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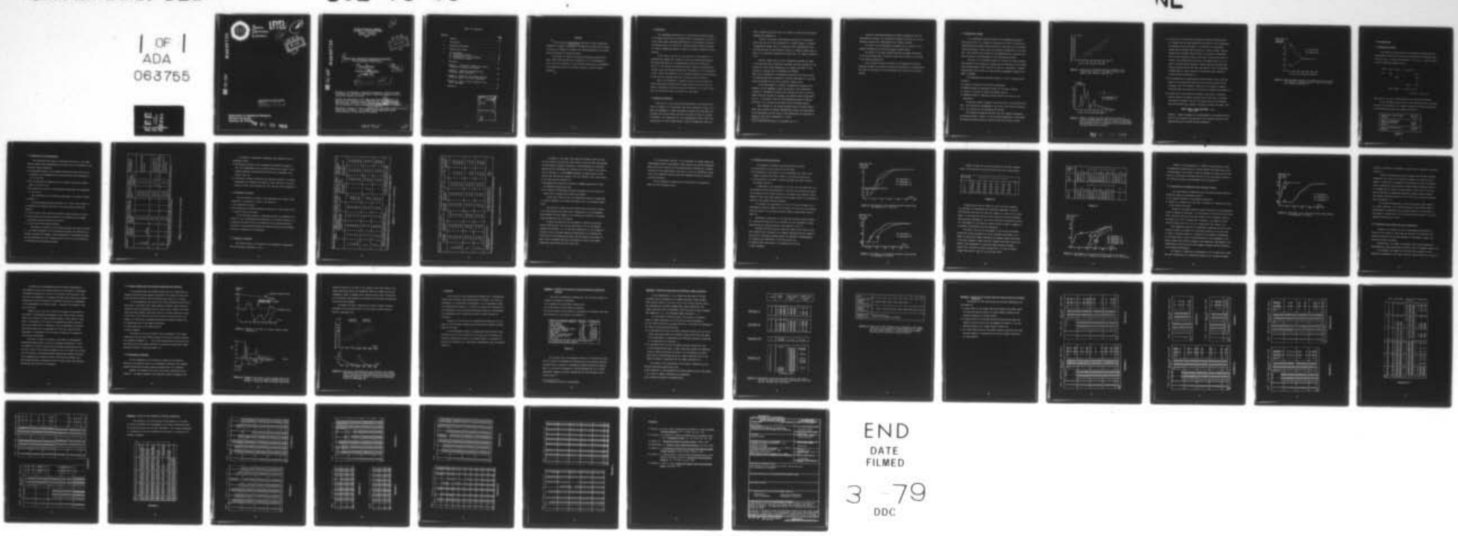

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Systems
Optimization
Laboratory

Department of Operations Research
Stanford University
Stanford, CA 94305

(12)

SYSTEMS OPTIMIZATION LABORATORY
DEPARTMENT OF OPERATIONS RESEARCH
Stanford University
Stanford, California
94305

(6)

COMPUTATIONAL EXPERIMENTS ON LARGE-SCALE OPTIMIZATION WITH THE DECOMPOSITION PRINCIPLE.

by

(10) Gerhard Schiefer*

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*Professor at the Department of Agricultural Economics, University of Kiel, West-Germany; on leave at the Department of Operations Research, Stanford University, Stanford, California.

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Abstract

In this paper, ~~we trace out~~ the solution of large optimization problems for regional planning are traced out by a decomposition program that is based on the use of standard LP programs. The underlying method is the decomposition principle of Dantzig and Wolfe.

The concept is tested by solving an optimization problem with about 1250 columns and 900 rows. Furthermore, it is investigated to what extent the efficiency of the solution procedure can be influenced by an appropriate choice of starting solutions or specific matrix divisions.

1. Introduction

The programming formulation of a large spatial equilibrium model of the agricultural sector of Germany, which is in preparation, results in a very large linear programming problem with a block-angular structure in the problem matrix. For solving it, existing standard computer programs for linear programming problems (standard LP programs) provide too little capacity.

In this paper, we trace out the solution of such problems by a decomposition program that is based on the use of Standard-LP-Programs. The underlying method is the decomposition principle of Dantzig and Wolfe [1]. The concept is tested by solving a similar but smaller problem with about 1250 columns and 900 rows. Furthermore, it is investigated to what extent the efficiency of the solution procedure can be influenced by the choice of starting solutions or specific matrix divisions.

Some of the results are of general interest for the solution of linear optimization problems with a similar matrix structure. They are based on experiments that have been partly reported in [5] and [6] and that are outlined in detail in section 4 of this paper.

2. Background information

Since 1970, a large spatial equilibrium model of the agricultural sector of Germany is in preparation. The first (completed) version of the model was designed as a linear optimization problem with about 8000 constraints. While this problem could still be solved by means of standard LP programs, the formulation of a more disaggregated version and/or the incorporation of other "Common Market" countries (suggestions which are under consideration) would result in a model too large for the available programs (see appendix 1).

However, the special (block-angular) structure of the problem matrix that was due to the distinction of a limited number of "regions" as aggregated economic units of production and/or consumption[1] would allow the solution of the problem by means of a computer program that is based on an appropriate decomposition method (see, for example, Geoffrion [2]).

However, despite the fact that decomposition methods are aimed at solving large-scale optimization problems, efficient programs for the solution of really large problems are not commonly available. Of the standard LP programs, only "LP 600/6000" and "OPTIMA" offer (as an alternative) a decomposition procedure that could be used for solving large-scale optimization problems. Unfortunately, these programs are designed for the use of specific computers only.

As the development of a specific decomposition program was not possible, it was suggested to base the solution of the optimization problem on a decomposition approach that allowed the use of one of the highly efficient standard LP programs. The approach that has been developed is, in principle, based on the decomposition method of Dantzig and Wolfe. It is outlined in more detail in the following chapter.

The efficiency of the concept has been tested in several experiments with a similar but smaller optimization problem. The organization of the experiments and the results of the computations are discussed in chapters 4 and 5 and in appendices 2, 3 and 4.

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1) For a detailed discussion of such models see [7].

During the following discussion, we assume a familiarity with the decomposition principle. Its application requires the division of the optimization problem into "subproblems" and a "master problem".

The matrix of each subproblem is formed by one or several of the diagonal "submatrices" of the block-angular problem matrix.

The matrix of the master problem includes the common constraints of the problem matrix and, in addition, might also include one or several of the diagonal submatrices.

In an iterative solution procedure, the decomposition principle generates columns for the master problem from solutions of the subproblems and objective functions for the subproblems from solutions of the master problem.

3. Computational concept

In an optimization process based on the decomposition method of Dantzig and Wolfe [1], standard LP programs can be used for solving the subproblems and the master problem in each of the cycles of the iterative solution procedure. In this case, only computer programs have to be developed that transform the solutions of the subproblems and the master problems according to the requirements of the decomposition method.

Like most of the available standard LP programs for large problems (see appendix 1), the standard LP program that was available for the experiments (program XDLA of ICL) could not be used as a subprogram in computer programs which were written in a user-oriented programming language like ALGOL or FORTRAN.

The experiments were, therefore, based on a cycle of programs which included:

1) standard LP program for solving the subproblems;
2) FORTRAN program for generating columns for the master problem;
3) standard LP program for solving the master problem;
4) FORTRAN program for computing revised objective functions for the subproblems.

During the solution procedure, this cycle had to be activated repeatedly. Each repetition corresponded to one cycle of the solution procedure (i.e., one iteration of the decomposition process).

The optimization approach differed from the original presentation by Dantzig and Wolfe insofar as the following suggestions for shortening the solution procedure could be realized within the outlined concept:

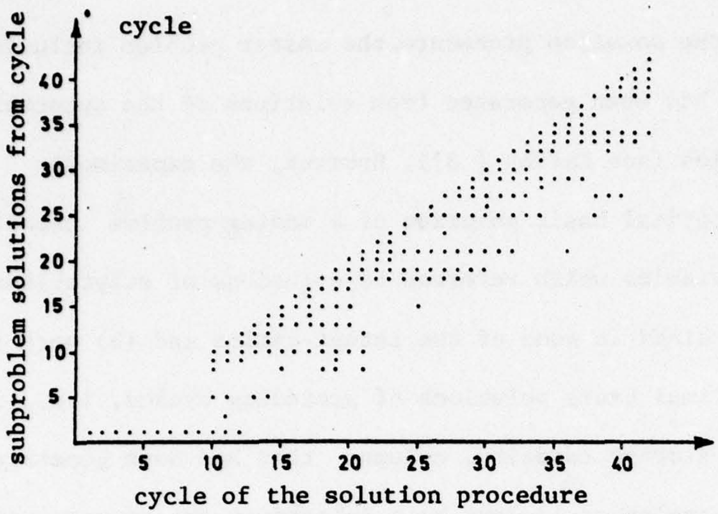

Exhibit 1: Solutions of subproblems that were included in the optimal basis solution of the master problem in different cycles (example: experiment I).

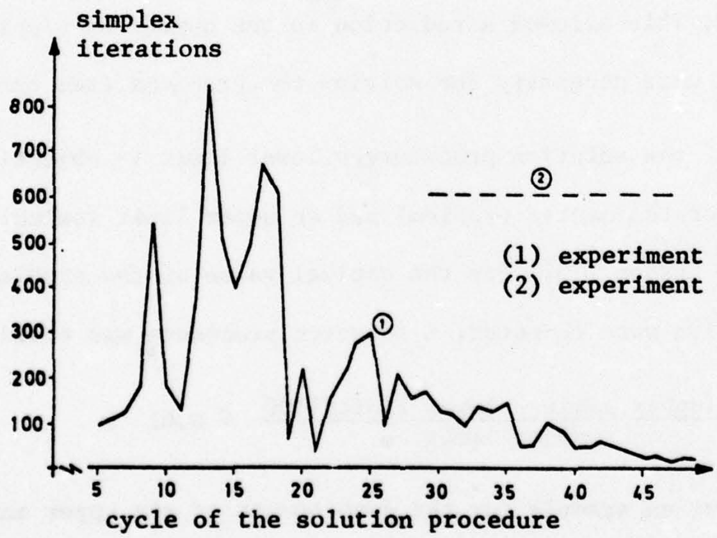

Exhibit 2: Number of simplex iterations that were necessary for solving the subproblems in different cycles. (In experiment VII, the optimizations were based on optimal basis solutions from preceding cycles; in experiment I, on the optimal basis solutions from the first cycle.)

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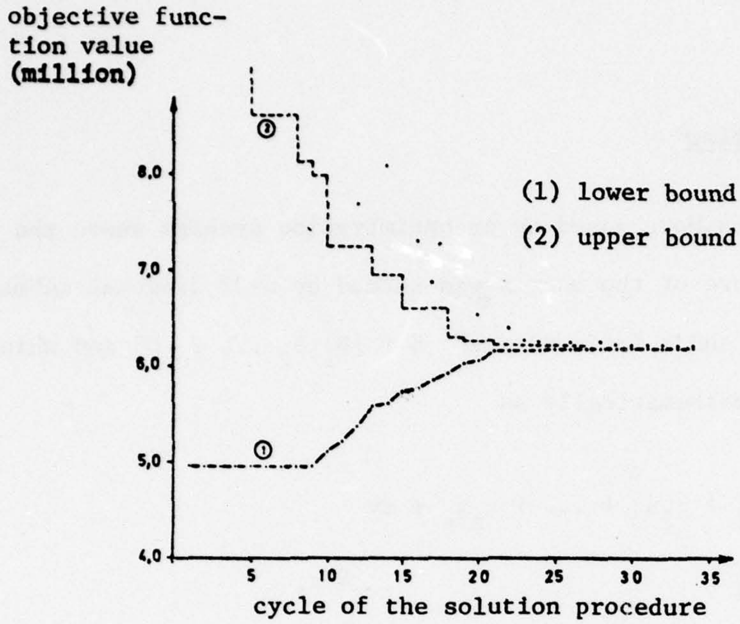
1) In any cycle of the solution procedure, the master problem included all columns that had been generated from solutions of the subproblems in preceding cycles (see Lasdon [3]). However, the experiments showed that the optimal basis solution of a master problem usually included only variables which referred to solutions of subproblems that (a) were obtained in some of the latest cycles and (b) were included in the optimal basis solutions of preceding cycles, i.e., in cases of limited storage capacity, columns that had been generated during preceding cycles could have been deleted if the corresponding variables were no longer realized in the optimal basis solution of a master problem (see exhibit 1).

