zeta = (1 - \tan\phi \cot\theta')^{-1} \qquad (2\text{-}54)$$

$$\xi = \frac{U}{v} \sin\phi \cot\gamma' \qquad (2\text{-}55)$$

After differentiating Equation (2-43) with respect to x' it is found that

$$\frac{\partial^2 \phi}{\partial x' \partial y'} = \frac{\tan \theta'}{G} \frac{\partial^2 G}{\partial x' \partial y'} - \frac{\tan \gamma'}{v} \frac{\partial^2 v}{\partial x' \partial y'} \tag{2-56}$$

When Equation (2-56) is substituted into (2-51) the simplified result becomes

$$\frac{\partial^2 G}{\partial x' \partial y'} = \rho \left(\frac{\partial^2 U}{\partial x' \partial y'} \cos \phi + \sigma \frac{\partial^2 v}{\partial x' \partial y'} \right) \tag{2-57}$$

2.3 Ray Curvature for Nonparallel Water Depth Contours. The ray curvature κ_G depends only on derivatives of the first order. By making the calculations in the x'y'-coordinate system the first derivatives in y' vanish. This results in a simplified ray curvature expression which is formally the same as presented in Part I for locally parallel water depth contours.

$$\kappa_G = \frac{\sin \theta'}{G} \frac{\partial G}{\partial x'} \tag{2-58}$$

2.4 Ray Separation Equation for Nonparallel Water Depth Contours. The ray separation equation is defined (Part I)

$$\frac{d^2 \beta}{dt^2} + p \frac{d\beta}{dt} + q\beta = 0 \tag{2-59}$$

where β is the ray separation factor, t is time, and

$$p = -2 \cos \theta' \frac{\partial G}{\partial x'} \tag{2-60}$$

$$q = G \left(\sin^2 \theta' \frac{\partial^2 G}{(\partial x')^2} - 2 \sin \theta' \cos \theta' \frac{\partial^2 G}{\partial x' \partial y'} + \cos^2 \theta' \frac{\partial^2 G}{(\partial y')^2} \right) \tag{2-61}$$

In the x'y'-coordinate system p is simplified as is κ_G. In q there is also a reduction in the expressions for the second order derivatives involving y'.

CHAPTER III SUBROUTINE SURFCE

3.1 Modifications to Subroutine SURFCE. Modifications have been made to the program subroutine SURFCE in Part I in order to remove the restriction that the water depth contours be locally parallel about each ray point and to simplify some of the computations. When the program is run with these modifications minor changes will occur in the printed output for the sample input data presented in Part I.

Referring to the program listing for SURFCE on pages 52-54 of Part I the following modifications have been made. In place of SURFACE 58 an alternative expression is used to compute $\partial h/\partial x'$ (DHDX). Between SURFACE 87 and SURFACE 88 $\partial I/\partial x'$ (DIDX) is computed as are the hyperbolic sine, cosine, and tangent of I. Statements SURFACE 91 through SURFACE 93 for computing $\partial U/\partial x'$ (DUDX) have been replaced by a simplified expression. Between SURFACE 107 and SURFACE 108 $\partial^2 h/(\partial y')^2$ (DHDYY) and $\partial^2 h/\partial x'\partial y'$ (DHDXY) are computed. SURFACE 111 has been replaced by expressions to compute $\partial^2 v/(\partial y')^2$ (DVDYY) and $\partial^2 v/\partial x'\partial y'$ (DVDXY). Statements SURFACE 114 through SURFACE 120 have been replaced by expressions for calculating $\partial^2 I/(\partial y')^2$ (DIDYY) and $\partial^2 I/\partial x'\partial y'$ (DIDXY). A simplified expression has been substituted for $\partial^2 U/(\partial x')^2$ (DUDXX). Also, the values of $\partial^2 U/(\partial y')^2$ (DUDYY) and $\partial^2 U/\partial x'\partial y'$ (DUDXY) are computed. Between SURFACE 122 and SURFACE 123 ζ (ZETA), ξ (XI), $\partial^2 G/(\partial y')^2$ (DGDYY), and $\partial^2 G/\partial x'\partial y'$ (DGDXY) are determined. Simplified expressions are used to compute $\partial^2 G/(\partial y')^2$ and $\partial^2 G/\partial x'\partial y'$ when $|\tan \theta'| \leq \tan 5^{\circ}$ and $|\tan \gamma'| \leq \tan 5^{\circ}$. Statement SURFACE 124 has been replaced by the complete expression for computing q (QOT).

3.2 Description of Subroutine SURFCE. SURFCE is called by RAYN and MOVE to calculate h, α, γ, G, p, q, k_G, and other ray particulars. At the first ray point twelve values of h from CMAT are selected about the point as shown in Figure (2-2) of Part I. A quadratic surface is fit to the set of water depths. At successive ray points the quadratic surface is determined only if there is a change in the set of twelve water depths. The water depth and its partial derivatives in the fixed xy-system, $\partial h/\partial x$, $\partial h/\partial y$, $\partial^2 h/\partial x^2$, $\partial^2 h/\partial y^2$, and $\partial^2 h/\partial x\partial y$, are determined at the ray point by interpolating on the quadratic surface.

If $h \leq 0$, NDP = 2 and there is a RETURN. If $h > 0$ the ratio of the water depth to the deep water wavelength is computed. If $h/\lambda_d > 0.64$, which defines deep water, NFK = 1. If $h/\lambda_d \leq 0.64$, NFK = 2. VELCTY is called, and after the return if NFK = 1, W = 0. If NFK = 2, CONDER is called to compute W. The values of $\partial v/\partial x$ and $\partial v/\partial y$ are calculated using W.

At each ray point calculations are made in a $x'y'$-coordinate system which is chosen with the positive x'-axis in the direction of the gradient of the water depth. The value of $\partial h/\partial x'$ is computed, and if it exceeds 0.00001 the angle α by which the x'-axis is rotated with respect to the x-axis is computed. If $|\partial h/\partial x'| \leq 0.00001$ the water depth is assumed to be constant and α remains constant.

If FLAG1 = 0, γ' is computed, and if necessary it is placed within the range $|\gamma'| \leq 360^{\circ}$. A check is made to determine if there is total reflection. If there is, FLAG2 = 1 and there is a RETURN. Otherwise, FLAG2 = 0 and the new γ' is computed using Snell's law with phase velocity following a set of rules. Using the values of γ', γ is computed. When FLAG 1 $\neq$ 0 these steps for computing the new wavelet direction and the test for total reflection are omitted.

The values of ϕ , G, and $\partial v/\partial x'$ are calculated. If NFK = 2, $\partial U/\partial x'$ is determined using its unsimplified expression. If NFK $\neq$ 2, the deep water formula is used to calculate $\partial U/\partial x'$. The value of $\partial U/\partial x'$ is used in computing $\partial G/\partial x'$.

If NFK $\neq$ 2, the coefficients of the ray separation equation and the ray curvature are set equal to zero. Then there is a RETURN. If NFK = 2, p, $\partial^2 h/(\partial x')^2$, $\partial^2 h/(\partial y')^2$, $\partial^2 h/\partial x'\partial y'$, $\partial^2 v/(\partial x')^2$, $\partial^2 v/(\partial y')^2$, $\partial^2 v/\partial x'\partial y'$, $\partial^2 U/(\partial x')^2$, $\partial^2 U/(\partial y')^2$, $\partial^2 U/\partial x'\partial y'$, $\partial^2 G/(\partial x')^2$, $\partial^2 G/(\partial y')^2$, $\partial^2 G/(\partial x'\partial y')$, q, and κ_G are computed. This is followed by a return.

