AD-A063 773 FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 1/3
THE UTILIZATION OF THE ANALYTIC GRAPH METHOD TO DETERMINE THE B--ETC(U)
JUN 78 A NASTASE, C NASTASE
UNCLASSIFIED FTD-ID(RS)T-0977-78 NL

| OF |
AD
A063773

END
DATE
FILMED
3-79
DDC

FTD-ID(RS)T-0977-78

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

THE UTILIZATION OF THE ANALYTIC GRAPH METHOD
TO DETERMINE THE BEST SHAPE OF THIN
DELTA WINGS IN SUPERSONIC FLOW

By

iana Nastase, C. Nastase, and Francine Atanasiu-Moldovan

FOREIGN TECHNOLOGY DIVISION

EDITED TRANSLATION

FTD-ID(RS)T-0977-78 29 June 1978

MICROFICHE NR: FTD-78-C-000875

THE UTILIZATION OF THE ANALYTIC GRAPH METHOD TO DETERMINE THE BEST SHAPE OF THIN DELTA WINGS IN SUPERSONIC FLOW

By: Adriana Nastase, C. Nastase, and Francine Atanasiu-Moldovan

English pages: 16

Source: Studii Si Cercetari De Mecanica Aplicata, Vol 32, nr 6, 1973, pp. 1179-1190

Country of Origin: Rumania
Translated by: LINGUISTIC SYSTEMS, INC.
F33657-76-D-0389
Patrick T. Nolan
Requester: FTD/TQTA

ACCESSION for	
NTIS	White Section ✓
DDC	Buff Section
UNANNOUNCED	
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SPECIAL
A	

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

STUDIES

THE UTILIZATION OF THE ANALYTIC GRAPH METHOD TO DETERMINE THE BEST SHAPE OF THIN DELTA WINGS IN SUPERSONIC ~~CURRENTS~~ FLOW*

by ADRIANA NASTASE**

C. NASTASE***

FRANCINE ATANASIU-MOLDOVAN****

In the present work an analytic graph method is used (by the first author) to determine the best shape of a thin delta wing from among the original thin delta wing class in which the distribution of the vertical speed of disturbance is expressed in the form of second and third order homogenous polynomials. In addition, the thin delta wing satisfies and meets the geometrical and aerodinamic conditions of natural law.

1. INTRODUCTION

The analytic graph method is utilized in the present work (1) for the effective determination, at a given Mach cruising speed M, the best optimum form of thin delta wings with subsonic edges, which belong to the original class of thin delta wings and which, in addition, satisfy the following conditions of law: the lift power and moment of pitch are given, and the (axis) of disturbance speed u is finite along the subsonic (leading) edge of the wings, in order to avoid the formation and falling of the vortexes which have the tendency to appear along that edge at the Mach number of cruising speed M. The incidents of the thin delta wing are presupposed to be sufficiently small, in such a manner that we are able to apply the results of disturbance theories.

The vertical speeds of disturbance on the wings, w, is analyzed by using second and third order homogenous polynomial tests.

---

* The authors thank Mrs. Denise Vallee-Guiraud, main scientific researcher and Mr. M. Bois, scientific researcher at ONERA (France) for valuable advice in laying out the calculus program.

** Bucharest Polytechnical Institute

*** Galati Polytechnical Institute (honor professor)

**** IMFCA

ST. CERC. MEC. APL., TOM. 32, NR. 6, 1973, P. 1179 - 1190

Thus we can apply the results of P. Germain's higher order cone flow theories, (3).

The axis of disturbance speeds u on the wing is obtained with the help of the hydrodynamic analogy method, through the superimposition of the contributions of the borders and edges on the wing (4), (5).

The study of the optimum form of a thin delta wing through the analytic graph method leads to the determination of the optimum values $\check{\gamma}_0$ of the parameter of similitude $\check{\gamma} = Bl$ of flow, which minimizes the wave resistance expression, $C_d = f(\check{\gamma})$. For that purpose an imaginary transformed thin delta wing (projected in a fixed plane) is used, which is the only function of the parameter of similitude $\check{\gamma}$.

The transformed thin delta wing is placed in a suitably chosen supersonic flow.

2. Axial and Vertical Disturbance Velocity Expressions

The thin delta wing is referred to a triorthogonal system of axis $Ox_1x_2x_3$, which has the origin in the peak of the wing, axis Ox_1 parallel with the velocity at infinity U_∞, again the wing is plotted and numbered in plane Ox_1x_2 (fig.1).

The vertical disturbance velocity $w_a = \frac{w}{U_\infty}$ on the initial thin delta wing is presupposed as expressed in the form of second and third order homogenous polynomials

$$w_a = \frac{w}{U_\infty} = x_1(w_{10} + w_{01}|y|) + x_1^2(w_{20} + w_{11}|y| + w_{02}y^2). \quad (1)$$

In the following similarity transformations (1),

$$\tilde{x}_1 = \frac{x_1}{h_1}, \quad \tilde{x}_2 = \frac{x_2}{l_1}, \quad \tilde{x}_3 = \frac{x_3}{h_1} \quad \left(\tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1}\right), \quad (2)$$

plane $\tilde{O}\tilde{x}_1\tilde{x}_2$, the designated transformed plane, corresponds to the initial plain Ox_1x_2.

The initial thin delta wing projection in plane Ox_1x_2 is an isosceles triangle OA_1A_2 in which the height corresponding to peak O is h_1, and its base is $2l_1$ (fig.1).

