AD-A063 930 YALE UNIV NEW HAVEN CT SYSTEMS AND INFORMATION SCIENCES F/G 12/1
A NEW ERROR MODEL FOR DISCRETE SYSTEMS AND ITS APPLICATION TO A--ETC(U)
OCT 78 Y LIN, K S NARENDRA N00014-76-C-0017
UNCLASSIFIED S/IS-7802 NL

| OF |
AD
A063930

END
DATE
FILMED
3-79
DDC

LEVEL II (12)

(6) A NEW ERROR MODEL FOR DISCRETE SYSTEMS AND ITS APPLICATION TO ADAPTIVE IDENTIFICATION AND CONTROL,

(10) Yuan-Hao/Lin ~~and~~ Kumpati S./Narendra

Contract / N00014-76-C-0017

(15) S / IS ~~Report No.~~ 7802

(11) Oct~~ober,~~ 1978

(12) 17p.

(14) S/IS-7802

A New Error Model for Discrete Systems and Its Application to Adaptive Identification and Control

Yuan-Hao Lin and Kumpati S. Narendra

1. Introduction: Model reference adaptive schemes for the identification and control of continuous and discrete systems have been discussed by numerous authors in recent years [1-9]. Among the many approaches that are currently in vogue, the stability approach using Lyapunov Theory and the Hyperstability Theory of Popov have found wide acceptance. The problem of convergence of the parameters of a dynamical system to desired values in identification and control problems is recast, using this approach, to the equivalent asymptotic stability problem of a set of error differential or difference equations.

In a recent paper [9] three such error models were described for continuous systems which arise frequently in problems of adaptation. Discrete versions of these models are also very desirable since in most applications which use a computer as a part of the controller, discrete rather than continuous algorithms are used. While the discrete versions of the first two models can be obtained in a relatively straightforward manner from their continuous counterparts, the third error model has proved to be considerably more difficult. In this paper, a simple structure is chosen for the third error model and a complete stability analysis is presented. The applications of the new model to adaptive identification and control are also briefly discussed.

In a forthcoming report [12] different versions of three discrete error models and their applications to adaptive observers and controllers will be treated extensively and the reader is referred to that report for further details. Only the principal result concerned with the third error model is presented in this paper. Its detailed analysis here is justified by its simplicity and frequent use in real applications.

2. <u>The Error Model for the Continuous Case</u>: For the sake of completeness as well as to provide the reader with a basis for comparison with the discrete case, the error model of the third prototype [9] for the continuous case is briefly discussed in this section. Figure (1) represents the error model in block

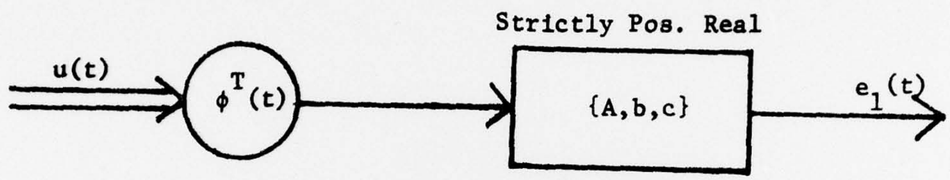

Figure (1)

diagram form. The model is described by the equations

$$\begin{aligned} \dot{e}(t) &= Ae(t) + b\phi^T(t)u(t) \\ e_1(t) &= c^T e(t) \end{aligned} \tag{1}$$

where $e(t)$ is an $(nx1)$ state vector, $\phi(t)$ and $u(t)$ are m dimensional vectors with the elements of $u(t)$ piecewise continuous and uniformly bounded. A is a stable (nxn) matrix, b and c are $(nx1)$ constant vectors, with (A,b) completely controllable, and the transfer function $c^T(sI-A)^{-1}b$ is strictly positive real. The elements of the vector $\phi(t)$ are unknown but the time derivative $\dot{\phi}(t)$ can be adjusted using the signals $u(t)$ and $e_1(t)$ which can be measured. The aim of the adjustment is to make $\lim_{t\to\infty} e_1(t) = 0$.

If the adaptive law

$$\dot{\phi}(t) = -\Gamma\, u(t)e_1(t) \qquad \Gamma = \Gamma^T > 0 \tag{2}$$

is chosen, it can be shown easily using the Kalman-Yacubovich Lemma that

(i) $e(t)$ and $\phi(t)$ are bounded

(ii) $\lim_{t\to\infty} e(t) = 0$

and (iii) if $u(t)$ is sufficiently rich [10]

$\lim_{t\to\infty} \phi(t) = 0$

<u>Proof</u>: Choosing $V(e,\phi) = e^TPe + \phi^T\Gamma^{-1}\phi > 0$ as a candidate for a Lyapunov function

$$\dot{V}(e,\phi) = e^T[A^TP + PA]e + 2e^TPb\phi^Tu + 2\phi^T\Gamma^{-1}\dot{\phi}$$

