AD-A063 917 TENNESSEE UNIV KNOXVILLE DEPT OF PSYCHOLOGY F/G 12/1

ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CA--ETC(U)

DEC 78 F SAMEJIMA N00014-77-C-0360

UNCLASSIFIED RR-78-4 NL

1 OF 2

AD A063917

RESEARCH REPORT 78-4

(12)

LEVEL II

ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES V: WEIGHTED SUM PROCEDURE IN THE CONDITIONAL P.D.F. APPROACH

FUMIKO SAMEJIMA

DEPARTMENT OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TENN. 37916

DECEMBER, 1978

Prepared under the contract number N00014-77-C-0360,
NR 150-402 with the
Personnel and Training Research Programs
Psychological Sciences Division
Office of Naval Research

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

14 RR-78-4 moves

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER Research Report 78-4	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER 9 Research rept.	
4. TITLE (and Subtitle) Estimation of the Operating Characteristics of Item Response Categories V. Weighted Sum Procedure in the Conditional P.D.F. Approach	5. TYPE OF REPORT & PERIOD COVERED Technical Report
	6. PERFORMING ORG. REPORT NUMBER 16
7. AUTHOR(s) 10 Dr. Fumiko Samejima	8. CONTRACT OR GRANT NUMBER(s) 17 PE: 61153N; PROJ: RR 042 04 TA: RR 042 04 01 WU: NR 150-402
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Psychology University of Tennessee Knoxville, TN 37916	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 15 N00014-77-C-0360
11. CONTROLLING OFFICE NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research (Code 458) Arlington, VA 22217	12. REPORT DATE 11 10 Dec[redacted]-78
	13. NUMBER OF PAGES 121
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 126p.	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited. Reproduction in whole or in part is permitted for any purpose of the United States government

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Operating Characteristic Estimation
Tailored Testing
Latent Trait Theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(Please see reverse side)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

404 225 Gen

Conditional P.D.F. Approach for estimating the operating characteristics of item response categories has been introduced earlier. In this study, a variation of this approach is introduced, which is called Weighted Sum Procedure, in contrast to the previous one, which is now called Simple Sum Procedure. The new method is applied to the same hypothetical data, i.e., the maximum likelihood estimates of ability of the five hundred hypothetical subjects and their responses to the ten binary items each of which follows the normal ogive model. The criterion item characteristic function for each binary item is obtained and compared with the one obtained in the simple sum procedure to find out if the new procedure has possibilities for a better estimation than the previous one. This is actually done using "pseudo" criterion item characteristic functions which adopt the approximated density functions of the maximum likelihood estimate by polynomials of degree 3, 4 and 5 by means of the method of moments. Also the Pearson System Method and the Two-Parameter Beta Method are used for both Degree 3 and 4 Cases in this variation of the Conditional P.D.F. Approach and the results are compared. The mean square errors are adopted in evaluating the resultant estimated item characteristic functions and probability density functions of ability. The two item parameters in the normal ogive model are also estimated for each item, and compared with the true parameter values.

ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CATEGORIES V: WEIGHTED SUM PROCEDURE IN THE CONDITIONAL P.D.F. APPROACH

ABSTRACT

Conditional P.D.F. Approach for estimating the operating characteristics of item response categories has been introduced earlier. In this study, a variation of this approach is introduced, which is called Weighted Sum Procedure, in contrast to the previous one, which is now called Simple Sum Procedure. The new method is applied to the same hypothetical data, i.e., the maximum likelihood estimates of ability of the five hundred hypothetical subjects and their responses to the ten binary items each of which follows the normal ogive model. The criterion item characteristic function for each binary item is obtained and compared with the one obtained in the simple sum procedure to find out if the new procedure has possibilities for a better estimation than the previous one. This is actually done using "pseudo" criterion item characteristic functions which adopt the approximated density functions of the maximum likelihood estimate by polynomials of degree 3, 4 and 5 by means of the method of moments. Also the Pearson System Method and the Two-Parameter Beta Method are used for both Degree 3 and 4 Cases in this variation of the Conditional P.D.F. Approach and the results are compared. The mean square errors are adopted in evaluating the resultant estimated item characteristic functions and probability density functions of ability. The two item parameters in the normal ogive model are also estimated for each item, and compared with the true parameter values.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked for her as assistants at various times include Robert L. Trestman, Philip S. Livingston and Paul S. Changas.

TABLE OF CONTENTS

I Introduction

Various methods and approaches have been introduced for the purpose of estimating the operating characteristics of item response categories (Samejima, 1977b, 1977d, 1978a, 1978b, 1978c). In the present study, a variation of the Conditional P.D.F. Approach, which has been introduced earlier, is presented and tried on a set of simulated data, and the results are compared with those of the earlier study. The simulated data adopted here are the same ones that were used in the previous studies, and their characteristics are summarized in Appendix I. Again in this study ten binary items, each of which follows the normal ogive model, are used, and the final goal is to estimate their item characteristic functions. In so doing, as usual, no prior mathematical forms are assumed for the item characteristic functions. After the estimation procedure has been completed, however, the two item parameters in the normal ogive model on the dichotomous response level, i.e., the discrimination and difficulty parameters, are estimated by a least square method, and the results are compared with the true parameter values.

In addition to the item parameter estimation, in the present study, the mean square error and its square rooted value of the estimated item characteristic function from the true item characteristic function is computed as an index of accuracy of estimation of each item characteristic function. The same index is used in evaluating the estimated probability density function of the latent trait, which is obtained in the process, against the true density function.

The asymptotic property of the maximum likelihood estimate, i.e., the normality of its conditional distribution, given the latent trait θ , with θ itself as the first parameter and the inverse of the test information function $I(\theta)$ as the second parameter (cf. Samejima, 1975, 1977a, 1977b), has an important role. The probability density function of the maximum likelihood estimate is approximated by a polynomial of degree 3, 4 or 5 by the method of moments (Elderton and Johnson, 1969; Johnson and Kotz, 1970), and these cases are called Degree 3, 4 and 5 Cases, respectively.

II Conditional P.D.F. Approach

The Conditional P.D.F. Approach has been introduced earlier as the Conditional P.D.F. Method, as a variation of the Two-Parameter Beta Method (Samejima, 1978a). Let $\phi(\theta|\hat{\theta})$ be the conditional density function of ability θ, given its maximum likelihood estimate $\hat{\theta}$. We can write

$$(2.1) \qquad \phi(\theta|\hat{\theta}) = \psi(\hat{\theta}|\theta)\ f(\theta)\ [\int_{-\infty}^{\infty}\psi(\hat{\theta}|\theta)\ f(\theta)\ d\theta]^{-1}$$
$$= \psi(\hat{\theta}|\theta)\ f(\theta)\ [g(\hat{\theta})]^{-1}\ ,$$

where $\psi(\hat{\theta}|\theta)$ is the conditional density function of $\hat{\theta}$, given θ, $f(\theta)$ is the probability density function of ability θ, and $g(\hat{\theta})$ is the probability density function of the maximum likelihood estimate $\hat{\theta}$.

By virtue of the asymptotic property of the maximum likelihood estimate and the constancy of the test information function of the Old Test, the conditional density $\psi(\hat{\theta}|\theta)$ is approximated by the normal density $n(\theta,\sigma^2)$, where $\sigma^2 = (21.63)^{-1} \doteq 0.046225$. Because of this fact, although the density function $\phi(\theta|\hat{\theta})$ given by (2.1) is not observable in the empirical situation, it is possible to estimate the conditional moments of ability θ, given the maximum likelihood estimate $\hat{\theta}$, provided that the density function $g(\hat{\theta})$ is estimated. That is to say, we can derive the following equations.

$$(2.2) \qquad E(\theta|\hat{\theta}) \doteq \hat{\theta} + \sigma^2\cdot\frac{d}{d\theta}\log g(\hat{\theta}) = \hat{\theta} + \sigma^2[\frac{d}{d\theta}g(\hat{\theta})][g(\hat{\theta})]^{-1}.$$

$$(2.3) \qquad Var.(\theta|\hat{\theta}) \doteq \sigma^2[1 + \sigma^2\frac{d^2}{d\theta^2}\log g(\hat{\theta})]$$
$$= \sigma^2[1 + \sigma^2\{\frac{d^2}{d\theta^2}g(\hat{\theta})\cdot g(\hat{\theta}) - [\frac{d}{d\theta}g(\hat{\theta})]^2\}\{g(\hat{\theta})\}^{-2}].$$

$$(2.4)\quad E[\{\theta - E(\theta|\hat\theta)\}^3|\hat\theta] \doteq -\sigma^6[\frac{d^3}{d\theta^3}\log g(\hat\theta)]$$

$$= -\sigma^6[\{g(\hat\theta)\}^2\cdot\frac{d^3}{d\theta^3}g(\hat\theta) - 3\,g(\hat\theta)\cdot\frac{d}{d\theta}g(\hat\theta)\cdot\frac{d^2}{d\theta^2}g(\hat\theta) + 2\{\frac{d}{d\theta}g(\hat\theta)\}^3][g(\hat\theta)\}^{-3}]\ .$$

$$(2.5)\quad E[\{\theta - E(\theta|\hat\theta)\}^4|\hat\theta] \doteq \sigma^4[3 + 6\sigma^2\{\frac{d^2}{d\theta^2}\log g(\hat\theta)\} + 3\sigma^4\{\frac{d^2}{d\theta^2}\log g(\hat\theta)\}^2 + \sigma^4\{\frac{d^4}{d\theta^4}\log g(\hat\theta)\}]$$

$$= \sigma^4[3 + 6\sigma^2\{[g(\hat\theta)\cdot\frac{d^2}{d\theta^2}g(\hat\theta) - \{\frac{d}{d\theta}g(\hat\theta)\}^2][g(\hat\theta)]^{-2}\} + 3\sigma^4\,[g(\hat\theta)\cdot\frac{d^2}{d\theta^2}g(\hat\theta) - \{\frac{d}{d\theta}g(\hat\theta)\}^2]^2[g(\hat\theta)]^{-1}$$
$$+ \sigma^4[\{g(\hat\theta)\}^3\cdot\frac{d^4}{d\theta^4}g(\hat\theta) - 4\{g(\hat\theta)\}^2\cdot\frac{d}{d\theta}g(\hat\theta)\cdot\frac{d^3}{d\theta^3}g(\hat\theta) - 3\{g(\hat\theta)\}^2\{\frac{d^2}{d\theta^2}g(\hat\theta)\}^2 + 12\,g(\hat\theta)\{\frac{d}{d\theta}g(\hat\theta)\}^2\frac{d^2}{d\theta^2}g(\hat\theta) - 6\{\frac{d}{d\theta}g(\hat\theta)\}^4][g(\hat\theta)]^{-4}]\ .$$

The density function $g(\hat\theta)$ is approximated by a polynomial of degree 3, 4 or 5, using the method of moments, and its first through fourth derivatives provide us with the estimates of the first through fourth conditional moments of θ, given $\hat\theta$, by means of (2.2) through (2.5). If we choose to use all these four estimated conditional moments, it leads us to the <u>Pearson System Method</u>, and, if we use only the first two estimated conditional moments, we will end up with using either the <u>Normal Approach Method</u> or the <u>Two-Parameter Beta Method</u>. That is to say, in the Pearson System Method, we compute Pearson's criterion κ using all the four estimated conditional moments, and, depending upon the value of κ, one of the Pearson's distributions is selected to approximate the conditional density $\phi(\theta|\hat\theta)$; in the Normal Approach Method, the first two estimated conditional moments are used as the two parameters of a normal distribution, and the resultant normal density function is used as the approximation to the conditional density $\phi(\theta|\hat{\theta})$; and in the Two-Parameter Beta Method the two parameters, which are the end-points of the interval for which the Beta density function assumes positive values, are a priori given, and the other two parameters are estimated from the estimated first two conditional moments, to approximate the conditional density $\phi(\theta|\hat{\theta})$ by a resultant Beta density function.

The estimated item characteristic function $\hat{P}_g(\theta)$ in the conditional P.D.F. Approach has been given by

(2.6) $$\hat{P}_g(\theta) = \sum_{s \in G} \hat{\phi}(\theta|\hat{\theta}_s)[\sum_{s=1}^{N} \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where G is the group of examinees who answered item g correctly, N is the total number of examinees, which is 500 in the present set of data, and $\hat{\phi}(\theta|\hat{\theta}_s)$ is the estimated conditional density of θ , given $\hat{\theta}_s$. If we use the true conditional density $\phi(\theta|\hat{\theta}_s)$ instead of $\hat{\phi}(\theta|\hat{\theta}_s)$ in (2.6), which is observable only in the simulation work, (2.6) will provide us with the criterion item characteristic function, which is the maximal attainable estimate of the item characteristic function we can hope for, using the same approach and set of data.

III Weighted Sum Procedure

We generalize (2.6) in the form

$$(3.1) \qquad \hat{P}_g(\theta) = \sum_{s\epsilon G} w(\hat{\theta}_s)\ \hat{\phi}(\theta|\hat{\theta}_s)\ [\sum_{s=1}^{N} w(\hat{\theta}_s)\ \hat{\phi}(\theta|\hat{\theta}_s)]^{-1} ,$$

where $w(\hat{\theta}_s)$ is a weight assigned to the maximum likelihood estimate of each examinee s. In this way, (2.6) can be regarded as a special case of (3.1) where $w(\hat{\theta}_s) = 1$ for every examinee s. To distinguish the current procedure from the one based on (2.6), however, we shall call the present one the Weighted Sum Procedure, as distinct from the previous one which will be called the Simple Sum Procedure, of the Conditional P.D.F. Approach.

We can define $w(\hat{\theta}_s)$ in any appropriate way in order to make the estimation of the operating characteristics, hopefully, more accurate, or as accurate as the one in the Simple Sum Procedure to provide us with more varieties of techniques. In the present study, the weight is defined and used as follows.

(1) All the 500 maximum likelihood estimates are arranged in the ascending order.

(2) To each maximum likelihood estimate, an interval of $\hat{\theta}$ which starts with the midpoint between the maximum likelihood estimate and its adjacent, lower maximum likelihood estimate, and ends with the midpoint between the maximum likelihood estimate and its adjacent, higher maximum likelihood estimate is assigned. For the two extreme values of maximum likelihood estimate, the missing endpoints are supplemented by extending, in each case, the same distance between the maximum likelihood estimate and the available endpoint to the other direction.

(3) The interval thus defined is multiplied by the estimated density, $\hat{g}(\hat{\theta}_s)$. This estimated density is provided by the polynomial of degree 3, 4 or 5, which has been obtained by the method of moments applied for the set of 500 maximum likelihood estimates.

Just as before, when we replace $\hat{\phi}(\theta|\hat{\theta}_s)$ in (3.1) by the true conditional density $\phi(\theta|\hat{\theta}_s)$, we obtain the criterion item characteristic function. It is also possible that we use the theoretical $g(\hat{\theta}_s)$ in the weight adopted for the criterion item characteristic function, through

$$(3.2) \qquad g(\hat{\theta}_s) = \int_{-\infty}^{\infty} \psi(\hat{\theta}_s|\theta)\ f(\theta)\ d\theta \quad ,$$

where $f(\theta) = 0.2$ in the interval of θ , $(-2.5, 2.5)$, and $f(\theta) = 0.0$ otherwise. Since the purpose of obtaining the criterion item characteristic function is to provide us with the maximal possible result obtainable from the current set of maximum likelihood estimates, however, it can be said that the use of (3.2) in the weight $w(\hat{\theta}_s)$ will contradict our purpose. For this reason, we adopt the same weight used for the estimated item characteristic functions for the criterion item characteristic functions. Thus we have obtained a set of three criterion item characteristic functions for each item, by using the polynomials of degree 3, 4 and 5 for $g(\hat{\theta})$ respectively. To distinguish them from one another, we shall call them <u>Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases</u>, respectively.

The three sets of weights used in Degree 3, 4 and 5 Cases are presented in Appendix II. The polynomials used as $g(\hat{\theta})$ in these three cases are as shown below.

$$
(3.3)\quad \begin{cases} g(\hat{\theta}) = 0.22416 - 0.00351\hat{\theta} - 0.01873\hat{\theta}^2 + 0.00095\hat{\theta}^3 \\ g(\hat{\theta}) = 0.19620 + 0.00238\hat{\theta} + 0.01319\hat{\theta}^2 - 0.00062\hat{\theta}^3 - 0.00427\hat{\theta}^4 \\ g(\hat{\theta}) = 0.19539 - 0.00638\hat{\theta} + 0.01449\hat{\theta}^2 + 0.00405\hat{\theta}^3 - 0.00449\hat{\theta}^4 \\ \qquad\qquad - 0.00048\hat{\theta}^5 \end{cases}
$$

As we have observed before (Samejima, 1977d), the approximated density functions are similar for Degree 4 and 5 Cases, but the one in Degree 3 Case is substantially different from the other two.

It should be noted that the weight adopted in the present study has the meaning of the approximated probability assigned to each value of the maximum likelihood estimate. In other words, instead of taking the area under the polynomial for the subinterval of $\hat{\theta}$, it is approximated by the area of the rectangle which is the product of the width of the subinterval and the value of the polynomial at $\hat{\theta} = \hat{\theta}_s$. For this reason, the denominator on the right hand side of (3.1) can be considered as the approximation to the density function of ability θ , i.e.,

$$
(3.4)\qquad \hat{f}(\theta) = \sum_{s=1}^{N} w(\hat{\theta}_s)\, \hat{\phi}(\theta|\hat{\theta}_s) \quad .
$$

This estimated density function of ability is the sum total of the numerator of (3.1) and $\sum_{s\epsilon\bar{G}} w(\hat{\theta}_s)\, \hat{\phi}(\theta|\hat{\theta}_s)$, where $\bar{G}$ indicates the group of examinees who did not answer item g correctly. Hereafter, we shall call them the <u>estimated shared density of</u> θ <u>by the success group</u>, and that <u>by the failure group</u>, respectively.

IV Two Methods Used for Approximating the Conditional Density $\phi(\theta|\hat{\theta}_s)$: Pearson System Method and Two-Parameter Beta Method

In the previous research, when we adopted the Simple Sum Procedure of the Conditional P.D.F. Approach, we used three different methods, i.e., Pearson System Method, Normal Approach Method, and Two-Parameter Beta Method, and in each method we distinguished two cases from each other, i.e., Degree 3 and 4 Cases (cf. Samejima, 1978a, 1978b, 1978c). Since the same approximated density functions can be used in the Weighted Sum Procedure, it will be meaningful to review several problems which we encountered in the previous research. All the 500 hypothetical subjects are numbered 1 through 500, and in the following descriptions these numbers are used as the subjects' identifications.

(1) The estimated density $\hat{g}(\hat{\theta})$ turned out to be negative:

Degree 3 Case: None.
Degree 4 Case: #2.

(2) The estimated conditional variance $\mathrm{Var.}(\theta|\hat{\theta}_s)$ turned out to be negative:

Degree 3 Case: None.
Degree 4 Case: #99, 101, 201, 296, 299, 300.

(3) The estimated fourth conditional moments $E[\{\theta - E(\theta|\hat{\theta}_s)\}^4|\hat{\theta}_s]$ turned out to be negative:

Degree 3 Case: #2.
Degree 4 Case: (Note all the examinees in (2) are also here.) #1, 3, 4, 98, 99, 101, 199, 201, 296, 299, 300, 401, 500.

For these reasons, in the Two-Parameter Beta Method, seven examinees mentioned in (1) and (2) are excluded in Degree 4 Case, whereas in Degree 3 Case all the five hundred examinees are used; in the Pearson

System Method, since we need up to the fourth conditional moments to compute Pearson's criterion κ (Samejima, 1977d), all the fourteen examinees mentioned in (1), (2) and (3) are excluded in Degree 4 Case, whereas only one examinee is excluded in Degree 3 Case; and, in the Normal Approach Method, although we need only up to the second conditional moments and, therefore, the exclusion can be only for the examinees mentioned in (1) and (2), the fourteen examinees mentioned in (1), (2) and (3) are excluded in Degree 4 Case, and the one examinee is excluded in Degree 3 Case, in order to make the results comparable with those of the Pearson System Method.

It happened later that, in the Pearson System Method, for three examinees, #100, 104 and 198, to which Beta density functions were assigned because of the negative values of the criterion κ, the two estimated parameters, $p_{\hat{\theta}}$ and $q_{\hat{\theta}}$, such that

$$p_{\hat{\theta}},\ q_{\hat{\theta}} = (r/2)[1 \pm (r+2)\{\beta_1[\beta_1(r+2)^2 + 16(r+1)]^{-1}\}^{1/2}] , \tag{4.1}$$

where

$$r = 6(\beta_2 - \beta_1 - 1)(6 + 3\beta_1 - 2\beta_2)^{-1} \tag{4.2}$$

and

$$\beta_1 = \{E([\theta - E(\theta|\hat{\theta})]^3|\hat{\theta})\}^2\{E([\theta - E(\theta|\hat{\theta})]^2|\hat{\theta})\}^{-3} , \tag{4.3}$$

$$\beta_2 = E([\theta - E(\theta|\hat{\theta})]^4|\hat{\theta})\{E[\theta - E(\theta|\hat{\theta})]^2|\hat{\theta})\}^{-2} , \tag{4.4}$$

turned out to be negative values, which are very close to zero. For this reason, these three examinees are excluded in the Pearson System Method, in addition to the fourteen subjects who were already excluded. As the result, the number of examinees used in each case of each method is as presented in Table 4-1.

In the Pearson System Method, it turned out that only three types of Pearson's distributions, i.e., the normal distribution, Pearson's Type I distribution, and Pearson's Type II distribution, are assigned to these examinees, as the result of the values of Pearson's criterion κ . The frequencies for these three categories are shown as Table 4-2.

As we can see in Table 4-2, in both Degree 3 and 4 Cases, the majority of the examinees belong to the category of the normal distribution. For this reason, it will be meaningless to use both Pearson System Method and Normal Approach Method in the present study, since we should expect that the results are practically the same. We shall adopt, therefore, only two methods for approximating the conditional density $\phi(\theta|\hat{\theta}_s)$, i.e., Pearson System Method and Two-Parameter Beta Method. (For the detail of these methods, see Samejima, 1978a, 1978b.)

TABLE 4-1

Number of Examinees Used in Each Case of Each Method

Method \ DGR.	3 Case	4 Case
Two-Parameter Beta	500	493
Normal Approach	499	486
Pearson System	499	483

TABLE 4-2

Number of Examinees Assigned to Each of the Three Types of Pearson's Distributions in Pearson System Method

Distribution \ DGR.	DGR. 3 Case	DGR. 4 Case
Normal	483	404
Type I	6	38 (41)
Type II	10	41
Total	499	483 (486)

V Results I: Estimated Density Functions of Ability

Figure 5-1 presents the three estimated density functions of ability θ obtained by the Weighted Sum Procedure, based on the true conditional density function of ability θ, given its maximum likelihood estimate $\hat{\theta}_s$, which also are the denominators of (3.1) for the Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, respectively. In the same figure, the corresponding estimated density function of ability θ obtained by the Simple Sum Procedure is drawn for comparison, together with the theoretical density function of θ. We can see that the estimated density functions of θ in Degree 4 and 5 Cases are almost identical, and the closest to the theoretical density, $f(\theta)$, whereas the one in Degree 3 Case is substantially different from these two and from the theoretical density function. The mean square error, which is given by

$$(5.1) \qquad \frac{1}{m} \sum_{j=1}^{m} [\hat{f}(\theta_j) - f(\theta_j)]^2 ,$$

where $m = 25$, and θ_j's are -2.4 through 2.4 with the step of 0.2, was calculated for each estimated density function, $\hat{f}(\theta)$. It turned out to be 0.00007 (0.00863) for Degree 5 Case, 0.00009 (0.00972) for Degree 4 Case, 0.00060 (0.02450) for Degree 3 Case, and 0.00017 (0.01316) for the Simple Sum Procedure. (The numbers in the parentheses are the square roots of the mean square errors.) It can be concluded, therefore, that there is some improvement in the accuracy of estimation of the density of ability θ in Degree 4 and 5 Cases of the Weighted Sum Procedure, in comparison with the estimation by the Simple Sum Procedure, although the same is not true in Degree 3 Case of the Weighted Sum Procedure.

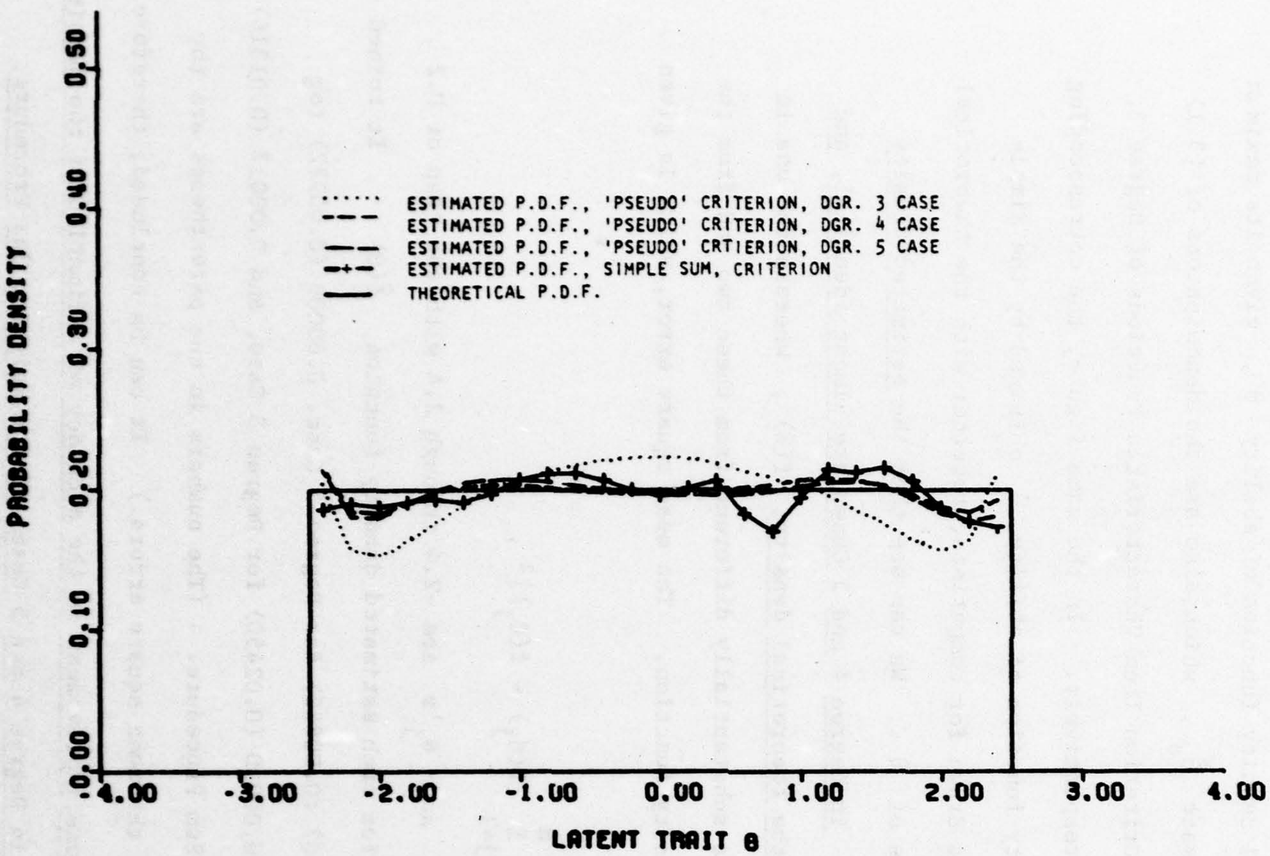

FIGURE 5-1

Three Estimated Density Functions of Ability θ Obtained by the Weighted Sum Procedure, Which Are Based on the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$. These Functions Are Also Used as the Denominators of the Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, Respectively. Also Drawn Are the Corresponding Function by the Simple Sum Procedure, Together with the Theoretical Density Function $f(\theta)$.

The above are the results when we use the theoretical conditional density $\phi(\theta|\hat{\theta}_s)$ in (3.4) in place of the estimated conditional density $\hat{\phi}(\theta|\hat{\theta}_s)$, and are observable only if we use simulated data. Since they indicate the maximal possible attainment that we can expect when we apply these procedures for the given data, however, we can say that these results suggest a positive prospect of the current method.

Figure 5-2 presents the estimated shared densities of θ by the failure and success groups, which were described in Section 3, for each of the ten binary items, using the theoretical conditional density $\phi(\theta|\hat{\theta}_s)$ in the process. In other words, for each item, the sum of these estimated shared densities by the failure and success groups makes the estimated density $\hat{f}(\theta)$ in each case, which is presented in Figure 5-1. In these graphs, the actual frequency ratios of θ for the separate item score groups are also shown as histograms, using 0.25 as the width of subintervals. The theoretical shared densities are given by

(5.2) $$f(\theta)\,P_g(\theta) \begin{cases} = 0.2\,P_g(\theta) & \text{for } -2.5 < \theta < 2.5 \\ = 0.0 & \text{otherwise} \end{cases}$$

for the success group, and

(5.3) $$f(\theta)[1 - P_g(\theta)] \begin{cases} = 0.2[1 - P_g(\theta)] & \text{for } -2.5 < \theta < 2.5 \\ = 0.0 & \text{otherwise} \end{cases}$$

for the failure group, where $P_g(\theta)$ is the item characteristic function in the normal ogive model which is defined by

(5.4) $$P_g(\theta) = [2\pi]^{-1/2} \int_{-\infty}^{a_g(\theta-b_g)} \exp[-t^2/2]\, dt \ .$$

The two parameters, a_g and b_g , for each item are shown in Tables 6-3, 6-5 and 6-7, and Tables 6-4, 6-6 and 6-8, respectively.

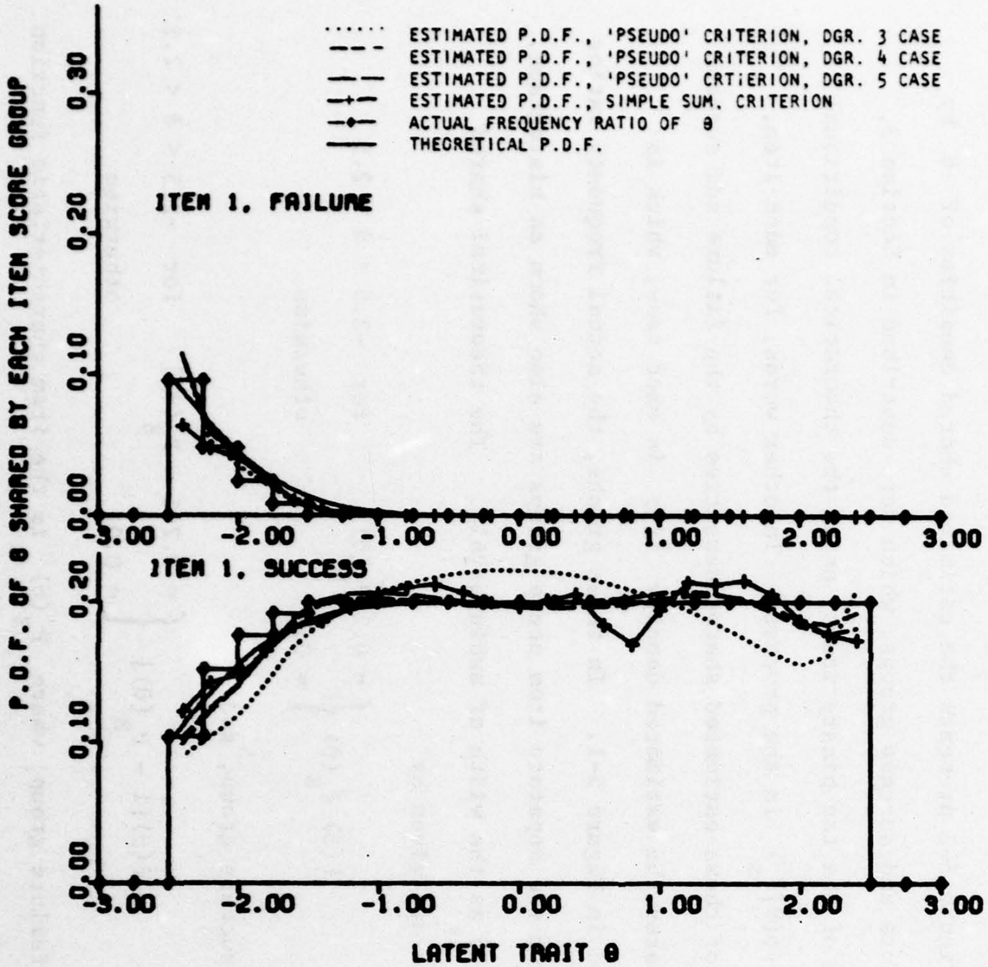

FIGURE 5-2

Comparison of the Three Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which Are Obtained by the Weighted Sum Procedure, Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, with the One Obtained by the Simple Sum Procedure. Theoretical Shared Density Function and Actual Frequency Ratios Are Also Presented.

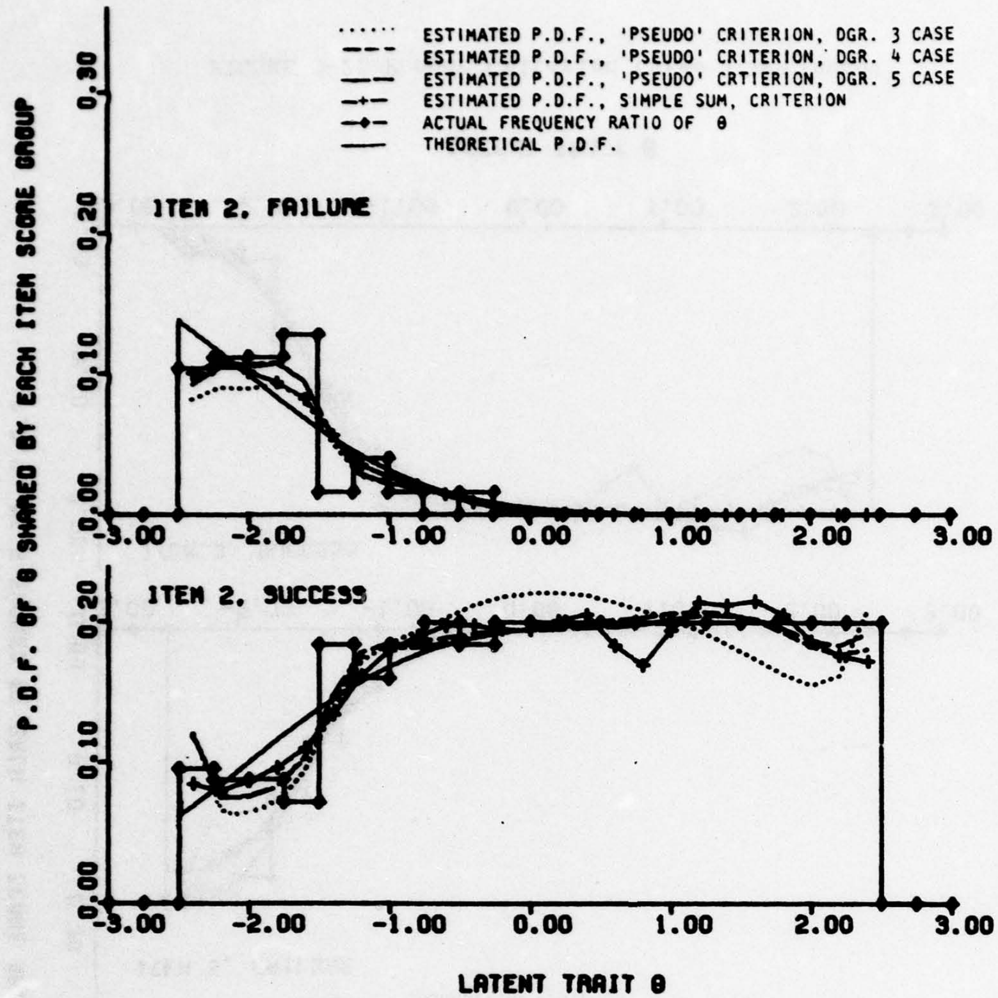

FIGURE 5-2: Pseudo Criterion Cases (Continued)

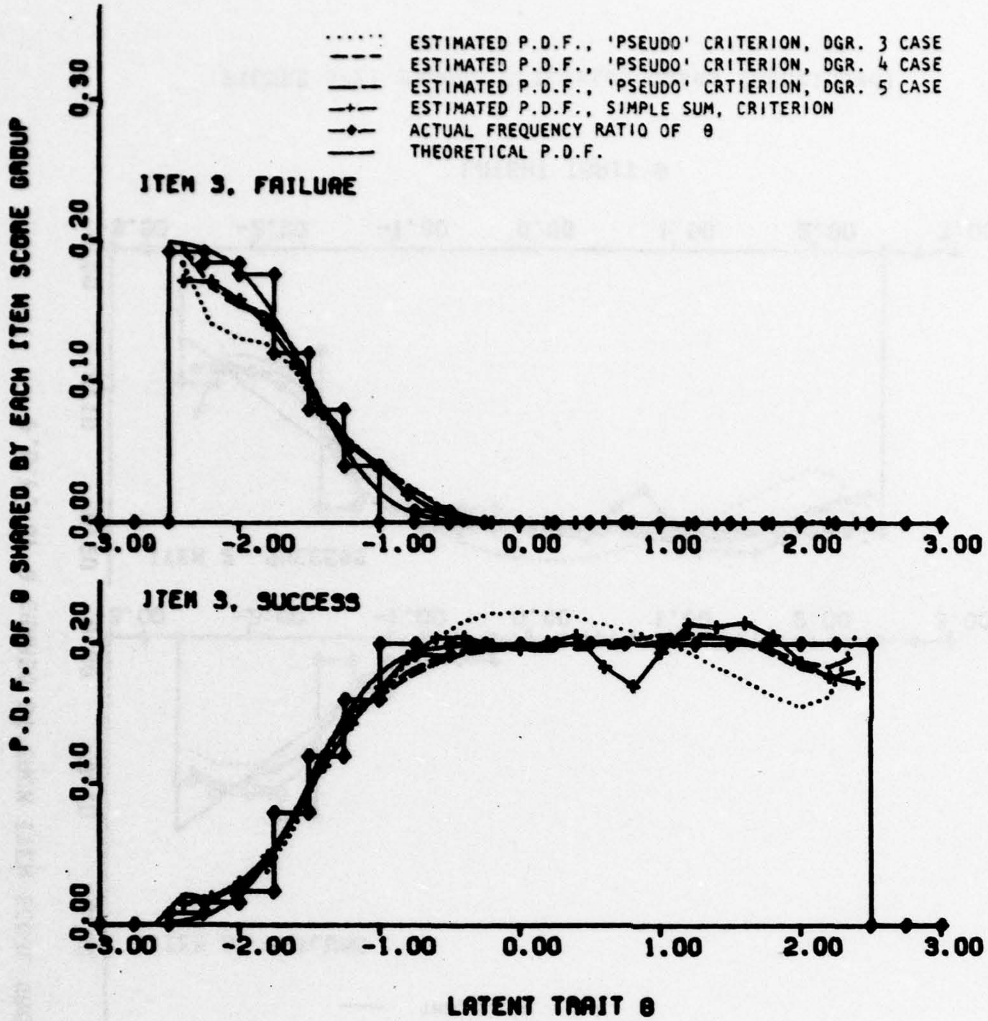

FIGURE 5-2: Pseudo Criterion Cases (Continued)

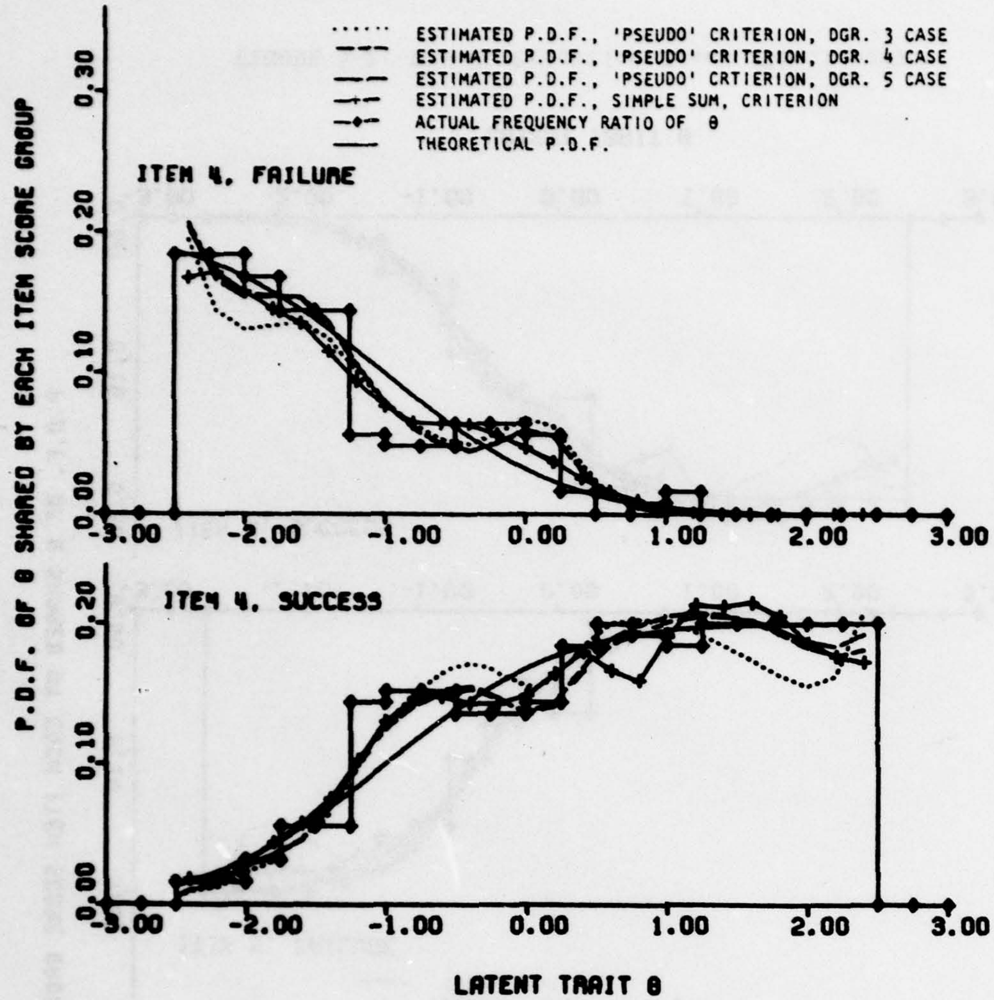

FIGURE 5-2: Pseudo Criterion Cases (Continued)