In the succeeding similarity transformation (2) the transformed thin delta wing projection in plane $\tilde{O}\tilde{x}_1\tilde{x}_2$ is an isosceles triangle $\tilde{O}\tilde{A}_1\tilde{A}_2$ which has height 1 and base 2 (fig.2), and the vertical disturbance velocity $w_a = \tilde{w}_a$ acquires the following form

$$\tilde{w}_a = \frac{\tilde{w}}{\tilde{U}_\infty} = \tilde{x}_1(\tilde{w}_{10} + \tilde{w}_{01}|\tilde{y}|) + \tilde{x}_1^2(\tilde{w}_{20} + \tilde{w}_{11}|\tilde{y}| + \tilde{w}_{02}\tilde{y}^2). \quad (3)$$

Coefficients w_{ij} and $\tilde{w}_{ij}$ from (1) and (3) are connected through the relations:

$$\tilde{w}_{10} = h_1 w_{10}, \quad \tilde{w}_{01} = h_1 l w_{01}, \quad (4)$$

$$\tilde{w}_{20} = h_1^2 w_{20}, \quad \tilde{w}_{11} = h_1^2 l w_{11}, \quad \tilde{w}_{02} = h_1^2 l^2 w_{02}. \quad (5)$$

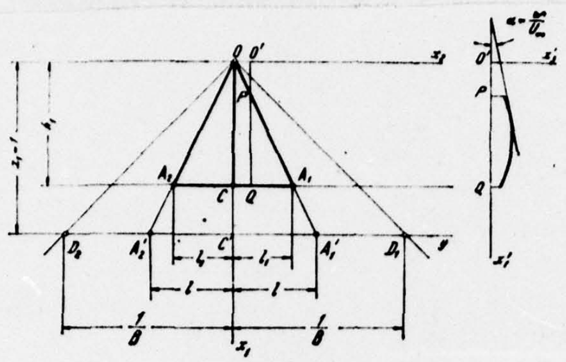

Fig. 1

The expression of the disturbance axis velocity u on the initial thin delta wing with subsonic leading edges, corresponding to the distribution (1) of the vertical disturbance speed w, is of the form:

$$\frac{u}{U_\infty} = x_1\left(\frac{A_{20}+A_{22}y^2}{\sqrt{l^2-y^2}}\right) + x_1^2\left(\frac{A_{30}+A_{32}y^2}{\sqrt{l^2-y^2}} + C_{32}y^2 \operatorname{argch}\sqrt{\frac{l^2}{y^2}}\right). \quad (6)$$

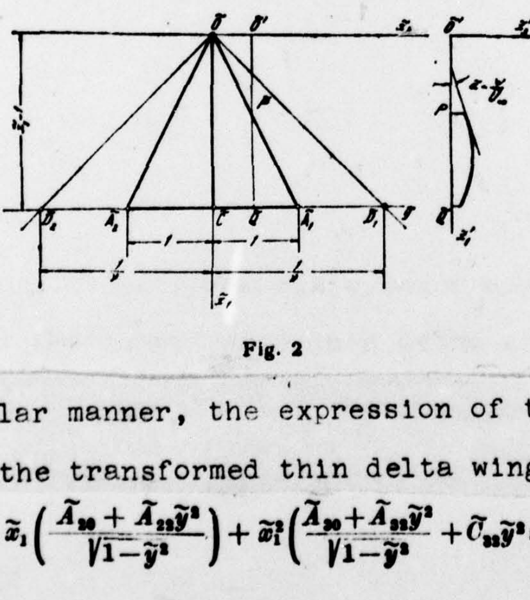

Fig. 2

In a similar manner, the expression of the disturbance axis velocity $\tilde{u}$ on the transformed thin delta wing is of the form

$$\frac{\tilde{u}}{\tilde{U}_\infty} = \tilde{x}_1\left(\frac{\tilde{A}_{20}+\tilde{A}_{22}\tilde{y}^2}{\sqrt{1-\tilde{y}^2}}\right) + \tilde{x}_1^2\left(\frac{\tilde{A}_{30}+\tilde{A}_{32}\tilde{y}^2}{\sqrt{1-\tilde{y}^2}} + \tilde{C}_{32}\tilde{y}^2 \operatorname{argch}\sqrt{\frac{1}{\tilde{y}^2}}\right). \quad (7)$$

The transformed thin delta wing is presupposed to be placed in an imaginary supersonic flow whose velocity is $\tilde{U}_\infty = \sqrt{1 + \nu^2}$.

The axis of disturbance speeds u and $\tilde{u}$ are connected through the relation (1)

$$u = l\tilde{u}, \tag{8}$$

and among the constants A_{ij}, C_{ij} and $\tilde{A}_{ij}$, $\tilde{C}_{ij}$ respectively of that axis of disturbance velocity exists relations of the form:

$$\tilde{A}_{20} = \frac{h_1}{l^2} A_{10}; \qquad \tilde{A}_{22} = h_1 A_{22}, \tag{9a}$$

$$\tilde{A}_{30} = \frac{h_1^2}{l^2} A_{30}; \qquad \tilde{A}_{32} = h_1^2 A_{32}, \qquad \tilde{C}_{32} = h_1^2 l C_{32}. \tag{9b}$$