By the Kalman-Yacubovich Lemma a matrix $P = P^T > 0$ exists such that

$$A^TP + PA = -qq^T - \varepsilon L$$
$$Pb = c$$

for some vector q, matrix $L = L^T > 0$ and $\varepsilon > 0$ iff $c^T(sI-A)^{-1}b$ is strictly positive real. In such a case,

$$\dot{V}(e,\phi) = -e^T(qq^T+\varepsilon L)e + 2\, e_1\phi^Tu + 2\,\phi^T\Gamma^{-1}\dot{\phi}$$

By choosing the adaptive law (2) $\dot{V}$ becomes

$$\dot{V}(e,\phi) = -e^T(qq^T+\varepsilon L)e \leq 0$$

so that the system is stable and (i) holds. If $u(t)$ is uniformly bounded it follows [8] that $\lim_{t\to\infty} e(t) = 0$. The convergence of $\phi(t)$ has been extensively investigated [10] and it has been shown that when the input $u(t)$ is "sufficiently rich" $\lim_{t\to\infty} \phi(t) = 0$.

3. <u>The New Error Model for the Discrete Case</u>: Figure (2) represents the error model of the third prototype for the discrete case and is

$$\begin{aligned} e(k+1) &= Ae(k) + bv(k) \\ e_1(k) &= c^Te(k) + dv(k) \qquad (3) \\ v(k) &= \phi^T(k)u(k) - \alpha u^T(k)\Gamma u(k)e_1(k);\ \alpha > \tfrac{1}{2},\ \Gamma = \Gamma^T > 0 \end{aligned}$$

where e(k) and u(k) are respectively n and m dimensional vectors and v(k), $e_1(k)$

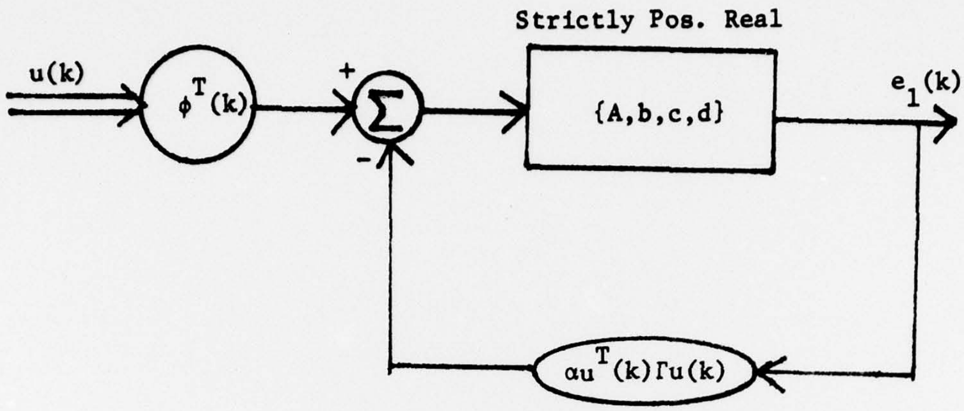

Figure (2)

scalars. A is a matrix, b and c are vectors of suitable dimensions and d is a positive scalar such that the z-transfer function $d + c^T(zI-A)^{-1}b$ is strictly positive real [12].

From Figure (2) it is seen that the model contains a known linear time-invariant part with a strictly positive real transfer function, a known time varying gain $\alpha u^T(k)\Gamma u(k)$, which depends on the input signal, in the feedback path and a time varying vector gain $\phi(k)$ in the feedforward path. The vector $\phi(k)$ is assumed to be unknown but changes in $\phi(k)$ can be made using the measured signals u(k) and $e_1(k)$. The objective is to determine suitable adaptive control laws for updating $\phi(k)$ such that in the limit $e_1(k) \to 0$ as $k \to 0$. From the previous section it is seen that this adaptive updating is quite simple in the continuous case. While relatively complex schemes are known in the literature [5], our aim is to develop a scheme for the discrete case which is comparable in its simplicity to that in the continuous model. The principal result of this paper is that the adaptive law

$$\Delta\phi(k) = \phi(k+1) - \phi(k) = -\Gamma\, e_1(k)u(k) \tag{4}$$

achieves the desired aim. While this adaptive law is similar to (2) in the continuous case, it is achieved by using an additional feedback signal in the discrete model.

<u>Lemma</u>: Given an (nxn) matrix A with all its eigenvalues within the unit circle, a symmetric positive definite matrix Γ, vectors $b,c \in \mathbb{R}^n$ with (A,b) completely controllable and $u(k): Z_+ \to \mathbb{R}^m$ whose elements are bounded, the equilibrium states of the set of n + m difference equations (3),(4) is stable and $\lim_{k\to\infty} e(k) = 0$ if the transfer function $d + c^T(zI-A)^{-1}b$ is strictly positive real. Further if u(k) is sufficiently rich, $\lim_{k\to\infty} \phi(k) = 0$.

<u>Proof</u>: From the discrete version of the Kalman-Yacubovich Lemma [11] it is known that if $d + c^T(zI-A)^{-1}b$ is strictly positive real, a matrix $P = P^T > 0$ exists such that

$$\begin{aligned} A^TPA - P &= -qq^T - \varepsilon L \\ A^TPb &= c/2 + \nu q \\ d - b^TPb &= \nu^2 \end{aligned} \qquad (5)$$

for some vector q, matrix $L = L^T > 0$ and $\varepsilon, \nu > 0$.

