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NRL Report 8274

Beam Optics of the Quadrupole Magnet State Selector

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MARK RUBINSTEIN

Magnetism Branch
Electronics Technology Division

November 15, 1978

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The hydrogen beam optics of the quadrupole magnet state selector designed by RCA for ultimate use in a satellite-borne hydrogen maser has been theoretically analyzed. With the aid of several simplifying assumptions, the polarization of the hydrogen atoms entering the storage bulb has been calculated as a function of the length of the quadrupole state selector.

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BEAM OPTICS OF THE QUADRUPOLE MAGNET STATE SELECTOR

INTRODUCTION

RCA has designed and constructed a quadrupole magnet state selector based on a scaled-down version of a similar state selector built by NASA. The purpose of the state selector is to introduce a polarized beam of hydrogen atoms into the storage bulb of the hydrogen maser. The beam-optics system is an axial one consisting of a dissociator bulb with an exit collimator tube, a quadrupole magnet state selector, and a storage cell with an entrance tube. A schematic of the system is shown in Fig. 1.

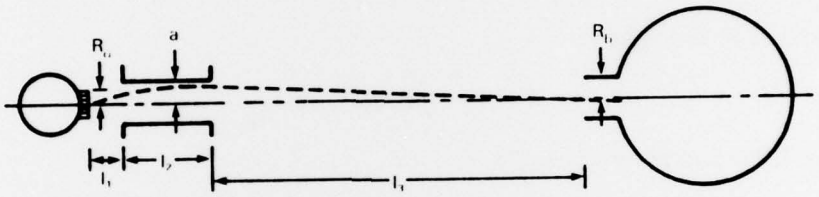

Fig. 1 – Beam optical system

In the RCA design, the collimator leading from the dissociator bulb is a tube 0.1016 cm (40 mils) long with an inside diameter of 0.0127 cm (5 mils). The end of this tube is situated as close as possible to the entrance bore of the state selector. We have assumed this distance to be 1 mm.

The state selector is of the quadrupole type, having four permanent magnets of alternating polarity symmetrically situated around the central bore. The unit has an outer diameter of 3.8 cm (1.5 in.) and a length of 3.2 cm (1.25 in.), with a bore diameter of 0.05 cm (0.020 in.). The field at the pole tips is between 8 and 10 kOe. The design of the state selector permits a distance of 8.26 cm (3.25 in.) between the state selector exit and the entrance to the storage bulb. We assume that the entrance tube to the storage bulb is situated 8.26 cm (3.25 in.) from the state selector, and has an inside diameter of 0.41 cm (0.16 in.). The quadrupole state selector is expected not to require a stopping disk on the beam axis, such as is used with hexapole selectors, to interrupt the on-axis atoms with zero transverse velocity, since a stronger deflection force is exerted by the quadrupole on hydrogen atoms that travel close to the axis.

As a good first approximation, the quadrupole design produces a linear variation of field strength with radius, resulting in a deflection force on the hydrogen atoms that is constant with radius. The magnetic field distribution inside the bore of the quadrupole magnet can be obtained (approximately) from a solution of Laplace's equation for an infinitely long magnet. For boundary conditions we assume four pole tips, with each tip occupying an angle

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of 45° on a cylinder of radius r_0. The magnetic potential φ is assumed to be $\pm V_0$ on these surfaces (alternating sign from one pole to the next) and zero in the gaps. The magnetic potential $\varphi\,(r,\theta)$ in the interior of the bore is of the form

$$\varphi(r,\theta) = \sum_{n=1}^{\infty} B_n r^n \sin(n\theta). \tag{1}$$

By symmetry, only the coefficients of $\sin(2\theta)$, $\sin(6\theta)$, $\sin(10\theta)$ . . . are nonzero, and

$$\varphi(r,\theta) = B_2 r^2 \sin(2\theta) + B_6 r^6 \sin(6\theta) + B_{10} r^{10} \sin(10\theta) + \ldots. \tag{2}$$

The coefficients B_n are found by matching boundary conditions at $r = r_0$, resulting in

$$\varphi(r,\theta) = \frac{4V_0}{\sqrt{2\pi}}\left[\left(\frac{r}{r_0}\right)^2 \sin 2\theta - \frac{1}{3}(r/r_0)^6 \sin 6\theta - \frac{1}{5}(r/r_0)^{10} \sin 10\theta + \ldots\right]. \tag{3}$$

The magnetic field H is given by

$$H = H_r^2 + H_\theta^2,$$

where $H_r = \partial\varphi/\partial r$ and $H_\theta = \frac{1}{r}\,\partial\varphi/\partial\theta$.