2) The optimization of subproblems and master problem in each cycle was based on the optimal basis solutions that had been obtained in the preceding cycle. This allowed a reduction in the number of simplex iterations that were necessary for solving the problems (see exhibit 2).

3) In each cycle of the solution procedure, a lower limit (= objective function value of the master problem) and an upper limit (calculated as described in Lasdon [3]) for the optimal value of the problem's objective function were computed. A solution procedure was ended when

$$\frac{(\text{upper limit} - \text{lower limit})*100}{\text{upper limit}} \leq 0.01 \ .$$

(Exhibit 3 shows an example for the development of the upper and lower limit for the objective function value of the optimal solution of the problem during the iterative solution procedure.)



<u>Exhibit 3</u>: Upper and lower bound for the optimal value of the objective function in different cycles of the solution procedure (example: experiment I).

4. The experiments

4.1 Optimization problem

The experiments were based on an optimization problem where the block-angular structure of the matrix was formed by n=32 diagonal submatrices A_i (i=1,..,n) and a "main matrix" $H = [B_1\ B_2\ \ldots\ B_n\ D]$ and which could be formulated mathematically as

$$
\begin{array}{llllll}
\max z = & c_1x_1 & + c_2x_2 & + \ldots & + c_nx_n & + dy \\
\text{s.t.} & A_1x_1 & & & & \leq a_1 \\
 & & A_2x_2 & & & \leq a_2 \\
 & & & \ddots & & \vdots \\
 & & & & A_nx_n & \leq a_n \\
 & B_1x_1 & + B_2x_2 & + \ldots & + B_nx_n & + Dy = b \\
 & & & & & x_i, y \geq 0 \quad (i=1,..,n)
\end{array}
$$

with x_i(i=1,..,n) and y representing vectors of decision variables.

In the following, we will refer to the constraints that are connected with the main matrix H as "common" constraints. The size and density of the different matrices are outlined in exhibit 4.

Matrix size (rows,columns) Density	(907,1265) 0.71
Number of submatrices Size (rows,columns) Density	32 (28,39) 18.58
Size of the main matrix Density	(11,1265) 7.73

Exhibit 4

4.2 Organization of the experiments

The experiments were aimed at testing the efficiency of the computational concept and investigating ways of improving the convergence of the iterative solution process by

a) an appropriate choice of the feasible starting solution that has to be formulated at the beginning of the solution process (see experiments I, II, III and VII),

b) the incorporation of submatrices into the master problem (see experiments I, IV and V) and

c) the variation of the size of distinguished subproblems (see experiments VI, VII and X).

The organization of the different experiments is outlined in detail in exhibit 5 .

The distinguished starting solutions A, B, C and D (see column 2 of the exhibit) were feasible solutions of the optimization problem that differed

a) in the values of their objective functions (except solutions A and D),

b) in the values of the dual variables that corresponded to the common constraints (except solutions A and B) and

c) in the values of the primal variables.

The number of constraints in the master problem (see column 6) refers to the common constraints of the optimization problem, the additional constraints necessary for forming convex combinations of subproblem proposals and (in experiment IV and V) constraints that are due to the incorporation of submatrices into the master problem.

Experiment	Identification of starting solution	Deviation of the objective function value*)	Number of subproblems	Matrix size of subproblem (m,n)	Number of constraints in the master problem	Main aspect of investigation
I	A	-20 %	4	224,312	11 + 4	
II	B	-40 %	4	224,312	11 + 4	Different starting
III	C	-50 %	4	224,312	11 + 4	solutions
VII	D	-20 %	4	224,312	11 + 4	
VIII	A+B+C+D	-16 %	4	224,312	11 + 4	
I	A	-20 %	4	224,312	11 + 4	Incorporation of
IV	A	-16 %	3	224,312	11 + 224 + 3	submatrices into
V	A	-14 %	2	224,312	11 + 448 + 2	the master problem
VI	D	-20 %	8	112,156	11 + 8	Different size of
VII	D	-20 %	4	224,312	11 + 4	subproblems
X	D	-20 %	1	896,1248	11 + 1	

*) Deviation of the master problem's optimal objective function value from the optimal value of z at the beginning of the iterative solution procedure.

Exhibit 5 : Organization of the basic experiments.

In addition to these basic experiments some computations were performed in which

a) the objective function of the optimization problem was changed in one of the experiments after an optimal solution with regard to the original objective function had been reached (see experiment IX in chapter 4.36) and

b) differences in common constraints that had been observed in the basic experiments were examined with regard to their influence on the optimization process (see experiments XI, XII, XIII and XIV in appendix 2).

4.3 Discussion of results

We will discuss the results of the experiments with regard to the usefulness of an approach in terms of its effects on

a) the computation time and

b) the generation of "good" suboptimal solutions at an early stage of the iterative solution procedure.

In the following context, a suboptimal solution is considered to be "good" if it is a sufficiently good approach to the optimal solution, i.e., an approach that would allow the termination of the iterative solution procedure if some deviation between the computed solution and the optimal solution is considered to be acceptable.

4.31 Relevance of results

The results that are of relevance for an experiment's computation time are outlined in exhibits 6 and 7 .

Experiment	Number of cycles	Number of generated subproblem solutions (total)	Number of simplex iterations in the subproblems (average)			Number of simplex iterations in the master problem		Calculations of the type a=b+c*d (in 1000)	
			per sub-problem	per cycle	total	per cycle (average)	total	per cycle (average)	total
I	42	168	150	600	- [1)]	5	210 [1)]	28.7	1205
II	40	160	48	192	7680	5	200	28.7	1148
III	42	168	52	210	8809	5	210	28.7	1205
VII	49	196	57	228	11172	5	245	28.7	1406
VIII	40	172	50	201	8024	5	200	28.7	1148
I	42	168	150	600	- [1)]	5	210 [1)]	28.7	1205
IV	13	39	59	178	2310	63	819	21.5	280
V	10	20	59	118	1185	175	1753	14.4	144
VI	35	280	29	233	8184	7	245	28.7	1005
VII	49	196	57	228	11172	5	245	28.7	1406
X	103	103	138	138	14214	2	206	28.7	2956
Number of simplex iterations necessary for solving the problem without decomposition:						a) with starting solution A: 1729 b) with starting solution D: 1582			

1) The optimization was not based on the optimal basis solution of a preceding cycle.

Exhibit 6 : Some results of the experiments.

Experiment	Average core time (min:sec)			Average transfer time (min:sec)			Total time (hrs:min)	
	per sub-problem	per master problem	per cycle	per sub-problem	per master problem	per cycle	core time	core+transfer time
I	3:00	0.09	14:00 *)	5:25	4:30	34:55	-	- *)
II	1:11	0.09	7:11	5:25	4:30	34:55	4:47	28:03
III	1:18	0.09	7:39	5:25	4:30	34:55	5:21	29:47
VII	1:22	0.09	7:55	5:25	4:30	34:55	6:27	34:57
VIII	1:11	0.09	7:11	5:25	4:30	34:55	4:47	28:03
I	3:00	0.09	14:99 *)	5:25	4:30	34:55	-	- *)
IV	1:27	2:04	8:09	5:25	6:40	29:45	1:45	8:12
V	1:24	12:06	16:03	5:25	9:10	25:00	2:40	6:50
VI	0:27	0.09	6:03	3:45	6:30	48:30	3:31	31:48
VII	1:22	0.09	7:55	5:25	4:30	34:55	6:27	34:57
X	11:35	0.09	14:02	21:20	2:50	31:55	24:05	78:52
Solving the problem without decomposition:				a) with starting solution A			4:14	
				b) with starting solution D			4:00	

*) The optimizations were not based on the optimal basis solution of a preceding cycle.

Exhibit 7 : Some results of the experiments.

In exhibit 7, the terms "core time" and "transfer time" are used. In this context, the term "core time" refers to the time that was necessary for performing the simplex iterations in the subproblems and the master problems and for performing the calculations of type a=b+c*d, the calculation of indices, etc. in the FORTRAN programs. An identification of the relevant CPU time was not possible. The term "transfer time" refers mainly to the time that was necessary for

a) exchanging data between LP programs and FORTRAN programs (with discs as temporary storage units) and
b) the transfer of subproblems and master problems from the central processor unit to storage units (discs) and reverse.

At any time, there was only one optimization problem (a subproblem or a master problem) in the central processor unit for performing computations.

The specified average core and transfer times are closely related to the used computer facilities and the organization of the programming cycle whereas the other figures in the exhibits are related to the optimization problem that had to be solved.

It should be noted that the percentage of computation time that could be specified as core time was relatively low in all experiments (between 11% and 39%), i.e., an efficient organization of the storage facilities is one of the most important possibilities for increasing the efficiency of the optimization process. This is especially true with regard to the LP programs which were responsible for most of the needed computation time (usually for more than 75%).

Of a more general interest are the variations of results within the distinguished groups of experiments. Their analysis can provide information which could be of interest for the optimization of problems with a similar structure in the problem matrix. The same is true for a comparative evaluation of the suboptimal solutions that were generated during the iterative solution procedures.

The more important observations and conclusions are discussed in detail in the following sections.

4.32 <u>Different starting solutions</u>

A comparison of different starting solutions was based

a) on differences in their objective fuction value,

b) on deviations of the values of the dual variables that refer to the common constraints from their values in the optimal solution and

c) on deviations of the values of primal variables from their values in the optimal solution.

A comparison of the experiments I, II, III, and VII showed that it is not possible to judge the usefulness of a specific starting solution merely on the basis of one of these criteria, not to mention the problem of determining at least some rough estimates for the optimal values of the objective function or the primal and dual variables.

For example, the experiments I, II and III could be finished in an approximately identical number of cycles despite the fact that the objective function values of the starting solutions differed significantly (see exhibit 6).

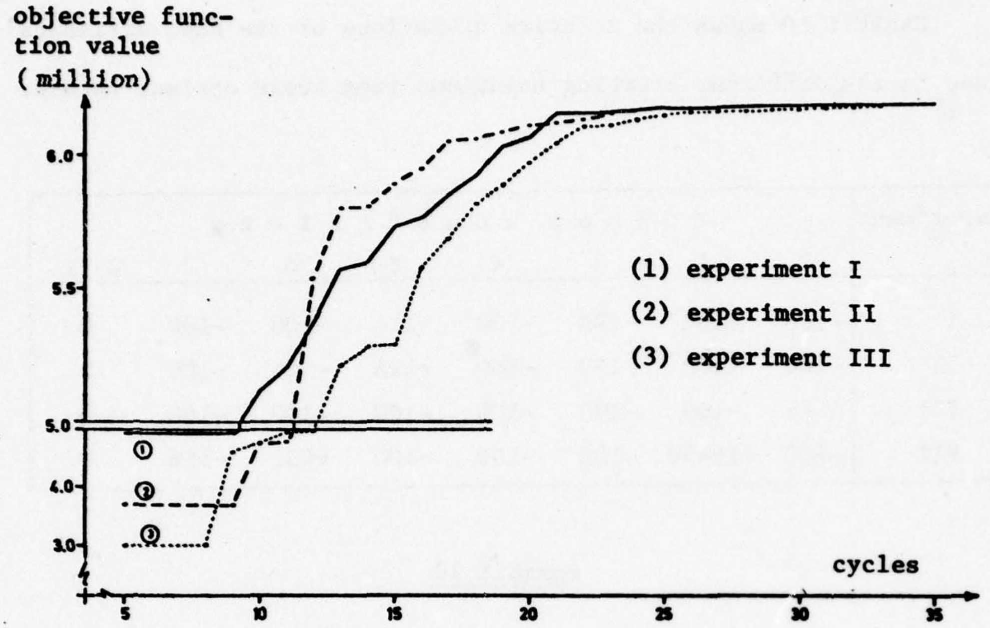
Furthermore, differences in the objective function values of different starting solutions were of no relevance for the development of these values during the solution procedure (see exhibits 8 and 9).