Defining a Lyapunov function candidate for the set of difference equations (3),(4) as:

$$V(e(k),\phi(k)) = 2e^T(k)Pe(k) + \phi^T(k)\Gamma^{-1}\phi(k)$$

we obtain

$$\begin{aligned} \Delta V(e(k),\phi(k)) &\triangleq \Delta V(k) = V(k+1) - V(k) \\ &= 2[e^T(k)(A^TPA-P)e(k) + 2e^T(k)A^TPbv(k) \\ &\quad + b^TPbv^2(k)] + \phi^T(k+1)\Gamma^{-1}\phi(k+1) - \phi^T(k)\Gamma^{-1}\phi(k) \end{aligned}$$

Choosing the matrix P given by the relations (5):

$$\begin{aligned} \Delta V(k) = &-2[e^T(k)q-\nu v(k)]^2 - 2\varepsilon e^T(k)Le(k) + 2e_1(k)v(k) \\ &+ 2\phi^T(k)\Gamma^{-1}\phi(k) + \Delta\phi^T(k)\Gamma^{-1}\Delta\phi(k) \end{aligned}$$

With the adaptive law (4) for $\Delta\phi(k)$ the sum of the last three terms may be expressed as

$$2e_1(k)v(k) - 2\phi^T(k)e_1(k)u(k) + e_1^2(k)u^T(k)\Gamma u(k)$$

or
$$(-2\alpha+1)u^T(k)\Gamma u(k)e_1^2(k)$$

Hence
$$\Delta V(k) = -2[e^T(k)q-\nu v(k)]^2 - 2\varepsilon e^T(k)Le(k) + (-2\alpha+1)u^T(k)\Gamma u(k)e_1^2(k)$$
$$\leq 0 \qquad \text{if } \alpha > \frac{1}{2} \tag{6}$$

The system (3),(4) is stable and $e(k)$ and $\phi(k)$ are bounded if $e(0)$ and $\phi(0)$ are bounded.

Having established the global stability of the discrete error model we now state some of its other stability properties:

(i) The boundedness of $e(k)$ and $\phi(k)$ are assured even when $u(k)$ is not bounded.

(ii) Furthermore, we can prove $\lim_{k\to\infty} e(k) \to 0$, $\lim_{k\to\infty} e_1(k) \to 0$ whether or not $u(k)$ is bounded. Defining

$$(2\alpha-1)u^T(k)\Gamma u(k) \triangleq \xi(k) \quad , \quad \xi(k) > 0$$

we have

$$\Delta V(k) = -2[e^T(k)q-\nu v(k)]^2 - 2\varepsilon e^T(k)Le(k) - \xi(k)e_1^2(k)$$

$$\left|\sum_{k=0}^{\infty} \Delta V(k)\right| = |V(\infty) - V(0)| < \infty \qquad \text{since } \Delta V(k) \lesssim 0$$

or
$$\sum_{k=0}^{\infty} \{2[e^T(k)q-\nu v(k)]^2 + 2\varepsilon e^T(k)Le(k) + \xi(k)e_1^2(k)\} < \infty$$

Hence we conclude $\lim_{k\to\infty} e(k) = 0$ for $u(k)$ bounded or not.

If $u(k)$ is unbounded, so is $\xi(k)$, $e_1(k)$ has to go to zero.

If $u(k)$ is bounded but does not tend to zero as $k \to \infty$, $\xi(k)$ does not tend to zero and hence $e_1(k) \to 0$. If however $u(k)$ does tend to zero as $k \to \infty$ then $e_1(k) \to 0$ by (3) and the result follows.

(iii) Since from (ii) $\xi(k)e_1^2(k) \to 0$, $\phi(k+1) - \phi(k)$ in (4) tends to zero or $\phi(k) \to \phi^*$, a constant vector. From (3) it follows that if $u(k)$ is "sufficiently rich" $\phi^* = 0$ or $\phi(k) \to 0$ asymptotically.

(iv) In Figure (2), if we just consider the loop with the strictly positive real transfer function in the forward path and $(\alpha u^T(k)\Gamma u(k))e_1(k)$ in the feedback path, it follows that this loop is uniformly asymptotically stable if u(k) is bounded. Therefore, if there is any input disturbance which tends to zero, the effect of this disturbance on the output will also tend to zero.

A Note: As pointed out in the introduction, a simple adaptive law is achieved by the use of the structure of the error model shown in Figure (2) which involves the use of the feedback term $(\alpha u^T(k)\Gamma u(k))e_1(k)$. In general, the output of the unknown gain (i.e. $\phi^T(k)u(k)$) must be assumed to be inaccessible for measurement since otherwise we would use the first prototype error model as discussed in [4]. Hence one may question the very validity of the model described so far. The answer to this lies in the following section on applications where it is shown that the above error model can be realized even when the output of the unknown gain is not measurable.

