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### I. INTRODUCTION

In 1970, Ward and Mansfield<sup>1</sup> made some hypersonic damping-in-pitch measurements of 9-degree cones by the free oscillation technique. Since an air bearing support was used, the models were free to spin as well as pitch and yaw. An apparently minor tip asymmetry was induced by a planar cut 40° off the X-Z plane. The projected frontal area of the cut was approximately 0.5% of the base area.

The free oscillations were then fitted by the Nicolaides tricyclic theory.<sup>2</sup> This theory assumes that the sole effect of small asymmetry is to add a missile-fixed constant-amplitude trim moment term to the moment expression for the unmodified symmetric cone. A missile that satisfies this assumption is said to have a slight configurational asymmetry. It was found, however, that the measured motion cannot be reasonably represented by the tricyclic theory. In 1972, Walchner, Sawyer, and Yelmgren<sup>3</sup> showed that the apparently very small asymmetry used in the AEDC tests was sufficient to make the pitch and yaw frequencies unequal and thus a more complex mathematical moment assumption is required to analyze the free oscillation data. References 4 and 5 discuss two other cases of apparently symmetric missiles whose motions seem to be poorly described by either the epicyclic motion performed by symmetric missiles or the tricyclic motion performed by slightly asymmetric missiles.

- 1. L.K. Ward and A.C. Mansfield, "Dynamic Characteristics of a 9-Degree Cone with and without Asymmetries at Mach Number 10," Arnold Engineering Development Center TR-70-1, March 1970.
- 2. J.D. Nicolaides, "On the Free Flight Motion of Missiles Having Slight Configurational Asymmetries," Ballistic Research Laboratories Report No. 858, June 1953, AD 26405. (See also IAS Preprint 395, 1952.)
- 3. O. Walchner, F.M. Sawyer and K.E. Yelmgren, "Aerodynamics of a Slender Cone with Asymmetric Nose Bluntness at Mach 14," <u>AIAA</u> Journal 10, August 1972, pp. 1121-1122.
- 4. P.F. Intrieri, D.B. Kirk, G.T. Chapman and J.E. Terry, "Ballistic Range Tests of Ablating and Nonablating Slender Cones," <u>AIAA</u> <u>Journal</u> 8, March 1970, pp. 558-564.
- 5. P. Jaffe, "Nonplanar Tests Using Wind Tunnel Free Flight Technique," <u>Journal of Spacecraft and Rockets</u> 10, July 1973, pp. 435-442.

The effect of constant axial spin on the stability of aircraft was first considered by Phillips.<sup>6</sup> Recently, Hodapp<sup>7-8</sup> has extended this work to unsymmetric re-entry vehicles in a most elegant fashion. Both Phillips and Hodapp make use of missile-fixed coordinates. These coordinates are convenient if the missile is not spinning or if on-board instrumentation is used. For the three experiments cited earlier, external photographic instrumentation was used and the nonrolling aeroballistic axes are much more convenient. A second difficulty of the Phillips and Hodapp papers is that the general pitching and yawing motion of unsymmetric missiles is described by five modal vectors and the results are given in a form in which it is difficult to see how the five vectors reduce to the three modal vectors appropriate to symmetric motion with trim.

In this paper, we make use of the complex variables that considerably simplify the analysis of symmetric missile motion.<sup>9</sup> These variables allow us to move easily from missile-fixed axes to nonspinning axes. They also allow us to obtain very simple expressions for an "almost" symmetric missile. All results are obtained for constant spin.

In Sections II through VI, the analysis is limited to a linear yaw moment and a linear pitch moment with  $C_{m_{\alpha}}$  not equal to  $-C_{m_{\beta}}$ .

Since the pitch and the yaw frequencies are different for zero spin, there is a resonance region for spin when the spin is between these frequencies. After deriving exact relations for the two frequencies of the combined pitching and yawing motion for a spinning missile, we obtain approximations for an "almost symmetric" missile. An almost symmetric missile is one whose zero-spin pitch frequency is very near its zero-spin yaw frequency. For these approximations, three regions of spin are considered separately: spin near zero, spin near resonance, and spin far from zero spin and resonance spin. For the third region, it is shown that in the aeroballistic system, two of the frequencies are approximated by the fast and slow rates  $\phi_{1S}$ ,  $\phi_{2S}$  appropriate to a symmetric missile with a static moment derivative

- 8. A.E. Hodapp, Ir., "Effects of Unsymmetrical Stability Derivative Characteristics on Re-Entry Vehicle Transient Angular Motion," Journal of Spacecraft and Rockets 13, February 1976, pp. 82-90.
- J. C.H. Murphy, "Free Flight Motion of Symmetric Missiles," Ballistic Research Laboratories Report No. 1216, July 1963, AD 442757.

<sup>6.</sup> W.H. Phillips, "Effect of Steady Rolling on Longitudinal and Directional Stability," NACA Technical Note No. 1627, June 1948.

A.E. Hodapp, Jr., "Effects of Unsymmetrical Stability Derivative Characteristics on Re-Entry Vehicle Trim Angle Behavior," <u>Journal of Spacecraft and Rockets</u> 11, May 1974, pp. 300-307.

$$C_{M_{\alpha}} = (C_{m_{\alpha}} - C_{n_{\beta}})/2$$
(1.1)

Two other modes exist with frequencies  $2p - \dot{\phi}_{1S}$ ,  $2p - \dot{\phi}_{2S}$ . The amplitude of these modes is shown to approach zero as  $C_{m_{\alpha}} \rightarrow -C_{n_{\beta}}$ .

Near resonance the fast mode and its associated frequency  $2p - \dot{\phi}_1$ both approach the spin rate, p. For spin rates in the resonance zone between the pitch and yaw frequencies, both  $\dot{\phi}_1$  and  $2p - \dot{\phi}_1$  become equal to the spin rate and one mode is exponentially damped while the other is exponentially undamped.

In Section VII, the effect of linear aerodynamic damping is studied. Its major effect is to reduce the region of spin rates for which exponential undamping exists. Outside the resonance region, the two damping rates are very similar to those for a completely symmetric missile.

#### II. EQUATIONS OF MOTION

The velocity equation for nearly horizontal flight can be easily written in terms of the aerodynamic force along the trajectory--the drag force:

$$mV = -(\rho V^2/2)SC_{\rm D}$$
 (2.1)

If the independent variable is changed from time to dimensionless arclength, s, this reduces to

$$\frac{V'}{V} = -\left(\frac{\rho S\ell}{2m}\right)C_{\rm D} \equiv -C_{\rm D}^{\star}$$
(2.2)

The usual coordinates to describe the angular motion of a rigid body are body-fixed coordinates. For the case of a symmetric missile, the aeroballistic axes, which pitch and yaw with the missile but have a zero spin rate, are more convenient. In this study of an "almost" symmetric missile, we will use missile-fixed axes to obtain our results and then transform the results to the aeroballistic axes.

