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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Simulation development methodology is presented as a succession of five closely related and often iterative stages. The stages are: (1) system analysis and requirements definition, (2) implementation, (3) verification, (4) validation, and (5) applications. The objectives for each of the development stages are detailed, and the analytical and investigative procedures for accomplishing those objectives are specified. Requirements for project documentation for each stage of simulation development are also presented.

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US ARMY TEST AND EVALUATION COMMAND
TEST OPERATIONS PROCEDURE

DRSTE-RP-702-104
*Test Operations Procedure 5-1-030 1 October 1978
AD No.

# ANALYTICAL MODELLING AND COMPUTER SIMULATION OF SYSTEMS TEST OPERATIONS PROCEDURE





*This TOP supersedes TOP 5-1-030, 30 November 1972 -

| Paragraph | | Page |
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## 1. SCOPE

a. This document presents methodology which should be followed by system analysts and simulation personnel in the development and application of simulation programs to support system test and evaluation missions. Simulation development is presented as a succession of the following five closely related and often iterative stages:

(1) System analysis and model requirements definition

(2) Implementation

(3) Verification

(4) Validation

(5) Project applications

The objectives for each of the five development stages are detailed, and the analytical and investigative procedures for accomplishing those objectives are specified. Requirements for project documentation for each stage of simulation development are also presented.

b. This TOP addresses the need for acceptance of standard terminology and methodology to be used in development activities across the simulation community, and the five-phase program presented in section 4 is recommended as a standard approach.

c. The detail to which this TOP should be applied depends on the size and complexity of the desired simulation.

## 2. INTRODUCTION

a. Simulation, as an aid to design, development, and test and evaluation of complex Army systems, is an essential resource which affords a means whereby analysts may examine system performance responses when actual system testing either cannot be performed or is impractical. During the system design and development stages, simulation can be used to assess the impact of design and implementation plans on system performance objectives. During system test and evaluation, simulation can be used to obtain data for making performance assessments when resource constraints and the complexity of the system itself make it impractical and often impossible to demonstrate performance objectives by exhaustive testing.

b. For simulation to be as effective as possible in supporting test and evaluation objectives, it is essential that a well-defined methodology be established and maintained — a methodology which requires close coordination among simulation personnel, system developers, system analysts, and system test and evaluation directors. This methodology involves thorough planning for model and simulation development and for system test design which will accommodate simulation validation.

c. The intent of this document is to describe the methodology which should be established by simulation groups to effectively support system test and evaluation. However, the procedures discussed are commonly applicable for simulation development in support of all system investigation activities (design, development, test and evaluation, capability assessment, etc.). Analogous to Mihram's (1) discussion, this methodology is presented as a five-stage simulation development program as follows:

(1) A thorough systems analysis investigation should be conducted jointly by simulation personnel, test directors, and systems analysts for

1. Mihram, G. Arthur (1972) Simulation: Statistical Foundations and Methodology, Academic Press

the purpose of identifying simulation requirements. This activity should also identify the system performance objectives which must be evaluated by using simulation.

(2) Once the detailed requirements have been established and documented (model functional requirements), the implementation stage should be initiated. This activity spans model development (development of math models, algorithms, etc., to depict system characteristics and relationships) and computer simulation program implementation (coding, installation, and checkout when development is required, modification and/or adaptation when existing programs or routines are available).

(3) Simulation verification is the third stage in the development cycle, and though it is a specific activity in and of itself, it cannot be divorced from any of the other development stages. The basic objectives of verification are to establish the integrity of the implemented simulation with respect to program design (system model) and to establish the integrity of the program design with respect to the system which has been modeled. As a result, verification should be a basic concern throughout initial system analyses, model design and development, simulation program checkout, and project applications.

(4) As soon as a simulation program has been implemented and checked out and performance data from actual system/subsystem tests are available, simulation validation can begin. The intent for this activity should be to establish the correspondence between system responses and simulation responses for comparable external stimuli. The objective for validation investigations is threefold:

(a) It serves to answer questions concerning adequacy of modeling detail which were uncovered during verification investigations by demonstrating acceptable (unacceptable) performance;

(b) It identifies errors in program design which often require a review of systems analyses for correction;

(c) It provides data for making an assessment of the credibility of the simulation and either certifies it for or disqualifies it from use in evaluating system performance in the project applications stage to follow.

It should further be noted that simulation validation comparisons should be made against the results from system tests which were carefully designed to establish critical values for the key system performance parameters. While all available system test data can be used for simulation validation, the correspondence with test data from system boundary tests does more to establish simulation credibility.

(5) The project applications stage of simulation development is a realization of the objectives for which the simulation development was initiated. Once an acceptable level of confidence is developed in the program's capacity to represent the actual system or to perform as the actual system would, the program can then be utilized for predicting system performance on subsequent tests, for further test design, and even for making inferences concerning system performance where tests cannot be conducted. This is not to say that the simulation is perfect when it reaches this stage. Often, during this phase, problems arise which require reiteration of the earlier development stages. Systems analyses may have to be reexamined, the program may have to be modified or corrected, and verification and validation may have to be performed on the revised program before it can again be used for system evaluations.

d. Throughout each of the five development stages, careful documentation is essential. Well-organized plans for all investigations with carefully developed, realistic schedules help to insure timely and effective results, and detailed specification and user documentation promotes effective program utilization and helps to deter redundant development efforts and program misapplication. Figure 1, which was adapted from a figure by Mirahm (Footnote 1), depicts the order and relationships for the five development stages and their documentation requirements.

e. As discussed above, simulation can be a highly effective tool in accomplishing a test and evaluation mission; however, everyone involved, from the project manager to the individual who codes a simulation program, must agree on objectives, limitations, and requirements (particularly validation requirements). Above all, complete cooperation is essential.

## 3. DEFINITION OF TERMS

a. System: a collection of regularly interacting or interdependent components acting as a unit in carrying out an implicitly or explicitly defined mission. (As an example, an Army air defense system might be comprised of a number of interdependent components such as

