

AD-A063 771

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO  
INVESTIGATION OF THE GEOID SHAPE IN THE TATRA MOUNTAINS. (U)  
JUN 78 M BARLIK

F/G 8/7

UNCLASSIFIED

FTD-ID(RS)T-0839-78

NL

| OF |  
AD  
A063771



END  
DATE  
FILMED  
3-79  
DDC

①

FOREIGN TECHNOLOGY DIVISION



INVESTIGATION OF THE GEOID SHAPE  
IN THE TATRA MOUNTAINS

by

Marcin Barlik

DDC  
RECEIVED  
JAN 22 1979  
F



Approved for public release;  
distribution unlimited.

AD-A063771

78 12 26 287

# EDITED TRANSLATION

FTD-ID(RS)T-0839-78 15 June 1978

MICROFICHE NR: *FD-78-C-000828*

INVESTIGATION OF THE GEOID SHAPE  
IN THE TATRA MOUNTAINS

By: Marcin Barlik

English pages: 13

Source: Geodezja i Kartografia, Warsaw, Volume 36,  
Number 3, 1977, pages 211-217.

Country of origin: Poland

Translated by: Linguistics Systems, Inc.  
F33657-76-D-0389  
F. Zaleski

Requester: DMAAC/SDDL  
Approved for public release;  
distribution unlimited.

SEARCHED BY	
INDEXED	White Card <input checked="" type="checkbox"/>
FILED	Red Card <input type="checkbox"/>
CLASSIFIED	<input type="checkbox"/>
EXTENDED	<input type="checkbox"/>
BY	
DISTRIBUTION/AVAILABILITY CODES	
GEN. AVAIL. and/or SPECIAL	
<b>A</b>	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:  
TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-afb, OHIO.

# Investigation of the Geoid Shape in the Tatra Mountains

Marcin Barlik

## 1. Introduction

In the elaboration below we offer several notes connected with the usefulness of the gravimetric data for investigating the geoid shape in the area of the Polish Tatra Mountains. They are connected with the manner of realizing basic models of astronomic-gravimetric levelling. The writer took part in studies connected with investigating the effect of the mountain environment on the results of geodesic measurements in an experimental net. This weave was composed of seven stations on which geodesic observations were performed (measurement of distance, level and vertical angle measurements) having as their purpose the designation of changes in atmospheric refraction, which consequently served to the connected adjustment of observations in this local triangular area related to an ellipsoid as a reference surface. The ellipsoid has been "applied" in ~~Figure~~ point No. 1 in the net presented in Figure 1. This Figure is cited here from publication (5). Its authors presented a connected adjustment of a trigonometric levelling net with regard to vertical deflection and refraction. On the drawing triangles indicate so-called intermediate points whose use is discussed in a later part of the article. Points of the test net designated by circles have considerable height above sea level. The lowest No. 1 has a height of  $H_1 = 1123$  m., the highest placed No. 7--  
 $H_7 = 2092$  m. From the patterns given in the publication previously cited, namely

to proceed according to the method of adjusting intermediate observations and for the adjustment of conditional observations, needs arise--first, finding components of relative vertical deflections in observational stations--second, determining ellipsoidal excesses. The latter are the total of the difference of the orthometric heights and geoid excesses over the ellipsoid between the traverse points.

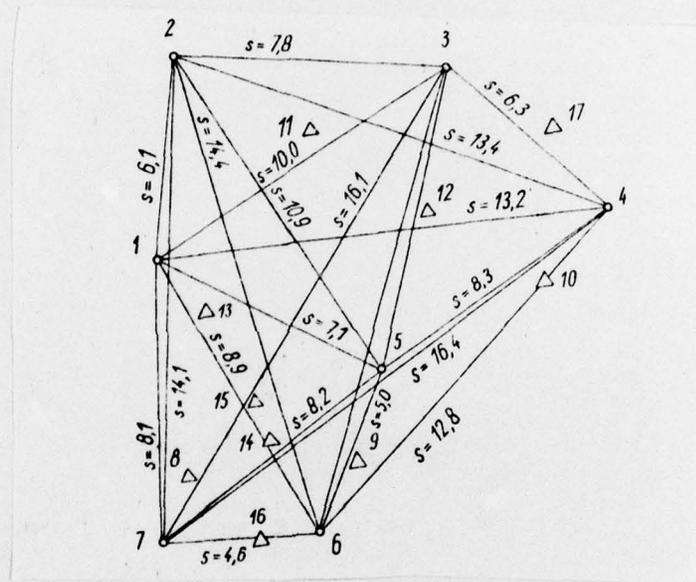


Figure 1.

