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SOUND TRANSMISSION IN A
HALF CHANNEL AND SURFACE DUCT,

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Naval Postgraduate School
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Abstract

$t_0 = t_{\infty} - 5 \text{ seconds}$ p. u.

The normal mode solution of sound transmission has been used as the basis of an approximate (WKB) calculation. The attenuation of low frequency signals in a surface duct is shown to be proportional to $f^{5/3}$ where f is the frequency. The high frequency attenuation is based upon experimental data and is proportional to $f^{1/2}$. The leakage of sound energy from the duct is used as a means to estimate the signal received beneath the duct. The theoretical results have compared with theoretical data. The surface duct attenuation formula fits the experimental data within the spread of the data.

Sqrt of f

I Introduction

The theories needed to describe sound propagation in the half channel and surface duct are essentially the same, so we have combined the discussions. The ray paths for these cases are shown on Fig. 1. At very high frequencies when the surface duct is hundreds of wavelengths deep, the ray theory gives a good description of signal transmission and the formation of shadow zones. At low frequencies when the duct (or half channel) is a few wavelengths thick, the wave diffraction effects are important. In a shallow duct, low frequency waves may not be trapped and thus have very high attenuation while a high frequency signal would be trapped and have low attenuation.

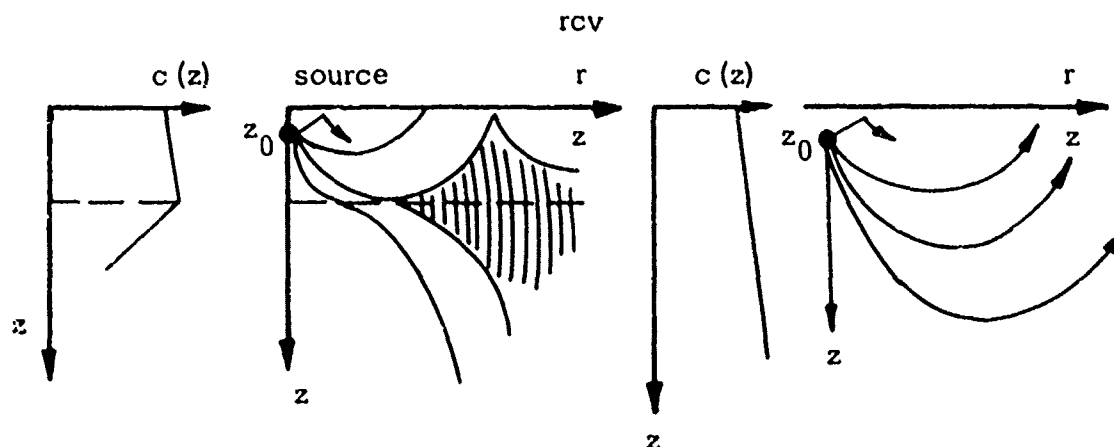


Fig. 1 Half channel and surface duct. The shadow is crosshatched.

A considerable body of theory and experiment has shown that the depth of the duct, the frequency and sound velocity gradients are needed to describe the sound transmission. Since we must consider frequencies lower than have been studied to any extent, our theory should be extrapolatable.

The answers to the following questions are needed to describe sound transmission in a half channel or surface duct.

1. When are signals trapped?
2. How much of the energy is trapped?
3. What is the attenuation?
4. What happens to the energy lost from the duct and can it be received?

We have tried several approaches to the problem: First, ray tracing procedures coupled with estimates of attenuation were tried; however, the results were not good and too much empirical adjustment was required. Wave theory in the form of an approximate normal mode calculation was found to give agreement with the experimental data of

Pederson and Gordon (1965). The development is given in Appendix A. The description of the simplified surface duct model in the following sections is an empirical adaptation of those results.

II Trapped Sound Energy

The model is sketched on Fig. 2. The duct is assumed to be many acoustic wavelengths deep. More exact specification can be obtained from the normal mode solution of the problem as given in Appendix A. A slight rearrangement of equation A20 yields for the lowest mode the following:

$$f > \left(\frac{c_0}{2h} \right)^{3/2} g_1^{-1/2}$$

$$c(z) = c_0 + g_1 z \quad z \leq h$$

where

- f is the frequency in cps
- h is the mixed-layer depth
- c_0 is the sound velocity
- g_1 is the gradient of sound velocity in the mixed layer

On assuming that part of the sound energy is trapped, the mean square pressure can be expressed as a cylindrically spreading wave. With the aid of Fig. 2, it is the following:

$$\left| p^2 \right| = p_0^2 A (rh)^{-1} \Delta \theta \exp - Br \quad (1)$$

where p_0 is the acoustical pressure at a unit distance. For convenience, A and B are regarded as being empirical constants which can be determined as a function of frequency, h and sound velocity gradients. Actually an

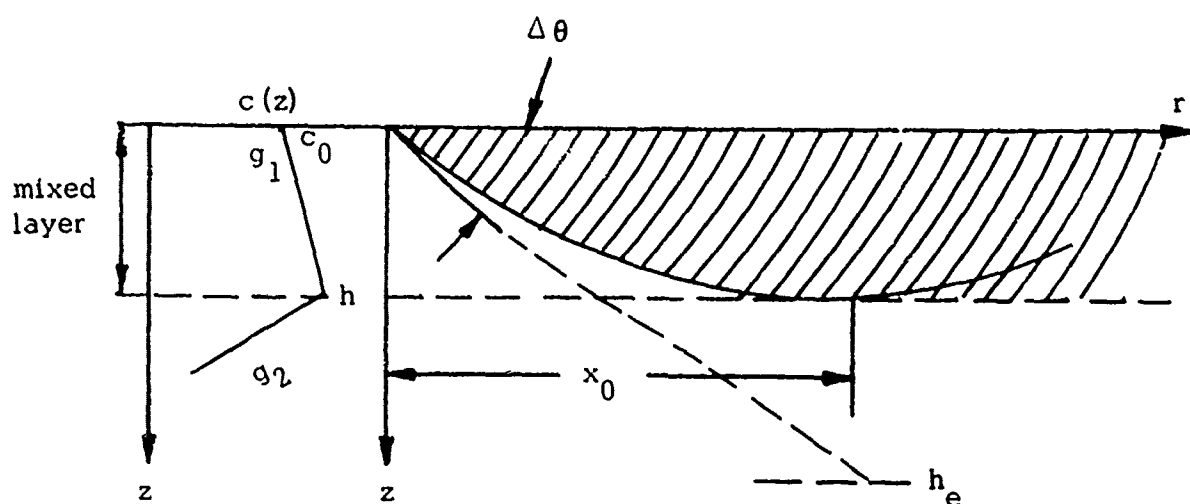


Fig. 2 Surface duct as a wave guide; x_0 is the distance required for the steepest turning ray to reach depth h . g_1 and g_2 are the sound velocity gradients above and below depth h . The effective depth h_e is also shown.

approximate wave theory solution of the problem yields the functions for both A and B.