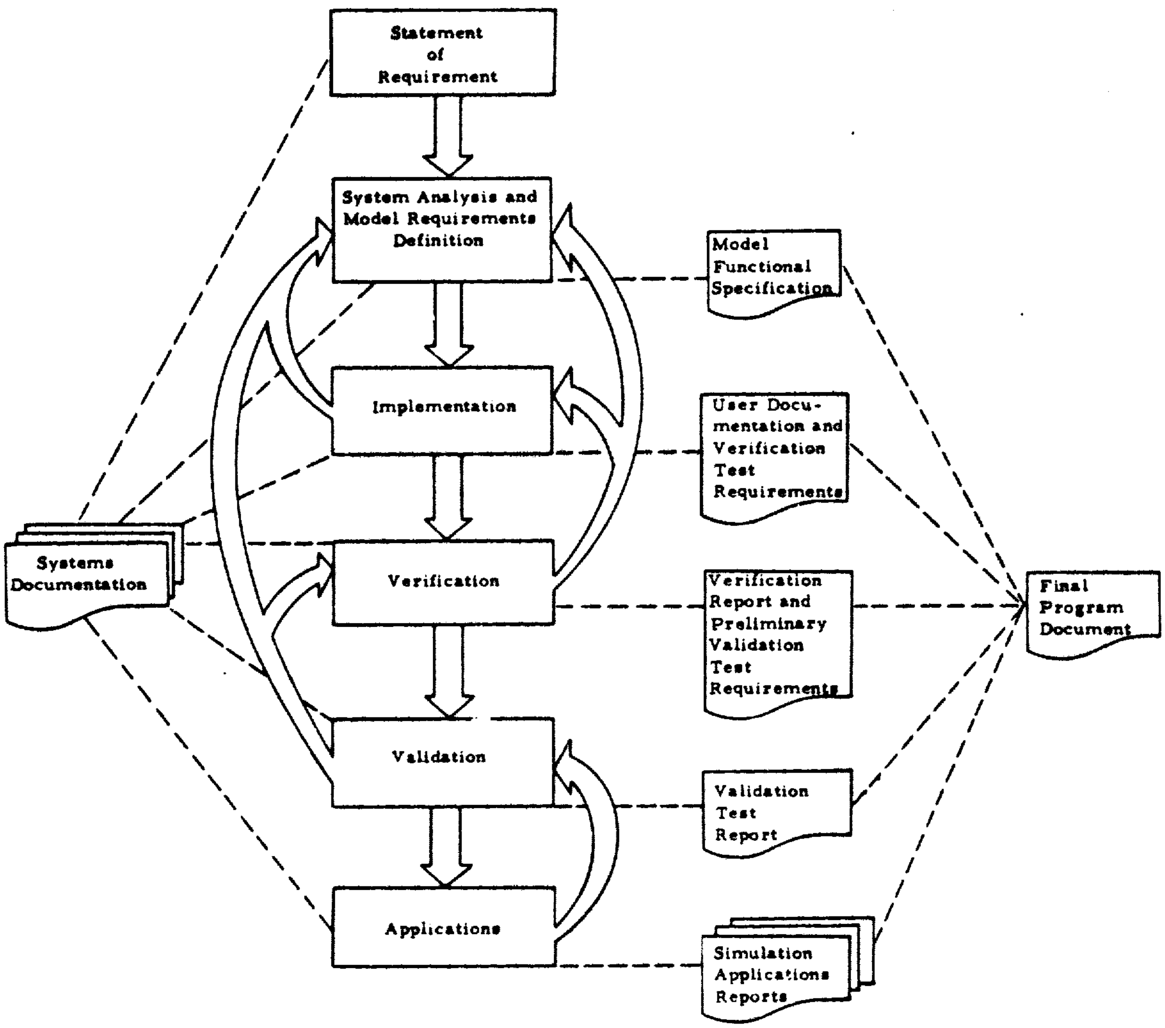



Fig. 1. Stages of Simulation Development

weapon groups, radar groups, weapon control computers, and command and control groups, all of which must interact to form a unit with the specific mission of detecting, engaging, and destroying enemy threats according to a predefined doctrine.)

b. System performance parameter: a measurable system output variable which may be examined to determine whether the system and/or its components (subsystems) are attaining performance levels consistent with mission objectives

c. System response: a measurement or a set of measurements on any of the system performance parameters triggered by external stimuli

d. System analysis: an activity which involves examination of a system (either planned or in existence) for the purpose of identifying all system components and the component interrelationships which are essential to accomplishing the system mission. (Of necessity, this involves conducting an investigation to identify all external stimuli and to determine how they influence the system, to identify the system's processes and to determine how they are performed, and to identify the system's performance parameters and to specify the means for examining system responses in evaluating the system.)

e. System model: a functional (often mathematical) description of a system. This functional description is established for the purpose of answering specific questions concerning the actual system's performance and, depending on application requirements, the amount of system detail in models will vary from problem to problem. However, the requirements common for all system models are that the expressions (equations, algorithms, etc.) in the functional description must maintain the interrelationships between all system components, and that input-to-output transformations for the processes modelled must be equivalent to those of the actual system.

f. Continuous system: a dynamic system whose state variables vary continuously as functions of time. This type of system can be modelled using time-dependent mathematical formulae (differential equations, etc.).

g. Discrete event systems: those systems whose state variables are altered only at discrete instants of time. The state variables may be either functions of time or logic functions (events the occurrence of which may depend on previous event outcomes and logic decision rules).

h. Computer program: either a set of machine-sensible instructions (digital programs) arranged in a proper sequence or specially organized electronic circuitry (analog program). Both analog and digital programs are designed to accept specific inputs and transform them into required outputs (problem solutions).

i. Simulation:

(1) A simulation program is a computer program (analog, digital, or hybrid) which embodies a system/subsystem model or models. The program is designed to accept external stimuli (input parameters), comparable to those which the system would experience, to process this information and to provide responses (outputs) comparable to those which would be provided by the system.

(2) The term simulation is also used when referring to the total five-stage simulation development activity.

(3) The execution of a simulation program for a given set of initial conditions (stimuli) and the responses provided from a computer run can be called a simulation.

j. Deterministic variable: a function whose value is exactly determinable, given the value(s) of its independent variable(s). This type of function, when used to represent some physical phenomenon, is also referred to as a deterministic math model of that phenomenon.

k. Deterministic process: a function of one or more deterministic variables (functions)

l. Random variable: a real-valued function which is associated with some observable phenomenon by a mapping rule which assigns a real number to an observation of that phenomenon. The values of a random variable may or may not be the same when the phenomenon is observed repeatedly under the same conditions.

m. Domain of a random variable: The set of all possible observations of a random phenomenon. (This set is usually called the probability sample space for the phenomenon.)

n. Probabilistic (stochastic) models: models of phenomena which employ random variables.

o. Sample record (sample function): a single time history, over a finite interval, representing measurements of some phenomenon

p. Random (stochastic) process: the totality of sample records which a random phenomenon might produce

q. Stochastic simulation: a simulation which produces time history records of random phenomena

r. Monte Carlo simulation: a technique whereby either random or nonrandom phenomenon occurrences (even highly complex, intractable mathematical function evaluations) are simulated by drawing values for random variables from either known or theorized probability distributions and then solving mathematical or probabilistic models

s. Statistical hypothesis: an assumption about the probability density function of a random variable

t. Statistical hypothesis test: a procedure for deciding whether or not to reject a statistical hypothesis

u. Type I error: the error of rejecting a statistical hypothesis when it is true

v. Type II error: the error of not rejecting a statistical hypothesis when it is false

w. Statistical inference: the process of making decisions or estimations about a population based on information contained in a sample

x. Simulation verification: an investigation activity conducted to establish both the integrity of the implemented simulation with respect to program design (the math model(s) which resulted from systems analysis) and the integrity of the program design with respect to the system which has been modelled

y. Simulation validation: an investigation conducted to demonstrate that the simulation produces responses acceptably comparable to responses produced by the actual system for equivalent stimuli and environmental conditions. (This activity often involves obtaining statistical samples of both simulation and system test data and testing the hypothesis that the samples belong to the same population.)

## 4. SIMULATION METHODOLOGY

a. The discipline of modeling and simulation is one which has been expanding rapidly into most areas of the physical and social sciences. It has gained acceptance as an invaluable tool for use in systems analysis and evaluation by providing scientists, engineers, systems analysts, and managers with a timely and cost-effective means of gaining insight into how actual systems would perform in specific environments when testing is either impractical or impossible. It follows that there are as many diverse considerations in model and simulation development as there are physical and social systems and as many as applications requirements dictate. However, the investigative procedures which must be followed and the order which must be maintained in model and simulation development remain invariant across all disciplines.

b. The intent of this TOP is to present the developmental procedures and the order of their pursuit as a conceptual guide to model and simulation development which could be followed in any field of application. Sections 4.1 through 4.5 present detailed discussions on the procedures for and requirements of each of the five stages of modeling and simulation development listed in paragraph 1a.

### 4.1 System Analysis and Model Requirements Definition

a. Since the fundamental objective for developing models and simulation programs is to establish a capability which will allow investigation of the performance characteristics of systems, either planned or in existence, it follows that the first phase of development should be a detailed study of the system in question. However, before proceeding with the development, a comprehensive statement of requirement for simulation development must be established. This is not to be confused with the detailed functional requirements of the simulation to be determined later; rather, it is a statement of need which identifies for whom the simulation is to be developed and which presents the questions concerning system performance which must be answered.