The X-axis is fixed in the body along the principal axis of inertia nearest to the desired flight direction. Since we retain the assumption of mass rotational symmetry, the transverse moments of inertia are equal and for any roll orientation of the Y and Z axes, these axes will also be along principal axes of inertia. The actual orientation of these axes will be selected to simplify the aerodynamic moment. (The effect of small mass asymmetry is given in Appendix A.) The equations of motion can be written in terms of the aerodynamic force and moment and the derivatives of the linear and angular momentum. (For simplicity the effect of gravity has been neglected. Its effect on the angular motion is usually quite small.)

$$\vec{W} = (\rho V^2/2) S (C_{\chi}, C_{\gamma}, C_{\chi})$$
 (2.3)

$$\vec{H} = (\rho V^2/2) S \ell (C_{\ell}, C_m, C_n)$$
 (2.4)

where

$$\vec{V} = (u, v, w)$$
  
 $\vec{H} = (I_x p, I_t q, I_t r)$ 

The vector derivatives can be computed by the well-known relation for any vector  $(U_x, U_y, U_z)$ :

$$(\mathbf{U}_{\mathbf{x}}, \mathbf{U}_{\mathbf{y}}, \mathbf{U}_{\mathbf{z}})^{\cdot} = (\dot{\mathbf{U}}_{\mathbf{x}}, \dot{\mathbf{U}}_{\mathbf{y}}, \dot{\mathbf{U}}_{\mathbf{z}}) + \vec{\omega} \times (\mathbf{U}_{\mathbf{x}}, \mathbf{U}_{\mathbf{y}}, \mathbf{U}_{\mathbf{z}})$$
(2.5)

where

$$\tilde{\omega} = (p, q, r)$$
.

Equations (2.3-2.4) can be expanded by use of Equation (2.5) and the independent variable changed from t to s. For small angles  $(u \sim V)$ , the transverse components of these vector equations become

$$\frac{\mathbf{v}'}{\mathbf{V}} + \frac{\mathbf{r}\ell}{\mathbf{V}} - \phi'\left(\frac{\mathbf{w}}{\mathbf{V}}\right) = \mathbf{C}_{\mathbf{Y}}^{\star}$$
(2.6)

$$\frac{w'}{V} + \phi'\left(\frac{v}{V}\right) - \frac{q\ell}{V} = C_Z^*$$
(2.7)

$$\frac{q' \ell}{V} - \left(1 - \frac{I_x}{I_t}\right) \phi' \left(\frac{r\ell}{V}\right) = k_t^{-2} C_m^*$$
(2.8)

$$\frac{\mathbf{r}'\,\mathfrak{L}}{V} + \left(\mathbf{1} - \frac{\mathbf{I}}{\mathbf{I}_{t}}\right)\phi'\left(\frac{q\ell}{V}\right) = k_{t}^{-2} C_{n}^{*} \qquad (2.9)$$

The complex notation that is so helpful for a symmetric missile can now be introduced. The complex angle of attack and transverse angular velocity have the usual definitions:

$$\xi = \frac{V + i w}{V} \stackrel{*}{=} \beta + i \alpha \qquad (2.10)$$

$$\mu = (q + i r) \ell / V$$
 (2.11)

By differentiating Equations (2.10) and (2.11) we obtain (with the aid of Equations (2.2) and (2.6-2.9)):

$$\xi' + i \phi' \xi - i \mu = (C_Y^* + i C_Z^*) + C_D^* \xi$$
 (2.12)

$$\mu' + i (\phi' - P)\mu = k_t^{-2} (C_m^* + i C_n^*) + C_D^* \mu$$
 (2.13)

where

$$P = (I_x/I_t)\phi'$$
$$k_t^2 = I_t/m\ell^2$$

#### III. LINEAR STATIC MOMENT

The most general moment components that are linear in  $\alpha$  and  $\beta$  are

$$C_{m} = C_{m_{0}} + C_{m_{\alpha}} \alpha + C_{m_{\beta}} \beta$$
(3.1)

$$C_{n} = C_{n} + C_{n} \alpha + C_{n} \beta$$
(3.2)

In complex form, this becomes

$$C_{\rm m} + i C_{\rm n} = C_{\rm m_0} + i C_{\rm n_0} + \left[C_{\rm MS_{\alpha}} - i C_{\rm M_{\alpha}}\right] \xi + \left[\hat{C}_{\rm MS_{\alpha}} + i \hat{C}_{\rm M_{\alpha}}\right] \bar{\xi}$$
(3.3)

where

$$C_{M_{\alpha}} = \left(C_{m_{\alpha}} - C_{n_{\beta}}\right)/2$$
$$\hat{C}_{M_{\alpha}} = \left(C_{m_{\alpha}} + C_{n_{\beta}}\right)/2$$
$$C_{MS_{\alpha}} = \left(C_{m_{\beta}} + C_{n_{\alpha}}\right)/2$$
$$\hat{C}_{MS_{\alpha}} = \left(C_{m_{\beta}} - C_{n_{\alpha}}\right)/2$$

The effect of rotating the Y and Z axes through the angle  $\theta_0$  can be easily calculated by multiplying Equation (3.3) by exp (i $\theta_0$ ).

$$\begin{bmatrix} C_{m} + i C_{n} \end{bmatrix} e^{i\theta_{0}} = \begin{bmatrix} C_{m_{0}} + i C_{n_{0}} \end{bmatrix} e^{i\theta_{0}} + \begin{bmatrix} C_{MS_{\alpha}} - i C_{M_{\alpha}} \end{bmatrix} \begin{bmatrix} \xi & e^{i\theta_{0}} \end{bmatrix} + \begin{bmatrix} (\hat{C}_{MS_{\alpha}} + i \hat{C}_{M_{\alpha}}) e^{2i\theta_{0}} \end{bmatrix} \begin{bmatrix} \xi & e^{i\theta_{0}} \end{bmatrix} + \begin{bmatrix} (\hat{C}_{MS_{\alpha}} + i \hat{C}_{M_{\alpha}}) e^{2i\theta_{0}} \end{bmatrix} \begin{bmatrix} \xi & e^{i\theta_{0}} \end{bmatrix}$$
(3.4)

According to Equation (3.4), the coefficient of the complex angle is unaffected by a rotation of coordinates but the coefficient of the <u>conjugate</u> of the complex angle is multiplied by exp  $(2i\theta_0)$ . This property allows us to choose a convenient orientation of the Y and Z axes. This orientation will be selected so that the coefficient of the conjugate complex angle is zero or a negative imaginary number  $(\hat{C}_{MS} = 0, \hat{C}_{M} \leq 0)$ . In terms of the coefficients defined in Equations (3.1-3.2).