If the primary interest is the surface duct propagation then the wave equations in the summary of Appendix A should be used. For order of magnitude estimates of surface duct transmission an empirical value of A is satisfactory.

III Attenuation

The measured attenuation of signal transmission in the surface duct has been shown to have large attenuations at low frequencies. These losses are caused by poor trapping or leakage of sound energy from the

duct. The attenuation also increases at high frequencies and this loss is assumed to be caused by the scattering of sound. On separating the effects, B can be expressed as the following:

$$B \approx 2 (\delta' + \delta'') \quad (2)$$

where δ' is the leakage attenuation (low frequency)

δ'' is the scattering attenuation (high frequency)

Experimental measurements of B have helped but theoretical description of the functional form of δ' is needed. Diffraction or leakage of energy can be estimated, with the aid of wave theory.

Normal mode descriptions of the wave field were used by Pederson and Gordon(1965) and more recently Bucker and Morris (1967). One could use their methods, however, we felt that amount of computational effort could be too large. A less accurate theory having simpler equations would be sufficient for our purpose. On the basis of these excellent comparisons of theory and experiment, we have chosen to use the normal mode method to develop the attenuation function B. In the development we will use their results as a guide in making many of our approximations and simplifications. The development and equations are given in Appendix A.

The attenuation due to leakage through the bottom of the surface duct is (Appendix Eq. A2 and A37)

$$\delta' \approx f^{-5/3} h_e^{-3} g_2^{-1/3} \times 10^6 \text{ ir. m}^{-1} \quad (3)$$

where f is frequency in cps

h_e is effective layer depth in m

g_2 is the sound velocity gradient beneath the duct

and h is much greater than λ , and generally of the order of 10λ .

In a duct having a positive sound gradient, h_e is approximately $2h$, Fig. 2. Both h_e and the mode number (here it is one) enter as somewhat arbitrary factors in Eq. (2). Thus it is reasonable to add an adjustable factor to Eq. (2) if necessary.

To this attenuation we should add the losses that have been presumed to be caused by sound scattering at the surface. The high frequency sound can also be scattered by the smaller inhomogenities that are generally found in mixed layers. We use the following empirical formulas given by Marsh and Shulkin (1967):

for sea state < 3

$$a_s = 4.5 (f/h)^{1/2} \quad \text{db/kyd}$$

for sea state > 3

$$a_s = 9 (f/h)^{1/2} \quad \text{db/kyd} \quad (4)$$

f in k cps

h in ft

or for exponential function,

$$\delta'' \approx \left\{ \begin{matrix} 3 \\ 6 \end{matrix} \right\} (f/h)^{1/2} \times 10^{-5} \quad \text{nepers/m}$$

for f in cps and h in m (5)

In the appendix, we estimated the surface reflection loss, Eq. (A39), and found it to be proportional to frequency squared. The role of the various scattering mechanisms in the mixed layer are not understood

and for the present, it is recommended that the empirical functions (4) or (5) be used. The total attenuation is

$$B = 2 (\delta' + \delta'')$$

$$B \approx 2 [f^{-5/3} h_3^{-3} g_2^{-1/3} \times 10^6 + \left\{ \frac{3}{6} \right\} (f/h)^{1/2} \times 10^{-5}] \quad (6)$$

Use 3 for sea state < 3

6 for sea state > 3

In normal use B is converted to db/kyd and the parameters are entered in kH_z and ft. On replacing h_e by $2h$, the attenuation is

$$B_{db} = 4 \times 10^5 f_{(kcps)}^{-5/3} h_{(ft)}^{-3} g_2^{-1/3} + \left\{ \frac{4.5}{9} \right\} \left(\frac{f}{h} \right)^{1/2} \text{ db/kyd} \quad (7)$$

for sea states < 3 and > 3

The dependence of B upon frequency is sketched on Fig. 3. The numerical constants in Eq. (6) are approximate and can be adjusted empirically.

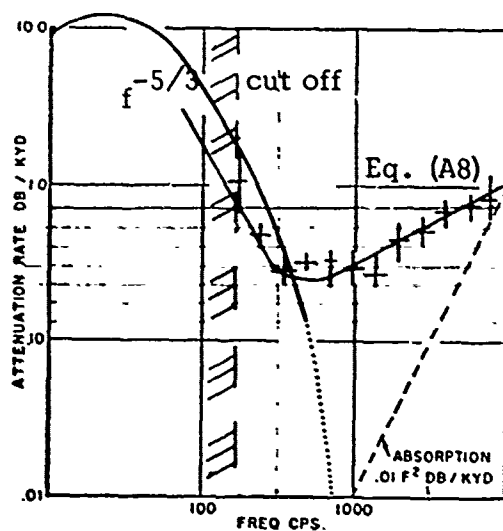


FIG. 10. Attenuation rate vs frequency for the leakage arrival for one refraction cycle at Bear, shallow hydrophone. Spectrum levels are given for a band width of 1 cps and a reference pressure level of 1 d/cm² rms for 1 sec. The height of the crosses gives the variation of the levels and the width of the crosses gives the band width of the filter. The theoretical attenuation rate vs frequency was calculated for a channel depth of 290' and a thermocline of 0.147 sec.

Fig. 3 Surface duct attenuation, B. The data is from T. Arase, J. Acoust. Soc. Am. 31, 588-595 (1959).