Without such a formalization of policy to constrain the analysis and model development activity, the program which results may be of little or no value in system evaluation. It should further be emphasized that the statement of requirement should be established jointly by system analysts, system developers, simulation personnel, and system test directors to preclude misconception by model developers of what is needed and misapplication by analysts and decision makers of what the simulation will provide.

b. Once a statement of requirement for modeling and simulation development has been formalized, systems analysts and model developers must analyze the system (or at least system design and requirements documentation) for the purpose of developing detailed modeling functional requirements. The key investigation activities in this effort involve studying the system and its subsystems to:

(1) Determine interactions

(2) Determine interrelationships

(3) Identify system and subsystem processes and their performance requirements

(4) Define the environment (inputs, external stimuli, etc.) within which the system must operate

(5) Characterize system and environmental stochastic elements

(6) Identify system/subsystem performance parameters (state variables, outputs, etc.) which can be examined to evaluate system performance

4.1.1 Analysis Methodology

a. The methodology which should be followed in the analysis and requirements definition phase is a top-down systems approach which involves developing Hierarchy plus Input, Process, Output (HIPO)[2]

2. Katzon, Harry Jr. (1976) System Design and Documentation, an Introduction to the HIPO Method, Van Nostrand Reinhold Company

descriptions of the system and its subsystems. This involves developing hierarchical functional decomposition descriptions of the system and its components and processes, and developing complete descriptions of the input (environmental stimuli, etc.) -process (internal functions, characteristics, constraints, etc.) -output (responses and state variables) sequence at each level of the system hierarchy. At the top level of the hierarchy, the task involves the following:

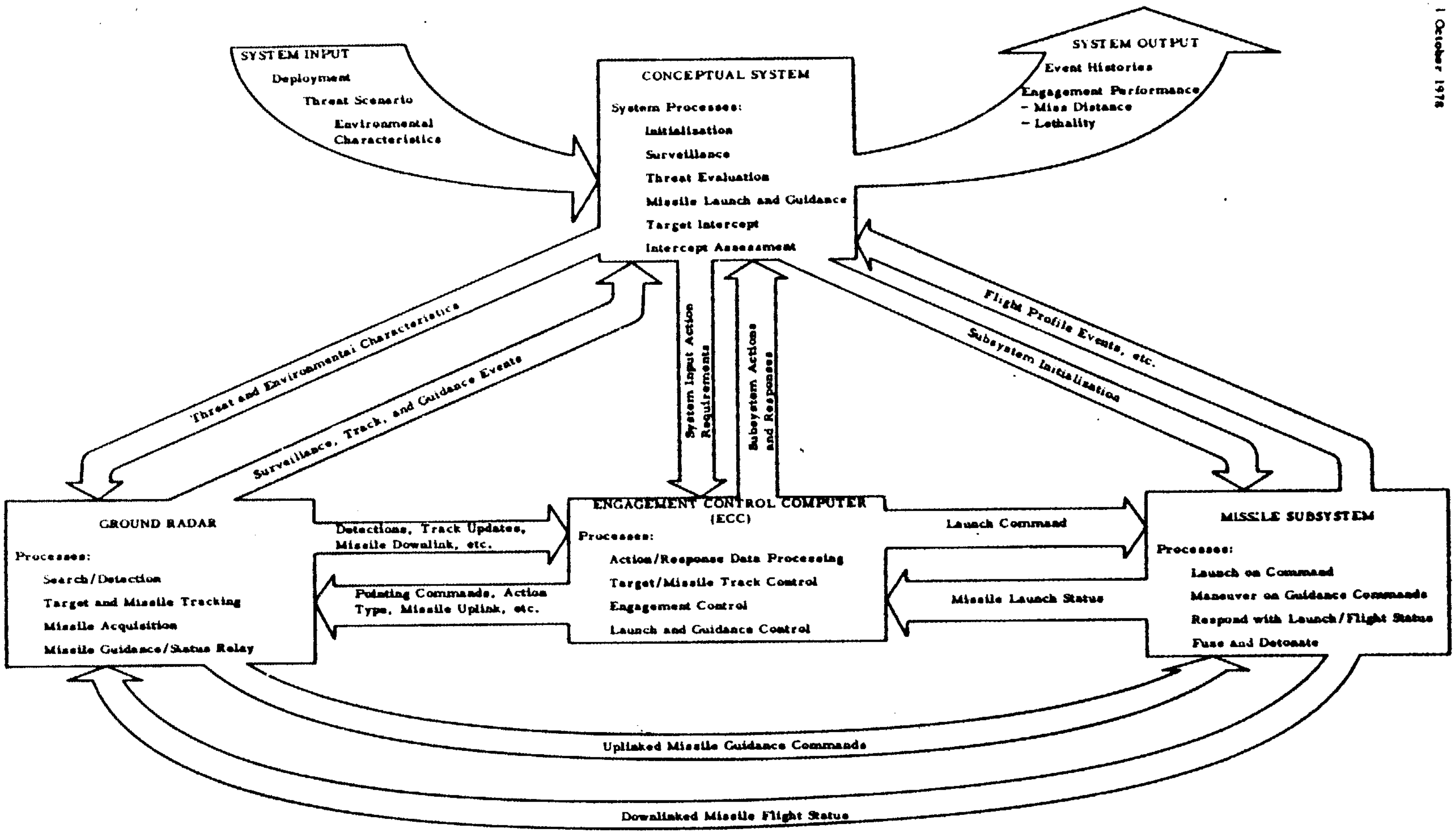
(1) Input descriptions: comprehensive descriptions of the environment within which the system must operate and identification of system input parameters with complete characterization as to timing, units, and possible parameter combinations

(2) Process description: a detailed functional description of the manner of accepting input, of the system mechanics for organizing the actions of its components (subsystems), and of the means whereby subsystem responses are organized to produce a system response. If one were to develop a hierarchical diagram of the system's functional decomposition such as the example in Figure 2, the system process would be the top node.

(3) Output description: a complete description of all possible system state/response variables. There must also be an association of response variables with the possible combinations of input parameters.

At lower levels in the hierarchy, the requirements for organizing analysis results are the same as those for the system level. However, particular attention must be given to functionally depicting the interactions and interrelationships between the system and its subsystems and between subsystems. It is important to note that each system component (subsystem, sub-subsystem, system process, etc.) is also a system and can be described as an input-process-output sequence. Subsystem input can come from either system external sources or from sources strictly internal to the system; however, the former are consider system input. The subsystem process may be either at a bottom level in the system hierarchy or a node which combines activities of even lower-level system components. Outputs from the subsystem process may be either inputs to other system processes or coordinated with outputs from other processes to formulate a system response (output). Figures 2 and 3 are examples of the system's hierarchical functional decomposition, depicting the input-process-output sequences.