For a symmetric missile,  $\hat{C}_{M_{\alpha}}$  is zero and  $C_{MS_{\alpha}}$  is an odd function of spin. (The side moment for this case is usually called a Magnus moment.) The addition of a complex constant  $(C_{m_{0}} + i C_{n_{0}})$  to the

moment of a symmetric missile introduces a trim angle and the possibility of resonance.<sup>2</sup>, 10-11

The primary effect of the  $\mathrm{C}_{\mathrm{MS}_\alpha}$  side moment coefficient on the

angular motion of a symmetric missile is on the damping rates and not on the frequencies. The damping rates are assumed to be small compared with the frequencies and will be calculated later as a perturbation of the zero-damped motion. Thus, the side moment will be grouped with the aerodynamic forces and the damping moments and neglected until the perturbations are introduced. Hence the static moment to be used in the first stage of our analysis is defined by

$$C_{m} + i C_{n} = C_{m_{0}} + i C_{n_{0}} - i \left[ C_{M_{\alpha}} \xi - \hat{C}_{M_{\alpha}} \bar{\xi} \right]$$

$$= C_{m_{0}} + C_{m_{\alpha}} \alpha + i \left[ C_{n_{0}} + C_{n_{\beta}} \beta \right]$$
(3.5)

- 10. R. L. Nelson, "The Motions of Rolling Symmetrical Missiles Referred to Body-Axis System," NACA Technical Note No. 1627, November 1956.
- 11. D.A. Price, Jr., and L. E. Ericsson, "A New Treatment of Roll-Pitch Coupling for Ballistic Re-Entry Vehicles," <u>AIAA Journal</u> 8, September 1970, pp. 1608-1615.

## IV. FREQUENCIES AND MODAL AMPLITUDES

The static moment of Equation (3.5) can now be inserted in Equation (2.13), the aerodynamic force neglected ( $C_D = C_Y = C_Z = 0$ ) and Equation (2.12) used to eliminate  $\mu$ . We obtain

$$\xi'' + i f \phi' \xi' - M_{\pi} \xi + \hat{M} \bar{\xi} = i A \qquad (4.1)$$

where

$$f = 2 - \frac{I_x}{I_t}$$

$$M_r = M + (f - 1)(\phi')^2$$

$$M = k_t^{-2} C_{M_{\alpha}}^*$$

$$\hat{M} = k_t^{-2} \hat{C}_{M_{\alpha}}^* \leq 0$$

$$A = k_t^{-2} \left( C_{m_0}^* + i C_{n_0}^* \right)$$

Since Equation (4.1) is a fourth-order differential system in the real variables  $\alpha$  and  $\beta$ , periodic solutions correspond to two pairs of conjugate roots of a fourth-degree polynomial.

For a symmetric missile with trim,  $\hat{M}$  is zero and the solution to Equation (4.1) can be written in terms of two rotating complex vectors plus a constant trim vector.

$$\xi = k_1 e^{i\omega_1 s} + k_2 e^{i\omega_2 s} + k_3$$
(4.2)

where

$$\omega_{j} = \frac{1}{2} \left[ -f \phi' \pm \left[ (f \phi')^{2} - 4 M_{r} \right]^{\frac{1}{2}} \right]$$
$$= -\phi' + \frac{1}{2} \left[ P \pm \left[ P^{2} - 4M \right]^{\frac{1}{2}} \right]$$
$$k_{3} = -i A/M_{r}$$

In the nonrolling aeroballistic coordinates, this becomes

 $\tilde{\xi} = \xi e^{i\phi's} = k_1 e^{i\phi'_1s} + k_2 e^{i\phi'_2s} + k_3 e^{i\phi's}$  (4.3)

where

$$\phi'_{j} = \omega_{j} + \phi' = \left[P \pm \left[P^{2} - 4M\right]^{\frac{1}{2}}\right]/2$$

Note that the trim vector  $k_3$  becomes infinite for  $M_r = 0$ . This is the usual spin-yaw resonance condition and the spin that makes  $M_r$  vanish is resonance spin.

Usually  $\phi'_1$  is defined to be the larger frequency and, if we limit the spin to positive values,  $\omega_1$  would then correspond to the positive square root and  $\omega_2$  to the negative square root. This means that for negative M,  $\omega_2$  is always negative and  $\omega_1$  is positive for M<sub>r</sub> negative and negative for M<sub>r</sub> positive. Thus,  $\omega_1$  changes sign at resonant spin and is always less in magnitude than  $\omega_2$ .

Since Equation (4.1) contains  $\bar{\xi},$  the solution should consist of complex vectors and their conjugates.

$$\xi = k_1 e^{i\omega_1 s} + k_2 e^{i\omega_2 s} + k_3$$
  
+ k\_4 e^{-i\omega\_1 s} + k\_5 e^{-i\omega\_2 s} (4.4)

By direct substitution into Equation (4.1), we see that the trim vector takes the very simple form

$$k_{3} = -\frac{i[M_{r} A - M \bar{A}]}{M_{r}^{2} - \hat{M}^{2}}$$
(4.5)

Resonance now occurs for the two values of spin that make the denominator in Equation (4.5) vanish. In terms of pitch and yaw zero-spin-rate frequencies  $\omega_{\alpha}$  and  $\omega_{\beta}$ , these spin rates are

$$\phi'_{\ell} = \omega_{\ell} [f - 1]^{-\frac{1}{2}} \qquad \qquad \ell = \alpha, \beta \qquad (4.6)$$

where

$$\omega_{\alpha} = \left[ -k_{t}^{-2} C_{m}^{*} \right]^{\frac{1}{2}}$$

 $\omega_{\beta} = \left[k_{t}^{-2} C_{n_{\beta}}^{*}\right]^{\frac{1}{2}}$ 

Note that the pitch and yaw axes were selected so that

$$\phi'_{\alpha} \ge \phi'_{\beta} \tag{4.7}$$

Direct substitution of Equation (4.4) into Equation (4.1) yields two pairs of equations, for  $k_1$  and  $k_4$  and for  $k_2$  and  $k_5$ . The first pair can be written in the form

$$k_1 [C_1 + 2 f \phi' \omega_1] - \bar{k}_4 \hat{M} = 0$$
 (4.8)

$$-\bar{k}_{1}\hat{M} + k_{4}C_{1} = 0 \qquad (4.9)$$

where

$$C_1 = \omega_1^2 - f \phi' \omega_1 + M_r$$
 (4.10)

The solution to Equations (4.8-4.9) is

$$k_{i_{4}} = \hat{M} \, \bar{k}_{1} \, C_{1}^{-1}$$
 (4.11)

$$C_1^2 + 2 \mathbf{f} \phi' \omega_1 C_1 - \hat{M}^2 = 0$$
 (4.12)