Performing these steps, and imposing the boundary condition $H(r_0) = H_0$ where H_0 is the magnetic field at the pole tips, we obtain after some algebra,

$$H = H_0\left(\frac{r}{r_0}\right)\left[1 - 2(r/r_0)^4 \cos 4\theta + (r/r_0)^8 (1 - 2\cos 8\theta) + \ldots\right]^{1/2} \tag{4}$$

For r sufficiently small compared to r_0, the magnetic field varies linearly with the radius r, independently of the polar angle

$$H \approx H_0\,(r/r_0). \tag{5}$$

In an inhomogeneous magnetic field, the hydrogen atoms of effective moment μ are subjected to a radial force F

$$\overline{F} = \mu\Delta H(r). \tag{6}$$

The atoms with positive μ are thus attracted towards the high magnetic field regions at the poles of the magnets; those with a negative magnetic moment are repelled by the regions of high magnetic fields and tend to be focused toward the central axis of the state selector. This effect allows the separation of these two groups of atoms, producing a high state of magnetic polarization in the storage bulb.

The force acting upon an atom in the state selector is then

$$\overline{F} = \pm\frac{\mu H}{r_0}\,\frac{\overline{r}}{r}. \tag{7}$$

This is a force directed radially inward or outward, depending on the sign of the magnetic moment. The equation of motion of hydrogen atoms in a magnetic field of quadrupolar symmetry is then

$$\ddot{r} - m\dot{r}\dot{\theta}^2 = \frac{\mu H}{r_0}, \tag{8}$$

where m is the mass of the hydrogen atoms ($m = 1.67 \times 10^{24}$ g), r_0 is the radius of the state selector ($r_0 = 2.54 \times 10^{-2}$ cm), and H_0 is the field measured at the pole tips (H_0 = 8—10 kOe). Assuming that skewed orbits are neglected, the angular momentum term of Eq. (8) vanishes, and the equation simplifies to

$$\ddot{r} = \mu H/mr_0. \tag{9}$$

The magnitude of the effective moment of the hydrogen atom varies with magnetic field for two of the four energy states involved, but is effectively equal to one Bohr magneton for all states ($\mu = 9.27 \times 10^{-21}$ erg G^{-1}), provided that the atoms reside in a sufficiently strong magnetic field.

In the present calculation, the following approximations are made.

1. The angular dependence of the quadrupolar magnetic field is neglected, and the radial dependence of the field is approximated by its leading term $H(r) = H_0\ (r/r_0)$.

2. The atoms are treated as if they were emitted from the dissociator collimator at a point on the central axis of the system, but with an angular distribution appropriate to the collimator tube dimensions. The angular momentum of the hydrogen atoms about the central axis of the state selector is thus assumed to be zero, resulting in a simplification of the equation of motion.

3. The effective magnetic moments of the hydrogen atoms are assumed to be independent of the magnetic field, and are approximated by the unit Bohr magneton value. Only two of the four hyperfine-split atomic states involved in the hydrogen maser actually possess atomic moments that are independent of external fields. The other two states have zero moment in zero applied field, and only attain a value of $\sim\mu_B$ in moderate fields in excess of ~500 Oe. For atoms residing in these two levels, it is assumed that the time spent near the central axis, in the region of small magnetic fields, is small enough that it will not seriously alter the final conclusions.

4. It is assumed that the emitted atoms will retain their initial atomic states while traversing the state selector.

5. The effect of the state selector's fringing fields is neglected, and it is assumed that the trajectories of the atoms upon leaving the state selector are linear.

The above assumptions are the same as for previous calculations involving the hexapolar state selector made by Hughes and others [1]. For a truly accurate calculation, each approximation should be removed. This would result in a highly nonlinear differential equation for the path of the hydrogen atoms. With the above assumptions, the atoms follow simple parabolic paths within the state selector, and linear trajectories outside; the resulting trajectories can easily be traced analytically. For present purposes, it is felt that the approximate procedure will at least give the correct order of magnitude for the resulting polarization of the beam.