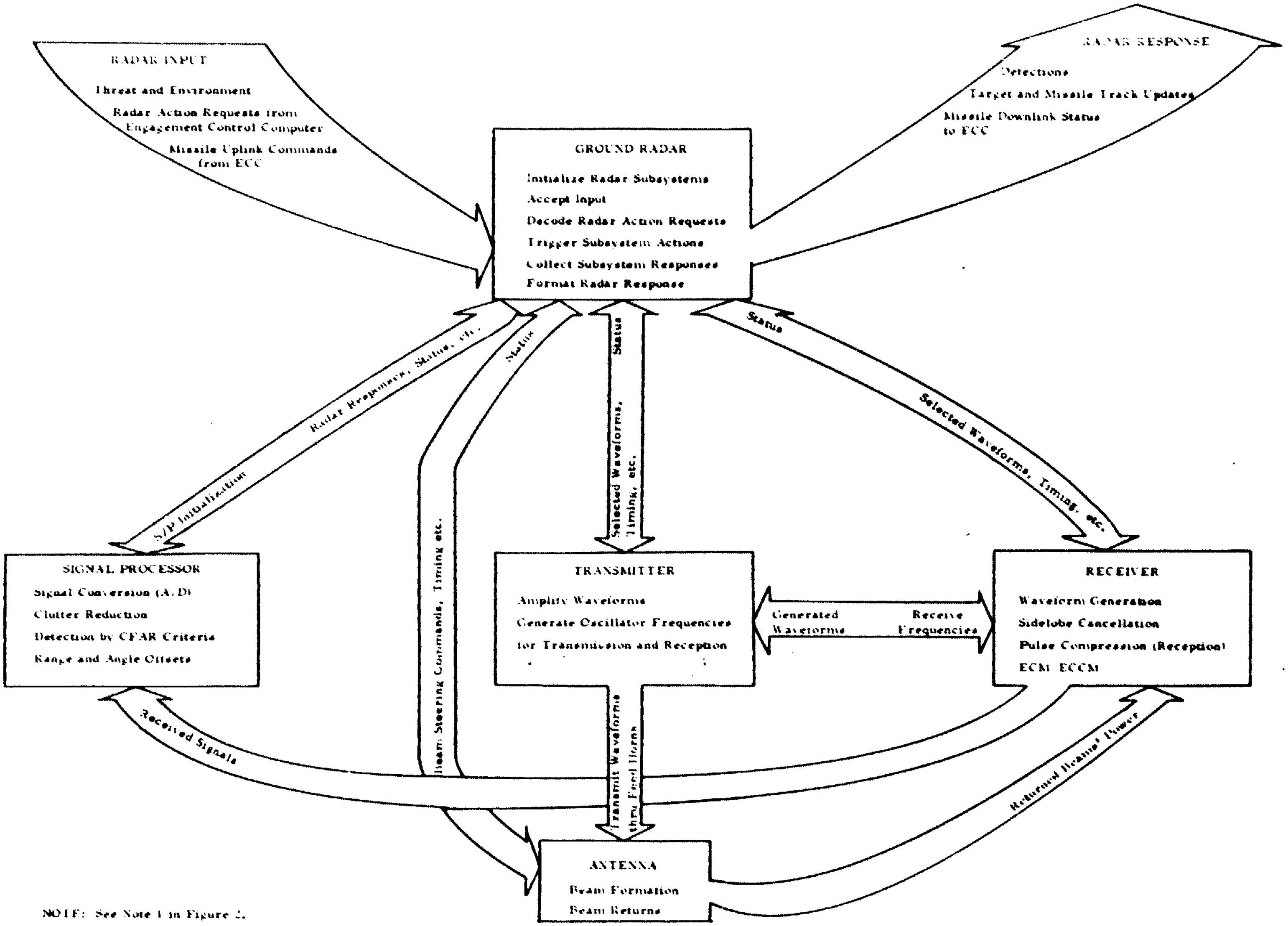
b. The system hierarchy is, in reality, a network in which data may be accepted almost anywhere and may flow in any direction.




NOTE:

1. Comprehensive descriptions of all input (type, format, etc.) -process (functions, logic, algorithms, etc.) -output (type, format, etc.) would be documented during system analysis.
2. The conceptual system block is usually not a physical reality. Its functions may be performed, all or in part, by the composite actions of subsystems. However, in the diagram, it represents the focal point for input-process-output actions.

Fig. 2. Simplified System Input-Process-Output Diagram for a Typical Air Defense System




NOTE: See Note 1 in Figure 2.

Fig. 3. Simplified Lower-Level Functional Decomposition (Ground Radar from Figure 2)

This complicates the analysts' task of developing detailed system model specifications. However, model and simulation development can succeed only if the analysis of the system has been rigorous and well documented. This does not mean that the model specifications will require complete detail correspondence between model components and elements in the system hierarchy. The model itself will have to be developed on the basis of a number of assumptions. It will contain many formalizations and mathematical idealizations, and it will be designed only to answer questions concerning system performance which were formalized in the original statement of need or which were agreed to later, during analysis investigations.

### 4.1.2 Analysis Documentation Requirements

a. The end result of the system analysis investigation should be a detailed model functional requirements document (see Fig. 1) which should contain the following:

(1) The original statement of need for simulation development

(2) Input-process-output descriptions, complete with diagrams depicting all system elements to be modelled. These descriptions must accurately represent the system's interaction with its environment and the interactions and interrelationships between system components.

(3) Events and event sequences over the system's operations cycle, with detailed information on combinations, options, timing, etc., to provide a basis for model executive logic development

(4) Data and plans for model testing with specific requirements for verification and validation tests which should be performed to determine the acceptability of any proposed model assumptions, simplifications, etc.

Within the input-process-output descriptions, careful attention must be given to describing all parameters as to data source, units, nominal values, role in system/subsystem performance evaluation, etc. Moreover, system and environmental random variables must be characterized (statistics, distributions, etc.) as accurately as possible.

b. It should be reemphasized that the success of the following model and simulation development stages is completely dependent on the results of the system analysis activity, and unless the analysis results are well documented in the model functional requirements document, the entire project could easily fail.

## 4.2 Implementation

There are two principal activities to be pursued in the implementation phase of simulation development: model development and simulation program development. Both endeavors are highly dependent on the results of system analyses and interdependent on each other. Model development involves designing and developing mathematical models of the system components' functions (processes), organization of the models into modules comparable to the system components which they must emulate, and development of the control logic required to interface the modules and effect the system's interactive and intra-active relationships in the system model. Simulation program development is the activity of transforming the system model into an operational computer program which is functionally equivalent to the system model. This effort involves selecting the appropriate computer hardware (analog, digital, or hybrid), translating detailed model descriptions into a computer program (coding in the selected simulation language, developing analog program circuitry diagrams, etc.), and then implementing the program. Sections 4.2.1 and 4.2.2 discuss model development and simulation program development in more detail.

### 4.2.1 Model Development

a. The model functional requirements document provides a comprehensive description of the conceptual system model, and often the document will completely specify many of the math models to be used (the system and subsystem hierarchical input-process-output descriptions are, themselves, most often, models of the system's operation). However, in cases where analysis results do not present specific models, they must then be designed in the model development stage.

b. The principal objective of model development is to establish computational procedures (equations, algorithms, logic, etc.) which will serve as input-to-output transformations which are functionally equivalent to their system/subsystem process counterparts. Again, this activity should be constrained by the model functional requirements document which serves as a prescription for methodology whereby the original simulation requirements may be satisfied.

4.2.1.1 Method for Model Development

a. The specific methodology for system model development should be pursued as follows:

(1) Organize system level input (environment, stimuli, etc.) information from the system analysis investigation for the purpose of completely characterizing all system model input scenarios.

(2) Associate the output requirements specified in the model functional requirements document with specific input scenarios.

(3) Based on the system process description as discussed under system analysis, design the system model executive module to accept input scenarios, to sequence events and establish action timing, to organize subsystem models actions, and to organize subsystem models responses to produce the system model responses (outputs). It is at this level that the system's interaction with its environment is modelled with the establishment of system level input and output procedures. The system-with-subsystem relationships are also reflected in the executive module by sequencing and commanding subsystem model process performance and by receiving and organizing subsystem model responses.

(4) Once the system model executive module is established, proceed to develop subsystem model modules for processes at successively lower levels in the system hierarchy descriptions which were established during system analysis. As was discussed earlier, each subsystem is in itself an input-process-output system. Hence, the methodology for developing subsystem model modules is the same as that for developing system modules. The primary considerations here, again, are that the subsystem modules must accept input from the system module and/or other subsystem modules, they must process information with the math models and algorithms, and they must organize actions of lower level modules and utilize their results in establishing subsystem model responses.

From the discussion above, it should be obvious that the system and subsystem model interfaces are accomplished by the input and output procedures.

b. As each module is developed, it should be analytically tested to insure that it will perform as intended on data from the system analysis investigation and on data which can be formulated during model development to demonstrate that performance objectives have been satisfied. Moreover, as modules which must interact with each other are completed, they should be tested collectively, and eventually the composite system model should be tested against specific system input scenarios.

c. The top-down structured approach to system model development discussed above is very effective in two respects: It completely defines the required level of system detail to be modelled, and the resulting detailed model descriptions facilitate the subsequent simulation program development.