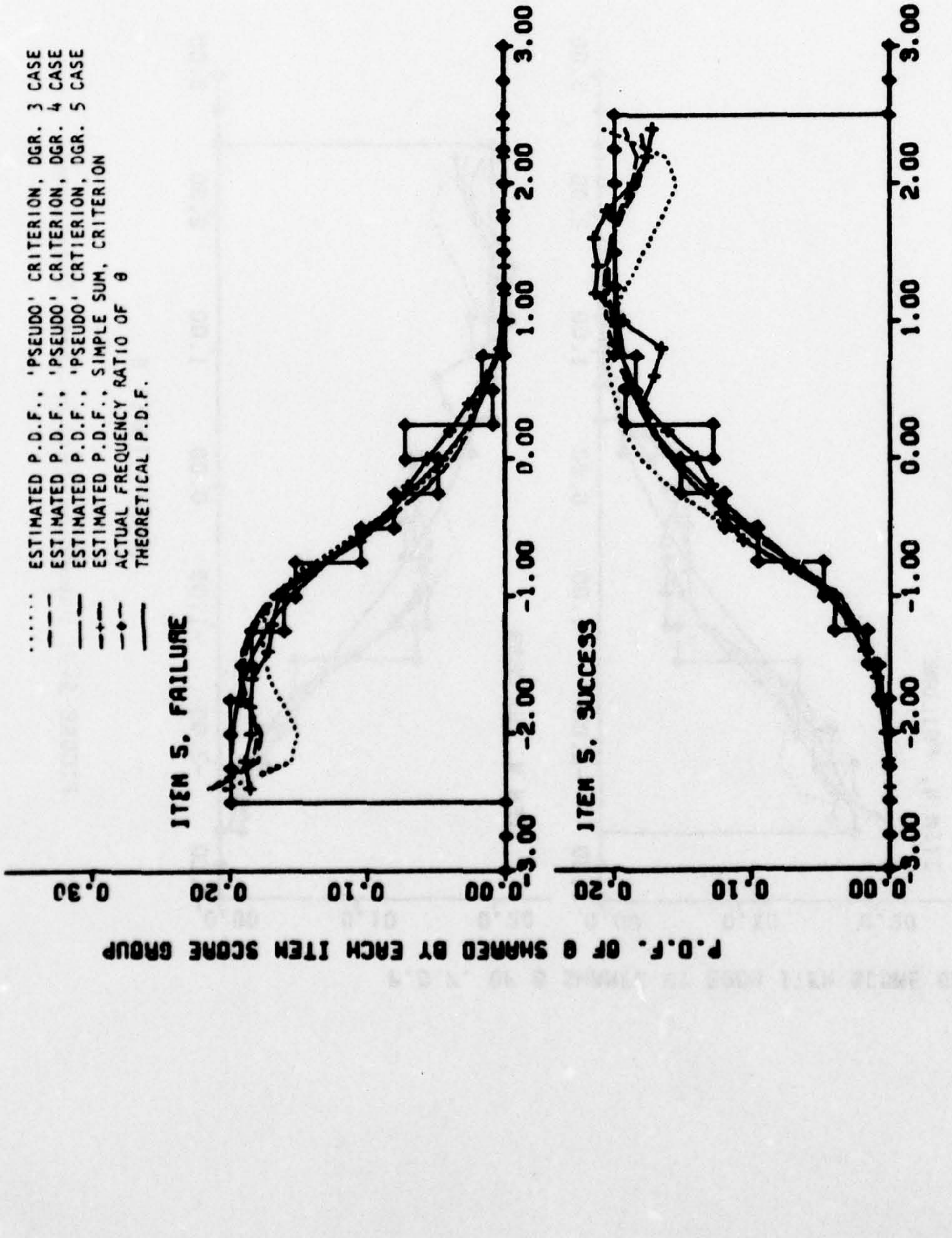

FIGURE 5-2: Pseudo Criterion Cases (Continued)

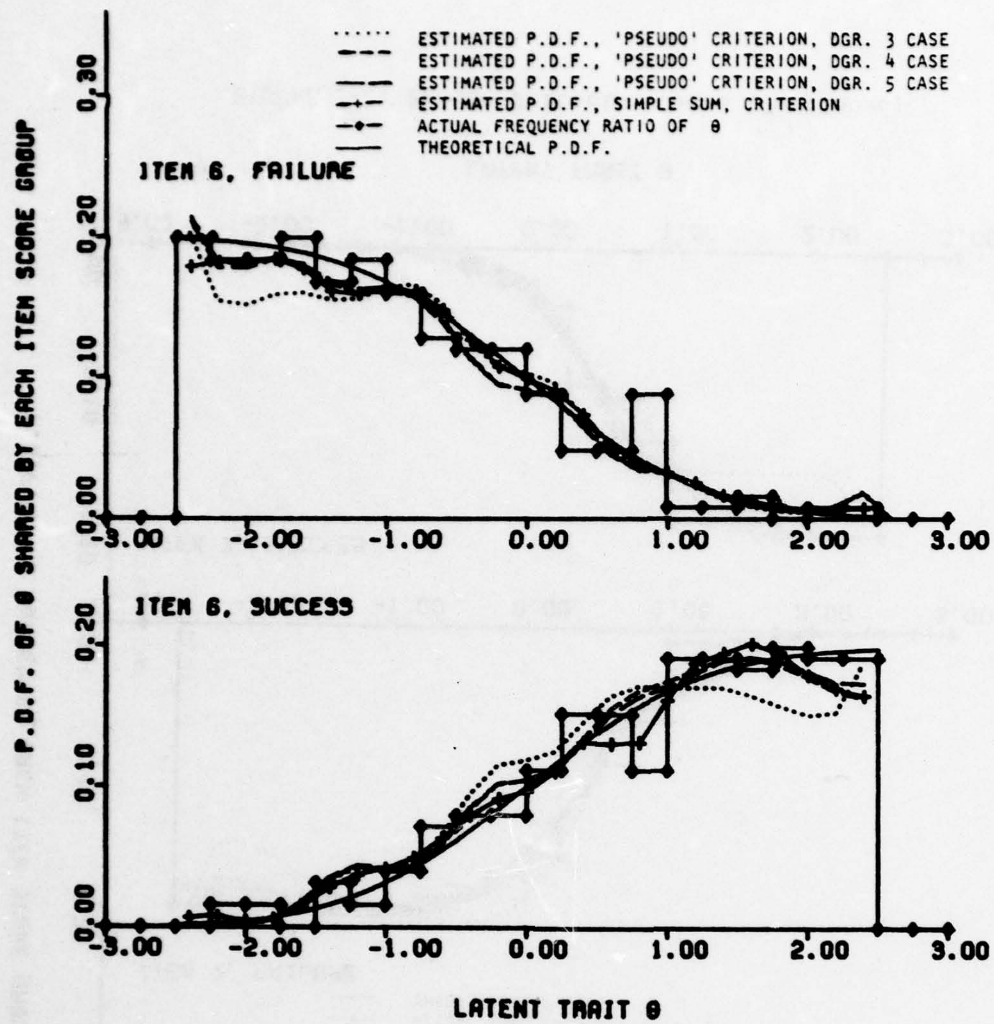

FIGURE 5-2: Pseudo Criterion Cases (Continued)

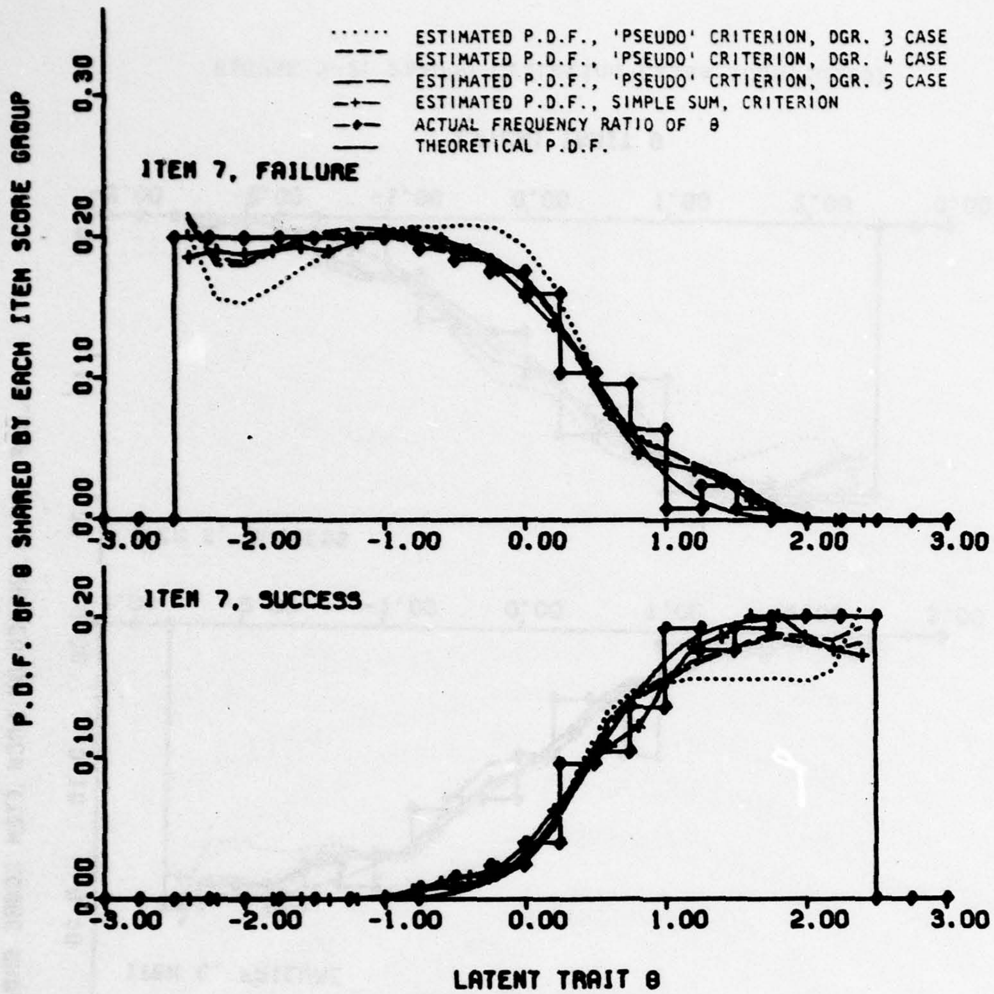

FIGURE 5-2: Pseudo Criterion Cases (Continued)

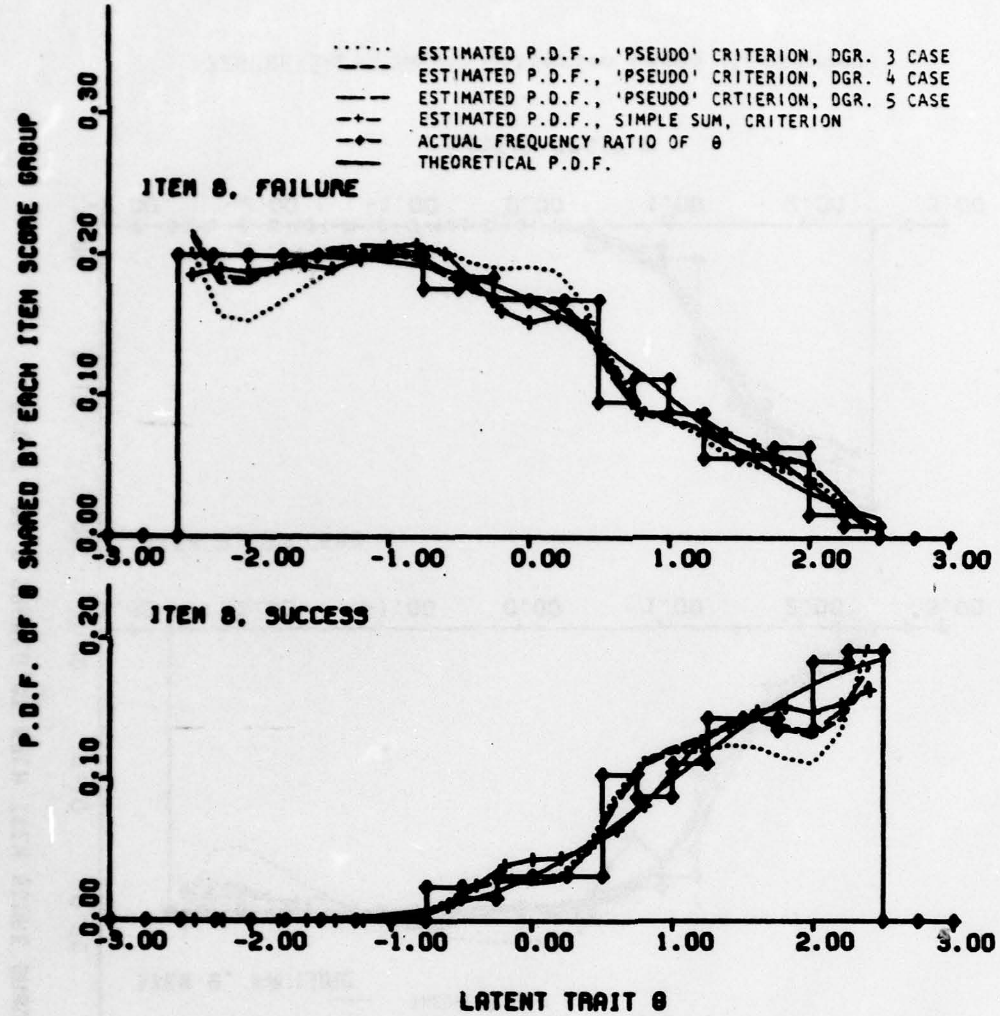

FIGURE 5-2: Pseudo Criterion Cases (Continued)

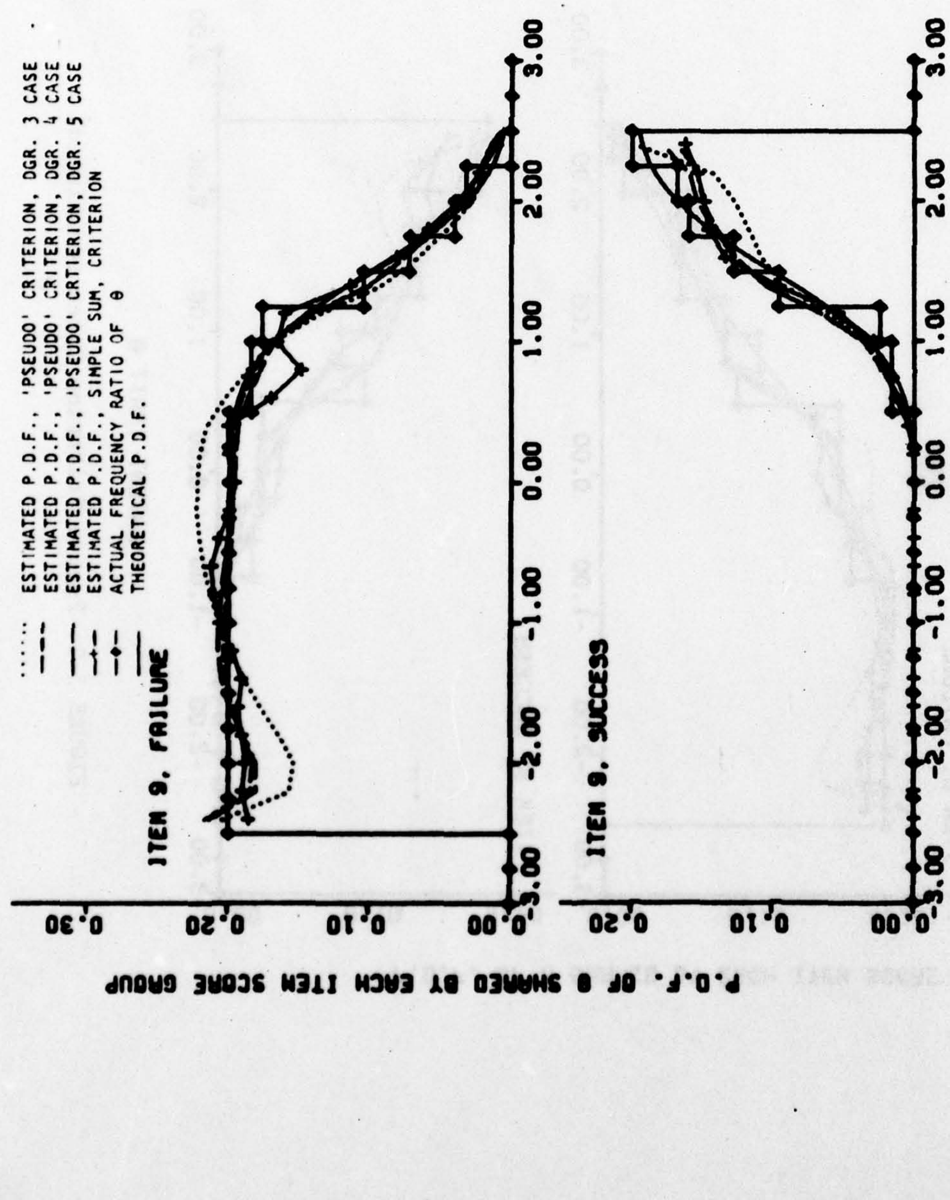

FIGURE 5-2: Pseudo Criterion Cases (Continued)

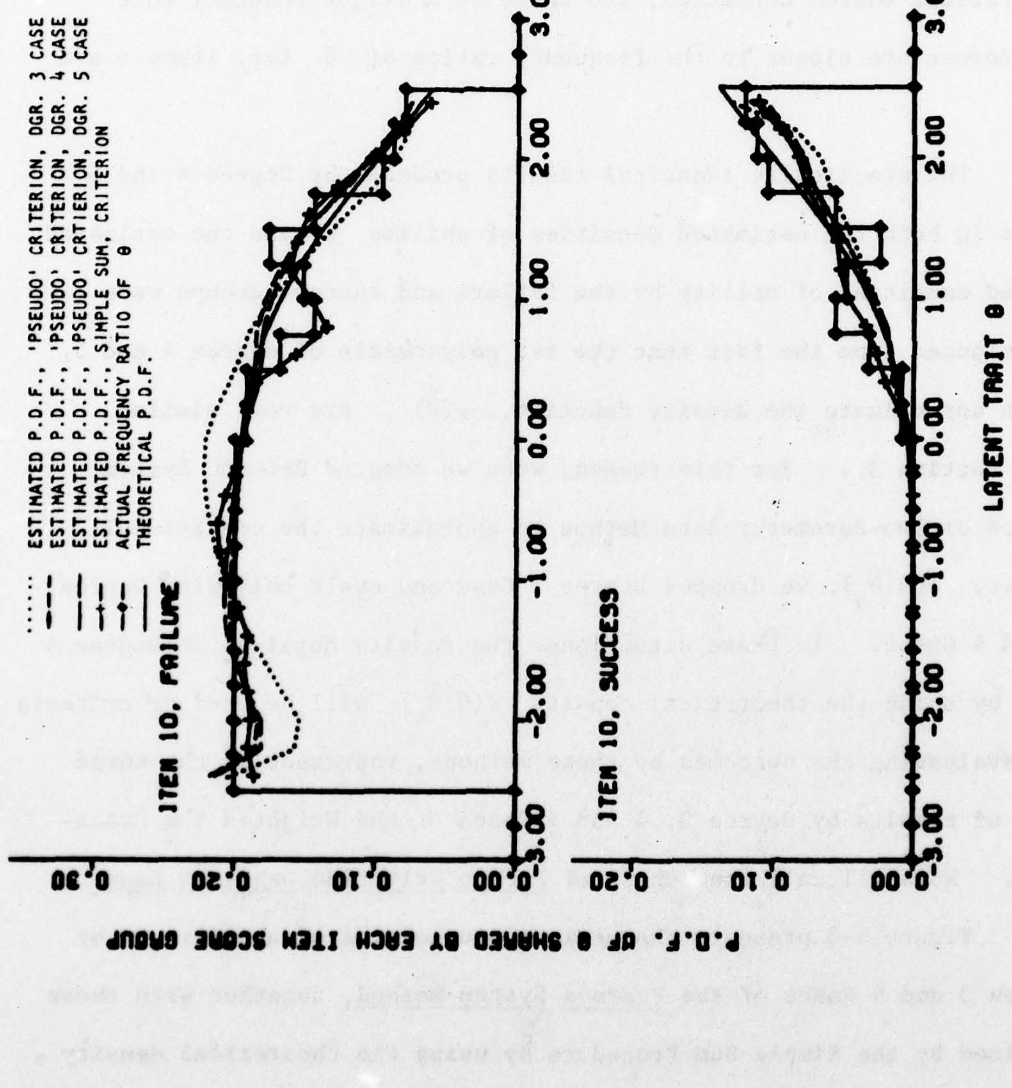

FIGURE 5-2 : Pseudo Criterion Cases (Continued)

These graphs show, as was expected, that the estimated shared densities are practically the same for Degree 4 and 5 Cases of the Weighted Sum Procedure, and they are close to the theoretical shared densities, in contrast with those in Degree 3 Case. Those obtained by the Simple Sum Procedure are, generally speaking, not as close as those in Degree 4 and 5 Cases of the Weighted Sum Procedure to the theoretical shared densities, and there is a slight tendency that the former are closer to the frequency ratios of θ (cf. items 6 and 10).

The practically identical results produced by Degree 4 and 5 Cases in both the estimated densities of ability θ and the estimated shared densities of ability by the failure and success groups were anticipated from the fact that the two polynomials of degree 4 and 5, which approximate the density function, $g(\hat{\theta})$, are very similar (cf. Section 3). For this reason, when we adopted Pearson System Method or Two-Parameter Beta Method to approximate the conditional density $\phi(\theta|\hat{\theta}_s)$, we dropped Degree 5 Case and dealt only with Degree 3 and 4 Cases. In these situations, the results obtained in Degree 4 Case by using the theoretical density $\phi(\theta|\hat{\theta}_s)$ will be used as criteria for evaluating the outcomes by these methods, representing the three sets of results by Degree 3, 4 and 5 Cases of the Weighted Sum Procedure. We shall call them those of Pseudo Criterion Degree 4 Case.

Figure 5-3 presents the estimated densities of ability θ by Degree 3 and 4 Cases of the Pearson System Method, together with those obtained by the Simple Sum Procedure by using the theoretical density $\phi(\theta|\hat{\theta}_s)$ and by the Pseudo Criterion Degree 4 Case, and the theoretical density $f(\theta)$.

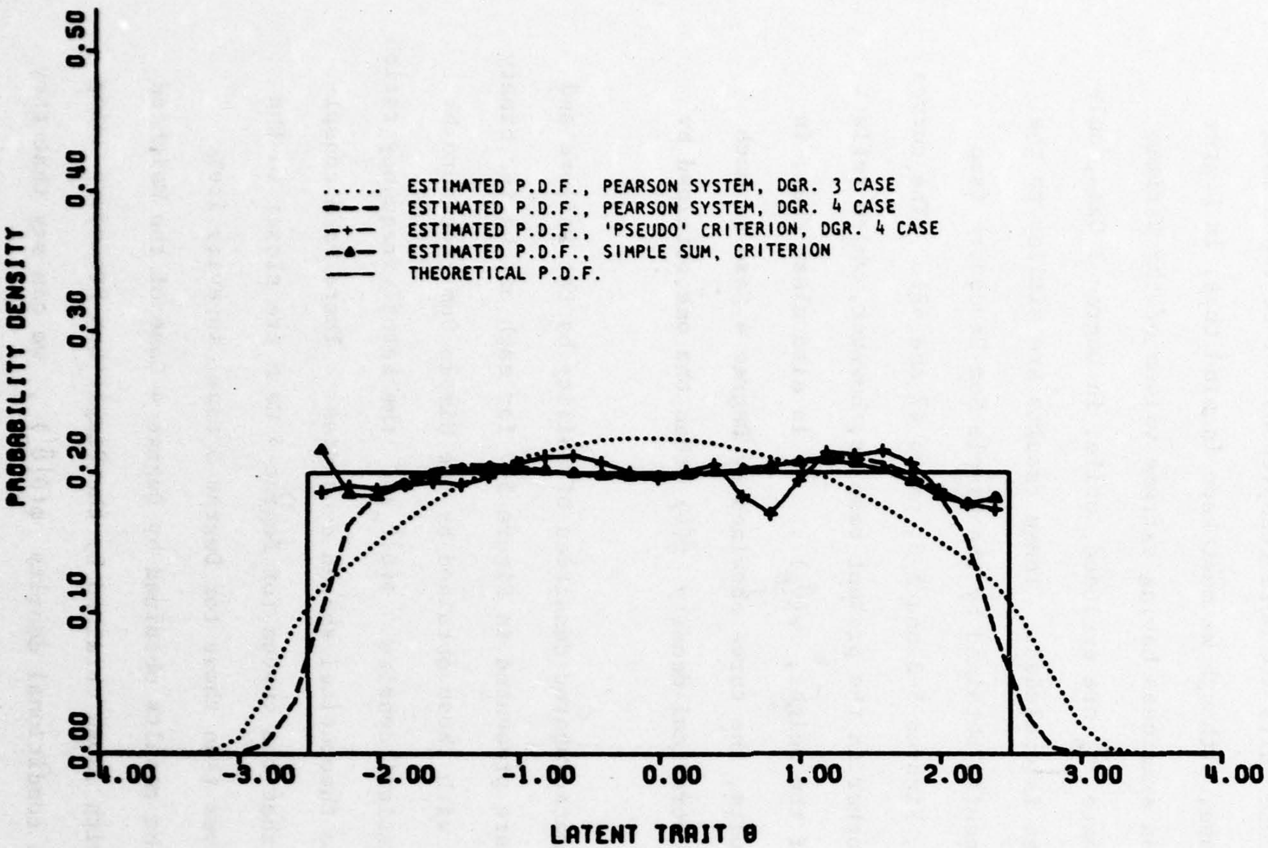

FIGURE 5-3

Estimated Density Functions of Ability θ Obtained by the Weighted Sum Procedure, Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Pearson System Method. Also Drawn Are Those Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by Degree 4 Case of the Weighted Sum Procedure, and by the Simple Sum Procedure, Respectively, Together with the Theoretical Density, $f(\theta)$.

We can see in this figure that, unlike those other curves which are obtainable only when we use simulated data, both results show "tails" outside the interval of θ , (-2.5, 2.5). We can also see that these tails are more conspicuous for Degree 3 Case than Degree 4 Case, although we must keep in mind that, in Degree 4 Case, seventeen examinees having extreme values of the maximum likelihood estimate $\hat{\theta}_s$ are excluded, while, in Degree 3 Case, only one such examinee is excluded. These results are similar to the corresponding results obtained by the Simple Sum Procedure (see Samejima, 1978b, Figures 5-2 and 5-3, pages 47 and 48). The curves are visibly smoother in the present results, however, which reflect the influence of the weight, $w(\hat{\theta}_s)$. It is also clear that, in the present results, the curve obtained by Degree 4 Case is much closer to the theoretical density $f(\theta)$ than the one obtained by Degree 3 Case.

The estimated shared densities of ability by the failure and success groups are presented in Figure 5-4 for each of the ten binary items, together with those obtained by the Simple Sum Procedure by using the theoretical density $\phi(\theta|\hat{\theta}_s)$, the actual frequency ratios of θ , and the theoretical shared densities. There is a conspicuous tendency that the curves for Degree 4 Case are closer to the theoretical curves than those for Degree 3 Case, in every item. If we compare the results obtained by Degree 4 Case of the Weighted Sum Procedure with those obtained by the Simple Sum Procedure using the theoretical conditional density $\phi(\theta|\hat{\theta}_s)$, we can say that they are competitive, and around the interval of θ , (0.5, 1.5), the former are even visibly closer to the theoretical curves than the latter in most cases.

The "tails" outside of the interval of θ , (-2.5, 2.5), are conspicuous especially in Degree 3 Case. In relation with this, the decline of the curve within the interval of θ , (-2.5, 2.5), is substantial around the endpoints of the interval, and this is more so for Degree 3 Case than Degree 4 Case.

The mean square errors and their square roots of the estimated density functions, $\hat{f}(\theta)$, which were obtained in the same manner as described earlier in this section, turned out to be 0.00136 (0.03688) in Degree 3 Case, and 0.00112 (0.03347) in Degree 4 Case, respectively. Although these values are substantially greater than the corresponding values for the Pseudo Criterion Degree 3 and 4 Cases, i.e., 0.00060 (0.02450) and 0.00009 (0.00972), still we can say that they are reasonably small, considering that the theoretical density is 0.2 for this interval of θ .

The results corresponding to Figures 5-3 and 5-4, which were obtained by using the Two-Parameter Beta Method for approximating the conditional density $\phi(\theta|\hat{\theta}_s)$, are presented as Figures 5-5 and 5-6 respectively. We can see in Figure 5-5 that the estimated density functions of ability, $\hat{f}(\theta)$, in Degree 3 and 4 Cases are very similar to those obtained by the Pearson System Method, as was the case with the Simple Sum Procedure of the Conditional P.D.F. Approach (cf. Samejima, 1978b, Figures 5-2 and 5-3, pages 47 and 48). Again the result obtained by Degree 4 Case is much closer to the theoretical density $f(\theta)$, compared with the one obtained by Degree

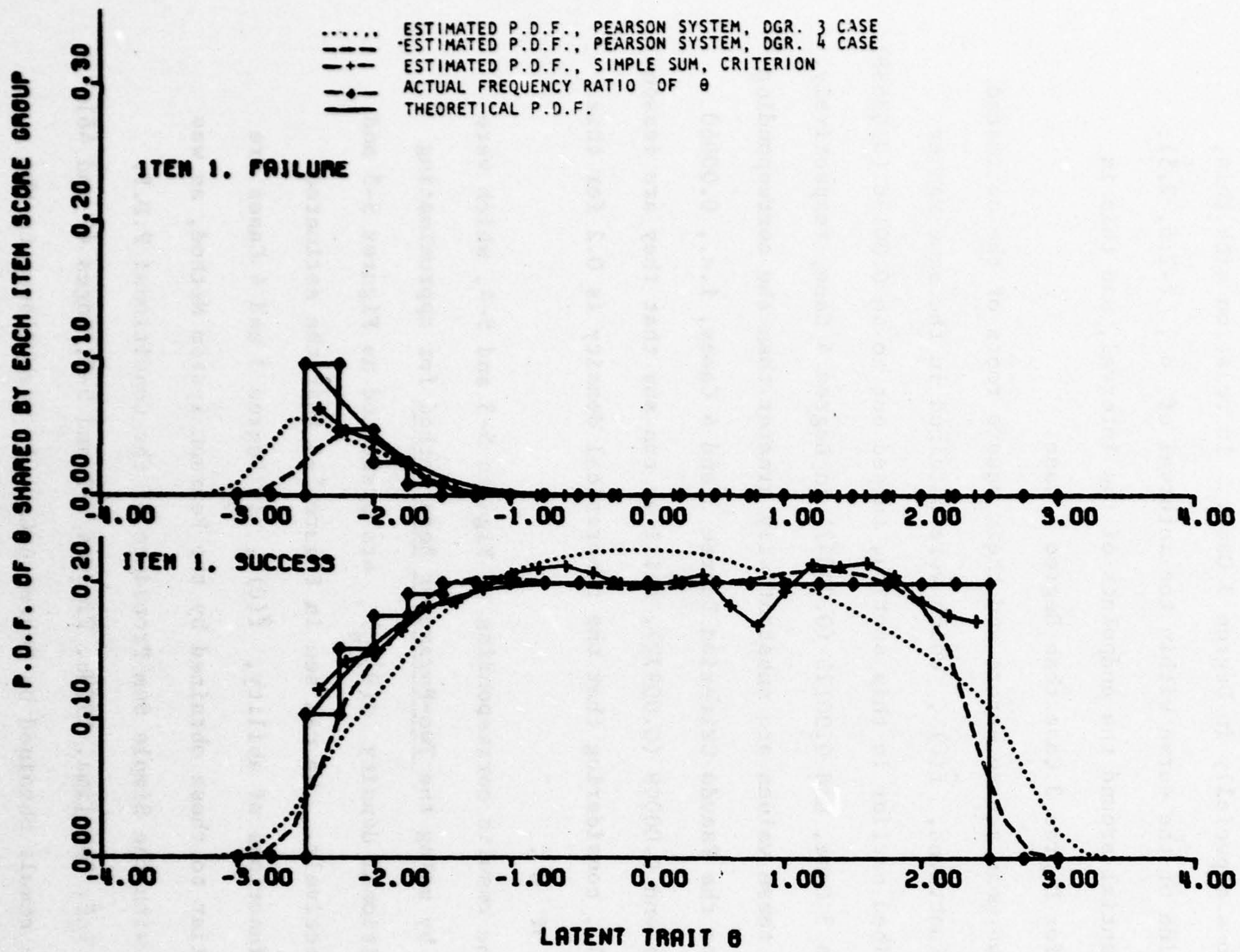

FIGURE 5-4

Two Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which Are Obtained by the Weighted Sum Procedure Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Pearson System Method. Also Presented Are the One Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by the Simple Sum Procedure, Actual Frequency Ratios of Ability θ_s, and Theoretical Shared Density Function.

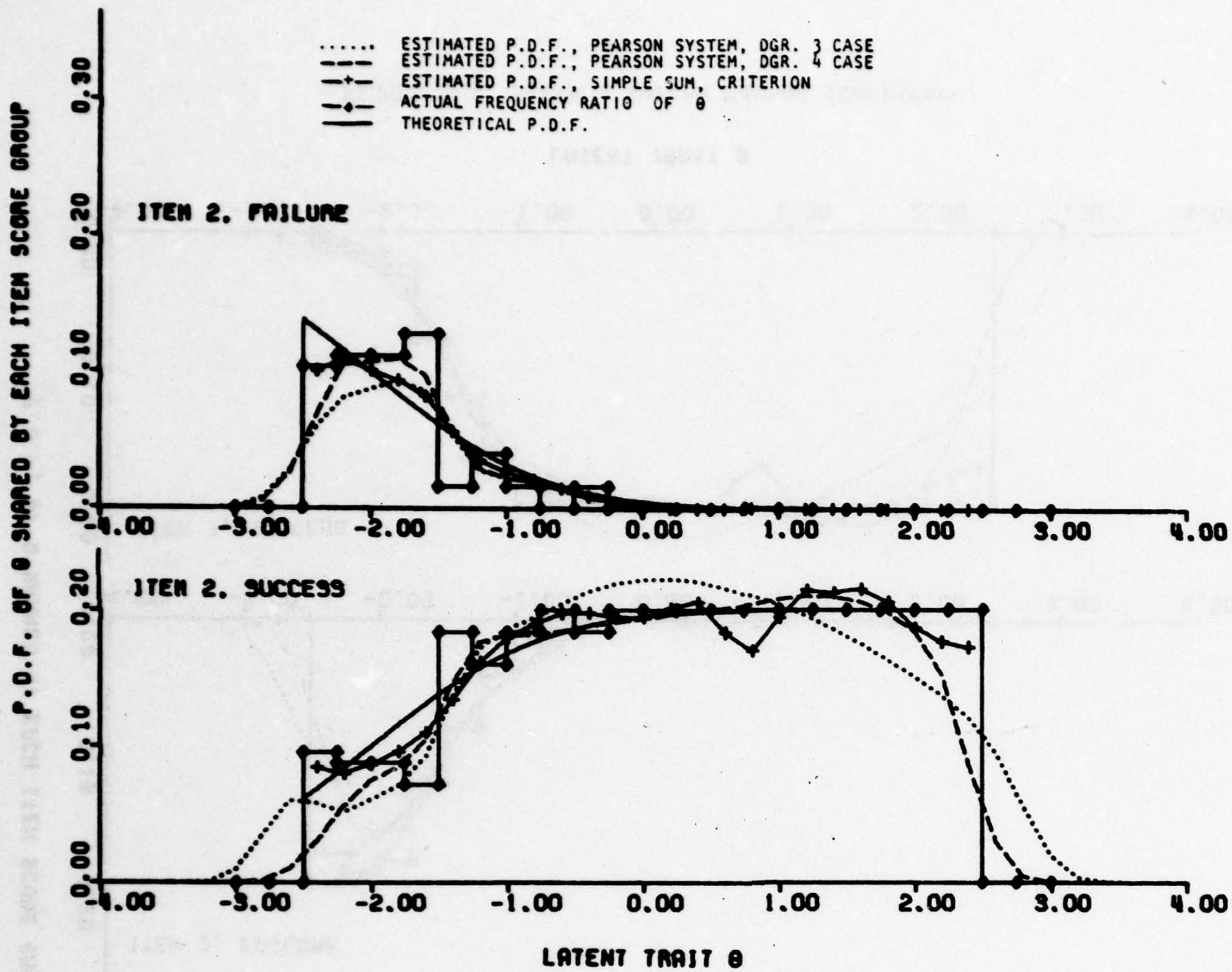

FIGURE 5-4: Pearson System Method (Continued)

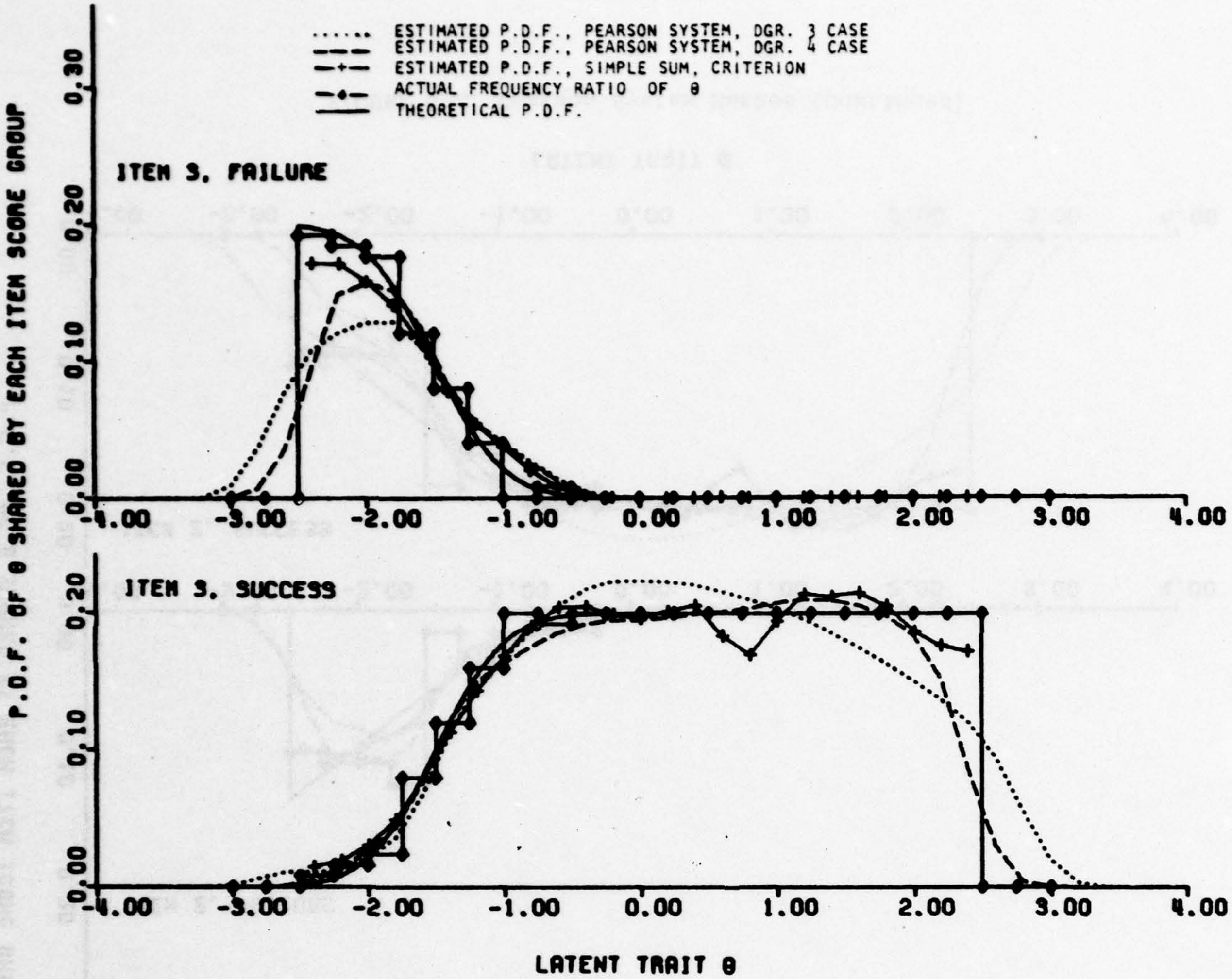

FIGURE 5-4: Pearson System Method (Continued)