IV Half Channel

The half channel condition generally refers to a deep ocean having the sound velocity minimum near or at the surface. The sound trapping condition applies as follows

$$f \gg \left(\frac{c_0}{2h} \right)^{3/2} g_1^{-1/2}$$

and the water is of the order of hundreds of wave lengths thick. The simplest approximation is to assume that all sound energy that does not interact with the bottom is trapped and spread cylindrically. Since the wave guide is very thick, leakage losses can be neglected and the signal level is

$$\left| p^2 \right| = \left| p_0^2 \right| A(rh)^{-1} \Delta \theta \exp - \alpha r$$

where α is the sound absorption loss in sea water. In this formula, the vertical distribution of sound energy is approximated as being constant whereas it actually decreases as a function of depth. For more accurate calculations, the normal mode solutions, Eq. (A8) and (A31), should be used.

If the water is too shallow for the trapping of sound, then the problem should be treated as a shallow water wave guide. Here the properties of the bottom are very important.

V Surface Duct Leakage

Sound energy can be scattered into and out of the surface duct. The only losses aside from the absorption are those caused by scattering and diffraction. As shown on Fig. 4, the flow of energy is horizontal and the losses flow downward.

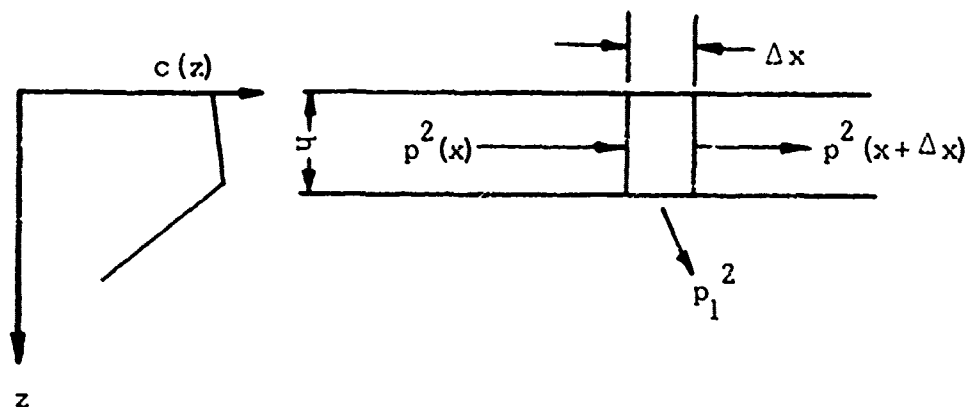


Fig. 4 Surface duct energy leakage

The energy flow per unit area is sound pressure times the particle velocity. If the ray paths are in the direction of energy flow then the energy flow per unit area is proportional to pressure squared. For our approximation we will use the pressure squared.

The energy loss from the length of guide Δx is

$$p_1^2 \Delta x = [p^2(x) - p^2(x + \Delta x)] h \quad (8)$$

The Taylor expansion for $p^2(x + \Delta x)$ is

$$p^2(x + \Delta x) = p^2(x) + \frac{\partial p^2(x)}{\partial x} \Delta x \quad (9)$$

and

$$-p_1^2 = \frac{\partial p^2(x)}{\partial x} h \quad (10)$$

The loss of p^2 per unit length is proportional to p^2 and is

$$\frac{\Delta p^2}{\Delta x} = -B p^2 \quad (11)$$

Thus the leakage pressure is

$$p_l^2 = B h p^2 \quad (12)$$

The transmission loss in going from a pressure in the duct to a receiver beneath the duct is

$$TL \text{ for leakage} = -10 \log Bh \quad (13)$$

The leakage signal is large for large duct attenuation and small for low duct attenuation. At large distances from the source the leakage pressure is proportional to the signal in the surface duct. From Urlick it appears that the transmission losses in going from within the duct to beneath the duct are of the order of 15 db. In the model that we have assumed, the leakage signal is essentially independent of the receiver depth beneath the duct. The direction of the leakage arrival is approximately that of an initially horizontal ray just beneath the duct.

The leakage signals are traveling in the forward direction. Thus, we can regard the leakage as being a set of directional sources just beneath the sound duct. As suggested by Hersey and Officer, the sound can follow the usual deep transmission paths and also be received at refraction or convergence zone distances. The transmission paths are shown on Fig. 5. Experiments have shown that the apparent broadening of the convergence zone (or "bounce" in this paper) can be ascribed to leakage signals, Fig. 6. The leakage signals following the convergence zone were about 20 db less than the main signal.

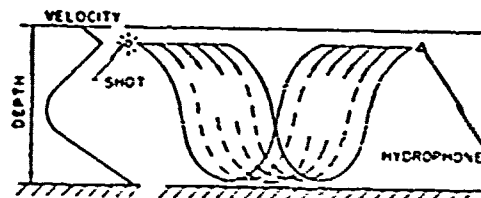


FIG. 2. Possible ray paths for one refraction cycle leakage propagation.

Fig. 5 From Arase (1959)

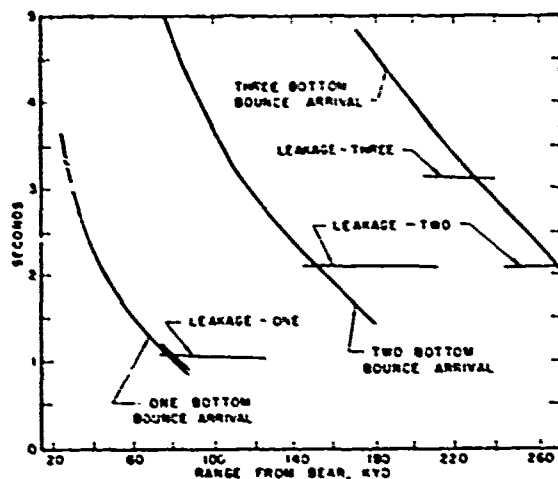


FIG. 7. Travel lag of the second and succeeding arrivals after the surface channel arrival as a function of range.