4.2.1.2 Model Documentation Requirement. As the system and subsystem models are developed, detailed descriptions of them should be written and organized into a simulation program specification document which must present:

a. Objectives

b. Input-process-output descriptions

c. Events and event sequences

d. Data and plans for program testing

This document must be specifically oriented toward computer implementation of a simulation program which will be functionally equivalent to the system model. In particular, the quantity, types, and formats of data (input and output) must be described in detail. Additionally, any further program verification and validation tests which should be performed to judge model simplifications and assumptions should be completely specified.

4.2.2 Simulation Program Development

Developing a system simulation computer program is analogous to fabricating the system itself from detailed design plans. The primary objective for this stage of development is to transform the functional system descriptions which resulted from system analysis and model development into an operational computer program which is functionally equivalent to the system model. Once its performance has been proven acceptable, the simulation program will be a practical experimental device which can be used to generate data predictive of the performance capabilities of those elements of the system which have been simulated.

4.2.2.1 Program Development Methodology

a. As indicated earlier, simulation program development involves three primary tasks:

(1) Analysis of model descriptions in the program specification document, to determine appropriate computer hardware (analog, digital, or hybrid) for model modules

(2) Selecting appropriate programming techniques (simulation language, etc.) and development of the program (coding, analog circuitry diagramming, etc.)

(3) Implementing, debugging, and testing the program

b. Considering each program development task in turn, based on detailed model descriptions and simulation specifications, each component (module) of the system model should be examined to identify the most appropriate computer equipment for implementation. Some of the factors to be considered in making the selections include:

(1) Proposed use of the program

(2) Number and types of runs to be required

(3) Available resources

(4) Level of model-to-system detail required

(5) Comparative degree of implementation difficulty

(6) Implementation schedule requirements

(7) Program flexibility requirements

c. After selection of the computer hardware to be used, program development should be pursued following top-down, structured approach analogous to model development. The program executive routine (most often the main program) should be developed (coded in an appropriate simulation language or diagrammed for analog implementation) to accomplish the following:

(1) Accept properly formatted system input scenarios and translate them into system/subsystem action requirements (logical options, input or stimuli to functions and subroutines, etc.)

(2) Sequence events and establish action timing

(3) Call on subroutines to perform operations functionally equivalent to subsystem model processes on their input

(4) Organize subroutine results (output) and combine them to establish program output

(5) Format and produce program results (output)

d. It is not mandatory that all of the executive functions be performed in a single routine. Input and output procedures, for example, may be established in separate routines which have the capacity for communicating data to and from the main program and/or any other subroutine. Following the development of the simulation executive routine, subroutines should be developed to perform functions corresponding to subsystem model processes at successively lower levels in the system model hierarchy. Specifically involved in subroutine development is the translation of subsystem model processes (math models, stochastic processes, algorithms, etc.) into computer-sensible procedures which will accept appropriate input and transform it into the required subsystem model response. The key steps to be followed are:

(1) Establish machine-sensible descriptions for program elements (cards, tapes, diagrams, etc.)

(2) Enter the program elements into (on) the computer

(3) Translate digital program routines in higher level languages by computer system software (compilers) into "machine language" (often relocatable binary code elements)

(4) Implement digital program by computer system software processing (collection) of the binary elements into an absolute executable program

The correction of program implementation errors and repetition of the step in which they were encountered are called program "debugging."

e. As each of the program elements is completed, it should be tested with sample inputs as prescribed in the simulation requirements specification document and, on completion and collection of all program elements, the simulation program should be tested as a whole with more comprehensive input scenarios. The purpose of these tests should be to completely debug and check out the program and to determine whether it will perform as intended in correspondence with the system model.

4.2.2.2 Simulation Program Documentation. During model and simulation program development, a simulation program description document should be prepared. This document is the primary source for program user information. Moreover, it presents a single source of information for the full development activity, from statement of requirement to operational program. The simulation program description document should present at least the following information:

a. An abstract statement of purpose and function (This should reflect the original statement of requirement for development and should provide a general description of the input-process-output sequence.)

b. Special features and requirements (program limits, restrictions, computer configuration, language requirements, etc.)

c. Detailed user information (all input requirements, such as files and variables with names, formats, units, etc.; output provided, such as sample listings, plots, etc.; and job control requirements for program execution)

d. A detailed program methodology section to present, on a routine-by-routine basis, a comprehensive description of the computational procedures used to simulate the various system elements. This section should also contain functional flow diagrams for each of the program routines.

e. Sample runs (input and output)

f. A preliminary assessment of the program on the basis of checkout test results

In addition to the simulation program description document, a comprehensive statement of program verification requirements should be established prior to the conclusion of the implementation stage. Also, any requirements for validation testing (to demonstrate acceptability of assumptions, simplifications, etc. in implementation) should be completely documented.

## 4.3 Verification

Once a simulation program has been developed and implemented, and prior to its use as a system evaluation tool, its capacity for representing its counterpart (the physical system) must be established. The simulation verification and the simulation validation stages which follow are conducted to establish program credibility, and both activities must be performed before adequate confidence in the program's performance can be achieved. Validation demonstrates comparability between the simulation program responses (outputs) and the responses from the modelled system for equivalent stimuli (inputs). Verification establishes the integrity of the system model with respect to the design of the actual system and establishes, analytically, the rectitude of the program with respect to the system model.

4.3.1 Verification Analysis and Testing. Verification is a basic concern throughout the simulation development cycle. The degree to which it must be a separate activity, once an operational simulation program is available, depends on whether the program was developed specifically to satisfy the objectives of the current development or whether it was a preexisting program which was adapted to satisfy those objectives. If, in response to the original statement of requirement, the program has evolved through the previously discussed simulation development stages, the major part of the verification investigation will already have been accomplished, and all that remains is to present the appropriate results from the analysis, model development, and implementation stages with pertinent test results in the program verification report. On the other hand, if the program was obtained, and must be evaluated for its applicability, its verification can be satisfactorily accomplished only if it is analyzed in parallel with the physical system, following the methodology specified in the system analysis and model requirements definition development stage (Section 4.1). Regardless of the program's origin, however, the key verification activities may be itemized as follows:

a. Ascertaining the degree of correspondence (functional equivalence) between the physical system and the system model and judging that correspondence on the basis of what the resulting simulation is to be used for (reference the original statement of requirement)

b. Ascertaining the rectitude of the operational simulation computer program with respect to the system model

c. Insuring that the program is syntactically free of errors

d. Running specific test cases with the program to obtain results that can be carefully checked against known relationships and against manually derived results from math models in the system model

e. Specifically documenting all identifiable simplifications or idealizations in either the system model or the simulation program which must be further evaluated in the program validation stage to follow

### 4.3.2 Program Verification Documentation

The results from verification analyses must be carefully examined by simulation personnel, analysts, project directors and decision makers alike to ascertain whether the program will be suitable for the prescribed applications (from the statement of requirement). The material to support the evaluation of these results should be presented in a simulation verification report and should consist of at least the following:

a. Statement of requirements (adapted and expanded from the original statement of requirement)

b. Parallel functional descriptions of the system, system model, and simulation structures (hierarchical input-process-output sequences) which depict cross-correspondence

c. Results from specific tests which have been conducted

d. Discussion of all assumptions, simplifications, etc., which must be evaluated by specific validation testing

From the evaluation of the information in the simulation verification report, a decision must be made to either reexamine earlier analyses and make revisions if the program is inadequate, or to proceed with validation testing.