3.3 Listing of Subroutine SURFCE

```
      SUBROUTINE SURFCE(X,Y,A,FK,NFK,NDP,AV)
      DIMENSION S(6,6),EM(6,12),C(12),YVW(6),E(6)
     $,CMAT(100,100),AX(2000),AY(2000),CONTUR(9)
      REAL KR,KF,KS,KRTOL,KFC
      INTEGER FLAG1,FLAG2
      COMMON S,EM,E,YVW,CMAT,C,AX,AY,CONTUR,PROJCT,GRID,DCON,FAN,DATE1
     $,DATE2/CIN,DIR,ROP,TT,WBCOP,MOE,DY,DELTAT,SDLTAT,D,HGT,HGTZ,SVX
     $,SVY,SDEP,W,DEF,WL,V,SAVV,PREV,SPREV,U,SAVU,GZERO,G,SG,SVG,DUD,KS
     $,DGDX,SVA,TPI,SAV,SVAV,PHI,ALFA,SVALFA,SSALFA,CNVRSA,DELA,DHDX
     $,SVFKB,SAVFK,FKBAR,MAXQ,SK,FCC,NUMT,INUM,IFLG,RCOUNT,AMM,ANN
     $,REFLCT,RFLBUM,REFPCT,RFRBUM,BRK,FLAG1,FLAG2,FLAG3,FLAGR,KFC,CF,BZ
     $,SBZ,BDZ,SBDZ,KRTOL,KR,POT,P1,P2,P3,P4,P5,QOT,Q1,Q2,Q3,Q4,Q5
C IN THIS SUBROUTINE THE WATER DEPTH, ROTATION ANGLE, WAVELET
C DIRECTION, GEOMETRIC GROUP VELOCITY, COEFFICIENTS OF THE RAY
C SEPARATION EQUATION, AND THE PACKET RAY CURVATURE ARE COMPUTED.
C SUBROUTINES VELCTY AND CONDER ARE CALLED.
      I=X $ J=Y $ FI=I $ FJ=J $ XL=X+1.-FI $ YL=Y+1.-FJ
      IF (MAXQ .LE. 1) GO TO 1
      IF (ZI .NE. FI) GO TO 1
      IF (ZJ .EQ. FJ) GO TO 3
1     ZI=FI $ ZJ=FJ
C SELECT 12 WATER DEPTHS ABOUT RAY POINT
      C(1)=CMAT(J+1,I) $ C(2)=CMAT(J+2,I) $ C(3)=CMAT(J,I+1)
      C(4)=CMAT(J+1,I+1) $ C(5)=CMAT(J+2,I+1) $ C(6)=CMAT(J+3,I+1)
      C(7)=CMAT(J,I+2) $ C(8)=CMAT(J+1,I+2) $ C(9)=CMAT(J+2,I+2)
      C(10)=CMAT(J+3,I+2) $ C(11)=CMAT(J+1,I+3) $ C(12)=CMAT(J+2,I+3)
C FIT QUADRATIC SURFACE TO 12 WATER DEPTHS
      DO 318 II=1,6
      YVW(II)=0.
      DO 318 L=1,12
      YVW(II)=YVW(II)+C(L)*EM(II,L)
 318  CONTINUE
      DO 319 II=1,6
      E(II)=0.
      DO 319 JJ=1,6
      E(II)=E(II)+S(JJ,II)*YVW(JJ)
 319  CONTINUE
C COMPUTE INTERPOLATED WATER DEPTH
 3    DEP=(E(1)+E(2)*XL+E(3)*YL+E(4)*XL**2+E(5)*XL*YL+E(6)*YL**2)*DCON
C COMPUTE PARTIAL DERIVATIVES OF WATER DEPTH IN FIXED XY-SYSTEM
      HX=(E(2)+2.*E(4)*XL+E(5)*YL)*DCON
      HY=(E(3)+E(5)*XL+E(6)*2.*YL)*DCON
      HXX=2.*E(4)*DCON $ HYY=2.*E(6)*DCON $ HXY=E(5)*DCON
      IF (DEP .GT. 0.) GO TO 324
      NDP=2
      GO TO 403
 324  IF (DEP/WL .GT. .64) GO TO 322
      NFK=2
      GO TO 323
 322  NFK=1
 323  CALL VELCTY(V,TT,MAXQ,DEP,NFK,U)
      IF (NFK .EQ. 2) GO TO 402
      W=0.
      GO TO 10
 402  DN=1.
      CALL CONDER(DN,TT,V,MAXQ,NFK)
      W=DN
```

```
10    VX=W*HX $ VY=W*HY $ DHDX=HX*COS(ALFA)+HY*SIN(ALFA)
      IF (ABS(DHDX/GRID) .GT. 0.00001) GO TO 8
      GO TO 9
C COMPUTE ROTATION ANGLE
8     ALFA=ATAN2(HY,HX)
9     IF (FLAG1 .NE. 0) GO TO 12
C COMPUTE WAVELET DIRECTION IN ROTATED XY-SYSTEM USING SNELLS LAW
C WITH V AND TEST FOR TOTAL REFLECTION DUE TO THE WAVELETS
      GP=SAV-ALFA
14    IF (ABS(GP) .LE. 6.2831853) GO TO 13
      IF (GP) 16,13,17
16    GP=GP+6.2831853
      GO TO 14
17    GP=GP-6.2831853
      GO TO 14
13    ARG1=V*SIN(GP)/SAVV
      IF (ABS(ARG1) .LE. 1.) GO TO 18
      FLAG2=1.
      GO TO 403
18    FLAG2=0. $ GPT=ASIN(ARG1)
      IF (ABS(GP) .LE. 4.7123889) GO TO 20
      AVP=6.2831853+GPT
      GO TO 22
20    IF (ABS(GP) .LE. 1.5707963) GO TO 23
      AVP=3.1415927-GPT
      GO TO 22
23    AVP=GPT
22    AV=AVP+ALFA
12    PHI=A-AV $ G=U*COS(PHI)
      DVDX=W*DHDX $ BAR3=12.5663708/TT $ BAR4=BAR3*DEP/V
      DIDX=BAR4*(DHDX/DEP-DVDX/V) $ SINHI=(EXP(BAR4)-EXP(-BAR4))/2.
      COSHI=(EXP(BAR4)+EXP(-BAR4))/2. $ TANHI=SINHI/COSHI
      IF (NFK .EQ. 2) GO TO 25
      DUDX=DVDX/2.
      GO TO 27
25    DUDX=(.5/SINHI)*((SINHI+BAR4)*DVDX+V*DIDX*(1.-BAR4/TANHI))
27    RHO=1./(1.+TAN(PHI)*TAN(A-ALFA)) $ SIGMA=U*SIN(PHI)*TAN(AV-ALFA)/V
      DGDX=RHO*(DUDX*COS(PHI)+SIGMA*DVDX)
      IF (NFK .EQ. 2) GO TO 28
      POT=0. $ QOT=0. $ FK=0.
      GO TO 403
C COMPUTE P IN ROTATED XY-SYSTEM
28    POT=(-2.*COS(A-ALFA)*DGDX)/GRID $ DAVDX=TAN(AV-ALFA)*DVDX/V
      DADX=TAN(A-ALFA)*DGDX/G $ DPHIDX=DADX-DAVDX
      DRHODX=-(RHO**2)*(TAN(A-ALFA)*DPHIDX/(COS(PHI)**2)+
     $TAN(PHI)*DADX/(COS(A-ALFA)**2))
      DSIGDX=SIGMA*(DUDX/U-DVDX/V)+U*(COS(PHI)*TAN(AV-ALFA)*DPHIDX+
     $SIN(PHI)*DAVDX/(COS(AV-ALFA)**2))/V
      DHDXX=(COS(ALFA)**2)*HXX+2.*SIN(ALFA)*COS(ALFA)*HXY+(SIN(ALFA)**
     $2)*HYY
      DHDYY=(SIN(ALFA)**2)*HXX-2.*SIN(ALFA)*COS(ALFA)*HXY
     $+(COS(ALFA)**2)*HYY
      DHDXY=(SIN(ALFA)*COS(ALFA))*((HYY**2)-(HXX**2))+
     $((COS(ALFA)**2)-(SIN(ALFA)**2))*HXY
      SMA=6.2831854/(32.2*TT) $ SMAB=1./64.4
      DVDXX=W*(DHDXX+(DHDX**2)*(-4.*SMAB*(V**2)/((2.*SMAB*(V**2)+DEP*
     $(1.-(SMA*V)**2))**2)))
      DVDYY=W*DHDYY $ DVDXY=W*DHDXY
```

```
      DIDXX=(DIDX**2)/BAR4+BAR4*((DHDXX-(DHDX**2)/DEP)/DEP-(DVDXX
     $-(DVDX**2)/V)/V)
      DIDYY=ØAR+*(DHDYY/DEP-DVDYY/V) $ DIDXY=BAR4*(DHDXY/DEP-DVDXY/V)
   30 DUDXX=(.5/SINHI)*(-((SINHI+BAR4)*DVDX+V*DIDX*(1.-BAR4/TANHI))*DIDX
     $/TANHI*(SINHI+BAR4)*DVDXX+DVDX*DIDX*(2.+COSHI-BAR4/TANHI)+V*(DIDXX
     $*(1.-BAR4/TANHI)+(DIDX**2)*(BAR4/SINHI-COSHI)/SINHI))
      DUDYY=(.5/SINHI)*((SINHI+BAR4)*DVDYY+V*DIDYY*(1.-BAR4/TANHI))
      DUDXY=(.5/SINHI)*((SINHI+BAR4)*DVDXY+V*DIDXY*(1.-BAR4/TANHI))
   32 DGDXX=RHO*(COS(PHI)*DUDXX+SIGMA*DVDXX)+(COS(PHI)*DRHODX-RHO*SIN
     $(PHI)*DPHIDX)*DUDX+(RHO*DSIGDX+SIGMA*DRHODX)*DVDX
      IF (ABS(TAN(A-ALFA)) .GT. 0.08748806 .AND. ABS(TAN(AV-ALFA)) .GT.
     $0.08748866) GO TO 35
      ZETA=1. $ XI=0.
      GO TO 36
   35 ZETA=1.0/(1.0-TAN(PHI)/TAN(A-ALFA)) $ XI=U*SIN(PHI)/V*TAN(AV-ALFA)
   36 DGDYY=ZETA*(DUDYY*COS(PHI)-XI*DVDYY)
      DGDXY=RHO*(DUDXY*COS(PHI)+SIGMA*DVDXY)
C COMPUTE Q IN ROTATED XY-SYSTEM
      QOT=(G*((SIN(A-ALFA)**2)*DGDXX-2.*SIN(A-ALFA)*COS(A-ALFA)*DGDXY+
     $(COS(A-ALFA)**2)*DGDYY))/(GRID**2)
C COMPUTE PACKET RAY CURVATURE IN ROTATED XY-SYSTEM
      FK=SIN(A-ALFA)*DGDX/G
  403 RETURN
      END
```