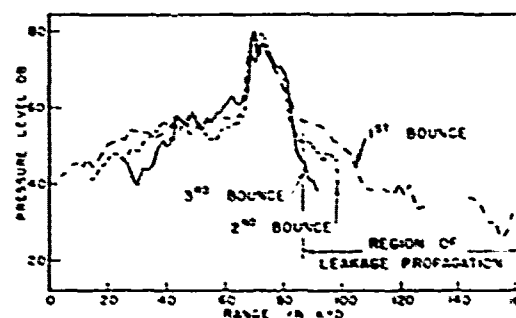


FIG. 8. Broad-band pressure levels vs range for 1st bounce arrivals at Bear, deep hydrophone. Pressure levels are corrected for spreading and are not corrected for absorption. The reference pressure level is 1 d/cm² rms pressure, for 1 sec.

Fig. 6 From Arase (1959)

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Appendix A

SOUND TRANSMISSION IN A HALF CHANNEL AND A SURFACE DUCT

The mathematical developments of the normal mode solution of the half channel and surface duct are a bit involved so at the outset we will state what we are after. In a wave guide the pressure field is

$$|p^2| \propto \frac{p_0^2}{rh} e^{-\delta'_m r} \quad (A1)$$

We would like to calculate the constant of proportionality. But it is even more important to derive the dependence of the attenuation on frequency, layer depth etc. Ignoring the loss due to surface roughness, we will show that the (rms pressure) attenuation is the following

$$\begin{aligned} \delta'_m &\approx \frac{14 m^2}{k_0^{5/3} h^3 (3a_2)^{1/3}} \\ &\approx f^{-5/3} h^{-3} g_2^{-1/3} 10^6 \text{ in nepers } m^{-1} \end{aligned} \quad (A2)$$

where

- $k_0 = \frac{\omega}{c_0}$
- $h = \text{layer depth}$
- $g_2 = \text{sound velocity gradient beneath the mixed layer}$
- $m = \text{mode number, } 1, 2, 3, \dots$

The most optimistic estimate for attenuation is to assume $m = 1$. The essential behavior is that δ'_m is proportional to $f^{-5/3} h^{-3}$.

The model that we used to estimate the surface duct attenuation is that of Brekhovskikh. As shown on Fig. A1, the sound velocity is constant in the mixed layer and has a sharp gradient beneath the layer. The sound is partially reflected by the change of the sound velocity gradient. It may be surprising, but comparison of the attenuations and transmission losses computed by means of Eq. (A2) are nearly the same as those given by Pedersen and Gordon (1965). This means that, except for very high frequencies, the essential part of the trapping in the surface duct is the reflection at the change of gradient.

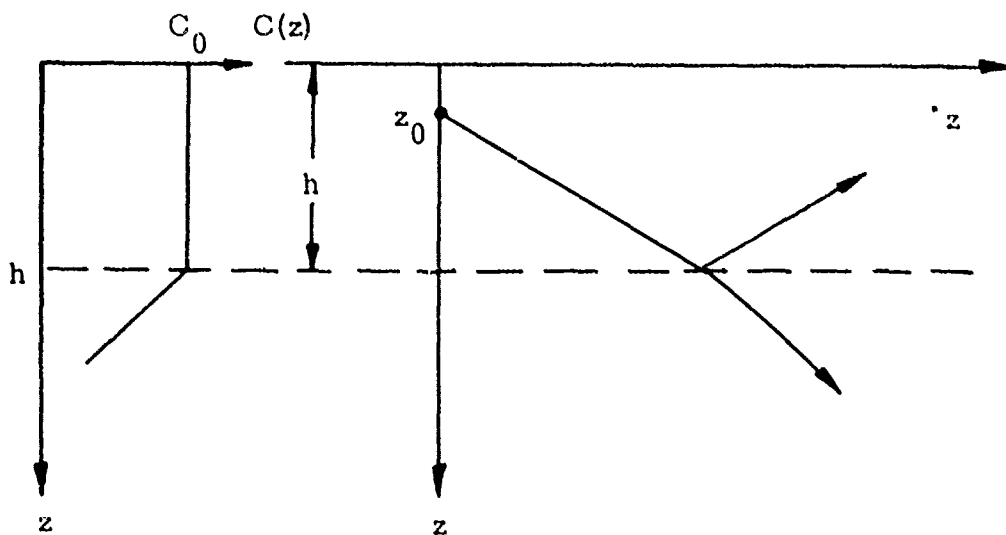


Fig. A1 Surface duct, $c(z) = \text{constant}$ in mixed layer

The acoustical pressure due to a continuous wave point source in a wave guide has been discussed in much detail by Tolstoy and Clay (1966). We will use their results and generally follow their notations. In the stratified wave guide, $c(z)$, the acoustical pressure at large range is (with the aid of their Eq 2.5 and 3.118) the following:

$$p = \frac{i\rho}{r^{1/2}} \sum_{m=1}^M P_m \exp - i (\kappa_m r - \omega t - \frac{\pi}{4}) - \delta_m r \quad (A3)$$

$$P_m \equiv p_m \varphi_m(z_0) \varphi_m(z) \rho_s^{-1} \quad (A4)$$

$$p_m \equiv 2\pi (\rho_s c_0 \mathcal{P})^{1/2} \rho_s \nu_m^{-1} \kappa_m^{-1/2} \quad (A5)$$

z_0 and z are the source and receiver depths (Fig. A1)

r is range in cylindrical coordinates

z is depth, positive downward

ω is angular frequency

κ_m is horizontal component of wave number

$\kappa_m = \omega/c_m$, c_m is phase velocity

γ_m = vertical component of wave number

$k = (\gamma_m^2 + \kappa_m^2)^{1/2} = \omega/c = 2\pi/\lambda$

$\mathcal{P}$ is source power

ρ_s and ρ are the densities at source and receiver

δ_m is the mode attenuation and is an addition to the original equation

φ_m are the eigen functions and are solutions of the separated wave equation:

$$\frac{\partial^2}{\partial z^2} \varphi_m + \gamma_m^2 \varphi = 0 \quad (A6)$$

$$\begin{aligned}
\nu_m &= \int \rho \varphi_m^2 dz \\
&= \frac{c_0^2}{2\omega^2 a_1} \left[\gamma_m^2 \varphi_m^2 + \left(\frac{\partial \varphi_m}{\partial z} \right)^2 \right]_0^{z_m}
\end{aligned} \tag{A7}$$

where $c = c_0 (1 - 2a_1 z)^{-1/2}$

The solutions of Eq. (A6) in a stratified wave guide (γ_m is a function of z) generally require extensive numerical computations. Our purpose in the following is to develop approximate solutions which can be used as a guide in devising a simple approximation such as Eq. (A1). But first we take the absolute square of Eq. (A3) and examine the result.