The same conclusion could be drawn with regard to the values of the dual variables that correspond to the common constraints and with regard to the values of the primal variables.which were examined by distinguishing

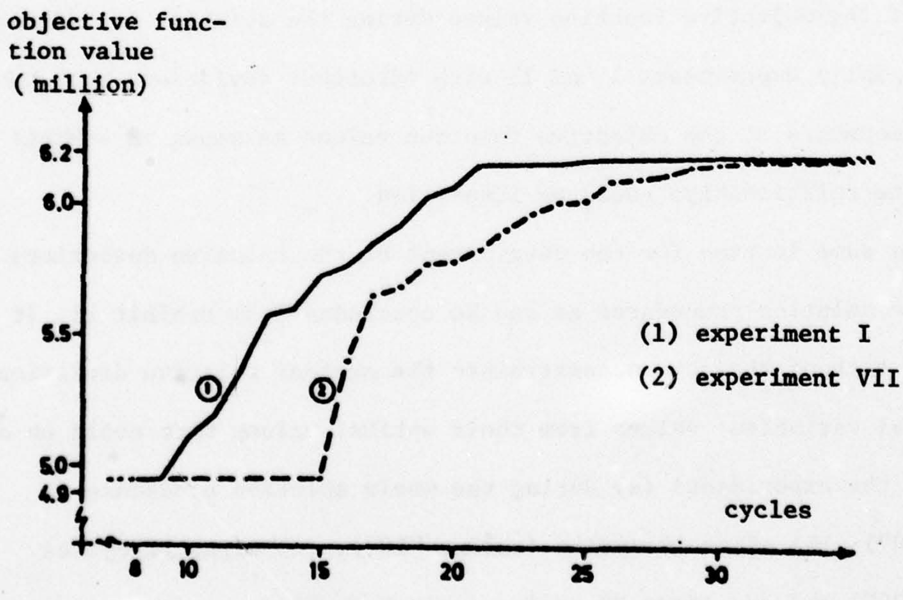
a) variables with coefficients in the main matrix only,

b) variables with coefficients in the submatrices only and

c) other variables.



<u>Exhibit 8</u>: Development of the objective function values during the experiments I, II and III.



<u>Exhibit 9</u>: Development of the objective function values during the experiments I and VII.

Exhibit 10 shows the relative deviations of the dual variables' values in the different starting solutions from their optimal values.

Experiment	Common constraints							
	1	2	3	4	5	6	7	8-11
I	-100	-100	-100	-100	+114	-100	-100	0
II	-100	-100	-100	-100	+114	-100	-100	0
III	+676	-100	-100	-100	-100	-100	-100	0
VII	-100	+15450	-100	-100	-100	+431	-196	0

Exhibit 10

Comparing them with the number of cycles that were necessary for performing the experiments (see especially experiments I and III with different deviations but identical numbers of cycles) and the development of the objective function values during the solution procedures (see especially experiments I and II with identical deviations but different developments of the objective function values as shown in exhibit 8) no definite relationships could be identified.

The same is true for the development of the relative deviations during the solution procedures as can be concluded from exhibit 11. It shows for each of the common constraints the maximal relative deviations of the dual variables' values from their optimal values that could be observed in the experiments (a) during the whole solution procedure (column "0"), (b) after 10 cycles (column "10"), (c) after 20 cycles (column "20") and (d) after 30 cycles (column "30").

Experiment	Common constraint 1				2				3				4			
	0	10	20	30	0	10	20	30	0	10	20	30	0	10	20	30
I	167	54	39	7	100	54	29	21	100	24	11	5	100	66	19	11
II	295	58	16	5	120	80	29	25	100	13	13	7	100	76	21	1
III	676	61	24	7	165	165	73	29	100	12	8	4	100	100	17	2
VII	199	199	34	8	15450/2329/29/29				100	63	10	10	100	100	45	3

	Common constraint 5				6				7				8-11
	0	10	20	30	0	10	20	30	0	10	20	30	
I	114	100	33	18	690	142	79	11	196	196	45	12	0
II	114	100	76	9	444	109	18	18	196	196	78	13	0
III	114	100	45	45	620	228	77	25	196	196	38	7	0
IV	114	114	100	21	690	690	132	38	196	196	99	31	0

Exhibit 11

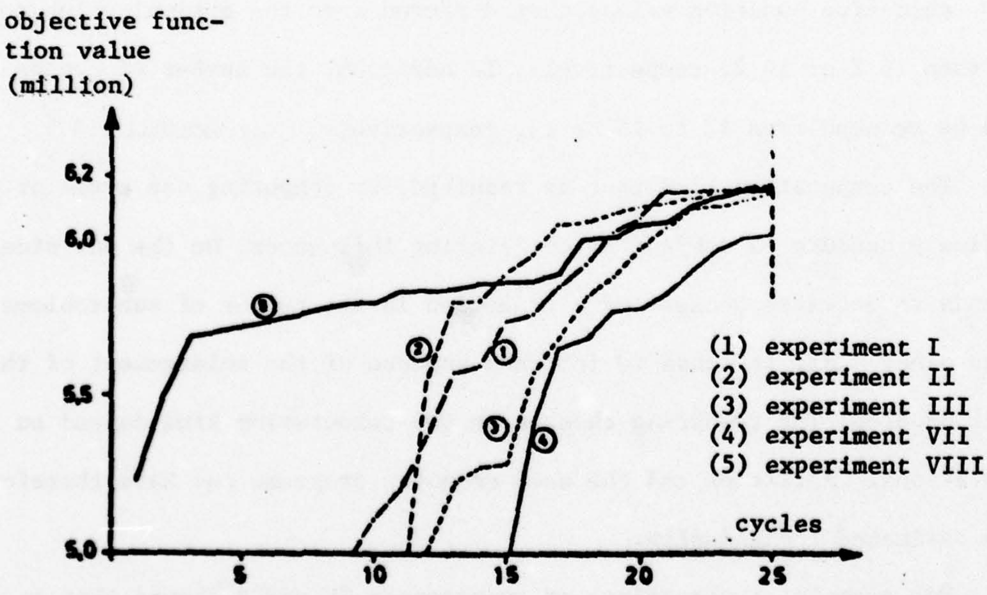


Exhibit 12: Development of the objective function value during experiment VIII (for a comparison see experiments I, II, III and VII).

However, the determination of a "useful" starting solution could be simplified by specifying not a single solution but several ones, each of which might be less useful or even not feasible but which could be combined to a more useful "starting solution" in the first master problem of the solution procedure (see experiment VIII and exhibit 12).

4.33 Incorporation of submatrices into the master problem

The incorporation of submatrices into the master problem (while reducing the number of subproblems) results

a) in a higher flexibility of the master problem and

b) usually in a change in the time that is required for computing one cycle of the solution procedure.

Because of the master problem's higher flexibility in the experiments IV and V compared to experiment I it was possible to reach in the first iteration objective function values that differed from the optimal value not more than 16 % or 14 %, respectively. In addition, the number of cycles could be reduced from 42 to 15 or 13, respectively. (See exhibit 13.)

The computation time that is required for computing one cycle of the solution procedure is subject to conflicting influences. On the one side, it tends to decrease because of a reduction in the number of subproblems, on the other side, it tends to increase because of the enlargement of the master problem. The resulting changes in the computation time depend on the computational facilities and the used computer programs and have, therefore, to be estimated individually.

For example, a comparison of experiments IV and V showed that in this case the incorporation of an additional submatrix into the master problem

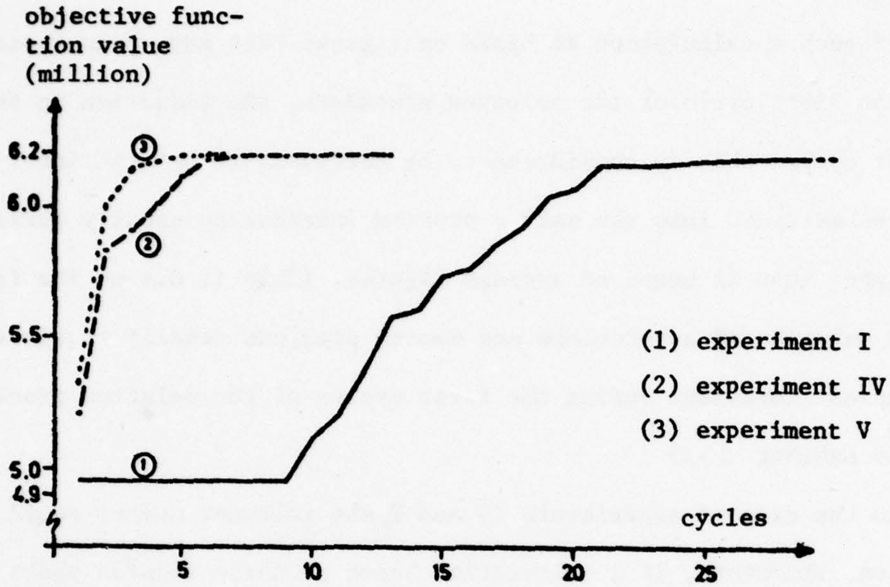


Exhibit 13: Development of the objective function values during the experiments I, IV and V.

would be of interest if the number of cycles could be reduced by (theoretically) 0.1.

If such a calculation is based on figures that have been obtained during the first cycle of the solution procedure, the reduction in the number of cycles that is considered to be necessary to make an incorporation of submatrices into the master problem interesting usually turns out to be higher than if based on average figures. (This is due to the fact that the solution of subproblems and master problems usually requires more simplex iterations during the first cycles of the solution procedure (see also exhibit 2).)

In the case of experiments IV and V, the relevant number would be 0.5 cycles. Therefore, if a calculation based on these figures shows an incorporation of submatrices into the master problem to be efficient, it can usually be implied that it will be efficient with regard to the (still unknown) average figures as well.

4.34 Variations in the number and size of subproblems

Changes in the number and size of distinguished subproblems influence (a) the number of cycles that are necessary for solving an optimization problem and (b) the time required for performing a single cycle within the solution procedure.

Theoretically, the number of necessary cycles can be minimized by distinguishing as many subproblems as possible (see, for example, Lasdon [3]).

As an example, the number of cycles that were necessary for performing the experiments X, VII and VI could be reduced from 103 to 49 and 35.

In addition, by distinguishing more but smaller subproblems it was possible to reduce the average time that was necessary for performing the simplex iterations in a cycle of the procedure. This was due to the fact that the computation of a simplex iteration takes less time in smaller problems and that this reduction in computation time was not compensated in the experiments by an observed slight increase in the number of simplex iterations.

However, due to the use of standard LP programs, the increase in the number of distinguished subproblems is only useful within a certain range. The activation of these programs requires a basic amount of time which is independent of the size of an optimization problem. Because of this, the average time for computing a cycle in experiment VI was about 500 seconds longer than in experiment VII, while the transition from experiment X to experiment VII still caused a reduction in computation time of about 250 seconds.

The extent to which an increase in the number of distinguished subproblems remains useful can only be estimated individually. If such an estimation is based on figures that have been obtained during one of the first cycles of the solution procedure, it has to be taken into account that the solving of subproblems usually requires more simplex iterations at the beginning than towards the end of the solution procedure, i.e., the possible savings in computation time are usually lower than indicated during the first cycles of the procedure.

4.35 General remarks about the values of primal and dual variables

In situations where the solution process has to be ended before an optimal solution has been reached, the values of the primal and dual variables and their deviations from the optimal values might be of interest.

