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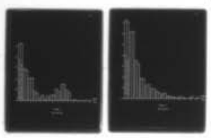

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A MODEL FOR THE DISTRIBUTION OF THE NUMBER OF BIDDERS IN AN AUCTION*

by

Richard Engelbrecht-Wiggans

Abstract

The distribution of the number of bidders in auctions with uncertain numbers is usually assumed to be Poisson. The observed distribution, for example in OCS Federal Offshore Oil Lease Sales, is apparently not Poisson. A simple model is presented showing that if the objects have different values and individuals tend to only bid on objects with high value, then the resulting distribution of the number of bidders will not be Poisson. The results of the model correspond closely to data observed in Federal Offshore Oil lease auctions and the model is simple enough so that it may be of practical use to an individual participating in such an auction.

Introduction

In many bidding models, there are a fixed number of individuals who submit bids. The models assume that each bidder knows precisely how many competing bids will be submitted. This assumption is at best only approximately true in many real world auction situations.

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The observation that the number of bids submitted on a particular object might be a random variable is not new. In his pioneering work in auction theory, Friedman (1956) suggests that the number of bidders might be Poisson distributed. A number of others have used Poisson models since; indeed, in surveying the literature on auctions and bidding models, Engelbrecht-Wiggans (1978a) observes that the Poisson model is essentially the only alternative to the common fixed bidder model ever considered.

The use of the Poisson distribution is often justified on theoretical grounds. If there are several similar individuals, each independently deciding whether or not to bid on a particular object (with each individual having the same probability of deciding to submit a bid), then the resulting number of bids has a binomial distribution. As the number of individuals becomes large while the probability that an individual's bid becomes small in such a way so the average number of bids remains constant, then the binomial distribution approaches a Poisson distribution.

If there are a number of similar independent auctions, then it may reasonably be assumed that each auction has the same probability distribution on the number of bids submitted. In such cases, the observed distribution of the number of bids serves as an estimate of the common distribution. Keller and Bor (1978) examine data for a number of similar construction contracts in the United Kingdom and conclude that the Poisson distribution is a reasonable approximation. Thus, there appears to be both theoretical and empirical support for the Poisson model.

The data reported in the United States Department of the Interior Outer Continental Shelf Statistical Summary 1976-1978 (1978) appears to refute a simple Poisson model. Indeed, the observed distribution of the number of bids on the various offshore oil leases within a particular sale may have a strongly bimodal distribution; for example, see Figure 1 for the distribution in OCS Sale #40. In such examples, a particular object is likely to receive either a relatively small or a relatively large number of bids. A particular object is relatively unlikely to receive a number of bids equal to the average number of bids.

Despite the bimodal nature of the observed distribution, the data is not inconsistent with a slightly modified Poisson model. If the leases within a sale are sufficiently dissimilar, then different leases will have different distributions for the number of bids submitted. The composite of several different Poisson distributions need not be Poisson; indeed a mixture of Poisson distributions with two different means can result in a bimodal distribution. This paper presents a simple model using assumptions similar to those for the Poisson model and resulting in predicted distributions consistent with the observed data.

The main goal of the model, however, is not so mucy to provide further evidence in favor of the Poisson model assumptions as to provide a mechanism for estimating the underlying distribution of the number of bids on each particular object. Work of Capen, Clapp and Campbell (1971) and Engelbrecht-Wiggans (1978b) indicates that the optimal multiplicative strategy results in larger bids when there are a relatively small number of competitors than when the number of competitors is either larger or very small. Thus the optimal multiplicative strategy in a situation where an object is likely to receive either a very small or a large number of bids can be substantially different (or, more specifically, less aggressive) than if the object is likely to receive an intermediate number of bids. Even if bidders are not restricted to multiplicative strategies, it appears likely that an individual should prefer to somehow estimate the distribution of the number of competitors on each particular object rather than simply use the average distribution.

Model

The model assumes that the objects being sold are not all similar. In this discussion, objects will be described by their actual value; for example the value of the oil and gas actually present under a particular OCS site. (It is not assumed that the potential bidders know this value.) For simplicity, the objects have one of three possible values; there are m_0 "worthless" objects, m_1 "doubtful" objects, and m_2 "promising" objects.