$$\begin{aligned}
|p|^2 &= \rho^2 r^{-1} \sum_m^M p_m^2 e^{-2\delta_m r} + \frac{2}{r} \sum_{m>n} p_m p_n e^{-(\delta_m + \delta_n) r} \\
&\quad \cos (\kappa_m - \kappa_n) r
\end{aligned} \tag{A8}$$

The first summation gives the average level as a function of range. The second set of terms is oscillatory and these terms describe the maximums and minimums often observed in experiments. These terms depend upon $(\kappa_m - \kappa_n)$ and great accuracy in the computation of the eigen functions, κ_m , is required to theoretically duplicate the actual maximums and minimums of experimental data. At very large range, the sound field is analyzed by measuring the mean level and correspondingly, we ignore the second summation or the fluctuations. Thus, approximate eigen values κ_m will be adequate.

We use the WKB approximation (Tolstoy and Clay, pp. 48-52) to determine the eigen functions. As shown on Fig. A2, the following sound velocity profiles are convenient to describe the surface duct and for computation:

$$\begin{aligned}
 c &= c_0 (1 - 2a_1 z)^{-1/2} & 0 \leq z \leq h \\
 c &= c_h [1 + 2a_2 (z - h)]^{-1/2} & z \geq h \\
 c_h &= c_0 (1 - 2a_1 h)^{-1/2}
 \end{aligned} \tag{A9}$$

To begin with, we are assuming a positive gradient in the mixed layer so that we can also apply the theory to the half channel.

For small gradients, Eq. (A9) are the approximate linear functions as follows:

$$\begin{aligned}
 c &\approx c_0 (1 + a_1 z) \\
 c &\approx c_h [1 - a_2 (z - h)]
 \end{aligned} \tag{A10}$$

The approximate eigen functions are

$$\varphi_m(z) \propto \gamma_m^{-1/2} \sin [s_m(z) + s_1] \tag{A11}$$

$$\begin{aligned}
 s_m(z) &= \int_0^z \gamma_m dz + s_0 = m\pi \\
 m &= 1, 2, 3, \dots M
 \end{aligned} \tag{A12}$$

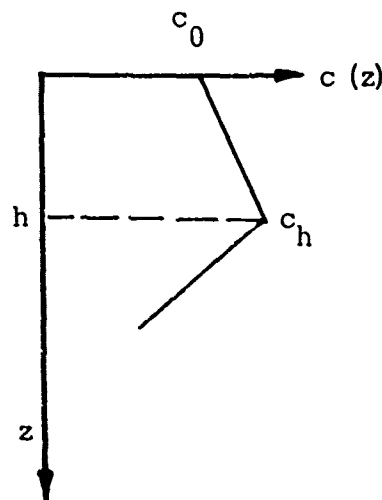


Fig. A2 Surface duct

The condition for validity of the WKB method is (Tolstoy and Clay, 2.213)

$$\frac{1}{\gamma} \left| \frac{d}{dz} \ln \gamma \right| \ll 1 \quad (\text{A13})$$

The approximate eigen functions, Eq. (A11), cannot be used at small γ , or at the turning depths. The mathematical techniques used to connect the WKB eigen functions through turning points are discussed in many quantum mechanics texts. We will not use ϕ_m near the turning depths and not calculate the connection functions.

Modes are trapped for those frequencies, or κ_m , for which Eq. (A12) is satisfied. Let us assume that the m th mode turns at depth z_m , has the wave number component κ_m and phase velocity c_m . γ_m is

$$\gamma_m^2 = k^2 - \kappa_m^2 = \omega^2 (c^{-2} - c_m^{-2}) \quad (\text{A14})$$

With the aid of the preceding equation, the integration of Eq. (A12) [for c given by Eq. (A9)] yields after manipulation

$$s_m(z_m) = \frac{k_0}{3a_1} (2a_1 z_m)^{3/2} + \frac{\pi}{4} = m\pi \quad (A15)$$

where (A9) or (A10) have been used to express the parameters as follows:

$$\begin{aligned} k_0 &= \omega/c_0 \\ s_m(z) &= (m - \frac{1}{4}) \pi [1 - (1 - z/z_m)^{3/2}] \\ \gamma_m &= k_0 (2a_1 z_m)^{1/2} (1 - z/z_m)^{1/2} \\ x_m &= k_0 (1 - 2a_1 z_m)^{1/2} \\ k &= h_0 (1 - 2a_1 z)^{1/2} \\ z_m &= [3a_1 (m - \frac{1}{4}) \pi k_0^{-1}]^{2/3} [2a_1]^{-1} \\ &= \frac{1}{2a_1} \left(1 - \frac{c_0^2}{c_m^2}\right) \end{aligned} \quad (A16)$$

(Note in Eq. (A14), let $\gamma_m = 0$ to determine c_m)

In summary:

WKB is valid for (by application of Eq. (A13))

$$2k_0 z_m (2a_1 z_m)^{1/2} (1 - z/z_m)^{3/2} \gg 1 \quad (A17)$$

For trapped modes

$$\frac{k_0}{3a_1} (2a_1 h)^{3/2} > (M - \frac{1}{4}) \pi \quad (A18)$$

M is an interger

For a given h, and a_1 , k_0 must satisfy (A18) for the waves to be trapped.