Similar equations follow for k , k , C and  $\omega_2$  . Equation (4.12) and the corresponding equation for C can be solved:

$$C_{j} = -f \phi' \omega_{j} \pm [(f \phi' \omega_{j})^{2} + \hat{M}^{2}]^{\frac{1}{2}}$$
 (4.13)

Since  $k_1$  and  $k_2$  are the only modes present for a symmetric missile,  $k_4$  and  $k_5$  should vanish for  $\hat{M} = 0$  and therefore  $C_j$  should not be zero for this value of  $\hat{M}$ . This means that the plus sign in Equation (4.13) is appropriate for  $C_1$  when  $\phi' > \phi'_{\alpha}$  and for  $C_2$  for all spins. The negative sign should be used for  $C_1$  when  $\phi' < \phi'_{\beta}$ . This can all be summarized as follows:

$$C_{1} = -f \phi' \omega_{1} + \delta [(f \phi' \omega_{1})^{2} + \hat{M}^{2}]^{\frac{1}{2}}$$

$$C_{2} = -f \phi' \omega_{2} + [(f \phi' \omega_{2})^{2} + \hat{M}^{2}]^{\frac{1}{2}}$$
(4.14)

where

$$\delta = \begin{cases} 1 \text{ if } \phi' > \phi'_{\alpha} \\ -1 \text{ if } 0 \leq \phi' < \phi'_{\beta} \end{cases}$$

The frequency equations follow from Equations (4.10, 4.12) and the corresponding equations for the second frequency.

$$\omega_{j}^{4} - 2 a \omega_{j}^{2} + b = 0$$
 (4.15)  
 $j = 1, 2$ 

$$a = (1/2) (f \phi')^2 - M_r$$
  

$$b = M_r^2 - \hat{M}^2$$
  

$$= (f - 1)^2 [(\phi')^2 - (\phi'_\alpha)^2] [(\phi')^2 - (\phi'_\beta)^2]$$

The solution to Equation (4.15) can be written in the form:

$$\omega_{j}^{2} = a \pm \sqrt{a^{2} - b}$$

$$= \left[ \left( \frac{a + \sqrt{b}}{2} \right)^{\frac{1}{2}} \pm \left( \frac{a - \sqrt{b}}{2} \right)^{\frac{1}{2}} \right]^{2} \quad (4.16)$$

Since a is always positive, Equation (4.16) gives positive values of  $\omega_1^2$  and  $\omega_2^2$  for b and (a<sup>2</sup> -b) positive. For negative b, only  $\omega_2^2$  is positive and a real  $\omega_1$  does not exist. Under these conditions, the precise relations for real  $\omega_i$  are:

$$\omega_{1} = \delta \left[ -\left(\frac{a + \sqrt{b}}{2}\right)^{\frac{1}{2}} + \left(\frac{a - \sqrt{b}}{2}\right)^{\frac{1}{2}} \right]$$
(4.17)

$$\omega_{2} = -\left(\frac{a + \sqrt{b}}{2}\right)^{\frac{1}{2}} - \left(\frac{a - \sqrt{b}}{2}\right)^{\frac{1}{2}}$$
(4.18)

If Equation (4.11) and its k counterpart are substituted in Equation (4.4) and if the k's are expressed in the polar form  $K_j \exp(i \phi_{j0})$ , the solution motion takes a very simple form in aeroballistic coordinates:

$$\tilde{\xi} = K_1 \left[ e^{i\phi_1} + \hat{M} C_1^{-1} e^{i(2\phi - \phi_1)} \right] + K_2 \left[ e^{i\phi_2} + \hat{M} C_2^{-1} e^{i(2\phi - \phi_2)} \right] (4.19) + k_3 e^{i\phi}$$

where

for  $0 \leq \phi' < \phi'_{\beta}$  or  $\phi' > \phi'_{\alpha}$ , where

$$\phi_{j} = \phi_{j0} + \phi_{j}'s$$

$$j = 1, 2$$

$$\phi_{j}' = \omega_{j} + \phi'$$

$$\phi = \phi's$$

### V. RESONANCE REGION

When the spin rate  $\phi'$  is between  $\phi'_{\beta}$  and  $\phi'_{\alpha}$ , b is negative and Equation (4.17) is invalid. (It should be noted that Equation (4.18) does give a real number for  $\omega_2$  although the calculation involves complex numbers.) For this resonance region, the terms with  $\omega_1$  in Equation (4.4) must be modified:

$$\xi = k_{1R}e^{\lambda S} + k_{4R}e^{-\lambda S} + k_{2}e^{i\omega_{2}S}$$

$$+ k_{5}e^{-i\omega_{2}S} + k_{3}$$
(5.1)

where

$$k_{jR} = K_{jR} \exp(i \phi_{jR}), j = 1, 4$$

λ > 0

Substitution of Equation (5.1) in Equation (4.1) yields two pairs of real equations for the coefficients of exp ( $\lambda$ s) and exp ( $-\lambda$ s).

$$(\lambda^2 - M_r + \hat{M}) \cos \phi_{1R} - f \phi' \lambda \sin \phi_{1R} = 0$$
 (5.2)

$$\mathbf{f} \phi' \lambda \cos \Phi_{1R} + (\lambda^2 - M_r - \hat{M}) \sin \Phi_{1R} = 0$$
 (5.3)

$$(\lambda^2 - M_r + \hat{M}) \cos \phi_{4R} + f \phi' \lambda \sin \phi_{4R} = 0 \qquad (5.4)$$

$$- f \phi' \lambda \cos \phi_{4R} + (\lambda^2 - M_r - \hat{M}) \sin \phi_{4R} = 0 \quad (5.5)$$
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According to these equations,

$$= \frac{-\mathbf{f} \phi' \lambda}{\lambda^2 - M_r - \hat{M}}$$
(5.6)

$$\lambda^{4} + 2a \ \lambda^{2} + b = 0 \tag{5.7}$$

The real positive solution to Equation (5.7) is

$$\lambda = \left[ -a + \left( a^2 - b \right)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$
 (5.8)

For an almost symmetric missile ( $\hat{M} \ll M$ ), b  $\ll a^2$  and

$$a \doteq (f \phi')^2/2$$
 (5.9)

$$\lambda \doteq \left[\hat{M}^2 - M_r^2\right]^{\frac{1}{2}} / (f \phi') = \sqrt{-b} / (f \phi')$$
 (5.10)

$$\tan \Phi_{1R} \doteq \left[ \frac{(\phi')^2 - (\phi'_{\beta})^2}{(\phi'_{\alpha})^2 - (\phi')^2} \right]^{\frac{1}{2}}$$
(5.11)

According to these approximations,  $\lambda$  vanishes for spin on the boundaries of the resonance region and is a maximum near the center (M<sub>r</sub> = 0). The

 $\tan \Phi_{1R} = -\tan \Phi_{4R}$ 

planes of the exponentially growing and decaying angles are near the pitch plane for spin near  $\phi'_{\alpha}$  and are near the yaw plane for spin near  $\phi'_{\beta}$ . For maximum growth/decay rates, the planes of the growing angle and the decaying angle are perpendicular to each other and both planes bisect the angle between the pitch and yaw planes.