CALCULATION

Using RF energy to excite the discharge, the dissociator produces an atomic hydrogen beam. This beam consists of atoms whose ground state ($J = 0$) and first excited states ($J = 1$) are in thermal equilibrium at a temperature determined by the walls of the dissociator chamber. The dissociator manufactured by Hughes runs quite cool, and we have assumed that the effective temperature of the atomic beam is $T = 310$ K. The beam is partially collimated by a long, thin exit tube. RCA uses a glass tube 0.1016 cm (0.040 in.) long with an inner diameter of 0.0127 cm (0.005 in.). Hughes utilizes a collimator of somewhat different design.

The beam emerges from the collimator with an angular distribution determined by the geometry of the collimator tube and a velocity distribution determined by the effective beam temperature. The velocity distribution I (v) is the modified Maxwellian velocity distribution and can be written [2]

$$I(v) = \frac{2I_0}{\alpha c^4} v^3 e^{-(v^2/\alpha^2)}$$

$$\alpha = (2KT/m)^{1/2}, \tag{10}$$

where v is the velocity of an atom and α is the average thermal velocity.

There is no simple analytic expression for the angular distribution of the beam emitted from the collimator for arbitrary angles, and a computer calculation is needed to generate these numbers. However, Dayton [3] presents a table of results for several values of the collimator aspect ratio, from which a function valid at small emission angles can be inferred. Since atoms emitted at large angles will be rejected by the state selector anyway, the small-angle approximation is usable.

The angular distribution of the emitted beam is given, in general, by the expression

$$J(\theta) = T(\theta) \cos\theta \sin\theta, \tag{11}$$

where θ is the angle (expressed in radians) measured from the collimator tube axis. $T(\theta)$ is a function that depends on the length-to-radius ratio of the collimator tube, and $T(\theta) = 1$ for a circular hole; i.e., no collimation at all. Examining the results of Dayton, we find that we can approximate the function $T(\theta)$ by a function $T(\theta) \approx \exp(-L\theta/R\pi)$ (where L is the length of the collimating tube and R is the bore radius) valid for small θ. The (unnormalized) angular distribution function is then assumed to be

$$J(\theta) = \exp(-L\theta/R\pi) \sin\theta \cos\theta. \tag{12}$$

For collimating tubes of the RCA design, the ratio $L/R = 16$.

We now discuss the trajectories of atoms within the bore of the state selector. An atom leaving the collimator tube at an angle θ with respect to the system's central axis and having a thermal velocity v will enter the bore of the state selector with the initial conditions

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$$v_{\parallel} = v\cos\theta$$

$$v_{\perp} = v\sin\theta$$

$$v_{\text{init}} = L_1 \tan\theta;$$

where L_1 is the distance between the collimator tube and the front of the state selector. Atoms with a positive magnetic moment will be deflected away from the central axis, whereas atoms with a negative magnetic moment will be deflected toward the central axis, tending to focus negative moment atoms at the entrance port of the storage bulb. This effect results in a net polarization of the stored atoms, permitting the maser to operate.

Atoms with a positive magnetic moment are governed by the following equation of motion while traveling in the uniform force field of the state selector:

$$r(t) = r_{\text{init}} + v_{\perp}t + \frac{1}{2}\left(\frac{\mu H_0}{mr_0}\right)t^2, \tag{13}$$

where t is the time spent in the state selector bore. Atoms that strike the sides of the state selector, i.e., with $r(t) > r_0$, are assumed to be lost from the system. The surviving atoms are collected in the storage bulb only if they emerge from the state selector with a resulting perpendicular component of velocity $v_{\perp f}$ small enough to allow them to be collected at the entrance port of the storage bulb. The criterion for collection is approximately

$$v_{\perp f} = (v\cos\theta)\ (R_3/L_3), \tag{14}$$

where R_3 is the radius of the entrance port of the storage bulb, and L_3 is the distance from entrance port to the state selector. Values used in this calculation are $R_3 = 0.2032$ cm (80 mils) and $L_3 = 7.62$ cm (3 in.).

Hydrogen atoms with negative magnetic moments are assumed to be always accelerated toward the central axis of the state selector bore with a constant acceleration $G = \mu H_0/mr_0$. Upon crossing the central axis, they are decelerated and returned again to the axis since the force reverses on each crossing. This cycle is repeated until the atoms leave the state selector bore, through which they are traveling at a constant axial speed $v\cos\theta$. The motion is described by a series of parabolic arcs oscillating about the central axis. Particles whose radial excursions are greater than the bore radius r_0 are again assumed to be eliminated from the system. Particles whose linear trajectories upon leaving the state selector cause them to miss the entrance hole of the storage bulb are also not counted.