The WKB eigen function

$$\varphi_m(z) \propto (1 - \frac{z}{z_m})^{-1/2} \sin [s_m(z) - \frac{\pi}{4}] \quad (A19)$$

(Note that $s_1 = -\pi/4$ because φ_m vanishes at the free surface or $z = 0$)

The first question: "When are signals trapped?" can be answered by application of condition (A18). A rearrangement gives

$$k_0 h > 3 (M - \frac{1}{4}) \pi (2a_1 h)^{-1/2}/2 \quad (A20)$$

An evaluation of (A19) for the surface duct of Pedersen and Gordon (1965) follows:

$$\text{sound velocity gradient} \approx 10^{-2} \text{ sec}^{-1}$$

$$h \approx 100 \text{ m}$$

$$a_1 \approx 7 \times 10^{-6} \text{ m}^{-1}$$

and

$$M \approx 1 \quad (A21)$$

Thus

$$k_0 h > 30$$

$$f = \omega/2\pi > 225 \text{ cps}$$

To trap the first mode, this duct must be more than 15λ thick. A similar calculation for second mode yields $f > 530$ cps. For 530 cps signals, they show fair trapping of the second mode and large leakage for higher modes.

Recalling Eq. (A3) and (A4), it is evident that P_m is constant for a fixed source and receiver depths, power and wave guide parameters. P_m depends upon the product $\phi_m(z_0) \phi_m(z)$ and thus upon the source and receiver depths. Since ϕ_m tends to zero as z tends to zero, the source and receiver depth dependence should be included. Incidentally, the value of $\phi_m(z)$ for small z is the same as that given by the dipole formula. To show this, we let γ_m be defined with the aid of the grazing angle θ as follows:

$$\gamma_m = k_0 \sin \theta \quad (\text{A22})$$

The substitution of Eq. (A22) in (A3) and (A7) yields

$$|p^2| \propto \sin^2 \gamma_m z \quad (\text{A23})$$

The evaluation of $s_m(z)$ for small z gives

$$\phi_m(z) \propto \sin \gamma_m z$$

$$\text{for } z/z_m \ll 1$$

$$P_m^2 \propto \sin^2 \gamma_m z \quad (\text{A24})$$

We observe that the dipole effect is included in the mode formulation and has the same form as it would have in a ray trace solution. Since $\max \gamma_m$ or $\sin \theta_{\max}$ can be determined very easily for a simple surface duct, it is easy to include the surface reflection for very shallow sources. The comments concerning deep dipoles and uncertain source depths would also apply to the eigen functions for deep source and receiver.

Eigen functions as a function of depth are sketched in Fig. A3. The approximate functions and exact ones from Bucker and Morris are compared. The limit of validity of the WKB approximation is indicated.

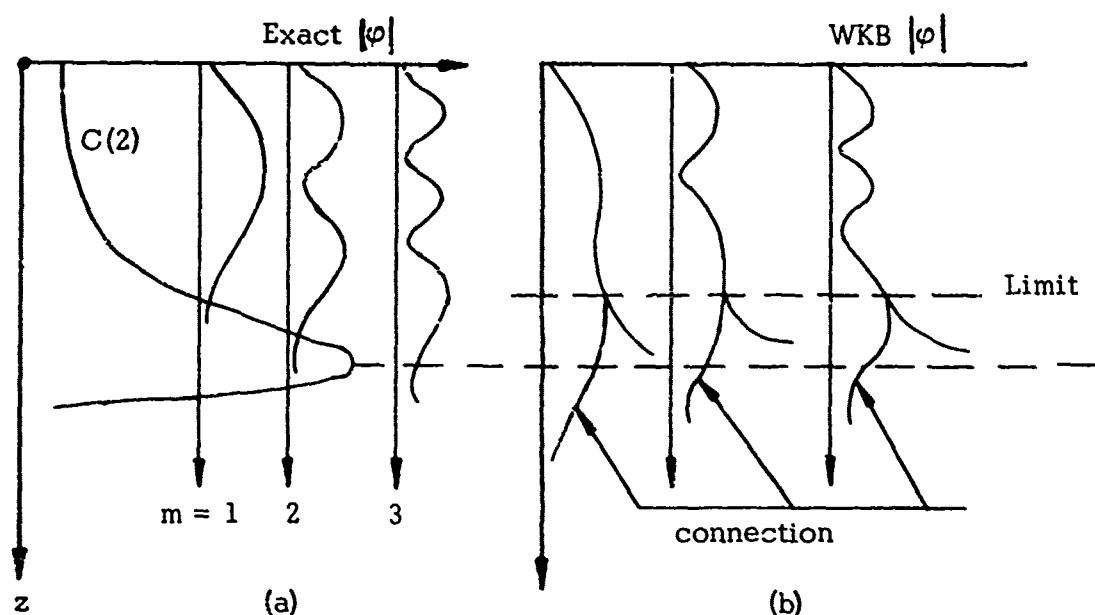


Fig. A3 Eigen functions

- a) From Bucker and Morris (1967)
- b) WKB

Approximate evaluation of the excitation functions can be made by using the WKB eigen functions (A11) and ignoring the dependence of γ_m upon z . (γ_m is slowly varying except near the turning depth.) In this, the infinite value of $\varphi_m(z)$ at the turning depth is eliminated. We also

approximate $\varphi_m(z)$ by letting $\varphi_m(z)$ be zero for depths larger than z_m .

The approximate φ_m and derivatives are

$$\varphi_m(z) \approx \gamma_m^{-1/2} \sin [s_m(z) + s_1] \quad 0 \leq z \leq z_m \quad (A25)$$

$$\gamma_m \approx \text{constant}$$

$$\varphi_m(z) = 0 \quad z > z_m \quad (A26)$$

$$\frac{\partial \varphi}{\partial z} \approx \gamma_m^{-1/2} \cos [s_m(z) + s_1]$$

by Eq. (A12)

$$\frac{d s_m(z)}{d z} = \gamma_m$$

In Eq. (A7), ν_m becomes after substitution of (A25) and (A26) the following

$$\nu_m \approx c_0^2 (2\omega^2 a_1)^{-1} [\gamma_m] \Big|_{z=0} \quad (A27)$$

We can express the source function as either a source power $\mathcal{J}$ or as an rms pressure at unit distance R_0 . The relation is from Tolstoy and Clay (2.46) - (2.50).

$$|p^2| = \rho \left(\frac{\mathcal{J} c}{4\pi} \right) \frac{1}{R^2} \quad (A28)$$

$$(\rho \mathcal{J} c)^{1/2} = 2\pi^{1/2} p_0 R_0 \quad (A29)$$

The substitution of (A27), (A29) and (A16) into (A5) yields

$$p_m = \frac{4 \pi^{3/2} k_0^{1/2} (2a_1)}{(2a_1 z_m)^{1/2} (1 - 2a_1 z_m)^{1/2}} p_0 R_0 \quad (A30)$$

$$p_m = p_0 R_0 4 \pi^{3/2} \rho^{-1} z_m^{-1} k_0^{-1/2} (1 - 2a_1 z_m)^{-1/2} \sin [s_m(z) - \frac{\pi}{4}] \sin [s_m(z_0) - \frac{\pi}{4}] \quad (A31)$$

Numerical values of P_m can be used in (A3) and (A8) for the calculation of the acoustical pressure.