4. Applications: All the adaptive observer and control problems which could be resolved using the error model for the third prototype in the continuous case have their counterparts in the discrete case as well. We merely describe here briefly two important applications to indicate how the error model described in the previous section (3) can be derived and used in the generation of the adaptive laws.

a) A Discrete Adaptive Observer:

The input u(k) and the output y(k) of an unknown stable linear time-invariant plant are given. If the z-transfer function of the plant is P(z)/Q(z) where P(z) is an n^{th} degree polynomial in z and Q(z) an n^{th} degree monic stable polynomial, the plant can be described by (2n+1) parameters which are the coefficients of

$P(z)$ and $Q(z)$. An adaptive observer is to be constructed which will estimate these parameters and evolve to the true values asymptotically.

Following [2] it is known that any transfer function with n poles and n zeros has an equivalent non-minimal canonical representation given by

$$\frac{P(z)}{Q(z)} = \frac{P_1(z)}{R(z)} \left[\frac{F_1(z)/F_2(z)}{1 - \frac{Q_1(z)}{R(z)} \cdot \frac{F_1(z)}{F_2(z)}} \right] \tag{6}$$

where P_1 and Q_1 are n^{th} and $(n-1)^{th}$ degree polynomials, $R(z)$ is any n^{th} degree monic stable polynomial, $F_1(z)$ and $F_2(z)$ are m^{th} degree monic polynomials ($m \leq n$) with $F_1(z)/F_2(z)$ strictly positive real and such that $R(z)$ contains $F_1(z)$ as a factor. (In the simplest case we can choose $F_1 = F_2 = 1$).

The representation of (6) in block diagram form is shown in Figure (3).

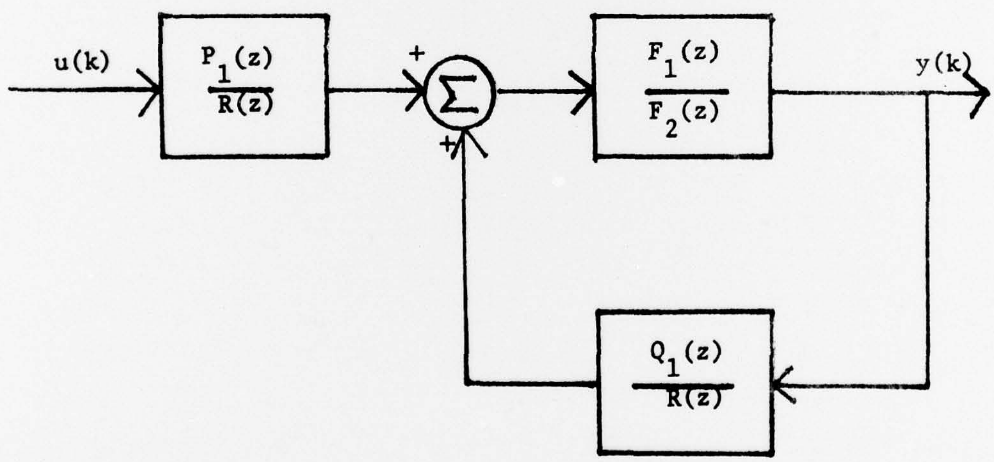

Figure (3)

The identification of the plant transfer function (i.e. the polynomials $P(z)$ and $Q(z)$) is now reduced to the identification of the polynomials $P_1(z)$ and $Q_1(z)$. Figure (4) shows the discrete version of an adaptive observer. The structure of the observer is identical to that in the continuous case with the exception of the feedback signal $(\alpha u^T(k)\Gamma u(k))e_1(k)$. The parameters $\hat{a}_i(k)$ and $\hat{b}_j(k)$ in the model are estimates of the corresponding parameters a_i and b_j in the plant ($i = 1,2,\ldots,n$; $j = 0,1,\ldots n$).

