Explicit Guidance Equations for a Variable Trim Reentry Vehicle.

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FOREWORD

The work covered in this report was performed in the FBM Geoballistics Division, with the exception of Appendix B which was contributed by Christopher Gracey of the Aeromechanics Branch of the Exterior Ballistics Division. Both divisions are in the Strategic Systems Department.

This report was reviewed by Carlton W. Duke, Jr., Head, FBM Geoballistics Division.

Released by:

Ralph A. Niemann

RALPH A. NIEMANN, Head
Strategic Systems Department

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METRIC CONVERSION TABLE

To convert from	To	Multiply by
feet (ft)	meters (m)	0.30480
feet/second (ft/sec)	meters/second (m/sec)	0.30480
degrees (angle)	radians (rad)	0.01745

INTRODUCTION

In this report equations are derived for guiding a variable lift reentry vehicle to impact with an earth fixed target. The concept of a dive line located at the target is used to meet preselected terminal conditions on impact flight path angle and approach azimuth. With a simplifying assumption the resulting equations can be put in a cross product form.

Preliminary simulation results are given but the report is circulated primarily to stimulate discussion of the ideas involved and demonstrate feasibility of the equations rather than present an in-depth design study.

The next section, the primary one, contains a derivation of the guidance equations. This is followed by a short section of simulation results and discussion and finally two appendices. The first describes the coordinate frames and notation used. The second, by Christoper Gracey, contains a derivation of a simplified cross product form of the guidance law.

DERIVATION OF THE GUIDANCE EQUATIONS

The guidance law derived in this section is a modification of a technique originated by the author in the early sixties for ascent guidance. The problem to be solved is as follows: given a dive line at the target, control the vehicle so that at target impact the vehicle's velocity vector is parallel to the dive line. The solution is to drive the horizontal plane and vertical plane components of the line of sight which are orthogonal to the dive line, and their time rates of change, to zero by impact time.

The procedure to accomplish this is simple. Two components of the commanded lift vector are prescribed as functions of the estimated time to go. The functions have a total of four undetermined parameters. These are chosen so that when time to go is zero the line of sight components of interest and their derivatives are zero. The third component is determined by imposing the physical constraint that the lift vector is normal to the vehicle's relative velocity vector.

Usually, derivatives of the vehicle motion with respect to inertial space are the ones available. In deriving the guidance law, however, derivatives are needed with respect to an earth fixed frame. These are related to inertial derivatives by the Coriolis formula which is used without comment where needed. In the following, a dot over a vector indicates its time derivative with respect to inertial space, while a prime indicates differentiation relative to the earth. For scalars the distinction does not exist and either symbol may be used.

The first step in the derivation is to compute components of the line of sight and their derivatives relative to the dive/turn frame. Appendix A contains definitions of the quantities used in the following.

The line of sight vector $\overline{r}$ which is defined as

$$\overline{r} = \overline{R}_T - \overline{R}$$

can be expressed in component form as

$$\overline{r} = r_D\overline{U}_D + r_P\overline{U}_P + r_N\overline{U}_N$$

from which it follows

$$\bar{r}' = r_D'\bar{U}_D + r_P'\bar{U}_P + r_N'\bar{U}_N$$

and

$$\bar{r}'' = r_D''\bar{U}_D + r_P''\bar{U}_P + r_N''\bar{U}_N$$

From the definitions of E, H, F and W given in the appendix it follows that

$$E = \bar{r} \cdot \bar{U}_P$$

$$H = E' = \bar{r}' \cdot \bar{U}_P$$

$$F = \bar{r} \cdot \bar{U}_N$$

$$W = F' = \bar{r}' \cdot \bar{U}_N$$

The derivative $\bar{r}'$ is computed by

$$\bar{r}' = \bar{\omega} \times \bar{R} - \dot{\bar{R}}$$

The second derivative $\bar{r}''$ will be needed also and is

$$\bar{r}'' = 2\bar{\omega} \times \dot{\bar{R}} - \bar{\omega} \times (\bar{\omega} \times \bar{R}) - \ddot{\bar{R}}$$

At this point it should be noted that $\bar{r}''$ is used only in the closed-loop guidance equations; so approximations are in order. The general philosophy in this report regarding approximations holds that they are acceptable, provided the errors engendered are nowhere too bad, and that they get smaller (or remain negligible) as terminal conditions are met.