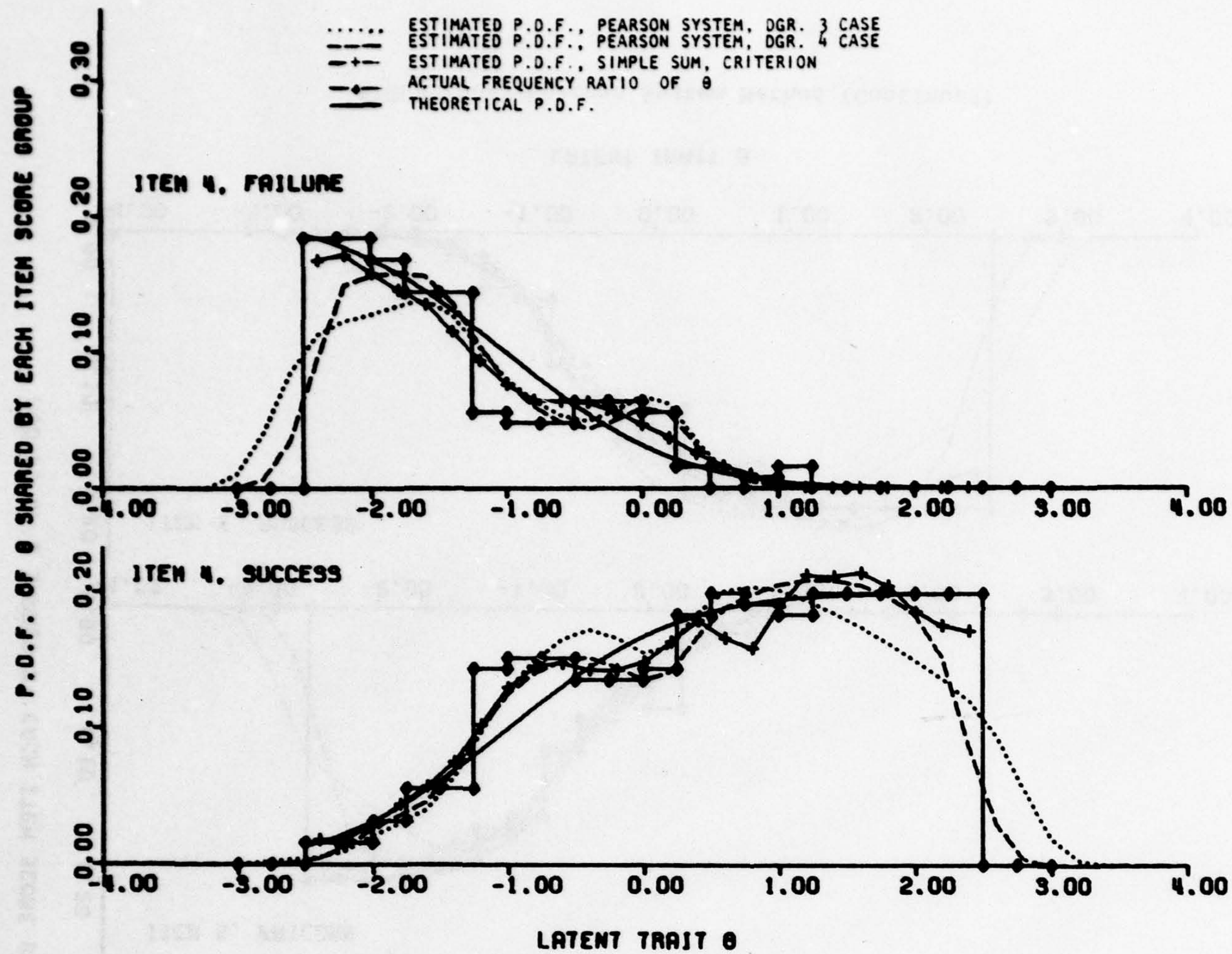

FIGURE 5-4: Pearson System Method (Continued)

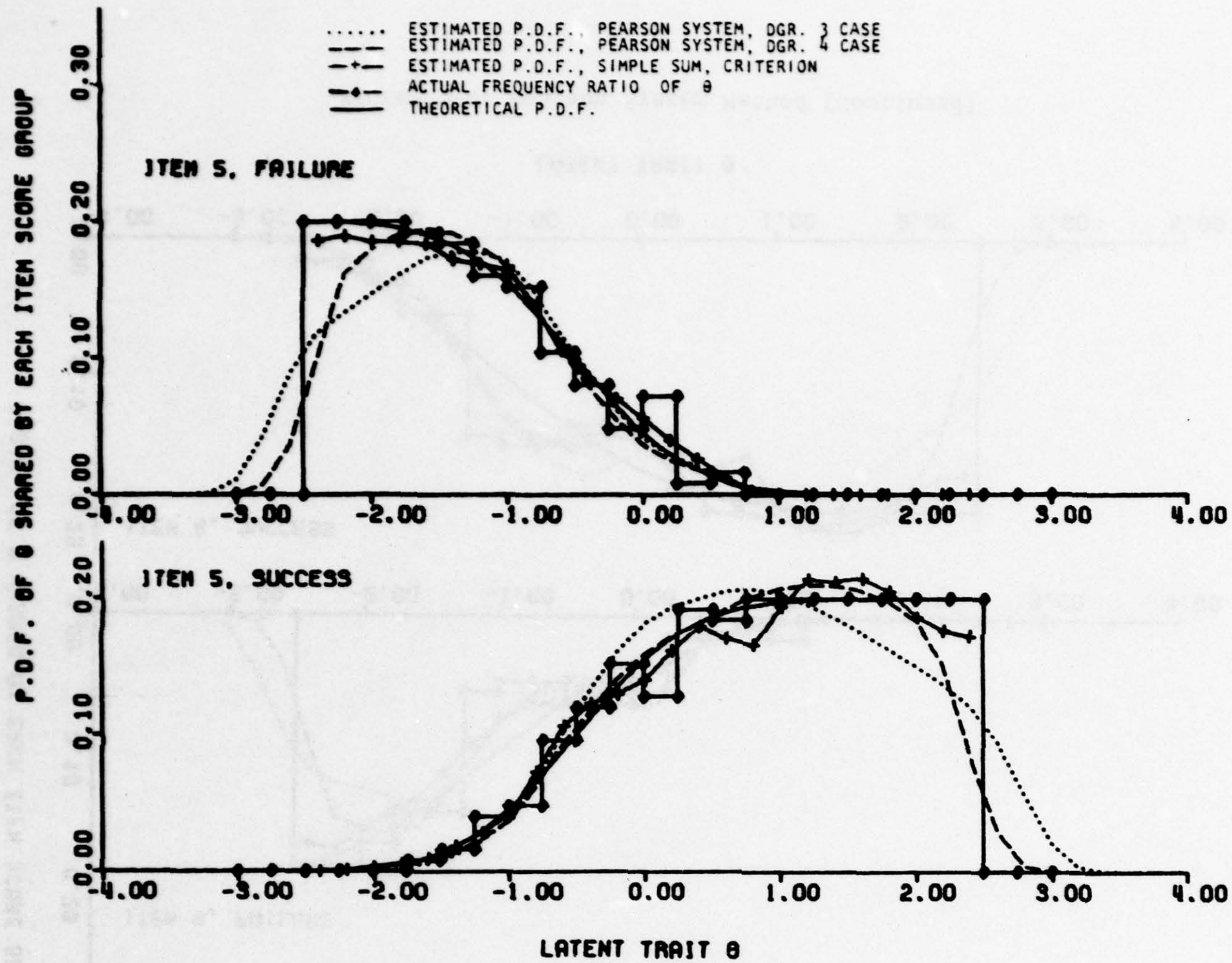

FIGURE 5-4: Pearson System Method (Continued)

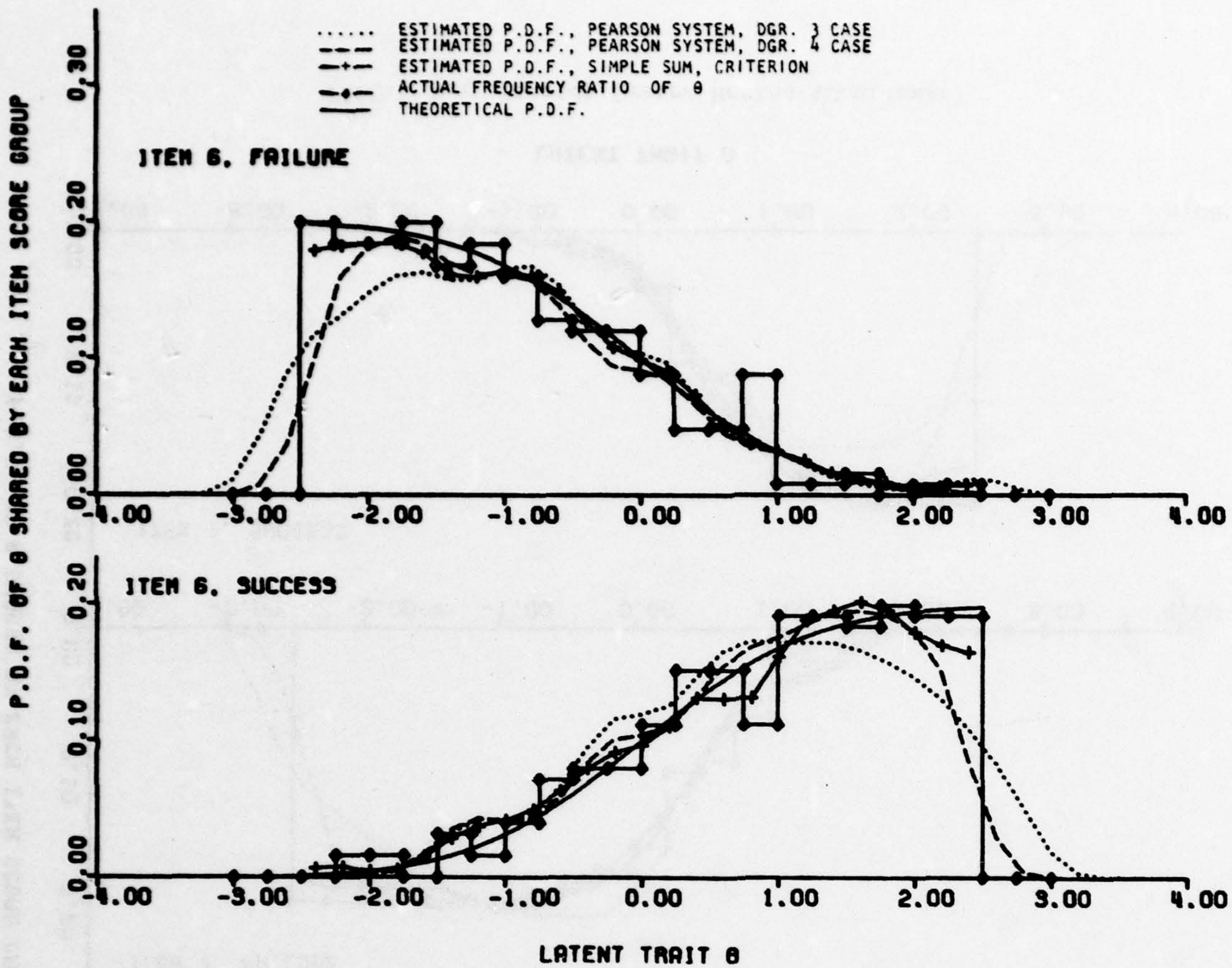

FIGURE 5-4: Pearson System Method (Continued)

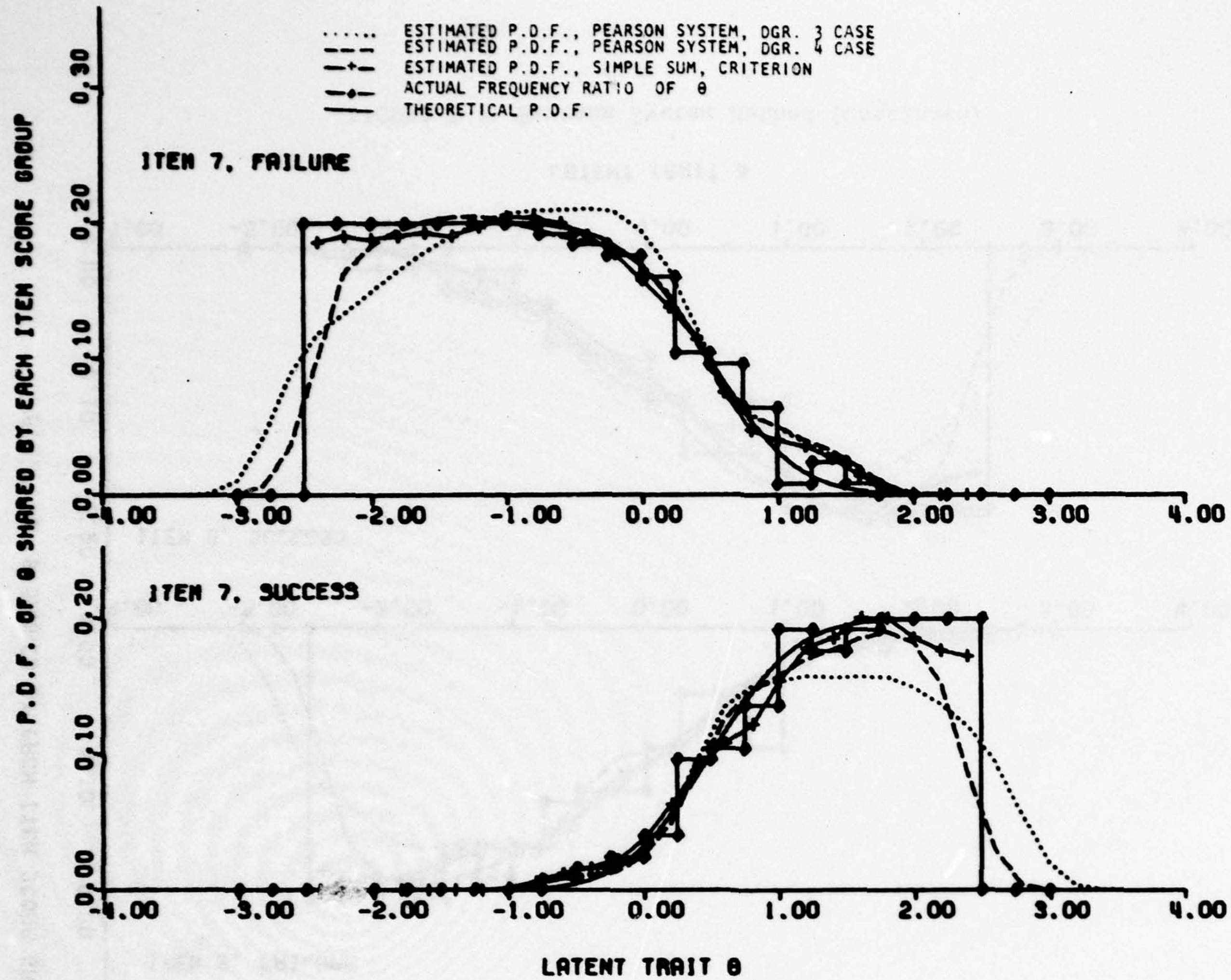

FIGURE 5-4: Pearson System Method (Continued)

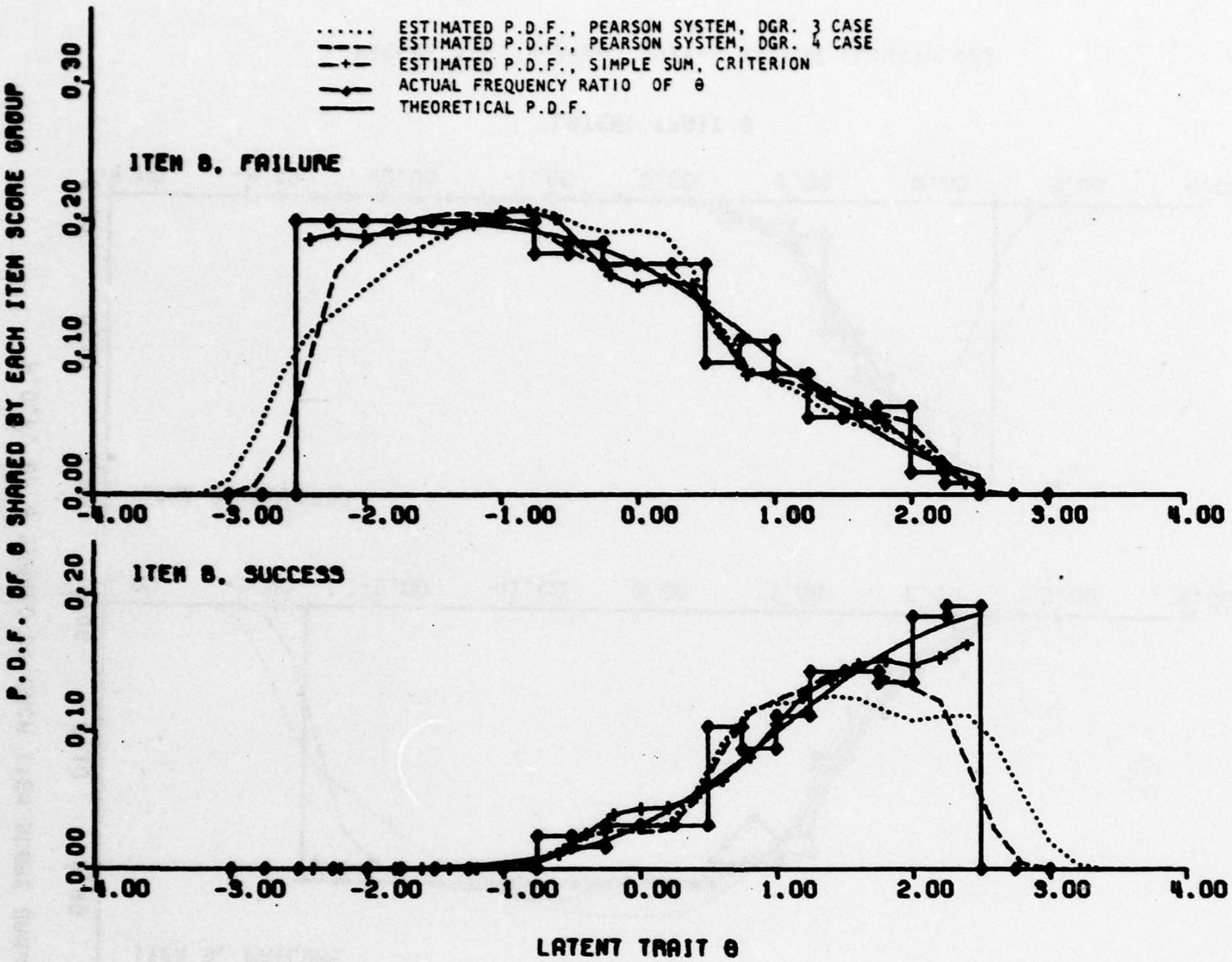

FIGURE 5-4: Pearson System Method (Continued)

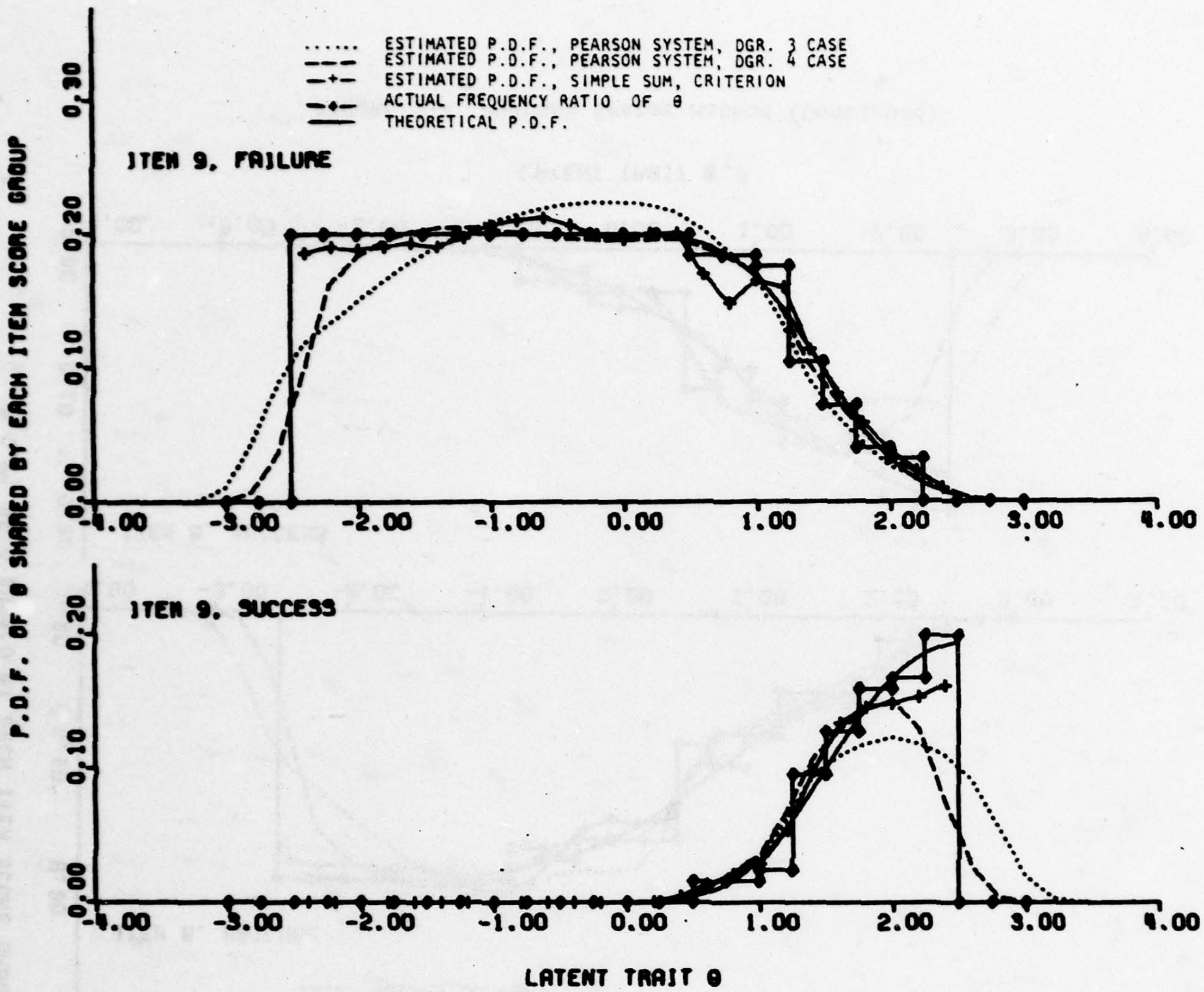

FIGURE 5-4: Pearson System Method (Continued)

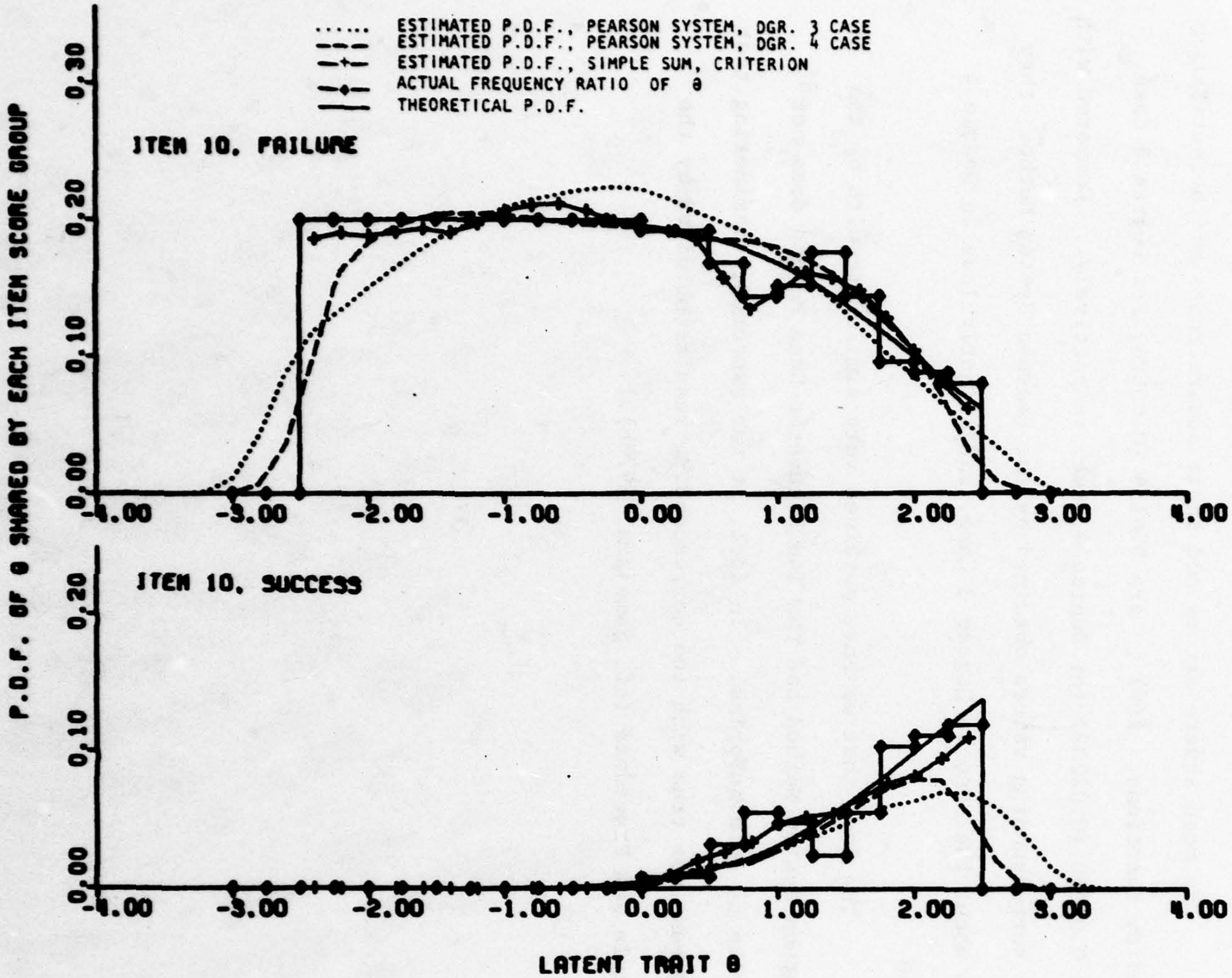

FIGURE 5-4: Pearson System Method (Continued)

3 Case. The estimated shared densities of ability by the failure and success groups are also very similar to those obtained by the Pearson System Method, and, therefore, the same comments made earlier also hold for the present results of the Two-Parameter Beta Method.

The mean square errors and their square roots of the estimated density functions, $\hat{f}(\theta)$, are 0.0136 (0.03684) for Degree 3 Case, and 0.00050 (0.02237) for Degree 4 Case, respectively. Compared with the corresponding values obtained by the Pearson System Method, they are about the same in Degree 3 Case, and slightly less in Degree 4 Case.

The fact that we have obtained very similar results by the Pearson System Method and the Two-Parameter Beta Method does not strike us as a surprise. In fact, it was expected, considering that the same was true with the corresponding results obtained by the Simple Sum Procedure (cf. Samejima, 1978b).

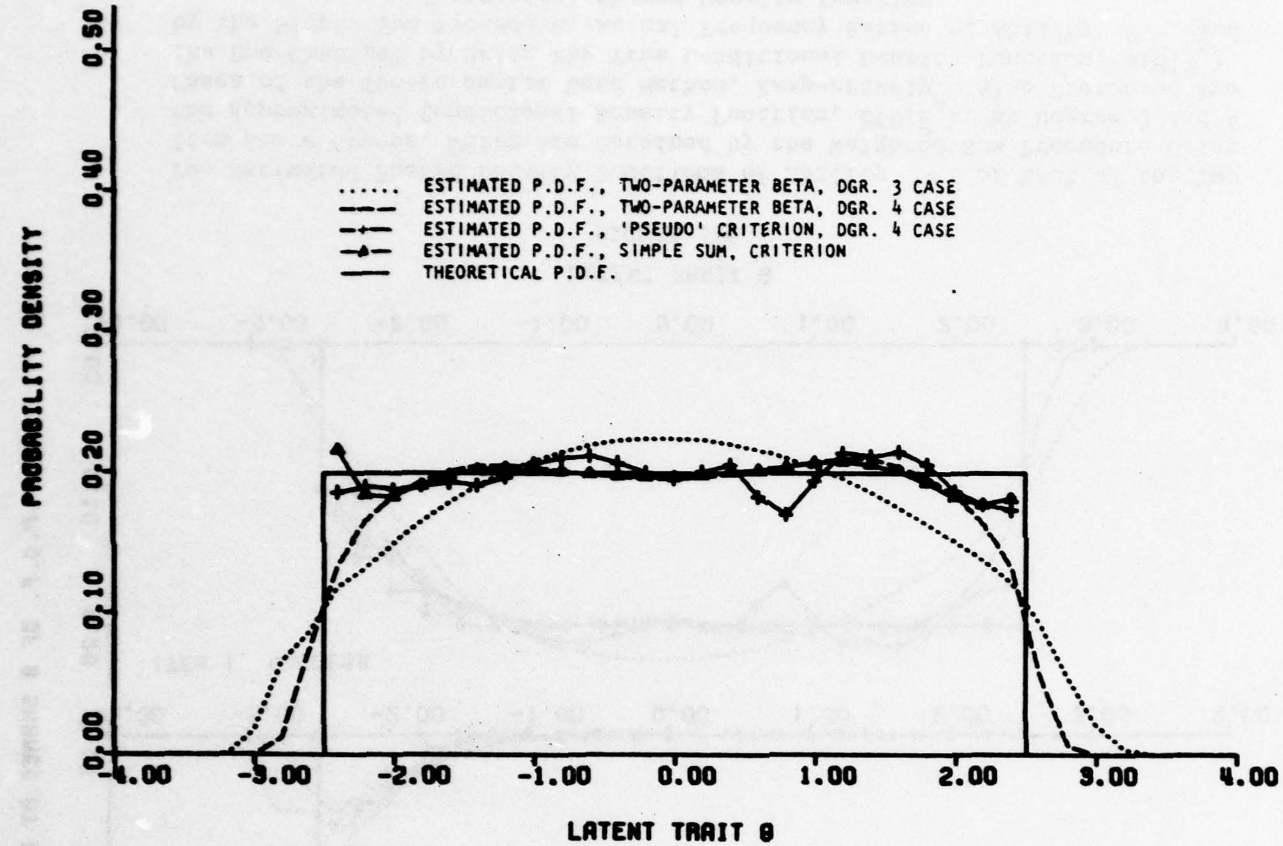

FIGURE 5-5

Estimated Density Functions of Ability θ Obtained by the Weighted Sum Procedure, Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Two-Parameter Beta Method. Also Drawn Are Those Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by Degree 4 Case of the Weighted Sum Procedure, and by the Simple Sum Procedure, Respectively, Together with the Theoretical Density, $f(\theta)$.

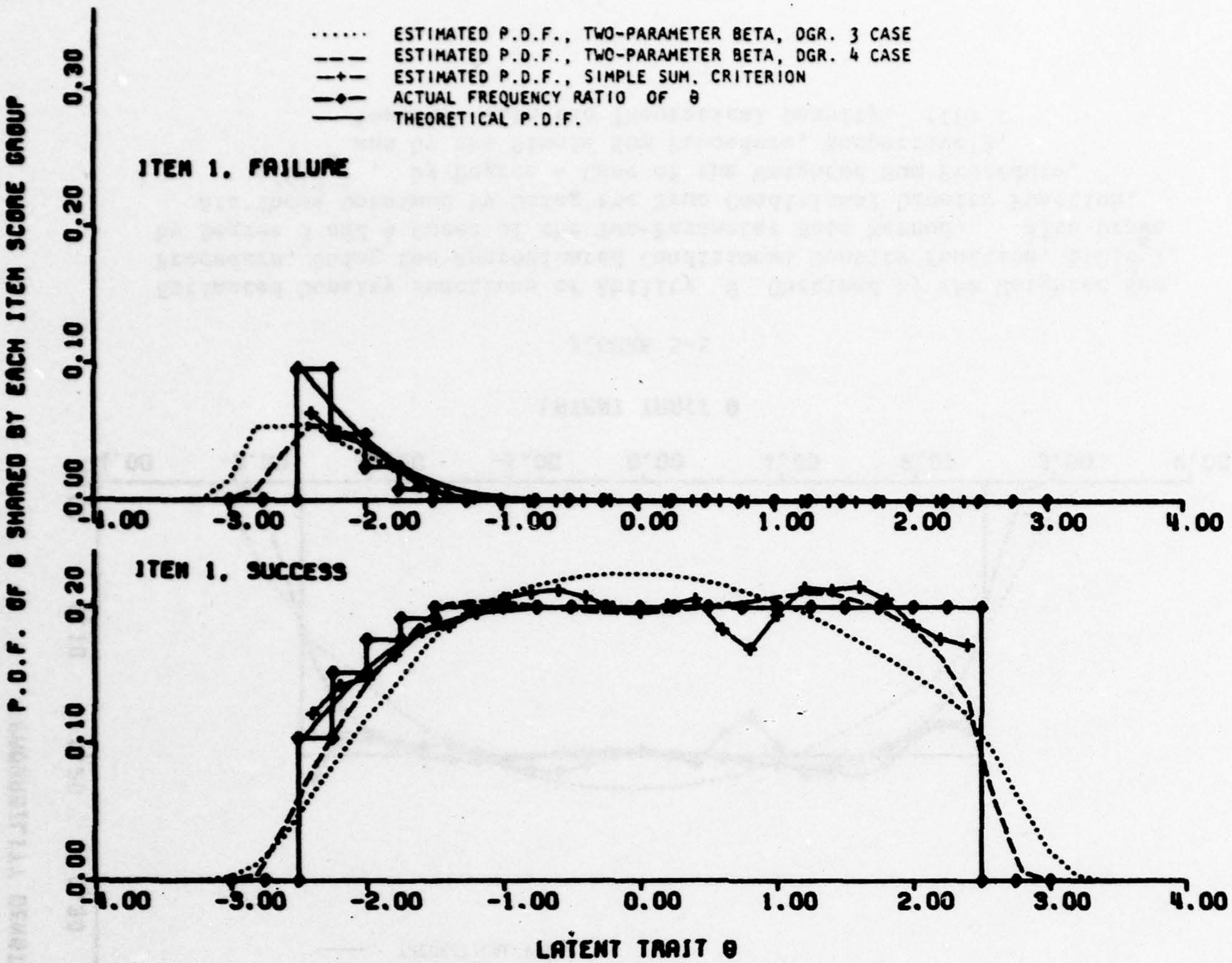

FIGURE 5-6

Two Estimated Shared Density Functions of Ability θ of Each of the Two Item Score Groups, Which Are Obtained by the Weighted Sum Procedure Using the Approximated Conditional Density Function, $\hat{\phi}(\theta|\hat{\theta}_s)$, by Degree 3 and 4 Cases of the Two-Parameter Beta Method, Respectively. Also Presented Are the One Obtained by Using the True Conditional Density Function, $\phi(\theta|\hat{\theta}_s)$, by the Simple Sum Procedure, Actual Frequency Ratios of Ability θ, and Theoretical Shared Density Function.

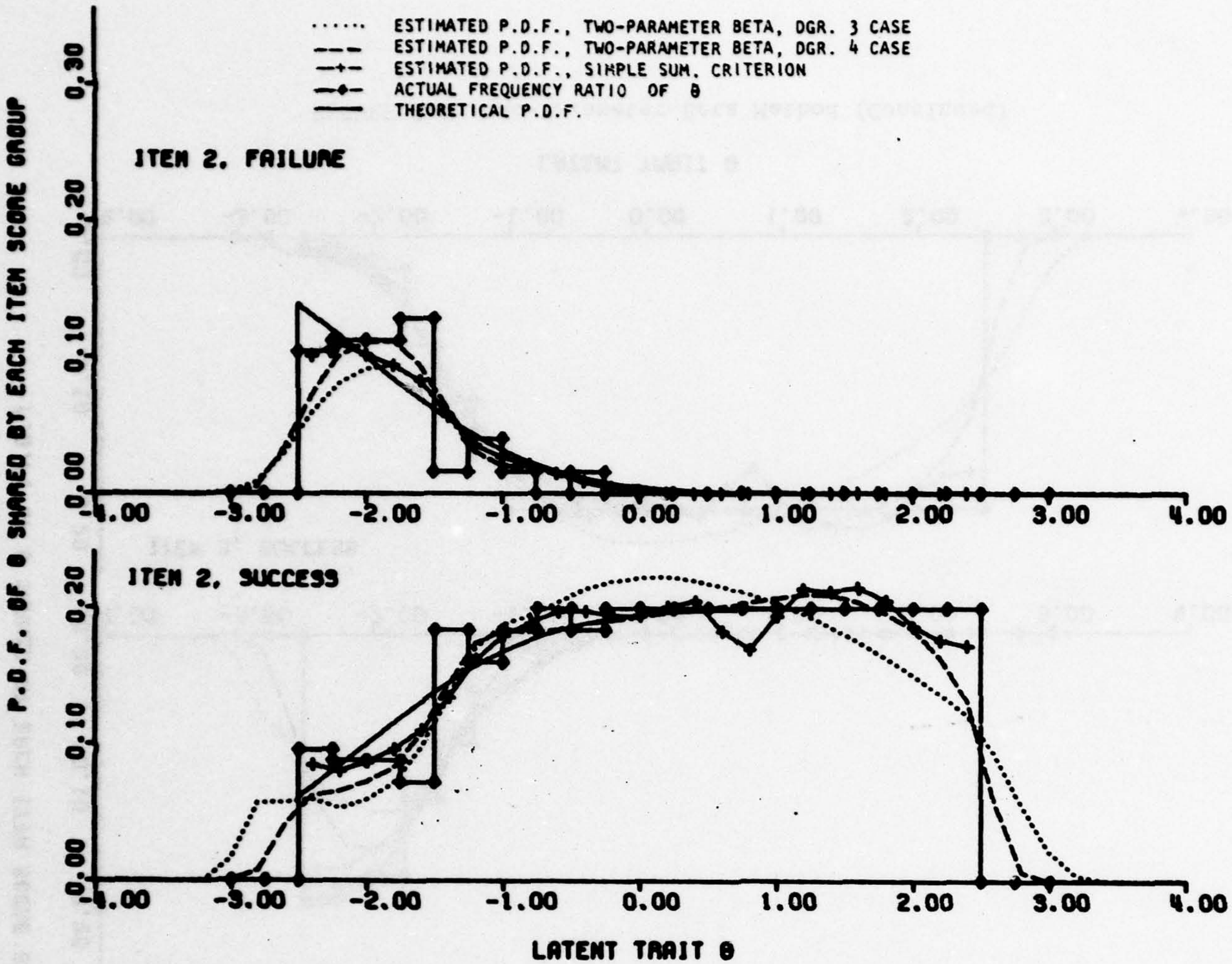

FIGURE 5-6: Two-Parameter Beta Method (Continued)

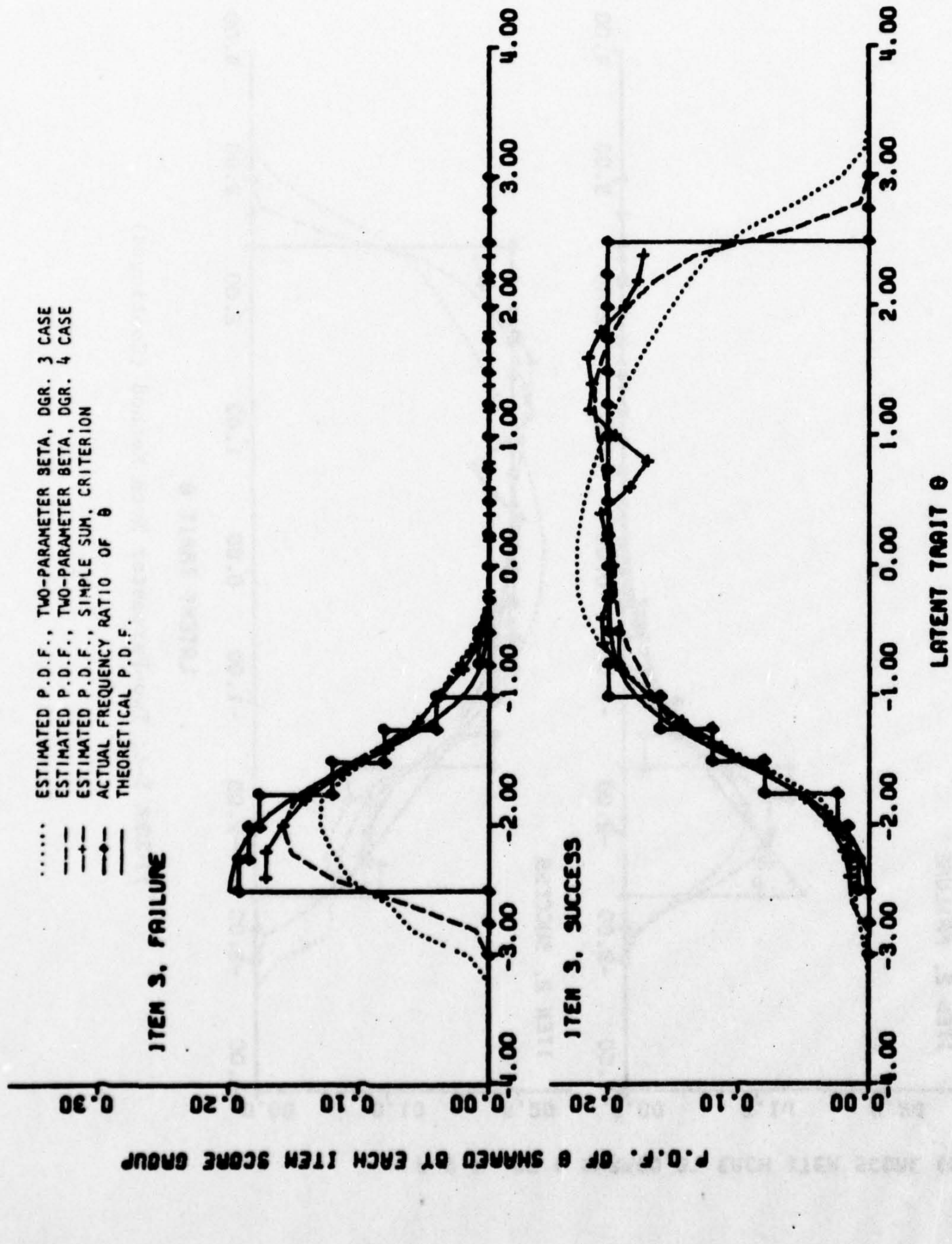

FIGURE 5-6: Two-Parameter Beta Method (Continued)