In the experiments, the values of the primal variables usually varied only within a relatively small range within the feasible range during the iterative process (disregarding the very first cycles). However, the deviations from their optimal values could still be relatively high (with regard to the value of the objective function) in almost optimal solutions (see exhibit 14). There were no significant differences between variables with

a) coefficients only in the common constraints,

b) coefficients only in the submatrices and

c) other variables.

The values of the dual variables (that corresponded to the common constraints) that had been observed during the iterative solution processes are outlined in appendix 4. They varied significantly around their optimal values and usually showed some kind of convergence properties, as demonstrated in exhibit 15 (see also exhibit 11).

4.36 Postoptimal calculations

For the examination of the effects of changes in the objective function or the capacity vector of an optimization problem on the computed optimal solution, the iterative solution procedure has to be restarted.

However, the columns of the last cycle's master problem can be included in the master problem of the restarted process if changes in the

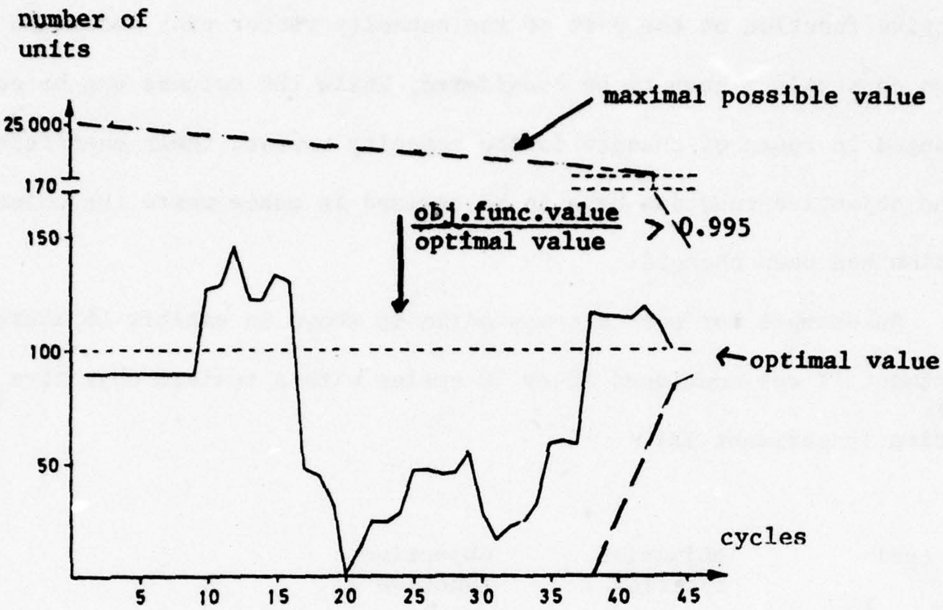


Exhibit 14: Changes in the value of a primal variable during experiment I.

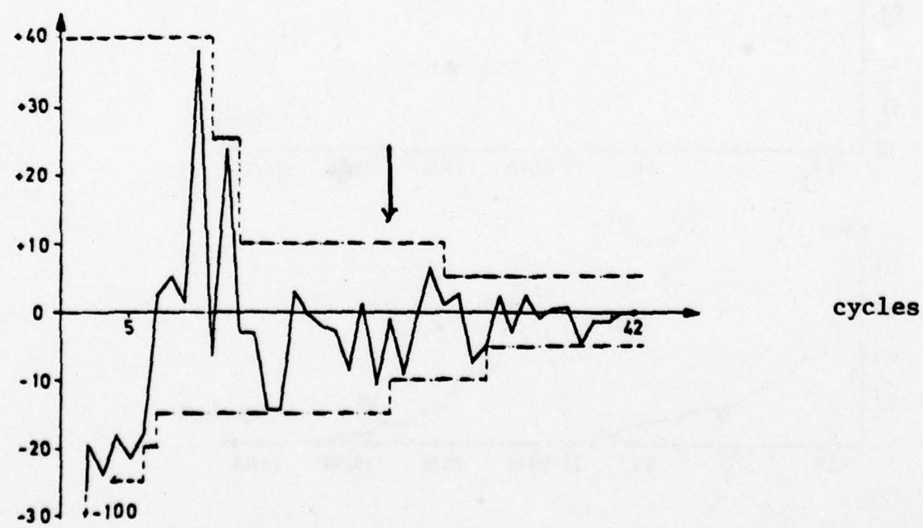


Exhibit 15: Changes in the value of a dual variable (that corresponded to one of the common constraints) during experiment I (deviations from the optimal value in %).

objective function or the part of the capacity vector that refers to the common constraints have to be considered. While the columns can be used unchanged in cases of changes in the capacity vector, their coefficients in the objective function have to be revised in cases where the objective function had been changed.

An example for such a computation is shown in exhibit 16, where experiment II was continued after 39 cycles with a revised objective function (experiment IX).

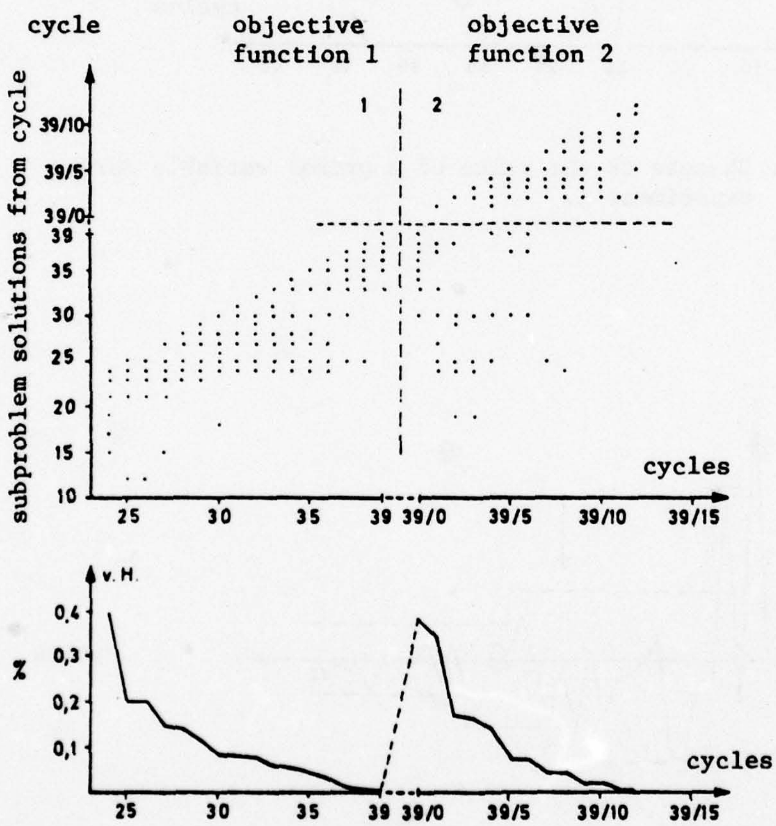


Exhibit 16: Solutions of subproblems that were included in the optimal basis solution of the master problem and the relative deviations of the objective functions' values from their optimal values when objective function 1 was replaced by objective function 2 (experiment IX).

5. Conclusion

The solution of large optimization problems with a decomposition program that is based on efficient standard LP programs is usually no alternative to a simultaneous optimization, i.e. an optimization without decomposition by means of a standard LP program. The realization of the computational concept requires additional efforts for establishing the programming cycle, organizing the data etc., which reduces its efficiency.

However, for the solution of optimization problems that are too large for the available computational facilities, the proposed procedure might be efficient.

As the experiments showed, an appropriate identification of subproblems could allow the reduction of core time that is necessary for solving an optimization problem to an amount similar to the amount of time that is necessary for a simultaneous optimization (see, for example, exhibit 5).

Appendix 1: Standard LP programs for solving large-scale optimization problems

The size of optimization problems that can be solved by means of standard LP programs is limited by

a) the capacity of the available computers and

b) the efficiency of the standard LP programs.

The crucial figure in this context is the number of constraints that have been formulated in the problem (see exhibit 17).

Name of the program (Company)	Rows (m)	Columns (n)
FMPS (UNIVAC)	8 192	unlimited
ILONA (UNIVAC)	m+n = 9 500	
LP 4004 (SIEMENS)*	6 000	32 767
LP 600/6000 (HONEYWELL BULL)*	4 095	262 000
MARIE CLAIRE (UNIVAC)	12 000	unlimited
MPSX (IBM)	16 383	unlimited
OPHELIE II (CONTROL DATA)*	10 000	unlimited
OPTIMA (CONTROL DATA)*	4 094	unlimited
XDLA (ICL)	25 000**	100 000

* Information from 1972, others 1973 (XDLA 1974)
** Problems with m=2500 reported

Exhibit 17

For problems with a block-angular structure in the matrix the standard LP programs "LP 600/6000" and "Optima" offer an alternative solution procedure which is based on a decomposition method developed by Orchard-Hays [4] and which is designed for solving problems with up to 50,000 constraints. However, practical experiences with this procedure seem to be very scarce.[1)]

1) Orchard-Hays in private correspondence.

Appendix 2: Additional experiments with different common constraints

In the experiments I - X, it turned out that some of the dual variables that corresponded to the common constraints always reached their optimal values in the first cycle of the solution procedure while the remaining dual variables changed their values iteratively. The common constraints that corresponded to the first group of dual variables were numbered 8 - 11, the remaining commom constraints 1 - 7.

For getting more information about the influence of different kinds of common constraints on the solution procedure, experiments XI, XII, XIII and XIV were outlined, basically as described for experiment X but with the following specifications:

1) In experiments XI and XII, the master problems included only one of the common constraints whereas the remaining common constraints were added to the subproblems. In experiment XI we considered explicitly constraint 11, in experiment XII constraint 1.
2) In experiments XIII and XIV, the master problems included only one of the distinguished groups of common constraints whereas the remaining group of common constraints was added to the subproblems. In experiment XIII we considered explicitly the common constraints 8-11 in the master problems, in experiment XIV the common constraints 1-7.

The results of the computations are outlined in exhibits 18 to 19. The most noteworthy results seem to be:

a) the similarity in the experiments XI and XII despite the fact that different kinds of common constraints were considered,
b) the solution procedure of experiment XIII.

Experiment	Cycle	Lower bound	Upper bound computed in cycle	"Lowest" computed upper bound
Experiment XI	1	4 955 266		
	2	.	6 268 201	6 268 201
	3	6 140 388	17 117 741	.
	4	6 181 931	6 481 359	.
	5	6 193 908	6 248 256	6 248 256
	6	6 194 023	6 194 531	6 194 531
	7	6 194 083	6 194 097	6 194 097
Experiment XII	1	4 955 266		
	2	.	8 559 708	8 559 708
	3	.	6 460 372	6 460 372
	4	6 038 806	6 279 672	6 279 672
	5	6 143 447	6 234 462	6 234 462
	6	6 187 820	6 195 371	6 195 371
	7	6 191 760	6 199 105	.
	8	6 193 735	6 194 327	6 194 327
	9	6 194 013	6 194 344	.
	10	6 194 082	6 194 083	6 194 083
Experiment XIII	1	4 955 266		
	2	6 194 083	6 194 083	6 194 083
Experiment XIV	1	4 955 266		
	2	.	108 019 791	108 019 791
	3	.	12 857 900	12 857 900
	4	.	15 858 291	.
	5	.	13 687 413	.
	6	.	16 955 389	.
	7	.	8 986 892	8 986 892
	8	.	12 328 003	.
	9	.	9 698 789	.
	10	.	9 230 287	.
	11	.	8 923 979	8 923 979
	12	.	9 760 913	.
	13	.	8 559 648	8 559 648
	14	.	7 945 981	7 945 981
	15	.	10 879 345	.
	16	.	12 621 243	.
	17	.	8 623 734	.
	18	.	8 204 922	.
	19	5 335 049	7 142 742	7 142 742
	20	5 425 695	14 211 884	.
	21	5 674 791	9 989 577	.
	22	5 678 287	7 737 021	.

experiment ended

Exhibit 18: **Development of the lower and upper bound for the optimal value of the objective function in experiments XI, XII, XIII and XIV. (Optimal value: 6,194,083).**

Cycle	1	2	3	4	5	6	7	8	9	10
Experiment XI: (common constraint 11)	0	-1073.6	-101.0	-68.2	-58.7	-58.6	-58.6	-	-	-
Experiment XII: (common constraint 1)	0	100.1	311.3	159.3	225.3	193.1	209.9	218.3	214.4	215.2
Experiment XIII: (common constraints 8-11)	optimal values in first cycle (remained unchanged)									

Exhibit 19: Values of the dual variables that corresponded to the common constraints in experiments XI (common constraint 11), experiment XII (common constraint 1) and experiment XIII (common constraints 8-11) during the solution procedure.