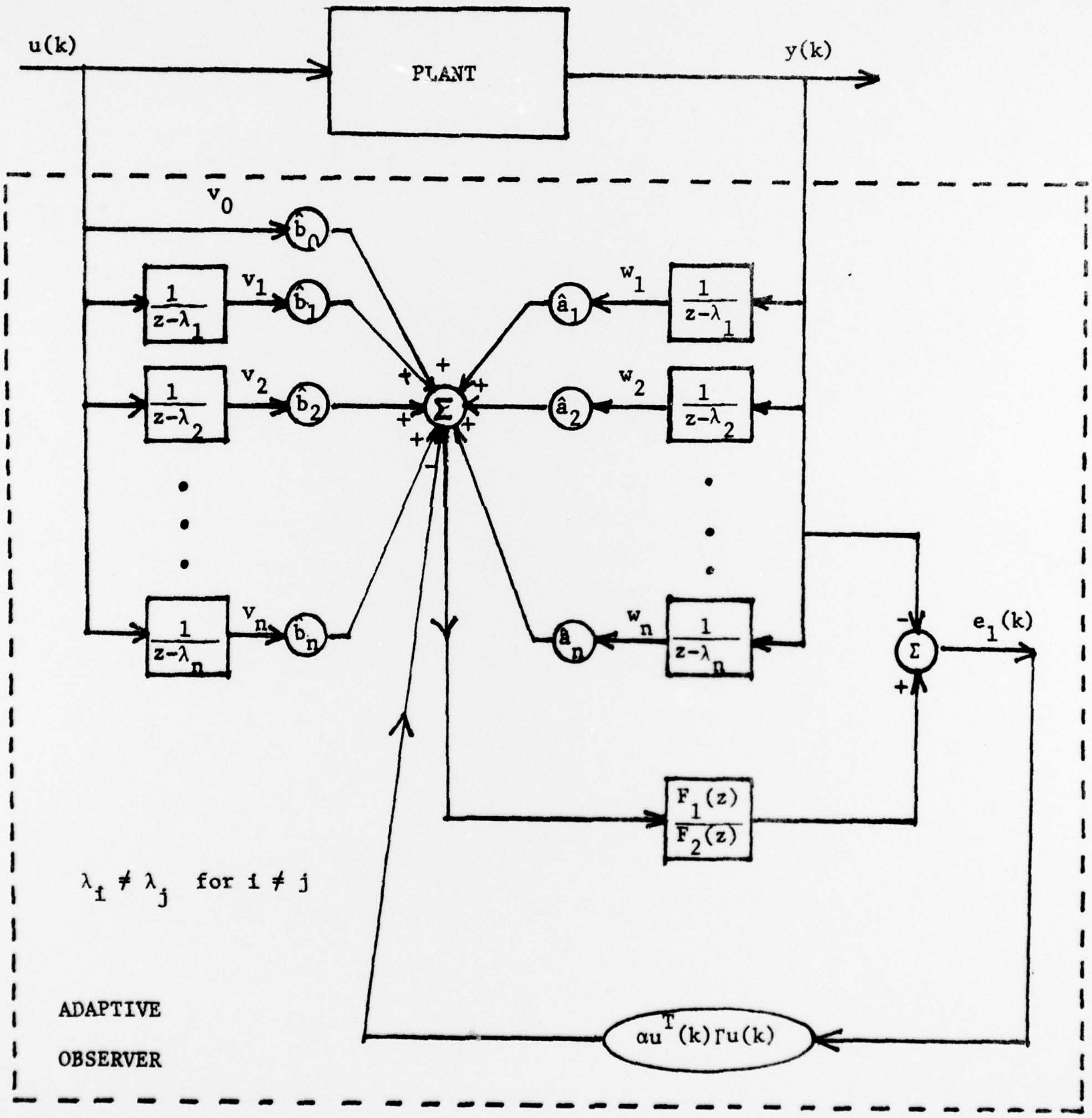

Figure (4)

If $w^T(k) = [w_1(k), w_2(k), \ldots w_n(k)]$; $v^T(k) = [v_0(k), v_1(k), \ldots v_n(k)]$ in Figure (4), defining $u^T(k) = [w^T(k), v^T(k)]$ the error equations can be represented in the form shown in Figure (2) and hence the adaptive laws can be written by inspection as

$$\begin{bmatrix} \hat{a}(k+1) \\ \hat{b}(k+1) \end{bmatrix} = \begin{bmatrix} \hat{a}(k) \\ \hat{b}(k) \end{bmatrix} - \Gamma\ e_1(k) \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}$$

It is worth pointing out that while the signals corresponding to w(k) and v(k) in the plant are not accessible, (and hence $\phi(k)^T u(k)$ in Figure (2) is not accessible) the desired error model is obtained by feeding back the signal $(\alpha u^T(k)\Gamma u(k))e_1(k)$ in the model as shown in Figure (4).

b) <u>The Adaptive Control Problem:</u>

We consider in this section the simplest of the control problems treated in [8] where an unknown linear time-invariant plant with a single input and single output is to be controlled to follow the output of a specified model with a strictly positive real transfer function. We shall treat only the case where a single parameter of the controller can be adjusted and extend the results directly to the general case where (2n+1) parameters are to be adjusted. For the latter case we shall merely indicate the structure of the adaptive controller and the form of the adaptive laws.

Let an unknown plant be represented by the difference equation

$$x_p(k+1) = Ax_p(k) + bv(k)$$
$$y_p(k) = c^T x_p(k) + dv(k)$$

where the matrix A, vectors b and c and scalars d are unknown. If $v(k) = r(k) +$ $+ \beta(k)y_p(k)$, then $\beta(k)$ represents the gain in the feedback path as shown in Figure (5), and r(k) the input to the overall system.