CHAPTER IV WAVE PREDICTION EXAMPLES FOR NONPARALLEL WATER DEPTH CONTOURS

Two examples of nonparallel water depth contours are used to illustrate the wave prediction program.

4.1 Sinusoidal to Parallel Water Depth Contours. Figure (4-1) shows a set of wave packet trajectories for a wave period of 7.0 seconds. The rays begin in deep water (initial water depth equal to or greater than one half the wavelength). The water depth contours are sinusoidal at the shoreline and gradually become parallel at a water depth of 38.6 meters. The amplitude of the water depth contour at the shoreline is 2.5 kilometers, and the contour wavelength is 10 kilometers. GRID, the distance between grid points, is 156.25 meters.

From the figure it is seen that the energy is fairly evenly distributed along the coastline with slightly more energy at the headland than in the bay. Figures (4-2) through (4-4) show the printed output for rays number 4, 8, and 16, respectively, of Figure (4-1). The computed refraction coefficients are in good agreement with values estimated from the plot.

4.2 Sinusoidal Water Depth Contours. In Figure (4-5) the period of the wave packet trajectories is 14.0 seconds and they begin in deep water. The water depth contours are sinusoidal with an amplitude of 5 kilometers and a wavelength of 20 kilometers. GRID has a value of 312.5 meters.

This example is quite interesting since there is decidedly more energy in the bay than at the headland. The opposite result would be expected for monochromatic trajectories. The refraction of wave packets could explain why there is more erosion in bays than at headlands for some coastlines. Figures (4-6) through (4-8) show the printed output for rays number 4, 8, and 16, respectively, of Figure (4-5). The computed refraction coefficients agree favorably with values estimated from the plot.

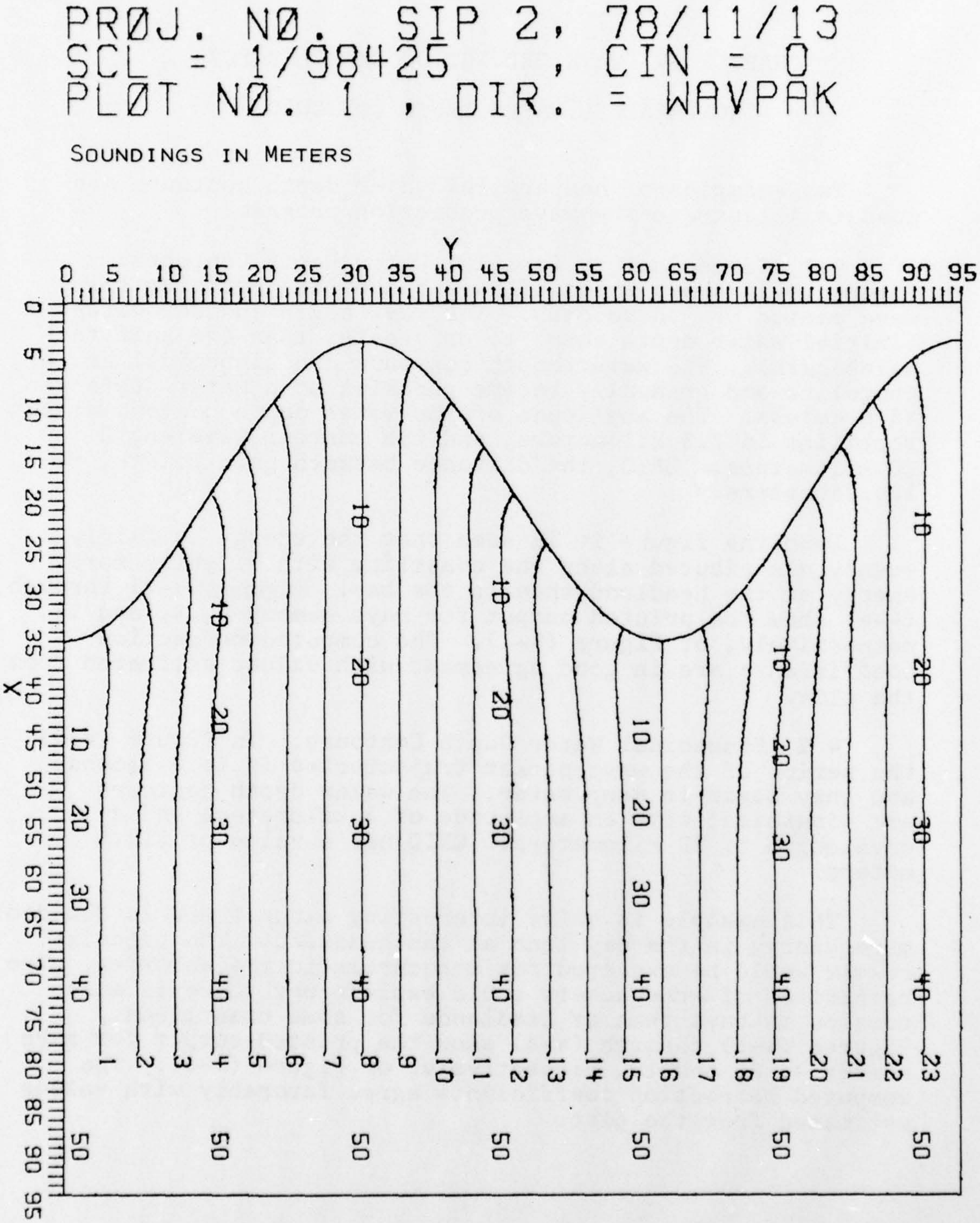

Figure (4-1). Plot for sinusoidal to parallel water depth contours.