Appendix 3: Bounds for the optimal objective function values in the basic experiments

The exhibits in this section show for the basic experiments the development of

a) a lower bound for the optimal objective function value which equals the objective function value of a cycle's master problem (in the exhibits referred to as "lower bound"),

b) an upper bound for the optimal objective function value which was computed in each cycle in the way suggested by Lasdon [3] (in the exhibits referred to as "upper bound in cycle") and

c) the lowest upper bound for the optimal objective function value that had been computed in preceding cycles (in the exhibits referred to as "upper bound").

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	4 955 266		
2	.	212 517 341	212 517 341
3	.	12 993 362	12 993 362
4	.	14 345 848	.
5	.	8 626 938	8 626 938
6	.	15 172 051	.
7	.	9 356 478	.
8	.	8 142 341	8 142 341
9	.	7 985 506	7 985 506
10	5 113 444	7 241 441	7 241 441
11	5 201 369	11 496 234	.
12	5 373 734	7 686 574	.
13	5 576 173	6 934 658	6 934 658
14	5 600 763	8 088 533	.
15	5 734 756	6 603 113	6 603 113
16	5 765 017	7 299 943	.
17	5 853 308	6 872 263	.
18	5 920 986	6 297 866	6 297 866
19	6 031 620	6 274 664	6 274 664
20	6 068 026	6 548 404	.
21	6 155 005	6 221 113	6 221 113
22	6 158 179	6 400 293	.
23	6 158 278	6 270 954	.
24	6 164 156	6 245 333	.
25	6 175 879	6 205 714	6 205 714
26	6 176 095	6 245 292	.
27	6 179 075	6 206 331	.
28	6 179 384	6 252 788	.
29	6 180 533	6 217 509	.
30	6 186 087	6 204 441	6 204 441
31	6 186 576	6 219 973	.
32	6 188 631	6 202 570	6 202 570
33	6 190 127	6 199 092	6 199 092
34	6 190 849	6 198 342	6 198 342
35	6 191 401	6 209 804	.
36	6 191 678	6 199 230	.
37	6 192 020	6 196 573	6 196 573
38	6 193 424	6 195 243	6 195 243
39	6 193 735	6 197 707	.
40	6 193 785	6 194 768	6 194 768
41	6 193 822	6 194 673	6 194 673
42	6 193 879	6 194 324	6 194 324
Opt.	6 194 083		

Experiment I

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	3 716 450		
2	.	212 517 341	212 517 341
3	.	12 886 689	12 886 689
4	.	14 259 235	.
5	.	11 605 810	11 605 810
6	.	8 665 219	8 665 219
7	.	16 196 403	.
8	.	8 314 422	8 314 422
9	.	9 784 374	.
10	4 750 103	8 022 513	8 022 513
11	4 785 851	8 258 896	.
12	5 540 575	6 741 136	6 741 136
13	5 803 604	7 152 522	.
14	5 807 867	6 936 295	.
15	5 893 770	6 696 114	6 696 114
16	5 952 703	6 481 977	6 481 977
17	6 053 890	6 538 358	.
18	6 060 872	6 455 696	6 455 696
19	6 087 795	6 269 133	6 269 133
20	6 110 669	6 245 987	6 245 987
21	6 120 900	6 245 849	6 245 849
22	6 133 251	6 410	.
23	6 152 305	6 242 246	6 242 246
24	6 169 005	6 211 482	6 211 482
25	6 181 561	6 234 677	.
26	6 181 561	6 310 339	.
27	6 184 824	6 221 685	.
28	6 185 401	6 214 304	.
29	6 186 883	6 214 457	.
30	6 188 790	6 198 919	6 198 919
31	6 188 941	6 248 248	.
32	6 189 139	6 205 687	.
33	6 190 104	6 196 542	6 196 542
34	6 190 460	6 214 937	.
35	6 191 343	6 197 037	.
36	6 192 019	6 198 797	.
37	6 193 250	6 195 664	6 195 664
38	6 193 491	6 197 723	.
39	6 193 683	6 194 589	6 194 589
40	6 194 027	6 194 457	6 194 457
Opt.	6 194 083		

Experiment II

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	2 997 090		
2	.	223 334 657	223 334 657
3	.	25 342 983	25 342 983
4	.	12 497 970	12 497 970
5	.	9 446 001	9 446 001
6	.	16 280 151	.
7	.	8 038 380	8 038 380
8	.	10 261 482	.
9	4 616 759	9 228 777	.
10	4 713 943	12 611 518	.
11	4 940 078	8 959 236	.
12	4 988 826	8 901 547	.
13	5 212 468	7 038 385	7 038 385
14	5 274 927	7 033 457	7 033 457
15	5 290 075	7 487 558	.
16	5 579 064	6 836 243	6 836 243
17	5 685 244	7 366 260	.
18	5 819 794	7 194 596	.
19	5 887 304	6 683 289	6 683 289
20	5 979 091	6 489 596	6 489 596
21	6 043 182	6 265 268	6 265 268
22	6 106 552	6 313 636	.
23	6 110 099	6 390 107	.
24	6 130 099	6 227 602	6 227 602
25	6 151 247	6 205 422	6 205 422
26	6 158 898	6 278 221	.
27	6 159 593	6 294 854	.
28	6 160 541	6 243 568	.
29	6 168 317	6 217 935	.
30	6 174 044	6 212 724	.
31	6 176 735	6 203 989	6 203 989
32	6 184 994	6 201 048	6 201 048
33	6 187 831	6 201 200	.
34	6 187 831	6 203 355	.
35	6 193 143	6 196 134	6 196 134
36	6 193 173	6 202 623	.
37	6 193 298	6 197 473	.
38	6 193 298	6 196 331	.
39	6 193 335	6 196 944	.
40	6 193 390	6 194 787	6 194 787
41	6 193 508	6 197 333	.
42	6 193 890	6 195 000	.
43	6 193 921	6 194 468	6 194 468
44	6 194 012	6 194 109	6 194 109
Opt.	6 194 083		

Experiment III

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	5 202 708		
2	5 833 010	6 255 145	6 255 145
3	5 902 600	6 414 367	.
4	6 014 362	6 288 313	.
5	6 111 826	6 460 578	.
6	6 176 351	6 203 931	6 203 931
7	6 184 451	6 227 643	.
8	6 186 787	6 202 398	6 202 398
9	6 188 251	6 208 535	.
10	6 190 315	6 195 003	6 195 003
11	6 191 645	6 203 620	.
12	6 193 618	6 195 482	.
13	6 193 812	6 195 923	.
14	6 193 930	6 194 612	6 194 612
15	6 194 047	6 194 128	6 194 128
Opt.	6 194 083		

Experiment IV

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	5 327 386		
2	5 991 934	6 229 154	6 229 154
3	6 135 900	6 246 032	.
4	6 163 200	6 252 072	.
5	6 181 622	6 202 128	6 202 128
6	6 192 884	6 199 038	6 199 038
7	6 192 896	6 196 491	6 196 491
8	6 193 470	6 194 571	6 194 571
9	6 193 653	6 194 505	6 194 505
10	6 194 048	6 194 227	6 194 227
11	6 194 068	6 194 232	.
12	6 194 082.6432	6 194 085	6 194 085
13	6 194 083.0314	6 194 085	.
14		6 194 083.0314	6 194 083.0314
Opt.	6 194 083.0314		

Experiment V

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	4 955 266		
2	.	275 169 947	275 169 947
3	.	85 975 784	85 975 784
4	.	72 415 707	72 415 707
5	.	18 910 950	18 910 950
6	.	15 495 504	15 495 504
7	.	12 791 332	12 791 332
8	.	11 505 378	11 505 378
9	5 003 687	15 003 472	.
10	5 348 966	7 905 338	7 905 338
11	5 771 999	7 995 469	.
12	5 807 214	11 282 227	.
13	5 834 155	7 791 450	7 791 450
14	5 834 392	7 928 540	.
15	5 856 340	6 947 648	6 947 648
16	5 862 133	7 053 991	.
17	5 938 393	6 494 368	6 494 368
18	5 967 046	7 482 284	.
19	6 017 127	6 446 273	6 446 273
20	6 019 396	6 466	.
21	6 027 602	6 379 788	6 379 788
22	6 044 830	6 518 158	.
23	6 106 586	6 276 437	6 276 437
24	6 165 907	6 215 197	6 215 197
25	6 172 528	6 246 574	.
26	6 173 751	6 270 626	.
27	6 178 467	6 234 426	.
28	6 184 612	6 220 223	.
29	6 188 013	6 203 378	6 203 378
30	6 188 216	6 210 289	.
31	6 191 121	6 198 954	6 198 954
32	6 192 560	6 199 019	.
33	6 192 847	6 195 925	6 195 925
34	6 193 776	6 194 559	6 194 559
35	6 193 984	6 194 362	6 194 362
36	6 194 041	6 194 320	6 194 320
Opt.	6 194 083		

Experiment VI

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	5 227 982		
2	5 501 163	228 715 619	228 715 619
3	5 690 727	85 691 850	85 691 850
4	5 704 696	13 372 397	13 372 397
5	5 724 102	11 580 588	11 580 588
6	5 740 149	8 918 861	8 918 861
7	5 756 676	8 707 237	8 707 237
8	5 786 609	6 850 761	6 850 761
9	5 799 482	8 560 009	.
10	5 800 453	9 039 583	.
11	5 838 345	7 163 590	.
12	5 840 019	7 103 635	.
13	5 846 760	8 393 268	.
14	5 864 841	6 666 933	6 666 933
15	5 867 116	7 071 689	.
16	5 868 711	6 522 057	6 522 057
17	5 896 414	6 352 371	6 352 371
18	6 008 318	6 580 268	.
19	6 014 415	8 192 655	.
20	6 056 013	6 538 546	.
21	6 061 601	6 391 291	.
22	6 116 450	6 264 739	6 264 739
23	6 139 180	6 222 478	6 222 478
24	6 157 722	6 243 964	.
25	6 157 943	6 300 824	.
26	6 164 836	6 238 665	.
27	6 172 897	6 211 148	6 211 148
28	6 183 812	6 199 922	6 199 922
29	6 183 812	6 226 700	.
30	6 186 051	6 217 184	.
31	6 188 192	6 200 194	.
32	6 188 248	6 244 468	.
33	6 188 764	6 201 977	.
34	6 190 059	6 199 279	6 199 279
35	6 190 894	6 197 149	6 197 149
36	6 191 895	6 196 854	6 196 854
37	6 192 493	6 194 614	6 194 614
38	6 193 207	6 195 602	.
39	6 193 430	6 194 608	6 194 608
40	6 193 877	6 194 679	.
41	6 194 001	6 194 134	6 194 134
Opt.	6 194 083		