With this in mind consider the magnitudes of the first two terms in the equation for $\bar{r}''$.

$$|2\bar{\omega} \times \dot{\bar{R}}| \sim 2 \times (7\times10^{-5}) \times (1\times10^{4}) = 1.4 \text{ ft/sec}^2$$

$$|\bar{\omega} \times (\bar{\omega} \times \bar{R})| \sim (7 \times 10^{-5})^2 \times 2 \times 10^7 = 0.1 \text{ ft/sec}^2$$

Both of these are small compared with the available acceleration due to lift; so, in all that follows, the approximation is made,

$$\bar{r}'' = -\ddot{\bar{R}}$$

Consistent with this approximation,

$$E'' = \bar{r}'' \cdot \bar{U}_P = -\ddot{\bar{R}} \cdot \bar{U}_P$$

$$F'' = \bar{r}'' \cdot \bar{U}_N = -\ddot{\bar{R}} \cdot \bar{U}_N$$

The next step, the pivotal one, is to require

$$E'' = A + BT_G$$

$$F'' = C + DT_G$$

where T_G is an estimate of time to go.

Three problems remain to be solved:

(a) Determine the parameters A, B, C and D by requiring satisfaction of the desired terminal conditions.

(b) Find an approximation for time to go.

(c) Relate components of the lift vector--the only quantity available for control--to the parameter values and T_G.

If all the approximations made were exact, A, B, C and D would not vary with time. In practice they do vary under continual recomputation, but this is ignored when computing their values at any time step.

In the dive/turn frame, the lift vector is expressed as

$$\bar{L} = L_D\bar{U}_D + L_P\bar{U}_P + L_N\bar{U}_N$$

The three components are determined by imposing the constraints,

(1) E and E' must approach zero as T_G goes to zero

(2) F and F' must approach zero as T_G goes to zero

(3) The lift vector is perpendicular to the relative velocity vector.

To impose condition (1), the equation for E'' is integrated twice with respect to T_G, noting the sign change and taking into account current values of E and E'. This procedure yields two equations for A and B, viz.

$$2A + BT_G = - 2H/T_G$$

$$3A + BT_G = GE/T_G^{\,2}$$

which have solutions.

$$A = \frac{2}{T_G}\left(\frac{3E}{T_G} + H\right)$$

$$B = -\frac{6}{T_G^{\,2}}\left(\frac{2E}{T_G} + H\right)$$

In exactly the same way, condition (2) implies

$$C = \frac{2}{T_G}\left(\frac{3F}{T_G} + W\right)$$

$$D = -\frac{6}{T_G^{\,2}}\left(\frac{2F}{T_G} + W\right)$$

Before implications of condition (3) are considered, it is perhaps as well to digress for a moment to find an expression for T_G and show that the constraints imposed so far have determined L_P and L_N.

First, an estimate for time to go is obtained simply by dividing the magnitude of the line of sight vector $\overline{r}$ by the negative of its time rate of change. This yields

$$T_G = -\frac{\overline{r} \cdot \overline{r}}{\overline{r}' \cdot \overline{r}}$$

or, equivalently,

$$T_G = \frac{\overline{r} \cdot \overline{r}}{\overline{V} \cdot \overline{r}}$$

where $\overline{V}$ is the relative velocity vector

$$\overline{V} = \dot{\overline{R}} - \overline{\omega} x \overline{R}$$

Next, to show that conditions (1) and (2) determine L_P and L_N, consider the total acceleration of the vehicle with respect to inertial space. It is just the sum of the accelerations due to lift, drag and gravitation.