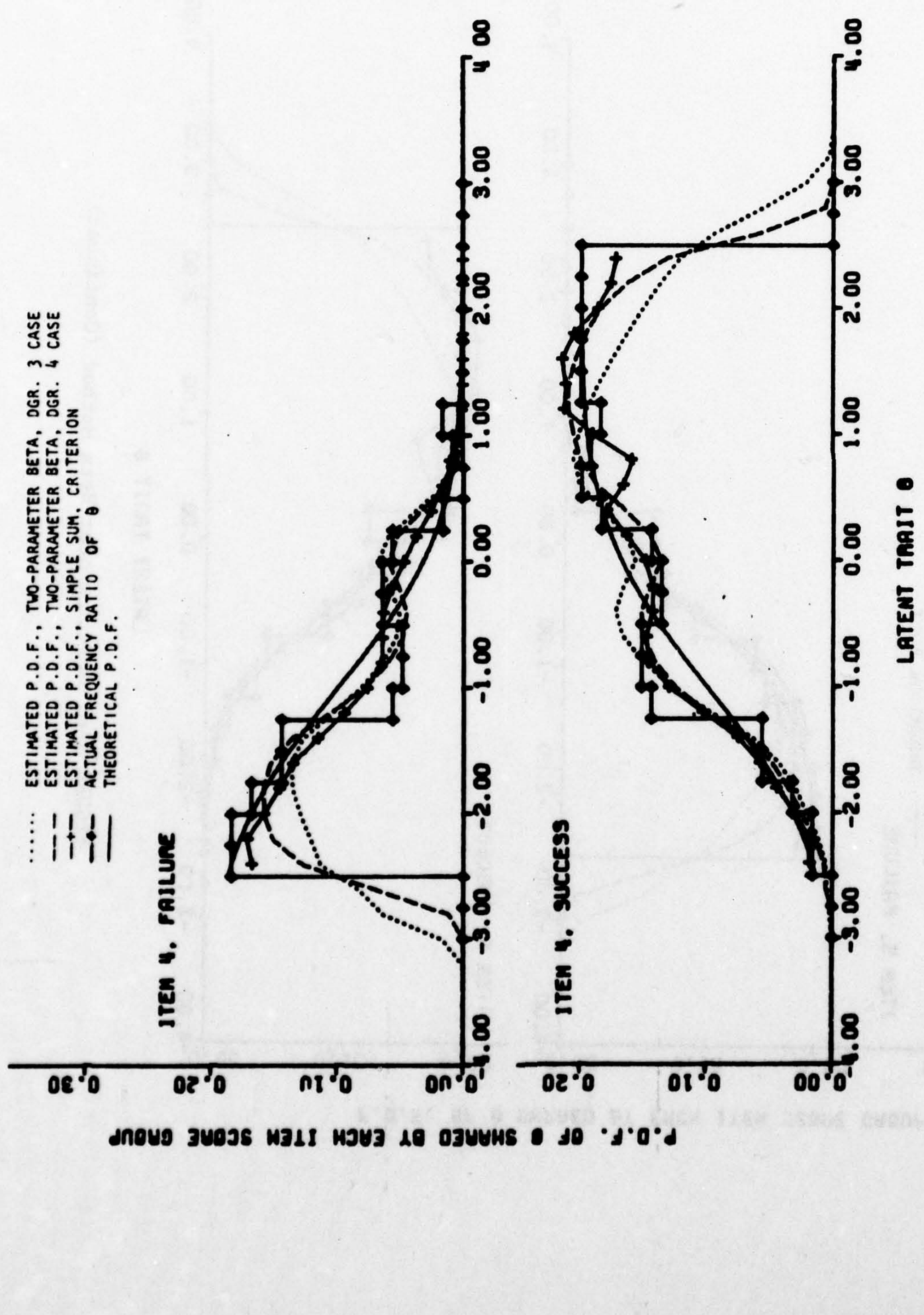

FIGURE 5-6: Two-Parameter Beta Method (Continued)

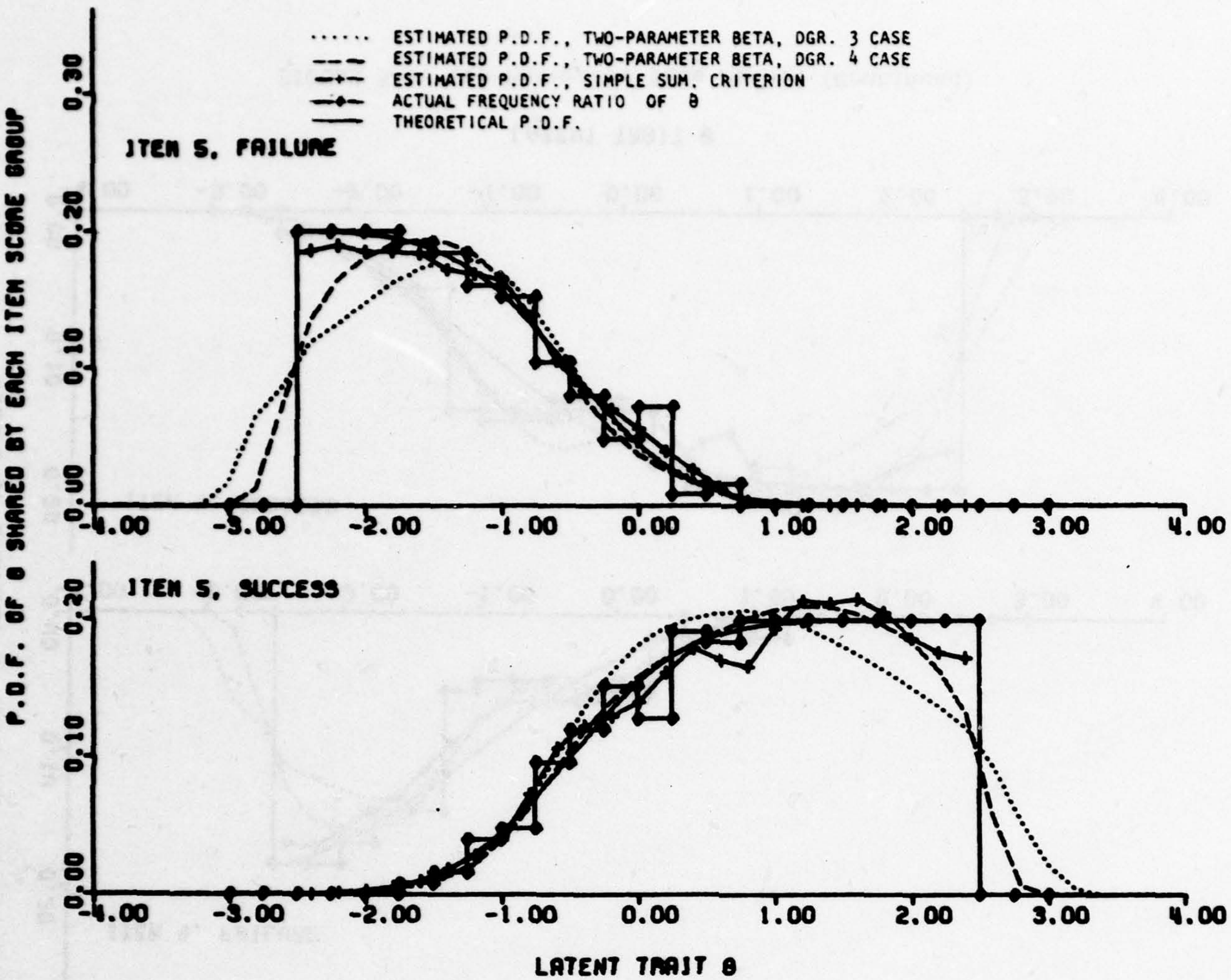

FIGURE 5-6: Two-Parameter Beta Method (Continued)

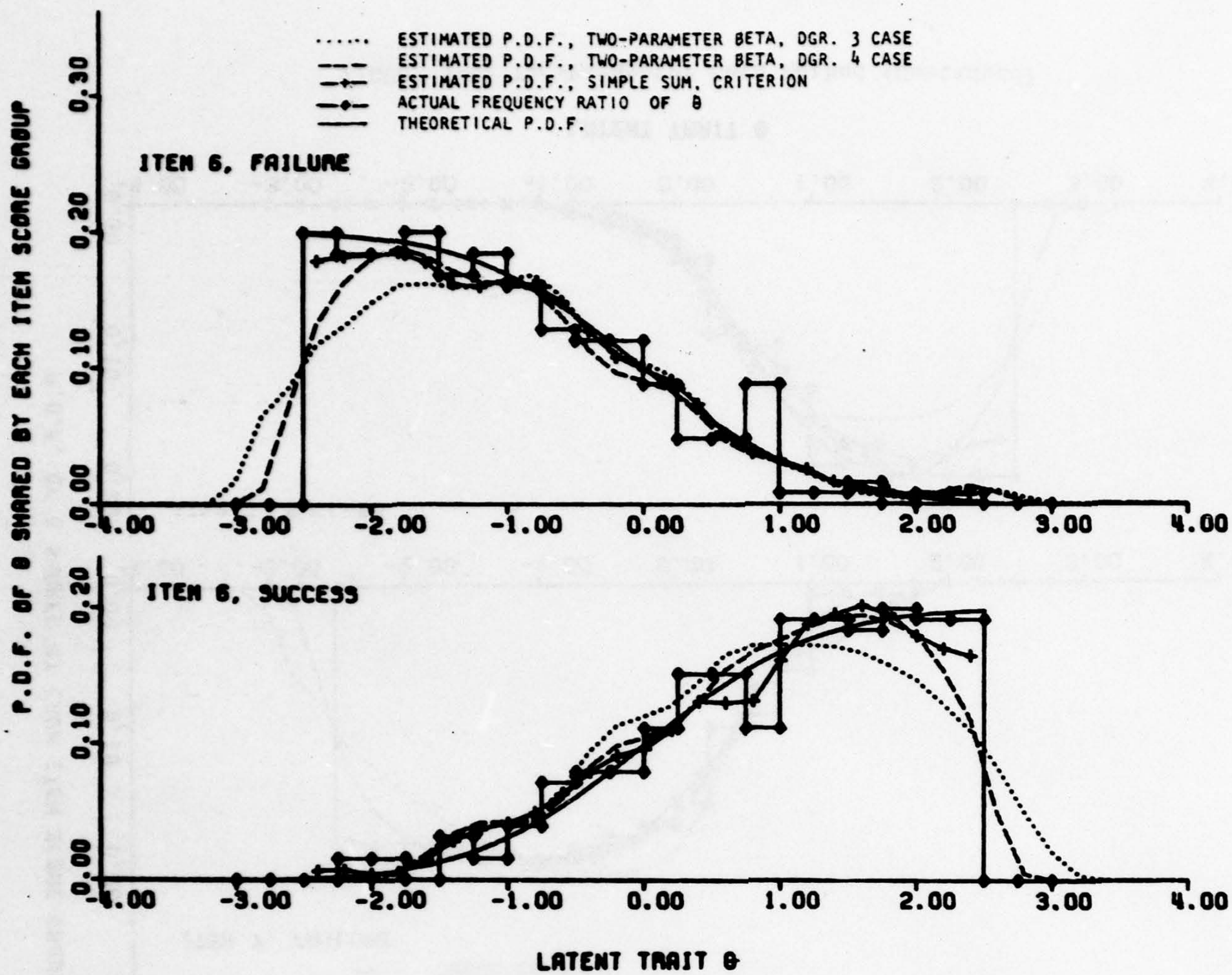

FIGURE 5-6: Two-Parameter Beta Method (Continued)

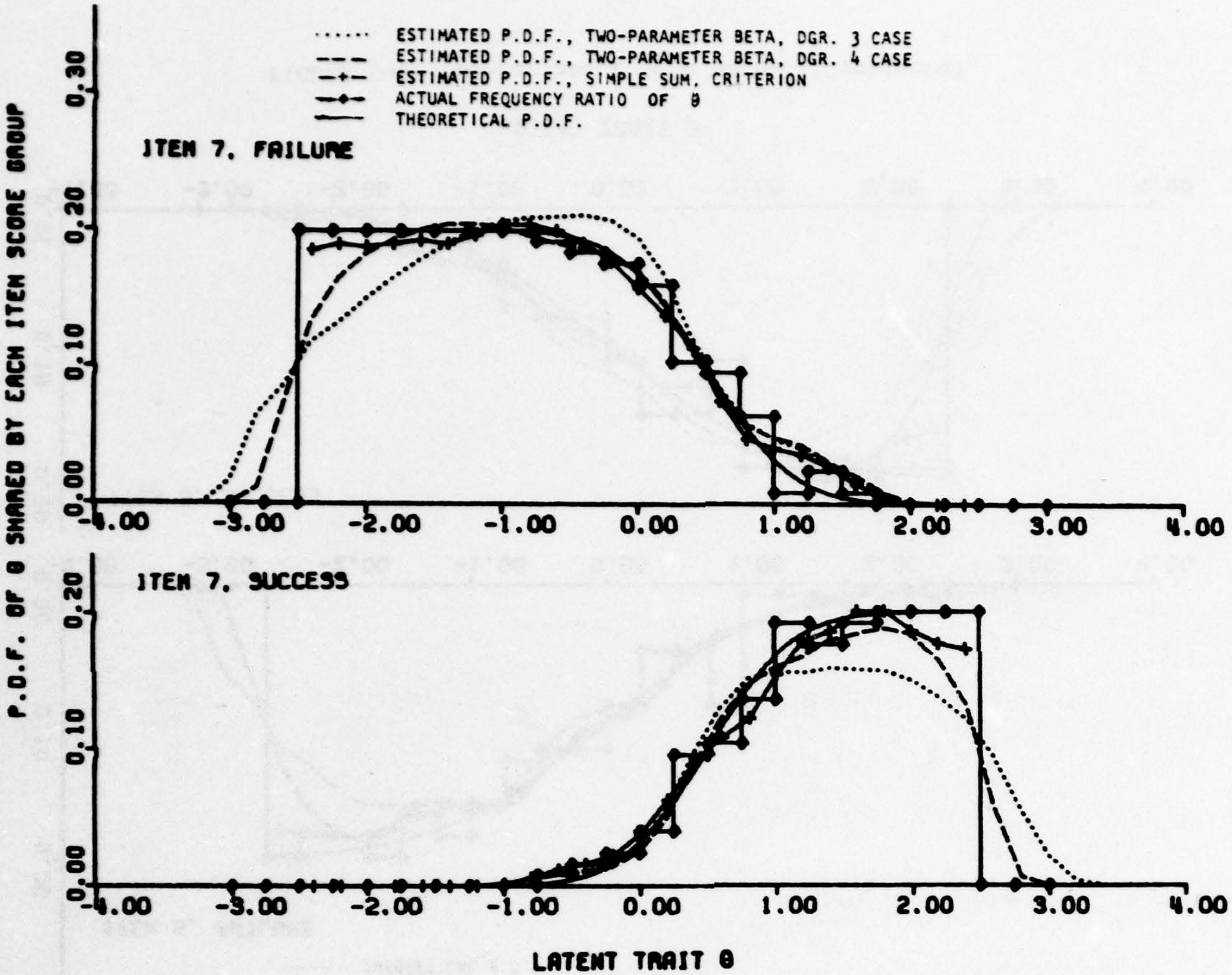

FIGURE 5-6: Two-Parameter Beta Method (Continued)

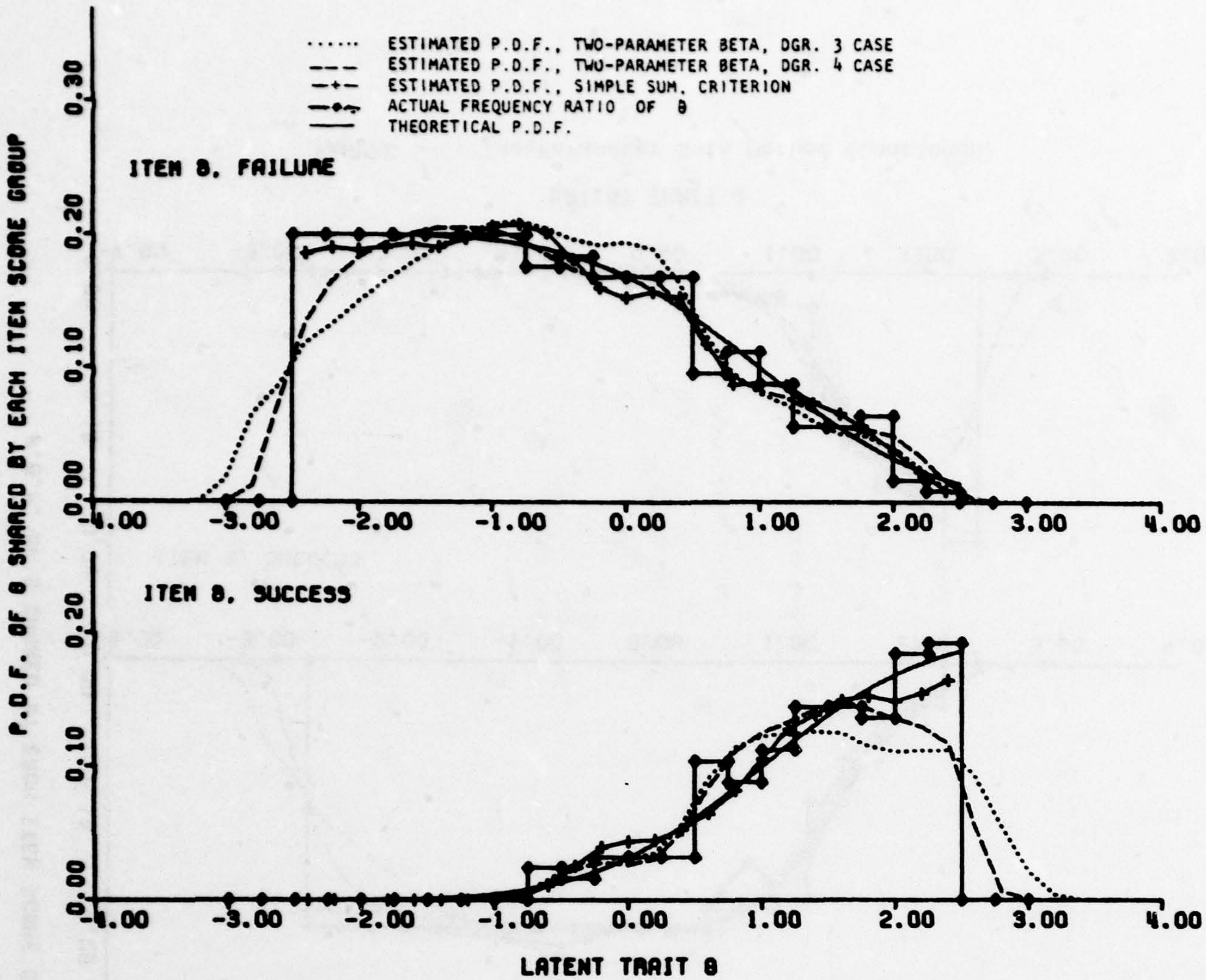

FIGURE 5-6: Two-Parameter Beta Method (Continued)

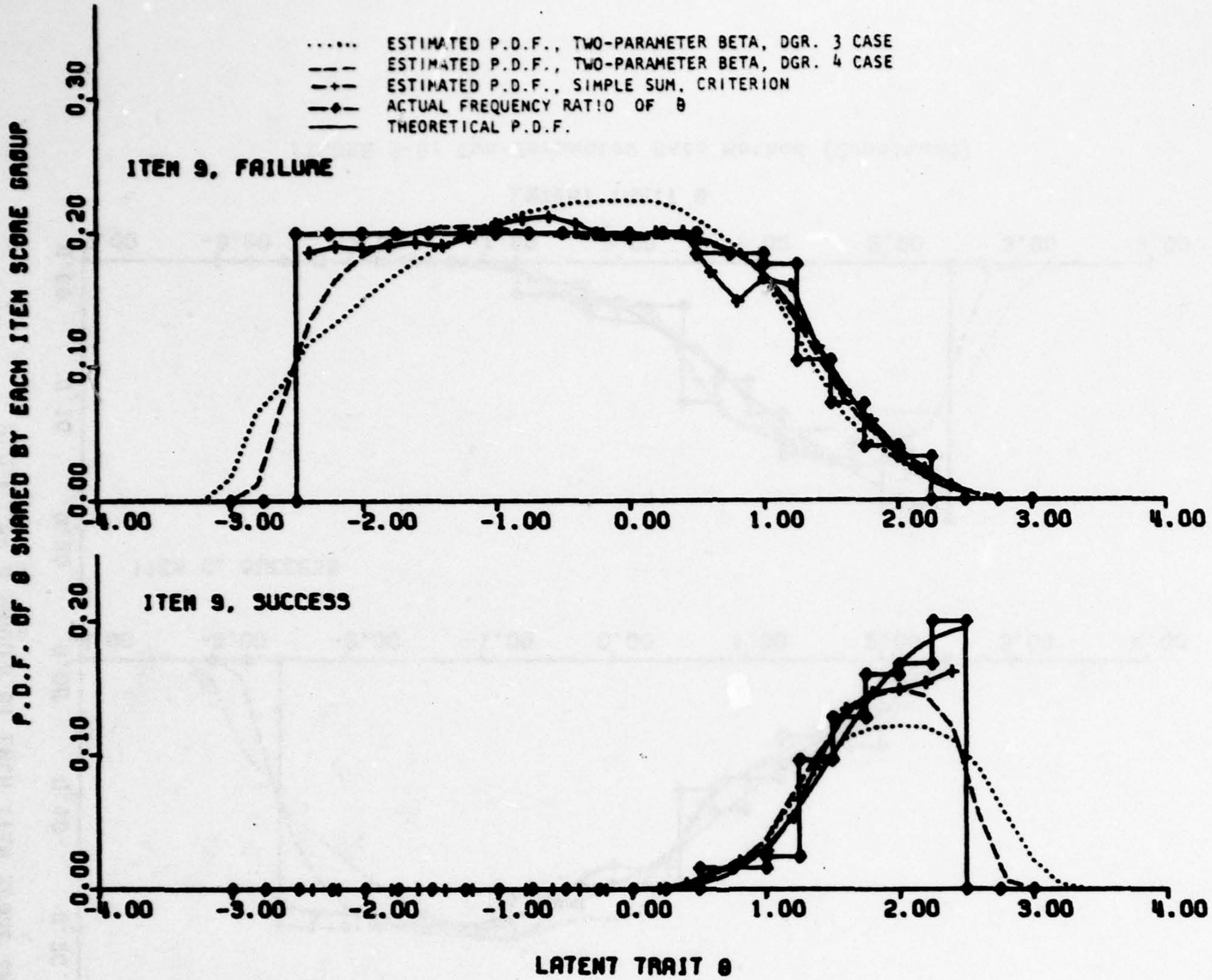

FIGURE 5-6: Two-Parameter Beta Method (Continued)

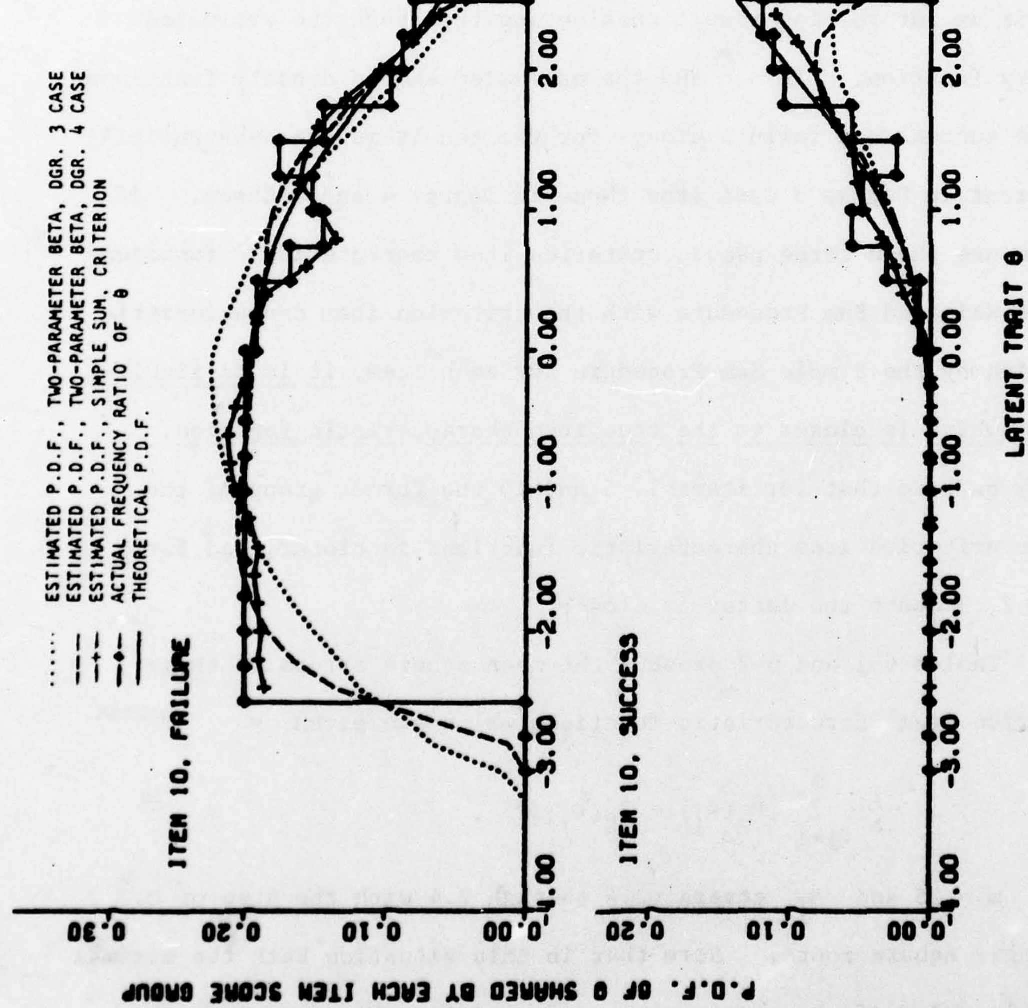

FIGURE 5-6: Two-Parameter Beta Method (Continued)

VI Results II: Estimated Item Characteristic Functions

Figure 6-1 presents the Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, which were described in Section 3, together with the criterion item characteristic function by the Simple Sum Procedure, the actual frequency ratios of θ , and the theoretical item characteristic function, $P_g(\theta)$, for each of the ten binary items. We can see that for all the ten items the three pseudo criterion item characteristic functions are almost identical. This is an interesting result considering that both the estimated density function, $\hat{f}(\theta)$, and the estimated shared density functions by the success and failure groups for the ten items are substantially different in Degree 3 Case from those in Degree 4 and 5 Cases. If we compare these three pseudo criterion item characteristic functions by the Weighted Sum Procedure with the criterion item characteristic function by the Simple Sum Procedure for each item, it is difficult to say which is closer to the true item characteristic function. It may be said that for items 1, 5 and 10 the former group of the pseudo criterion item characteristic functions is closer, and for items 2, 3 and 6 the latter is closer.

Tables 6-1 and 6-2 present the mean square errors of these criterion item characteristic functions which are given by

$$(6.1) \qquad \frac{1}{m} \sum_{j=1}^{m} [\hat{P}_g(\theta_j) - P_g(\theta_j)]^2 ,$$

where $m = 25$ and θ_j covers -2.4 through 2.4 with the step of 0.2 , and their square roots. Note that in this situation both the maximal possible value of the mean square error and that of its square root are unity. If we compare the three columns in these tables which are labeled "PSEUDO" CRITERION DGR.3 DGR.4 DGR.5 with the column that is labeled SSP (Simple Sum Procedure) CRITERION, the rough observations made for Figure 6-1 are confirmed. In general, these values are close among the four cases, although they tend to be slightly less for the Simple Sum Procedure. We also notice that the fairly large values in the column labeled AFR (Actual Frequency Ratio) are somewhat ameliorated in these four columns for the criterion item characteristic functions, even though there is a tendency that, if the value is large for the actual frequency ratio it is also large for the other four cases.

The estimated item parameters, $\hat{a}_g$ and $\hat{b}_g$, in the normal ogive model are shown in Tables 6-3 and 6-4 for these four types of criterion item characteristic functions, for each of the ten binary items. These values were obtained using the least square principle, applied for, at most, $m = 25$ values of $\hat{P}_g(\theta)$, excluding those greater than 0.95 or less than 0.05 . We can see that these four sets of results are close, and there is a tendency that the estimates of the difficulty parameter, $\hat{b}_g$, are slightly better for the criterion item characteristic functions of the Simple Sum Procedure than for the three sets of pseudo item characteristic functions, whereas no such a tendency is observed for the discrimination parameter estimates, $\hat{a}_g$. Generally speaking, the difficulty parameters are more accurately estimated than the discrimination parameters in all cases. It is also noticed that, when the discrimination parameter takes a large value, its estimate tends to be much less than the true value, no matter what procedure is used. It is noted, moreover, that the parameter estimation tends to be more accurate for the items of intermediate difficulties than for those which are either very difficult or very easy.

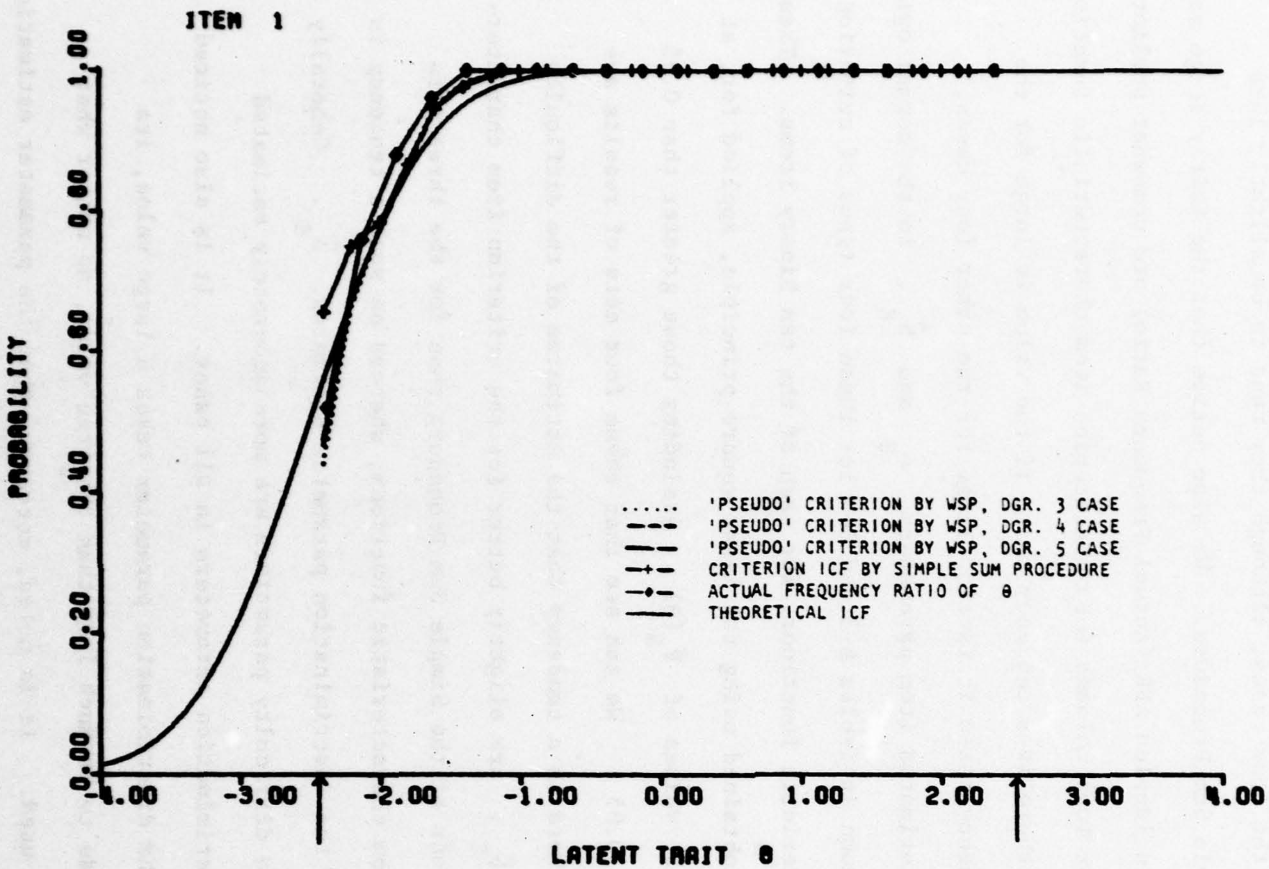

FIGURE 6-1

Comparison of the Three Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases by the Weighted Sum Procedure of Conditional P.D. F. Approach with the Criterion Item Characteristic Function by the Simple Sum Procedure. Actual Frequency Ratios and Theoretical Item Characteristic Function Are Also Presented for Comparison.

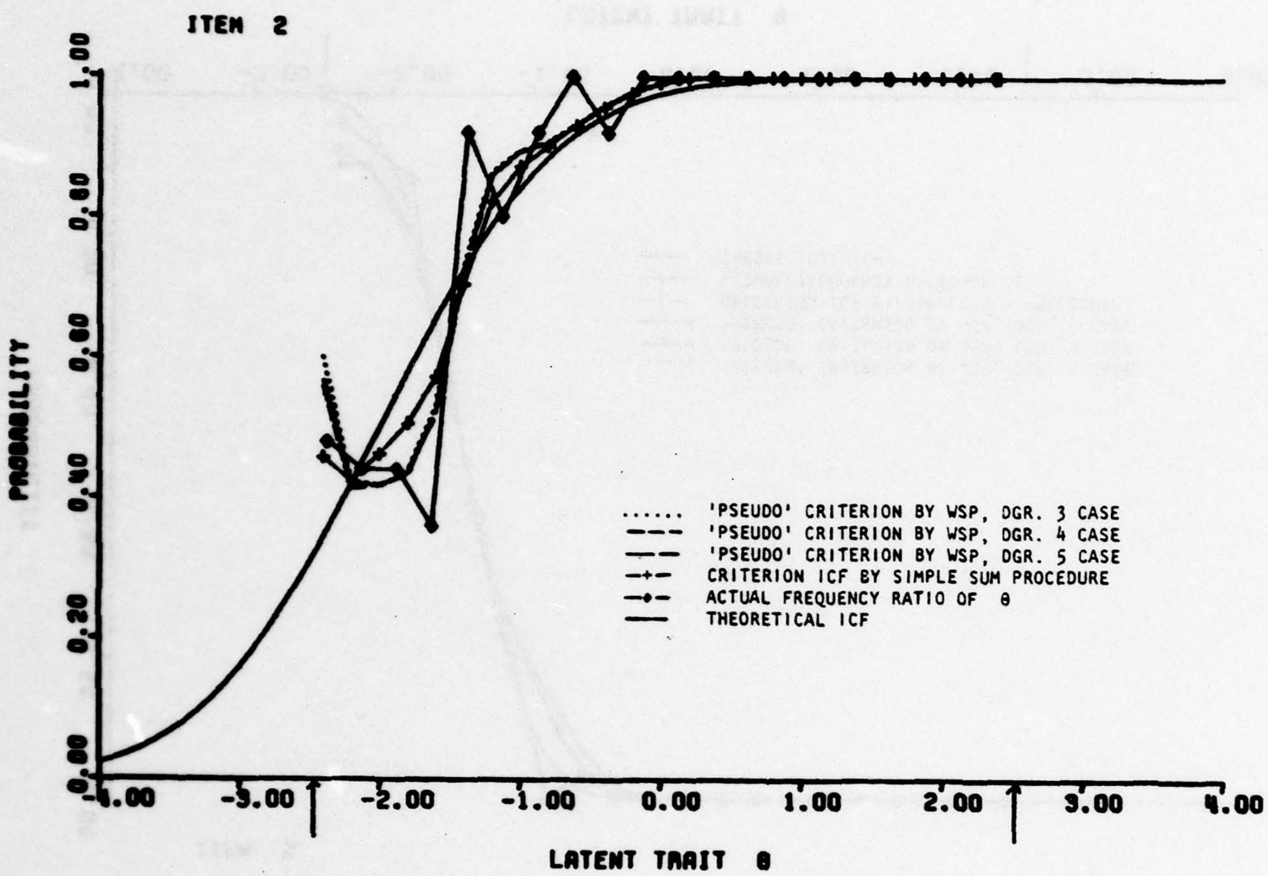

FIGURE 6-1: Pseudo Criterion Cases (Continued)

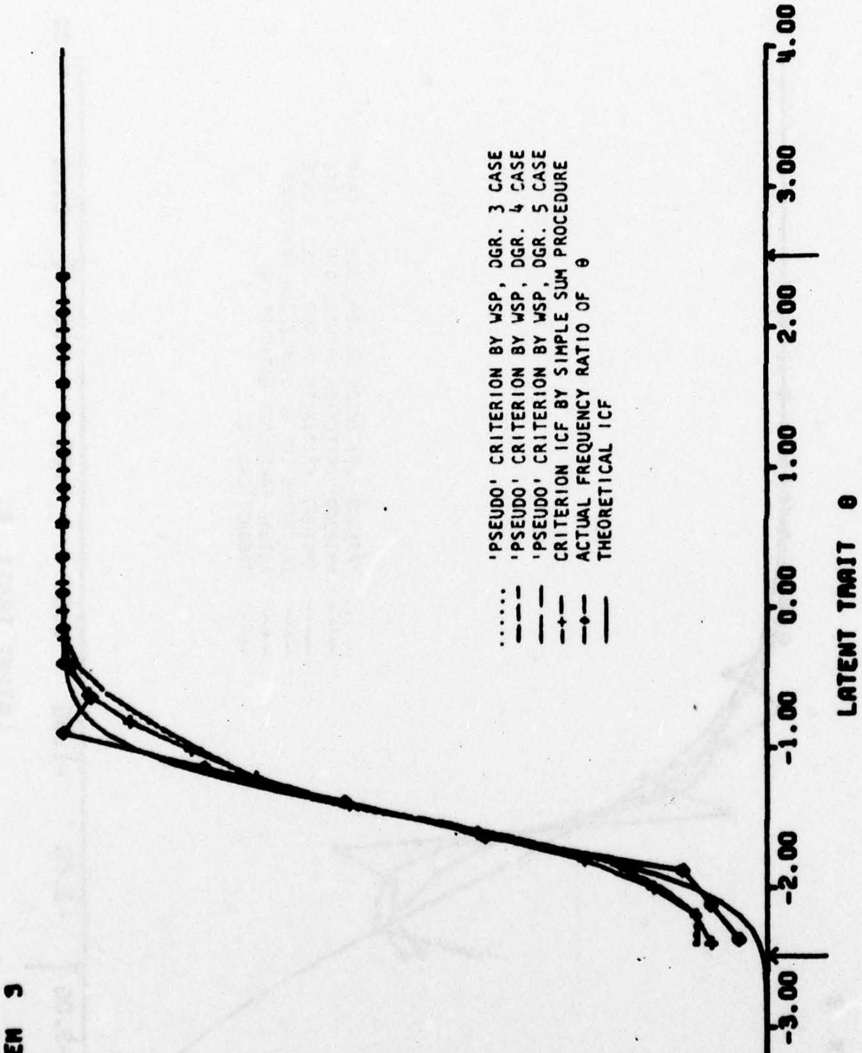

FIGURE 6-1: Pseudo Criterion Cases (Continued)