Experiment VIII

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	4 955 266		
2	.	420 978 653	420 978 653
3	.	18 889 849	18 889 849
4	.	32 725 385	.
5	.	13 258 772	13 258 772
6	.	19 824 079	.
7	.	8 907 348	8 907 348
8	.	9 131 240	.
9	.	24 040 963	.
10	.	12 142 211	.
11	4 955 266	11 347 691	8 907 348
12	.	8 962 272	.
13	.	15 737 733	.
14	.	16 463 484	.
15	.	11 436 065	.
16	5 406 210	9 452 740	.
17	5 656 894	7 812 594	7 812 594
18	5 686 546	7 792 453	7 792 453
19	5 776 902	7 063 807	7 063 807
20	5 783 511	6 603 430	6 603 430
21	5 830 713	6 419 268	6 419 268
22	5 909 971	6 494 145	.
23	5 972 189	6 494 856	.
24	6 011 365	6 636 554	.
25	6 019 900	6 411 896	6 411 896
26	6 095 565	6 343 410	6 343 410
27	6 101 241	6 285 538	6 285 538
28	6 118 197	6 226 617	6 226 617
29	6 152 758	6 270 220	.
30	6 160 182		.
31	6 162 637	6 237 298	.
32	6 176 106	6 211 897	6 211 897
33	6 176 160	6 369 322	.
34	6 178 110	6 241 221	.
35	6 178 376	6 257 250	.
36	6 178 946	6 204 284	6 204 284
37	6 179 477	6 207 337	.
38	6 180 367	6 220 992	.
39	6 183 673	6 198 519	6 198 519
40	6 187 123	6 204 489	.
41	6 188 282	6 196 300	6 196 300
42	6 189 385	6 201 608	.
43	6 191 879	6 195 268	6 195 268
44	6 191 998	6 196 583	.
45	6 192 380	6 195 239	6 195 239
46	6 192 684	6 195 963	.
47	6 192 909	6 196 245	.
48	6 193 279	6 194 475	6 194 475
49	6 193 876	6 194 362	6 194 362
50	6 193 934	6 194 562	.
51	6 193 955	6 194 273	6 194 273
52	6 194 008	6 194 229	6 194 229
53	6 194 037	6 194 149	6 194 149
54	6 194 067	6 194 119	6 194 119
55	6 194 082	6 194 096	6 194 096
56	6 194 082	6 194 083	6 194 083
57	6 194 082	6 194 082	6 194 082
Opt.	6 194 082		

Experiment VII

Cycle	Lower bound	Upper bound in cycle	Upper bound
1	4 955 266		
2	.	217 047 863	217 047 863
3	.	.	.
4	.	17 830 734	17 830 734
5	.	16 319 236	16 319 236
6	.	17 238 340	.
7	.	24 226 868	.
8	.	12 415 236	12 415 236
9	.	9 309 526	9 309 526
10	.	19 185 116	.
11	.	11 619 119	.
12	.	8 799 343	8 799 343
13	.	7 954 440	7 954 440
14	.	7 755 565	7 755 565
15	.	7 263 815	7 263 815
16	.	7 776 014	.
17	.	7 123 213	7 123 213
18	.	7 243 302	.
19	.	6 600 664	6 600 664
20	.	7 425 629	.
21	.	10 117 763	.
22	.	8 447 956	.
23	.	8 021 950	.
24	5 236 008	42 740 620	.
25	5 277 788	9 113 509	.
26	5 502 446	7 961 892	.
27	5 639 805	6 793 281	.
28	5 711 358	7 103 785	.
29	5 896 987	6 538 908	6 538 908
30	5 925 485	6 605 551	.
31	5 926 247	6 505 713	6 505 713
32	5 941 105	6 481 943	6 481 943
33	5 945 921	6 457 796	6 457 796
34	5 965 896	6 335 940	6 335 940
35	5 978 318	–	–
36	5 994 012	6 323 979	6 323 979
37	6 010 747	6 306 364	6 306 364
38	6 044 436	6 479 959	.
39	6 077 282	6 250 551	6 250 551
40	6 104 511	6 254 105	.
41	6 105 355	6 521 463	.
42	6 117 422	6 238 111	6 238 111
43	6 122 593	6 300 784	.
44	6 122 906	6 301 882	.
45	6 124 460	6 301 657	.
46	6 132 621	6 241 720	.
47	6 140 659	6 272 025	.
48	6 141 374	6 277 734	.
49	6 147 933	6 257 774	.
50	6 153 862	6 238 557	.

Cycle	Lower bound	Upper bound in cycle	Upper Bound
51	6 155 479	6 217 055	6 217 055
52	6 156 325	6 246 694	.
53	6 159 980	6 233 950	.
54	6 162 453	6 207 439	6 207 439
55	6 165 530	6 219 665	.
56	6 166 276	6 255 475	.
57	6 167 304	6 203 156	6 203 156
58	6 174 252	6 204 795	.
59	6 174 285	6 232 446	.
60	6 174 658	6 216 015	.
61	6 179 234	6 206 756	.
62	6 179 734	6 216 195	.
63	6 184 267	6 203 395	.
64	6 184 400	6 204 908	.
65	6 184 812	6 222 934	.
66	6 185 156	6 208 211	.
67	6 187 253	6 196 791	6 196 791
68	6 187 304	6 203 073	.
69	6 188 305	6 200 163	.
70	6 188 313	6 198 050	.
71	6 188 832	6 197 506	.
72	6 188 832	6 212 718	.
73	6 189 363	6 206 746	.
74	6 189 611	6 205 447	.
75	6 190 138	6 201 875	.
76	6 190 377	6 196 474	6 196 474
77	6 190 402	6 196 962	.
78	6 191 317	6 195 509	6 195 509
79	6 191 822	6 202 651	.
80	6 191 852	6 201 116	.
81	6 191 966	6 199 467	.
82	6 192 466	6 198 687	.
83	6 192 596	6 205 027	.
84	6 192 596	6 200 974	.
85	6 192 606	6 199 687	.
86	6 192 668	6 198 395	.
87	6 192 698	6 197 971	.
88	6 192 825	6 196 784	.
89	6 192 840	6 197 349	.
90	6 192 995	6 197 053	
91	6 193 053	6 195 434	6 195 434
92	6 193 082	6 197 490	.
93	6 193 095	6 195 789	.
94	6 193 334	6 195 478	.
95	6 193 354	6 195 307	6 195 307
96	6 193 407	6 196 583	.
97	6 193 514	6 195 140	6 195 140
98	6 193 559	6 196 744	.
99	6 193 579	6 194 652	6 194 652
100	6 193 588	6 195 310	.
101	6 193 623	6 195 043	.
102	6 193 623	6 195 121	.
103	6 193 671	6 194 319	6 194 319

Experiment X

Appendix 4: Values of dual variables in the basic experiments

The exhibits in this section show the development of the values of the dual variables that corresponded to the common constraints during the solution procedures of the basic experiments (only common constraints 1-7 considered, as the dual variables that referred to constraints 8-11 remained unchanged).

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	0	0	0	7.90	0	0
2	0	0	532.5	0	7.90	0	0
3	296.6	0	506.0	0	7.90	0	0
4	275.4	206.8	542.7	0	7.90	0	0
5	120.0	110.8	520.5	9.96	7.90	-15.90	0
6	114.0	217.7	542.6	9.03	7.90	-14.34	0
7	132.2	89.0	678.2	1.88	7.90	- 5.94	0
8	135.1	225.1	694.3	0.22	7.90	- 5.38	0
9	265.0	2219.4	671.3	3.81	7.90	- 5.35	0
10	572.0	123.3	914.3	18.0	0	20.0	0
11	131.8	236.3	619.0	18.0	0	- 0.88	-50.0
12	330.6	249.3	818.7	18.0	0	1.44	-39.35
13	269.5	182.9	641.4	6.90	0	1.21	126.50
14	269.5	182.9	641.4	6.90	0	1.21	34.28
15	177.5	104.0	567.7	18.0	0	- 7.96	59.45
16	179.6	143.6	568.1	18.0	0	- 7.76	24.61
17	229.5	115.3	681.6	9.32	1.22	- 3.39	57.63
18	216.4	114.6	658.8	11.92	3.05	- 2.67	1.21
19	127.5	128.4	648.4	6.38	1.43	- 6.85	-29.23
20	223.6	182.0	642.2	10.74	4.10	- 3.36	14.83
21	131.8	115.4	604.7	10.62	4.92	- 6.07	61.68
22	160.3	115.8	667.1	10.71	3.29	- 5.86	56.98
23	160.4	116.0	591.5	10.66	3.23	- 5.81	56.83
24	198.0	115.4	655.0	10.73	3.42	- 4.42	53.94
25	229.9	124.5	603.5	10.64	4.43	- 2.57	57.49
26	232.5	127.5	651.5	10.67	4.30	- 2.53	54.08
27	205.5	115.4	704.6	10.53	3.20	- 3.15	29.68
28	205.5	116.1	668.1	10.49	3.12	- 3.10	28.71
29	219.0	123.1	678.7	11.07	4.01	- 3.33	47.07
30	230.6	151.1	613.3	11.04	3.77	- 2.97	58.00
31	228.7	150.8	629.4	11.00	3.86	- 3.02	56.42
32	225.6	144.7	675.6	10.74	3.81	- 3.18	51.36
33	211.4	139.0	641.3	10.94	4.11	- 3.35	54.54
34	215.6	128.2	677.2	10.87	3.20	- 3.47	58.05
35	218.7	127.1	656.3	10.84	3.04	- 3.37	55.06
36	215.8	163.6	663.7	10.84	3.80	- 3.23	56.40
37	216.8	153.5	666.8	10.87	3.65	- 3.24	54.54
38	216.4	141.2	630.0	10.74	4.06	- 3.18	53.06
39	220.0	163.1	653.3	10.83	3.78	- 3.10	51.96
40	221.1	155.5	652.3	10.83	3.72	- 3.12	52.03
41	214.3	162.7	662.3	10.81	3.75	- 3.48	51.95
42	214.6	160.5	661.2	10.80	3.75	- 3.41	52.16
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment I

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	0	0	0	7.90	0	0
2	0	0	539.7	0	7.90	0	0
3	402.0	0	506.6	0	7.90	0	0
4	285.3	0	795.7	0	7.90	0	0
5	388.2	324.7	540.8	0	7.90	0	0
6	154.3	282.5	652.2	0	7.90	-18.0	0
7	199.2	308.3	756.5	0	7.90	- 9.71	0
8	312.2	291.2	875.2	18.0	0	5.92	0
9	848.0	355.5	1002.4	18.0	0	11.65	0
10	188.6	159.7	647.7	1.49	0	- 4.49	126.50
11	169.0	156.5	643.8	2.63	0	- 4.35	-17.71
12	232.3	122.7	590.5	18.0	7.68	- 6.42	56.98
13	212.1	107.6	711.0	18.0	0.85	- 3.87	-50.0
14	210.0	148.3	709.6	18.0	0.78	- 3.98	-50.0
15	338.9	134.4	636.9	10.25	5.49	- 0.77	-50.0
16	309.8	291.7	604.8	11.78	1.92	- 2.11	-50.0
17	277.8	214.6	708.6	10.71	3.87	- 0.59	68.78
18	284.3	219.8	718.8	10.82	3.88	0.31	44.10
19	176.5	115.5	619.0	10.66	3.84	- 5.84	57.60
20	176.8	115.5	618.7	10.54	3.75	- 5.93	32.12
21	179.5	120.3	616.2	13.10	5.62	- 3.19	54.63
22	205.6	117.6	644.6	11.73	2.74	- 3.08	55.52
23	227.5	115.8	669.1	10.65	4.38	- 3.18	46.27
24	219.8	135.7	663.8	10.79	0.87	- 3.10	11.27
25	183.7	115.2	575.4	10.87	3.94	- 3.99	32.11
26	183.7	115.2	627.5	10.87	3.94	- 3.99	32.11
27	209.1	115.6	684.3	11.03	3.36	- 3.92	53.16
28	234.1	124.8	652.8	11.20	3.51	- 3.01	42.53
29	224.6	141.5	678.0	11.01	3.41	- 2.83	49.41
30	219.2	119.3	603.2	10.80	3.63	- 3.20	60.13
31	216.9	121.7	613.0	10.82	3.59	- 3.39	57.17
32	214.2	122.1	625.6	10.89	3.71	- 3.44	55.03
33	207.8	157.1	664.2	10.83	3.72	- 3.12	58.82
34	204.3	155.8	649.3	10.87	3.72	- 3.46	54.71
35	204.9	157.9	632.3	10.92	3.89	- 3.99	51.82
36	218.0	144.0	639.9	10.89	4.03	- 3.20	52.29
37	211.6	122.0	672.0	10.80	3.80	- 3.61	53.42
38	209.1	122.6	648.4	10.82	3.80	- 3.41	51.89
39	209.1	142.8	638.7	10.85	3.73	- 3.40	52.11
40	214.7	129.4	639.5	10.81	3.63	- 3.23	51.86
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment II