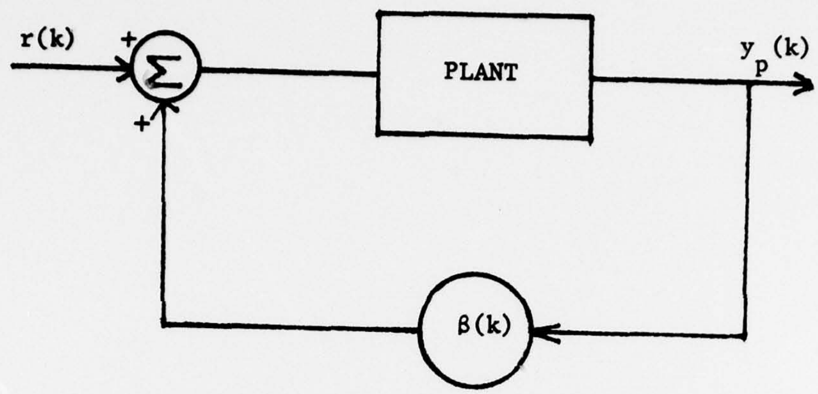

Figure (5)

Let the specified model have a strictly positive real transfer function $W_M(z)$. We assume that a constant β^* exists such that when $\beta \equiv \beta^*$ the transfer function of the plant matches exactly that of the model. The plant and model are subjected to the same input r(k) and the aim of the adaptive procedure is to update $\beta(k)$ using input-output data such that

$$|e_1(k)| \triangleq |y_p(k) - y_M(k)| \to 0 \quad \text{as } k \to \infty$$

where $y_M(k)$ is the output of the model.

It follows immediately that the error model can be represented in the form shown in Figure (6).

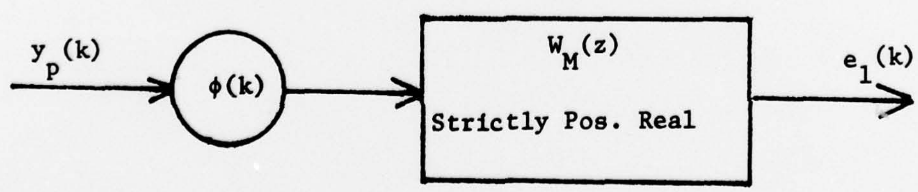

Figure (6)

where $\beta(k) - \beta^* = \phi(k)$ and $y_p(k) - y_M(k) = e_1(k)$.

Our aim is now to adjust $\phi(k)$ (or equivalently $\beta(k)$) such that $e_1(k) \to 0$ as $k \to \infty$. To reduce the error model (6) to the standard form described in section (3) we have to use a feedback with a gain $\alpha\gamma y_p^2(k)$ as shown in Figure (7) or in the plant as shown in Figure (8).