Since drag changes the direction of the relative velocity little compared with lift, it will be neglected in computing the commanded lift. With this comment, $\overline{r}''$ becomes

$$\overline{r}'' = -(\overline{L} + \overline{G})$$

where $\overline{G}$ is the gravitation vector. It follows immediately that

$$L_P = -(A + BT_G)$$

$$L_N = -(C + DT_G) - \overline{G} \cdot \overline{U}_N$$

where a small component of gravitation has been dropped in the expression for L_P.

We have to find only one more component of the lift vector, namely L_D, and this is determined by condition (3)

$$\bar{L} \cdot \bar{V} = 0$$

If $\bar{V}$ is written

$$\bar{V} = V_D\bar{U}_D + V_P\bar{U}_P + V_N\bar{U}_N$$

then L_D is seen to be

$$L_D = -\frac{L_P V_P + L_N V_N}{V_D}$$

provided V_D is not zero. If, due to some earlier maneuvering, V_D is zero or negative, the commanded lift vector is taken to have the direction $\bar{U}_D$ and be of some large, nominal value until V_D becomes positive.

This completes the guidance law derivation. The equations are summarized below for convenience. As a final comment, note that a multiplicity of (false) target vectors and associated dive lines can be given sequentially, if desired, for trajectory shaping.

GUIDANCE EQUATION SUMMARY

$$\bar{r} = \bar{R}_T - \bar{R}$$

$$\bar{V} = \dot{\bar{R}} - \bar{\omega} \times \bar{R}$$

$$E = \bar{r} \cdot \bar{U}_P$$

$$H = -\bar{V} \cdot \bar{U}_P$$

$$F = \bar{r} \cdot \bar{U}_N$$

$$W = -\bar{V} \cdot \bar{U}_N$$

$$T_G = \frac{\bar{r} \cdot \bar{r}}{\bar{V} \cdot \bar{r}}$$

$$A = \frac{2}{T_G}\left(\frac{3E}{T_G} + H\right)$$

$$B = -\frac{6}{T_G{}^2}\left(\frac{2E}{T_G} + H\right)$$

$$C = \frac{2}{T_G}\left(\frac{3F}{T_G} + W\right)$$

$$D = -\frac{6}{T_G{}^2}\left(\frac{2F}{T_G} + W\right)$$

$$L_P = -(A + BT_G)$$

$$L_N = -(C + DT_G) - \overline{G} \cdot \overline{U}_N$$

$$L = -\frac{L_P V_P + L_N V_N}{V_D}$$

DISCUSSION AND SIMULATION RESULTS

In an actual implementation of the guidance law derived in the preceding section, a number of factors suddenly become of increased importance. The most obvious one is that when time to go approaches zero, the coefficients A, B, C and D become singular. This is not surprising since if one has only a very small correction to make but no time in which to make it, the required acceleration becomes infinite. This difficulty is easy to overcome in the original form of the guidance law: simply stop recomputing (freeze) the coefficients when time to go becomes small, in the case at hand somewhere around one to one-half second. This is potentially important for realistic, noisy systems. It is less clear how to accomplish this freezing in the simplified form derived in Appendix B since indeterminacy arises from the cross product itself. In the noise-free cases, however, both forms lead to impact errors which are very close, differing at most by a few feet.

The order in which computations are performed is important if a fixed-point computer is used. Care can greatly reduce the dynamic range of the variables involved. This must be considered together with the computation of factors common to several expressions. If airborne computations become a real problem, which seems rather unlikely with today's technology, it may well be possible to compute successive values of the coefficients by perturbation techniques, thereby reducing the computational load.

For the simulation results shown in Table 1, a nominal reentry trajectory and three nonstandard trajectories are considered. The initial conditions are as shown. For the disturbed trajectories, only the condition which differs from the nominal is given. Linerized aerodynamics were used for all runs.* The angular error recorded is the angle between the dive line and the terminal velocity vector. Differences of only a few feet are not considered significant.