Each of the n individuals will bid only on objects he considers of sufficient value to justify any costs associated with the bidding and with the development of the site. Different individuals could have different threshholds for deciding when an object is worthwhile bidding on. We, however, consider the simplest case in which all individuals will bid only on objects they believe to be "promising"; individuals will bid on a randomly chosen fraction q of all objects believed to be "promising."

Since the individuals are uncertain about the true value of each object, it is possible that an individual bids on a low valued object which was incorrectly believed to be of higher value. In this model it is assumed that individuals know precisely which objects are "worthless"; each individual can correctly classify any of the remaining objects as either "doubtful" or "promising" with probability p.

"Worthless" objects will never receive any bids. The number of bids on a "doubtful" or a "promising" object will have a binomial distribution with parameters n and qp, or parameters n and $q(1-p)$, respectively.

The observed distribution of the number of bids will be a mixture of these three distributions; the precise mixture depends on the ratios $r_2 = m_2/(m_0 + m_1 + m_2)$, $r_1 = m_1/(m_0 + m_1 + m_2)$, and $r_0 = 1 - r_1 - r_2$.

The model has five unknown parameters: n , p , q , r_1 , and r_2 . If q can be assumed to be small, then for large n , the binomial distributions are each approximately Poisson and the model has only four unknown parameters: $u_1 = nqp$, $u_2 = nq(1-p)$, r_1 and r_2 . However, an individual using the model as an aid in determining the distribution of the number of competitors on a particular object need not know the values of either r_1 or r_2 ; the individual need only estimate the number of potential bidders, the probability he misclassifies a "doubtful" or a "promising" object, and on what fraction of objects believed to be "promising" he would bid.

Validation

The model's assumptions and predictions are consistent with a number of different observed data. The United States Geological Survey (undated b) considers at least two classes of sites; "noneconomic" sites receive a minimal pre-sale value; the remaining sites receive higher pre-sale estimates. While only 54% of the noneconomic sites received any bids, fully 93% of the other sites received at least one bid. Furthermore, the United States Department of the Interior statistical summary (1978) indicates that approximately 90% of the noneconomic sites received less than four bids; about three fourths of the remaining sites received at least four bids. These observations appear consistent with the assumption that each bidder only bids on objects whose value is estimated to exceed some critical value.

In order to compare the predicted and observed distributions of number of bids on an object, the parameters of the model must be determined. Let

$$A_k = \frac{n!}{k!(n-k)!}[r_1(qp)^k(1-qp)^{n-k} + r_2(q(1-p))^k(1-q(1-p))^{n-k}]$$

and let $P_0 = r_0 + A_0$, $P_k = A_k$ for $1 \leq k \leq n$, and $P_k = 0$ for $k < 0$ and $k > n$. Then the number of objects with k bids has a binomial distribution with parameters $M = m_0 + m_1 + m_2$ and P_k . Thus, the parameters can be choosen to minimize the squared error:

$$E = \sum_{k=0}^{k=n} \frac{(OBS_k - P_k M)^2}{P_k(1-P_k)M} + \sum_{k=n+1}^{k=\infty} (OBS_k)^2$$

where OBS_k denotes the number of objects observed to receive k bids and M is the total number of objects being sold.

The model predicts an average of $P_k M$ objects with k bids. Using values of $m_1 = 80$, $m_2 = 40$ ($m_0 = M - m_1 - m_2 = 154 - 120 = 34$) , $p = .15$, $q = 1.0$ and $n = 9$, the model predicts average numbers of objects with k bids as plotted in Figure 1. Figure 1 also plots the observed distribution of bids in OCS Sale #40. The model clearly provides a more accurate prediction than the traditional Poisson model could.

In fitting the model to the data, the number of individuals was set at nine. Since there are many joint bids in OCS Sale #40, and the companies represented in joint bids varied over the bids on different objects, it is difficult to determine exactly how many potential bidders actually participated. However, at least one of the following nine firms had an interest in approximately 90% of the bids submitted; Exxon, Chevron, BP, Mobil, Tenneco, General Crude, Conoco, Murphy, and Freeport Minerals.