For each initial value of v and θ, the computer calculates the trajectories of atoms with both positive and negative moment. The computer is instructed not to count those particles that hit the walls of the state selector bore or those particles whose final free-space trajectories cause them to miss being collected by the storage bulb. The (unnormalized) flux F of either kind of atom is given by the expression

$$F = \int_0^{\infty}\int_0^{\theta_0}\left(x^3 e^{-x}\sin\theta\cos\theta\, e^{-\frac{L\theta}{kT}}\, d\theta\, dx\right), \tag{15}$$

where the prime on the integrand indicates that only particles that enter the storage bulb are to be counted. The angle θ_0 is the angle subtended by the state selector bore radius at the exit port of the collimator tube; $\theta_0 = \arctan(r_0/L_1)$, where r_0 is the bore radius; and L_1 is the distance between the bore and the collimator tube. In the above equation $x = v/v_{av}$, where $v_{av} = (2kT/m)^{1/2}$. The total flux incident on the state selector bore is given again by Eq. (16) but without the prime on the integrand, indicating that all incident particles are counted whether or not they eventually are collected by the storage bulb.

The following parameters were used in the calculation.

Distance of collimator tube to state selector: $L_1 = 0.1$ cm.

Radius of state selector: $R_2 = 10$ mils (0.0254 cm).

Length of state selector: $L_2 = 1.25$ in. (3.175 cm), or arbitrary.

Distance of state selector to storage bulb entrance: $L_3 = 8.25$ cm (3.25 in.).

Radius of storage bulb entrance: $R_3 = 0.203$ cm (80 mils).

Field strength at pole tips: $H_0 = 9$ kOe.

Hydrogen atom mass: 1.673×10^{-24} g.

Magnetic moment: $\pm 0.927 \times 10^{-20}$ erg G^{-1}.

The calculation first computes the unnormalized flux of atoms that enters the bore of the state selector. This flux is called TF in the program. The program next computes the unnormalized flux of atoms that enters the storage bulb, both for atoms of positive magnetic moment and for atoms of negative magnetic moment. This is done by eliminating from the total flux all atoms whose trajectories cause them to strike the walls of the state selector, or whose final trajectories cause them to miss the entrance of the storage bulb. In the Fortran program, these fluxes are named GF for fhe flux of atoms with negative moment and BF for the positive moment flux. The ratios GF/TF and BF/TF are then computed, yielding the percentages of negative moment flux and positive moment flux entering the storage bulb. We have computed these values for several values of the length of the state selector, including the length presently being used in the RCA design, 3.175 cm. These values are shown in Table 1. The result of this calculation shows that a state selector of length 1.25 in. (3.175 cm) is sufficiently long to produce a high state of polarization within the storage bulb.

Table 1 - Computer Program Results for Beam Polarization as a Function of State Selector

Length of State Selector (cm)	GF/TF: Ratio of Negative Moment Hydrogen Flux Entering Storage Bulb to Flux Entering Bore of State Selector (percent)	BF/TF: Ratio of Positive Moment Hydrogen Flux Entering Storage Bulb to Flux Entering Bore of State Selector (percent)	Polarization
0.5	4.9	0.57	0.79
1.0	3.8	0.21	0.89
1.5	3.2	0.58×10^{-1}	0.96
2.0	3.2	0.83×10^{-2}	0.99
2.5	3.2	0.76×10^{-3}	0.99
3.175	3.2	0.14×10^{-4}	0.99
3.5	3.2	0.17×10^{-5}	0.99
4.0	3.2	0.03×10^{-7}	0.99

Note: Calculations assume the distance of the state selector to the storage bulb entrance to be 8.25 and the radius of the storage bulb entrance bore, 0.203 cm.