## VI. C, AND $\omega_1$ FOR ALMOST SYMMETRIC MISSILES

We will now obtain special relations for an almost symmetric missile for which  $\hat{M}$  is much smaller than M. For this case, the radical in Equations (4.14) for C<sub>j</sub> can be expanded for three regions of spin-near zero, near resonance and otherwise. These expansions yield the results in Table I.

Far from resonance,  $M_r^2 >> \hat{M}^2$  and a good estimate of the frequencies can be obtained from the approximation

$$\sqrt{b} \doteq \delta \left( M_r - \hat{M}^2 / 2M_r \right)$$
(6.1)

Equations (4.17-4.18) can now be expanded in terms of M to yield:

$$\phi'_{j} = \phi'_{jS} + g_{j} \hat{M}^{2}$$
 (6.2)

$$\phi'_{jS} = \frac{1}{2} \left[ P \pm \sqrt{P^2 - 4M} \right]$$
 (6.3)

where the  $g_j$ 's are given in Table II. According to these equations, very good approximations to the frequencies of an almost symmetric missile are the symmetric frequencies of a missile with the average moment coefficient  $C_{M_{\alpha}}$  of the almost symmetric missile. This is correct to within a term in  $\hat{M}^2$ . Note that the coefficients  $g_j$  are unbounded for zero spin but only  $g_1$  has difficulty near resonance.  $(g_2$  has the limit -  $[(f \phi')^{-3}]/2$  as  $M_r \neq 0$ .)

Near resonance, Equation (4.17) can be expanded for small b to give the relation in Table II. A little additional algebra yields the expansion for frequencies near zero spin given in Table II.

According to this table,  $\phi'_{1}$  at zero spin is the zero-spin yaw frequency and  $\phi'_{1}$  increases as the spin increases. At frequencies well away from zero or  $\phi'_{\beta}$ ,  $\phi'_{1}$  is well approximated by  $\phi'_{1S}$ . In the resonance region, it is the spin frequency and at spins well above the resonance region, it is once again well approximated by the symmetric missile fast frequency.

¢′	C <sub>1</sub>	С <sub>2</sub>
$0 \leq \phi' < \epsilon$	$\hat{M} - f \phi' \omega_{1}$	$-\hat{M} - f \phi' \omega_2$
$\phi'_{\beta} - \varepsilon < \phi' \leq \phi'_{\beta}$	$\hat{M} - f \phi' \omega_1$	B <sub>2</sub>
$\phi'_{\alpha} \leq \phi' < \phi'_{\alpha} + \varepsilon$	$-\hat{M} - f \phi' \omega_1$	B <sub>2</sub>
otherwise	B <sub>1</sub>	B 2

Table I. Approximations for  $C_j$ 

where  $0 < \varepsilon < < \phi'_{\alpha} - \phi'_{\beta}$ 

 $B_{j} = -2 f \phi' \omega_{j} - \hat{M}^{2} [2 f \phi' \omega_{j}]^{-1}$ , j = 1, 2

 $\phi'_2$  has a much simpler behavior. At zero spin, it is the negative of the zero-spin pitch frequency and increases as spin increases. Well away from zero spin, it is well approximated by  $\phi'_{2S}$ .

In Table III, the various approximations for  $C_j$  are used to construct expressions for the various motions for an almost symmetric missile with no trim moment ( $k_3 = 0$ ). This catalog of motions starts at zero spin with the expected Lissajous figure formed by combining the pitching and yawing motion with different frequencies (Case A). For slow spin, we have the more complicated form of Case B.

Case C is the general case for spin well away from zero or resonance. It is a four-mode motion with two modes quite similar to those associated with the symmetric missile epicyclic motion with frequencies  $\phi'_1$  and  $\phi'_2$ . The two additional modes have much smaller amplitudes that go to zero as the missile becomes symmetric  $(\hat{M} \neq 0)$ . Their frequencies have the rather odd form  $2\phi' - \phi'_i$ .

As the spin approaches the resonance region,  $\phi'_1$  and  $2\phi' - \phi'_1$ become equal and the motion associated with  $K_1$  changes character. The amplitude of the  $2\phi' - \phi'_1$  mode approaches the amplitude of the  $\phi'_1$ mode (Case D). At the singular spin value of  $\phi'_\beta$ , a very special secular solution is found (Case E). In the resonance region (Case F), the two modes associated with  $\phi'_1$  both rotate at the spin rate but one mode grows exponentially and the other decays exponentially. The exponent,  $\lambda$ , depends on the size of the resonance region and how far the spin rate is from the boundaries. Case G is the secular solution for the upper boundary of the resonance region and Case H is the motion for slightly higher spins.

In order to show this spectrum of motions in a more specific way, the frequencies, modal amplitudes and resonance damping rate have been computed for an almost symmetric missile with  $\hat{C}_{M_{\alpha}} = 0.095 C_{M_{\alpha}}$  and

f = 1.9. For these values,

 $\phi'_{\alpha}/\phi'_{\beta} = 1.10$  $\omega_{\alpha}/\phi'_{\beta} = 1.04$  $\omega_{\beta}/\phi'_{\beta} = 0.95$ 

Table III. Aeroballistic Motion

$$\begin{split} \frac{\mathbf{A}_{n} \cdot \mathbf{e}' + \mathbf{0}}{\mathbf{k} + \mathbf{l} + \mathbf{2} + \mathbf{K}_{n} \cos \phi_{1} + 2 \pm \mathbf{K}_{n} \sin \phi_{2}} \\ \frac{\mathbf{B}_{n} \cdot \mathbf{0} + \mathbf{e}' + \mathbf{1}}{\mathbf{k}_{n} \cdot \mathbf{1}} + \mathbf{1} + \mathbf{1} + \mathbf{1} + \mathbf{k}_{n} \cdot \mathbf{1} + \mathbf{1}$$

Same as Case C

where

$$\tan \phi_{1R} = \left\{ \left[ (\phi')^2 - (\phi'_\beta)^2 \right] / \left[ (\phi'_\alpha)^2 - (\phi')^2 \right] \right\}^{\frac{1}{2}}$$

and where  $K_{1R}^{}$ ,  $K_{4R}^{}$ ,  $\phi_{\alpha}^{}$ ,  $\phi_{\beta}^{}$ ,  $K_{1\alpha}^{}$ ,  $K_{1\beta}^{}$  are parameters determined by initial conditions.