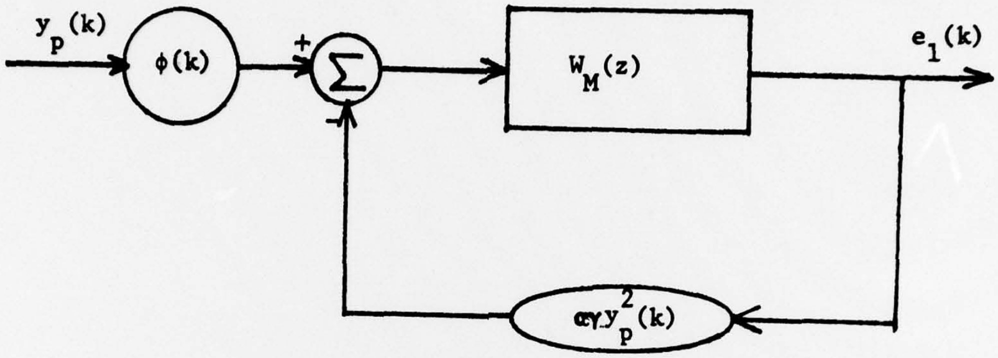

Figure (7) Modified Error Model

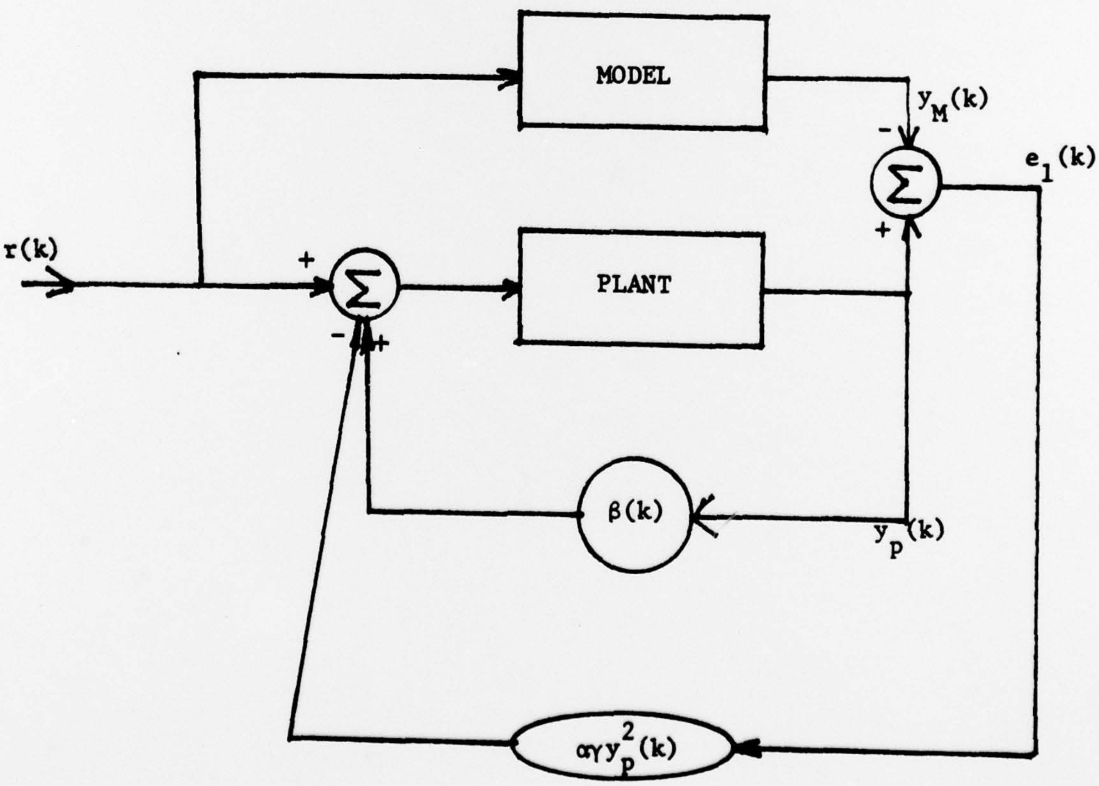

Figure (8) Modified Plant

It is the fact that the error $e_1(k)$ can be fed back into the plant that allows the new error model to be realized. The adaptive law can now be written down by inspection as:

$$\Delta\beta(k) = \Delta\phi(k) = -\gamma\, e_1(k)y_p(k) \qquad \gamma > 0 .$$

For the general case of an unknown plant with transfer function $\frac{P(z)}{Q(z)}$ where $P(z)$ is an n^{th} degree stable polynomial and $Q(z)$ is a monic n^{th} degree polynomial whose output is to follow the output of a given model with strictly positive real transfer function, it can be shown that a controller of the form shown in Figure (9) can be used to achieve the above objective.

The controller consists of $(2n+1)$ adjustable parameters $\hat{k}_0, \hat{c}_1, \hat{c}_2, \ldots$ $\ldots, \hat{c}_n, \hat{d}_1, \hat{d}_2, \ldots, \hat{d}_n$ which are the elements of a parameter vector $\hat{\theta}$ and two identical auxiliary signal generators with stable transfer functions whose inputs are the plant input and output respectively. If reference input $r(k)$ together with the $2n$ signals $v_1(k), v_2(k), \ldots v_n(k)$, $w_1(k), \ldots w_n(k)$ is defined as $u(k)$:

$$u(k)^T \triangleq [r(k), v_1(k), v_2(k), \ldots v_n(k), w_1(k), \ldots w_n(k)]$$

and a signal $[\alpha u(k)^T \Gamma u(k)]$ is fed back as shown in Figure (9), the total input to the plant may be expressed as

$$\hat{\theta}^T(k)u(k) - [\alpha u^T(k)\Gamma u(k)]e_1(k)$$

If a constant θ^* exists such that Model and Plant transfer functions are identical when $\hat{\theta} = \theta^*$, then we obtain an error model of the form shown in Figure (2). Once again the adaptive laws can be written by inspection as