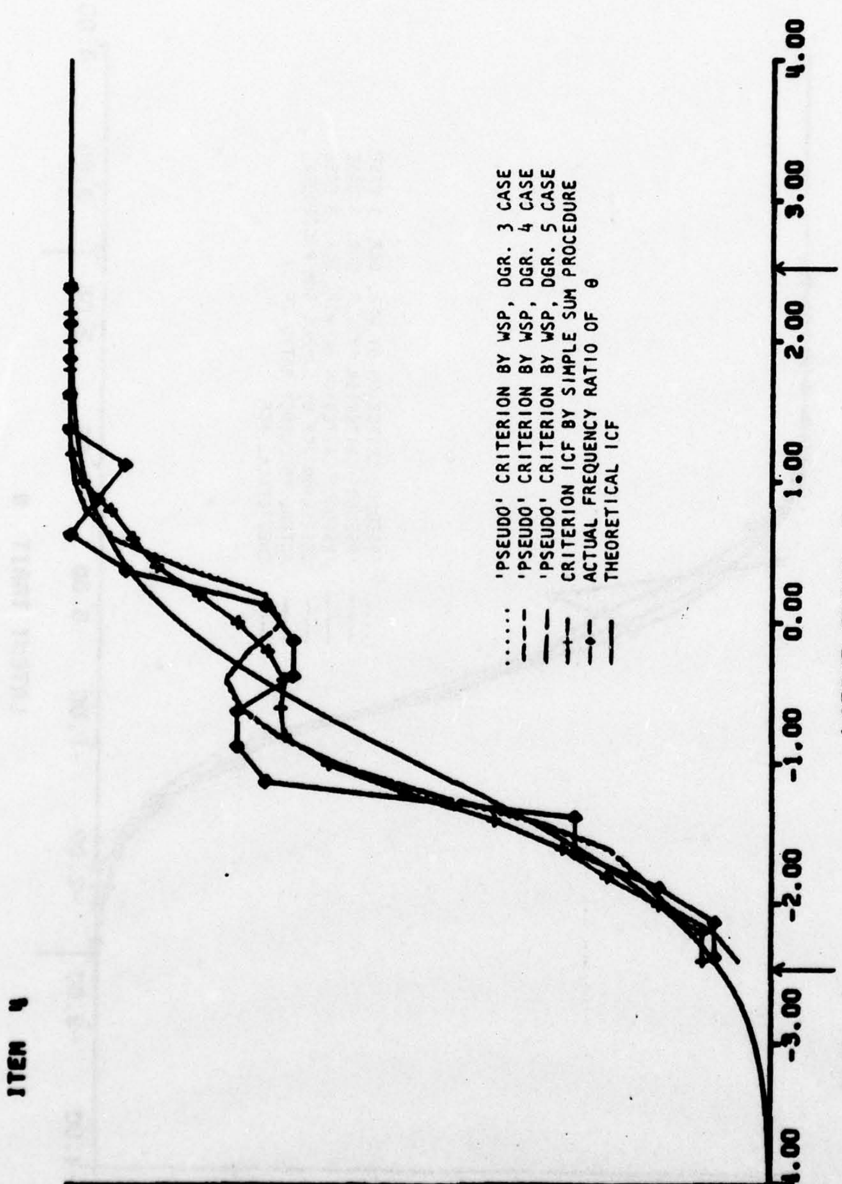

FIGURE 6-1: Pseudo Criterion Cases (Continued)

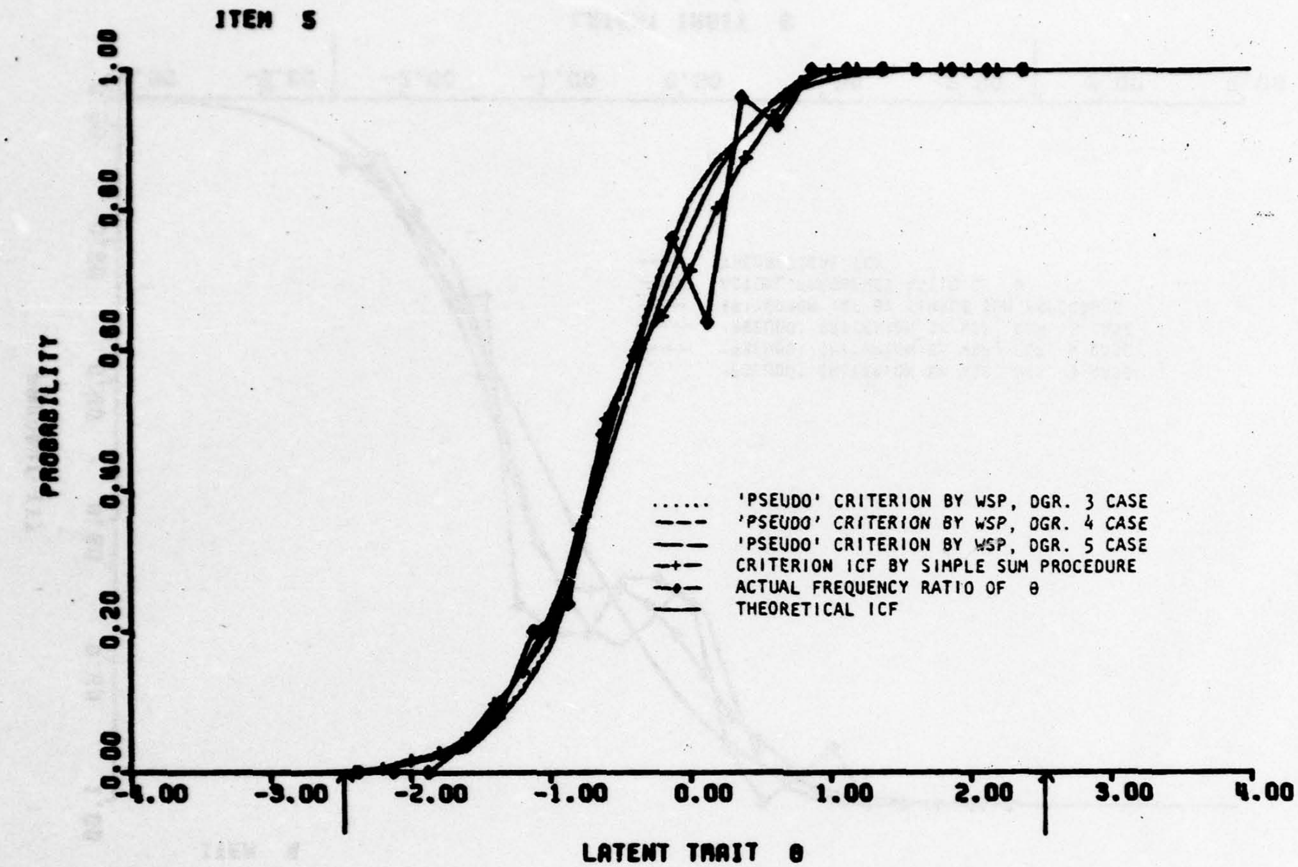

FIGURE 6-1: Pseudo Criterion Cases (Continued)

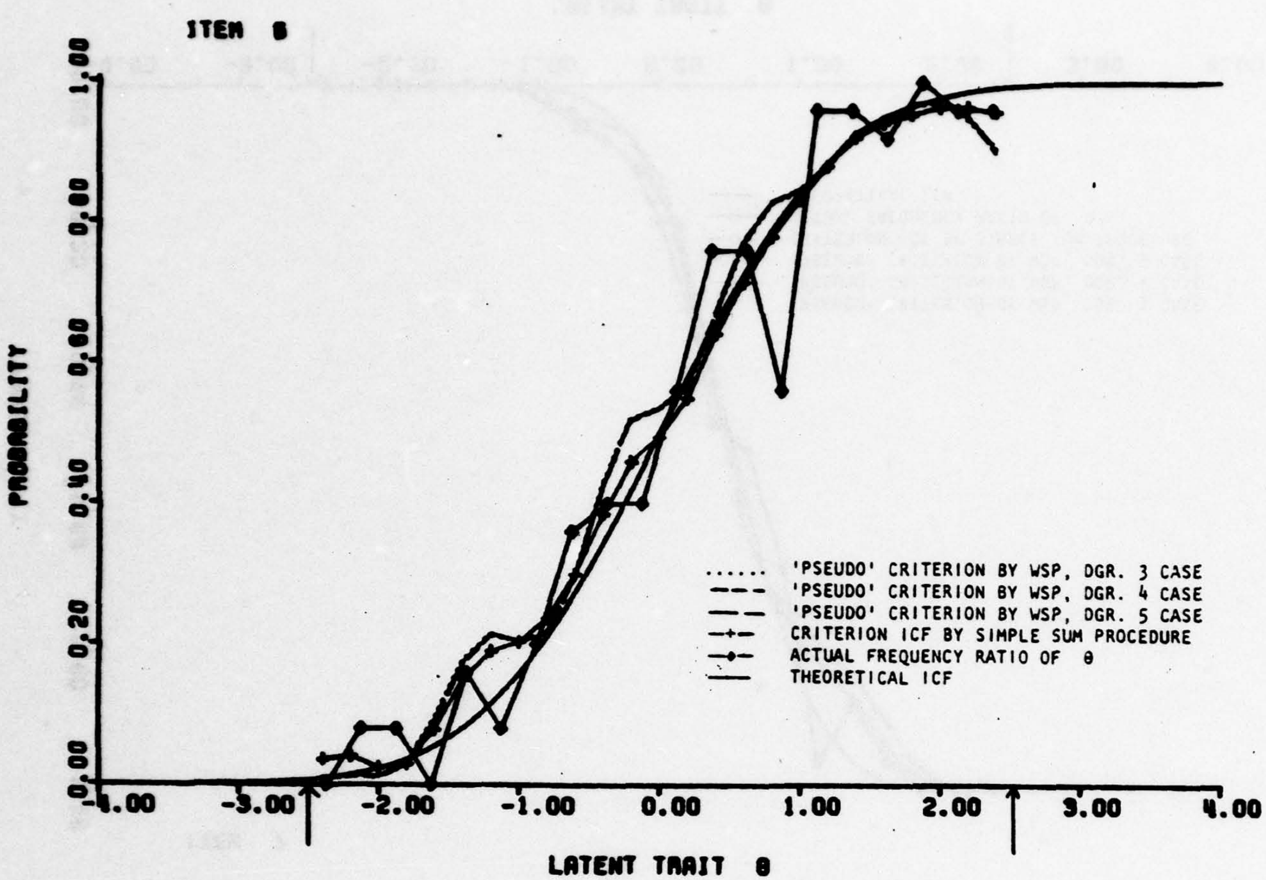

FIGURE 6-1: Pseudo Criterion Cases (Continued)

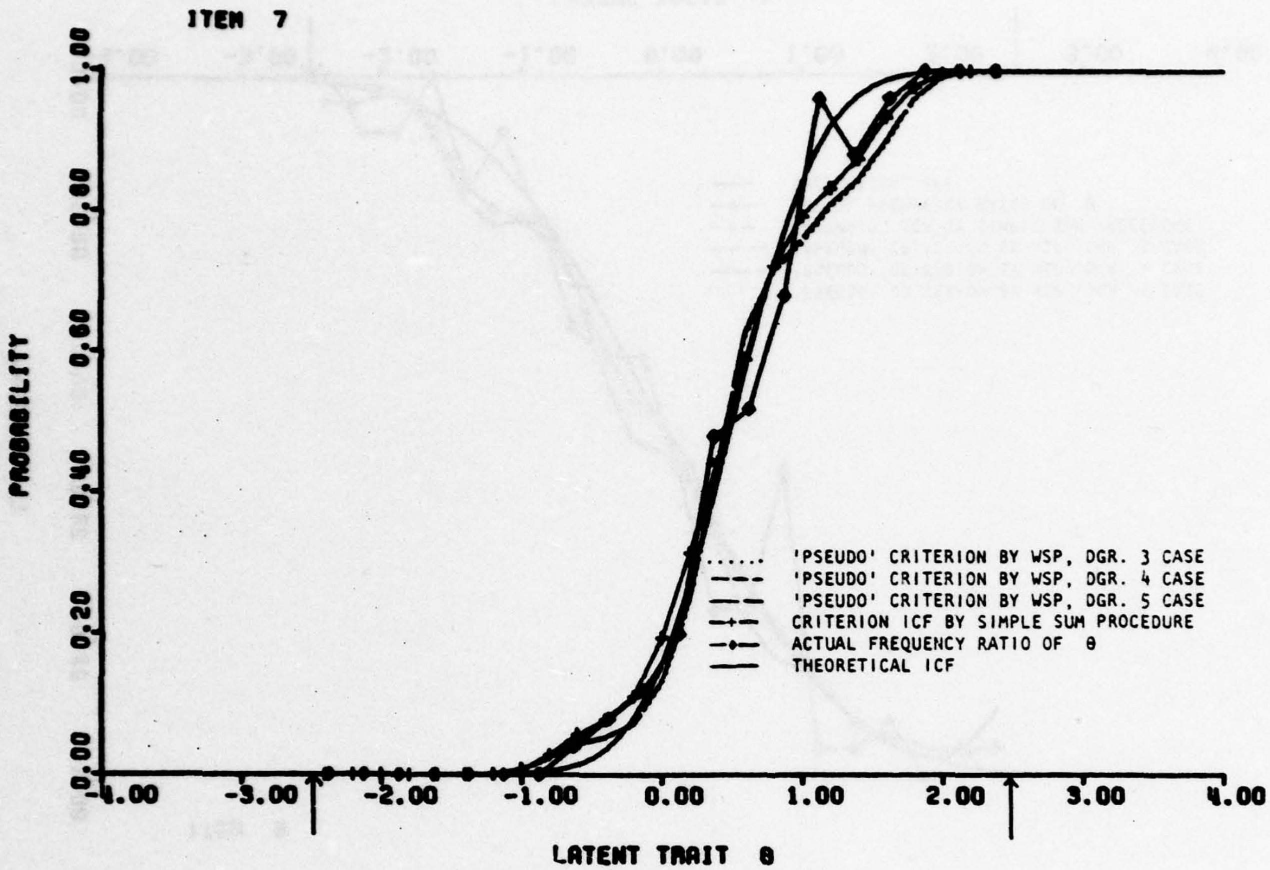

FIGURE 6-1: Pseudo Criterion Cases (Continued)

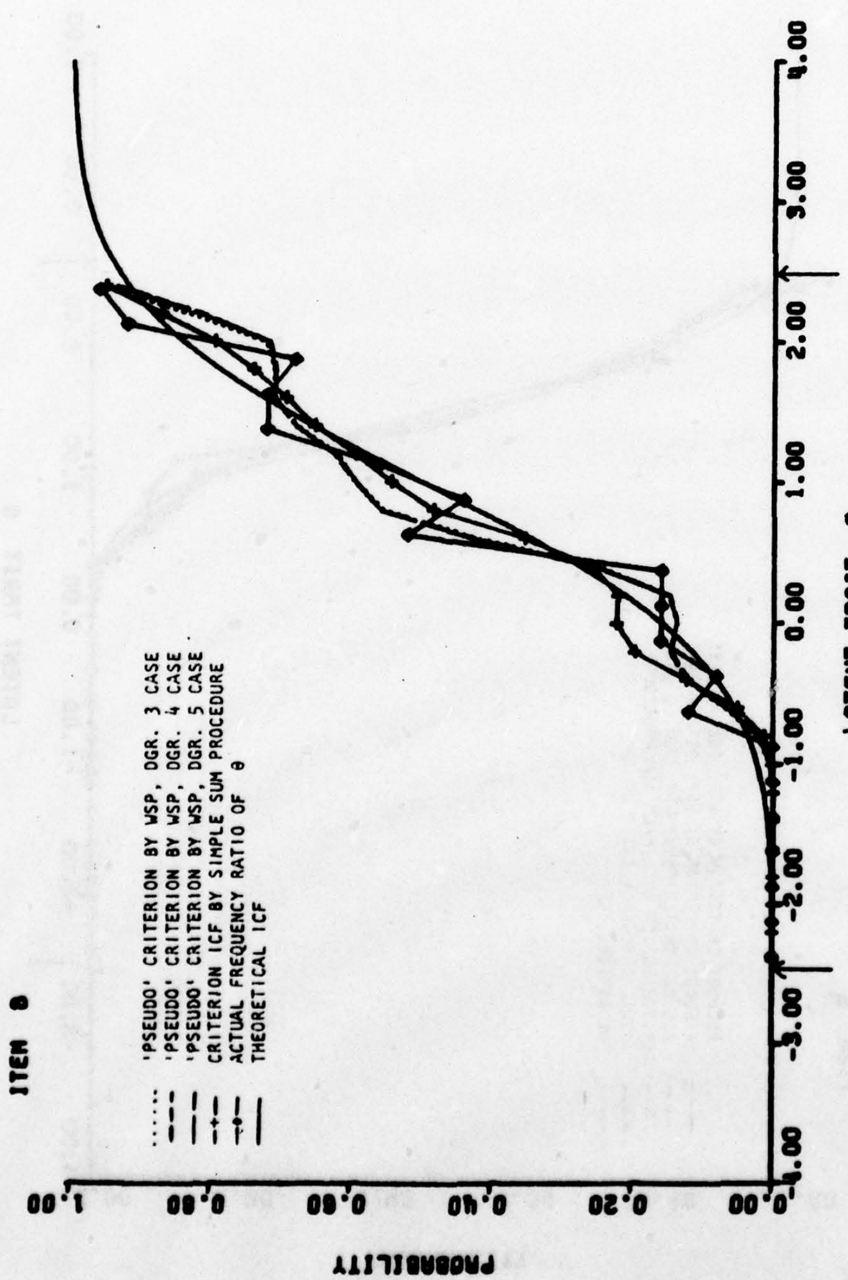

FIGURE 6-1: Pseudo Criterion Cases (Continued)

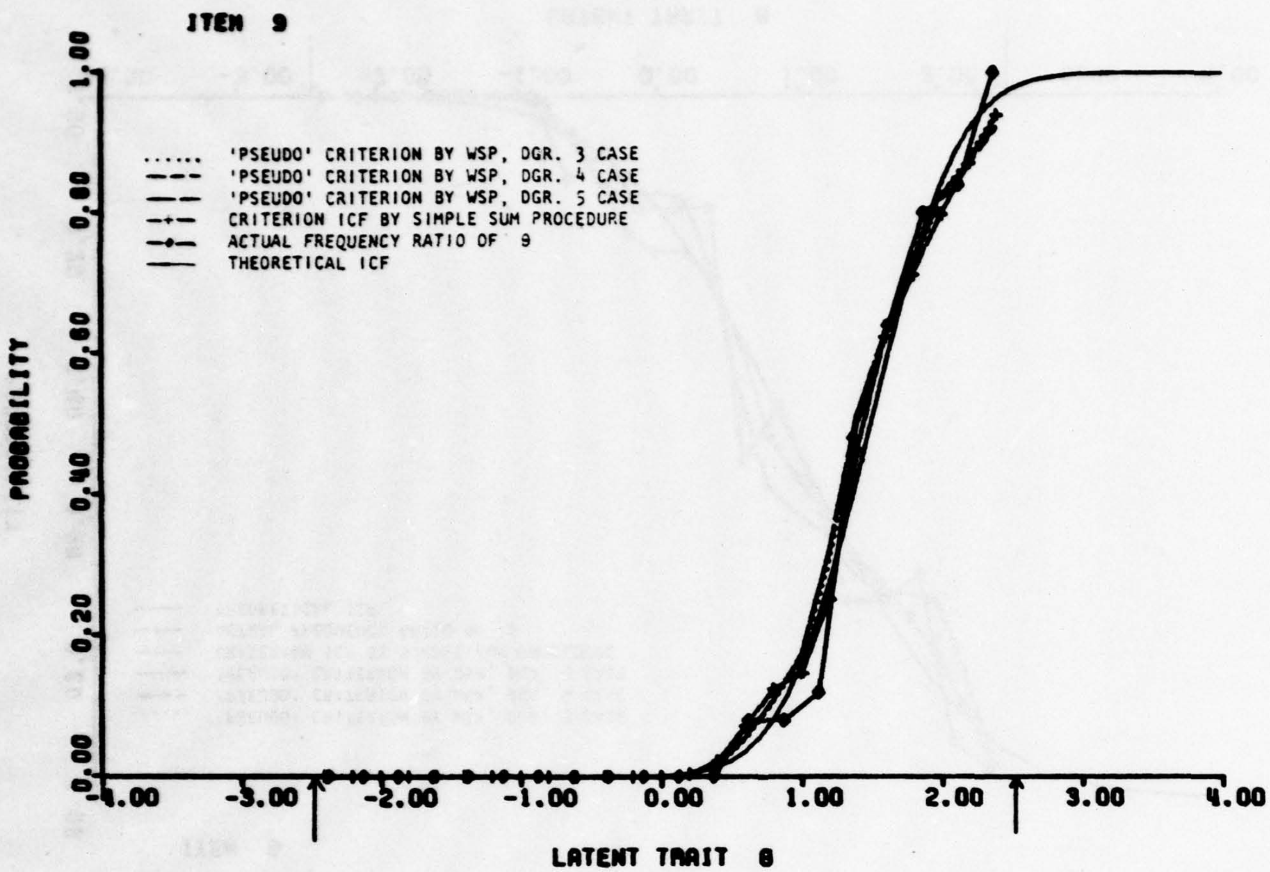

FIGURE 6-1: Pseudo Criterion Cases (Continued)

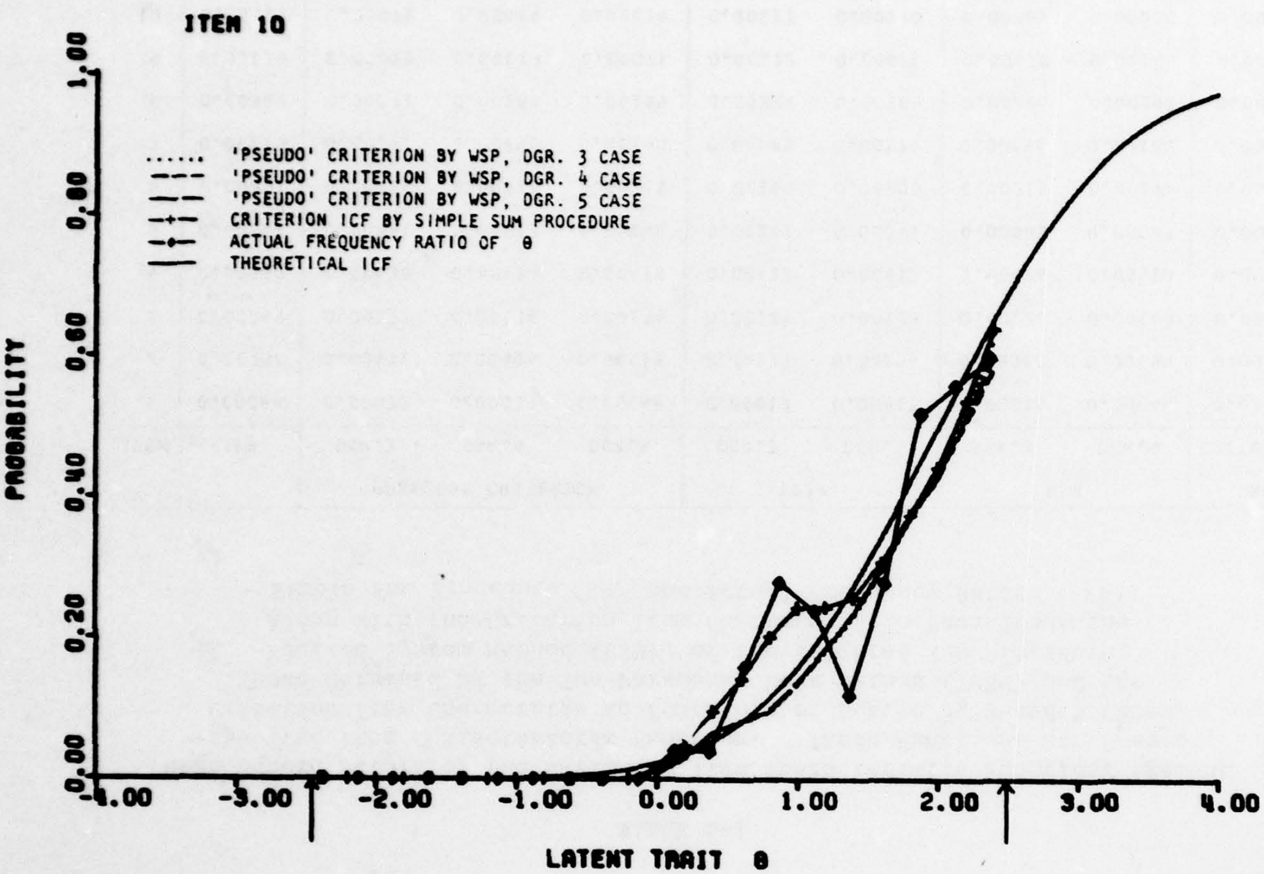

FIGURE 6-1: Pseudo Criterion Cases (Continued)

TABLE 6-1

Mean Square Errors of the Estimated Item Characteristic Functions Against the True Item Characteristic Function. These Functions Are Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, Those Obtained by the Two-Parameter Beta Method (TPBM) and the Pearson System Method (PSM), of the Weighted Sum Procedure, Along with the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP) and Actual Frequency Ratios (AFR).

		"PSEUDO" CRITERION			TPBM		PSM		SSP
ITEM	AFR	DGR.3	DGR.4	DGR.5	DGR.3	DGR.4	DGR.3	DGR.4	CRITERION
1	0.00068	0.00070	0.00039	0.00046	0.00013	0.00015	0.00016	0.00044	0.00070
2	0.00782	0.00474	0.00406	0.00418	0.00327	0.00294	0.00320	0.00258	0.00121
3	0.00049	0.00133	0.00134	0.00134	0.00127	0.00133	0.00128	0.00103	0.00098
4	0.00940	0.00438	0.00424	0.00419	0.00412	0.00413	0.00426	0.00428	0.00269
5	0.00242	0.00041	0.00043	0.00040	0.00041	0.00041	0.00043	0.00043	0.00059
6	0.00601	0.00220	0.00215	0.00215	0.00199	0.00205	0.00215	0.00176	0.00089
7	0.00179	0.00201	0.00190	0.00190	0.00183	0.00179	0.00191	0.00186	0.00120
8	0.00368	0.00272	0.00289	0.00289	0.00264	0.00269	0.00285	0.00298	0.00109
9	0.00135	0.00053	0.00073	0.00071	0.00072	0.00071	0.00078	0.00106	0.00051
10	0.00325	0.00071	0.00054	0.00079	0.00087	0.00170	0.00083	0.00065	0.00124

TABLE 6-2

Square Roots of the Mean Square Errors of the Estimated Item Characteristic Functions Against the True Item Characteristic Function. These Functions Are Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases, Those Obtained by the Two-Parameter Beta Method (TPBM) and Those by the Pearson System Method (PSM), of the Weighted Sum Procedure, Along with the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP) and Actual Frequency Ratios (AFR).

		"PSEUDO" CRITERION			TPBM		PSM		SSP
ITEM	AFR	DGR.3	DGR.4	DGR.5	DGR.3	DGR.4	DGR.3	DGR.4	CRITERION
1	0.02610	0.02640	0.01985	0.02137	0.01151	0.01210	0.01275	0.02108	0.02650
2	0.06843	0.06888	0.06375	0.06463	0.05721	0.05420	0.05659	0.05077	0.03473
3	0.02223	0.03643	0.03662	0.03666	0.03557	0.03644	0.03577	0.03216	0.03138
4	0.09656	0.06618	0.06509	0.06470	0.06417	0.06429	0.06526	0.06544	0.05187
5	0.04923	0.02034	0.02074	0.02012	0.02017	0.02016	0.02076	0.02077	0.02421
6	0.07754	0.04687	0.04641	0.04632	0.04464	0.04529	0.04635	0.04193	0.02982
7	0.04236	0.04481	0.04355	0.04360	0.04276	0.04233	0.04369	0.04317	0.03458
8	0.06070	0.05215	0.05372	0.05377	0.05137	0.05187	0.05336	0.05455	0.03306
9	0.03671	0.02310	0.02693	0.02671	0.02677	0.02667	0.02793	0.03250	0.02260
10	0.05701	0.02670	0.03068	0.02802	0.02952	0.04119	0.02886	0.02540	0.03526

TABLE 6-3

Estimated Item Discrimination Parameter, $\hat{a}_g$, Obtained from the Three Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		"PSEUDO" CRITERION			SSP
ITEM	TRUE	DGR.3	DGR.4	DGR.5	CRITERION
1	1.5	2.113_5	1.978_5	2.014_5	1.400_5
2	1.0	1.013	1.040	1.031	1.024
3	2.5	1.707	1.711	1.710	1.788
4	1.0	0.896	0.902	0.902	0.868
5	1.5	1.635	1.651	1.643	1.368
6	1.0	0.760	0.779	0.779	0.895
7	2.0	1.530	1.537	1.537	1.473
8	1.0	0.880	0.910	0.910	0.886
9	2.0	1.775	1.714	1.723	1.716
10	1.0	0.921	0.888	0.898	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-4

Estimated Item Difficulty Parameter, $\hat{b}_g$, Obtained from the Three Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based On the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		"PSEUDO" CRITERION			SSP
ITEM	TRUE	DGR.3	DGR.4	DGR.5	CRITERION
1	-2.5	-2.354_5	-2.385_5	-2.379_5	-2.651_5
2	-2.0	-2.046	-2.009	-2.019	-2.002
3	-1.5	-1.491	-1.483	-1.483	-1.507
4	-1.0	-0.954	-0.950	-0.952	-1.005
5	-0.5	-0.503	-0.499	-0.497	-0.472
6	0.0	-0.100	-0.104	-0.105	-0.075
7	0.5	0.621	0.615	0.615	0.527
8	1.0	1.022	1.001	1.001	0.981
9	1.5	1.486	1.488	1.488	1.502
10	2.0	2.151	2.191	2.170	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

The estimated item characteristic functions in Degree 3 and 4 Cases of the Pearson System Method for each of the ten binary items are presented in Figure 6-2, in comparison with the true item characteristic function, the actual frequency ratios of θ , and the criterion item characteristic function by the Simple Sum Procedure. Just like the case of their corresponding pseudo item characteristic functions, these two curves for each item are almost identical with each other for the interval of θ , (-2.5, 2.5). In addition to this, the comparison of Figure 6-2 with Figure 6-1 makes us realize that these two curves are practically the same as those of the Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases. We recall that the same finding resulted between the set of estimated item characteristic functions obtained by the Pearson System Method, and also those by the Normal Approach Method and the Two-Parameter Beta Method as well, and the criterion item characteristic function, for each binary item, when we applied the Simple Sum Procedure of the Conditional P.D.F. Approach (cf. Samejima, 1978a, 1978b). The mean square errors and their square roots to evaluate these estimated item characteristic functions, which were obtained in the same way that was described earlier, using (6.1), are presented in Tables 6-1 and 6-2, in the columns titled PSM (Pearson System Method) DGR. 3 DGR. 4 . A close examination of these numbers confirms that they are very close to the corresponding values for the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 Cases. It should be added that the comparison of the present results with those obtained by the criterion item characteristic functions by the Simple Sum Procedure also provides us with the comparison with those obtained by the Pearson System Method in the Simple Sum Procedure, since, as was mentioned earlier, the latter two sets of results are practically identical.

Tables 6-5 and 6-6 present the estimated item parameters, $\hat{a}_g$ and $\hat{b}_g$, respectively. These values were obtained in the same manner as described for the pseudo criterion item characteristic functions. We can see that these estimated parameters are as good as those obtained from the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 Cases, and also those obtained from the criterion item characteristic functions by the Simple Sum Procedure. Again, the estimated difficulty parameters, $\hat{b}_g$, turned out to be closer to the true parameter values, compared with the estimated discrimination parameters, $\hat{a}_g$. Also there is a tendency that for large values of the discrimination parameters, like those of items 3, 7 and 9, the estimated parameter values are smaller than the true parameter values.

It is interesting to note that some estimated parameter values are even closer to the true values than those obtained from the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 Cases. We should not put an emphasis on this fact, however, since the differences are very small.

From the observations made so far, we can conclude that Pearson System Method in the Weighted Sum Procedure with the weight described in Section 3 works as accurately as the one in the Simple Sum Procedure, in both Degree 3 and 4 Cases, as far as the present simulated data are concerned.

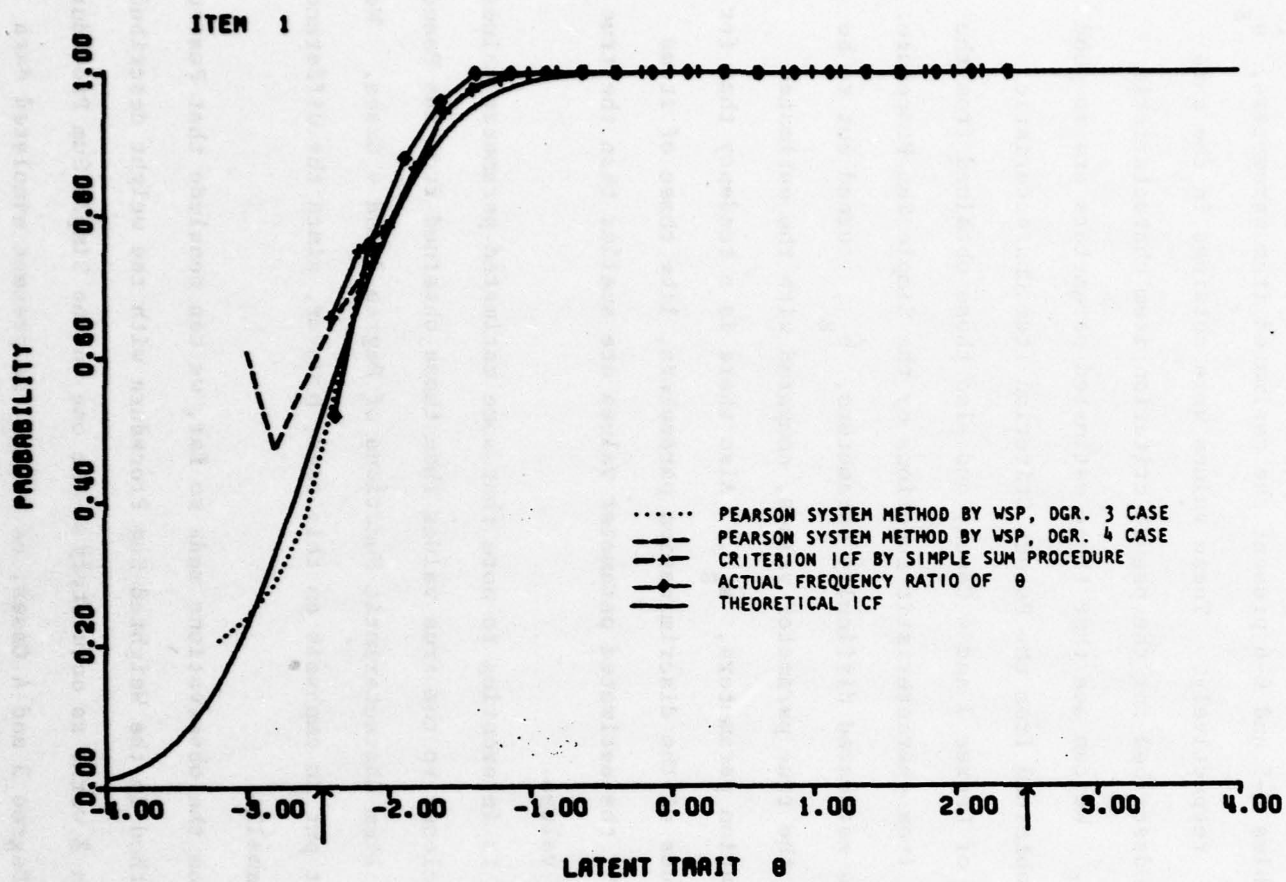

FIGURE 6-2

Comparison of the Two Estimated Item Characteristic Functions of Degree 3 and 4 Cases by the Pearson System Method of the Weighted Sum Procedure with the Criterion Item Characteristic Function by the Simple Sum Procedure. Actual Frequency Ratios and Theoretical Item Characteristic Function Are Also Presented for Comparison.

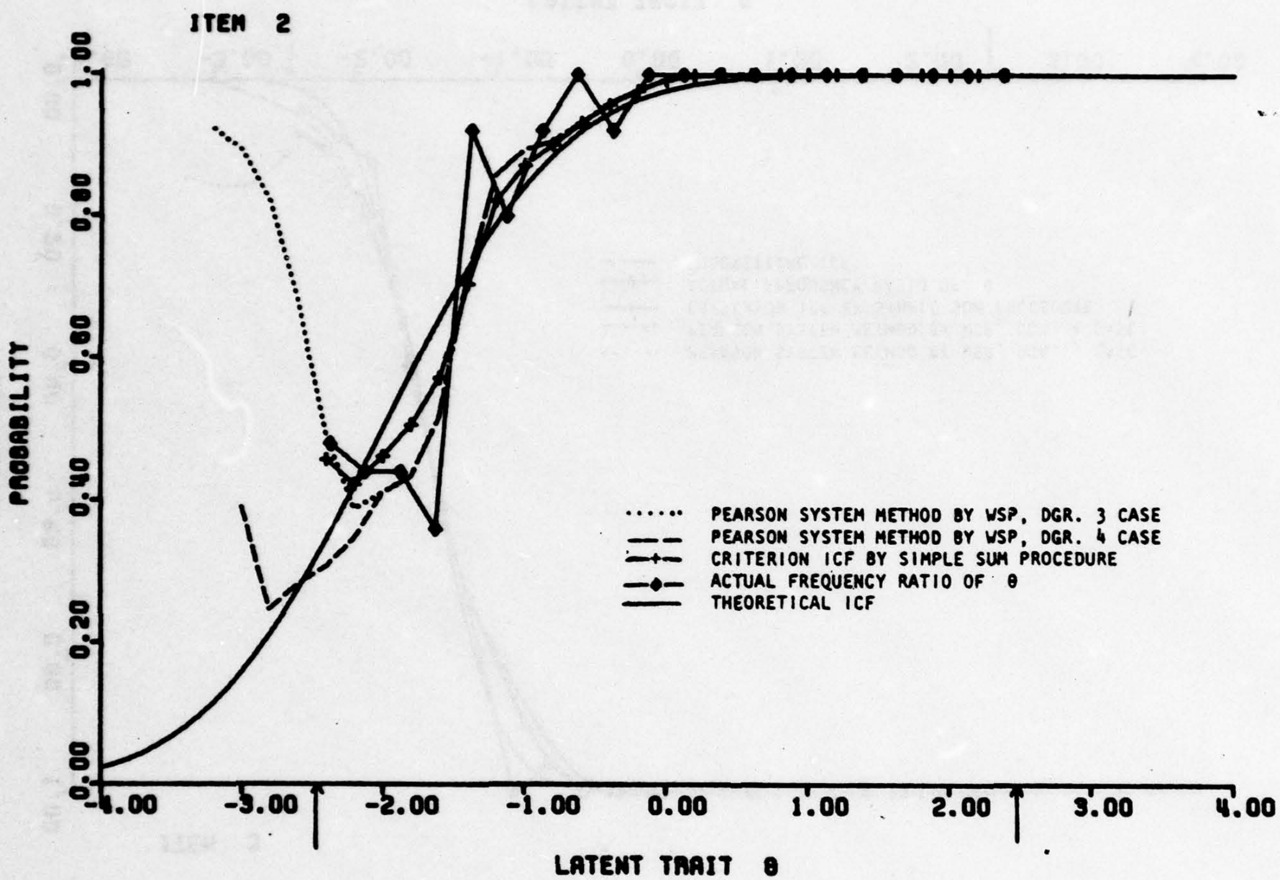

FIGURE 6-2: Pearson System Method (Continued)

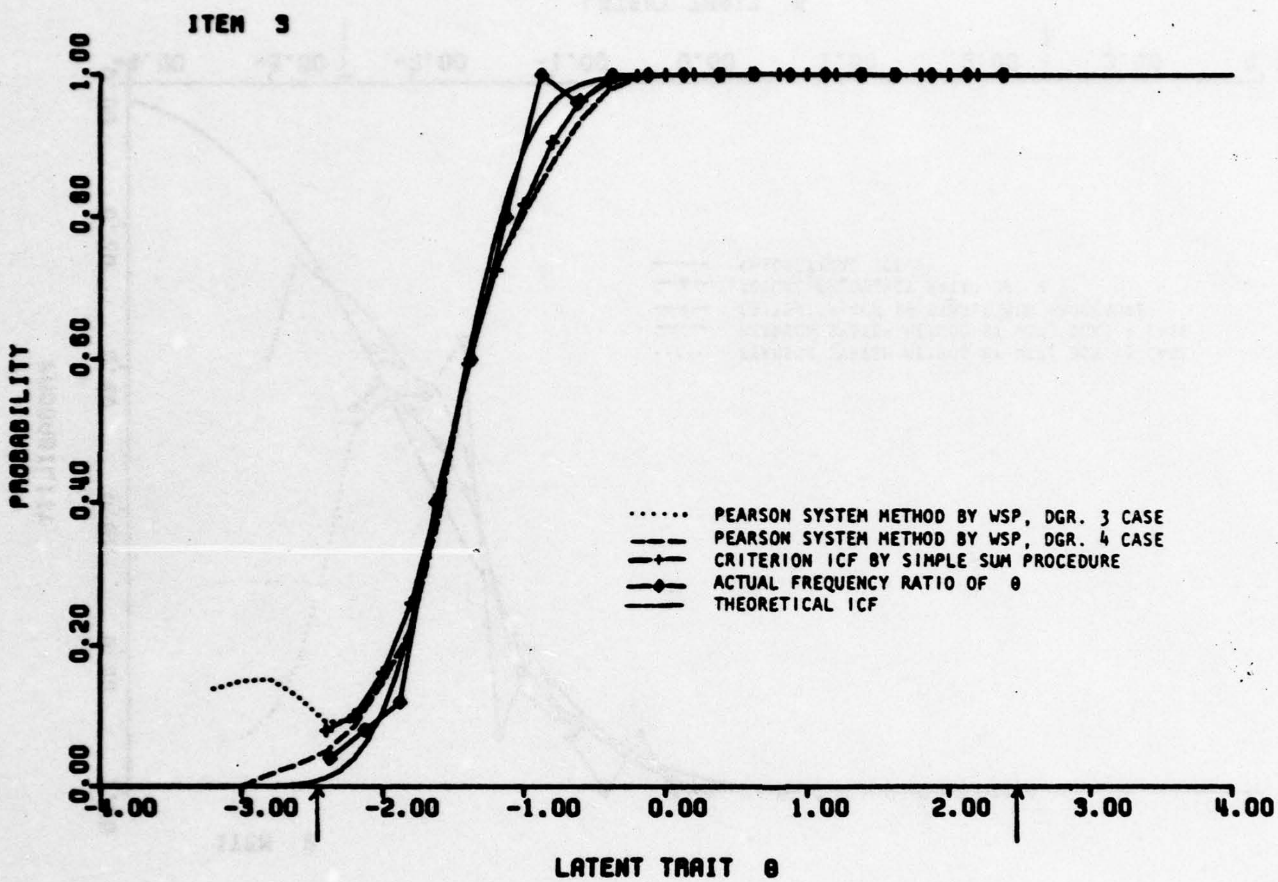

FIGURE 6-2: Pearson System Method (Continued)