Figures 1 and 2 give the actual frequencies as functions of  $\phi'/\phi'_{\beta}$ and compare them with the corresponding frequencies for a symmetric missile with moment coefficient  $C_{M_{\alpha}}$ .  $\phi'_{1}$  is essentially  $\phi'_{1S}$  except near zero spin and resonance and  $\phi'_{2}$  is essentially  $\phi'_{2S}$  except near zero spin. At zero spin,  $\phi'_{1}$  and  $\phi'_{2}$  reduce to  $\omega_{\beta}$  and  $-\omega_{\alpha}$ , respectively.

Figures 3 and 4 show the variation of the "asymmetric" modal amplitudes  $K_4$  and  $K_5$  in comparison with the corresponding "symmetric" modal amplitudes  $K_1$  and  $K_2$ .  $K_4$  is quite small except near zero spin and near resonance and  $K_5$  is quite small except near zero spin. It is important to note that  $K_4$  is always a larger fraction of  $K_1$ than  $K_5$  is of  $K_2$ . This is due to the presence of  $\omega_j$  in  $C_j$  and the fact that  $\omega_1$  is always smaller in amplitude than  $\omega_2$ . Thus we would expect to observe the presence of the  $2\phi' - \phi'_1$  mode before the  $2\phi' - \phi'_2$  mode has a noticeable effect.

Finally, the resonance damping rate is plotted in Figure 5. The peak value of .047 corresponds to about a 30% increase in amplitude during one revolution.

### VII. EFFECT OF DAMPING

In order to complete our study of an almost symmetric missile, we will now assume asymmetric normal force and damping moments in addition to the side moment we omitted in section III. These terms are generalizations of those of Reference 9.

$$C_{Y} + i C_{Z} = -C_{N_{\alpha}} \xi - \hat{C}_{N_{\alpha}} \xi$$
(7.1)  

$$C_{m} + i C_{n} = C_{m_{0}} + i C_{n_{0}} + [C_{MS_{\alpha}} - i C_{M_{\alpha}}]\xi$$

$$+ i \hat{C}_{M_{\alpha}} \bar{\xi} + C_{M_{q}} \mu + \hat{C}_{M_{q}} \bar{\mu}$$

$$- i C_{M_{\alpha}} [\xi' + i \phi' \xi]$$

$$+ i \hat{C}_{M_{\alpha}} [\bar{\xi}' - i \phi' \bar{\xi}]$$
(7.2)

The new coefficients are defined in Table IV. This assumed force and moment can now be inserted in Equations (2.12-2.13) and  $\mu$  eliminated between the resulting equations. (Due to size considerations, products of C\* terms are ignored.) We obtain

$$\xi'' + [H + i f \phi']\xi' - [M_r + i(M_s - \phi'H)]\xi$$
  
= i A -  $\hat{H} \bar{\xi}' - [\hat{M} + i (\hat{M}_s - \phi'\hat{H})]\bar{\xi}$  (7.3)

where

$$H = C_{N_{\alpha}}^{*} - 2 C_{D}^{*} - k_{t}^{-2} (C_{M_{q}}^{*} + C_{M_{\alpha}}^{*})$$

$$M_{S} = P (C_{N_{\alpha}}^{*} - C_{D}^{*}) + k_{t}^{-2} C_{MS_{\alpha}}^{*}$$

$$\hat{H} = \hat{C}_{N_{\alpha}}^{*} + k_{t}^{-2} (\hat{C}_{M_{q}}^{*} + \hat{C}_{M_{\alpha}}^{*})$$

$$\hat{M}_{S} = f \phi' \hat{C}_{N_{\alpha}}^{*}$$

$$29$$

Table IV. Aerodynamic Coefficients

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$$C_{N_{\alpha}}^{n} = -(C_{Y_{\beta}} + C_{Z_{\alpha}})/2 \qquad (C_{Y_{\beta}} - C_{Z_{\alpha}})/2 \qquad (C_{M_{\alpha}} = -(C_{Y_{\beta}} - C_{Z_{\alpha}})/2 \qquad (C_{M_{\alpha}} - C_{N_{\beta}})/2 \qquad (C_{M_{\alpha}} - C_{M_{\alpha}})/2 \qquad (C_{M_{\alpha}} - C_{M_{\alpha}})/2$$

where

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The solution is assumed to have the form:

$$\xi = k_1 e^{(\lambda_1 + i \omega_1)s} + k_2 e^{(\lambda_2 + i \omega_2)s}$$
  
+  $k_3 + k_4 e^{(\lambda_1 - i \omega_1)s} + k_5 e^{(\lambda_2 - i \omega_2)s}$  (7.4)

Substituting in Equation (7.3), we have

$$k_{3} = \frac{-i\left\{ \left[M_{r} - i\left(M_{S} - \phi' H\right)\right] A - \left[\hat{M} + i\left(\hat{M} - \phi'\hat{H}\right)\right]\hat{A} \right\}}{M_{r}^{2} - \hat{M}^{2} + \left(M_{S} - \phi' H\right)^{2} - \left(\hat{M}_{S} - \phi' \hat{H}\right)^{2}}$$
(7.5)

$$k_{1}\left[C_{1} + 2 f \phi' \omega_{1} - i (2 \phi'_{1} - P)(\lambda_{1} - \lambda_{1S})\right] - \bar{k}_{\mu}\left[\hat{M} + i [(\omega_{1} - \phi')\hat{H} + \hat{M}_{S}]\right] = 0$$
(7.6)

$$= k_{1} \left[ \hat{M} + i \left[ (\omega_{1} + \phi') \hat{H} - \hat{M}_{S} \right] \right]$$

$$+ \bar{k}_{4} \left[ C_{1} + i \left[ (2\phi'_{1} - P) (\lambda_{1} - \lambda_{1S}) - 2 \omega_{1} (H + 2 \lambda_{1}) \right] \right] = 0$$

$$(7.7)$$

where

$$\lambda_{jS} = \frac{-[H\phi'_j - M_S]}{2\phi'_j - P}$$

and similar equations hold for  $k_2$  and  $k_5$ . Outside the resonance

region, the real part of the determinate of the coefficients in Equations (7.6-7.7) set equal to zero gives a slightly modified relation for the frequency  $\phi'_1$ . The imaginary part of this determinate set equal to zero gives a good approximation for the damping. With the approximation

$$C_{1} \doteq -2 f \phi' \omega_{1}$$
 (7.8)

the damping exponent has the following simple approximation for spins far from zero and the resonance region:

$$\lambda_{1} = \lambda_{1S} + \frac{\hat{M}\hat{H}}{f\phi'(2\phi'_{1} - P)}$$
(7.9)

Similarly

$$\lambda_2 = \lambda_{2S} + \frac{\hat{M}\hat{H}}{f\phi'(2\phi'_2 - P)}$$
(7.10)

Thus the damping exponent differs only a little from the damping for a symmetric missile with the average moment coefficients.