The constants $\tilde{A}_{ij}$ and $\tilde{C}_{ij}$ of the axis of disturbance velocity $\tilde{u}$ on the transformed thin delta wing are connected by coefficients $\tilde{w}_{ij}$ of the vertical disturbance speed $\tilde{w}$ on that wing through homogenous and linear relations deduced from the compatibility conditions of P. Germain (3):

$$\tilde{A}_{20} = \tilde{a}_{00}^{(2)} \tilde{w}_{10} + a_{01}^{(2)} \tilde{w}_{01}, \tag{10a}$$

$$\tilde{A}_{22} = \tilde{a}_{20}^{(2)} \tilde{w}_{10} + a_{21}^{(2)} \tilde{w}_{01}, \tag{10b}$$

$$\tilde{A}_{30} = \tilde{a}_{00}^{(3)} \tilde{w}_{20} + \tilde{a}_{01}^{(3)} \tilde{w}_{11} + \tilde{a}_{02}^{(3)} \tilde{w}_{02}, \tag{10c}$$

$$\tilde{A}_{31} = \tilde{a}_{20}^{(3)} \tilde{w}_{20} + \tilde{a}_{21}^{(3)} \tilde{w}_{11} + \tilde{a}_{22}^{(3)} \tilde{w}_{02}, \tag{10d}$$

$$\tilde{C}_{32} = \tilde{\sigma}_{21}^{(3)} \tilde{w}_{11}. \tag{10e}$$

If $\nu = Bl$ as noted earlier and the eliptic integrals of the first and second cases E(k) and K(k), by the mode

$$k = \sqrt{1 - \nu^2}, \qquad (11)$$

then the coefficients $a_{ij}^{(2)}, a_{ij}^{(3)}, c_{ij}^{(3)}$ which intervene in the expressions (10 a, b, c, d, e) of the constants of the axis of disturbance velocity $\tilde{u}$ on the transformed thin delta wing with subsonic leading edges are , respectively, of the form (6), (7), (8):

$$\tilde{a}_{00}^{(2)} = -\frac{2(1-\nu^2)}{N_2}, \qquad (12a)$$

$$\tilde{a}_{01}^{(2)} = -\frac{2[E(k) - \nu^2 K(k)]}{\pi N_2}, \qquad (12b)$$

$$a_{20}^{(2)} = \frac{1-\nu^2}{N_2}, \qquad (12c)$$

$$a_{21}^{(2)} = \frac{2\nu^2[E(k) - K(k)]}{\pi N_2}, \qquad (12d)$$

The following expression is noted with N_2:

$$N_2 = (1 - 2\nu^2)\,E(k) + \nu^2 K(k), \qquad (13)$$

and

$$\tilde{a}_{00}^{(3)} = \frac{2}{N_2}[2(3 - 5\nu^2 + \nu^4)\,E(k) - \nu^2(3 - 5\nu^2)K(k)]. \qquad (14a)$$

$$\tilde{a}_{01}^{(3)} = \frac{6}{\pi N_2}[(4 + \nu^2)\,E^2(k) - 8\nu^2 E(k)\,K(k) + 3\nu^4 K^2(k)], \qquad (14b)$$

$$\tilde{a}_{02}^{(3)} = -\frac{2}{N_2}[(1 + \nu^2)\,E(k) - 2\nu^2 K(k)], \qquad (14c)$$

$$\tilde{a}_{20}^{(3)} = -\frac{2}{N_3}[(4 - 7\nu^2 + \nu^4)E(k) - 2(1 - 2\nu^2)K(k)], \tag{14d}$$

$$\tilde{a}_{21}^{(3)} = -\frac{2}{\pi N_3}[(12 - 17\nu^2 + 10\nu^4)E^2(k) -$$

$$- 4\nu^2(2 - \nu^2 + \nu^4)E(k)K(k) + \nu^4(1 + 2\nu^2)K^2(k)], \tag{14e}$$

$$\tilde{a}_{22}^{(3)} = \frac{2\nu^2}{N_3}[2(2 - \nu^2)E(k) - (3 - \nu^2)K(k)], \tag{14f}$$

$$c_{21}^{(3)} = -\frac{1}{\pi}, \tag{14g}$$

The expression noted with N_3

$$N_3 = (4 - 19\nu^2 + 4\nu^4)E^2(k) + 8\nu^2(1 + \nu^2)E(k)K(k) - 5\nu^4K^2(k). \tag{15}$$

The explicit forms of the relations of connections which occur in problems of the optimum transformed thin delta wing are the following:

- Lift conditions of the transformed thin delta wing to be given as

$$\tilde{C}_l = 8\int \tilde{u}_a\tilde{x}_1 d\tilde{x}_1 d\tilde{y} = \tilde{\Lambda}_{20}\tilde{w}_{10} + \tilde{\Lambda}_{21}\tilde{w}_{01} + \tilde{\Lambda}_{30}\tilde{w}_{20} +$$

$$+ \tilde{\Lambda}_{31}\tilde{w}_{11} + \tilde{\Lambda}_{32}\tilde{w}_{02} = \frac{2\pi}{3}[(2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)})\tilde{w}_{10} +$$

$$+ (2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)})\tilde{w}_{01}] + \frac{\pi}{2}\Big[(2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)})\tilde{w}_{20} + \Big(2\tilde{a}_{01}^{(3)} +$$

$$+ \tilde{a}_{21}^{(3)} + \frac{c_{21}^{(3)}}{3}\Big)\tilde{w}_{11} + (2a_{02}^{(3)} + a_{21}^{(3)})\tilde{w}_{02}\Big] = \tilde{C}_{l_0} = \frac{C_{l_0}}{l}. \tag{16}$$