FORTRAN PROGRAM

```
0001        PROGRAM GFL
0002        REAL L1,L2,L3
0003   1    TYPE 10
0004        ACCEPT20,N,M,TEMP,H0,L1,L2,L3,R3,Z
0005   10   FORMAT(1X,'N,M,TEMP,H0,L1,L2,L3,R3,Z?')
0006   20   FORMAT(2I4,7E14.0)
0007        R2=2.54E-2
0008        AMASS=1.673E-24
0009        AMU=0.927E-20
0010        B=1.38054E-16
0011        VAV=SQRT(2.*B*TEMP/AMASS)
0012        G=AMU*H0/(AMASS*R2)
0013        ANGLE=ATAN(R2/L1)
0014        AN=N
0015        AM=M
0016        DELV=5./AN
0017        DELT=ANGLE/AM
0018        VP=DELV
0019        GF=0
0020        BF=0
0021        TF=0
```

```
0022          DO 200 I=1,N
0023          T=DELT
0024          DO 100 J=1,M
0025          V=VAV*VP
0026          VU=V*SIN(T)
0027          VZ=V*COS(T)
0028          TD=L2/VZ
0029          R0=L1*TAN(T)
0030          TQ=SQRT(2.*R0/G+(VU/G)**2.)
0031          TAU=(TD+TQ-VU/G)
0032          TH=2.*TQ
0033          T1=AMOD(TAU,TH)
0034          VY=ABS(SQRT(2.*R0*G+VU**2.)-G*T1)
0035          XTVP=EXP(-Z*T)*SIN(T)*COS(T)*VP**3.*EXP(-VP**2.)
0036          TF=TF+XTVP
0037          TFL=L2/(VZ)
0038          T1M=VU/G
0039          R0=L1*TAN(T)
0040          RM=R0+(0.5*VU**2.)/G
0041          RTFL=R0+VU*TFL-0.5*G*TFL**2.
0042          VYC=VZ*R3/L3
0043          IF(TFL.GT.T1M.AND.RM.GT.R2)GO TO 98
0045          IF(TFL.LT.T1M.AND.RTFL.GT.R2)GO TO 98
0047          IF(VY.GT.VYC)GO TO 98
0049          GF=GF+XTVP
0050    98    VYB=VU+G*TD
0051          RMB=R0+VU*TD+0.5*G*TD**2.
0052          IF(RMB.GT.R2)GO TO 99
0054          IF(VYB.GT.VYC)GO TO 99
0056          BF=BF+XTVP
0057    99    T=T+DELT
0058   100    CONTINUE
0059          VP=VP+DELV
0060   200    CONTINUE
0061          PERG=GF/TF
0062          PERB=BF/TF
0063          TYPE 300
0064          TYPE 400, PERG,PERB
0065   300    FORMAT(1X,'PERG       PERB')
0066   400    FORMAT(1X,2E14.7)
0067          GO TO 1
0068          STOP
0069          END
```

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2. N.F. Ramsay, *Molecular Beams*, Clarendon Press, Oxford, 1956.

3. B.B. Dayton, *National Symposium on Vacuum Technology, Trans.* pp 5-11, 1956.

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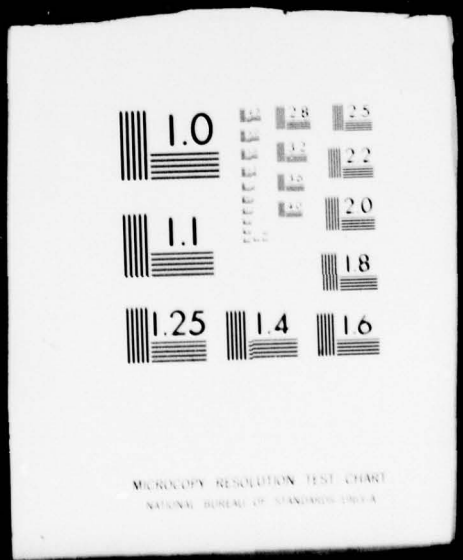

SUPPLEMENTARY

INFORMATION

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ERRATA

BEAM OPTICS OF THE QUADRUPOLE MAGNET STATE SELECTOR,
Mark Rubinstein, NRL Report 8274

The text following Equation (3), page 2 should read:

The magnetic field H is given by

$$H^2 = H_r^{\,2} + H_\theta^{\,2},$$

Equation (6), page 2 should read

$$\overline{F} = \mu \nabla H(r). \tag{6}$$

Equation (7), page 2 should read

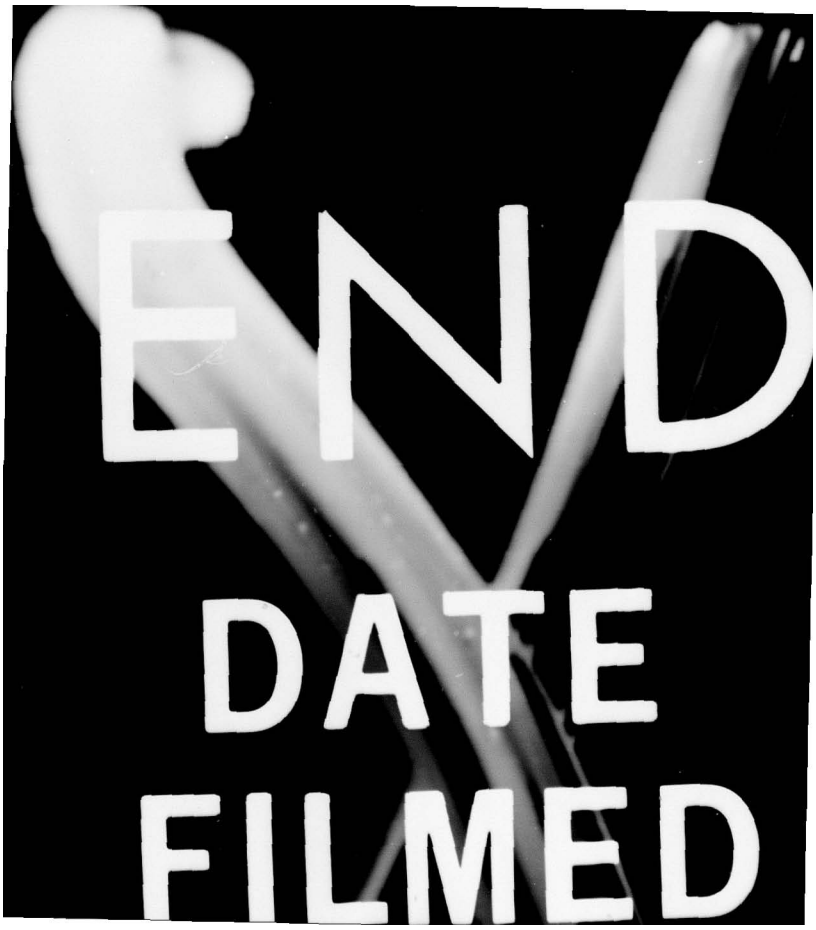
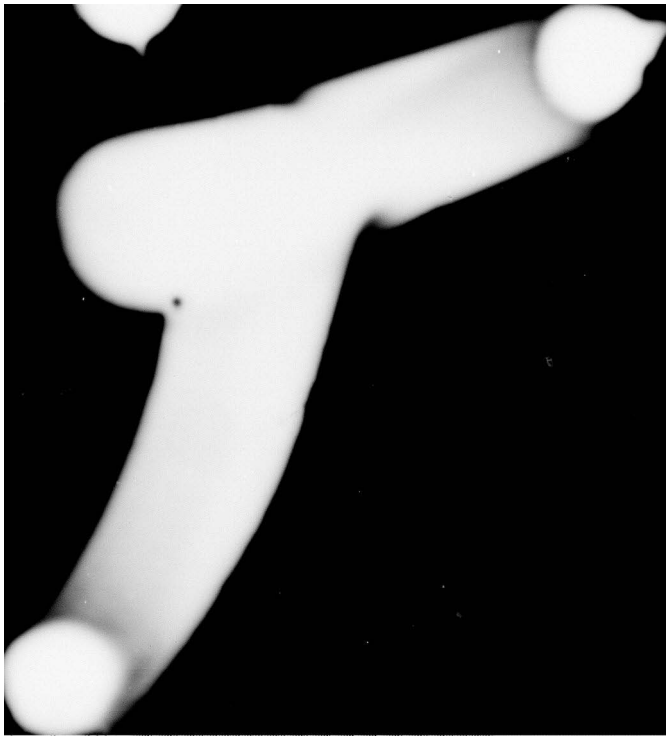
$$\overline{F} = \pm \frac{\mu H_0}{r_0} \frac{\overline{r}}{r}. \tag{7}$$

Equation (8), page 3 should read

$$\ddot{r} - mr\dot{\theta}^2 = \frac{\mu H_0}{r_0}, \tag{8}$$

Equation (9), page 3 should read

$$\ddot{r} = \mu H_0 / m r_0. \tag{9}$$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