Cycle	Common constraint 1	2	3	4	5	6	7
1	1664.7	0	0	0	0	0	0
2	0	0	466.9	0	0	0	0
3	0	0	544.6	0	0	0	0
4	206.5	387.4	509.7	0	0	0	0
5	88.5	246.1	619.7	0	0	- 18.00	0
6	149.2	276.9	668.9	0	0	- 11.34	0
7	328.8	387.8	822.3	18.00	0	15.82	0
8	714.8	386.5	804.0	18.00	0	17.62	0
9	808.7	76.9	598.0	18.00	0	- 5.25	126.50
10	877.6	139.5	670.6	1.03	7.90	- 1.24	-50.00
11	117.8	191.6	586.9	18.00	0	- 4.39	126.50
12	120.1	189.8	586.9	18.00	0	- 4.37	-50.00
13	346.3	129.3	584.3	18.00	0	- 2.20	60.85
14	337.0	97.0	676.6	18.00	0	- 2.06	-50.00
15	335.1	173.1	672.0	18.00	0	- 2.09	-50.00
16	311.2	179.2	669.4	0.15	0	4.33	38.34
17	268.4	428.8	668.2	0.05	0	2.33	60.69
18	165.7	148.2	693.7	6.16	1.34	- 4.32	-50.00
19	191.5	160.6	586.0	8.03	3.69	- 2.59	-31.34
20	151.7	116.4	629.9	10.43	4.47	- 5.50	24.22
21	267.0	152.0	693.1	9.02	2.67	- 0.77	32.16
22	206.4	118.4	712.4	11.20	3.31	- 4.17	63.44
23	207.8	120.2	685.9	11.22	3.43	- 4.05	54.80
24	210.4	134.9	610.6	10.86	3.48	- 4.00	47.50
25	212.7	279.4	676.7	10.48	4.17	- 3.21	41.86
26	205.7	113.2	653.9	10.83	4.10	- 3.20	61.13
27	204.4	119.0	653.7	10.88	4.07	- 3.22	60.99
28	206.3	114.2	660.3	10.96	4.14	- 3.18	51.36
29	201.4	159.5	652.4	11.21	3.73	- 3.19	54.37
30	237.9	140.2	661.1	10.87	3.55	- 2.54	57.07
31	201.5	126.7	635.4	10.59	4.03	- 3.82	52.08
32	228.9	161.2	680.4	10.98	2.04	- 2.82	48.51
33	216.7	176.9	638.9	10.88	3.32	- 3.30	53.63
34	216.7	146.6	638.9	10.88	3.32	- 3.30	53.63
35	203.1	115.5	663.6	10.87	3.61	- 4.24	52.71
36	202.3	149.1	653.9	10.86	3.71	- 3.84	55.31
37	210.9	163.9	663.4	10.77	3.79	- 3.35	54.65
38	210.8	115.6	657.4	10.77	3.79	- 3.36	54.48
39	210.8	117.4	657.1	10.83	3.72	- 3.44	52.31
40	217.7	127.4	658.1	10.81	3.76	- 3.55	49.48
41	211.7	133.4	646.5	10.89	3.62	- 3.15	51.45
42	217.4	136.7	664.1	10.87	3.69	- 3.08	51.75
43	215.0	133.4	659.3	10.84	3.69	- 3.36	51.82
44	213.6	127.7	661.4	10.83	3.67	- 3.39	51.75
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment III

Cycle	Common constraint 1	2	3	4	5	6	7
1	249.3	162.5	684.4	11.19	5.62	-1.32	52.48
2	146.1	114.2	574.6	9.33	0	-7.45	56.21
3	179.7	162.5	651.6	10.82	0	-5.90	51.77
4	321.8	238.3	753.8	10.79	0	4.17	23.26
5	224.2	138.5	682.5	10.64	3.43	-2.45	49.07
6	213.3	118.5	650.8	10.87	4.78	-4.39	53.28
7	214.1	128.8	662.2	10.81	4.13	-3.83	56.46
8	213.6	128.3	661.4	11.10	4.76	-3.20	50.40
9	214.3	129.0	662.1	10.87	3.96	-3.42	52.34
10	214.0	128.8	662.2	10.81	2.72	-3.88	52.21
11	214.3	129.0	661.9	10.81	3.43	-3.43	54.64
12	215.3	130.0	633.2	10.81	3.79	-3.03	54.11
13	214.9	129.6	639.0	10.81	3.73	-3.31	53.50
14	214.3	129.0	662.0	10.83	3.72	-3.43	52.12
15	214.4	129.1	662.1	10.85	3.70	-3.36	51.75
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment IV

Cycle	Common constraint 1	2	3	4	5	6	7
1	246.9	160.3	656.5	11.06	5.23	-1.87	52.48
2	197.4	161.7	660.2	10.82	0	-4.41	54.66
3	241.4	152.4	631.5	10.90	2.36	-1.47	-50.00
4	213.7	128.4	635.8	10.81	3.92	-3.86	42.27
5	214.1	128.9	664.9	10.81	2.87	-3.02	52.43
6	214.7	129.4	662.3	10.83	3.84	-3.44	56.46
7	214.7	129.4	662.3	10.83	3.84	-3.44	53.12
8	214.7	129.4	662.3	10.83	3.50	-3.44	50.91
9	215.0	129.7	662.4	10.87	3.76	-3.22	51.99
10	214.4	129.1	662.1	10.85	3.60	-3.36	51.71
11	214.3	129.1	662.0	10.84	3.71	-3.30	51.88
12	214.4	129.1	662.0	10.84	3.70	-3.40	51.85
13	214.6	129.3	662.1	10.84	3.70	-3.40	51.85
14	214.6	129.3	662.1	10.84	3.70	-3.40	51.85
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment V

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	6800.1	0	0	7.90	-18.00	-50.00
2	2636.8	.	0	0	0	20.00	126.50
3	0	2246.8	362.2	0	0	20.00	126.50
4	0	0	566.1	0	0	20.00	126.50
5	585.4	585.4	509.0	0	0	20.00	126.50
6	607.3	159.2	800.8	0	0	20.00	126.50
7	730.5	1876.9	1001.8	0	0	20.00	126.50
8	178.7	244.8	594.6	18.00	0	-18.00	-30.44
9	186.0	259.6	563.7	18.00	7.90	- 9.70	-50.00
10	195.0	274.8	563.7	0	0	0.46	-50.00
11	108.4	0	752.7	0	0	- 5.65	-50.00
12	154.1	156.4	606.5	0	0	- 8.84	91.79
13	90.7	163.5	601.8	1.93	0	- 9.25	-23.53
14	163.8	183.2	644.8	0.42	0	- 7.03	-22.25
15	141.4	119.6	582.7	19.00	5.26	- 4.91	71.78
16	200.9	117.5	566.2	13.75	3.97	- 6.02	45.67
17	152.2	109.8	636.4	11.39	7.90	- 0.49	- 8.50
18	185.0	131.2	631.4	11.53	0	- 1.61	26.17
19	227.8	110.9	629.0	9.14	4.86	- 2.08	59.61
20	245.9	129.7	634.1	9.21	2.68	- 1.23	61.92
21	246.7	128.8	631.3	9.06	6.56	- 0.90	26.32
22	261.1	181.1	652.5	11.23	0.15	- 1.52	30.06
23	189.6	116.3	658.1	10.33	3.38	- 4.14	46.00
24	202.6	116.7	609.9	10.98	4.97	- 4.08	59.60
25	218.0	113.9	692.5	11.49	5.19	- 3.32	53.75
26	234.2	121.6	653.0	10.80	5.14	- 3.24	52.45
27	220.6	131.8	687.1	11.31	4.69	- 2.73	57.36
28	213.6	129.8	622.8	10.97	4.25	- 3.11	38.36
29	225.0	150.7	680.1	10.86	3.67	- 3.32	58.22
30	224.7	133.7	655.4	10.86	3.68	- 3.33	54.72
31	215.8	124.8	663.8	10.73	4.13	- 3.21	47.22
32	220.1	140.0	665.7	10.90	3.35	- 2.92	51.53
33	211.8	123.1	653.9	10.82	3.56	- 3.46	51.31
34	214.9	128.2	662.4	10.82	3.68	- 3.19	51.68
35	214.5	129.4	662.2	10.83	3.76	- 3.48	52.04
36	214.1	128.6	661.8	10.84	3.64	- 3.39	51.69
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment VI

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	25144.8	0	0	0	-18.00	-50.00
2	0	0	579.6	0	0	-18.00	-50.00
3	582.9	0	528.0	0	0	-18.00	-50.00
4	219.1	0	568.0	0	0	-13.56	-50.00
5	450.4	387.4	570.3	0	0	-16.29	-50.00
6	127.1	273.7	529.4	0	0	-12.46	-50.00
7	468.2	279.3	574.9	18.00	0	- 9.37	-50.00
8	0	83.5	469.3	18.00	0	- 1.63	-50.00
9	0	83.5	537.3	18.00	0	- 1.01	-50.00
10	325.8	83.4	511.1	18.00	0	4.13	-50.00
11	253.2	83.4	669.9	18.00	0	4.38	-50.00
12	419.0	3927.0	511.1	0	7.90	20.00	126.50
13	309.4	3558.4	898.7	0	7.90	20.00	126.50
14	637.7	1987.6	811.6	18.00	7.90	20.00	126.50
15	285.7	283.2	597.0	0	0	9.39	-50.00
16	640.7	273.3	1076.6	0	0	8.39	-50.00
17	139.0	131.2	601.5	18.00	7.90	- 5.36	87.14
18	139.0	131.2	601.5	18.00	7.90	- 5.36	-24.06
19	124.4	110.1	579.1	9.90	0	- 7.71	59.39
20	131.7	143.2	581.7	10.29	0	- 7.41	59.61
21	206.5	115.3	598.2	13.78	0	- 5.66	0.49
22	162.2	119.7	599.8	5.94	4.04	- 5.53	8.38
23	224.4	144.8	657.7	13.22	0	1.10	50.61
24	287.1	195.1	656.9	11.43	0	0.95	63.71
25	287.1	195.1	656.9	11.43	0	0.95	31.70
26	204.7	190.5	696.0	11.71	3.77	- 3.34	58.68
27	248.2	178.8	694.1	10.48	3.24	- 2.51	53.18
28	194.6	117.7	663.0	12.00	3.98	- 3.24	7.18
29	234.4	115.0	659.1	10.67	1.80	- 2.23	10.18
30	213.6	115.4	592.3	10.57	4.07	- 3.84	44.48
31	208.2	115.2	653.5	10.48	3.98	- 4.01	43.64
32	216.9	128.2	727.5	10.50	4.12	- 2.09	42.27
33	214.5	125.8	706.1	10.51	4.11	- 2.22	45.77
34	199.7	135.2	604.0	10.75	3.50	- 3.54	60.98
35	201.3	140.6	632.1	10.78	3.38	- 3.55	57.40
36	217.3	139.6	609.9	10.93	3.44	- 3.07	50.61
37	230.9	116.1	691.4	11.19	2.94	- 3.07	54.40
38	220.7	164.1	678.1	10.90	4.05	- 2.73	54.20
39	218.2	151.9	681.8	10.98	3.43	- 3.21	35.54
40	205.8	144.3	654.2	10.92	3.93	- 3.50	50.66
41	232.1	125.4	666.2	10.95	3.73	- 2.63	55.83
42	222.2	147.6	668.1	10.79	3.58	- 2.99	51.43
43	219.9	163.5	627.5	10.80	3.82	- 3.21	53.17
44	219.3	150.6	638.8	10.82	3.88	- 3.20	53.18
45	209.5	160.2	666.4	10.81	3.62	- 3.66	53.43
46	212.6	157.8	638.7	10.77	3.48	- 3.62	50.94
47	212.8	162.8	665.2	10.84	3.68	- 3.46	51.60
48	213.1	157.9	658.8	10.88	3.76	- 3.23	52.37
49	216.5	156.9	667.1	10.85	3.59	- 3.36	51.72
50	215.2	162.5	657.6	10.82	3.78	- 3.44	52.10
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment VII