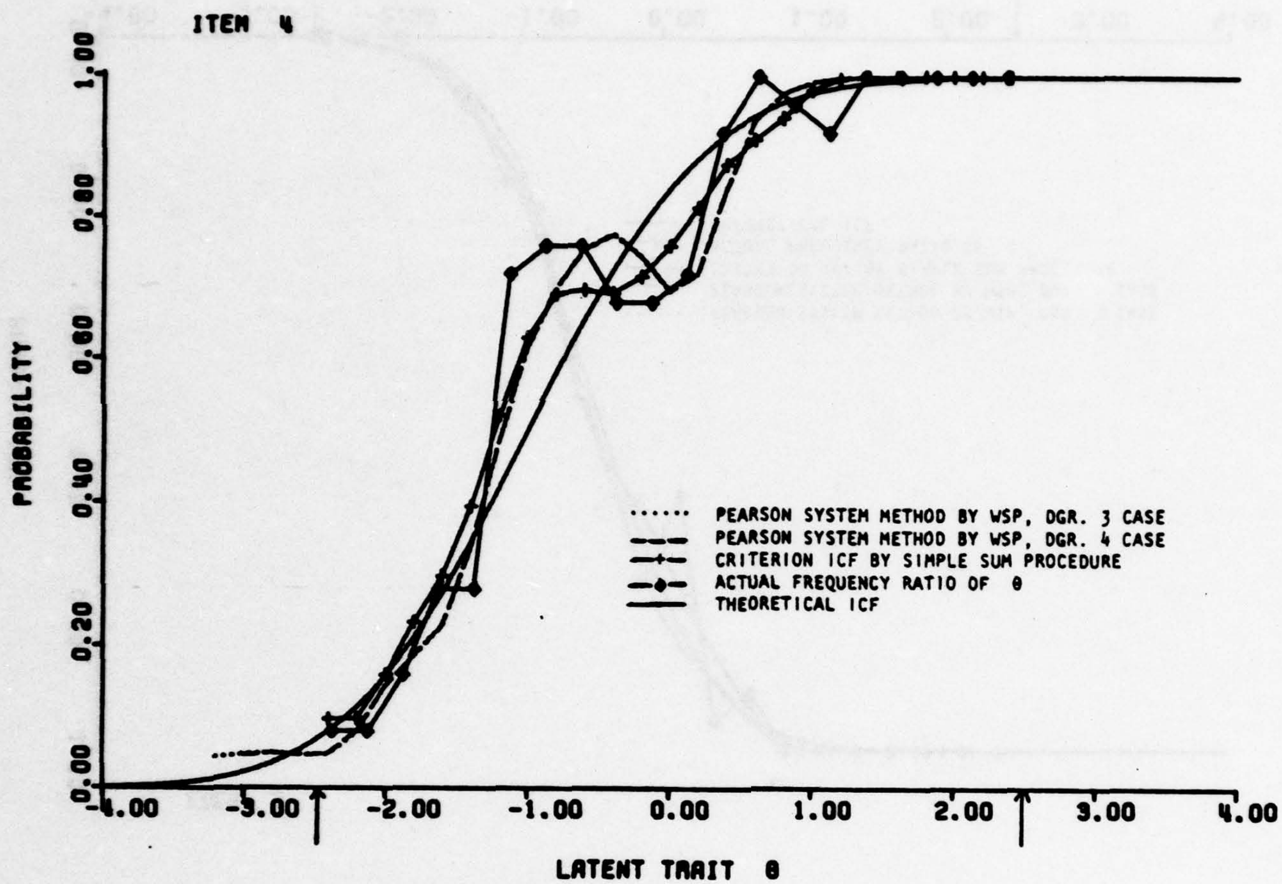

FIGURE 6-2: Pearson System Method (Continued)

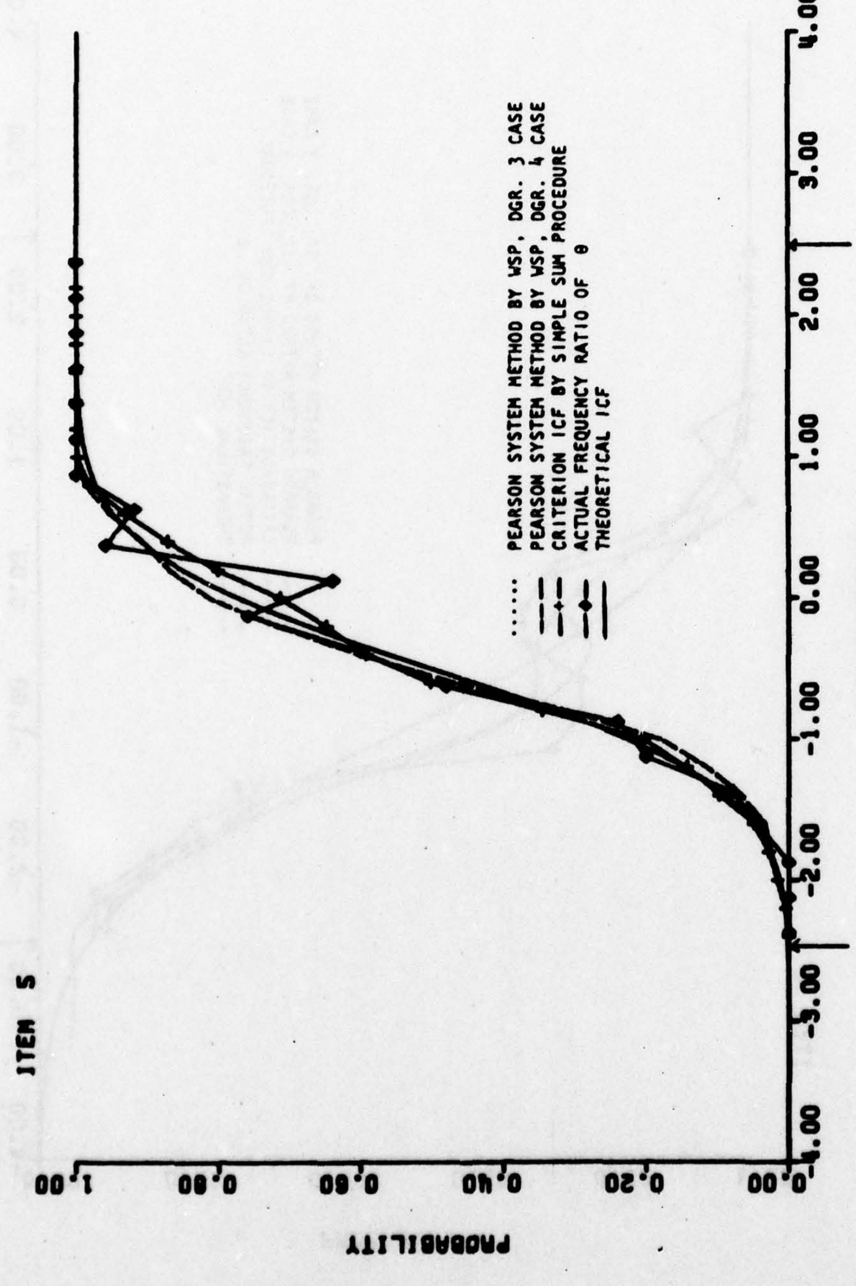

FIGURE 6-2: Pearson System Method (Continued)

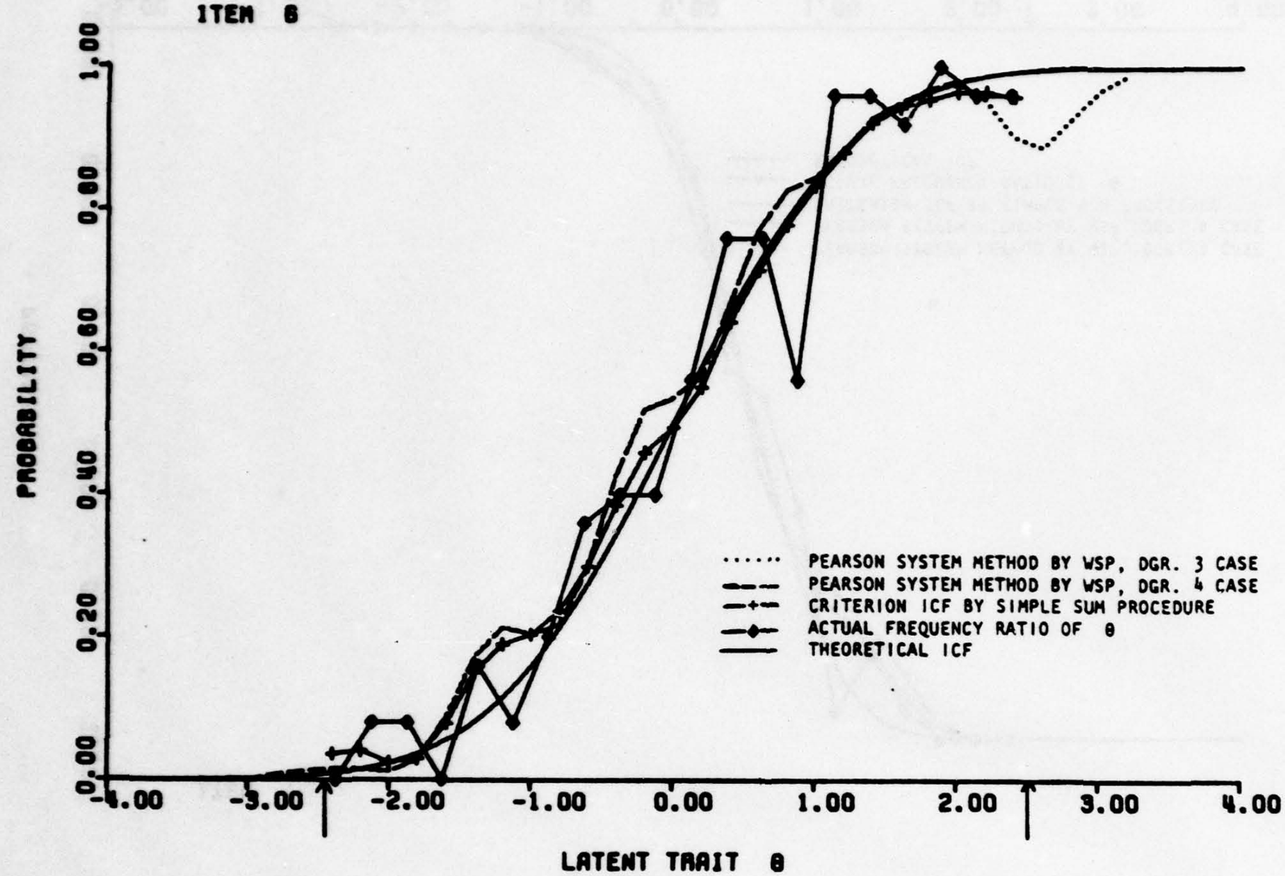

FIGURE 6-2: Pearson System Method (Continued)

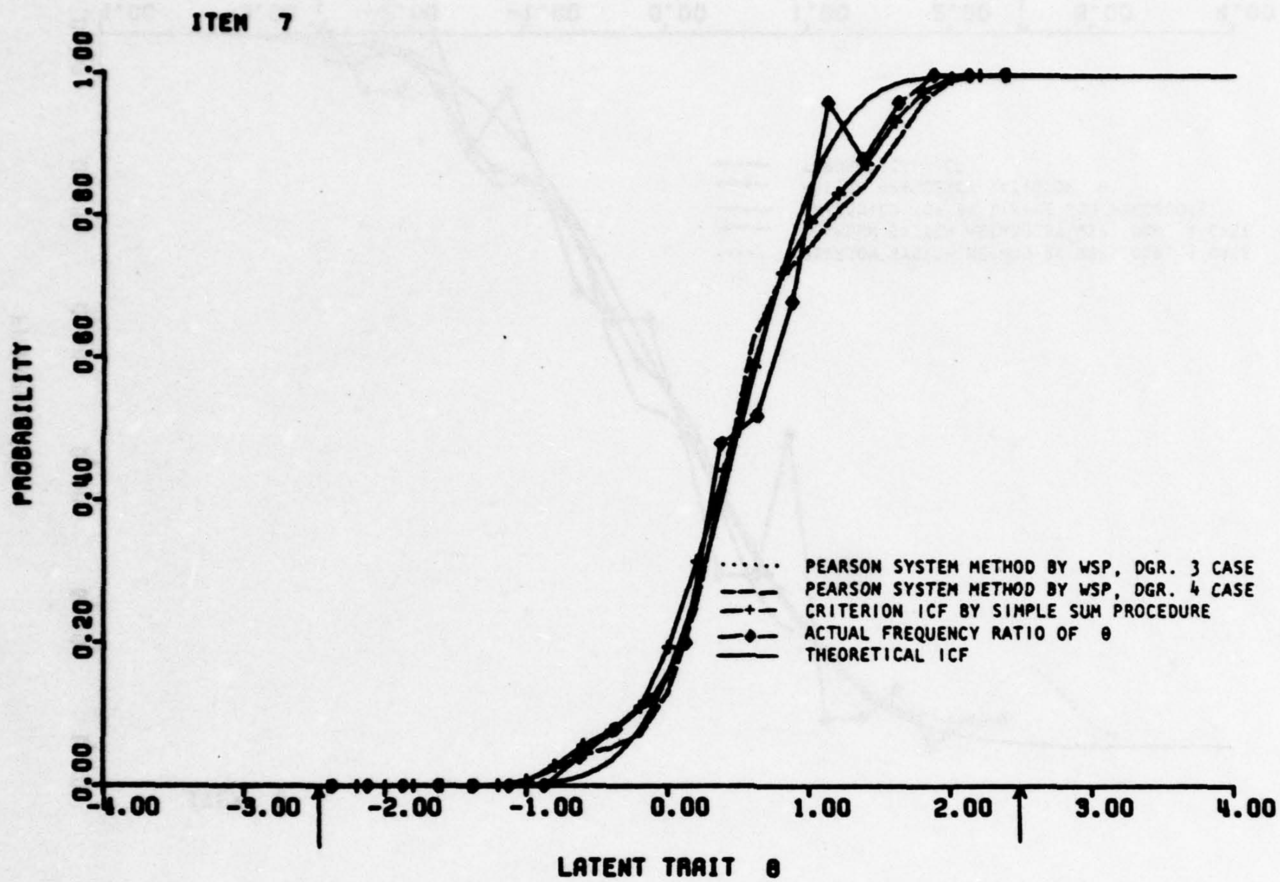

FIGURE 6-2: Pearson System Method (Continued)

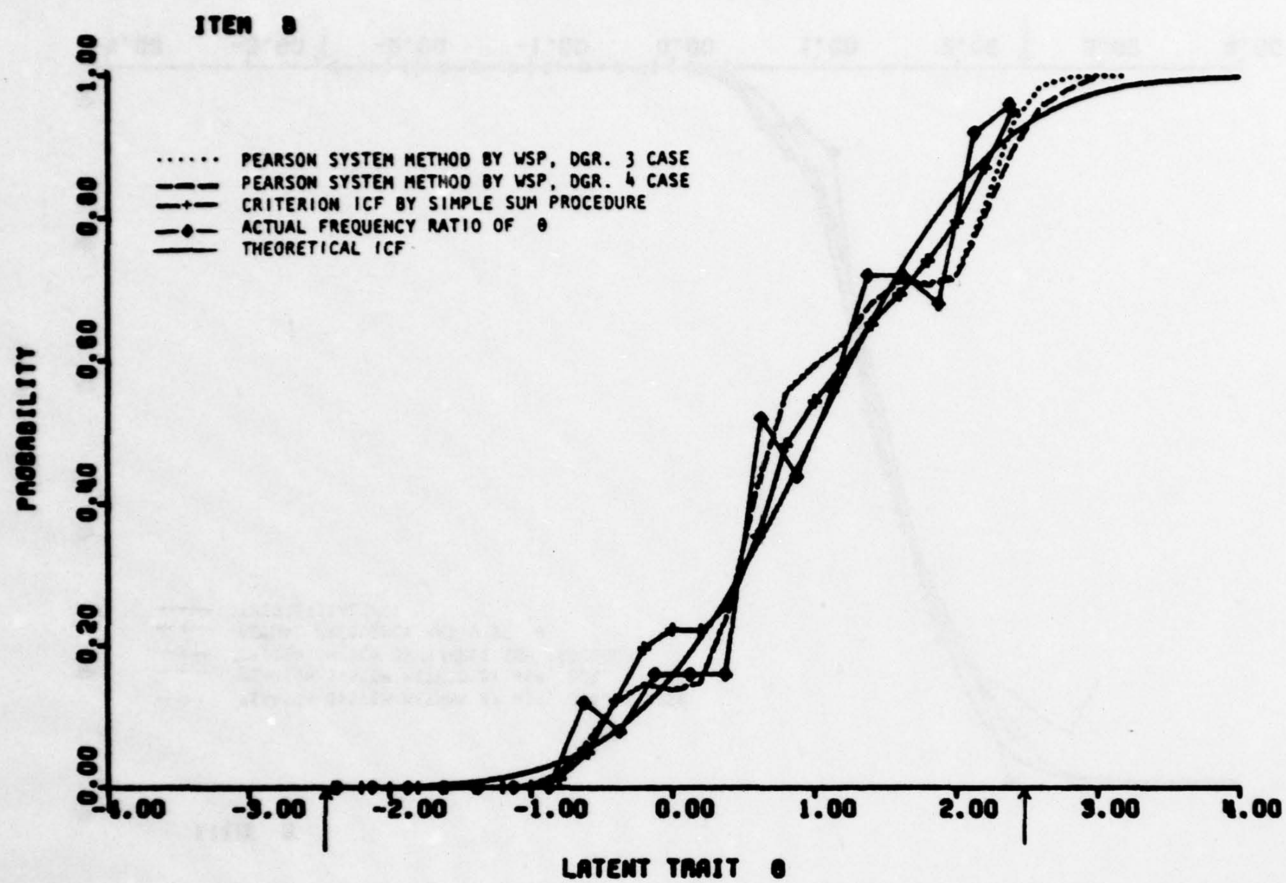

FIGURE 6-2: Pearson System Method (Continued)

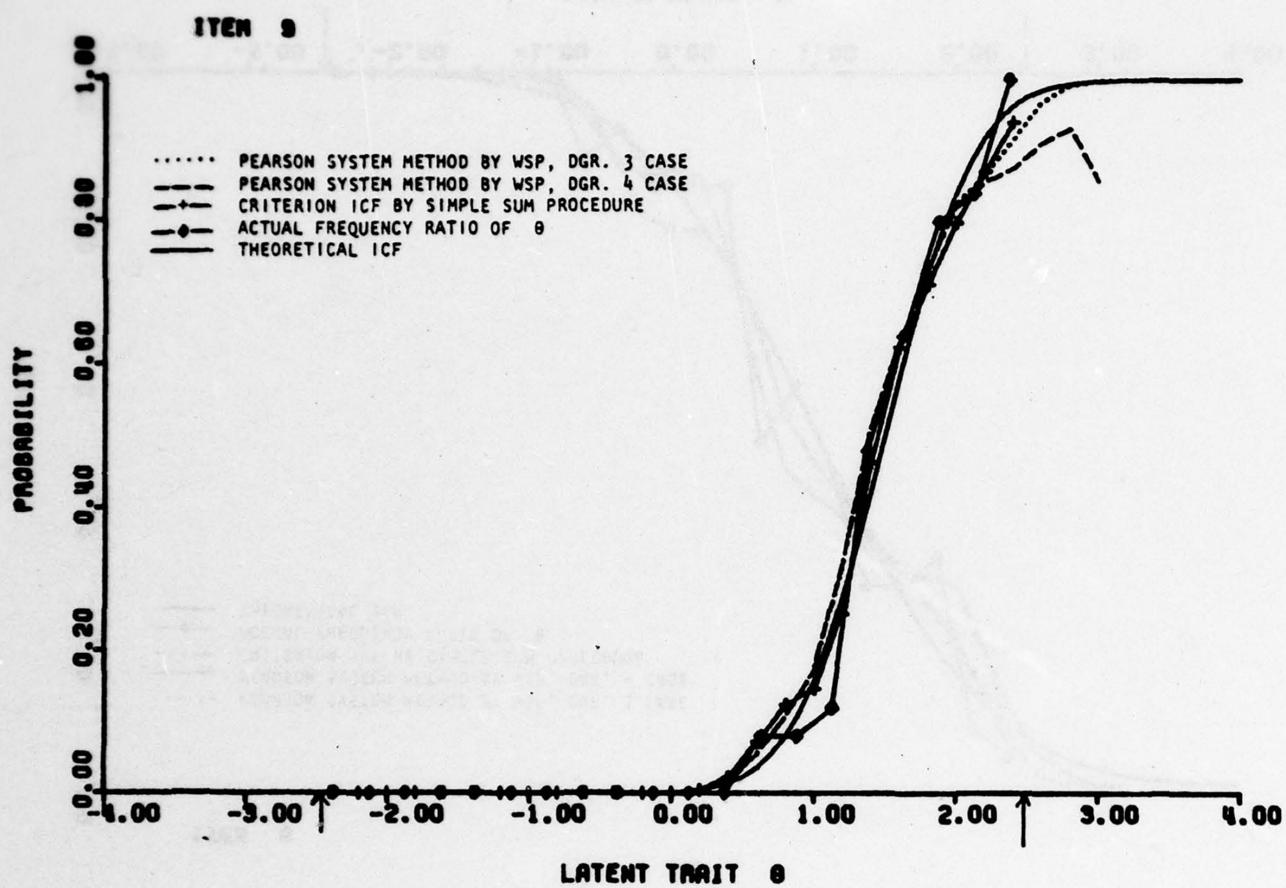

FIGURE 6-2: Fearson System Method (Continued)

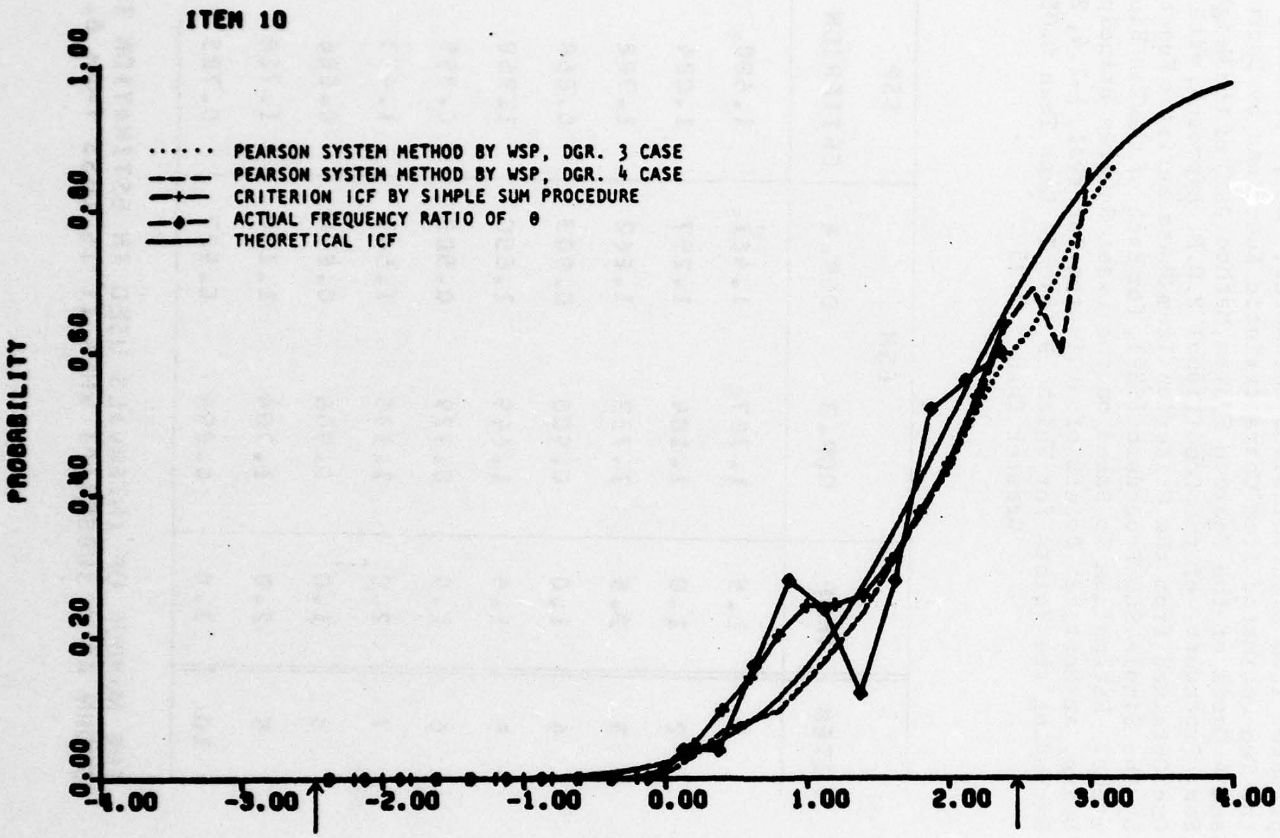

FIGURE 6-2: Pearson System Method (Continued)

TABLE 6-5

Estimated Item Discrimination Parameter, $\hat{a}_g$, Obtained from the Two Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Pearson System Method (PSM) of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4] Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95 .

		PSM		SSP
ITEM	TRUE	DGR.3	DGR.4	CRITERION
1	1.5	1.787_5	1.461_5	1.400_5
2	1.0	1.104	1.257	1.024
3	2.5	1.733	1.862	1.788
4	1.0	0.906	0.903	0.868
5	1.5	1.649	1.650	1.368
6	1.0	0.779	0.903	0.895
7	2.0	1.539	1.541	1.473
8	1.0	0.906	0.876	0.886
9	2.0	1.704	1.630	1.716
10	1.0	0.894	0.927	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-6

Estimated Item Difficulty Parameter, $\hat{b}_g$, Obtained from the Two Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Pearson System Method (PSM) of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less than 0.05 or Greater Than 0.95 .

		PSM		SSP
ITEM	TRUE	DGR.3	DGR.4	CRITERION
1	-2.5	-2.442_5	-2.594_5	-2.651_5
2	-2.0	-1.957	-1.854	-2.002
3	-1.5	-1.475	-1.453	-1.507
4	-1.0	-0.945	-0.948	-1.005
5	-0.5	-0.498	-0.499	-0.472
6	0.0	-0.103	-0.148	-0.075
7	0.5	0.616	0.615	0.527
8	1.0	1.001	1.027	0.981
9	1.5	1.491	1.507	1.502
10	2.0	2.178	2.140	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

Figure 6-3 presents the estimated item characteristic functions of the ten binary items obtained by the Two-Parameter Beta Method of the Weighted Sum Procedure of the Conditional P.D.F. Approach, for both Degree 3 and 4 Cases, along with the criterion item characteristic functions by the Simple Sum Procedure, the actual frequency ratios of θ , and the theoretical item characteristic functions. We can see that, although there are some discrepancies around the endpoints, the two curves provided by Degree 3 and 4 Cases are almost the same for the interval of θ , (-2.5, 2.5), for which the theoretical density function, $f(\theta)$, assumes the positive constant, 0.2 . Also a close comparison of Figure 6-3 with Figure 6-1 reveals that the two estimated item characteristic functions by the Two-Parameter Beta Method are practically identical with the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4, for every binary item. The relationship between the present results and those obtained by the Two-Parameter Beta Method of the Simple Sum Procedure of the Conditional P.D.F. Approach is, therefore, very similar to the one between the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 and the criterion item characteristic functions by the Simple Sum Procedure, which was described earlier in this section.

The mean square errors and their square roots are presented in Tables 6-1 and 6-2 under the title TPBM (Two-Parameter Beta Method) DGR. 3 DGR. 4 , which were calculated in the same manner as described earlier. We can see that these values are very close to the corresponding values for the Pearson System Method, and those for the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 Cases, which are shown in the same tables.

Tables 6-7 and 6-8 present the estimated discrimination parameters, $\hat{a}_g$, and difficulty parameters, $\hat{b}_g$, respectively, for the ten binary items, together with the true parameters and their estimates from the criterion item characteristic functions by the Simple Sum Procedure. Again these results are very similar to those obtained by the Pearson System Method, and also those from the Pseudo Criterion Item Characteristic Functions of Degree 3 and 4 Cases, as was expected. It is also observed that these estimated parameters are competitive with those obtained from the criterion item characteristic functions by the Simple Sum Procedure. This implies that they are competitive with those obtained by the Two-Parameter Beta Method in the Simple Sum Procedure as well. Again, it is observed that the estimated difficulty parameters are closer to the true difficulty parameters, than the estimated discrimination parameters to the true discrimination parameters. Also it is noticed that the estimated discrimination parameters tend to be less for high true discrimination parameter values, such as those of items 3, 7 and 9, as we have observed in the results of the Pearson System Method.

From these observations, we can make the same conclusion as we did for the Pearson System Method earlier in this section.

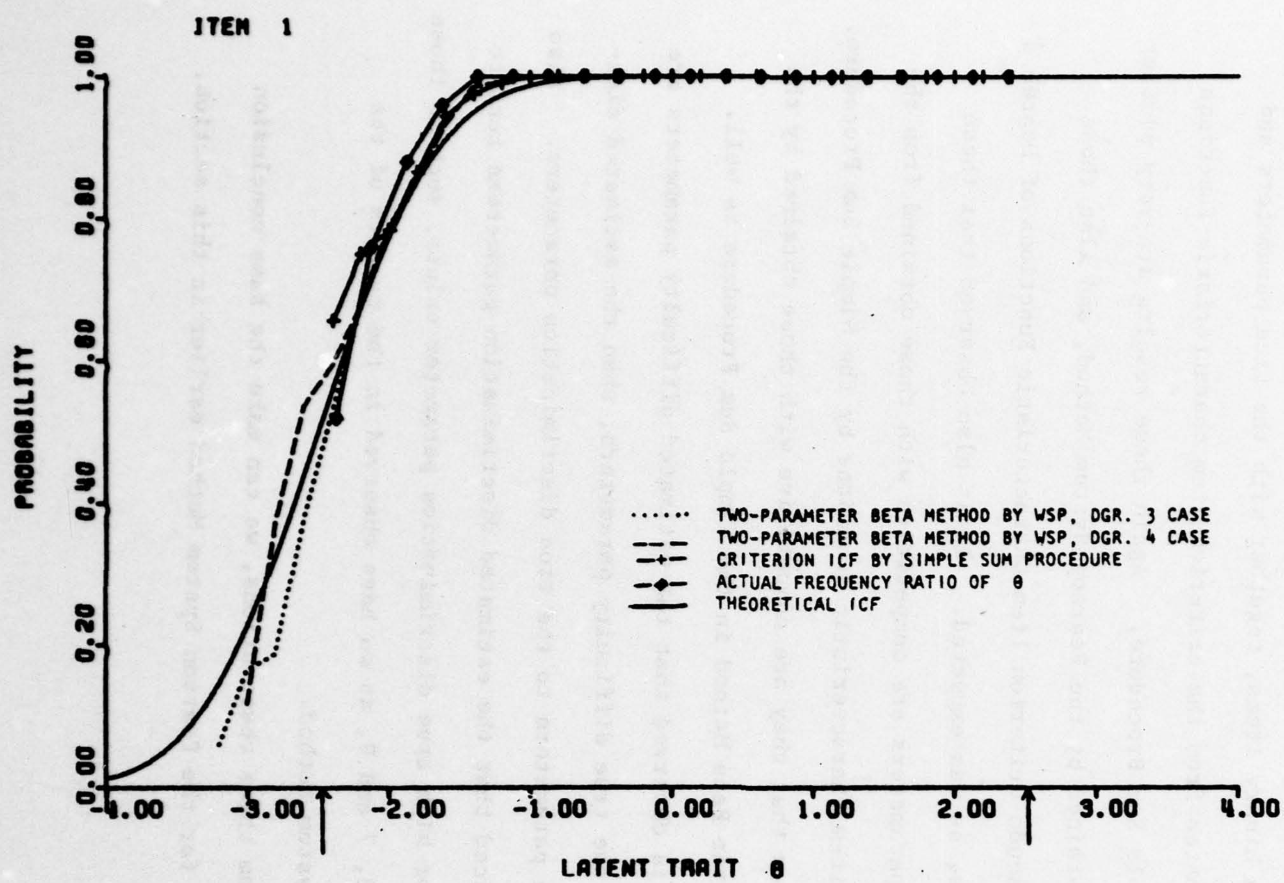

FIGURE 6-3

Comparison of the Two Estimated Item Characteristic Functions of Degree 3 and 4 Cases by the Two-Parameter Beta Method of the Weighted Sum Procedure with the Criterion Item Characteristic Function by the Simple Sum Procedure. Actual Frequency Ratios and Theoretical Item Characteristic Function Are Also Presented for Comparison.

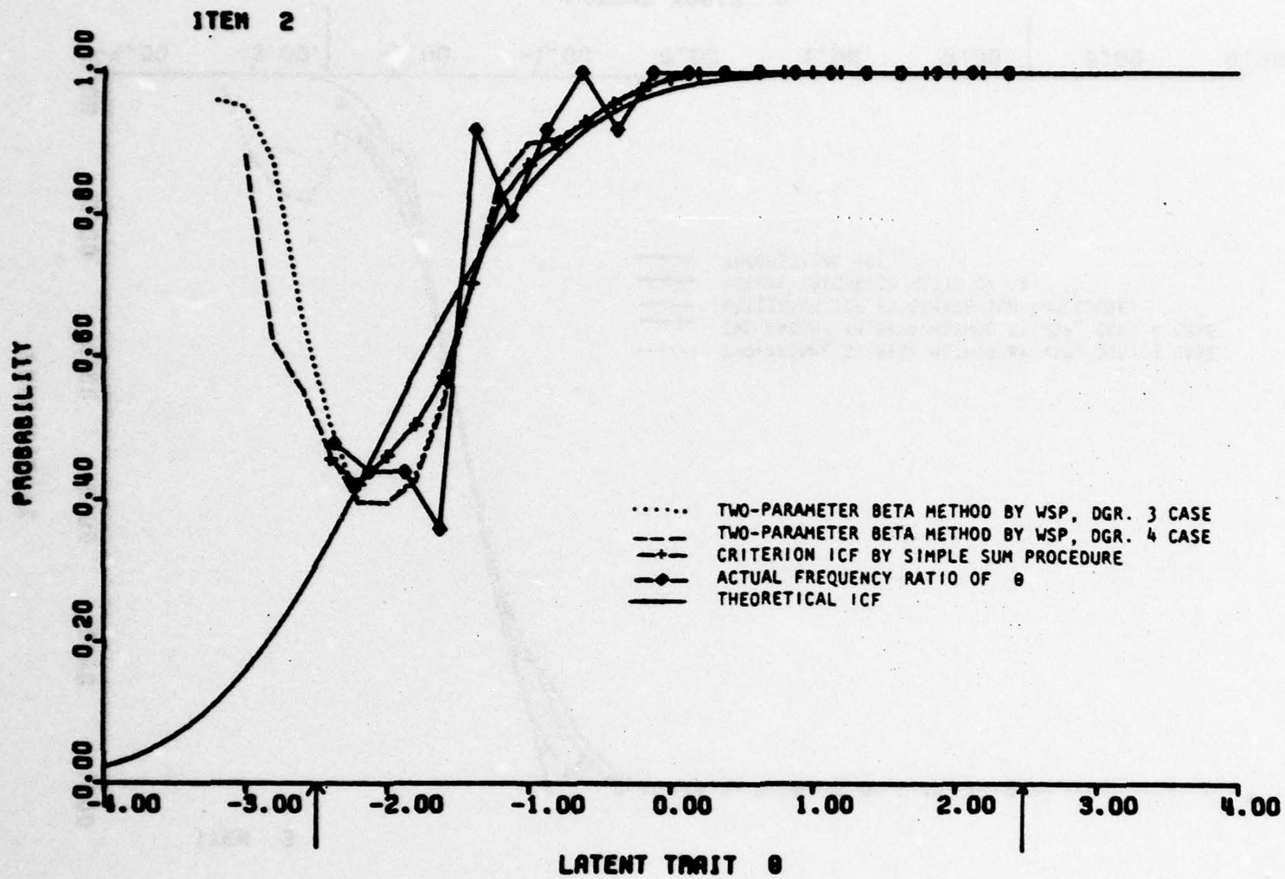

FIGURE 6-3: Two-Parameter Beta Method (Continued)

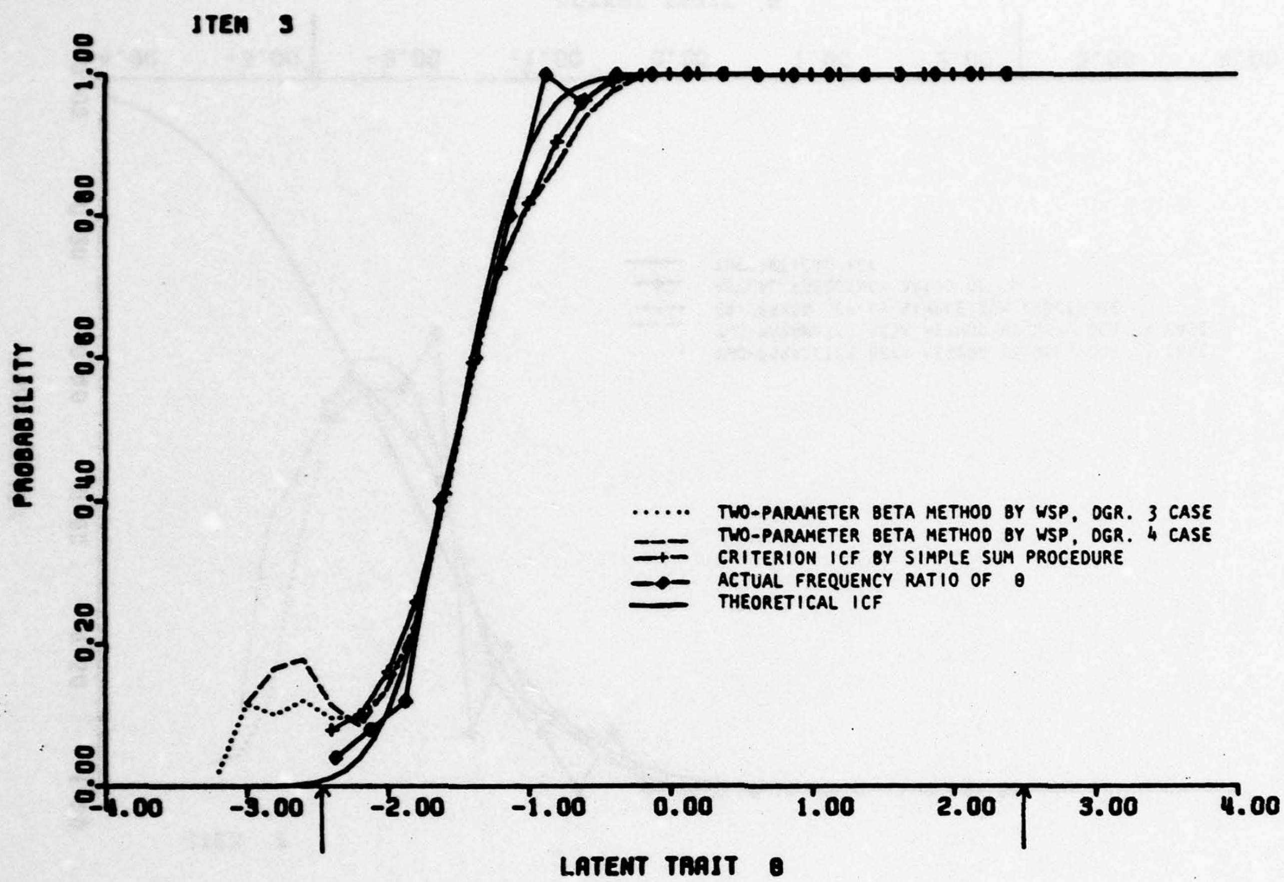

FIGURE 6-3: Two-Parameter Beta Method (Continued)