PROJECT NO. SIP 2, 78/11/13 , PLOT NO. 1, PERIOD= 7.0SEC., RAY NO. 4, DELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -1.845E-11

MAX	X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO	KR	PCTDIF
1	80.00	16.00	45.10	180.00	180.00	3.53E-01	5.75E-07	-.05	5.51	5.51	10.92	1.0000	1.0000	1.0000		1.0000	.01
5	78.59	16.00	44.25	180.00	180.00	3.53E-01	6.71E-07	-.05	5.51	5.51	10.92	.9995	.9995	1.0000		1.0000	.01
10	76.82	16.00	43.18	180.00	180.00	3.53E-01	7.71E-07	-.06	5.52	5.52	10.92	.9989	.9989	1.0000		1.0000	.04
15	75.06	16.00	42.20	180.00	180.00	3.54E-01	7.87E-07	-.05	5.53	5.53	10.91	.9982	.9982	1.0000		1.0000	.07
20	73.28	16.00	41.17	180.00	180.00	3.54E-01	8.98E-07	-.05	5.54	5.54	10.91	.9973	.9973	1.0000		1.0000	.01
25	71.51	16.00	40.10	180.00	180.00	3.55E-01	1.02E-06	-.05	5.55	5.55	10.90	.9963	.9963	1.0000		1.0000	.01
30	69.73	16.00	39.04	180.00	180.00	3.56E-01	1.16E-06	-.05	5.56	5.56	10.90	.9952	.9952	1.0000		1.0000	.01
35	67.95	16.00	37.97	180.00	180.00	3.57E-01	1.31E-06	-.05	5.58	5.58	10.89	.9939	.9939	1.0000		1.0000	.01
40	66.17	16.00	36.90	180.00	180.00	3.58E-01	1.34E-06	-.04	5.59	5.59	10.88	.9925	.9925	1.0000		1.0000	.01
45	64.37	16.00	35.82	180.00	180.00	3.59E-01	1.50E-06	-.04	5.61	5.61	10.88	.9909	.9909	1.0000		1.0000	.01
50	62.57	16.00	34.74	180.00	180.00	3.60E-01	1.68E-06	-.04	5.63	5.63	10.86	.9890	.9890	1.0000		1.0000	.01
55	60.77	16.00	33.65	180.00	180.00	3.62E-01	1.86E-06	-.04	5.66	5.66	10.85	.9869	.9869	1.0000		1.0000	.05
60	58.96	16.00	32.66	180.00	180.00	3.63E-01	2.04E-06	-.05	5.68	5.68	10.84	.9848	.9848	1.0000		1.0000	.05
65	57.13	16.00	31.58	180.00	180.00	3.65E-01	2.04E-06	-.04	5.71	5.71	10.82	.9823	.9823	1.0000		1.0000	.01
70	55.30	16.00	30.52	180.00	180.00	3.67E-01	-1.65E-04	2.81	5.74	5.74	10.80	.9797	.9795	1.0000	1*	1.0002	.26
75	53.46	16.00	29.33	180.04	179.97	3.70E-01	-1.23E-03	15.64	5.78	5.78	10.77	.9778	.9761	1.0000	2*	1.0017	.37
80	51.60	16.00	27.89	180.26	179.88	3.73E-01	-2.86E-03	25.54	5.84	5.84	10.73	.9766	.9716	1.0000	1*	1.0052	.22
85	49.73	16.02	26.21	180.63	179.71	3.77E-01	-4.00E-03	30.90	5.91	5.91	10.67	.9759	.9657	1.0000	1*	1.0105	.15
90	47.83	16.05	24.56	181.11	179.46	3.82E-01	-5.12E-03	34.32	5.98	5.99	10.60	.9758	.9594	1.0000		1.0171	.17
95	45.90	16.09	22.88	181.71	179.10	3.87E-01	-5.74E-03	36.39	6.07	6.08	10.50	.9766	.9527	1.0000		1.0250	.19
100	43.95	16.16	21.24	182.37	178.63	3.93E-01	-6.24E-03	39.40	6.15	6.17	10.39	.9785	.9461	1.0000		1.0342	.19
105	41.97	16.26	19.58	183.08	178.01	3.98E-01	-6.17E-03	39.46	6.24	6.26	10.25	.9816	.9397	1.0000	1*	1.0446	.23
110	39.97	16.38	17.92	183.77	177.22	4.03E-01	-5.68E-03	42.47	6.32	6.36	10.07	.9861	.9339	1.0000	1*	1.0559	.22
115	37.94	16.52	16.27	184.38	176.21	4.08E-01	-4.44E-03	42.88	6.38	6.44	9.86	.9917	.9294	1.0000	2*	1.0670	.25
120	35.90	16.69	14.63	184.77	174.92	4.10E-01	-2.07E-03	45.70	6.42	6.51	9.60	.9982	.9266	1.0000		1.0772	.24
125	33.86	16.86	13.02	184.82	173.33	4.11E-01	1.30E-03	47.10	6.42	6.55	9.30	1.0041	.9263	1.0000		1.0841	.28
130	31.81	17.03	11.46	184.42	171.45	4.09E-01	5.98E-03	48.83	6.39	6.55	8.94	1.0083	.9288	1.0000		1.0856	.39
135	29.78	17.17	9.64	183.36	169.07	4.04E-01	1.27E-02	49.88	6.30	6.50	8.50	1.0097	.9354	1.0000		1.0794	.43
140	27.79	17.25	8.28	181.44	166.24	3.95E-01	2.14E-02	51.17	6.14	6.36	7.99	1.0067	.9471	1.0000		1.0629	.58
145	25.36	17.25	6.62	178.35	162.66	3.76E-01	3.60E-02	52.26	5.88	6.11	7.33	1.0012	.9681	1.0000	2*	1.0342	.66
150	24.03	17.13	4.95	173.44	158.44	3.50E-01	6.10E-02	54.30	5.46	5.67	6.49	.9965	1.0042	1.0000	2*	.9923	.72
155	22.41	16.84	3.18	165.72	151.71	3.04E-01	1.09E-01	55.06	4.75	4.89	5.34	1.0091	1.0772	1.0000	4*	.9368	2.33
160	21.13	16.39	1.52	153.96	143.29	2.28E-01	2.17E-01	56.06	3.57	3.63	3.79	1.0878	1.2425	1.0000	4*	.8755	1.61
											BREAKUP	TIME STEP	LESS THAN	0.5	SECOND		
164	20.50	16.00	.45	140.50	134.28	1.31E-01	5.06E-01	56.57	2.04	2.06	2.08	1.3712	1.6420	1.0000	8*	.8351	999.00

Figure (4-2). Printed output for Ray No. 4 in Figure (4-1).

PROJECT NO. SIP 2, 78/11/13 , PLOT NO. 1, PERIOD= 7.0SEC., RAY NO. 8, DELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -1.834E-11