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*Internal Memorandum: Maneuvering Reentry Vehicle Formulation for the Advanced System Simulation, 10 Nov 1977, Code CK-21

INITIAL CONDITIONS FOR SIMULATIONS

Case A (Nominal)

100K ft altitude
200K ft up range from the target
800 ft crossrange to the right of the vertical plane containing the dive line
20K ft/sec speed (in plane)
-24° flight path angle
Lift vector in plane
Dive line: 74° up from the horizontal

Case B (Position Disturbance)

1800 ft crossrange

Case C (Velocity Disturbance)

Velocity vector 20° out of plane

Case D (Acceleration Disturbance)

Lift vector 90° out of plane

TABLE 1

PRELIMINARY RESULTS

CASE	DOWNRANGE MISS (FEET)	CROSSRANGE MISS (FEET)	ANGULAR ERROR (DEG.)
A	-1.1	0	0.08
B	1.0	1.0	0.01
C	-3.6	4.6	0.27
D	-1.1	0	0.08

The miss distances shown in Table 1 are trivial and demonstrate feasibility of the proposed guidance equations. A much more detailed study considering noise effects, computer word length limitations, and similar problems is required to obtain accuracy for a realistic system.

APPENDIX A

COORDINATE FRAMES AND DEFINITIONS

This appendix provides definitions for primary quantities involved in deriving the guidance equations. Less important terms are defined as they occur in the text.

COORDINATE FRAME

Earth fixed dive/turn:

Origin - at the target

X axis - defined by the unit vector $\overline{U}_D$ parallel to the dive line and positive into the earth. The dive line is specified by the relative approach azimuth and flight path angle.

Z axis - specified by the unit vector $\overline{U}_N$ in the geodetic vertical plane at the target, positive into the earth

Y axis - specified by the unit vector $\overline{U}_P$ to complete right-handed system

See Figure A-1 for a sketch of the geometry. The slight distinction between geodetic and astronomic vertical has been ignored for simplicity in drawing the figure.

Guidance:

Origin - center of the earth

Axes - inertial, specified by the guidance system

VECTORS

$\overline{R}$ - from the earth center to the reentry vehicle

$\overline{R}_T$ - from the earth center to the earth fixed target

$\overline{r}$ - line of sight vector from reentry vehicle to the earth fixed target

$\overline{V}$ - derivative of $\overline{R}$ with respect to an earth fixed frame