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \Gamma\, e_1(k)u(k)$$

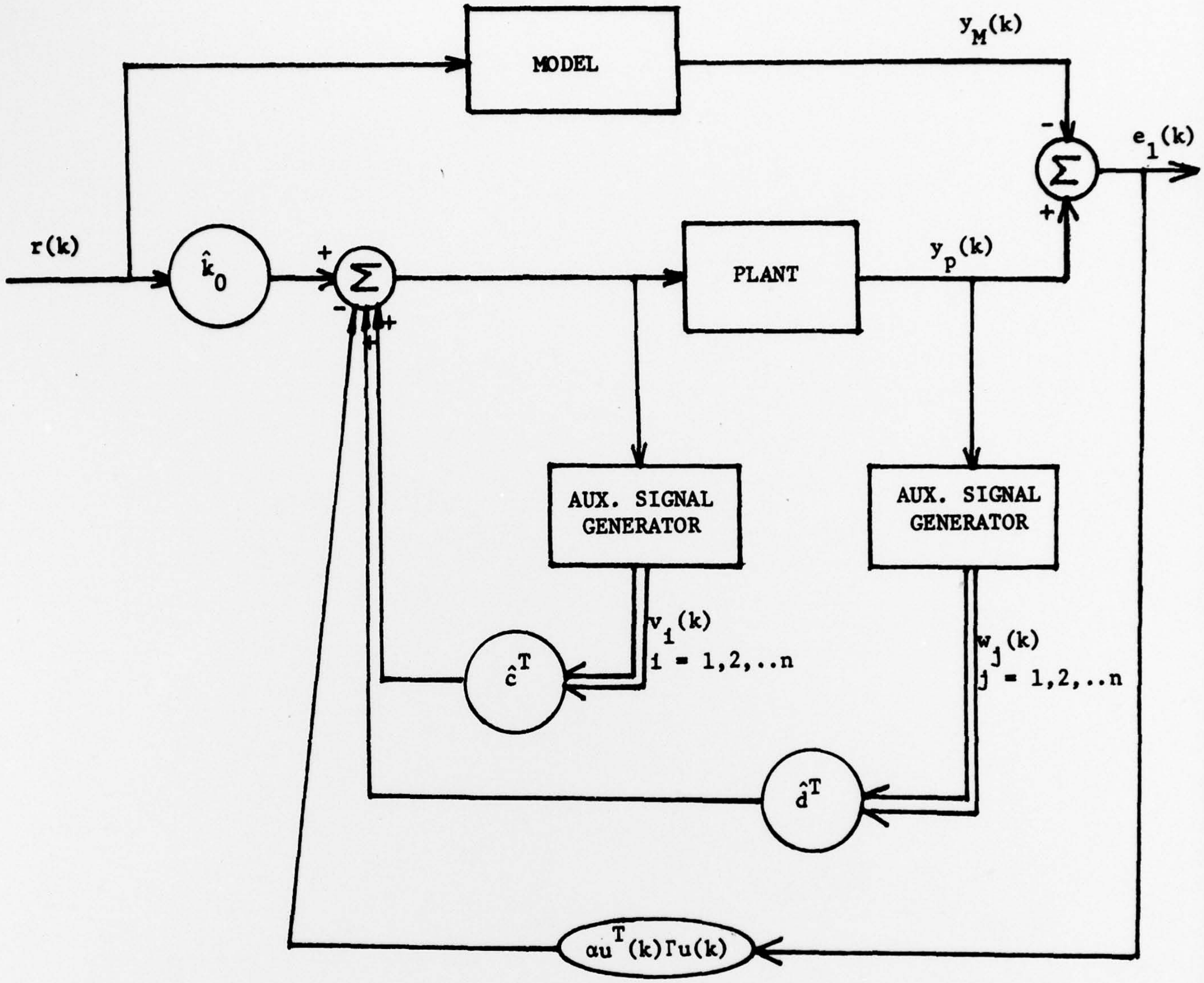

Figure (9)

<u>Conclusion</u>: A new error model for discrete adaptive systems is presented in this paper. By introducing a feedback term in the corresponding error model for the continuous case it is found possible to retain the same form for the adaptive laws. It is demonstrated that the output error tends to zero whether or not the input vector is bounded.

The new error model has wide applications in both adaptive identification and adaptive control. Two applications are outlined towards the end of the paper to indicate how the error model is derived and the corresponding adaptive laws generated.

References

[1] R. L. Carroll and D. P. Lindorff, "An Adaptive Observer for Single-Input Single-Output Linear System ," IEEE Trans. Automat. Contr., vol. AC-18, pp. 428-435, Oct. 1973.

[2] G. Lüders and K. S. Narendra, "Stable Adaptive Schemes for State Estimation and Identification of Linear Systems," IEEE Trans. Automat. Contr., vol. AC-19, pp. 841-847, Dec. 1974.

[3] K. S. Narendra and P. Kudva, "Stable Adaptive Schemes for System Identification and Control - Parts I, II," IEEE Trans. Systs., Man, & Cybern., vol. SMC-4, pp. 542-560, Nov. 1974.

[4] P. Kudva and K. S. Narendra, "An Identification Procedure for Discrete Multivariable Systems," IEEE Trans. Automat. Contr., vol. AC-19, pp. 549-552, Oct. 1974.

[5] I. D. Landau and H. M. Silveira, "A Stability Theorem with Applications to Adaptive Control," (To appear in the IEEE Transactions on Automatic Control).

[6] R. V. Monopoli, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. Automat. Contr., vol. AC-19, pp. 474-484, Oct. 1974.

[7] T. Ionescu and R. V. Monopoli, "Discrete Model Reference Adaptive Control with an Augmented Error Signal," Automatica, vol. 13, no. 5, pp. 507-518, Sept. 1977.

[8] K. S. Narendra and L. S. Valavani, "Stable Adaptive Controller Design - Direct Control," IEEE Trans. Automat. Contr., vol. AC-23, pp 570-583, Aug. 1978.

[9] K. S. Narendra, "Stable Identification Schemes," System Identification: Advances and Case Studies, New York, Academic Press, 1976.

[10] A. P. Morgan and K. S. Narendra, "On the Stability of Non-Autonomous Differential Equations $\dot{x} = [A + B(t)]x$, with Skew-Symmetric Matrix B(t)," SIAM J. Contr. and Optimization, vol. 15, pp. 163-176, Jan. 1977.

[11] C. P. Neumann, "Frequency Domain Stability Criteria in Nonlinear Automatic Control," Doctoral Dissertation, Harvard University, 1967.

[12] K. S. Narendra and Yuan-Hao Lin, "Stable Adaptive Identification and Control of Discrete Systems," Becton Center, Yale University, Technical Report, 1978.

Acknowledgement

The work reported here was supported by the Office of Naval Research under contract

N00014-~~67~~ 76-C-0017