MAX	X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO	KR	PCTDIF
1	80.00	32.00	45.10	180.00	180.00	3.53E-01	5.73E-07	-.05	5.51	5.51	10.92	1.0000	1.0000	1.0000		1.0000	.01
5	78.59	32.00	44.25	180.00	180.00	3.53E-01	6.69E-07	-.05	5.51	5.51	10.92	.9995	.9995	1.0000		1.0000	.01
10	76.82	32.00	43.18	180.00	180.00	3.53E-01	7.69E-07	-.06	5.52	5.52	10.92	.9989	.9989	1.0000		1.0000	.04
15	75.06	32.00	42.20	180.00	180.00	3.54E-01	7.85E-07	-.05	5.53	5.53	10.91	.9982	.9982	1.0000		1.0000	.07
20	73.28	32.00	41.17	180.00	180.00	3.54E-01	8.95E-07	-.05	5.54	5.54	10.91	.9973	.9973	1.0000		1.0000	.01
25	71.51	32.00	40.11	180.00	180.00	3.55E-01	1.02E-06	-.05	5.55	5.55	10.90	.9963	.9963	1.0000		1.0000	.01
30	69.73	32.00	39.04	180.00	180.00	3.56E-01	1.16E-06	-.05	5.56	5.56	10.90	.9952	.9952	1.0000		1.0000	.01
35	67.95	32.00	37.97	180.00	180.00	3.57E-01	1.31E-06	-.05	5.58	5.58	10.89	.9939	.9939	1.0000		1.0000	.01
40	66.17	32.00	36.90	180.00	180.00	3.58E-01	1.33E-06	-.04	5.59	5.59	10.88	.9925	.9925	1.0000		1.0000	.01
45	64.37	32.00	35.82	180.00	180.00	3.59E-01	1.50E-06	-.04	5.61	5.61	10.88	.9909	.9909	1.0000		1.0000	.01
50	62.57	32.00	34.74	180.00	180.00	3.60E-01	1.67E-06	-.04	5.63	5.63	10.86	.9890	.9890	1.0000		1.0000	.01
55	60.77	32.00	33.65	180.00	180.00	3.62E-01	1.86E-06	-.04	5.66	5.66	10.85	.9869	.9869	1.0000		1.0000	.05
60	58.96	32.00	32.66	180.00	180.00	3.63E-01	2.03E-06	-.05	5.68	5.68	10.84	.9848	.9848	1.0000		1.0000	.05
65	57.13	32.00	31.58	180.00	180.00	3.65E-01	2.03E-06	-.04	5.71	5.71	10.82	.9823	.9823	1.0000		1.0000	.01
70	55.30	32.00	30.48	180.00	180.00	3.67E-01	2.22E-06	-.04	5.74	5.74	10.80	.9794	.9794	1.0000		1.0000	.01
75	53.46	32.00	29.38	180.00	180.00	3.69E-01	2.41E-06	-.04	5.78	5.78	10.77	.9763	.9763	1.0000		1.0000	.01
80	51.61	32.00	28.26	180.00	180.00	3.72E-01	2.59E-06	-.03	5.82	5.82	10.74	.9728	.9728	1.0000		1.0000	.01
85	49.74	32.00	27.14	180.00	180.00	3.75E-01	2.76E-06	-.03	5.87	5.87	10.71	.9690	.9690	1.0000		1.0000	.01
90	47.85	32.00	26.01	180.00	180.00	3.78E-01	2.90E-06	-.03	5.92	5.92	10.66	.9649	.9649	1.0000		1.0000	.01
95	45.95	32.00	24.87	180.00	180.00	3.81E-01	3.02E-06	-.03	5.97	5.97	10.61	.9605	.9605	1.0000		1.0000	.01
100	44.03	32.00	23.73	180.00	180.00	3.85E-01	2.82E-06	-.03	6.03	6.03	10.55	.9559	.9559	1.0000		1.0000	.06
105	42.10	32.00	22.67	180.00	180.00	3.89E-01	2.87E-06	-.03	6.09	6.09	10.49	.9514	.9514	1.0000		1.0000	.06
110	40.14	32.00	21.48	180.00	180.00	3.93E-01	2.86E-06	-.03	6.15	6.15	10.41	.9462	.9462	1.0000		1.0000	.01
115	38.16	32.00	20.30	180.00	180.00	3.97E-01	2.78E-06	-.02	6.22	6.22	10.31	.9410	.9410	1.0000		1.0000	.01
120	36.16	32.00	19.10	180.00	180.00	4.02E-01	2.52E-06	-.02	6.29	6.29	10.20	.9358	.9358	1.0000		1.0000	.01
125	34.14	32.00	17.88	179.99	180.00	4.06E-01	2.37E-06	-.02	6.36	6.36	10.07	.9307	.9307	1.0000		1.0000	.01
130	32.10	32.00	16.66	179.99	180.00	4.10E-01	2.04E-06	-.02	6.42	6.42	9.91	.9260	.9260	1.0000		1.0000	.01
135	30.03	32.00	15.42	179.99	180.00	4.14E-01	1.60E-06	-.02	6.48	6.48	9.73	.9219	.9219	1.0000		1.0000	.01
140	27.95	32.00	14.19	179.99	180.01	4.17E-01	6.03E-05	-1.32	6.53	6.53	9.52	.9188	.9188	1.0000		1.0000	.41
145	25.86	32.00	13.00	179.99	180.02	4.19E-01	1.08E-06	-.04	6.55	6.55	9.29	.9171	.9169	1.0000		1.0001	.39
150	23.76	32.00	11.77	179.99	180.02	4.20E-01	2.65E-06	.21	6.56	6.56	9.02	.9171	.9166	1.0000		1.0005	.12
155	21.67	32.00	10.51	179.99	180.01	4.18E-01	1.26E-05	.22	6.53	6.53	8.69	.9192	.9186	1.0000		1.0007	.14
160	19.59	32.00	9.27	179.99	180.00	4.15E-01	2.60E-05	.22	6.46	6.46	8.32	.9238	.9236	1.0000		1.0002	.16
165	17.53	32.00	8.03	179.98	179.99	4.07E-01	4.37E-05	.22	6.34	6.34	7.90	.9312	.9325	1.0000		.9986	.18
170	15.53	32.00	6.83	179.98	179.98	3.96E-01	6.71E-05	.23	6.15	6.15	7.42	.9419	.9467	1.0000		.9950	.21
175	13.59	31.99	5.67	179.97	179.96	3.80E-01	9.85E-05	.23	5.88	5.88	6.88	.9569	.9677	1.0000		.9889	.26
180	11.75	31.99	4.59	179.95	179.93	3.55E-01	-4.41E-04	-.61	5.54	5.54	6.29	.9769	.9972	1.0000	4*	.9797	1.02
185	10.04	31.99	3.60	179.98	179.96	3.34E-01	-1.22E-04	-.10	5.12	5.12	5.65	1.0015	1.0376	1.0000	1*	.9652	.96
190	8.47	31.99	2.65	180.01	180.01	3.00E-01	-5.36E-04	-.36	4.57	4.57	4.92	1.0355	1.0980	1.0000		.9431	1.18
195	7.08	31.99	1.82	180.06	180.07	2.60E-01	-7.52E-04	-.37	3.92	3.92	4.12	1.0844	1.1852	1.0000		.9150	1.60
200	5.92	31.99	1.12	180.12	180.12	2.13E-01	-2.78E-04	-.18	3.17	3.17	3.26	1.1658	1.3192	1.0000		.8837	8.06
205	5.01	32.00	.58	180.35	180.40	1.60E-01	-1.06E-02	-1.63	2.33	2.33	2.37	1.3111	1.5370	1.0000		.8530	8.06
210	4.37	32.00	.23	180.23	180.23	1.06E-01	6.27E-03	.05	1.48	1.48	1.48	1.5974	1.9324	1.0000	1*	.8266	999.00
215	4.03	32.00	.03	179.98	179.96	3.28E-02	3.85E-02	.22	.51	.51	.51	2.6557	3.2810	1.0000	2*	.8094	999.00
										RAY REACHED SHORE							
217	3.97	32.00	.01	180.70	180.38	1.92E-02	-5.20E-01	-1.65	.30	.30	.30	3.4514	4.2798	1.0000	2*	.8064	999.00

Figure (4-3). Printed Output for Ray No. 8 in Figure (4-1).

PROJECT NO. SIP 2, 78/11/13 , PLOT NO. 1, PERIOD= 7.0SEC., RAY NO. 16, DELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -1.813E-11

MAX	X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NC	KR	PCTOIF
1	80.00	64.00	45.10	180.00	180.00	3.53E-01	5.70E-07	-.05	5.51	5.51	10.92	1.0000	1.0000	1.0000		1.0000	.01
5	78.59	64.00	44.25	180.00	180.00	3.53E-01	6.66E-07	-.05	5.51	5.51	10.92	.9995	.9995	1.0000		1.0000	.01
10	76.82	64.00	43.18	180.00	180.00	3.53E-01	7.64E-07	-.06	5.52	5.52	10.92	.9989	.9989	1.0000		1.0000	.04
15	75.06	64.00	42.20	180.00	180.00	3.54E-01	7.80E-07	-.05	5.53	5.53	10.91	.9982	.9982	1.0000		1.0000	.07
20	73.28	64.00	41.17	180.00	180.00	3.54E-01	8.89E-07	-.05	5.54	5.54	10.91	.9973	.9973	1.0000		1.0000	.01
25	71.51	64.00	40.11	180.00	180.00	3.55E-01	1.01E-06	-.05	5.55	5.55	10.90	.9963	.9963	1.0000		1.0000	.01
30	69.73	64.00	39.10	180.00	180.00	3.56E-01	-7.79E-07	.03	5.56	5.56	10.90	.9953	.9953	1.0000		1.0000	.21
35	67.95	64.00	37.38	180.00	180.00	3.57E-01	1.04E-05	-.19	5.59	5.59	10.89	.9931	.9932	1.0000		1.0000	.11
40	66.16	64.00	35.27	180.00	180.00	3.59E-01	-6.02E-05	.85	5.62	5.62	10.87	.9896	.9899	1.0000		.9997	.12
45	64.35	64.00	33.16	180.00	180.00	3.62E-01	6.06E-05	-.69	5.67	5.67	10.84	.9848	.9859	1.0000	1*	.9989	.14
50	62.53	64.00	31.04	180.00	180.00	3.66E-01	-2.37E-05	.22	5.73	5.73	10.81	.9783	.9809	1.0000		.9974	.11
55	60.69	64.00	28.86	179.99	180.00	3.70E-01	4.58E-05	-.35	5.80	5.80	10.76	.9697	.9747	1.0000	1*	.9949	.15
60	58.83	64.00	26.71	179.99	180.00	3.75E-01	-1.27E-04	.84	5.89	5.89	10.69	.9588	.9675	1.0000		.9910	.17
65	56.93	64.00	24.49	179.99	180.00	3.82E-01	8.08E-05	-.47	5.99	5.99	10.59	.9451	.9590	1.0000		.9856	.15
70	55.00	64.00	22.22	180.00	180.00	3.90E-01	1.10E-04	-.59	6.11	6.11	10.46	.9288	.9494	1.0000	1*	.9782	.21
75	53.02	64.00	19.91	179.99	180.01	3.98E-01	1.31E-04	-.68	6.24	6.24	10.28	.9099	.9393	1.0000	1*	.9688	.23
80	51.01	64.00	17.54	179.99	180.00	4.07E-01	-9.11E-05	.54	6.38	6.38	10.03	.8895	.9294	1.0000	1*	.9571	.18
85	48.95	64.00	15.15	179.99	180.01	4.14E-01	-6.54E-05	.53	6.49	6.49	9.69	.8686	.9212	1.0000		.9430	.22
90	46.86	64.00	12.71	179.99	180.01	4.19E-01	1.58E-05	-.55	6.56	6.56	9.23	.8501	.9167	1.0000		.9274	.39
95	44.77	64.00	10.24	179.99	180.00	4.17E-01	-3.23E-05	-.22	6.52	6.52	8.62	.8386	.9194	1.0000	2*	.9121	.54
100	42.70	64.00	7.86	180.00	180.02	4.08E-01	2.04E-04	.46	6.31	6.31	7.83	.8409	.9342	1.0000		.9002	.33
105	40.74	64.00	5.53	179.99	180.02	3.81E-01	-1.54E-04	-.15	5.84	5.84	6.81	.8695	.9709	1.0000	1*	.8956	.84
110	38.96	64.00	3.45	179.99	179.99	3.23E-01	7.86E-04	.38	5.04	5.04	5.54	.9444	1.0455	1.0000	2*	.9033	.81
115	37.49	64.00	1.75	180.01	180.01	2.64E-01	2.13E-04	.04	3.85	3.85	4.04	1.1066	1.1958	1.0000		.9254	3.11
120	36.45	64.00	.55	179.93	179.94	1.69E-01	-1.41E-03	-.01	2.28	2.28	2.31	1.4907	1.5549	1.0000		.9587	999.00
										RAY	REACHED	SHORE					
124	36.03	64.00	.05	180.16	180.17	4.66E-02	-3.23E-02	-.33	.73	.73	.73	2.6958	2.7504	1.0000	2*	.9802	999.00