4.4 Validation

The objective of simulation validation is to demonstrate that the responses (outputs) generated by a simulation correspond, within an allowable tolerance, to actual system performance measurements on the key parameters (system state variables) which must be used to evaluate the system. Before validation investigations are conducted, the following three criteria must be satisfied:

a. System performance data must be obtained from tests which are designed to establish critical values for the system state variables.

b. All environmental conditions which directly influence system performance should be measured and recorded for specification of simulation initial conditions (inputs).

c. The simulation to be validated must accommodate input specification of all critical test initial conditions.

The simulation runs must be constrained to the known system conditions for the test(s) to which they are to be compared; otherwise, conclusions drawn from validation testing may be completely erroneous. Special care must be taken to numerically characterize all test environmental conditions. This must be done to preclude superior (inferior) performance by the simulation which could be caused by too favorable (unfavorable) initial conditions. Where it is impossible to quantify some of the test conditions exactly (errors in observation, etc.), the quantities should be depicted distributionally, and Monte Carlo techniques should be used to select the corresponding simulation initial conditions.

4.4.1 Validation Analysis and Testing

a. The decision as to which kinds of procedures should be used to validate a simulation is constrained by more than the limitations on testing the system. The kind of model which the simulation embodies must also be taken into consideration. Deterministic models which do not incorporate random environmental effects on the system or the random effects within the system can only be judged subjectively, since one can, at best, proceed through the simulation verification stage to ascertain design integrity and then make a judgment concerning validity by examining the simulation results in comparison with the results from a large number of system tests. Dynamic stochastic simulations differ from purely deterministic simulations in that they also include random effects, and their responses can be viewed as random variables. Moreover, the methods which can be employed to validate this class of simulations are well-established statistical procedures for comparing random variables.

b. Considering the responses from a dynamic stochastic simulation as stochastic processes or random variables, there are two basic categories to be treated:

(1) Singular or event-oriented responses: variables for which one or a discrete number of values are generated on any given simulation run or system test (e.g., miss distance, event time occurrences, etc.)

(2) Time series responses: system state variables which are considered to be time dependent functions (e.g., missile or target positions, velocities, accelerations, etc.)

c. The statistical procedures for comparing system and simulation responses which are from the first category above are straightforward and relatively easy to apply. There are a number of useful statistical tests, both parametric and nonparametric, which can be used to establish confidence in comparability between simulation and system responses of this type. Time series responses pose more of a problem. Though the comparison tests for variables from the first class are still applicable, the time series responses will have to be compared discretely at selected points in time over their total duration.

d. In cases where system test data are too limited to permit statistical validation comparison, the analyst should examine as much subjective comparison data as can be obtained. The most frequently used method is to analyze overlay plots of the system test and simulated responses.

e. Appendix A to this TOP provides a detailed discussion on a number of commonly used statistical tests which may be used for simulation validation.

f. A note of qualification is in order regarding application of the procedures for validating simulations. In a complete, abstract sense, simulation validation cannot be accomplished. At best, one can infer simulation-system correspondence based on the lack of sufficient contradictive evidence (e.g., in statistics, when testing a null hypothesis, one either rejects or does not reject the hypothesis based on whether the computed value of a test statistic is judged to be significant). The simulation analyst should, therefore, be very cautious when claiming validity.

### 4.4.2 Program Validation Documentation

Model and simulation credibility are basic concerns throughout all five stages of simulation development. While validation test results alone cannot completely establish program credibility, they provide a basis for making the major decision to utilize the simulation for system performance capability evaluation. Because of their impact on that decision, the validation test results should be presented in a formal report which completely and accurately depicts the simulation's performance capability. The following information should be included in such a report:

a. Objectives. System performance features which have been exercised (relatable to original statement of requirement)

b. Test conditions. Complete description of all system input (including environmental conditions) and its correspondence with simulation input and initial conditions.

c. System data collection procedures. Methods used in measuring system input and response data

d. Methods of response comparison. Methods used for system-simulation response comparison including response (parameter) identification, comparison results (statistics, plots, etc.)

e. Conclusions. Complete analysis of the validation test results, including statistical significance of deviations, identification of inadequacies, recommendations for program improvement, etc.

4.5 Applications

a. The applications stage of model development is the realization of the objectives for which the simulation development was initiated. It is in this stage that the verified and validated simulation program is used by analysts and decision makers to gain insight into how the actual system would probably perform in response to a wide variety of conditions where system performance demonstration is impractical. The word "probably" has been stressed as a reminder that conclusions regarding system performance capability which are based on simulation results can only be inferences, with their attendant statements of significance or confidence.

b. There are many simulation applications possibilities, among which are the following:

(1) Test planning. Simulation results often identify potential system problems which should be examined on the basis of specific system tests.

(2) System design. Simulation may be used to evaluate the impact of alternative design plans on system performance.

(3) Performance prediction. The simulation may be used to predict system performance for specific conditions under which the system is actually to be tested (safety considerations, test adequacy, etc.).

(4) Sensitivity analysis. Inputs to the simulation may be varied parametrically to identify system sensitivities.

(5) System performance evaluation. On the basis of simulation results, analysts can make inferences concerning the system's capability to meet performance objectives (specifications, requirements, etc.).

(6) System capability assessment. Where impractical or impossible by system testing, probable system performance boundaries can be established on the basis of simulation results.

4.5.1 Comments on Applications Methodology

Even though simulation application requirements vary widely, it is extremely important that certain basic procedures be followed in conducting simulation investigations. The key ingredients for a successful, timely, and effective simulation study are as follows:

a. Goals. The objectives for the study must be carefully formalized and judged to be within the simulation's domain of applicability.

b. Schedules and milestones. There must be complete agreement among analysts, simulation personnel, and decision makers on what can be accomplished and in what time frame.

c. Data collection and presentation of results. Well-organized procedures must be established for collection of pertinent system test data and for presenting an appropriate interpretation of simulation results.

4.5.2 Documentation of Simulation Applications

4.5.2.1 Simulation Applications Reports. The results from specific simulation applications investigations should be documented in detail in simulation applications reports to be provided to analysts and decision makers for making system assessments. Such reports should contain the following information:

a. Objectives. A concise review of what the study was supposed to accomplish with a summary assessment of accomplishments and significance of results

b. Investigation details. A complete description of the investigation including data collection, simulation case run descriptions and results (outputs), and problems encountered

c. Conclusions. A comprehensive assessment of the significance of the results from the investigation in an interpreted form, to facilitate use of the results by decision makers

4.5.2.2 Final Program Documentation. Throughout the five simulation development stages, appropriate documentation should be maintained and updated to reflect any changes in project direction, changes in model or program implementations, key milestone accomplishments, etc. In addition, at the conclusion of a specific system evaluation project, a final report should be prepared to reflect project progress through the development cycle. The report should provide a single composite source for information on a simulation program, beginning with its inception and concluding with significant system evaluation accomplishments which were realized from its use. The final project document should be organized in a format similar to the simulation program description document (Section 4.2.2.2), with complete development details presented in the methodology section and the results of all simulation case studies presented in the applications section. The final report should also provide a complete assessment of project accomplishments, along with all relevant management data (personnel, cost, schedules, etc.).

## 5. SUMMARY

The discipline of modelling and simulation is one which can be employed very effectively to support the analysis and evaluation of systems from almost any field of endeavor. However, to insure an effective contribution, it is imperative that simulation development be pursued as a five-stage activity with the objective of satisfying applications requirements which should be established jointly by system analysts, system developers, simulation personnel, and project managers. The five simulation development stages are:

a. System analysis and model requirements definition

b. Implementation

c. Verification

d. Validation

e. Applications

The development stages, while often iterative, reflect the logical succession of activities from program inception through system evaluation. The requirements for detailed documentation development during each of the five stages can be easily satisfied by formally describing investigation activities, and are essential to successful project accomplishment.