The moment of pitch condition of the transformed thin delta wing to be given as:

$$\widetilde{C}_m = 8\int \widetilde{u}_a \widetilde{x}_1^2 d\widetilde{x}_1 d\widetilde{y} = \widetilde{\Gamma}_{20}\widetilde{w}_{10} + \widetilde{\Gamma}_{21}\widetilde{w}_{01} + \widetilde{\Gamma}_{30}\widetilde{w}_{20} + \widetilde{\Gamma}_{31}\widetilde{w}_{11} +$$

$$+ \widetilde{\Gamma}_{32}\widetilde{w}_{02} = \frac{3}{4}(\widetilde{\Lambda}_{20}\widetilde{w}_{10} + \widetilde{\Lambda}_{21}\widetilde{w}_{01}) + \frac{4}{5}(\widetilde{\Lambda}_{30}\widetilde{w}_{20} + \widetilde{\Lambda}_{31}\widetilde{w}_{11} +$$

$$+ \widetilde{\Lambda}_{32}\widetilde{w}_{02}) = \widetilde{C}_{m_o} = \frac{C_{m_o}}{l}. \qquad (17)$$

The disturbance axis velocity condition $\widetilde{u}$ is to be finite along the subsonic leading edges of the transformed wings (1) leading in this case to the relation of connection of the form:

$$F_2 \equiv \widetilde{A}_{20} + \widetilde{A}_{22} = 0, \qquad (18a)$$

$$F_3 = \widetilde{A}_{30} + \widetilde{A}_{32} = 0. \qquad (18b)$$

If relations (10a,b,c,d,e) are taken into consideration, these relations are written in the form:

$$F_2 \equiv \widetilde{T}_{20}\widetilde{w}_{10} + \widetilde{T}_{21}\widetilde{w}_{01} = (\widetilde{a}_{00}^{(2)} + \widetilde{a}_{20}^{(2)})\widetilde{w}_{10} + (\widetilde{a}_{01}^{(2)} + \widetilde{a}_{21}^{(2)})\widetilde{w}_{01} = 0, \qquad (19)$$

$$F_3 \equiv \widetilde{T}_{30}\widetilde{w}_{20} + \widetilde{T}_{31}\widetilde{w}_{11} + \widetilde{T}_{32}\widetilde{w}_{02} = (\widetilde{a}_{00}^{(3)} + \widetilde{a}_{20}^{(3)})\widetilde{w}_{20} +$$

$$+ (\widetilde{a}_{01}^{(3)} + \widetilde{a}_{21}^{(3)})\widetilde{w}_{11} + (\widetilde{a}_{02}^{(3)} + \widetilde{a}_{22}^{(3)})\widetilde{w}_{02} = 0. \qquad (20)$$

The expression of the coefficient of wave resistance of the transformed thin delta wing is in the form:

$$C_d = 8\int_{\widetilde{\sigma}\widetilde{A}_1\widetilde{\sigma}} \widetilde{w}_a \widetilde{u}_a \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y} = \widetilde{\Omega}_{2200}\widetilde{w}_{10}^2 + (\widetilde{\Omega}_{2210} +$$

$$+ \widetilde{\Omega}_{2201})\widetilde{w}_{10}\widetilde{w}_{01} + \widetilde{\Omega}_{2211}\widetilde{w}_{01}^2 + (\widetilde{\Omega}_{2300} + \widetilde{\Omega}_{3200})\widetilde{w}_{10}\widetilde{w}_{01} +$$

$$+ (\widetilde{\Omega}_{2301} + \widetilde{\Omega}_{3210})\widetilde{w}_{01}\widetilde{w}_{20} + (\widetilde{\Omega}_{2311} + \widetilde{\Omega}_{3211})\widetilde{w}_{01}\widetilde{w}_{11} +$$

$$+ (\widetilde{\Omega}_{2321} + \widetilde{\Omega}_{3212})\widetilde{w}_{01}\widetilde{w}_{02} + \widetilde{\Omega}_{3300}\widetilde{w}_{20}^2 + \widetilde{\Omega}_{3311}\widetilde{w}_{11}^2 +$$

$$+\tilde{\Omega}_{3322}\tilde{w}_{02}^2 + (\tilde{\Omega}_{3310} + \tilde{\Omega}_{3301})\tilde{w}_{20}\tilde{w}_{11} + (\tilde{\Omega}_{3320} +$$

$$+\tilde{\Omega}_{3302})\tilde{w}_{20}\tilde{w}_{02} + (\tilde{\Omega}_{3321} + \tilde{\Omega}_{3312})\tilde{w}_{11}\tilde{w}_{02},$$

$$\left(\tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1}\right). \tag{21}$$

Continuing, the following definite integral is marked with $\tilde{\mathcal{J}}_k$:

$$\tilde{\mathcal{J}}_k = \int_0^1 \frac{\tilde{y}^k d\tilde{y}}{\sqrt{1-\tilde{y}^2}}, \tag{22a}$$

$$\tilde{\mathcal{J}}_{2t} = \frac{\pi(2t)!}{2^{2t+1}(t!)^2} \qquad (k = 2t), \tag{22b}$$

$$\mathcal{J}_{2t+1} = \frac{2^{2t}(t!)^2}{(2t+1)!} \qquad (k = 2t+1). \tag{22c}$$