If one assumes that the United States Geological Survey estimates (undated a) of actual values are approximately as accurate as the private firms' estimates, then the expected number of objects receiving government estimates of "promising" would be $pm_1 + (1-p)m_2$, or about 46. Actually 43 of the sites receiving bids has estimates in excess of the minimum estimates (\$142,848); three additional sites with non-minimal estimates received no bids. It would also be expected that approximately twelve (pm_1) "doubtful" sites would receive relatively high estimates. There were indeed eight sites where the highest bid was rejected as being too low when compared to the government estimate, and these eight sites tended to receive very few bids. These eight, together with the previously mentioned three bidless sites, suggests that the government misclassified at least eleven sites.

Finally, we consider a second, more recent, federal offshore oil lease auction with a large number of leases, OCS Sale #47. Although the distribution of the number of bids in this sale is essential unimodal, the observed distribution has considerably more variance than would be consistent with a Poisson distribution. Figure 2 plots the observed distribution along with the average number of objects receiving various number of bids predicted by the model with $m_2 = 50$, $m_1 = 160$, $m_0 = M - m_1 - m_2 = 223 - 210 = 13$, $u_1 = 1.0$, and $u_2 = 4.5$; the binomial distributions are approximated by Poisson distributions. Again, the distribution predicted by the model fits the observed data substantially better than a simple Poisson distribution would.

Application

The above observations support a model in which individuals bid only on, but not necessarily on all, objects which are estimated to have at least some critical value. In the simple case considered, there are few enough parameters to be estimated such that the model can be of use in practical applications. In symmetric auctions, the use of the general model is sufficiently simple to be practical.

Consider a symmetric auction modelled as a game with incomplete information. Nature chooses the true value Z of the object using a probability distribution known to all individuals. Although the choice is not revealed, individuals gain some information about the choice through the observation of an informational random variable whose distribution depends on the true value chosen by nature. Assume that all the individual's information variables X_i are identically distributed and that for any $x^* > x$, the probability (Z is at least z when X is observed to be equal to x^*) is at least the probability (Z is at least z when X is observed to be equal to x), and that this condition is true for all possible values of z. Finally, assume that all individuals will use the same monotonically increasing (in x) equilibrium bidding strategy $b(x)$, where the bid of individual i is given by $b(x_i)$, and that all individuals have the same critical value c for deciding whether or not to submit a bid.

Under these conditions, a bid is submitted if and only if it is at least equal to $b(c)$. Since, however, $b(x)$ is a monotonically increasing function, whether or not bids less than $b(c)$ are submitted has no affect on the probability of a bid B greater than $b(c)$ winning. Thus, the analysis is unaffected by assuming that such small bids are actually submitted. The analysis proceeds assuming that the number of bidders has a binomial distribution with parameters n and q. Of course, if $q = 1$, the analysis used a fixed number of bidders; in actuality, the number of bids submitted will be a random variable because some of the bids may be less than the critical value and thus will not actually be submitted. Note that even if q is less than one, the variance in the number of competing bidders is less than the variance in the number of bids exceeding the critical value (and thus actually submitted in practical situations).

Thus, in order to apply the general model in symmetric auctions, individuals need only estimate two parameters. In particular, an individual need only estimate the total number of individuals who might possibly bid and the probability q that any of these will be interested in bidding on any particular object. For asymmetric auctions, the application of the general model will, however, be considerably more difficult.

Conclusion

A simple model has been presented to predict the number of dissimilar objects which will receive any specified number of bids. The assumptions of the model are quite similar to those for the more traditional Poisson model except that individuals are assumed to only bid on objects that they estimate to have at least a critical value. The distribution of the number of bids on objects, however, corresponds much more closely with the observed data than a Poisson distribution would; the model is also consistent with a number of other data.