## 2. Investigation of geoid shape relative to local reference ellipsoid

### 2.1. Concept of determining relative vertical deflections

In this section we shall touch on the two following problems. The first of them is the method of determining gravimetric vertical deflections connected with

the use of proper gravimetric reduction and field of integration of anomalies for passage to relative vertical deflections. The second is the suitable selection, in mountain conditions, of points of astronomic and gravimetric levelling to achieve the required error of geoid excesses.

The choice of the most suitable gravimetric reduction in mountain areas to designate vertical deflections has already been treated earlier in publications (1) and (2). There it was shown that for a procedure compatible with Stokes' theory the truest is Faye's exact reduction. Because of the difficulties encountered in investigating the gravitational field in mountains (sculpture, rare gravimetric photos) we use an intermediate procedure, supported on the use of maps of Bouguer's anomalies and topographic maps. In this manner in the process of anomaly integration the equation realized is

$$A_F = A_B + 2\pi k\sigma H \quad (1)$$

between Faye's anomaly ( $A_F$ ) and Bouguer's anomaly ( $A_B$ ). In the above equation the symbol  $\sigma$  signifies the density of subsurface layers while H is the height of the terrain. Using Faye's reduction requires consideration of geoid deformation in the calculated vertical deflections. This is accomplished by introducing an additional term in the formulae of Vening Meinesz, namely

$$\begin{aligned} \Delta\xi &= -0.024 \frac{\partial H}{\partial x} H, \\ \Delta\eta &= -0.024 \frac{\partial H}{\partial y} H \sec\varphi, \end{aligned} \quad (2)$$

where the symbol  $\varphi$  signifies geographic width. The differentiation runs in a direction to the north and east.

At this point it is worth citing the work of A. Elmiger (3) on the determination of vertical deflections in the triangulation in Switzerland. There, a procedure is used supported on the principle of the existence of isostations in a mountain area according to the Pratt-Hayford hypothesis. We did not use this method in investigating the geoid shape in the area of experimental traverse in the Tatra Mountains as shown in Figure 1.

Next we shall discuss the specifics in the selection of limits of anomaly integration to calculate vertical deflections in the stations of an experimental traverse. This problem requires division of the field of anomaly integration into a series of zones and sectors. The division into rectangular elements, true to calculations by using computers, does not hasten the determination of vertical gravimetric deflections in the mountains and requires great concentration of dots on the diagram around the stations. For a field of anomalies in a mountain area an important matter is the division of the so-called central zone and further zones of integration. This projects on requirements having to do with concentration of gravimetric points in the central zone and accuracy of measurements. In mountain areas difficult land conditions ~~sometimes~~<sup>sometimes</sup> bluntly prevent measurement by a gravimeter in peak parts. On the other side the course of anomaly is more complex than on flat terrains. Using a whole series of practical tests in the area of

the Polish Tatra Mountains it was found that the greatest influence is noted in a zone up to about 11 km. from the station. The influence of further parts of the field of anomaly on components of vertical deflections is already considerably less and variable in a linear manner only at a distance of about 6 km. from the gravimetric point. From the sketch in Figure 1 we see that the majority of sight lines is longer by 6 km. since the stations are placed on the tops of the elevations in the Tatra Mountains. From this arises the necessity of introducing the so-called intermediate points for the purpose of increasing accuracy in the interpolation process of relative vertical deflections. These points must be located in the bands of valleys separating the stations of the experimental traverse.