Figure (4-4). Printed ouput for Ray No. 16 in Figure (4-1).

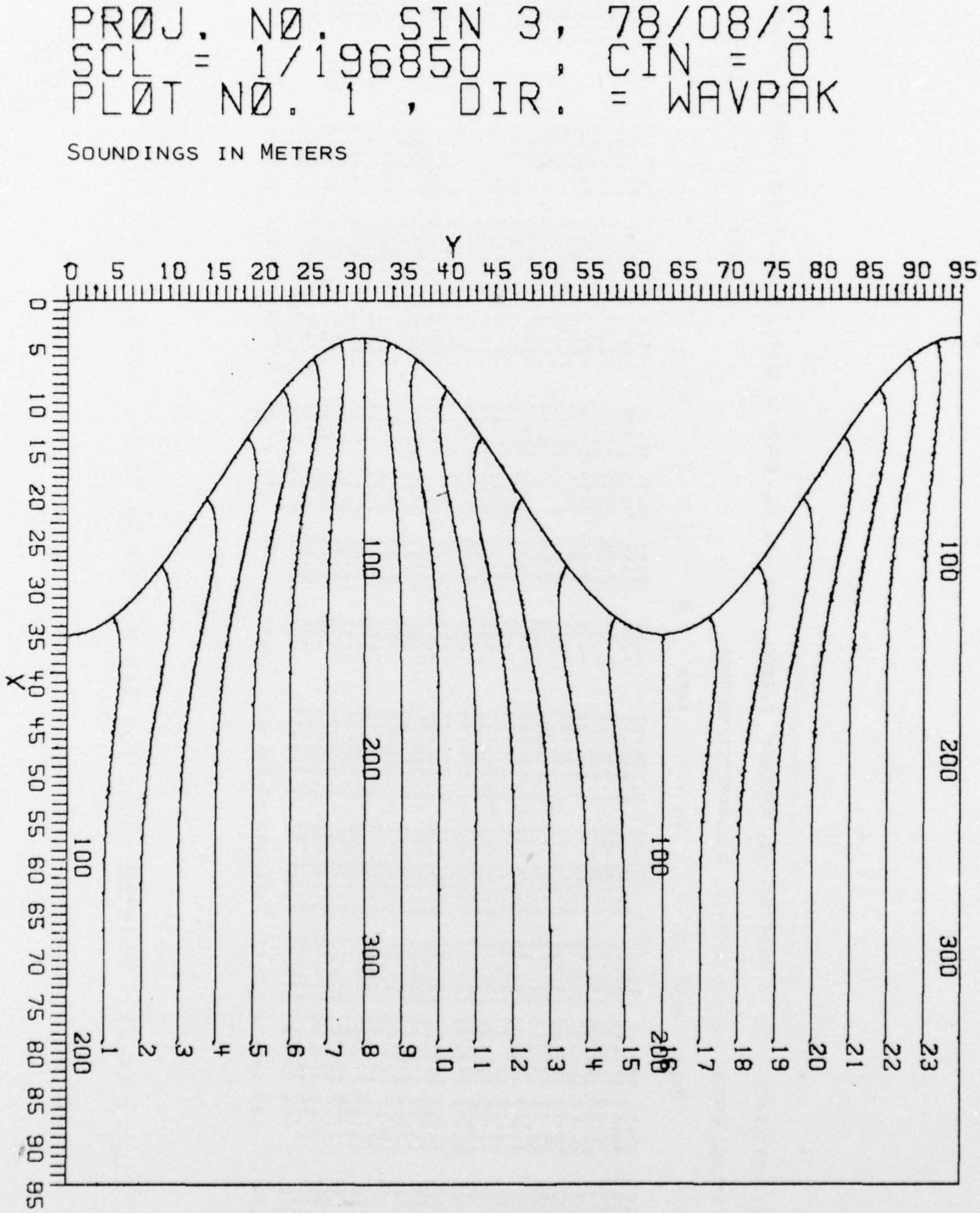

Figure (4-5). Plot for sinusoidal water depth contours.

PROJECT NO. SIN 3, 78/08/31 . PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 4, DELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -0.