Although the model is analyzed mainly for a very simple case, a more general model may be considered. In symmetric auctions, use of the general model requires only that two parameters be estimated. Thus, the model not only helps explain the apparent non-Poisson distribution of the number of bids in certain auctions, but should also be of considerable use to individuals considering bidding in such auctions; if, as the model suggests, the number of bids actually submitted depends in part on the number of individuals receiving sufficiently large estimates of the object's value, then it is inappropriate to simply use the distribution of the number of bidders in past auctions to predict the number of competitors on a particular object.

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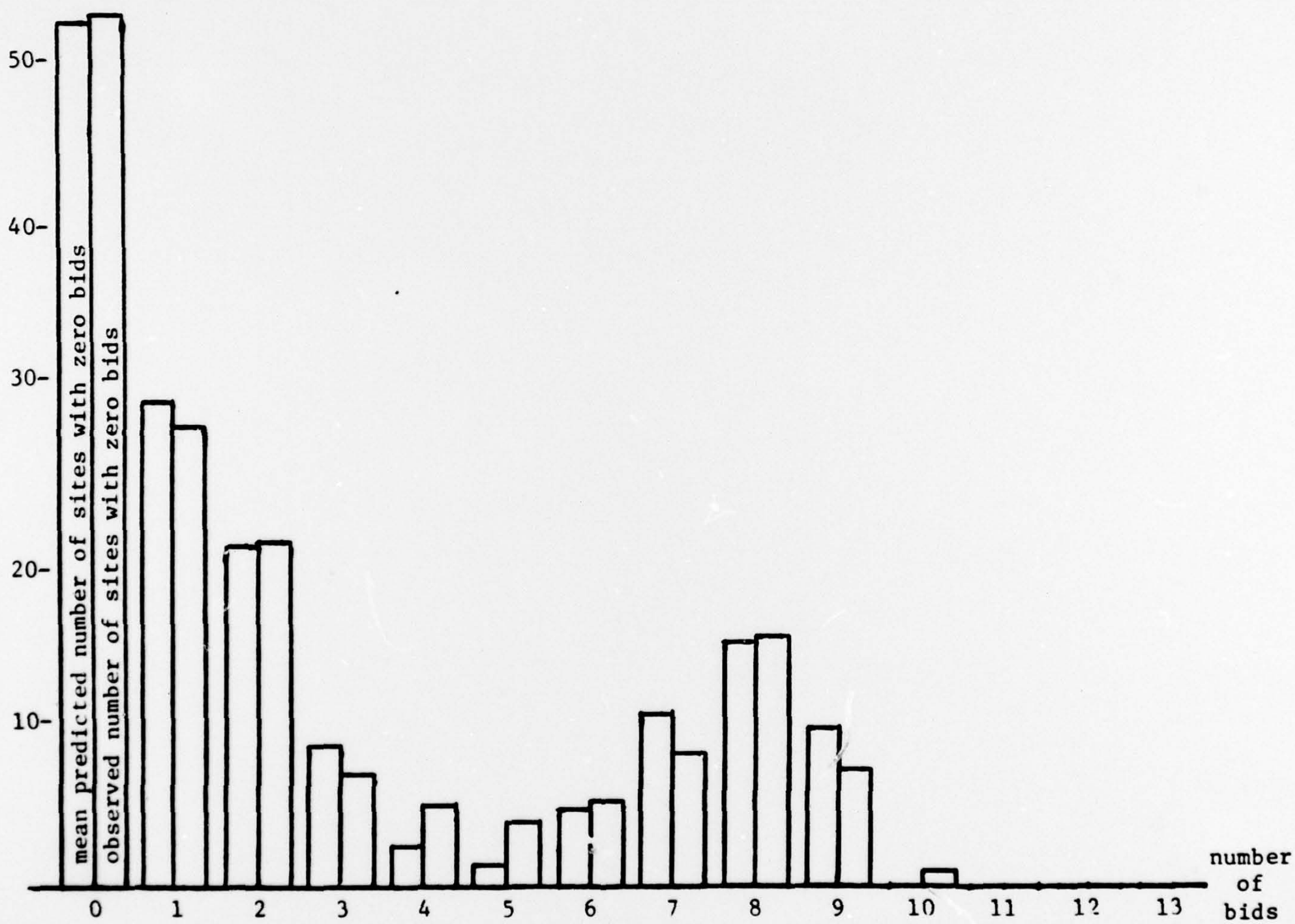