From which for k = 0,1,2,3,4,5 the following values for $\mathcal{J}_k$ result:

$$\mathcal{J}_0 = \frac{\pi}{2}, \qquad \mathcal{J}_1 = 1, \tag{23a}$$

$$\mathcal{J}_2 = \frac{\pi}{4}, \qquad \mathcal{J}_3 = \frac{2}{3}, \tag{23b}$$

$$\mathcal{J}_4 = \frac{3\pi}{16}, \qquad \mathcal{J}_5 = \frac{8}{15}. \tag{23c}$$

With these notations $\tilde{\Omega}_{nmkj}$, the constants, which appear in the expression (21) of wave resistance $\tilde{C}_d$ of the transformed thin delta wing can be written under the form

$$\tilde{\Omega}_{2mkj} = \frac{8}{m+2}(\tilde{a}_{0j}^{(2)}\tilde{\mathcal{J}}_k + \tilde{a}_{2j}^{(2)}\tilde{\mathcal{J}}_{k+2}) \tag{24}$$

$$(m = 2,3, \quad k = 0,1,\ldots,(m-1), \quad j = 0,1\ )$$

and, respectively,

$$\tilde{\Omega}_{3mkj} = \frac{8}{m+3}\left[\tilde{a}_{0j}^{(3)}\tilde{\mathfrak{I}}_k + \left(\tilde{a}_{2j}^{(3)} + \frac{\tilde{\sigma}_{2j}^{(3)}}{k+3}\right)\tilde{\mathfrak{I}}_{k+2}\right] \qquad (25)$$

$$(m = 2,3 \quad k = 0,1,\ldots,(m-1), \quad j = 0,1,2).$$

To explain by means of example, in this case, the coefficients are in the form:

$$\Omega_{2200} = \frac{\pi}{2}(2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}), \qquad \tilde{\Omega}_{2210} = \frac{2}{3}(3\tilde{a}_{00}^{(2)} + 2\tilde{a}_{20}^{(2)}), \qquad (26a)$$

$$\tilde{\Omega}_{2201} = \frac{\pi}{2}(2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}), \qquad \tilde{\Omega}_{2211} = \frac{2}{3}(3\tilde{a}_{01}^{(2)} + 2\tilde{a}_{21}^{(2)}) \qquad (26b)$$

for n = 2 and m = 2,

$$\tilde{\Omega}_{2300} = \frac{2\pi}{5}(2\tilde{a}_{00}^{(2)} + \tilde{a}_{20}^{(2)}), \qquad \tilde{\Omega}_{2310} = \frac{8}{15}(3\tilde{a}_{00}^{(2)} + 2\tilde{a}_{20}^{(2)}), \qquad (27a)$$

$$\tilde{\Omega}_{2301} = \frac{2\pi}{5}(2\tilde{a}_{01}^{(2)} + \tilde{a}_{21}^{(2)}), \qquad \tilde{\Omega}_{2311} = \frac{8}{15}(3\tilde{a}_{01}^{(2)} + 2\tilde{a}_{21}^{(2)}), \qquad (27b)$$

$$\tilde{\Omega}_{2320} = \frac{\pi}{16}(4\tilde{a}_{00}^{(2)} + 3\tilde{a}_{20}^{(2)}), \qquad (27c)$$

$$\tilde{\Omega}_{2321} = \frac{3\pi}{10}(4a_{01}^{(2)} + 3a_{21}^{(2)}) \qquad (27d)$$

for n = 2 and m = 3,

$$\tilde{\Omega}_{3200} = \frac{2\pi}{5}(2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}), \qquad \tilde{\Omega}_{3210} = \frac{8}{15}(3\tilde{a}_{00}^{(3)} + 2\tilde{a}_{20}^{(3)}), \qquad (28a)$$

$$\tilde{\Omega}_{3201} = \frac{2\pi}{15}(6\tilde{a}_{01}^{(3)} + 3\tilde{a}_{11}^{(3)} + \tilde{\sigma}_{21}^{(3)}), \qquad \tilde{\Omega}_{3211} = \frac{4}{15}(6\tilde{a}_{01}^{(3)} + 4\tilde{a}_{21}^{(3)} + \tilde{\sigma}_{21}^{(3)}), \qquad (28b)$$

$$\tilde{\Omega}_{3202} = \frac{2\pi}{5}(2\tilde{a}_{00}^{(3)} + \tilde{a}_{21}^{(3)}), \qquad \tilde{\Omega}_{3212} = \frac{8}{15}(3\tilde{a}_{02}^{(3)} + 2\tilde{a}_{22}^{(3)}), \qquad (28c)$$

for n = 3 and m = 2 respectively,

$$\tilde{\Omega}_{3300} = \frac{2\pi}{5}(2\tilde{a}_{00}^{(3)} + \tilde{a}_{20}^{(3)}), \qquad \tilde{\Omega}_{3310} = \frac{4}{9}(3\tilde{a}_{00}^{(3)} + 2\tilde{a}_{20}^{(3)}), \qquad (29a)$$

$$\tilde{\Omega}_{3302} = \frac{\pi}{3}(2\tilde{a}_{02}^{(3)} + \tilde{a}_{22}^{(3)}), \qquad \tilde{\Omega}_{3311} = \frac{2}{9}(6\tilde{a}_{01}^{(3)} + 4\tilde{a}_{21}^{(3)} + \tilde{\sigma}_{21}^{(3)}), \qquad (29b)$$

$$\tilde{\Omega}_{3320} = \frac{\pi}{12}(4\tilde{a}_{00}^{(3)} + 3\tilde{a}_{20}^{(3)}), \qquad (29d)$$

$$\tilde{\Omega}_{3321} = \frac{\pi}{60}\left(20\tilde{a}_{01}^{(3)} + 15\tilde{a}_{21}^{(3)} + 3\tilde{\sigma}_{21}^{(3)}\right), \tag{29e}$$

$$\tilde{\Omega}_{3322} = \frac{\pi}{12}\left(4\tilde{a}_{02}^{(3)} + 3\tilde{a}_{22}^{(3)}\right) \tag{29f}$$

for n = 3 and m = 3.