MAX	X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO	KR	PCTDIF
1	80.00	16.00	281.26	180.00	180.00	3.50E-01	0.	57.53	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
5	78.60	16.00	274.70	180.00	180.00	3.50E-01	0.	57.42	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
10	76.85	16.00	266.49	180.00	180.00	3.50E-01	0.	57.51	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
15	75.10	16.00	258.31	180.00	180.00	3.50E-01	0.	57.51	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
20	73.35	16.00	250.08	180.00	180.00	3.50E-01	0.	57.51	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
25	71.60	16.00	241.91	180.00	180.00	3.50E-01	0.	57.57	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
30	69.85	16.00	233.68	180.00	180.00	3.50E-01	0.	57.39	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
35	68.10	16.00	225.49	180.00	180.00	3.50E-01	0.	57.56	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
40	66.35	16.00	217.29	180.00	180.00	3.50E-01	0.	57.38	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
45	64.60	16.00	209.09	180.00	180.00	3.50E-01	0.	57.57	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
50	62.86	16.00	200.90	180.00	180.00	3.50E-01	0.	57.41	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
55	61.10	16.00	192.67	180.04	179.93	3.52E-01	-1.29E-03	57.54	10.99	10.99	21.85	.9975	.9975	1.0000	1*	1.0000	.01
60	59.34	16.00	184.48	180.18	179.91	3.52E-01	-1.68E-03	57.54	11.01	11.01	21.85	.9971	.9967	1.0000		1.0005	.03
65	57.58	16.01	176.24	180.37	179.87	3.53E-01	-2.18E-03	57.49	11.03	11.03	21.84	.9972	.9956	1.0000		1.0016	.02
70	55.81	16.03	168.10	180.61	179.82	3.54E-01	-2.81E-03	57.60	11.06	11.06	21.83	.9977	.9942	1.0000		1.0035	.02
75	54.04	16.05	159.95	180.93	179.75	3.55E-01	-3.58E-03	57.41	11.10	11.10	21.81	.9988	.9925	1.0000		1.0064	.01
80	52.26	16.09	151.67	181.33	179.66	3.56E-01	-4.50E-03	57.54	11.15	11.15	21.78	1.0009	.9903	1.0000		1.0106	.03
85	50.48	16.13	143.85	181.83	179.53	3.58E-01	-5.52E-03	57.37	11.21	11.22	21.75	1.0040	.9877	1.0000		1.0165	.03
90	48.68	16.20	135.92	182.45	179.36	3.61E-01	-6.67E-03	57.53	11.28	11.30	21.71	1.0084	.9845	1.0000		1.0243	.03
95	46.87	16.29	128.10	183.18	179.13	3.63E-01	-7.84E-03	57.36	11.37	11.39	21.65	1.0145	.9808	1.0000		1.0343	.02
100	45.05	16.40	120.39	184.05	178.83	3.66E-01	-8.94E-03	57.51	11.46	11.51	21.58	1.0224	.9766	1.0000		1.0468	.02
105	43.22	16.55	112.89	185.03	178.44	3.70E-01	-9.82E-03	57.37	11.57	11.65	21.48	1.0325	.9722	1.0000		1.0620	.04
110	41.37	16.73	105.50	186.11	177.95	3.73E-01	-1.04E-02	57.44	11.68	11.80	21.36	1.0446	.9675	1.0000		1.0797	.04
115	39.50	16.95	98.40	187.23	177.35	3.77E-01	-1.04E-02	57.30	11.79	11.97	21.20	1.0586	.9629	1.0000		1.0994	.03
120	37.63	17.20	91.45	188.34	176.63	3.80E-01	-9.89E-03	57.23	11.90	12.15	21.01	1.0738	.9586	1.0000		1.1202	.04
125	35.74	17.50	84.77	189.38	175.77	3.83E-01	-8.76E-03	57.25	11.99	12.34	20.77	1.0891	.9548	1.0000		1.1406	.06
130	33.84	17.83	78.30	190.26	174.76	3.86E-01	-7.09E-03	57.07	12.07	12.53	20.50	1.1030	.9517	1.0000		1.1590	.04
135	31.94	18.18	71.93	190.93	173.59	3.88E-01	-4.90E-03	56.77	12.13	12.71	20.16	1.1137	.9494	1.0000		1.1731	.02
140	30.03	18.56	65.66	191.32	172.24	3.89E-01	-2.04E-03	56.76	12.17	12.87	19.76	1.1193	.9480	1.0000		1.1806	.03
145	28.12	18.94	59.46	191.36	170.69	3.89E-01	1.40E-03	56.36	12.17	13.01	19.28	1.1178	.9479	1.0000		1.1792	.08
150	26.22	19.32	53.16	190.97	168.88	3.89E-01	5.82E-03	55.97	12.14	13.10	18.70	1.1076	.9492	1.0000		1.1668	.03
155	24.31	19.68	46.69	190.01	166.75	3.86E-01	1.18E-02	55.76	12.05	13.11	17.99	1.0876	.9527	1.0000		1.1416	.12
160	22.42	19.98	39.92	188.33	164.19	3.80E-01	2.03E-02	55.36	11.87	13.01	17.09	1.0577	.9597	1.0000	2*	1.1021	.17
165	20.55	20.21	32.76	185.50	161.03	3.70E-01	3.34E-02	55.08	11.57	12.71	15.91	1.0187	.9722	1.0000	2*	1.0478	.12
170	18.74	20.32	24.99	180.94	156.95	3.52E-01	5.68E-02	54.95	11.01	12.05	14.32	.9754	.9964	1.0000	2*	.9789	.14
175	17.05	20.25	16.58	173.45	151.40	3.18E-01	1.05E-01	55.15	9.95	10.73	12.03	.9408	1.0484	1.0000	4*	.8974	.27
180	15.64	19.95	7.91	160.59	143.35	2.48E-01	2.34E-01	55.49	7.76	8.12	8.57	.9616	1.1874	1.0000	4*	.8098	2.70
185	14.83	19.51	1.11	138.67	131.49	1.04E-01	9.04E-01	55.83	3.24	3.27	3.29	1.3583	1.6365	1.0000	16*	.7396	6.74
										RAY REACHED SHORE							
186	14.77	19.45	.41	133.04	128.63	6.41E-02	1.55E+00	55.89	2.00	2.01	2.01	1.7109	2.3369	1.0000	16*	.7321	6.74

Figure (4-6). Printed Output for Ray No. 4 in Figure (4-5).

PROJECT NO. SIN 3, 78/08/31 , PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 8, CELTAT= 10.00, CF=0.000000, KRTOL= .001000

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -0.

MAX X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NC	KR	PCTDIF
1	80.00	32.00	356.26	180.00	180.00	3.50E-01	0.	-.20	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
5	78.60	32.00	349.70	180.00	180.00	3.50E-01	0.	-.03	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
10	76.85	32.00	341.46	180.00	180.00	3.50E-01	0.	-.09	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
15	75.10	32.00	333.27	180.00	180.00	3.50E-01	0.	-.02	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
20	73.35	32.00	325.07	180.00	180.00	3.50E-01	0.	-.11	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
25	71.60	32.00	316.87	180.00	180.00	3.50E-01	0.	-.04	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
30	69.85	32.00	308.68	180.00	180.00	3.50E-01	0.	-.02	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
35	68.10	32.00	300.45	180.00	180.00	3.50E-01	0.	.20	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
40	66.35	32.00	292.29	180.00	180.00	3.50E-01	0.	-.13	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
45	64.60	32.00	284.06	180.00	180.00	3.50E-01	0.	-.14	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
50	62.86	32.00	275.89	180.00	180.00	3.50E-01	0.	.11	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
55	61.11	32.00	267.67	180.00	180.00	3.50E-01	0.	-.01	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
60	59.36	32.00	259.47	180.00	180.00	3.50E-01	0.	-.03	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
65	57.61	32.00	251.27	180.00	180.00	3.50E-01	0.	-.10	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
70	55.86	32.00	243.07	180.00	180.00	3.50E-01	0.	.04	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
75	54.11	32.00	234.89	180.00	180.00	3.50E-01	0.	-.03	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
80	52.36	32.00	226.65	180.00	180.00	3.50E-01	0.	.12	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.01
85	50.61	32.00	218.48	180.00	180.00	3.50E-01	0.	-.17	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
90	48.86	32.00	210.28	180.00	180.00	3.50E-01	0.	.04	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
95	47.11	32.00	202.10	180.00	180.00	3.50E-01	0.	-.02	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
100	45.36	32.00	193.87	180.00	180.00	3.51E-01	4.64E-08	-.00	10.99	10.99	21.85	.9976	.9976	1.0000		1.0000	.01
1.5	43.60	32.00	185.61	180.00	180.00	3.52E-01	6.69E-07	-.04	11.00	11.00	21.85	.9969	.9968	1.0000		1.0002	.02
110	41.84	32.00	177.36	180.00	180.00	3.53E-01	2.03E-06	-.09	11.03	11.03	21.84	.9963	.9957	1.0000		1.0005	.02
115	40.07	32.00	169.07	180.00	180.00	3.54E-01	6.64E-06	-.22	11.06	11.06	21.83	.9956	.9944	1.0000		1.0013	.02
120	38.30	32.00	160.79	180.00	180.00	3.55E-01	-7.16E-07	.02	11.10	11.10	21.81	.9950	.9926	1.0000		1.0024	.02
125	36.52	32.00	152.42	180.00	180.00	3.56E-01	-3.26E-06	.06	11.15	11.15	21.79	.9944	.9903	1.0000		1.0042	.02
130	34.73	32.00	144.07	180.00	180.00	3.58E-01	1.18E-05	-.18	11.22	11.22	21.75	.9940	.9874	1.0000		1.0067	.03
135	32.93	32.00	135.60	180.00	180.00	3.61E-01	4.22E-06	-.05	11.30	11.30	21.71	.9936	.9836	1.0000		1.0102	.03
140	31.12	32.00	127.12	180.00	180.00	3.64E-01	1.35E-05	-.13	11.41	11.41	21.65	.9936	.9790	1.0000		1.0150	.03
145	29.28	32.00	118.52	180.00	180.00	3.68E-01	-9.29E-07	.01	11.54	11.54	21.56	.9942	.9733	1.0000		1.0215	.02
150	27.43	32.00	109.81	180.00	180.00	3.74E-01	3.47E-06	-.02	11.71	11.71	21.43	.9957	.9664	1.0000		1.0303	.04
155	25.54	32.00	100.97	180.00	180.00	3.80E-01	-6.42E-06	.04	11.91	11.91	21.26	.9986	.9583	1.0000		1.0420	.03
160	23.62	32.00	91.97	180.00	180.00	3.87E-01	5.75E-06	-.03	12.14	12.14	21.02	1.0036	.9491	1.0000		1.0575	.04
165	21.66	32.00	82.80	180.00	180.00	3.95E-01	7.73E-06	-.04	12.40	12.40	20.70	1.0122	.9391	1.0000		1.0778	.02
170	19.60	32.00	73.41	180.00	180.00	4.04E-01	3.26E-07	.00	12.67	12.67	20.24	1.0259	.9291	1.0000		1.1042	.06
175	17.62	32.00	63.64	180.00	180.00	4.12E-01	-6.67E-06	.05	12.92	12.92	19.63	1.0470	.9201	1.0000		1.1379	.05
180	15.54	32.00	54.09	180.00	180.00	4.18E-01	2.15E-07	-.00	13.09	13.09	18.80	1.0782	.9141	1.0000		1.1796	.07
185	13.44	32.00	44.27	180.00	180.00	4.20E-01	-2.40E-06	-.03	13.09	13.09	17.69	1.1227	.9139	1.0000		1.2285	.04
190	11.36	32.00	34.50	180.00	180.01	4.13E-01	-3.35E-06	-.01	12.80	12.80	16.22	1.1835	.9241	1.0000		1.2807	.12
195	9.35	32.00	25.09	180.00	179.99	3.92E-01	3.97E-05	.06	12.06	12.06	14.34	1.2637	.9520	1.0000		1.3273	.12
200	7.50	32.00	16.42	179.99	179.99	3.53E-01	1.14E-05	.02	10.70	10.70	11.98	1.3697	1.0110	1.0000		1.3548	.22
205	5.92	32.00	9.01	180.00	180.00	2.90E-01	-6.17E-05	-.02	8.56	8.56	9.11	1.5267	1.1299	1.0000	1*	1.3511	.31
210	4.73	32.00	3.42	180.00	180.00	2.00E-01	4.35E-05	-.00	5.59	5.59	5.73	1.8443	1.3981	1.0000	1*	1.3192	999.00
215	4.06	32.00	.36	180.11	180.14	5.01E-02	-1.63E-02	-.26	1.88	1.88	1.88	3.0984	2.4127	1.0000	2*	1.2842	999.00
										RAY REACHED SHORE						
216	4.03	32.00	.11	180.19	180.20	3.34E-02	-3.33E-02	-.28	1.04	1.04	1.04	4.1452	3.2380	1.0000	2*	1.2802	999.00