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# APPENDIX A

## STATISTICAL TESTS AND PROCEDURES FOR SIMULATION VALIDATION

1. The Kolmogorov-Smirnov Distribution Equivalence Tests

a. The Kolmogorov-Smirnov one-sample and two-sample tests (referred to as K-S1 and K-S2) are nonparametric (i.e., "distribution-free") tests which can be used to infer equivalence between the distribution functions for two random variables and hence that they belong to the same statistical population. Both the K-S1 and the K-S2 tests involve comparison (at a chosen level of significance) of test statistics, computationally based on the maximum deviation between two distribution functions, with standard table values which are based on sample sizes. The K-S1 test can be used for comparing the computed estimates for the distribution function from one sample with a theoretical or a previously determined distribution function, whereas the K-S2 test can be used for comparing the computed estimates for the distribution functions from two samples with each other. The K-S1 test should be used when a variable is to be examined against a postulated distribution or when much more data is available from one source (simulation or system tests) than can be obtained from the other, since larger samples provide a better characterization of the theoretical distribution function.

b. Two assumptions must be satisfied before applying the K-S tests:

(1) That the sample observations are independently selected

(2) That the distribution functions being compared are continuous

c. The computational procedures for conducting a K-S2 test are as follows (K-S1 is a special case of K-S2):

(1) For random variables $X_S$ (simulation response) and $X_T$ (system test response), establish the hypothesis

$$Ho:\ F_{X_S}(a) = G_{X_T}(a) \text{ for all a,}$$

where F and G are the distribution functions for the simulation response and the system test response, respectively.

This hypothesis may be tested against:

(a) All alternatives, i.e., $F_{X_S}(a) \neq G_{X_T}(a)$

or

(b) The alternative that one of the distribution functions is stochastically greater than the other (e.g., Ha: $P(X_S \leq a) > P(X_T \leq a)$)

(2) Proceed by rank ordering the combined sample of $N = m + n$ observations $X_{S1}, X_{S2}, \ldots, X_{Sm}$ and $X_{T1}, X_{T2}, \ldots, X_{Tn}$, and determine values for the empirical distribution functions F and G corresponding to each of the N combined sample values. Then compute the differences $Fi - Gi$, $Gi - Fi$, and $|Fi - Gi|$ for $i = 1, N$. Example: Assume that the following samples have been collected for the random variable X from simulation and system tests.

$$X_{S1} = 1.7,\ X_{S2} = 2.1,\ X_{S3} = 0.9,\ X_{S4} = 0.7,$$

and

$$X_{T1} = 2.0,\ X_{T2} = 1.9,\ X_{T3} = 0.8,\ X_{T4} = 2.3.$$

After following steps 1 and 2 above, we have

$$Ho:\ F_{X_S}(a) = G_{X_T}(a) \text{ for all a, against say}$$

$$Ha:\ F_{X_S}(a) \neq G_{X_T}(a),$$

and the rank order table for i, i = 1, 8:

| i | Sample | Xi | $F_{Xi}$ | $G_{Xi}$ | Fi - Gi | Gi - Fi | \|Fi - Gi\| |
|---|---|---|---|---|---|---|---|
| 1 | S | 0.7 | 1/4 | 0/4 | 1/4 | -1/4 | 1/4 |
| 2 | T | 0.8 | 1/4 | 1/4 | 0 | 0 | 0 |
| 3 | S | 0.9 | 2/4 | 1/4 | 1/4 | -1/4 | 1/4 |
| 4 | S | 1.7 | 3/4 | 1/4 | 2/4 | -2/4 | 2/4 |
| 5 | T | 1.9 | 3/4 | 2/4 | 1/4 | -1/4 | 1/4 |
| 6 | T | 2.0 | 3/4 | 3/4 | 0 | 0 | 0 |
| 7 | S | 2.1 | 4/4 | 3/4 | 1/4 | -1/4 | 1/4 |
| 8 | T | 2.3 | 4/4 | 4/4 | 0 | 0 | 0 |

Note that both $F_i$ and $G_i$ values are determined by dividing the numbers of values from their respective samples which are less than or equal to the $i^{th}$ value in the combined sample by the appropriate sample size.

(3) Three different test statistics may now be computed as follows:

(a) $D_1 = \frac{mn}{d} \max (Gi - Fi)$

(b) $D_2 = \frac{mn}{d} \max (Fi - Gi)$

(c) $D_3 = \frac{mn}{d} \max (|Fi - Gi|)$

where d is the greatest common divisor of m and n.

In the example above, the null hypothesis is to be tested against all alternatives, so we would be interested in $D_3$ only. For the example,

$$D_3 = \frac{(4)(4)}{4} \max (|Fi - Gi|) = 4\left(\frac{2}{4}\right) = 2.$$

This value may now be compared, at say a level of significance corresponding to a type I error of size $\alpha = 0.1$, to the value $\mathcal{D}(0.1, 4, 4)$ from the K-S tables (Reference 2). (Note: The constant $\mathcal{D}(\alpha, m, n)$ satisfies the relation $P[D \geq \mathcal{D}(\alpha, m, n)] = \alpha$.) From the tables, we find that $P(X \geq 2) > 0.1$, and we would not reject Ho.

(4) In general, reject Ho at the $\alpha$ level of significance in favor of

(a) Ha: $F_{X_S}(a) \neq G_{X_T}(a)$

if $D_3 \geq \mathcal{D}(\alpha, m, n)$,

(b) Ha: $F_{X_S}(a) < G_{X_T}(a)$

if $D_1 \geq \mathcal{D}(\alpha, m, n)$,

(c) Ha: $F_{X_S}(a) > G_{X_T}(a)$

if $D_2 \geq \mathcal{D}(\alpha, m, n)$.

d. The quantity $\alpha$ was introduced in the previous discussion and called the size of the type I error, or error of the first kind, for the test. More descriptively, $\alpha$ is the probability an investigator will allow for rejecting a valid hypothesis. A second error, the type II error, or error of the second kind, of size $\beta$ must also be considered when testing hypotheses. Its value for the test is the probability of not rejecting a false hypothesis. This quantity is often called the analyst's risk. $\beta$ is usually depicted as operating characteristics curves obtained by plotting this probability as a function of significance level and sample size. Owen (1) presents a formula for determining sample size for the K-S1 test to attain a given power. For small samples, the K-S tests are more powerful than the Wilcoxon test which is discussed in the next section. For large samples, however, the reverse is true. Hollander and Wolfe (2) provide an excellent description of the K-S2 test, and they also include a discussion on large sample approximation. For a more detailed discussion on the efficiency of K-S testing, see Bradley (3).

2. The Wilcoxon Rank Sum Test. The nonparametric Wilcoxon rank sum test is applicable for comparing the locations (medians) from two random samples. The test hypothesis is that the two populations from which the samples were drawn have the same location. (For a detailed discussion on the Wilcoxon rank sum test, see Reference 2.)

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1. Owen, Donald B. (1962) Handbook of Statistical Tables, Addison-Wesley Publishing Company, Inc. (15.6, 16.2)

2. Hollander, Miles and Wolfe, Douglas A. (1973) Nonparametric Statistical Methods, John Wiley and Sons, Inc.

3. Bradley, James V. (1968) Distribution-Free Statistical Tests, Prentice-Hall International, Inc.

a. The assumptions which must be satisfied are as follows:

(1) The observations are mutually independent.

(2) The observations in each sample are obtained from the same continuous population.

b. The computational procedures for conducting the Wilcoxon test follow.

(1) Define the test hypothesis in terms of D, the difference in the respective population locations, to be Ho: $D = 0$.

The null hypothesis may be tested against:

(a) All alternatives, i.e., $D \neq 0$,

(b) The alternative that $D > 0$,

or

(c) The alternative that $D < 0$.

(2) Proceed, as with the K-S2 technique, by rank ordering the $N = m + n$ observations $X_{S_1}, X_{S_2}, \ldots, X_{S_m}$ and $X_{T_2}, \ldots, X_{T_n}$ from least to greatest, and assign rank values Ri, $i = 1, 2, \ldots, n$, to coincide with the rank of the $X_{Ti}$ in the ordering.