According to the analysis of the procedure presented by W. Jeremiejew (4) a variant is selected to calculate components of vertical deflection which is supported by numerical integration according to Gauss' points. This method however, must undergo modification. The radius of the zones was chosen in such a way that Gauss' five points used by Jeremiejew lie in the centres of gravity of each of the segments of the constructed diagram. The length of the radii equal respectively in the first zone--469 m. (12 sectors), in the second zone--1839 m. (12 sectors), in the third zone--3160 m. (24 sectors), in the fourth--5000 m. (24 sectors), in the fifth--7.31 km. (32 sectors) and 10.69 km. in the sixth zone which is divided into 32 sectors. The values of coefficients used to calculate the influence of anomalies in sequential zones equal:  $-0''.044$ ;  $-0''.018$ ;  $-0''.005$ ;  $-0''.0025$ ;  $-0''.0025$ . We have left the division of the further part of the field of anomalies

compatibly with Jeremiejew's concept. In eight points out of 17 presented in Figure 1 the following empirical determination was made. The effect of the field of anomalies integrated within limits of 11 to about 100 km. does not change in a linear way in a traverse area and thus we should designate it for each point. The effect of the field of anomalies from about 100 to about 300 km. changes in the area of the experimental net in a field of from  $0''.12$  to  $0''.59$  for the component  $\xi$  and from  $1''.05$  to  $1''.73$  for the  $\eta$  component of the vertical deflection. Thus their values for the station points numbered from 1 to 8 are designated. For the points numbered from 9 to 17 the effect of  $\Delta\xi_{100-300}$  and  $\Delta\eta_{100-300}$  is interpolated in a linear way between the stations of the traverse. The average error of the interpolation of relative vertical deflection served as an evaluation of the validity of such a procedure. If we interrupt the integration of anomalies on the border of 100 km. from the station then for our net in the Tatra Mountains the mentioned average error equals  $\pm 1''.85$ . Thus it exceeds the desirable value twice. At this point we should mention the unsuitability of empirical formulae elaborated by Molodienski (6) and Pellinen (7) engaging the dependence of the error of defining the excess of the quasigeoid on the extent of the borders of the integration of anomalies with respect to the specifics of the shape of anomalies in the mountains and the distance between astronomic stations. The authors of the publications (6) and (7) use the average anomaly ( $\overline{\Delta g_m}$ ) near the boundary of the field of anomaly. One of the mentioned formulae used for mountain areas has the form

$$m_c = \pm 21 \cdot 0''.11 \frac{\overline{\Delta g_m}}{\rho^2 - 1} \quad (3)$$

where  $l$  denotes the distance of levelling points expressed in kilometers,  $e$  is the relation of the extent of the limits of integration to the distance  $l$ . The quantity  $\overline{\Delta g_m}$  is expressed in milligals. In the mountains for the dense astronomical net as in our experimental field and for the application of the range of integration (300 km.) we would obtain an excess error of about  $\pm 9$  cm. which is not ~~covered~~ <sup>covered</sup> with the results of estimations. However to achieve an excess error of the geoid  $m_N = \pm 0.02$  m. requirements on the basis of these formulae would extend the limits of integration to about 500-600 km.

Here is the estimation of the average error the components of the gravimetric deflection of the vertical. . It was calculated on the basis of the two-time designations at the seven basic traverse stations. Only the errors of the representation of Faye's anomalies and the errors of frequency anomalies in the segments of the diagram are characterized.

Composed of the following parts are:

- a) error of designating the effect of Bouguer's field of anomalies to 11 km. from the point evaluated on  $m_1 = m_{\xi_1} = m_{\eta_1} = \pm 0'16,$
- b) error of designating the effect of the sculpture of the terrain to about 11 km. from the point compatible with the formula (1), equalling  $m_1 = m_{\xi_1} = m_{\eta_1} = \pm 0'23,$
- c) error of designating the effect of the field of Faye's anomalies from 11 to about 100 km, evaluated at  $m_2 = m_{\xi_2} = m_{\eta_2} = \pm 0'13,$

d) error of  $\epsilon$  designating the effect of the field of anomalies within limits of 100 to 300 km. from the point  $m_s = m_{\epsilon_s} = m_{\eta_s} = \pm 0''18$ .

The complete error of the p components of the gravimetric vertical deflection in view of the above values equals

$$m_{\xi} = m_{\eta} = \sqrt{m_1^2 + m_2^2 + m_3^2} = \pm 0''36.$$

For a real evaluation of the accuracy of the values of the error of components of the gravimetric deviation of the vertical it would be necessary to increase it to about  $\pm 0''5$ , which we accept for further estimations. In this way we consider roughly the effect of the errors of gravimetric and height measurements and the error interpolation of isoanomalies.