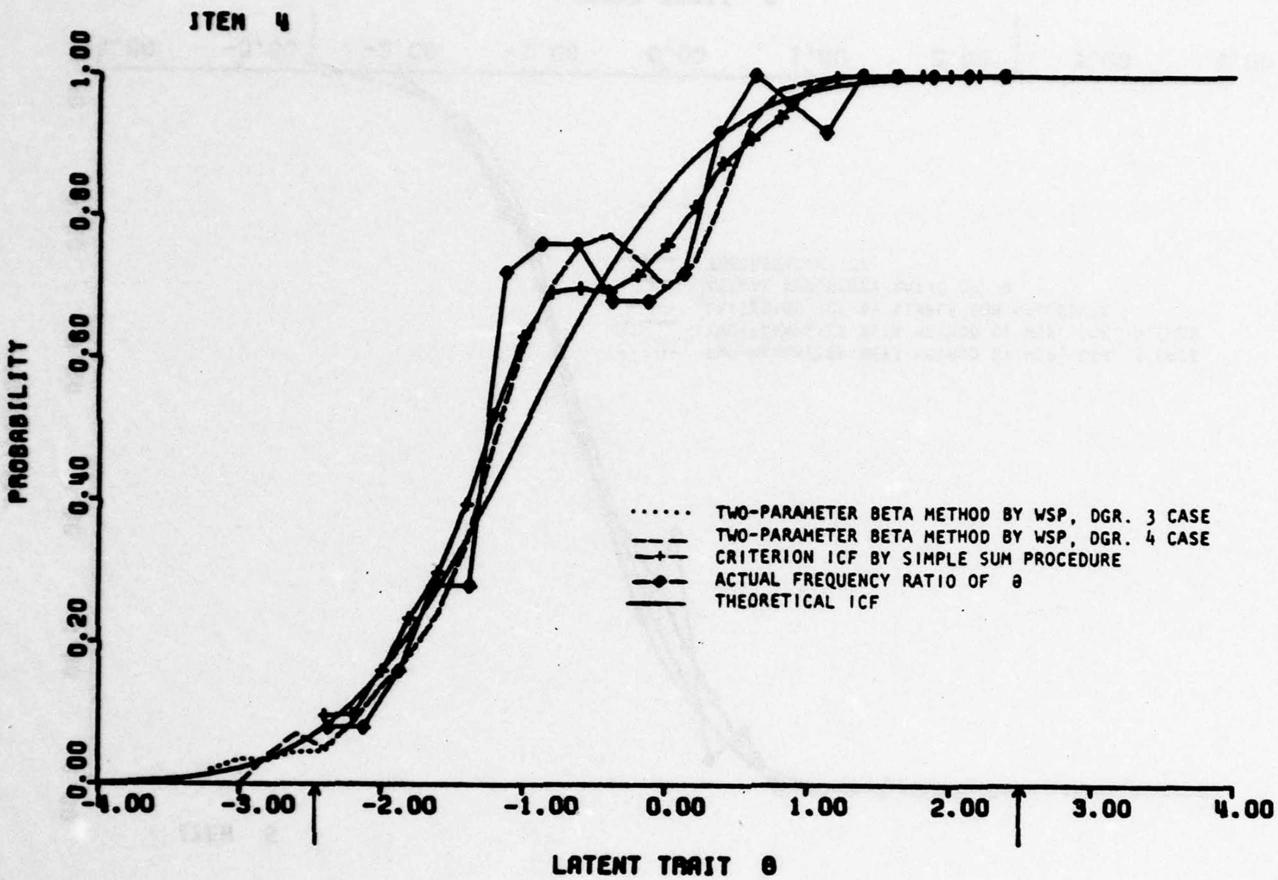

FIGURE 6-3: Two-Parameter Beta Method (Continued)

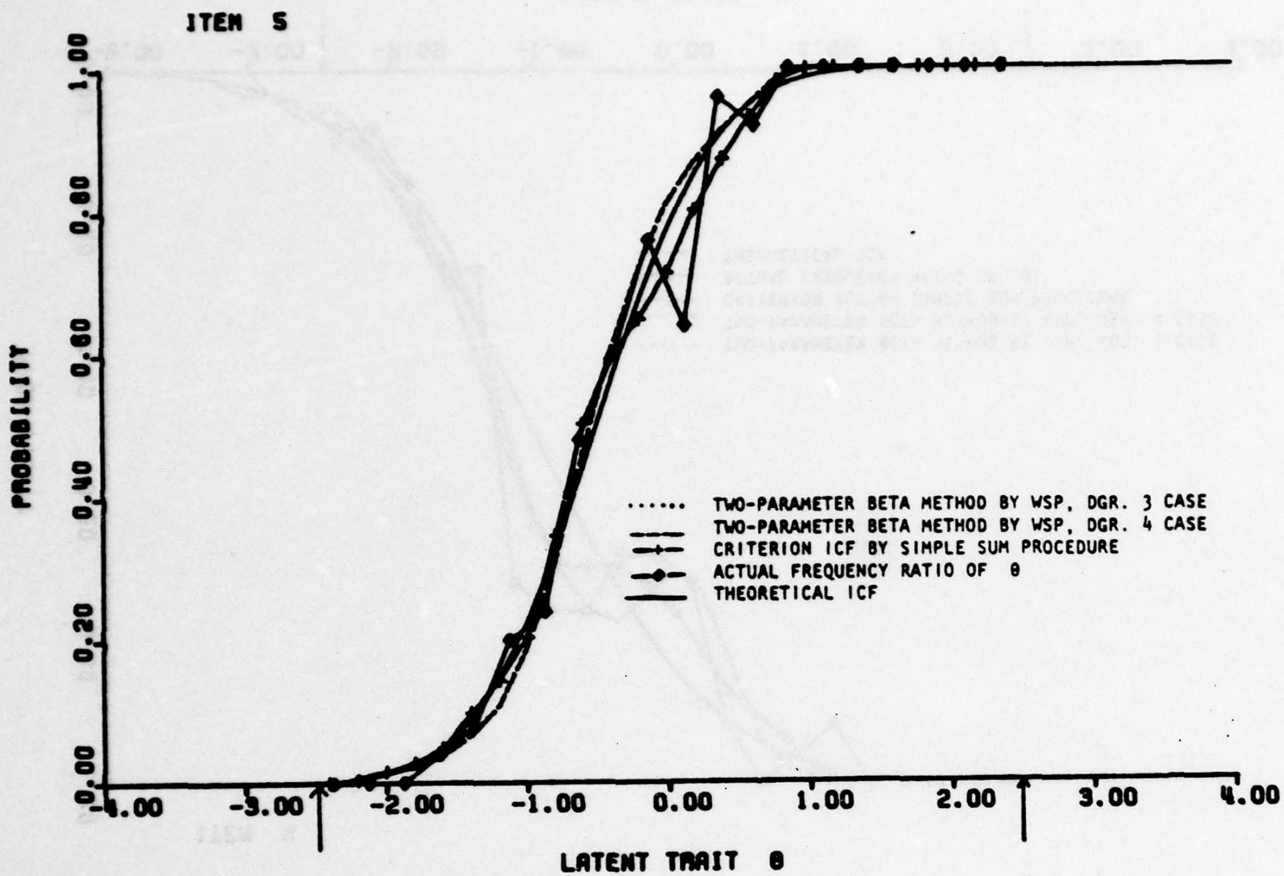

FIGURE 6-3: Two-Parameter Beta Method (Continued)

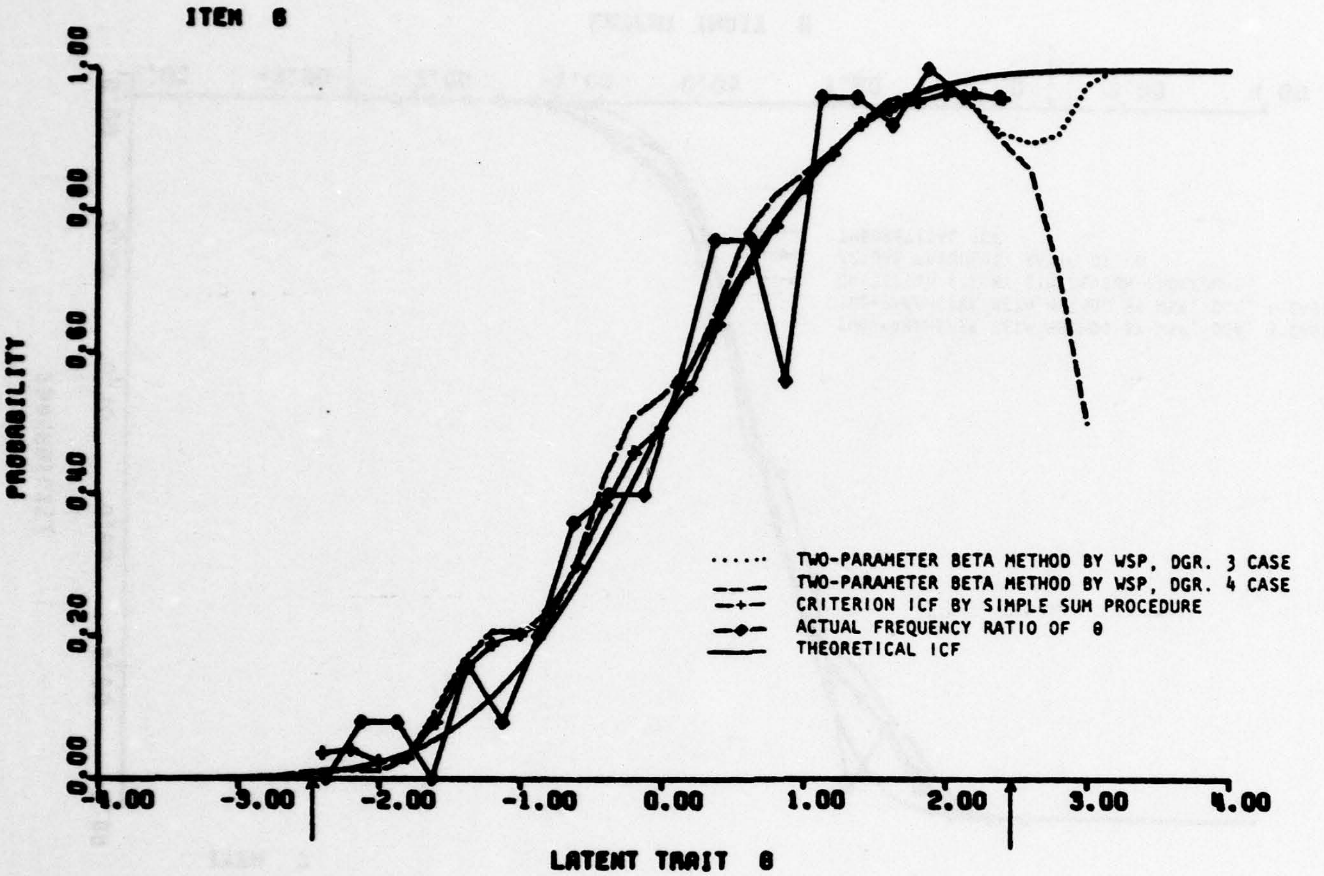

FIGURE 6-3: Two-Parameter Beta Method (Continued)

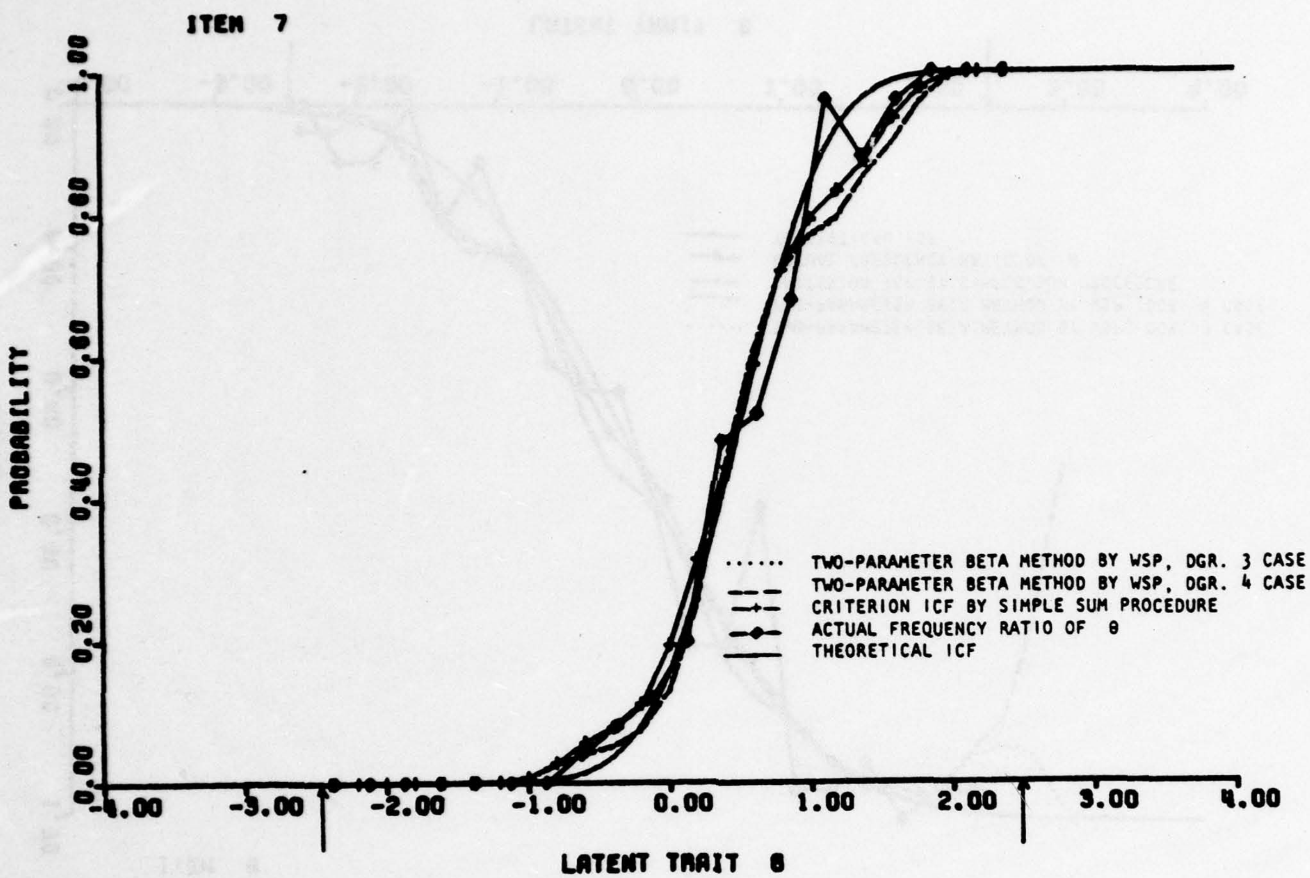

FIGURE 6-3: Two-Parameter Beta Method (Continued)

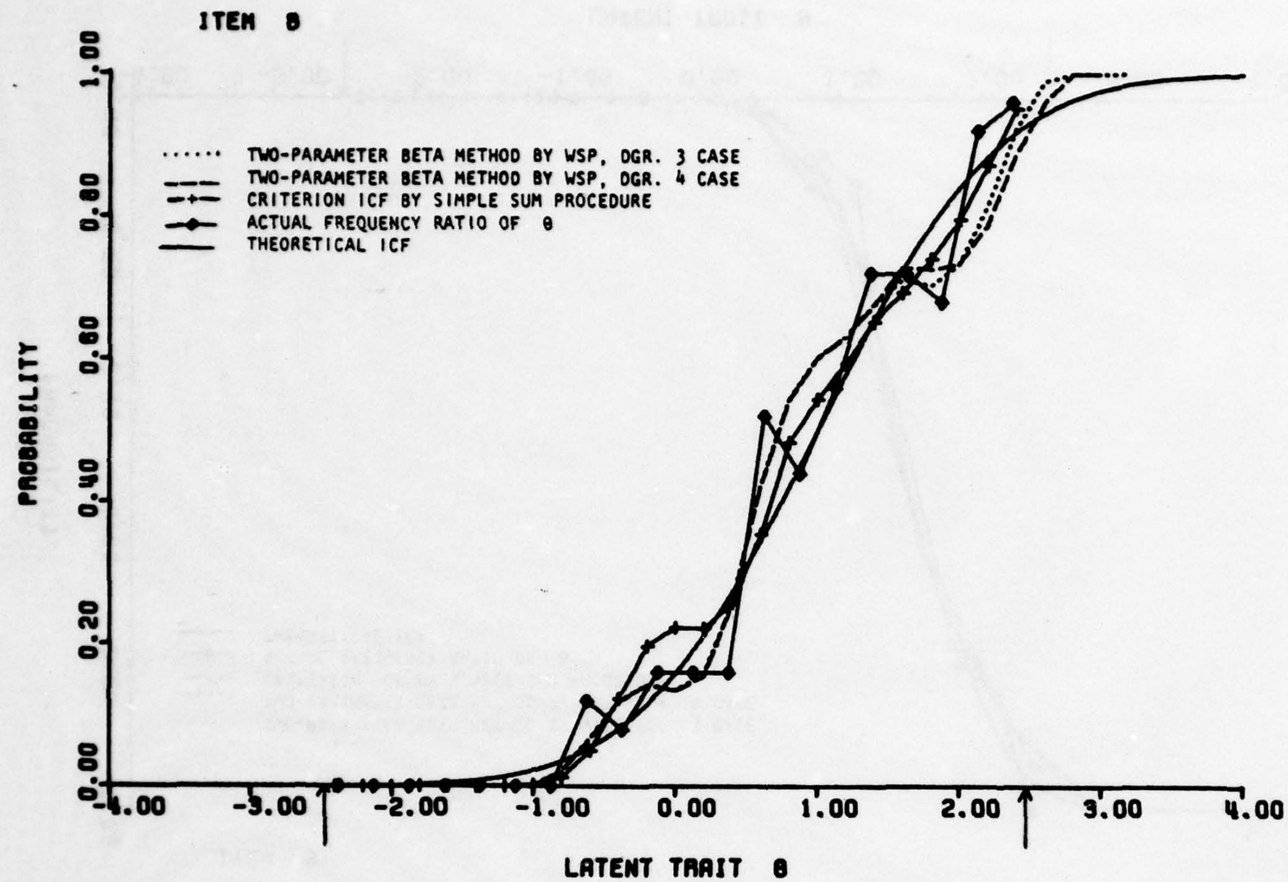

FIGURE 6-3: Two-Parameter Beta Method (Continued)

AD-A063 917 TENNESSEE UNIV KNOXVILLE DEPT OF PSYCHOLOGY F/G 12/1
ESTIMATION OF THE OPERATING CHARACTERISTICS OF ITEM RESPONSE CA--ETC(U)
DEC 78 F SAMEJIMA N00014-77-C-0360
UNCLASSIFIED RR-78-4 NL

2 OF 2
AD
A063917

END
DATE
FILMED
4 -79
DDC

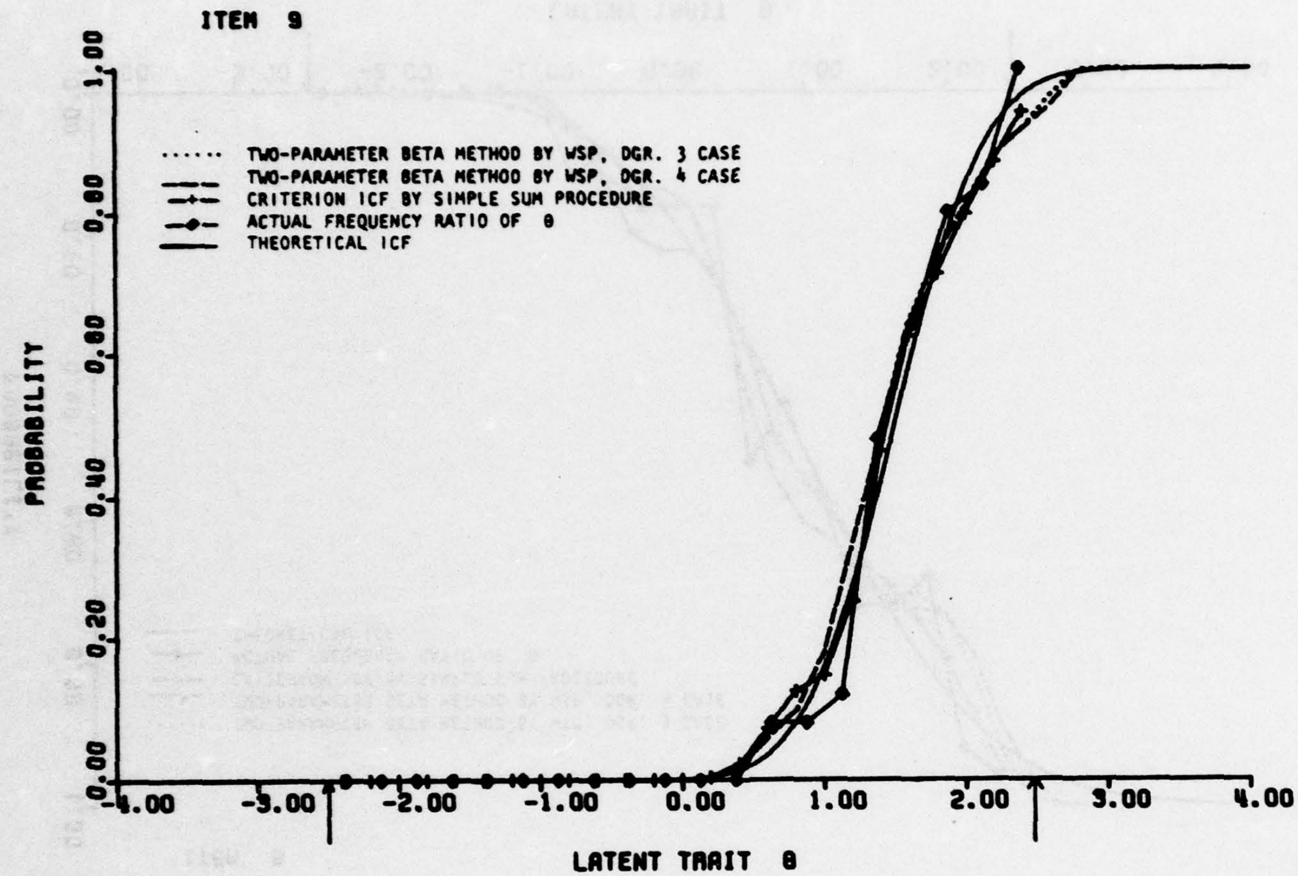

FIGURE 6-3: Two-Parameter Beta Method (Continued)

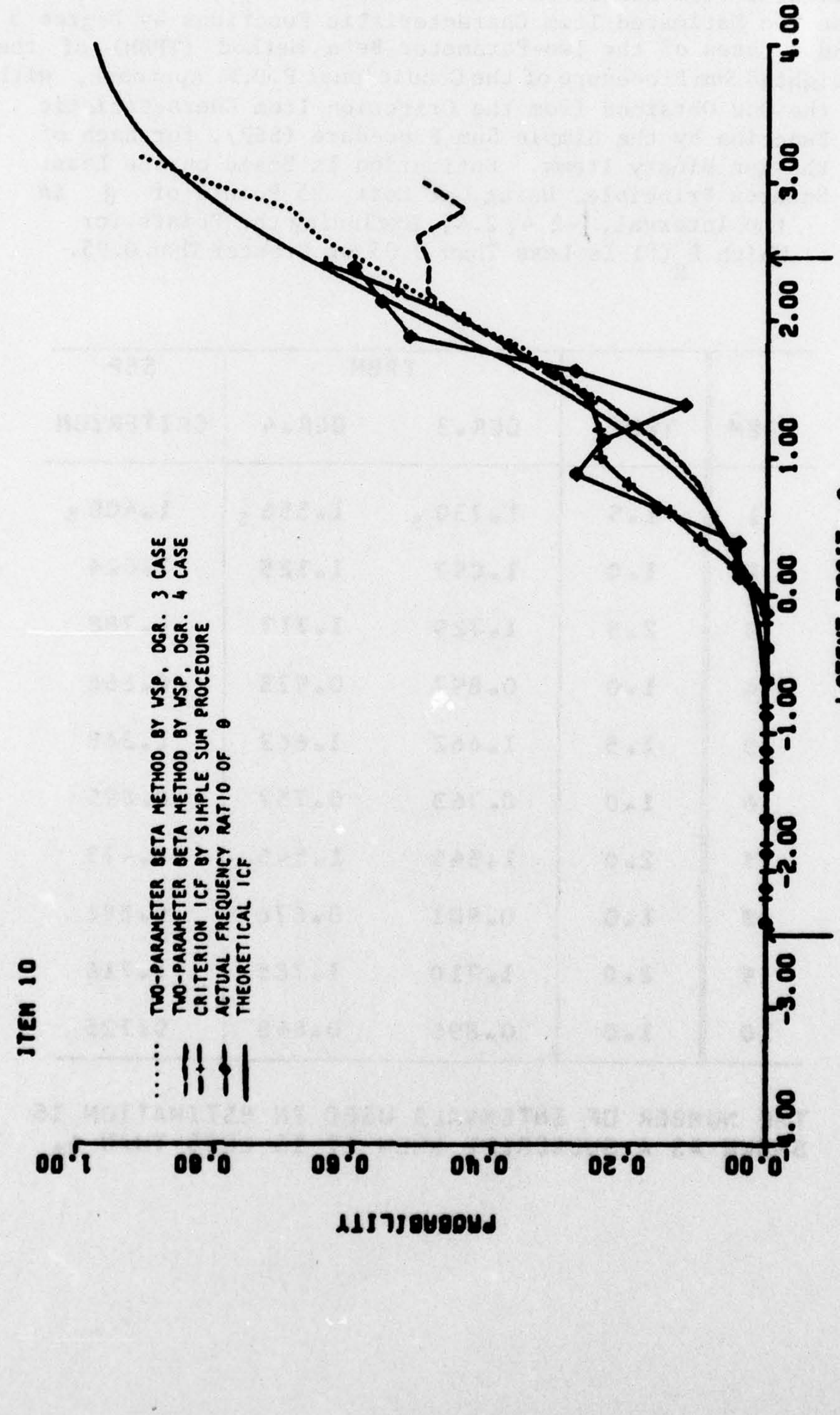

FIGURE 6-3: Two-Parameter Beta Method (Continued)

TABLE 6-7

Estimated Item Discrimination Parameter, $\hat{a}_g$, Obtained from the Two Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Two-Parameter Beta Method (TPBM) of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		TPBM		SSP
ITEM	TRUE	DGR.3	DGR.4	CRITERION
1	1.5	1.730_5	1.556_5	1.400_5
2	1.0	1.097	1.125	1.024
3	2.5	1.729	1.717	1.788
4	1.0	0.897	0.933	0.868
5	1.5	1.662	1.663	1.368
6	1.0	0.763	0.757	0.895
7	2.0	1.545	1.545	1.473
8	1.0	0.901	0.876	0.886
9	2.0	1.710	1.705	1.716
10	1.0	0.896	0.848	0.725

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

TABLE 6-8

Estimated Item Difficulty Parameter, $\hat{b}_g$, Obtained from the Two Estimated Item Characteristic Functions by Degree 3 and 4 Cases of the Two-Parameter Beta Method (TPBM) of the Weighted Sum Procedure of the Conditional P.D.F. Approach, with the One Obtained from the Criterion Item Characteristic Function by the Simple Sum Procedure (SSP), for Each of the Ten Binary Items. Estimation Is Based on the Least Squares Principle, Using, at Most, 25 Points of θ in the Interval, [-2.4, 2.4], Excluding the Points for Which $\hat{P}_g(\theta)$ Is Less Than 0.05 or Greater Than 0.95.

		TPBM		SSP
ITEM	TRUE	DGR.3	DGR.4	CRITERION
1	-2.5	-2.449_5	-2.522_5	-2.651_5
2	-2.0	-1.963	-1.939	-2.002
3	-1.5	-1.478	-1.483	-1.507
4	-1.0	-0.946	-0.922	-1.005
5	-0.5	-0.499	-0.500	-0.472
6	0.0	-0.097	-0.094	-0.075
7	0.5	0.615	0.614	0.527
8	1.0	1.004	1.022	0.981
9	1.5	1.490	1.491	1.502
10	2.0	2.181	2.246	2.118

THE NUMBER OF INTERVALS USED IN ESTIMATION IS SHOWN AS A SUBSCRIPT WHEN IT IS LESS THAN 6.

VII <u>Discussion and Conclusion</u>

A variation of the Conditional P.D.F. Approach was introduced, and called Weighted Sum Procedure, in contrast to the original one, which is now called Simple Sum Procedure. As the weight, an approximated proportion of the area under the density function of the maximum likelihood estimate $\hat{\theta}$, which is approximated by a polynomial of degree 3, 4 or 5, using the method of moments, was adopted. Pseudo Criterion Item Characteristic Functions of Degree 3, 4 and 5 Cases were defined as the indicator of the maximal possible attainment of the present method with the given data. Pearson System Method and Two-Parameter Beta Method are used to approximate the conditional density of ability θ , given its maximum likelihood $\hat{\theta}$. The estimated density functions of ability θ and the estimated shared density functions of θ by the success and failure groups of each of the ten binary items are observed, as well as the estimated item characteristic functions. Both the mean square errors, and their square roots, and the estimated item parameters were used in evaluating the resulting estimated item characteristic functions, and the former were also used in the evaluation of the estimated density functions of θ .

It is interesting to note that, in spite of the decline in accuracy of Degree 3 Case in estimating the density function of ability θ , it provided us with just as good estimated item characteristic functions as Degree 4 Case, in all the situations. This is the same finding that we obtained in the previous study in which the Simple Sum Procedure was adopted, and it looks as if the precision in approximating the density function of $\hat{\theta}$ were not very important. We should not jump to the conclusion, however, since these are the results on one set of data, and the use of different types of data may contradict this finding. It should also be pointed out that, in spite of the difference in approximating the conditional density $\phi(\theta|\hat{\theta})$, Pearson System Method and Two-Parameter Beta Method provided us with almost identical results. We should not generalize this fact too far, however, and wait until more results have been accumulated on different types of data.

From the present study, we cannot say which procedure, Simple Sum or Weighted Sum, gives a more accurate estimation than the other. At this moment, all we can say is that the present procedure has provided us with more varieties of methods which are useable in this type of estimation. They will be tested when we use these varieties of methods in estimating the operating characteristics of graded item response categories (Samejima, 1969, 1972), which are more complicated than the item characteristic functions of binary items. The concept of weakly parallel tests (Samejima, 1977c) will make it possible for us to compare the results obtained on different types of data directly.

REFERENCES

[1] Elderton, W. P. and N. L. Johnson. Systems of frequency curves. Cambridge University Press, 1969.

[2] Johnson, N. L. and S. Kotz. Continuous univariate distributions. Vol. 2. Houghton Mifflin, 1970.

[3] Samejima, F. Estimation of latent ability using a response pattern of graded scores. Psychometrika Monograph, No. 17, 1969.

[4] Samejima, F. A general model for free-response data. Psychometrika Monograph, No. 18, 1972.

[5] Samejima, F. Graded response model of the latent trait theory and tailored testing. Proceedings of the First Conference on Computerized Adaptive Testing, 1975, Civil Service Commission and Office of Naval Research, 1975, pages 5-17.

[6] Samejima, F. A use of the information function in tailored testing. Applied Psychological Measurement, 1, 1977a, pages 233-247.

[7] Samejima, F. A method of estimating item characteristic functions using the maximum likelihood estimate of ability. Psychometrika, 42, 1977b, pages 163-191.

[8] Samejima, F. Weakly parallel tests in latent trait theory with some criticisms of classical test theory. Psychometrika, 42, 1977c, pages 193-198.

[9] Samejima, F. Estimation of the Operating Characteristics of item response categories I: Introduction to the Two-Parameter Beta Method. Office of Naval Research, Research Report 77-1, 1977d.

[10] Samejima, F. Estimation of the operating characteristics of item response categories II: Further development of the Two-Parameter Beta Method. Office of Naval Research, Research Report 78-1, 1978a.

[11] Samejima, F. Estimation of the operating characteristics of item response categories III: The Normal Approach Method and the Pearson System Method. Office of Naval Research, Research Report 78-2, 1978b.

[12] Samejima, F. Estimation of the operating characteristics of item response categories IV: Comparison of the different methods. Office of Naval Research, Research Report 78-3, 1978c.

APPENDIX I

A-I Simulated Data

The simulated data used in the present study are characterized as follows.

(1) There are 500 hypothetical examinees.

(2) Their ability, or latent trait, distributes uniformly for the interval of θ , (-2.5, 2.5). Actually, we use 100 discrete points of θ , such as -2.475, -2.425, -2.375, -2.325, ..., 2.375, 2.425 and 2.475, i.e., the midpoints of the 100 subintervals with the width of 0.05, and at each point five examinees are located.

(3) There is a hypothetical test of 35 graded items, each of which has four item score categories, and which provides us with an approximately constant test information function, 21.63, for the interval of θ , [-3.0, 3.0], following the normal ogive model of the graded response level (Samejima, 1969, 1972). The test is called the Old Test, to distinguish from the New Test, which will be described later.

(4) Each of the 500 examinees is assumed to have taken the Old Test, and his response pattern on the 35 graded items has been calibrated by the Monte Carlo method. The score categories of each item are 0, 1, 2 and 3, and a typical response pattern looks like: (3,3,3,2,3,3,2,2,2,2,2,2,1,2,2,2,1,2,1,1,1,0,1,1, 1,0,1,0,1,1,0,0,0,0,0).

(5) From each response pattern, the maximum likelihood estimate of the examinee's ability has been obtained, using a computer program written for this purpose. In this process, out of 140 basic functions (Samejima, 1969, 1972), an appropriate set of 35 basic

functions are chosen depending upon the item scores in the response pattern, and, using the Newton-Ralphson procedure, the point of θ at which the sum total of these 35 basic functions equals zero is searched.

(6) There is another hypothetical test of 10 binary items, each of which follows the normal ogive model of the dichotomous response level. This is called the New Test.

(7) Each of the 500 examinees is assumed to have taken the New Test also, and his response pattern on the New Test has been calibrated by the Monte Carlo method. A typical response pattern looks like: (1,1,1,0,1,0,1,0,0,0).

(8) The item characteristic functions of the test items of the New Test are assumed to be unknown, and they are the target of estimation. Each method of estimation is evaluated by the the "closeness" of the resultant estimated item characteristic functions to the true item characteristic functions.

APPENDIX II

TABLE A-2-1

Weights, $W(\hat{\theta}_s)$, Assigned to the 500 Examinees Arranged in the Ascending Order of Their Maximum Likelihood Estimates, Which Are Used in the Weighted Sum Procedure of the Conditional P.D.F. Approach

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
1	2	-2.4250	-3.0555	0.0052	0.0	0.0
2	201	-2.4750	-2.7417	0.0115	0.0120	0.0130
3	101	-2.4750	-2.7417	0.0013	0.0011	0.0011
4	4	-2.3250	-2.7061	0.0014	0.0013	0.0012
5	401	-2.4750	-2.7057	0.0003	0.0003	0.0002
6	1	-2.4750	-2.6589	0.0013	0.0012	0.0012
7	3	-2.3750	-2.6723	0.0053	0.0051	0.0049
8	104	-2.3250	-2.5688	0.0077	0.0084	0.0080
9	102	-2.4250	-2.5074	0.0041	0.0046	0.0044
10	302	-2.4250	-2.4875	0.0016	0.0019	0.0018
11	109	-2.0750	-2.4760	0.0008	0.0009	0.0009
12	202	-2.4250	-2.4726	0.0005	0.0005	0.0005
13	105	-2.2750	-2.4672	0.0012	0.0014	0.0013
14	204	-2.3250	-2.4500	0.0025	0.0030	0.0029
15	206	-2.2250	-2.4193	0.0043	0.0051	0.0049
16	404	-2.3250	-2.3713	0.0032	0.0039	0.0037
17	5	-2.2750	-2.3634	0.0011	0.0013	0.0013
18	406	-2.2250	-2.3525	0.0007	0.0008	0.0008
19	305	-2.2750	-2.3517	0.0006	0.0007	0.0007
20	308	-2.1250	-2.3423	0.0017	0.0020	0.0020
21	209	-2.0750	-2.3230	0.0013	0.0015	0.0015
22	108	-2.1250	-2.3210	0.0005	0.0006	0.0005
23	409	-2.0750	-2.3153	0.0005	0.0006	0.0006
24	303	-2.3750	-2.3129	0.0003	0.0003	0.0003
25	210	-2.0250	-2.3110	0.0034	0.0041	0.0040
26	208	-2.1250	-2.2569	0.0042	0.0051	0.0049
27	6	-2.2250	-2.2448	0.0011	0.0013	0.0013
28	110	-2.0250	-2.2402	0.0006	0.0007	0.0007
29	301	-2.4750	-2.2354	0.0003	0.0004	0.0004
30	405	-2.2750	-2.2352	0.0007	0.0008	0.0008
31	203	-2.3750	-2.2251	0.0008	0.0009	0.0009
32	403	-2.3750	-2.2235	0.0002	0.0003	0.0002
33	10	-2.0250	-2.2219	0.0002	0.0002	0.0002
34	12	-1.9250	-2.2207	0.0001	0.0002	0.0002
35	407	-2.1750	-2.2197	0.0005	0.0006	0.0006
36	306	-2.2250	-2.2135	0.0008	0.0010	0.0010
37	118	-1.6250	-2.2070	0.0006	0.0007	0.0007
38	7	-2.1750	-2.2048	0.0015	0.0018	0.0018
39	113	-1.8750	-2.1842	0.0035	0.0043	0.0042
40	13	-1.8750	-2.1519	0.0043	0.0052	0.0051
41	14	-1.8250	-2.1213	0.0021	0.0026	0.0026
42	107	-2.1750	-2.1210	0.0016	0.0020	0.0019
43	11	-1.9750	-2.0980	0.0025	0.0031	0.0030

TABLE A-2-1 (Continued)

	SUBJECT			WEIGHT		
	ID	θ	MLE	DGR.3	DGR.4	DGR.5
44	103	-2.3750	-2.0847	0.0014	0.0017	0.0017
45	106	-2.2250	-2.0780	0.0007	0.0009	0.0009
46	313	-1.8750	-2.0743	0.0003	0.0003	0.0003
47	205	-2.2750	-2.0743	0.0	0.0	0.0
48	408	-2.1250	-2.0740	0.0013	0.0016	0.0016
49	304	-2.3250	-2.0554	0.0045	0.0055	0.0054
50	402	-2.4250	-2.0113	0.0033	0.0041	0.0040
51	211	-1.9750	-2.0102	0.0006	0.0007	0.0007
52	114	-1.8250	-2.0036	0.0014	0.0017	0.0017
53	307	-2.1750	-1.9910	0.0014	0.0016	0.0016
54	8	-2.1250	-1.9855	0.0016	0.0019	0.0019
55	119	-1.5750	-1.9657	0.0018	0.0022	0.0021
56	312	-1.9250	-1.9620	0.0008	0.0010	0.0010
57	412	-1.9250	-1.9591	0.0005	0.0007	0.0006
58	309	-2.0750	-1.9549	0.0005	0.0006	0.0006
59	116	-1.7250	-1.9525	0.0051	0.0062	0.0061
60	311	-1.9750	-1.8880	0.0051	0.0061	0.0060
61	413	-1.8750	-1.8879	0.0005	0.0006	0.0005
62	207	-2.1750	-1.8821	0.0013	0.0016	0.0016
63	115	-1.7750	-1.8711	0.0009	0.0010	0.0010
64	18	-1.6250	-1.8711	0.0	0.0	0.0
65	111	-1.9750	-1.8710	0.0009	0.0010	0.0010
66	418	-1.6250	-1.8603	0.0009	0.0011	0.0011
67	216	-1.7250	-1.8593	0.0018	0.0022	0.0022
68	219	-1.5750	-1.8372	0.0020	0.0023	0.0023
69	310	-2.0250	-1.8350	0.0009	0.0010	0.0010
70	9	-2.0750	-1.8263	0.0044	0.0052	0.0052
71	212	-1.9250	-1.7808	0.0052	0.0060	0.0060
72	416	-1.7250	-1.7641	0.0032	0.0038	0.0038
73	411	-1.9750	-1.7420	0.0020	0.0024	0.0024
74	414	-1.8250	-1.7400	0.0006	0.0007	0.0007
75	215	-1.7750	-1.7347	0.0007	0.0008	0.0008
76	319	-1.5750	-1.7316	0.0004	0.0005	0.0005
77	315	-1.7750	-1.7294	0.0006	0.0007	0.0007
78	415	-1.7750	-1.7249	0.0007	0.0009	0.0009
79	321	-1.4750	-1.7207	0.0004	0.0005	0.0005
80	15	-1.7750	-1.7199	0.0005	0.0005	0.0005
81	314	-1.8250	-1.7152	0.0045	0.0052	0.0052
82	17	-1.6750	-1.6669	0.0047	0.0053	0.0054
83	19	-1.5750	-1.6614	0.0006	0.0006	0.0006
84	120	-1.5250	-1.6604	0.0013	0.0015	0.0015
85	218	-1.6250	-1.6462	0.0013	0.0014	0.0014
86	318	-1.6250	-1.6461	0.0005	0.0005	0.0005
87	123	-1.3750	-1.6409	0.0009	0.0010	0.0010
88	317	-1.6750	-1.6360	0.0023	0.0026	0.0026
89	16	-1.7250	-1.6149	0.0019	0.0022	0.0022
90	410	-2.0250	-1.6141	0.0015	0.0016	0.0016
91	316	-1.7250	-1.5985	0.0015	0.0017	0.0017
92	217	-1.6750	-1.5973	0.0010	0.0012	0.0012
93	222	-1.4250	-1.5868	0.0016	0.0018	0.0018
94	124	-1.3250	-1.5794	0.0013	0.0014	0.0014
95	417	-1.6750	-1.5728	0.0023	0.0026	0.0026
96	419	-1.5750	-1.5534	0.0040	0.0045	0.0045
97	213	-1.8750	-1.5281	0.0027	0.0029	0.0030
98	117	-1.6750	-1.5243	0.0009	0.0010	0.0010
99	112	-1.9250	-1.5181	0.0013	0.0015	0.0015
100	121	-1.4750	-1.5096	0.0022	0.0024	0.0025
101	214	-1.8250	-1.4941	0.0042	0.0046	0.0047
102	20	-1.5250	-1.4638	0.0029	0.0031	0.0032
103	425	-1.2750	-1.4632	0.0001	0.0001	0.0001
104	224	-1.3250	-1.4632	0.0	0.0	0.0