Before we estimate the attenuation, let us consider our results in the context of Eq. (A1) and Fig. 2. Radiation into the M modes corresponds to signals leaving the source at a succession of angles $\theta_1, \theta_2, \dots, \theta_M$. θ_M is approximately $\Delta\theta$ in Fig. 2. The factor $r^{-1} p_m^2$ includes $(rh)^{-1}$. As mentioned earlier, the dipole or surface reflection is contained in $\sin \gamma[]$.

The half channel can be evaluated by means of Eq. (A8) and (A31). The attenuation would be small and the measured attenuation of signals in deep water can be used.

The attenuation in the surface duct is caused by leakage of waves through the bottom of the duct. The amount of leakage depends upon gradient of sound velocity beneath the duct in addition to the parameters that describe the duct and the signal. The velocity gradient below the duct is of the order of 40 times the gradient in the duct. For a rough approximation to the form of the attenuation function, we will follow Brekhovskikh (English ed 1960) and let the gradient in the duct be zero, $a_1 = 0$.

We give here, simplified results that are valid for trapped signals. The actual computation is for the modes that are reflected by the change of the velocity gradient. After considerable manipulation between pp 528 and 539, he gives the following. (Where we have used q for his s , m for his l and δ_m for his β_m on pp 537-539):

$$q \equiv \frac{1}{3} k_0 h \left(\frac{3a_2}{k_0} \right)^{1/3} \quad (A32)$$

for $q \gg 1$ (trapped modes)

$$x_m \approx \frac{m \pi}{3q + i\sqrt{3} (1.57)/2} \quad (A33)$$

$$x_m = x'_m - i x''_m \quad (A34)$$

$$\delta'_m \approx (9a_2^2 k_0)^{1/3} x'_m x''_m \quad (A35)$$

$$\delta'_m \approx 14 m^2 [k_0^{5/3} h^3 (3a_2)^{1/3}]^{-1} \quad (A36)$$

Brekhovskikh gives a table of x' and x'' as function of q (or his s) for high and low frequency, p. 540, and it is our Table 3.1.

The attenuation of a 530 cps signal in a 100m duct above a negative gradient ($a_2 = 3 \times 10^{-4} \text{ m}^{-1}$) is about $4 \times 10^{-5} \text{ m}^{-1}$ or about 0.3 db/km. These attenuations are somewhat higher than reported by Pedersen and Gordon but it should be emphasized that we have assumed no trapping of energy in the surface duct other than that reflected by the change of gradient of c at $z = h$.

Table 3.1

540

THE FIELD OF A CONCENTRATED SOURCE

TABLE 13

l	$\varepsilon = 0$		$\varepsilon = 0.1$		$\varepsilon = 0.2$		$\varepsilon = 0.4$		$\varepsilon = 0.6$		$\varepsilon = 0.8$		$\varepsilon = 1$		$\varepsilon = 1.5$	
	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1
1	0.6653	-1.1525	0.703	-1.0874	0.736	-1.024	0.813	-0.8975	0.8845	-0.740	0.8322	-0.4013	0.7425	-0.2911	0.5608	-0.1499
2	0.8825	-1.5285	0.9107	-1.4794	0.937	-1.430	0.9964	-1.3335	1.050	-1.229	1.1053	-1.1248	1.1589	-1.0144	1.2579	-0.6898
3	1.0259	-1.777	1.051	-1.733	1.072	-1.689	1.123	-1.607	1.168	-1.519	1.2183	-1.4344	1.2661	-1.3449	1.3808	-1.1049
4	1.1377	-1.9706	1.156	-1.924	1.176	-1.885	1.225	-1.639	1.268	-1.739	1.3125	-1.6637	1.3552	-1.3540	1.4606	-1.3782
5	1.231	-2.1322	1.264	-2.177	1.277	-2.046	1.312	-1.817	1.356	-1.923	1.3925	-1.849	1.4327	-1.7771	1.5303	-1.5905

l	$\varepsilon = 2$		$\varepsilon = 3$		$\varepsilon = 4$		$\varepsilon = 5$		$\varepsilon = 6$		$\varepsilon = 7$		$\varepsilon = 8$		$\varepsilon = 9$	
	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1	x'_1	x''_1
1	0.4161	-0.0912	0.3156	-0.0443	0.2431	-0.0260	0.1976	-0.0171	0.1664	-0.0121	0.1439	-0.0090	0.1266	-0.0070	0.1129	-0.0056
2	0.9021	-0.1805	0.6346	-0.0907	0.4876	-0.0536	0.3958	-0.0344	0.3331	-0.0243	0.2881	-0.0181	0.2532	-0.0140	0.2250	-0.0111
3	1.4913	-0.8198	0.9607	-0.1415	0.7940	-0.0804	0.5952	-0.0321	0.5004	-0.0366	0.4325	-0.0272	0.3801	-0.0210	0.3391	-0.0167
4	1.5627	-1.1520	1.2972	-0.1987	0.9868	-0.1037	0.7963	-0.0705	0.6686	-0.0492	0.5774	-0.0365	0.5073	-0.0281	0.4524	-0.0223
5	1.6286	-1.3921	1.8111	-0.9164	1.2332	-0.1312	0.9998	-0.0397	0.8376	-0.0622	0.7230	-0.0459	0.6349	-0.0353	0.5639	-0.0280

From Brekhovskikh--pg. 540, $s = \frac{1}{3} k_0 h \left(\frac{3a_2}{k_0} \right)^{1/3}$

A simple way to approximate the effect of a duct having a positive gradient is to replace h in Eq. (A36) by an effective h_e . To estimate the effective depth, let us assume that the ray leaving the surface at θ_m travels straight, reflects at the depth h_e and returns to the surface at the same distance as the actual curved ray. For very small θ_m , the effective depth is approximately $2h$. Thus (A36) can be written as

$$\delta'_m \approx 14 m^2 [k_0^{5/3} h_e^3 (3a_2)^{1/3}]^{-1} \quad (A37)$$

The value of h_e should be regarded as adjustable.