Let us continue on to the next problem mentioned at the beginning of this section. We shall describe below the specifics of defining the relative vertical deflection. Designations of ~~astronomic~~ astronomic co-ordinates were made at four points of the net. These are the points numbered 1, 3, 6 and 7. The average error of the designations of astronomic co-ordinates were estimated by elaboration of the observation on  $m_{\varphi} = m_{\lambda} = \pm 0''45$ .

These points also have geodesic co-ordinates B and L. On these points (on the geoid) the components of vertical deflections are  $\xi$  designated which create differences in the observed astronomic co-ordinates  $\varphi$  and  $\lambda$ ; reduced to a geoid and in the geodesic co-ordinated B and L. The sizes of the reducing expressions,

considering the angular effect of the curvature of the vertical line, calculated by using Bouguer's anomaly maps and a topographic map (equation (1) was used) varied within limits of  $0''.5-0''.1$  for the northern component and  $0''.7-0''.1$  for the eastern component. The interpolation expressions for calculating relative vertical deflections for the remaining points of the traverse were accepted in the form of a polynomial of the first order

$$\xi^{ns} = \xi^{nr} + bx_1 + ly_1 + z_1 \quad (4a)$$

for the northern component and

$$\eta^{ns} = \eta^{nr} + bx_2 + ly_2 + z_2 \quad (4b)$$

for the eastern component. Coefficients  $b$  and  $l$  signify the differences: of width of  $s$  astronomic point and pole of transformation, or

$$b_i = \varphi_i - \varphi_0$$

and of length of these same points, or

$$l_i = \lambda_i - \lambda_0$$

Using the method compatible with the method of the smallest sum of the square of corrections of the unknowns  $x_1, y_1, z_1$  and  $x_2, y_2, z_2$  were found. As we already mentioned above, the reference point was station No. 1 where  $\xi^{ns} = \eta^{ns} = 0$ .

The measure of accuracy of interpolation is the average error of the designation of the components of the relative vertical deflection. Its value in our net equals

$$m_{A\xi} = \pm 0''.43 \quad ; \quad m_{A\eta} = \pm 0''.48. \quad \text{For further estimations we take } m_{A\xi}^1 = m_{A\eta}^1 = \pm 0''.45.$$

Despite only one supernumerary observation (4 astronomic points minus 3 unknown interpolation coefficients = 1 supernumerary point) the average error of interpolation should be counted with the small ones. The chief reason of such a

useful solution is above all the relatively small span of the net. Summing the values of errors  $m_{\xi} = m_{\eta} = \pm 0''.50$ , the error of astronomic observations  $m_{\varphi} = m_{\lambda} = \pm 0''.45$  and  $m_{\Delta\xi} = m_{\Delta\eta} = \pm 0''.45$  we obtain the average error defining the components of the relative vertical deflection in the traverse area

$$M_{\xi} = M_{\eta} = \pm 0''.80$$

## 2. 2. Designation of geoid distances from local ellipsoid

The essence of astronomic-gravimetric levelling depends on the realization of the formula

$$\begin{aligned} \Delta N_{ik} &= -\frac{1}{2} (\xi_i \cos A_{ik} + \eta_i \sin A_{ik} + \xi_k \cos A_{ik} + \eta_k \sin A_{ik}) \Delta S_{ik} \\ &= -0,2424 \cdot \Delta S_{ik} [(\xi_i + \xi_k) \cos A_{ik} + (\eta_i + \eta_k) \sin A_{ik}], \end{aligned} \quad (5)$$

where  $\Delta S_{ik}$  denotes the distance of points in the profile of the sight line between stations in kilometers and  $\xi_i, \xi_k, \eta_i, \eta_k$  are the components of relative vertical deflections at the starting and ending points. The symbol  $A_{ik}$  signifies the azimuth of the profile. An analysis of the pattern (5) for the conditions answering to the test field shows that the error of defining the excess of the geoid at point No. 4 to be the most distant from point No. 1 can reach a value of about  $\pm 5$  cm. With that, the error of defining the components of the relative vertical deflection has the greatest influence. Errors of designating the distance between the points and the azimuth cause the effect of one order less for the longest sight line. However, it must be proven that the definition of geoid excesses directly from the point of application to the traverse stations in order is laden with great uncertainty, with respect to

undulations of the local geoid type. Thus intermediate points were chosen in the Tatra Mountain valleys separating the basins of elevations on which the stations of the experimental traverse were established.