$\overline{L}$ - commanded lift

$\overline{G}$ - gravitation vector

SCALARS

E - projection of $\overline{r}$ onto $\overline{U}_P$

H - time rate of change of E

F - projection of $\overline{r}$ onto $\overline{U}_N$

W - time rate of change of F

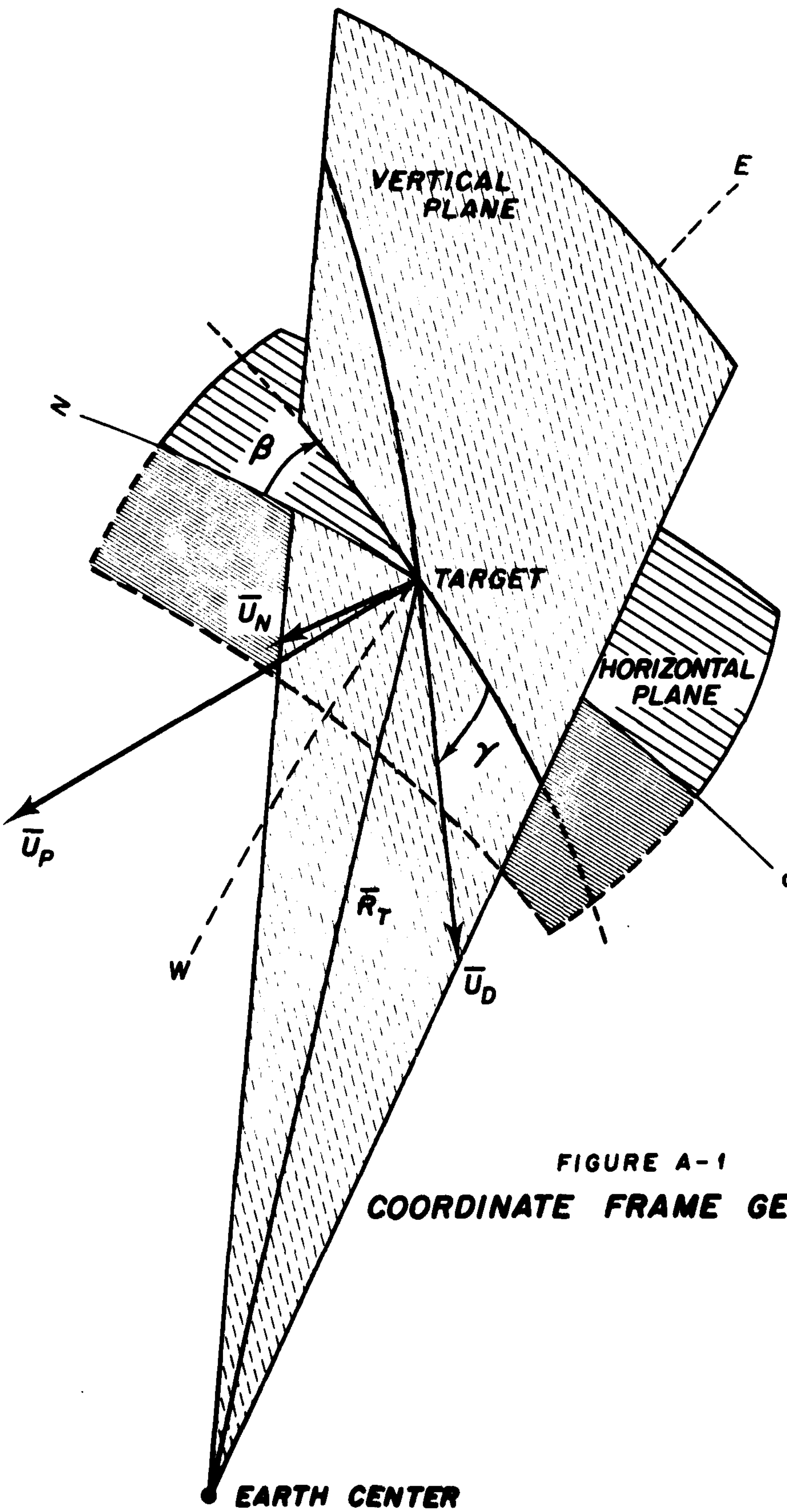

FIGURE A-1

COORDINATE FRAME GEOMETRY

APPENDIX B

CROSS PRODUCT FORM OF SIMPLIFIED GUIDANCE LAW

Christopher Gracey

CROSS PRODUCT FORM OF SIMPLIFIED GUIDANCE LAW

We seek to show that an equivalent "cross product" form of the guidance law can be obtained when gravity is neglected. To that end we start with the cross product form of the guidance law, namely

$$\bar{A}_C = K\alpha\left\{\bar{V} \times \left[(\bar{r} - K\phi\ T_G\ \bar{V}) \times \bar{U}_D\right]\right\} \qquad \text{B.1}$$

with

$$K\alpha = \frac{6}{T_G^2\ \ \bar{V}\cdot\bar{U}_D}, \quad K\phi = 2/3.$$

Expanding the cross product, we obtain

$$\bar{A}_C = \left\{(\bar{V}\cdot\bar{U}_D)(\bar{r} - K_\phi\ T_G\ \bar{V}) - \bar{V}\cdot(\bar{r} - K\ T_G\ \bar{V})\bar{U}_D\right\}K_\alpha \qquad \text{B.2}$$