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	0	0	16.50	0	-18.00	0
2	0	0	327.4	18.00	0	20.00	-50.00
3	0	0	525.0	18.00	0	- 9.36	126.50
4	76.7	0	520.4	18.00	0	- 5.85	-50.00
5	74.8	190.0	520.7	18.00	0	- 5.63	-50.00
6	193.3	193.3	507.8	18.00	0	- 9.36	52.53
7	129.6	129.6	547.3	18.00	0	- 6.24	-50.00
8	128.8	94.5	543.3	0	7.90	- 9.95	-50.00
9	89.5	208.4	554.2	0	7.90	-10.69	-50.00
10	151.2	155.0	549.0	0	2.69	- 9.13	-50.00
11	113.0	119.0	615.0	9.10	7.90	- 4.38	- 1.15
12	154.6	119.0	607.5	9.64	7.90	- 4.35	126.49
13	156.0	119.1	614.1	9.59	7.90	- 4.65	21.39
14	184.5	99.0	631.4	9.01	0.73	- 3.40	68.81
15	186.0	142.1	628.8	9.02	0.85	- 3.75	68.92
16	181.0	115.5	634.6	8.87	0.31	- 3.56	20.92
17	284.5	247.7	625.8	11.27	3.91	- 5.45	33.76
18	418.7	128.0	721.7	11.83	4.07	8.23	37.55
19	350.8	243.7	679.5	11.57	4.58	4.87	45.61
20	326.5	256.8	689.5	10.01	4.80	2.14	48.29
21	220.1	117.5	645.4	12.00	4.53	- 2.22	55.08
22	191.2	136.0	670.7	10.57	3.58	- 4.90	54.56
23	191.1	115.8	616.2	10.98	2.79	- 5.00	-18.64
24	218.7	115.9	587.5	10.08	4.19	- 3.47	2.23
25	218.1	116.9	659.4	10.09	4.14	- 3.16	- 7.85
26	240.4	168.0	649.3	10.79	2.56	- 2.60	51.13
27	211.2	130.0	644.5	10.76	2.87	- 3.17	52.11
28	236.8	127.9	702.0	11.12	3.88	- 3.05	42.87
29	236.8	127.9	623.4	11.12	3.88	- 3.05	42.87
30	220.4	151.3	679.8	11.02	4.06	- 2.98	50.37
31	215.6	154.6	609.3	10.93	4.02	- 3.31	60.62
32	214.7	148.4	614.0	10.87	3.97	- 3.46	56.27
33	212.6	134.7	624.5	10.73	3.82	- 3.79	51.28
34	213.1	163.3	634.3	10.98	3.86	- 3.43	55.53
35	225.3	144.5	639.0	10.89	3.90	- 2.93	54.38
36	210.9	152.7	655.4	10.86	3.84	- 3.52	52.04
37	213.8	156.5	666.3	10.78	3.66	- 3.45	54.47
38	218.8	163.1	661.3	10.82	3.78	- 3.13	51.95
39	219.6	157.9	661.4	10.86	3.54	- 3.11	51.59
40	214.7	161.8	661.8	10.83	3.75	- 3.42	51.89
41	214.4	156.9	666.1	10.85	3.70	- 3.34	51.89
Opt.	214.6	161.7	662.1	10.84	3.70	- 3.39	51.85

Experiment VIII

Cycle	Common constraint 1	2	3	4	5	6	7
1	0	0	0	0	7.90	20.00	11.02
2	0	7855.3	0	0	7.90	20.00	11.02
3	0	0	530.2	0	7.90	20.00	11.02
4	366.7	366.7	497.2	0	7.90	20.00	11.02
5	353.7	0	544.9	0	7.90	20.00	11.02
6	100.5	120.9	508.6	0	7.90	-18.00	126.50
7	139.6	154.9	514.2	0	7.90	-12.12	126.50
8	170.1	181.4	518.6	0	7.90	-7.53	126.50
9	68.4	56.6	626.1	0	7.90	-18.00	126.50
10	100.5	84.6	629.9	0	7.90	-13.20	126.50
11	171.3	146.8	638.4	0	7.90	-2.60	126.50
12	143.3	148.5	547.2	0	7.90	-10.40	12.18
13	103.4	111.6	531.3	18.00	0	-8.05	71.11
14	144.7	150.3	535.0	18.00	7.04	-5.51	39.40
15	106.5	85.5	633.4	18.00	5.24	-6.25	41.30
16	106.5	172.3	633.4	18.00	5.24	-6.25	41.30
17	154.7	158.4	597.6	18.00	1.26	-7.67	76.05
18	137.7	139.6	584.3	5.23	1.61	-5.15	31.31
19	343.6	152.6	657.4	18.00	7.90	-5.30	11.27
20	591.6	125.6	727.2	18.00	0	11.26	90.96
21	612.7	223.2	734.9	18.00	0	12.06	-50.00
22	612.7	569.5	734.9	18.00	0	12.06	-50.00
23	830.2	372.6	2256.7	0	0	10.76	-50.00
24	738.2	336.9	1178.5	0	0	6.14	113.54
25	738.6	278.9	1179.2	0	0	6.15	-45.57
26	291.3	224.3	618.9	18.00	0	-0.18	51.20
27	203.3	129.1	735.9	18.00	0	-3.38	40.17
28	212.6	121.8	695.2	15.59	0	-4.56	41.42
29	273.7	165.6	687.4	11.49	7.90	1.11	59.18
30	274.8	112.8	656.4	10.26	6.16	-0.95	-9.57
31	274.4	131.4	667.3	10.69	6.77	-0.23	25.82
32	257.4	167.7	622.2	9.33	4.58	-3.40	65.26
33	253.1	220.5	640.1	10.12	5.65	-2.25	43.29
34	274.7	117.2	658.9	10.36	2.62	-0.78	51.74
35	264.7	220.5	687.7	11.71	3.66	1.04	56.78
36	258.1	174.6	629.8	9.61	1.66	-2.88	47.87
37	252.8	208.9	694.7	12.53	4.73	1.71	-50.00
38	184.1	117.9	598.7	11.42	4.68	-4.10	35.74
39	219.4	138.5	709.1	10.37	4.30	-1.83	39.00
40	199.1	106.1	648.8	10.73	1.53	-3.36	54.12
41	206.1	118.0	656.0	10.93	1.13	-2.72	43.96
42	271.6	125.8	680.6	11.77	4.81	-2.23	58.15
43	261.1	114.2	672.6	11.56	4.77	-2.76	38.65
44	272.8	124.9	681.0	11.78	4.82	-2.20	36.80
45	197.2	116.1	639.1	10.53	4.48	-5.24	56.45
46	244.4	221.0	686.2	11.67	4.62	-2.39	44.77
47	182.8	114.4	631.1	10.29	4.42	-5.82	0.56
48	185.3	121.5	638.8	10.26	4.39	-5.60	4.38
49	258.4	145.9	672.8	10.41	4.55	-1.35	57.10
50	243.4	133.0	695.5	10.81	3.00	-2.95	52.18

Cycle	Common constraint 1	2	3	4	5	6	7
51	215.0	201.8	658.5	10.33	3.95	-2.89	58.65
52	223.4	194.4	656.3	10.30	4.23	-2.47	48.93
53	210.4	125.0	652.5	11.05	4.77	-3.21	54.65
54	195.4	176.0	608.4	10.90	2.88	-4.51	53.23
55	209.7	126.3	596.4	10.51	3.49	-3.64	55.94
56	202.3	137.2	625.7	10.56	3.38	-3.94	55.69
57	212.4	147.0	622.8	10.94	3.67	-3.63	23.67
58	223.9	114.4	679.3	10.61	4.11	-2.59	39.83
59	224.3	117.4	680.8	10.64	4.08	-2.57	39.95
60	229.8	152.5	697.7	10.95	3.70	-2.25	50.45
61	242.7	159.1	647.8	11.07	3.80	-1.37	56.40
62	222.5	145.9	648.2	10.86	3.92	-2.31	47.11
63	221.9	160.1	668.8	10.73	4.40	-3.49	54.20
64	206.4	148.0	672.1	10.29	2.97	-4.12	28.07
65	230.2	180.9	671.0	10.80	3.33	-2.93	51.73
66	224.1	144.4	670.3	10.83	3.45	-3.14	52.23
67	194.8	167.6	660.3	10.85	4.00	-4.16	51.00
68	195.0	142.5	659.3	10.84	3.99	-4.15	51.36
69	206.0	163.6	645.8	10.89	4.10	-3.68	55.37
70	205.7	132.7	645.4	10.87	4.09	-3.70	54.63
71	203.3	114.9	630.9	10.46	3.79	-3.99	38.20
72	203.3	117.6	630.9	10.46	3.79	-3.99	38.20
73	195.6	163.6	649.7	10.85	2.83	-3.82	44.83
74	200.1	123.6	644.7	10.87	2.97	-3.65	46.78
75	214.2	162.6	626.0	10.80	3.60	-3.26	50.27
76	214.1	115.6	636.1	10.86	3.65	-3.27	48.53
77	213.7	132.7	640.5	10.86	3.69	-3.32	47.05
78	224.1	131.7	638.4	10.73	4.04	-2.86	57.31
79	217.0	162.4	680.0	10.71	3.94	-3.07	55.69
80	221.5	164.7	663.7	10.71	4.04	-2.95	54.85
81	221.1	147.8	665.5	10.72	4.06	-2.93	54.56
82	210.4	159.1	669.3	10.81	3.39	-3.34	58.00
83	209.3	115.5	668.6	10.76	3.44	-3.49	56.54
84	203.3	116.4	668.5	10.76	3.44	-3.49	56.54
85	210.5	162.8	663.1	10.77	3.47	-3.46	56.64
86	212.8	116.5	651.8	10.79	3.54	-3.44	56.36
87	215.6	136.0	639.7	10.82	3.60	-3.39	56.48
88	209.6	139.8	665.9	10.72	3.58	-3.62	54.52
89	210.9	153.8	652.1	10.72	3.60	-3.65	54.41
90	217.2	162.4	668.5	10.86	3.83	-3.38	54.78
91	215.5	127.8	665.6	10.76	3.28	-3.07	54.29
92	220.3	124.8	641.9	10.84	3.66	-3.16	54.40
93	220.4	141.1	639.6	10.83	3.58	-3.12	54.32
94	211.5	121.9	657.3	10.79	3.85	-3.40	53.45
95	206.8	157.5	650.3	10.76	3.73	-3.41	54.13
96	212.4	148.3	657.4	10.76	3.73	-3.34	52.67
97	220.4	140.4	655.8	10.79	3.91	-3.31	50.86
98	216.3	126.8	663.0	10.84	3.78	-3.41	52.52
99	215.4	142.6	645.1	10.85	4.01	-3.31	52.86
100	219.8	129.1	669.4	10.81	3.72	-3.12	51.34
101	213.5	124.6	666.9	10.87	3.67	-3.43	53.61
102	215.9	137.5	664.8	10.84	3.70	-3.37	52.58
103	219.8	159.2	659.4	10.80	3.75	-3.07	51.03
Opt.	214.6	161.7	662.1	10.84	3.70	-3.39	51.85

Experiment X

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In this paper, we trace out the solution of large optimization problems for regional planning by a decomposition program that is based on the use of standard LP programs. The underlying method is the decomposition principle of Dantzig and Wolfe.

The concept is tested by solving an optimization problem with about 1250 columns and 900 rows. Furthermore, it is investigated to what extent the efficiency of the solution procedure can be influenced by an appropriate choice of starting solutions or specific matrix divisions.

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