The equations (1), (9) obtained through canceling all of the variation of coefficients $\delta\tilde{w}_{0q}$ are, in this case, of the form:

$$2\tilde{\Omega}_{2200}\tilde{w}_{10} + (\tilde{\Omega}_{2201} + \tilde{\Omega}_{2210})\tilde{w}_{01} + (\tilde{\Omega}_{3200} + \tilde{\Omega}_{2300})\tilde{w}_{20} + (\tilde{\Omega}_{3201} + \tilde{\Omega}_{2310})\tilde{w}_{11} + (\tilde{\Omega}_{3202} + \tilde{\Omega}_{2320})\tilde{w}_{02} + \lambda^{(1)}\tilde{\Lambda}_{20} + \lambda^{(2)}\tilde{\Gamma}_{20} + \tilde{\lambda}_2\tilde{T}_{20} = 0, \tag{30}$$

$$(\tilde{\Omega}_{2210} + \tilde{\Omega}_{2201})\tilde{w}_{10} + 2\tilde{\Omega}_{2211}\tilde{w}_{01} + (\tilde{\Omega}_{3212} + \tilde{\Omega}_{2312})\tilde{w}_{20} + (\tilde{\Omega}_{3211} + \tilde{\Omega}_{2311})\tilde{w}_{11} + (\tilde{\Omega}_{3212} + \tilde{\Omega}_{2321})\tilde{w}_{02} + \lambda^{(1)}\tilde{\Lambda}_{21} + \lambda^{(2)}\tilde{\Gamma}_{21} + \tilde{\lambda}_2\tilde{T}_{21} = 0, \tag{31}$$

$$(\tilde{\Omega}_{2300} + \tilde{\Omega}_{3200})\tilde{w}_{10} + (\tilde{\Omega}_{2301} + \tilde{\Omega}_{3210})\tilde{w}_{01} + 2\tilde{\Omega}_{3300}\tilde{w}_{20} + (\tilde{\Omega}_{3301} + \tilde{\Omega}_{3310})\tilde{w}_{11} + (\tilde{\Omega}_{3302} + \tilde{\Omega}_{3320})\tilde{w}_{02} + \lambda^{(1)}\tilde{\Lambda}_{30} + \lambda^{(2)}\tilde{\Gamma}_{30} + \tilde{\lambda}_3\tilde{\Gamma}_{30} = 0, \tag{32}$$

$$(\tilde{\Omega}_{2310} + \tilde{\Omega}_{3201})\tilde{w}_{10} + (\tilde{\Omega}_{2311} + \tilde{\Omega}_{3211})\tilde{w}_{01} + (\tilde{\Omega}_{3310} + \tilde{\Omega}_{3301})\tilde{w}_{20} + 2\tilde{\Omega}_{3311}\tilde{w}_{11} + (\tilde{\Omega}_{3312} + \tilde{\Omega}_{3321})\tilde{w}_{02} + \lambda^{(1)}\tilde{\Lambda}_{31} + \lambda^{(2)}\tilde{\Gamma}_{31} + \tilde{\lambda}_3\tilde{T}_{31} = 0, \tag{33}$$

$$(\tilde{\Omega}_{2320} + \tilde{\Omega}_{3202})\tilde{w}_{10} + (\tilde{\Omega}_{2311} + \tilde{\Omega}_{3211})\tilde{w}_{01} + (\tilde{\Omega}_{3320} + \tilde{\Omega}_{3302})\tilde{w}_{20} + (\tilde{\Omega}_{3321} + \tilde{\Omega}_{3312})\tilde{w}_{11} + 2\tilde{\Omega}_{3322}\tilde{w}_{02} + \lambda^{(1)}\tilde{\Lambda}_{32} + \lambda^{(2)}\tilde{\Gamma}_{32} + \tilde{\lambda}_3\tilde{T}_{32} = 0. \tag{34}$$

The equations (30 - 34), together with the relations of connection (16), (17), (18a,b), (19) and (20), form a linear system of algebraic equations from which is determined optimum values of the coefficients $\tilde{w}_{10}, \tilde{w}_{01}, \tilde{w}_{20}, \tilde{w}_{11}$ and $\tilde{w}_{02}$ of the vertical velocity of disturbance $\tilde{w}$, as well as the values of the multipliers $\lambda^{(1)}, \lambda^{(2)}, \tilde{\lambda}_2$ and $\tilde{\lambda}_3$ for a given value of the parameter of similitude $\nu = B\ell$.