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
105	24	-1.3250	-1.4632	0.0002	0.0002	0.0002
106	420	-1.5250	-1.4613	0.0012	0.0013	0.0013
107	328	-1.1250	-1.4507	0.0020	0.0022	0.0022
108	23	-1.3750	-1.4398	0.0017	0.0018	0.0018
109	127	-1.1750	-1.4331	0.0013	0.0014	0.0014
110	22	-1.4250	-1.4259	0.0014	0.0015	0.0015
111	27	-1.1750	-1.4183	0.0015	0.0016	0.0016
112	422	-1.4250	-1.4104	0.0009	0.0009	0.0010
113	423	-1.3750	-1.4090	0.0016	0.0018	0.0018
114	424	-1.3250	-1.3931	0.0071	0.0077	0.0078
115	122	-1.4250	-1.3340	0.0090	0.0095	0.0097
116	231	-0.9750	-1.2997	0.0057	0.0059	0.0060
117	430	-1.0250	-1.2759	0.0029	0.0030	0.0031
118	125	-1.2750	-1.2699	0.0012	0.0012	0.0013
119	322	-1.4250	-1.2639	0.0012	0.0013	0.0013
120	129	-1.0750	-1.2576	0.0019	0.0020	0.0021
121	326	-1.2250	-1.2442	0.0017	0.0018	0.0018
122	323	-1.3750	-1.2403	0.0017	0.0017	0.0018
123	38	-0.6250	-1.2274	0.0022	0.0023	0.0023
124	131	-0.9750	-1.2182	0.0010	0.0011	0.0011
125	29	-1.0750	-1.2171	0.0002	0.0002	0.0002
126	30	-1.0250	-1.2159	0.0003	0.0003	0.0003
127	221	-1.4750	-1.2141	0.0009	0.0010	0.0010
128	25	-1.2750	-1.2066	0.0014	0.0014	0.0015
129	325	-1.2750	-1.2000	0.0010	0.0011	0.0011
130	334	-0.8250	-1.1963	0.0008	0.0008	0.0009
131	320	-1.5250	-1.1918	0.0008	0.0008	0.0008
132	324	-1.3250	-1.1888	0.0032	0.0032	0.0033
133	31	-0.9750	-1.1601	0.0050	0.0051	0.0052
134	21	-1.4750	-1.1387	0.0028	0.0028	0.0028
135	421	-1.4750	-1.1329	0.0020	0.0021	0.0021
136	130	-1.0250	-1.1185	0.0023	0.0023	0.0024
137	126	-1.2250	-1.1100	0.0017	0.0017	0.0017
138	331	-0.9750	-1.1020	0.0030	0.0030	0.0030
139	428	-1.1250	-1.0808	0.0031	0.0031	0.0032
140	437	-0.6750	-1.0714	0.0015	0.0015	0.0015
141	223	-1.3750	-1.0664	0.0007	0.0007	0.0007
142	230	-1.0250	-1.0650	0.0021	0.0021	0.0021
143	226	-1.2250	-1.0458	0.0021	0.0021	0.0021
144	228	-1.1250	-1.0445	0.0003	0.0003	0.0003
145	227	-1.1750	-1.0431	0.0014	0.0014	0.0014
146	330	-1.0250	-1.0312	0.0024	0.0024	0.0025
147	232	-0.9250	-1.0195	0.0013	0.0012	0.0013
148	233	-0.8750	-1.0190	0.0026	0.0026	0.0026
149	35	-0.7750	-0.9941	0.0026	0.0025	0.0026
150	33	-0.8750	-0.9941	0.0001	0.0001	0.0001
151	234	-0.8250	-0.9931	0.0005	0.0005	0.0005
152	220	-1.5250	-0.9890	0.0009	0.0009	0.0009
153	229	-1.0750	-0.9844	0.0021	0.0021	0.0021
154	429	-1.0750	-0.9684	0.0027	0.0026	0.0026
155	327	-1.1750	-0.9590	0.0025	0.0024	0.0025
156	335	-0.7750	-0.9448	0.0033	0.0032	0.0032
157	237	-0.6750	-0.9279	0.0025	0.0024	0.0024
158	240	-0.5250	-0.9212	0.0025	0.0024	0.0024
159	34	-0.8250	-0.9045	0.0031	0.0029	0.0030
160	136	-0.7250	-0.8923	0.0029	0.0028	0.0028
161	225	-1.2750	-0.8773	0.0026	0.0024	0.0025
162	128	-1.1250	-0.8681	0.0022	0.0021	0.0022
163	434	-0.8250	-0.8565	0.0014	0.0013	0.0014
164	333	-0.8750	-0.8550	0.0004	0.0004	0.0004
165	432	-0.9250	-0.8523	0.0006	0.0005	0.0005

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
166	36	-0.7250	-0.8458	0.0009	0.0009	0.0009
167	426	-1.2250	-0.8435	0.0015	0.0014	0.0015
168	139	-0.5750	-0.8356	0.0040	0.0037	0.0038
169	431	-0.9750	-0.8064	0.0040	0.0038	0.0039
170	427	-1.1750	-0.7981	0.0028	0.0026	0.0027
171	132	-0.9250	-0.7801	0.0019	0.0018	0.0019
172	32	-0.9250	-0.7801	0.0013	0.0012	0.0013
173	133	-0.8750	-0.7678	0.0022	0.0021	0.0021
174	26	-1.2250	-0.7593	0.0015	0.0014	0.0014
175	442	-0.4250	-0.7537	0.0024	0.0022	0.0023
176	238	-0.6250	-0.7372	0.0022	0.0021	0.0021
177	246	-0.2250	-0.7330	0.0018	0.0017	0.0017
178	337	-0.6750	-0.7203	0.0018	0.0017	0.0017
179	28	-1.1250	-0.7163	0.0005	0.0005	0.0005
180	438	-0.6250	-0.7153	0.0028	0.0026	0.0027
181	436	-0.7250	-0.6904	0.0052	0.0048	0.0049
182	137	-0.6750	-0.6673	0.0025	0.0023	0.0024
183	134	-0.8250	-0.6672	0.0017	0.0015	0.0016
184	439	-0.5750	-0.6521	0.0020	0.0018	0.0019
185	339	-0.5750	-0.6487	0.0013	0.0012	0.0012
186	235	-0.7750	-0.6400	0.0026	0.0024	0.0024
187	435	-0.7750	-0.6247	0.0017	0.0015	0.0016
188	140	-0.5250	-0.6247	0.0004	0.0004	0.0004
189	338	-0.6250	-0.6207	0.0024	0.0022	0.0022
190	332	-0.9250	-0.6028	0.0024	0.0022	0.0022
191	346	-0.2250	-0.5990	0.0023	0.0021	0.0022
192	247	-0.1750	-0.5816	0.0023	0.0021	0.0021
193	141	-0.4150	-0.5783	0.0014	0.0013	0.0013
194	135	-0.7750	-0.5686	0.0016	0.0015	0.0015
195	343	-0.3750	-0.5633	0.0022	0.0020	0.0020
196	241	-0.4750	-0.5486	0.0017	0.0015	0.0015
197	329	-1.0750	-0.5480	0.0024	0.0022	0.0022
198	236	-0.7250	-0.5268	0.0024	0.0021	0.0022
199	445	-0.2750	-0.5265	0.0	0.0	0.0
200	433	-0.8750	-0.5265	0.0	0.0	0.0
201	145	-0.2750	-0.5265	0.0	0.0	0.0
202	142	-0.4250	-0.5265	0.0003	0.0002	0.0002
203	242	-0.4250	-0.5242	0.0008	0.0008	0.0008
204	449	-0.0750	-0.5188	0.0028	0.0025	0.0026
205	39	-0.5750	-0.4985	0.0051	0.0045	0.0046
206	40	-0.5250	-0.4730	0.0044	0.0039	0.0040
207	244	-0.3250	-0.4587	0.0025	0.0022	0.0022
208	446	-0.2250	-0.4507	0.0042	0.0038	0.0038
209	41	-0.4750	-0.4207	0.0038	0.0034	0.0034
210	336	-0.7250	-0.4164	0.0009	0.0008	0.0008
211	45	-0.2750	-0.4129	0.0012	0.0011	0.0011
212	245	-0.2750	-0.4053	0.0033	0.0029	0.0030
213	243	-0.3750	-0.3831	0.0030	0.0027	0.0027
214	340	-0.5250	-0.3780	0.0012	0.0011	0.0011
215	150	-0.0250	-0.3724	0.0006	0.0006	0.0006
216	37	-0.6750	-0.3724	0.0004	0.0004	0.0004
217	42	-0.4250	-0.3684	0.0017	0.0015	0.0015
218	146	-0.2250	-0.3574	0.0013	0.0012	0.0012
219	441	-0.4150	-0.3564	0.0002	0.0002	0.0002
220	440	-0.5250	-0.3552	0.0019	0.0016	0.0017
221	147	-0.1150	-0.3397	0.0022	0.0019	0.0020
222	444	-0.2250	-0.3356	0.0017	0.0015	0.0016
223	250	-0.0250	-0.3241	0.0056	0.0050	0.0050
224	138	-0.6250	-0.2851	0.0053	0.0047	0.0047
225	342	-0.4250	-0.2764	0.0020	0.0017	0.0017
226	149	-0.0750	-0.2675	0.0021	0.0019	0.0019

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
227	256	0.2750	-0.2573	0.0017	0.0015	0.0015
228	341	-0.4750	-0.2519	0.0008	0.0007	0.0007
229	46	-0.2250	-0.2501	0.0003	0.0002	0.0002
230	144	-0.3250	-0.2494	0.0002	0.0002	0.0002
231	44	-0.3250	-0.2484	0.0007	0.0006	0.0007
232	239	-0.5750	-0.2428	0.0059	0.0051	0.0052
233	344	-0.3250	-0.1961	0.0081	0.0071	0.0071
234	351	0.0250	-0.1764	0.0053	0.0047	0.0047
235	248	-0.1250	-0.1485	0.0049	0.0043	0.0043
236	447	-0.1750	-0.1268	0.0033	0.0029	0.0029
237	450	-0.0250	-0.1189	0.0013	0.0011	0.0011
238	345	-0.2750	-0.1153	0.0006	0.0005	0.0005
239	448	-0.1250	-0.1139	0.0013	0.0011	0.0011
240	452	0.0750	-0.1038	0.0019	0.0017	0.0017
241	443	-0.3750	-0.0969	0.0020	0.0017	0.0017
242	352	0.0750	-0.0861	0.0016	0.0014	0.0014
243	47	-0.1750	-0.0822	0.0008	0.0007	0.0007
244	48	-0.1250	-0.0794	0.0003	0.0003	0.0003
245	148	-0.1250	-0.0793	0.0022	0.0019	0.0019
246	251	0.0250	-0.0602	0.0038	0.0033	0.0033
247	143	-0.3750	-0.0456	0.0025	0.0022	0.0022
248	252	0.0750	-0.0379	0.0022	0.0019	0.0019
249	57	0.3250	-0.0261	0.0020	0.0018	0.0018
250	347	-0.1750	-0.0200	0.0011	0.0010	0.0010
251	56	0.2750	-0.0159	0.0009	0.0008	0.0008
252	151	0.0250	-0.0121	0.0009	0.0008	0.0008
253	353	0.1250	-0.0078	0.0006	0.0005	0.0005
254	50	-0.0250	-0.0066	0.0006	0.0005	0.0005
255	253	0.1250	-0.0027	0.0010	0.0009	0.0009
256	454	0.1750	0.0023	0.0012	0.0011	0.0011
257	348	-0.1250	0.0083	0.0008	0.0007	0.0007
258	152	0.0750	0.0094	0.0010	0.0009	0.0009
259	153	0.1250	0.0175	0.0009	0.0008	0.0008
260	55	0.2250	0.0176	0.0010	0.0009	0.0009
261	359	0.4250	0.0264	0.0010	0.0009	0.0009
262	43	-0.3750	0.0266	0.0105	0.0092	0.0092
263	51	0.0250	0.1203	0.0109	0.0096	0.0095
264	354	0.1750	0.1241	0.0015	0.0013	0.0013
265	355	0.2250	0.1339	0.0011	0.0010	0.0010
266	349	-0.0750	0.1339	0.0010	0.0009	0.0009
267	53	0.1250	0.1431	0.0047	0.0042	0.0041
268	154	0.1750	0.1762	0.0057	0.0050	0.0049
269	156	0.2750	0.1939	0.0045	0.0040	0.0039
270	157	0.3250	0.2167	0.0063	0.0056	0.0055
271	254	0.1750	0.2503	0.0043	0.0039	0.0038
272	259	0.4250	0.2557	0.0010	0.0009	0.0009
273	160	0.4750	0.2594	0.0016	0.0014	0.0014
274	65	0.7250	0.2702	0.0016	0.0014	0.0014
275	255	0.2250	0.2738	0.0011	0.0010	0.0010
276	59	0.4250	0.2803	0.0012	0.0010	0.0010
277	451	0.0250	0.2843	0.0008	0.0007	0.0007
278	453	0.1250	0.2876	0.0017	0.0016	0.0015
279	249	-0.0750	0.3000	0.0018	0.0016	0.0016
280	49	-0.0750	0.3038	0.0010	0.0009	0.0008
281	58	0.3750	0.3087	0.0006	0.0006	0.0005
282	455	0.2250	0.3094	0.0013	0.0012	0.0012
283	350	-0.0250	0.3209	0.0018	0.0016	0.0016
284	456	0.2750	0.3255	0.0018	0.0016	0.0016
285	69	0.9250	0.3369	0.0016	0.0014	0.0014
286	360	0.4750	0.3398	0.0012	0.0011	0.0011
287	262	0.5750	0.3478	0.0015	0.0014	0.0014

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
288	258	0.3750	0.3538	0.0023	0.0021	0.0020
289	457	0.3250	0.3686	0.0019	0.0017	0.0017
290	465	0.7250	0.3711	0.0004	0.0004	0.0004
291	63	0.6250	0.3723	0.0002	0.0002	0.0002
292	257	0.3250	0.3729	0.0003	0.0002	0.0002
293	159	0.4250	0.3748	0.0006	0.0006	0.0006
294	52	0.0750	0.3786	0.0020	0.0018	0.0018
295	161	0.5250	0.3929	0.0029	0.0026	0.0026
296	458	0.3750	0.4051	0.0017	0.0016	0.0015
297	356	0.2750	0.4085	0.0051	0.0046	0.0045
298	155	0.2250	0.4514	0.0056	0.0051	0.0050
299	158	0.3750	0.4598	0.0028	0.0026	0.0025
300	261	0.5250	0.4771	0.0028	0.0025	0.0025
301	461	0.5250	0.4852	0.0009	0.0008	0.0008
302	358	0.3750	0.4852	0.0011	0.0011	0.0010
303	357	0.3250	0.4957	0.0011	0.0011	0.0010
304	54	0.1750	0.4957	0.0004	0.0004	0.0004
305	170	0.9750	0.4996	0.0007	0.0007	0.0006
306	62	0.5750	0.5022	0.0003	0.0003	0.0003
307	166	0.7750	0.5028	0.0010	0.0009	0.0009
308	269	0.9250	0.5114	0.0027	0.0025	0.0024
309	462	0.5750	0.5275	0.0022	0.0020	0.0020
310	265	0.7250	0.5313	0.0005	0.0005	0.0005
311	260	0.4750	0.5325	0.0006	0.0005	0.0005
312	459	0.4250	0.5366	0.0042	0.0039	0.0038
313	460	0.4750	0.5714	0.0058	0.0054	0.0053
314	61	0.5250	0.5905	0.0031	0.0029	0.0029
315	466	0.7750	0.6005	0.0023	0.0022	0.0021
316	266	0.7750	0.6121	0.0015	0.0014	0.0014
317	267	0.8250	0.6142	0.0008	0.0008	0.0007
318	268	0.8750	0.6197	0.0015	0.0014	0.0013
319	168	0.8750	0.6278	0.0017	0.0016	0.0016
320	361	0.5250	0.6357	0.0013	0.0012	0.0012
321	374	1.1750	0.6400	0.0014	0.0013	0.0013
322	60	0.4750	0.6487	0.0014	0.0014	0.0013
323	165	0.7250	0.6535	0.0110	0.0104	0.0102
324	264	0.6750	0.7516	0.0122	0.0118	0.0115
325	162	0.5750	0.7691	0.0023	0.0022	0.0022
326	362	0.5750	0.7735	0.0007	0.0007	0.0006
327	270	0.9750	0.7755	0.0005	0.0005	0.0005
328	371	1.0250	0.7787	0.0005	0.0004	0.0004
329	164	0.6750	0.7799	0.0007	0.0006	0.0006
330	463	0.6250	0.7849	0.0009	0.0009	0.0009
331	276	1.2750	0.7889	0.0028	0.0028	0.0027
332	263	0.6250	0.8120	0.0112	0.0109	0.0106
333	364	0.6750	0.8554	0.0094	0.0094	0.0091
334	464	0.6750	0.9030	0.0056	0.0055	0.0054
335	368	0.8750	0.9493	0.0056	0.0057	0.0055
336	64	0.6750	0.9578	0.0011	0.0011	0.0010
337	163	0.6250	0.9596	0.0006	0.0006	0.0006
338	74	1.1750	0.9639	0.0004	0.0005	0.0004
339	363	0.6250	0.9640	0.0007	0.0007	0.0007
340	475	1.2250	0.9706	0.0008	0.0008	0.0008
341	366	0.7750	0.9715	0.0006	0.0006	0.0006
342	171	1.0250	0.9761	0.0013	0.0013	0.0013
343	471	1.0250	0.9842	0.0019	0.0019	0.0019
344	68	0.8750	0.9947	0.0012	0.0012	0.0012
345	365	0.7250	0.9962	0.0002	0.0002	0.0002
346	76	1.2750	0.9962	0.0017	0.0017	0.0017
347	367	0.8250	1.0126	0.0021	0.0021	0.0021
348	66	0.7750	1.0166	0.0018	0.0019	0.0018

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
349	469	0.9250	1.0306	0.0024	0.0024	0.0024
350	172	1.0750	1.0401	0.0011	0.0012	0.0011
351	167	0.8250	1.0419	0.0018	0.0018	0.0018
352	477	1.3250	1.0575	0.0049	0.0051	0.0050
353	470	0.9750	1.0910	0.0039	0.0040	0.0039
354	468	0.8750	1.0962	0.0010	0.0010	0.0010
355	169	0.9250	1.1006	0.0022	0.0023	0.0023
356	271	1.0250	1.1183	0.0026	0.0027	0.0027
357	467	0.8250	1.1269	0.0009	0.0009	0.0009
358	67	0.8250	1.1270	0.0001	0.0001	0.0001
359	473	1.1250	1.1275	0.0004	0.0004	0.0004
360	369	0.9250	1.1307	0.0011	0.0012	0.0011
361	372	1.0750	1.1387	0.0020	0.0021	0.0020
362	476	1.2750	1.1506	0.0013	0.0014	0.0014
363	273	1.1250	1.1524	0.0002	0.0003	0.0003
364	373	1.1250	1.1531	0.0002	0.0003	0.0003
365	472	1.0750	1.1549	0.0031	0.0033	0.0033
366	272	1.0750	1.1850	0.0029	0.0031	0.0031
367	72	1.0750	1.1850	0.0011	0.0012	0.0012
368	177	1.3250	1.1964	0.0012	0.0013	0.0013
369	71	1.0250	1.1974	0.0015	0.0016	0.0015
370	377	1.3250	1.2115	0.0019	0.0021	0.0021
371	376	1.2750	1.2174	0.0007	0.0008	0.0007
372	175	1.2250	1.2188	0.0014	0.0015	0.0015
373	375	1.2250	1.2318	0.0013	0.0014	0.0013
374	75	1.2250	1.2318	0.0017	0.0019	0.0018
375	183	1.6250	1.2496	0.0019	0.0021	0.0021
376	70	0.9750	1.2520	0.0004	0.0004	0.0004
377	383	1.6250	1.2537	0.0003	0.0004	0.0003
378	174	1.1750	1.2554	0.0004	0.0004	0.0004
379	379	1.4250	1.2575	0.0005	0.0005	0.0005
380	73	1.1250	1.2606	0.0018	0.0020	0.0020
381	277	1.3250	1.2766	0.0025	0.0028	0.0027
382	370	0.9750	1.2872	0.0021	0.0023	0.0023
383	176	1.2750	1.2985	0.0034	0.0038	0.0037
384	279	1.4250	1.3234	0.0044	0.0048	0.0048
385	82	1.5750	1.3451	0.0024	0.0026	0.0026
386	482	1.5750	1.3486	0.0017	0.0019	0.0019
387	83	1.6250	1.3637	0.0014	0.0016	0.0016
388	79	1.4250	1.3639	0.0018	0.0020	0.0020
389	181	1.5250	1.3833	0.0018	0.0020	0.0020
390	85	1.7250	1.3833	0.0020	0.0022	0.0022
391	280	1.4750	1.4048	0.0037	0.0042	0.0042
392	275	1.2250	1.4238	0.0024	0.0027	0.0027
393	479	1.4250	1.4309	0.0011	0.0013	0.0013
394	283	1.6250	1.4361	0.0031	0.0034	0.0034
395	274	1.1750	1.4642	0.0043	0.0049	0.0049
396	180	1.4750	1.4832	0.0032	0.0036	0.0036
397	182	1.5750	1.4991	0.0025	0.0029	0.0029
398	381	1.5250	1.5110	0.0013	0.0015	0.0015
399	184	1.6750	1.5137	0.0017	0.0019	0.0019
400	77	1.3250	1.5297	0.0023	0.0026	0.0027
401	480	1.4750	1.5395	0.0010	0.0011	0.0011
402	481	1.5250	1.5407	0.0001	0.0001	0.0001
403	178	1.3750	1.5407	0.0012	0.0014	0.0014
404	483	1.6250	1.5545	0.0023	0.0027	0.0027
405	486	1.7750	1.5668	0.0020	0.0024	0.0024
406	188	1.8750	1.5776	0.0029	0.0034	0.0034
407	179	1.4250	1.6001	0.0026	0.0030	0.0030
408	86	1.7750	1.6069	0.0013	0.0016	0.0016
409	80	1.4750	1.6155	0.0013	0.0016	0.0016

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
410	287	1.8250	1.6222	0.0007	0.0008	0.0008
411	282	1.5750	1.6234	0.0005	0.0006	0.0006
412	281	1.5250	1.6283	0.0006	0.0007	0.0007
413	78	1.3750	1.6301	0.0003	0.0003	0.0003
414	484	1.6750	1.6315	0.0011	0.0013	0.0013
415	87	1.8250	1.6430	0.0015	0.0018	0.0018
416	478	1.3750	1.6490	0.0007	0.0008	0.0008
417	378	1.3750	1.6506	0.0003	0.0004	0.0004
418	290	1.9750	1.6529	0.0009	0.0011	0.0011
419	285	1.7250	1.6615	0.0020	0.0024	0.0024
420	385	1.7250	1.6763	0.0015	0.0017	0.0018
421	173	1.1250	1.6789	0.0017	0.0020	0.0021
422	380	1.4750	1.6966	0.0023	0.0027	0.0027
423	286	1.7750	1.7056	0.0008	0.0009	0.0010
424	284	1.6750	1.7061	0.0017	0.0020	0.0020
425	474	1.1750	1.7252	0.0022	0.0026	0.0026
426	382	1.5750	1.7324	0.0010	0.0011	0.0011
427	485	1.7250	1.7366	0.0012	0.0015	0.0015
428	388	1.8750	1.7472	0.0009	0.0011	0.0011
429	89	1.9250	1.7478	0.0002	0.0002	0.0002
430	489	1.9250	1.7493	0.0006	0.0008	0.0008
431	490	1.9750	1.7555	0.0011	0.0013	0.0013
432	185	1.7250	1.7626	0.0006	0.0007	0.0007
433	84	1.6750	1.7627	0.0010	0.0012	0.0013
434	278	1.3750	1.7753	0.0011	0.0013	0.0013
435	487	1.8250	1.7759	0.0004	0.0005	0.0005
436	88	1.8750	1.7799	0.0013	0.0015	0.0015
437	190	1.9750	1.7912	0.0013	0.0016	0.0016
438	81	1.5250	1.7958	0.0035	0.0042	0.0042
439	91	2.0250	1.8339	0.0039	0.0047	0.0048
440	297	2.3250	1.8449	0.0020	0.0024	0.0025
441	384	1.6750	1.8591	0.0027	0.0032	0.0033
442	196	2.2750	1.8784	0.0023	0.0028	0.0028
443	90	1.9750	1.8884	0.0012	0.0015	0.0015
444	386	1.7750	1.8943	0.0005	0.0007	0.0007
445	191	2.0250	1.8954	0.0004	0.0005	0.0006
446	289	1.9250	1.9000	0.0035	0.0042	0.0043
447	390	1.9750	1.9401	0.0070	0.0085	0.0087
448	488	1.8750	1.9916	0.0042	0.0051	0.0053
449	389	1.9250	1.9966	0.0005	0.0006	0.0006
450	189	1.9250	1.9978	0.0012	0.0014	0.0014
451	387	1.8250	2.0121	0.0013	0.0016	0.0016
452	291	2.0250	2.0156	0.0007	0.0008	0.0008
453	193	2.1250	2.0212	0.0008	0.0010	0.0010
454	195	2.2250	2.0269	0.0018	0.0022	0.0022
455	187	1.8250	2.0455	0.0017	0.0020	0.0020
456	295	2.2250	2.0494	0.0003	0.0004	0.0004
457	394	2.1750	2.0497	0.0001	0.0001	0.0001
458	93	2.1250	2.0506	0.0029	0.0035	0.0036
459	292	2.0750	2.0895	0.0037	0.0044	0.0046
460	391	2.0250	2.1021	0.0009	0.0011	0.0011
461	197	2.3250	2.1021	0.0002	0.0002	0.0002
462	92	2.0750	2.1047	0.0008	0.0010	0.0010
463	95	2.2250	2.1136	0.0021	0.0025	0.0026
464	496	2.2750	2.1341	0.0018	0.0022	0.0022
465	186	1.7750	2.1396	0.0025	0.0029	0.0030
466	497	2.3250	2.1692	0.0023	0.0027	0.0028
467	393	2.1250	2.1727	0.0005	0.0006	0.0006
468	491	2.0250	2.1761	0.0015	0.0017	0.0018
469	492	2.0750	2.1940	0.0013	0.0015	0.0016
470	493	2.1250	2.1953	0.0001	0.0002	0.0002

TABLE A-2-1 (Continued)

	SUBJECT ID	θ	MLE	WEIGHT DGR.3	DGR.4	DGR.5
471	399	2.4250	2.1959	0.0016	0.0019	0.0019
472	298	2.3750	2.2188	0.0018	0.0021	0.0022
473	200	2.4750	2.2231	0.0013	0.0015	0.0016
474	288	1.8750	2.2381	0.0020	0.0023	0.0024
475	97	2.3250	2.2527	0.0012	0.0014	0.0014
476	400	2.4750	2.2563	0.0008	0.0009	0.0009
477	396	2.2750	2.2645	0.0010	0.0012	0.0012
478	94	2.1750	2.2720	0.0014	0.0016	0.0017
479	495	2.2250	2.2866	0.0046	0.0052	0.0054
480	392	2.0750	2.3434	0.0036	0.0039	0.0041
481	494	2.1750	2.3440	0.0009	0.0009	0.0010
482	96	2.2750	2.3570	0.0010	0.0011	0.0011
483	293	2.1250	2.3599	0.0017	0.0018	0.0019
484	395	2.2250	2.3841	0.0024	0.0025	0.0026
485	294	2.1750	2.3984	0.0026	0.0028	0.0029
486	499	2.4250	2.4271	0.0022	0.0023	0.0024
487	498	2.3750	2.4361	0.0016	0.0016	0.0017
488	398	2.3750	2.4535	0.0012	0.0012	0.0012
489	194	2.1750	2.4563	0.0028	0.0028	0.0029
490	192	2.0750	2.5017	0.0032	0.0031	0.0032
491	397	2.3250	2.5133	0.0009	0.0008	0.0009
492	100	2.4750	2.5177	0.0003	0.0003	0.0003
493	198	2.3750	2.5188	0.0065	0.0061	0.0062
494	500	2.4750	2.6346	0.0066	0.0050	0.0049
495	199	2.4250	2.6483	0.0007	0.0005	0.0005
496	98	2.3750	2.6483	0.0033	0.0024	0.0024
497	296	2.2750	2.7137	0.0032	0.0018	0.0017
498	99	2.4250	2.7142	0.0068	0.0040	0.0073
499	300	2.4750	2.8564	0.0066	0.0009	0.0
500	299	2.4250	2.8718	0.0006	0.0001	0.0

DISTRIBUTION LIST

Navy

1 Dr. Ed Aiken
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Jack R. Borsting
Provost & Academic Dean
U.S. Naval Postgraduate School
Monterey, CA 93940

1 Dr. Richard Elster
Department of Administrative Sciences
Naval Postgraduate School
Monterey, CA 93940

1 DR. PAT FEDERICO
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152

1 CDR John Ferguson, MSC, USN
Naval Medical R&D Command (Code 44)
National Naval Medical Center
Bethesda, MD 20014

1 Dr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Steve Harris
Code L522
NAMRL
Pensacola FL 32508

1 CDR Robert S. Kennedy
Naval Aerospace Medical and Research Lab
Box 29407
New Orleans, LA 70189

1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054

1 Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

Navy

1 CHAIRMAN, LEADERSHIP & LAW DEPT.
DIV. OF PROFESSIONAL DEVELOPMMENT
U.S. NAVAL ACADEMYY
ANNAPOLIS, MD 21402

1 Dr. James McBride
Code 301
Navy Personnel R&D Center
San Diego, CA 92152

1 DR. WILLIAM MONTAGUE
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Commanding Officer
Naval Health Research Center
Attn: Library
San Diego, CA 92152

1 Naval Medical R&D Command
Code 44
National Naval Medical Center
Bethesda, MD 20014

1 CAPT Paul Nelson, USN
Chief, Medical Service Corps
Code 7
Bureau of Medicine & Surgery
U. S. Department of the Navy
Washington, DC 20372

1 DR. RICHARD J. NIEHAUS
CODE 301
OFFICE OF CIVILIAN PERSONNEL
NAVY DEPT
WASHINGTON, DC 20390

1 Library
Navy Personnel R&D Center
San Diego, CA 92152

5 Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

Navy

6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

1 OFFICE OF CIVILIAN PERSONNEL
(CODE 26)
DEPT. OF THE NAVY
WASHINGTON, DC 20390

1 JOHN OLSEN
CHIEF OF NAVAL EDUCATION &
TRAINING SUPPORT
PENSACOLA, FL 32509

1 Psychologist
ONR Branch Office
495 Summer Street
Boston, MA 02210

1 Psychologist
ONR Branch Office
536 S. Clark Street
Chicago, IL 60605

1 Office of Naval Research
Code 200
Arlington, VA 22217

1 Code 436
Office of Naval Research
Arlington, VA 22217

5 Personnel & Training Research Programs
(Code 458)
Office of Naval Research
Arlington, VA 22217

1 Psychologist
OFFICE OF NAVAL RESEARCH BRANCH
223 OLD MARYLEBONE ROAD
LONDON, NW, 15TH ENGLAND

1 Psychologist
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101

Navy

1 Scientific Director
Office of Naval Research
Scientific Liaison Group/Tokyo
American Embassy
APO San Francisco, CA 96503

1 Head, Research, Development, and Studies
(OP102X)
Office of the Chief of Naval Operations
Washington, DC 20370

1 Scientific Advisor to the Chief of
Naval Personnel (Pers-Or)
Naval Bureau of Personnel
Room 4410, Arlington Annex
Washington, DC 20370

1 DR. RICHARD A. POLLAK
ACADEMIC COMPUTING CENTER
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402

1 Mr. Arnold Rubenstein
Naval Personnel Support Technology
Naval Material Command (08T244)
Room 1044, Crystal Plaza #5
2221 Jefferson Davis Highway
Arlington, VA 20360

1 A. A. SJOHOLM
TECH. SUPPORT, CODE 201
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr. Robert Smith
Office of Chief of Naval Operations
OP-987E
Washington, DC 20350

1 Dr. Alfred F. Smode
Training Analysis & Evaluation Group
(TAEG)
Dept. of the Navy
Orlando, FL 32813

1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

Navy

1 CDR Charles J. Theisen, JR. MSC, USN
Head Human Factors Engineering Div.
Naval Air Development Center
Warminster, PA 18974

1 W. Gary Thomson
Naval Ocean Systems Center
Code 7132
San Diego, CA 92152

1 Dr. Ronald Weitzman
Department of Administrative Sciences
U. S. Naval Postgraduate School
Monterey, CA 93940

1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

Army

1 Technical Director
U. S. Army Research Institute for the Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 HQ USAREUE & 7th Army
ODCSOPS
USAAREUE Director of GED
APO New York 09403

1 DR. RALPH CANTER
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Dr. Milton S. Katz
Individual Training & Skill Evaluation Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Frederick Steinheiser
U. S. Army Reserch Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Joseph Ward
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

1 Air Force Human Resources Lab
AFHRL/PED
Brooks AFB, TX 78235

1 Air University Library
AUL/LSE 76/443
Maxwell AFB, AL 36112

1 Dr. Philip De Leo
AFHRL/TT
Lowry AFB, CO 80230

1 CDR. MERCER
CNET LIAISON OFFICER
AFHRL/FLYING TRAINING DIV.
WILLIAMS AFB, AZ 85224

1 Dr. Roger Pennell
AFHRL/TT
Lowry AFB, CO 80230

1 Personnel Analysis Division
HQ USAF/DPXXA
Washington, DC 20330

1 Research Branch
AFMPC/DPMYP
Randolph AFB, TX 78148

1 Dr. Malcolm Ree
AFHRL/PED
Brooks AFB, TX 78235

1 Dr. Marty Rockway (AFHRL/TT)
Lowry AFB
Colorado 80230

1 Jack A. Thorpe, Capt, USAF
Program Manager
Life Sciences Directorate
AFOSR
Bolling AFB, DC 20332

1 Brian K. Waters, LCOL, USAF
Air University
Maxwell AFB
Montgomery, AL 36112

Marines

1 Director, Office of Manpower Utilization
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134

1 MCDEC
Quantico Marine Corps Base
Quantico, VA 22134

1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

Coast Guard

1 MR. JOSEPH J. COWAN, CHIEF
PSYCHOLOGICAL RESEARCH (G-P-1/62)
U.S. COAST GUARD HQ
WASHINGTON, DC 20590

1 Dr. Thomas Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Other DoD

1 Dr. Stephen Andriole
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209

12 Defense Documentation Center
Cameron Station, Bldg. 5
Alexandria, VA 22314
Attn: TC

1 Dr. Dexter Fletcher
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209

1 Dr. William Graham
Testing Directorate
MEPCOM
Ft. Sheridan, IL 60037

1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defense
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301

1 MAJOR Wayne Sellman, USAF
Office of the Assistant Secretary
of Defense (MRA&L)
3B930 The Pentagon
Washington, DC 20301

Civil Govt

1 Dr. Susan Chipman
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. William Gorham, Director
Personnel R&D Center
U.S. Civil Service Commission
1900 E Street NW
Washington, DC 20415

1 Dr. John Mays
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Arthur Melmud
National Intitute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Andrew R. Molnar
Science Education Dev.
and Research
National Science Foundation
Washington, DC 20550

1 Dr. Lalitha P. Sanathanan
Environmental Impact Studies Division
Argonne National Laboratory
9700 S. Cass Avenue
Argonne, IL 60439

1 Dr. Jeffrey Schiller
National Institute of Education
1200 19th St. NW
Washington, DC 20208

1 Dr. H. Wallace Sinaiko
Program Director
Manpower Research and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

Civil Govt

1 Dr. Thomas G. Sticht
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Vern W. Urry
Personnel R&D Center
U.S. Civil Service Commission
1900 E Street NW
Washington, DC 20415

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Non Govt

1 1 psychological research unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600, Australia

1 Dr. Alan Baddeley
Medical Research Council
Applied Psychology Unit
15 Chaucer Road
Cambridge CB2 2EF
ENGLAND

1 Dr. Nicholas A. Bond
Dept. of Psychology
Sacramento State College
600 Jay Street
Sacramento, CA 95819

1 Dr. David G. Bowers
Institute for Social Research
University of Michigan
Ann Arbor, MI 48106

1 Dr. John B. Carroll
Psychometric Lab
Univ. of No. Carolina
Davie Hall 013A
Chapel Hill, NC 27514

1 Charles Myers Library
Livingstone House
Livingstone Road
Stratford
London E15 2LJ
ENGLAND

1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

1 Dr. William Coffman
Iowa Testing Programs
University of Iowa
Iowa City, IA 52242

Non Govt

1 Dr. Meredith Crawford
Department of Engineering Administration
George Washington University
Suite 805
2101 L Street N. W.
Washington, DC 20037

1 Dr. Hans Cronbag
Education Research Center
University of Leyden
Boerhaavelaan 2
Leyden
The NETHERLANDS

1 MAJOR I. N. EVONIC
CANADIAN FORCES PERS. APPLIED RESEARCH
1107 AVENUE ROAD
TORONTO, ONTARIO, CANADA

1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240

1 Dr. Victor Fields
Dept. of Psychology
Montgomery College
Rockville, MD 20850

1 Dr. Edwin A. Fleishman
Advanced Research Resources Organ.
8555 Sixteenth Street
Silver Spring, MD 20910

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 DR. ROBERT GLASER
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Dr. Ron Hambleton
School of Education
University of Massechusetts
Amherst, MA 01002

Non Govt

1 Library
HumRRO/Western Division
27857 Berwick Drive
Carmel, CA 93921

1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105

1 DR. LAWRENCE B. JOHNSON
LAWRENCE JOHNSON & ASSOC., INC.
SUITE 502
2001 S STREET NW
WASHINGTON, DC 20009

1 Dr. Roger A. Kaufman
203 Dodd Hall
Florida State Univ.
Tallahassee, FL 32306

1 Dr. Steven W. Keele
Dept. of Psychology
University of Oregon
Eugene, OR 97403

1 LCOL. C.R.J. LAFLEUR
PERSONNEL APPLIED RESEARCH
NATIONAL DEFENSE HQS
101 COLONEL BY DRIVE
OTTAWA, CANADA K1A OK2

1 Dr. Michael Levine
Department of Psychology
University of Illinois
Champaign, IL 61820

1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

1 Dr. Frederick M. Lord
Educational Testing Service
Princeton, NJ 08540

Non Govt

1 Dr. Robert R. Mackie
Human Factors Research, Inc.
6780 Cortona Drive
Santa Barbara Research Pk.
Goleta, CA 93017

1 Dr. Scott Maxwell
Department of Psychology
University of Houston
Houston, TX 77025

1 Dr. Sam Mayo
Loyola University of Chicago
Chicago, IL 60601

1 Richard T. Mowday
College of Business Administration
University of Oregon
Eugene, OR 97403

1 Dr. Melvin R. Novick
Iowa Testing Programs
University of Iowa
Iowa City, IA 52242

1 Dr. Robert Pachella
Department of Psychology
Human Performance Center
330 Packard Road
Ann Arbor, MI 48104

1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207

1 MR. LUIGI PETRULLO
2431 N. EDGEWOOD STREET
ARLINGTON, VA 22207

1 DR. STEVEN M. PINE
4950 Douglas Avenue
Golden Valley, MN 55416

1 DR. DIANE M. RAMSEY-KLEE
R-K RESEARCH & SYSTEM DESIGN
3947 RIDGEMONT DRIVE
MALIBU, CA 90265

Non Govt

1 MIN. RET. M. RAUCH
P II 4
BUNDESMINISTERIUM DER VERTEIDIGUNG
POSTFACH 161
53 BONN 1, GERMANY

1 Dr. Mark D. Reckase
Educational Psychology Dept.
University of Missouri-Columbia
12 Hill Hall
Columbia, MO 65201

1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007

1 DR. WALTER SCHNEIDER
DEPT. OF PSYCHOLOGY
UNIVERSITY OF ILLINOIS
CHAMPAIGN, IL 61820

1 Dr. Edwin Shirkey
Department of Psychology
Florida Technological University
Orlando, FL 32816

1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305

1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520

1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305

1 Dr. Brad Sympson
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

Non Govt

1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

1 Dr. Kikumi Tatsuoka
Computer Based Education Research Laboratory
252 Engineering Research Laboratory
University of Illinois
Urbana, IL 61801

1 Dr. Benton J. Underwood
Dept. of Psychology
Northwestern University
Evanston, IL 60201

1 DR. THOMAS WALLSTEN
PSYCHOMETRIC LABORATORY
DAVIE HALL 013A
UNIVERSITY OF NORTH CAROL
CHAPEL HILL, NC 27514

1 Dr. John Wannous
Department of Management
Michigan University
East Lansing, MI 48824

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 DR. SUSAN E. WHITELY
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
LAWRENCE, KANSAS 66044

Navy

1 Donald Calder
Office of Naval Research
325 Hinman Research Building
Atlanta, GA 30332

Army

1 Dr. Randall M. Chambers, Ph.D.
U.S. Army Research Institute
for the Behavioral and Social
Sciences
Fort Sill Field Unit
P.O.Box 3066
Fort Sill, OK 73503

Non Govt

1 Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, Mass. 01002

1 Dr. William E. Coffman
Iowa Testing Programs
334 Lindquist Center
Iowa City, IA 52242

1 Mr. Isaac I. Bejar
Department of Psychology
Elliott Hall
75 East River Road
Minneapolis, Minnesota 55455

1 Mr. George Woods
1106 Newport Ave.
Victoria, B.C.
V8S 5E4 Canada

1 Dr. P. Mengal
Faculte' de Psychologie
et des Sciences de l'Education
Universite' de Geneve
3 fl. de l'Universite
1201 Geneva SWITZERLAND

1 Dr. Wim J. van der Linden
Vakgroep Onderwisjskunde
Postbus 217
7500 EA Enschede
The Netherlands