In general the calculations of geoid excesses were performed on 15 profiles between stations. Intermediate points entered the sight line profiles: point 11 for sight 1-3, point 12 for sight 1-4, point 13 for sight 1-5, points 13, 15, 14 for sight 1-6, intermediate points 13, 15, 8 for sight 1-7. Moreover, geoid excesses were calculated along the remaining sight lines of the test net, namely: for the sight 6-7 intermediate point 16 was considered, for sight 5-6 point 9, for sight 4-5 point 10, for sight 3-4 point 17, for sight 2-3 point 11, for profile 3-5 point 12, for sight 5-7 points 8 and 14, for sight 2-5 point 11 and finally for profile 2-4 points 11 and 12. From the calculated geoid excesses between stations 10 conditional equations were established describing the closing of the traverses from the astronomic-gravimetric levelling span. The maximum term allowed in the set of conditional equations equalled 0.068 m. for the length of a traverse of about 14 km. After adjustment the average error of the levelling span equalled  $m_{\Delta N} = \pm 0.012$  m., which becomes about  $\pm 3$  mm. for 1 km. of circuit. This error characterizes the influence of the faultiness of designations of elements in the realization of pattern (5) on the calculated geoid height over the local reference ellipsoid applied in point No. 1. However, it has the character of evaluation of interior compatibility of the designation of excesses.

## BIBLIOGRAPHY

- [1] Barlik M., *Pewne aspekty badania rzeczywistych odchylen pionu w terenach górskich*, Geodezja i Kartografia, t. XXIII, z. 4/1974.
- [2] Barlik M., *Wyznaczanie rzeczywistych odchylen pionu w górach*, Geodezja i Kartografia, t. XXV, z. 3/1976.
- [3] Elmiger A., *Lotabweichungen im Schweizerischen Triangulationsnetz 1. Ordnung*, Vermessung-Photogrammetrie-Kulturtechnik, nr 3/1972, Zürich.
- [4] Jeremiejew W. F., *Raszcziot palietki dla wyczislenija wysot kwazigieoida i uklonienij otwiesia po formulam Stoksa i Wening-Mejnesa*, Trudy CNIIGAiK, wyp. 121, Moskwa 1957.
- [5] Makowska A., Zorski Z., *Analiza wyrównania trygonometrycznej sieci wysokościowej z uwzględnieniem odchylen pionu i refrakcji*, Geodezja i Kartografia, t. XXV, z. 1/1976.
- [6] Molodiński M. S., Jeremiejew W. E., Jurkina M. I., *Metody izuczenija wnieszniewo grawimetriczeskogo pola i figury ziemi*, Trudy CNIIGAiK, wyp. 131, Moskwa 1960.
- [7] Pellin L. P., *Triebowanija k grawimetriczeskoj sjemkie, swjazannyje s obrabotkoj astronomo-geodieziczeskich i niwielirnych sietiej*, Trudy CNIIGAiK, wyp. 139, Moskwa 1960.

### 3. Summary

In this study we emphasized the qualities differentiating the investigation of the geoid in mountainous terrains using an astronomic-gravimetric levelling method from a procedure in a flat terrain. The most important feature of this process is the necessity of individual adjustment of the sphere of integration of the field of anomalies and the method of division into segments in the designated borders. The so-called central zone was enlarged to about 11 km. The necessity of such location of points used for astronomic-gravimetric levelling was also pointed out in order to best consider the shape of the geoid. Thus the placing of so-called intermediate points is indicated, which are not traverse stations in the mountain valleys. This improves the results of astronomic-gravimetric levelling. The investigation of the shape of a geoid in the mountains should be preceded by an investigation of the variability of the effect of the field of anomalies depending on the distance of the astronomic points and the individual qualities of the field. Thus we cannot use the empirically shown patterns which do not fulfill completely the purpose of estimating the accuracy designating the geoid shape for mountain areas.

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION	MICROFICHE	ORGANIZATION	MICROFICHE
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	8	E404 AEDC	1
C043 USAMIA	1	E408 AFWL	1
C509 BALLISTIC RES LABS	1	E410 ADTC	1
C510 AIR MOBILITY R&D LAB/FIO	1	E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
		ASD/FTD/NICD	3
C591 FSTC	5	NIA/PHS	1
C619 MIA REDSTONE	1	NICD	2
D008 NISC	1		
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P055 CIA/CRS/ADD/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		