Using the fact that

$$\bar{V} = (\bar{V}\cdot\bar{U}_D)\bar{U}_D + (\bar{V}\cdot\bar{U}_N)\bar{U}_N + (\bar{V}\cdot\bar{U}_P)\bar{U}_P$$

$$\bar{r} = (\bar{r}\cdot\bar{U}_D)\bar{U}_D + (\bar{r}\cdot\bar{U}_N)\bar{U}_N + (\bar{r}\cdot\bar{U}_P)\bar{U}_P$$

We obtain for equation B.2

$$\bar{A}_C = K_\alpha\Big\{\bar{V}\cdot\bar{U}_D\Big[(\bar{r} - K_\phi\ T_G\ \bar{V})\cdot\bar{U}_D\bar{U}_D + (\bar{r} - K_\phi\ T_G\ \bar{V})\cdot\bar{U}_N\bar{U}_N$$
$$+ (\bar{r} - K_\phi\ T_G\ \bar{V})\cdot\bar{U}_P\bar{U}_P\Big] - \bar{V}\cdot(\bar{r} - K_\phi\ T_G\ \bar{V})\bar{U}_D\Big\} \qquad \text{B.3}$$

For $K_\phi = 2/3$, equation B.3 becomes

$$\bar{A}_C = K_\alpha\left\{\frac{A_N\bar{V}\cdot\bar{U}_D\bar{U}_N}{6/T_G^2} + \frac{A_P\bar{V}\cdot\bar{U}_D\bar{U}_P}{6/T_G^2}\right.$$

$$+ \bar{V}\cdot\bar{U}_D(\bar{r} - 2/3\ T_G\bar{V})\cdot\bar{U}_D\bar{U}_D - \bar{V}\cdot(\bar{r} - 2/3\ T_G\bar{V})\bar{U}_D \Big\} \qquad \text{B.4}$$

where,

$$A_N = \frac{6}{T_G^2}\left[\bar{r} - 2/3\ T_G\ \bar{V}\right]\cdot\bar{U}_N$$

$$A_P = \frac{6}{T_G^2}\left[\bar{r} - 2/3\ T_G\ \bar{V}\right]\cdot\bar{U}_P.$$

This proves the equivalence of the cross product form of the guidance law, equation B.1, and the form of the guidance law used in the main text when the gains for the cross product law are properly selected.

Rearranging terms and simplifying, equation B.4 can be written as

$$\bar{A}_C = K_\alpha\Big\{\frac{A_N\ \bar{V}\cdot\bar{U}_D\bar{U}_N}{6/T_G^2} + \frac{A_P\ \bar{V}\cdot\bar{U}_P\bar{U}_P}{6/T_G^2} - \Big[(\bar{V}\cdot\bar{U}_N)(\bar{r}\cdot\bar{U}_N - 2/3\ T_G\bar{V}\cdot\bar{U}_N)$$

$$+ (\bar{V}\cdot\bar{U}_P)(\bar{r}\cdot\bar{U}_P - 2/3\ T_G\bar{V}\cdot\bar{U}_P)\Big]\bar{U}_D\Big\}$$

$$= \frac{K\alpha}{6/T_G^2}\Big\{A_N\bar{V}\cdot\bar{U}_D\bar{U}_N + A_P\ \bar{V}\cdot\bar{U}_D\bar{U}_P - \left[A_N\bar{V}\cdot\bar{U}_N + A_P\ \bar{V}\cdot\bar{U}_P\right]\bar{U}_D\Big\} \qquad \text{B.5}$$

Finally, equation B.5 can be put in the required form

$$\bar{A}_C = \frac{K\alpha}{6/T_G^2}\ \bar{V}\cdot\bar{U}_D\Big\{A_N\bar{U}_N + A_P\bar{U}_P + A_D\bar{U}_D\Big\}$$

where

$$A_D = -\left[\frac{A_P\bar{U}_P\cdot\bar{V} + A_N\bar{U}_N\cdot\bar{V}}{\bar{U}_D\cdot\bar{V}}\right]$$

$$K_\alpha = \frac{6}{T_G^2\ \ \bar{U}_D\cdot\bar{V}}$$

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