FIGURE 1

OCS Sale #40

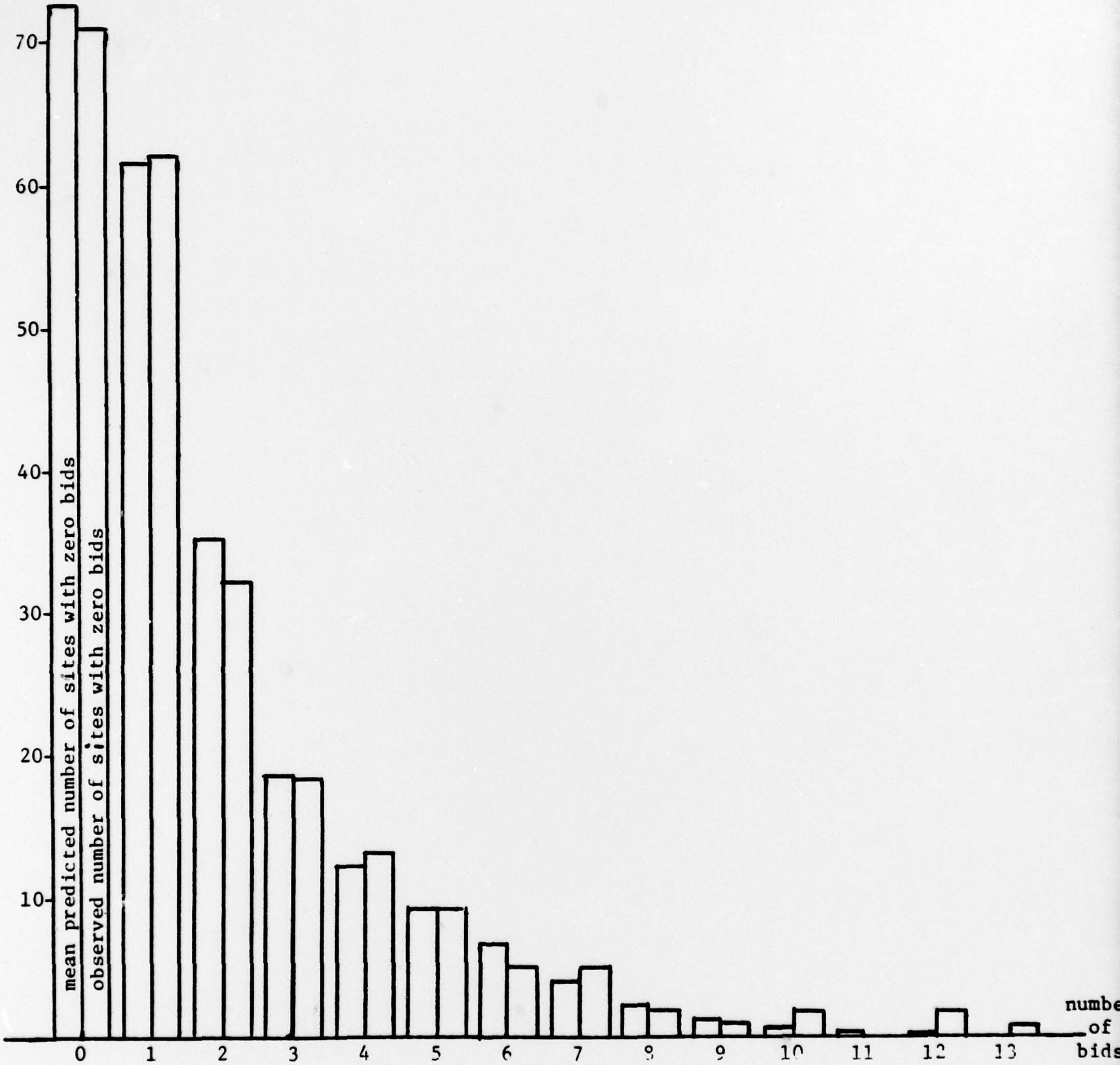

FIGURE 2

OCS Sale #47