(3) Compute the test statistic W as follows:

$$W = \sum_{i=1}^{n} R_i$$

(4) Compare the test statistic W, at a desired significance level, with the table value (Reference 2) for $\mathcal{W}(\alpha, m, n)$, where $P[W \geq \mathcal{W}(\alpha, m, n)] = \alpha$.

Based on the result of this comparison, reject Ho at the $\alpha$ level of significance in favor of

(a) Ha: $D \neq 0$

if $W \geq \mathcal{W}(\alpha_2, m, n)$ or $W \leq [n(m + n + 1) - \mathcal{W}(\alpha_1, m, n)]$,

where $\alpha_1 = \alpha_2 = \frac{\alpha}{2}$,

(b) Ha: $D > 0$

if $W \geq \mathcal{W}(\alpha, m, n)$,

(c) Ha: $D < 0$

if $W \leq [n(m + n + 1) - \mathcal{W}(\alpha, m, n)]$.

Example: In the example which was used to illustrate the K-S2 test, we could assign rank values to the test ob ervations as follows:

| $i$ | $X_{Ti}$ | $R_i$ |
|---|---|---|
| 1 | 2.0 | 6 |
| 2 | 1.9 | 5 |
| 3 | 0.8 | 2 |
| 4 | 2.3 | 8 |

and then

$$W = \sum_{i=1}^{4} R_i = 21 .$$

For a hypothesis Ho that there is no location difference, and for $\alpha = 0.1$ $\mathcal{W}(0.1, 4, 4) = 23$,

$$[n(m + n + 1) - \mathcal{W}(0.1, m, n)] = 36 - 23 = 13 ,$$

$$[n(m + n + 1) - \mathcal{W}(0.05, m, n)] \cong 36 - 24 = 12 ,$$

and

$$\mathcal{W}(0.05, 4, 4) \cong 24 .$$

Then for alternative hypothesis (4)(a) defined above, since

$$21 \not\geq \mathcal{W}(0.05, 4, 4) \simeq 24$$

and

$$21 \not\leq [4(4 + 4 + 1) - \mathcal{W}(0.05, 4, 4)] \cong 12$$

we do not reject Ho.

Similarly, for hypotheses (4)(b) and (4)(c), we see that

$$21 \not\leq W(0.1, 4, 4) = 23$$

and

$$21 \not\leq [4(4 + 4 + 1) - W(0.1, 4, 4)] = 13$$

and we would reject Ho in neither case.

c. As indicated earlier, the Wilcoxon test is stronger than the K-S tests when the sample size is large. There are two considerations which should be mentioned, however. First, the Wilcoxon test is not valid if the empirical distribution functions from the two sample sources differ significantly in form. Second, while it is possible to apply this test to smaller samples, as the example illustrates, the K-S tests are still to be preferred for small samples. Moreover, the K-S tests ascertain distributional equivalence, as opposed to just location equivalence. For a more comprehensive discussion on the strength of the Wilcoxon test, refer to Bradley (3).

3. Parametric Comparison Methods. There are a number of classical statistical techniques which are applicable for comparing two random variables, and while they are usually somewhat stronger than nonparametric methods, they are often more restrictive in that they are only applicable when comparing specific parameters (usually distributional moments). The distribution of the population is usually known and of a classical statistical form (e.g., normal, Student's t, Chi Square, etc.). There are, however, a number of simulation validation parameters to which the techniques of this section can be applied with the objective of increasing confidence in the simulations' performance. Since the classical procedures which will be referenced in this section are, in general, familiar to the simulation community, they will not be expanded in detail as were the K-S and Wilcoxon tests. Rather, their applications methodology will be discussed in general in the following subsections.

a. The Chi-Square Goodness-of-Fit Test

(1) The Chi-Square goodness-of-fit test is applicable for testing the hypothesis that a specific sample has been drawn from a known (usually normal) distribution. Its primary advantage over purely parametric methods is that it is a test for complete distributional equivalence. The assumptions which must be satisfied before applying

this test are that each of the observations in the sample are mutually independent, and that they are drawn from the same population.

(2) The application of this procedure involves dividing the total range of the sample into k classes and computing a value for the test statistic $X^2$ based on the difference between the observed and expected class frequencies. The test statistic $X^2$ may then be compared to table values for a theoretical Chi-square distribution with k - 1 degrees of freedom.

(3) The primary disadvantage of the Chi-square goodness-of-fit test is that it requires sample sizes which are much larger than the corresponding distribution-free K-S techniques. Bendat and Piersol (4) provide an excellent discussion of the Chi-square goodness-of-fit test and establish criteria for determination of sample size and of the number of subintervals to use.

b. The Normal and Student t Tests

(1) These tests are equally appropriate for testing the hypothesis that there is no difference between two populations for a given distributional parameter (e.g., mean) or the hypothesis that a particular sample has been drawn from a theoretical distribution with the same value for that parameter. The basic difference between the tests is that the Student t test should be used when the sample size is less than 30 observations. The assumptions which must be satisfied before the test can be applied are as follows:

(a) The test statistic must be normally distributed (student t distributed for the t test).

(b) The sample observations must be mutually independent.

(2) Procedurally, when applying the Normal test for a given population parameter with estimated value $P_S$ (computed from the sample), and with hypothesized value $P_H$, one computes the statistic

$$Z = \frac{P_S - P_H}{\sigma_{P_S}}$$

4. Bendat, Julius, and Piersol, Allan G., (1966), Measurement and Analysis of Random Data, John Wiley and Sons, Inc.

where $\sigma_P$ is the value of the standard deviation for that parameter. The statistic Z is then compared at an $\alpha$ level of significance with tabulated values for the standardized normal variate $Z_\alpha$. If it is larger, the test hypothesis should be rejected.

(3) The procedure for applying the Student t test is the same. However, the statistic

$$t = \frac{P_S - P_H}{\sigma_{P_S}}$$

is compared to values from a standard t distribution table.

(4) As mentioned above, when the sample size is less than 30 observations, the Student t test should be preferred over the normal. For cases where use of these tests is applicable, such as tests for means (M), we can use techniques from parametric estimation to establish an appropriate sample size. Specifically, we choose X units of allowable error in estimating the parameter and an associated confidence level $1 - \alpha$, and then solve for n in the relationship

$$Z_{\alpha/2} \cdot S/\sqrt{n} = X,$$

where $Z_{\alpha/2}$ is the value of the standardized Normal or t variate, as appropriate, at the $\alpha/2$ level of significance, and S is the sample standard deviation for the test parameter.

4. Comparison Procedures for Minimal Sample Sizes. While the modeller and the systems analyst would like to develop the highest level of confidence possible in a simulation, it is often impossible to perform complete rigorous statistical tests, due to the lack of sufficient test data. When this is the case, one can only obtain enough simulation data to distributionally characterize the parameters in question and then comment on comparability of a test result based on whether it falls within desired confidence limits. The following procedures are the more commonly used:

a. Plot the test response over the simulation response (with confidence bounds or response limits if the simulation is a Monte Carlo simulation).

b. Analyze the differences between the test response and the expected response (from analysis of simulation results)

c. Compute time-averaged integral square error values or the variance equivalent between the test response and the response predicted by the simulation

## 5. Comments

a. The methods which have been discussed for validation do not constitute a completely exhaustive set of simulation validation procedures. There are numerous methods which could be employed; however, the collection of methods which has been discussed here is certainly considered appropriate for qualifying simulations as instruments for use in assessing system performance capabilities.

b. Every effort must be made to collect adequate data samples so that some of the more rigorous statistical comparison techniques can be employed. The preferred validation tests procedure should be supplemented by the conduct of some of the alternative procedures (comparison plots, difference analyses, testing of hypotheses about additional distributional statistics, etc.) so that the highest possible level of confidence may be achieved.