The bounds of the resonance region are set by the zeros of the denominator of Equation (7.5). If we neglect the side moment coefficient, this condition has the simple form.

$$M_r^2 - \hat{M}^2 + (\phi')^2 F = 0$$
 (7.11)

where

$$F = [H - (2 - f)(C_{N_{\alpha}}^{*} - C_{D}^{*})]^{2}$$
$$- [\hat{H} - f \hat{C}_{N_{\alpha}}^{*}]^{2}$$

If we denote the damping-modified resonance spins as  $\phi'_{\beta R}$  and  $\phi'_{\alpha R}$  and assume F to be small, the modified resonance spins can be computed as perturbations of the zero-damping resonance spins  $\phi'_{\beta}$  and  $\phi'_{\alpha}$ .

$$\phi'_{\beta R} = \left[1 - \frac{F}{4 (f - 1) \hat{M}}\right] \phi'_{\beta}$$
 (7.12)

$$\phi'_{\alpha R} = \left[ 1 + \frac{F}{4 (f - 1) \hat{M}} \right] \phi'_{\alpha}$$
 (7.13)

Since  $\tilde{M}$  was selected to be negative, a positive F reduces the size of the resonance region and negative F increases it.

#### VIII. SUMMARY

1. The general motion of an almost symmetric missile is well approximated by a symmetric missile with average coefficients.

2. Far from zero spin or resonance spin rates, the first observable modification of the usual tricyclic motion for an almost symmetric missile is the appearance of a  $2\phi' - \phi'_1$  frequency followed by the appearance of a  $2\phi' - \phi'_2$  frequency as the asymmetry becomes greater.

3. Near zero spin, both of these additional frequencies have substantial amplitudes and near resonance, the  $2\phi' - \phi'_1$  frequency has a substantial amplitude.

4. For spin in the resonance region, large trims and exponential undamping are possible.





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Figure 2. Frequencies  $\phi_2'/\phi_\beta'$  and  $\phi_{2S}'/\phi_\beta'$  versus Spin Rate  $\phi'/\phi_\beta'$ 









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LIST OF SYMBOLS

A 
$$k_t^{-2} (C_{m_0}^* + i C_{n_0}^*)$$
  
a  $(f \phi')^2/2 - M_r$   
b  $M_r^2 - \hat{M}^2$   
C<sub>D</sub>  $\frac{drag \ force}{\rho \ S \ V^2/2}$   
C<sub>k</sub>, C<sub>m</sub>, C<sub>n</sub> missile-fixed components of  $\left(\frac{aerodynamic \ moment}{\rho \ S \ V^2/2}\right)$   
C<sub>k</sub>, C<sub>y</sub>, C<sub>z</sub> missile-fixed components of  $\left(\frac{aerodynamic \ force}{\rho \ S \ V^2/2}\right)$   
C<sub>1</sub>, C<sub>2</sub>  $\omega_j^2 - f \phi' \ \omega_j + M_r$ ,  $j = 1, 2$   
C(j),  $\hat{C}$  see Table IV for all C definitions not given above  
f 2 -  $(I_x/I_t)$   
g<sub>1</sub>  $-\omega_{2S} [2 \ f \phi' \ M_r \ (\omega_{1S} - \omega_{2S})]^{-1}$   
H  $C_{N_{\alpha}}^* - 2 \ C_D^* - k_t^{-2} \ (C_{M_q}^* + C_{M_{\alpha}}^*)$   
 $\hat{H}$  angular momentum  
I<sub>t</sub> transverse moment of inertia  $(=I_y = I_z)$   
I<sub>x</sub> axial moment of inertia  
K<sub>j</sub> absolute value of k<sub>j</sub>  
K<sub>jR</sub>

# LIST OF SYMBOLS (Continued)

k <sub>j</sub>	complex yaw modes, $j = 1-5$
<sup>k</sup> jR	complex yaw modes in the resonance region, $j = 1, 2$
k <sup>2</sup> t	I <sub>t</sub> /m <sup>2</sup>
L	reference length
М	$k_t^{-2} C_{M_{\alpha}}^{\star}$
ĥ	$k_t^{-2} \hat{C}_{M_{\alpha}}^*$
M <sub>r</sub>	M + $(f - 1)(\phi')^2$
м <sub>S</sub>	$P(C_{N_{\alpha}}^{*} - C_{D}^{*}) + k_{t}^{-2} C_{MS_{\alpha}}^{*}$
м <sub>s</sub>	$f \phi' \hat{C}_{N_{\alpha}}^{*}$
m	mass
Р	$(I_x/I_t)\phi'$
p	spin rate (rad/s)
q, r	transverse angular velocity components in the missile-fixed system
S	reference area
S	dimensionless arclength along the trajectory
t	time
u, v, w	missile-fixed components of $\vec{V}$
V	missile velocity
V	magnitude of $ec{V}$
X, Y, Z	missile-fixed axes in which the X-axis is along the principal axis of inertia nearest to the flight direction

## LIST OF SYMBOLS (Continued)

α	$\tan^{-1}$ (w/u), the angle of attack
β	$\sin^{-1}$ (v/V), the angle of sideslip
8	$\begin{cases} 1 \text{ if } \phi' > \phi'_{\alpha} \\ -1 \text{ if } 0 \le \phi' \le \phi'_{\beta} \end{cases}$
ε	any positive number small in comparison with $\phi_{\alpha}'$ - $\phi_{\beta}'$
λ	exponential damping coefficient in the resonance-region solution (5.1) of the yaw equation (4.1)
$\lambda_1, \lambda_2$	exponential damping coefficients in the solution $(7.4)$ of the yaw equation $(7.3)$
<sup>λ</sup> jS	- $(H\phi'_j - M_S)/(2\phi'_j - P)$ , $j = 1, 2$ (the symmetric-missile value of $\lambda_j$ )
μ	(q + ir) l/V
ξ	$\frac{v + i w}{V}$ , missile-fixed complex angle of attack
~ گر	$\xi e^{i\phi}$ , aeroballistic complex angle of attack
ρ	air density
φ	$\phi'$ s, the roll angle
<sup>∲</sup> jR	argument of k <sub>jR</sub>
<sup>φ</sup> j0	argument of k. j
φ'	pl/V = spin rate (rad/cal)
φ'j	j-th yaw arm frequency in the aeroballistic system, $j = 1, 2$
<sup>φ</sup> 'jS	$\left( p \pm \sqrt{p^2 - 4M} \right) / 2$ , $j = 1, 2$ (the symmetric-missile value of $\phi'_j$ )
$\phi'_{\alpha}$	upper boundary of the resonance spin region