$$
\begin{vmatrix}
2\Omega_{2200} & \Omega_{2201}+\Omega_{2210} & \Omega_{2300}+\Omega_{3200} & \Omega_{2310}+\Omega_{3201} & \Omega_{2320}+\Omega_{3202} & \Lambda_{20} & \Gamma_{20} & 1 & 0 \\
\Omega_{2201}+\Omega_{2210} & 2\Omega_{2211} & \Omega_{2301}+\Omega_{3210} & \Omega_{2311}+\Omega_{3211} & \Omega_{2321}+\Omega_{3212} & \Lambda_{21} & \Gamma_{21} & 0 & 0 \\
\Omega_{2300}+\Omega_{3200} & \Omega_{2301}+\Omega_{3210} & 2\Omega_{3300} & \Omega_{3301}+\Omega_{3310} & \Omega_{3302}+\Omega_{3320} & \Lambda_{30} & \Gamma_{30} & 0 & 1 \\
\Omega_{2310}+\Omega_{3201} & \Omega_{2311}+\Omega_{3211} & \Omega_{3301}+\Omega_{3310} & 2\Omega_{3311} & \Omega_{3312}+\Omega_{3321} & \Lambda_{31} & \Gamma_{31} & 0 & 0 \\
\Omega_{2320}+\Omega_{3202} & \Omega_{2321}+\Omega_{3212} & \Omega_{3302}+\Omega_{3320} & \Omega_{3312}+\Omega_{3321} & 2\Omega_{3322} & \Lambda_{32} & \Gamma_{32} & 0 & 0 \\
\Lambda_{20} & \Lambda_{21} & \Lambda_{30} & \Lambda_{31} & \Lambda_{32} & 0 & 0 & 0 & 0 \\
\Gamma_{20} & \Gamma_{21} & \Gamma_{30} & \Gamma_{31} & \Gamma_{32} & 0 & 0 & 0 & 0 \\
T_{20} & T_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & T_{30} & T_{31} & T_{32} & 0 & 0 & 0 & 0
\end{vmatrix}
$$

APPENDIX 1

Ifthe order of unknowns $\tilde{w}_{10}, \tilde{w}_{01}, \tilde{w}_{20}, \tilde{w}_{11}, \tilde{w}_{02}, \lambda^{(1)}, \tilde{\lambda}^{(2)}, \tilde{\lambda}_1$ and $\tilde{\lambda}_3$ is chosen presupposing the following order of equations in system: (30) - (34), (16),(17), (19) and (20), then the quadratic matrix for rank m = 9 of the system of coefficients formed from the liniar equations mentioned above has the form in appendix(1), and different independent single terms for zero appear in the sixth (16) and seventh (17) equations and have the values $\frac{C_{l_0} B}{\nu}$ and $\frac{C_{m_0} B}{\nu}$. respectively. Because the solution of the system is not changed for diverse values of the initial givens (C_{l_0} and C_{m_0}) the matrix of independent terms is considered to be a rectangular matrix R of the type m x n = 9 x 2,

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ \vdots & \vdots \\ R_{91} & R_{92} \end{pmatrix}. \qquad (35)$$

in which $R_{ij} = 0$ with the exception of the terms $R_{61} = 1$ and $R_{72} = 1$. If the solutions of the system obtained for the two sets of values for liniar terms R_{i1} and R_{i2} is noted with $\tilde{w}'_{ij}$ and $\tilde{w}''_{ij}$ (i = 1, ...,9) then the optimum values of the coefficients $\tilde{w}_{ij}$ have the values given by the relation

$$\tilde{w}_{ij} = \frac{C_{L_0} B}{\nu} \tilde{w}'_{ij} + \frac{C_{m_0} B}{\nu} \tilde{w}''_{ij}. \qquad (36)$$

For the determination of the optimum form of projection in a plane at the Mach flight number of M = 2, the range of admissible values of the span of the wing $l = \frac{l_1}{h_1}$ the range of values contained between $0{,}2 \leq l \leq 0{,}55$ is considered (the value $l = 0.5774$ corresponds to the sonic leading edge for Mach cruizing speed M = 2).

Therefore, the following range of values admissible for ν result :

$$0{,}3464 \leq \nu \leq 0{,}9526. \qquad (37)$$

A set of values of the parameter of similitude ν contained in this range of admissible values is considered, with the help of the following, (21) $(C_d)_{opt}$eight is calculated and the curve $(C_d)_{opt}$eight = f(v) (for B = constant) which has the aspect in figure 3, is drawn.

With the help of this curve the optimal value of the parameter of similitude $\check{\chi}$ is graphically determined (for which this curve touches its minimum).

The optimum value $\check{\chi}_0$ of the parameter of similitude $\check{\chi}$, together with the given area of projection in the plane, determines the optimum form of the initial thin delta wings' projection on the plane.