Figure (4-7). Printed output for Ray No. 8 in Figure (4-5).

PROJECT NO. SIN 3, 78/08/31 , PLOT NO. 1, PERIOD= 14.0SEC., RAY NO. 16, DELTAT= 10.00, CF=0.000080, KRTOL= .001080

THE OUTPUT IS IN METRIC UNITS. H,HGT(METER). G,U,V(METER/SECOND). D(BETA)/DT = -0.

MAX	X	Y	H	PACK	WAVE	D	FK	ALFA	G	U	V	HGT	KS	KF	NO	KR	PCTDIF
1	80.00	64.00	206.25	180.00	180.00	3.50E-01	0.	-.15	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
5	78.60	64.00	199.69	180.00	180.00	3.50E-01	0.	.02	10.93	10.93	21.87	1.0000	1.0000	1.0000		1.0000	.02
10	76.85	64.00	191.47	180.00	180.00	3.52E-01	7.85E-07	-.05	10.99	10.99	21.85	.9973	.9974	1.0000		1.0000	.02
15	75.09	64.00	183.23	180.00	180.00	3.52E-01	-6.90E-07	.03	11.01	11.01	21.85	.9963	.9965	1.0000		.9998	.03
20	73.32	64.00	174.93	180.00	180.00	3.53E-01	1.38E-06	-.05	11.04	11.04	21.84	.9946	.9954	1.0000		.9993	.01
25	71.56	64.00	166.65	180.00	180.00	3.54E-01	-1.00E-07	.00	11.07	11.07	21.82	.9923	.9939	1.0000		.9984	.03
30	69.78	64.00	158.34	180.00	180.00	3.55E-01	-1.38E-06	.03	11.11	11.11	21.80	.9891	.9920	1.0000		.9971	.02
35	68.00	64.00	150.01	180.00	180.00	3.57E-01	-6.04E-07	.01	11.17	11.17	21.78	.9847	.9895	1.0000		.9951	.03
40	66.21	64.00	141.60	180.00	180.00	3.59E-01	3.36E-06	-.05	11.24	11.24	21.74	.9789	.9864	1.0000		.9924	.01
45	64.41	64.00	133.12	180.00	180.00	3.62E-01	1.05E-05	-.12	11.33	11.33	21.69	.9713	.9824	1.0000		.9887	.02
50	62.59	64.00	124.53	180.00	180.00	3.65E-01	-1.16E-05	.11	11.44	11.44	21.62	.9615	.9774	1.0000		.9837	.04
55	60.75	64.00	116.00	180.00	180.00	3.70E-01	4.27E-06	-.03	11.59	11.59	21.52	.9491	.9714	1.0000		.9771	.04
60	58.88	64.00	107.26	180.00	180.00	3.75E-01	6.65E-06	-.04	11.76	11.76	21.39	.9338	.9641	1.0000		.9686	.02
65	56.99	64.00	98.37	180.00	180.00	3.82E-01	-2.12E-05	.12	11.97	11.97	21.20	.9157	.9557	1.0000		.9577	.05
70	55.06	64.00	89.35	180.00	180.00	3.89E-01	-2.41E-06	.01	12.21	12.21	20.94	.8934	.9463	1.0000		.9441	.05
75	53.09	64.00	80.08	180.00	180.00	3.98E-01	-3.43E-05	.18	12.48	12.48	20.58	.8685	.9361	1.0000		.9278	.05
80	51.07	64.00	70.66	180.00	180.00	4.06E-01	-1.03E-05	.06	12.74	12.74	20.09	.8417	.9263	1.0000		.9087	.06
85	49.02	64.00	61.01	180.00	180.00	4.14E-01	2.34E-05	-.18	12.98	12.98	19.41	.8145	.9179	1.0000		.8873	.04
90	46.93	64.00	51.25	180.00	180.00	4.19E-01	-1.68E-06	.05	13.11	13.11	18.51	.7897	.9132	1.0000	1*	.8648	.06
95	44.83	64.00	41.41	180.00	180.00	4.19E-01	-3.52E-06	-.03	13.05	13.05	17.30	.7720	.9155	1.0000		.8433	.10
100	42.77	64.00	31.72	180.00	180.00	4.05E-01	-1.30E-05	-.03	12.64	12.64	15.72	.7682	.9299	1.0000	2*	.8261	.05
105	40.8[illegible]	64.00	22.48	180.00	180.00	3.76E-01	8.71E-05	.09	11.74	11.74	13.71	.7889	.9651	1.0000	2*	.8174	.23
110	39.01	64.00	14.14	179.99	179.99	3.37E-01	1.50E-05	.02	10.17	10.17	11.21	.8516	1.0370	1.0000	1*	.8215	.22
115	37.53	64.00	7.15	179.99	180.00	2.66E-01	-5.67E-04	-.11	7.78	7.78	8.17	.9961	1.1856	1.0000		.8401	.80
120	36.48	64.00	2.24	180.01	180.02	1.69E-01	2.64E-04	.00	4.59	4.59	4.66	1.3425	1.5442	1.0000		.8693	999.00
												PACKET CURVATURE ITERATION NOT CONVERGING					
124	36.05	64.00	.25	179.99	179.99	4.97E-02	3.99E-03	.03	1.55	1.55	1.56	2.3579	2.6527	1.0000	2*	.8889	999.00

Figure (4-8). Printed output for Ray No. 16 in Figure (4-5).

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Block 20. ABSTRACT

In Part I a method for calculating wave packet trajectories and wave heights is based on the assumption that the water depth contours are locally parallel in the vicinity of each ray point. This method is extended in order to predict the modification to surface gravity water waves in shoaling water when the water depth contours are not parallel. The calculations are greatly simplified by choosing a coordinate system at each ray point in which one axis is aligned parallel with the direction of the gradient of the water depth. Example printouts and plots are presented to illustrate the wave prediction method. It is discovered that when waves initially approach sinuous water depth contours symmetrically with respect to the beach there can be more energy in the bays than at the headlands.