## LIST OF SYMBOLS (Continued)

Φ <sup>′</sup> β	lower boundary of the resonance spin region
ωj	j-th yaw arm turning rate in the missile-fixed system, $j = 1, 2$
ω <sub>α</sub>	$(-k_t^{-2} C_m^*)^{\frac{1}{2}}$ , the pitch frequency for zero spin
<sup>ω</sup> β	$(k_t^{-2} C_n^*)_{\beta}^{\frac{1}{2}}$ , the yaw frequency for zero spin
→ ω	angular velocity of the missile-fixed system with respect to an inertial system

# Superscripts

(	= d ( )/dt
()'	$= d ()/ds = ()\ell/V$
()*	$= \frac{\rho S l}{2m} ()$
( <sup>-</sup> )	= complex conjugate of ( )

#### APPENDIX A. EFFECT OF SMALL MASS ASYMMETRY

In this report, we have restricted the allowable asymmetries to the aerodynamic force and moment. Mass asymmetries can change the center of mass and the inertia properties. A transverse shift of the center of mass introduces a trim moment due to drag and a roll moment due to lift; these moments can be easily handled.<sup>12</sup> The effects of unequal transverse moments of inertia and of products of inertia are more complicated and will be considered here. We will outline the procedure to handle small inertia asymmetries and show their impact on the results of this report.

For asymmetric inertia properties, the angular momentum vector has missile-fixed components

$$H_{x} = I_{x} p - J_{xy} q - J_{xz} r$$
(A1)

$$H_{y} = -J_{xy} p + I_{t}[(1 + d_{1}) q + d_{2}r]$$
(A2)

$$H_z = -J_{xz} p + I_t [d_2 q + (1 - d_1) r]$$
 (A3)

where

$$d_{1} = (I_{y} - I_{z})/(2 I_{t})$$
$$d_{2} = -J_{yz}/I_{t}$$
$$I_{t} = (I_{y} + I_{z})/2$$

 D.A. Price, Jr., "Sources, Mechanisms, and Control of Roll Resonance Phenomena for Sounding Rockets," Journal of Spacecraft and <u>Rockets 4</u>, November 1967, pp. 1516-1525.

$$J_{xy}, J_{yz}, J_{xz}$$
 are products of inertia.

Equation (2.13) now takes on the form

$$\mu' + i (\phi' - P)\mu + (d_1 + i d_2)(\bar{\nu}' + i \phi' \bar{\mu})$$

$$= k_t^{-2} (C_m^* + i C_n^*) \qquad (A4)$$

$$+ C_D^* [\mu + (d_1 + i d_2)\bar{\mu}]$$

$$+ i (J_{xy} + i J_{xz})(\phi')^2/I_t$$

 $C_{D}$  can now be neglected and the static moment of Equation (3.5) can be inserted to obtain a new form of Equation (4.1):

$$\xi'' + i f \phi' \xi' - M_r \xi + \hat{M} \bar{\xi}$$
(A5)  
-  $(d_1 + i d_2) [\bar{\xi}'' + (\phi')^2 \bar{\xi}] = i A$ 

where

$$A = k_t^{-2} (C_{m_0}^* + i C_{n_0}^*) + i (J_{xy} + i J_{xz}) (\phi')^2 / I_t$$

If the pentacycle solution, Equation (4.4), is substituted in Equation (A5), we obtain the following relations for the modal amplitudes:

$$k_{3} = -i \left[ M_{r} A - [\hat{M} - (d_{1} + i d_{2})(\phi')^{2}]\bar{A} \right] b^{-1}$$
 (A6)

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and

$$k_{4} = \left[\hat{M} + (d_{1} + i d_{2})[\omega_{1}^{2} - (\phi')^{2}]\right]\tilde{k}_{1} C_{1}^{-1}$$
(A7)

$$k_{5} = \left[\hat{M} + (d_{1} + i d_{2})[\omega_{2}^{2} - (\phi')^{2}]\right]\tilde{k}_{2} C_{2}^{-1}$$
(A8)

$$C_{j}^{2} + 2 f \phi' \omega_{j} C_{j} - \left[\hat{M} + d_{1} [\omega_{j}^{2} - (\phi')^{2}]\right]^{2} - d_{2}^{2} [\omega_{j}^{2} - (\phi')^{2}]^{2} = 0$$
(A9)

where

$$C_{j} = \omega_{j}^{2} - f \phi' \omega_{j} + M_{r}$$
  
b =  $M_{r}^{2} - [\hat{M} - d_{1} (\phi')^{2}]^{2} - d_{2}^{2} (\phi')^{4}$ 

It should be noted that for general motion away from zero spin or the resonance region, a good approximation for C<sub>j</sub> is still - 2 f  $\phi' \omega_j$ .

The new frequency equation can be obtained from Equation (A9) by eliminating C<sub>j</sub> and assuming small mass asymmetry (i.e. neglecting  $d_j^2$  in comparison with one).

$$\omega_{j}^{4} - 2 a \omega_{j}^{2} + b = 0$$
 (A10)

where

$$a = (f\phi')^2/2 - M_r + \hat{M} d_1 - (d_1^2 + d_2^2)(\phi')^2$$

The pitch and yaw frequencies for zero spin are given by

$$\omega_{\alpha}^{2} = -k_{y}^{-2} C_{m_{\alpha}}^{*}$$
(A11)

$$\omega_{\beta}^2 = k_z^{-2} C_{n_{\beta}}^*$$
(A12)

where

$$k_{y} = \sqrt{I_{y}/m \ell^{2}}$$
$$k_{z} = \sqrt{I_{z}/m \ell^{2}}$$

The coefficients of the frequency equation can be expressed in terms of these zero-spin frequencies.

$$a = \frac{\omega_{\alpha}^{2} + \omega_{\beta}^{2} + (\phi')^{2} (f^{2} - 2 f + 2)}{2}$$
(A13)

$$b = \left[ \omega_{\alpha}^{2} - \left( \frac{I_{z} - I_{x}}{I_{y}} \right) (\phi')^{2} \right] \left[ \omega_{\beta}^{2} - \left( \frac{I_{y} - I_{x}}{I_{z}} \right) (\phi')^{2} \right]$$
$$- d_{2}^{2} (\phi')^{4}$$
(A14)

The resonance spin region is bounded by the zeros of b. For no product of inertia  $(d_2 = 0)$ , we see that these zeros are

$$\phi'_{\alpha} = \left[ \mathbf{I}_{y} / (\mathbf{I}_{z} - \mathbf{I}_{x}) \right]^{\mathbf{I}_{2}} \omega_{\alpha}$$
(A15)

$$\phi'_{\beta} = \left[ \mathbf{I}_{z} / (\mathbf{I}_{y} - \mathbf{I}_{x}) \right]^{\mathbf{I}_{z}} \omega_{\beta}$$
(A16)

In summary, we see that asymmetric mass terms complicate the algebra but do not change the four conclusions of the report.

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