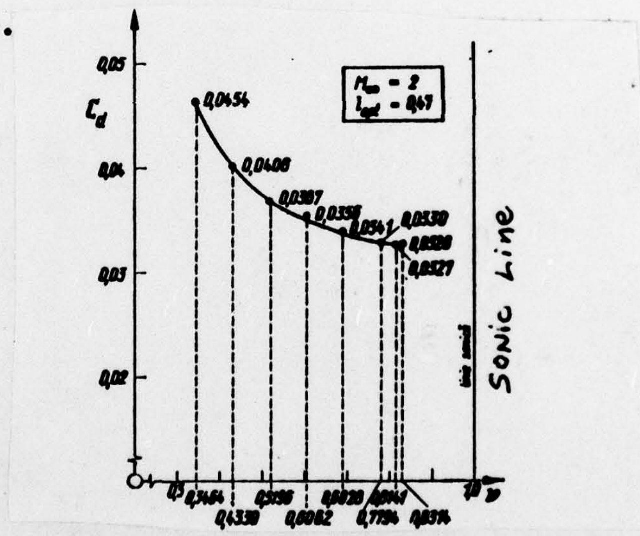

Figure 3

To make it clear, consider if you will, the following given initially:

- the number of Mach speed, $M_\infty = 2$,
- the coefficient of lift, $C_{\chi_l} = 0,3$,
- the coefficient of the moment of pitch, $C_m = 0,23$,
- the area of projection of the plane, $A_0 = 75$ m^2,

the optimum form of the projection on the plane of the initial thin delta wing is the isosceles triangle which has a height of $h_1 = 12.63$ m and a base of $2l_1 = 11.86$ m, and the optimum unit span is $l = \frac{l_1}{h_1} \approx 0.47$.

The minimum value of wave resistance of the initial thin delta wing is given by the relation:

$$C_d = l\tilde{C}_d = 0.327. \quad (3\ldots)$$

Accepted by the editors on Dec. 29, 1972

BIBLIOGRAPHY

1) NĂSTASE ADRIANA, *Eine graphisch-analitische Methode zur Bestimmung der optimum optimorum-Form des dünnen Deltaflügels in Überschallströmung*, RRST Méc. Appl. **18**, *4* (1973).
2) LEGENDRE R., *Nappes de tourbillons en cornets aux bords d'attaque d'une aile en delta*, La Rech. Aeron. *70*, Mai 1959.
3) GERMAIN P., *La théorie des mouvements homogènes et son application au calcul de certaines ailes delta en régime supersonique*. Rech. Aeron. *7*, 3—16 (1949).
4) CARAFOLI E., IONESCU M., *Ecoulements coniques d'ordre supérieur autours des ailes triangulaires où à épaisseur symétrique*, Rev. Méc, Appl. **2**, *1* (1957).
5) CARAFOLI E., MATEESCU D., NĂSTASE ADRIANA, *Wing Theory in Supersonic Flow*, cap. II Pergamon Press (1969).
6) NĂSTASE ADRIANA, *Forme aerodinamice optime prin metoda variaţională* (cap. IV). Edit. Academiei, 1969.
7) CARAFOLI E., NĂSTASE ADRIANA, *Étude des ailes triangulaires minces à symétrie forcée*, Rev. Méc. Appl. **3**, *4* (1958).
8) CARAFOLI E., NĂSTASE ADRIANA, *Minimum Drag Thin Delta Wing with Finite Velocity at the Leading Edges, in Supersonic Flow*, Rev. Méc. Appl. **11**, *2* (1967).
9) CARAFOLI E., NĂSTASE ADRIANA, *Minimum Drag Thin Triangular Wing in Supersonic Flow*, Rev. Méc. Appl. **5**, *4* (1960).

1) NASTASE ADRIANA, A graphical-analysis method of determining the best shape of the thin delta wing in supersonic flow, RRST Mech. Appl. 18, 4 (1973).

2) LEGENDRE R., Rolled-up vortex sheets at the leading edges of a delta wing, La Rech. Aeron. 70, May 1959.

3) GERMAINE P., The theory of homogeneous movements and its application to the calculation of certain delta wings at supersonic speed. Rech. Aeron. 7, 3-16 (1949).

4) CARAFOLI E., IONESCU M., High-order conical flows about the triangular or symmetrically thick wing. Rev. Mec. Appl. 2, (1957).

5) CARAFOLI E., MATEESCU D, NASTASE ADRIANA, Wing Theory in Supersonic Flow, cap. 11., Pergamon Press (1969).

6) NASTASE ADRIANA, Optimal Aerodynamic Form Using the Variational Mehtod.

7) CARAFOLI E., NASTASE ADRIANA, Study of Thin Triangular Forced Symmetry Wings, Rev. Mec. Appl 3, 4 (1958).

8) CARAFOLI E., NASTASE ADRIANA, Minimum Dray Thin Delta Wing with Finite Velocity at the Leading Edges, in Supersonic Flow, Rev. Mech Appl. 11, 2 (1967).

9) CARAFOLI E., NASTASE ADRIANA, Minimum Drag Thin Triangular Wing in Supersonic Flow, Rev. Mec. Appl. 5, 5 (1960).

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION		MICROFICHE
A205	DMATC	1
A210	DMAAC	2
B344	DIA/RDS-3C	8
C043	USAMIIA	1
C509	BALLISTIC RES LABS	1
C510	AIR MOBILITY R&D LAB/FIO	1
C513	PICATINNY ARSENAL	1
C535	AVIATION SYS COMD	1
C591	FSTC	5
C619	MIA REDSTONE	1
D008	NISC	1
H300	USAICE (USAREUR)	1
P005	ERDA	1
P005	CIA/CRS/ADB/SD	1
NAVORDSTA (50L)		1
NASA/KSI		1
AFIT/LD		1

ORGANIZATION		MICROFICHE
E053	AF/INAKA	1
E017	AF/RDXTR-W	1
E403	AFSC/INA	1
E404	AEDC	1
E408	AFWL	1
E410	ADTC	1
E413	ESD	2
	FTD	
	CCN	1
	ASD/FTD/NICD	3
	NIA/PHS	1
	NICD	2