We need to include a second form of attenuation, the loss at the rough sea surface. On the basis of reflection of signals at a rough surface, Tolstoy and Clay have estimated the attenuation in a wave guide having irregular boundaries. Their equation (6.120) is

$$\delta''_m = \gamma_m^3 \sigma^2 (h_e x_m)^{-1} \quad (A38)$$

and with the aid of (A16)

$$\delta''_m = k_0^2 \sigma^2 (2a_1 z_m)^{3/2} h_e^{-1} (-2a_1 z_m)^{-1/2}$$

In approximation, we can let z_m and h_e both be equal to h , so that δ''_m is

$$\delta''_m \approx k^2 \sigma^2 (2a_1 h)^{3/2} h^{-1} \quad (A39)$$

The total attenuation number is

$$\delta_m = \delta'_m + \delta''_m \quad (A40)$$

Summary:

For waves to be trapped

$$k_0 h > \frac{3}{2} (M - \frac{1}{4}) \pi (2a_1 h)^{-1/2} \quad (A19)$$

$$|p|^2 \approx \frac{\rho^2}{r} \sum_{m=1}^M p_m^2 e^{-2\delta_m r} + \text{cross terms} \quad (A8)$$

$$p_m \approx (p_0 R_0) \frac{4\pi^{3/2}}{k_0^{1/2} z_m \rho (1 - 2a_1 z_m)^{1/2}} \sin [k_0 (2a_1 z_m)^{1/2} z_0]$$

$$\sin [k_0 (2a_1 z_m)^{1/2} z] \quad (A31)$$

$$q = \frac{1}{3} k_0 h_0 (3a_2/k_0)^{1/3} \quad (A32)$$

$$\delta'_m \approx 14 m^2 [k_0^{5/3} h_0^3 (3a_2)^{1/3}]^{-1} \quad (A36)$$

$$\delta''_m \approx k_0^2 \sigma^2 (2a_1 z_m)^{3/2} [h_0 (1 - 2a_1 z_m)^{1/2}]^{-1} \quad (A39)$$

in which

$$c = c_0 (1 - 2a_1 z)^{-1/2} \quad 0 \leq z \leq h$$

$$c = c_h [1 + 2a_2 (z - h)]^{-1/2} \quad z \geq h \quad (A9)$$

$$z_m = [3a_1 (m - \frac{1}{4}) \pi k_0^{-1}]^{2/3} [2a_1]^{-1}$$

σ = rms sea surface roughness

h_e = effective depth of corresponding constant velocity surface duct, $h_e \approx 2h$.

p_0 is the source pressure at distance R_0 .

For one mode, the transmission loss in db is relative to p_0 at R_0 the following:

$$\begin{aligned}
 -10 \log (|p|^2 / p_0^2) &= 10 \log (r/R_0) \\
 &+ 1.2 \times 10^2 m^2 [k_0^{5/3} h_e^3 (3a_2)^{1/3}]^{-1} r \\
 &+ 8.6 k_0^2 \sigma^2 (2a_1 z_m)^{3/2} [h_e (1 - 2a_1 z_m)^{1/2}]^{-1} r \\
 &+ Ar + 10 \log (k_0 z_m^2 / R_0) \\
 &- 20 \log \left\{ \sin k_0 (2a_1 z_m)^{1/2} z_0 \right\} \\
 &- 20 \log \left\{ \sin k_0 (2a_1 z_m)^{1/2} z \right\} \\
 &+ 10 \log (1 - 2a_1 z_m) - 10 \log 16 \pi^3
 \end{aligned} \tag{A41}$$

and where A is the absorption loss for sound propagation in sea water.

As a numerical example, we assume conditions that are approximately those of Pedersen and Gordon:

$$f = 530 \text{ cps}, \quad h = 100\text{m}, \quad a_1 = 7 \times 10^{-6} \text{ m}^{-1}$$

$$a_2 = 3 \times 10^{-4} \text{ m}^{-1}, \quad \sigma = 0, \quad z_0 = z = 16\text{m}$$

and ignore A. The results of calculation are given on Table 1. Two values of the effective depth are given to demonstrate the effect.

Table 1 Estimates of mode attenuation δ'_m
 $M = 2, k_0 \approx 2\text{m}^{-1}, (2a_1 z_m)^{1/2} \approx 4 \times 10^{-2},$
 $(z_m = 100), q \approx 5$

<u>Mode</u>	<u>Effective depth</u>	<u>Estimated attenuation</u>	<u>Pedersen and Gordon</u>
for m = 1	$h_e = 100 \text{ m}$	$8.6 \delta'_m \approx 0.3 \text{ db/km}$	$7 \times 10^{-3} \text{ db/kyd}$
	$h_e = 200\text{m}$	$8.6 \delta'_m \approx 0.04 \text{ db/km}$	
for m = 2	$h_e = 100\text{m}$	$8.6 \delta'_m \approx 1.2 \text{ db/km}$	0.3 db/kyd
	$h_e = 200\text{m}$	$8.6 \delta'_m \approx 0.16 \text{ db/km}$	
roughness			
$\sigma = 0$		$\delta''_m = 0$	
$\sigma = 2\text{m}$	$h_e = 100\text{m}$	$8.6 \delta''_m \approx 0.2 \text{ db/km}$	

Pedersen and Gordon show data for a relatively short range of 10 kyds and it is difficult to measure the attenuation coefficients of the low modes in this distance. The estimated attenuation are probably with the right order of magnitude.

Evaluation of the rest of the constants in Eq. (A41) yields $m = 1$, $h_e = 100\text{m}$, $\sigma = 0$, and r in km, i.e. $R_1 = 1\text{ km}$

$$-10 \log (|p|^2 / p_0^2) \approx 52 + 10 \log (r/R_1) + 0.3r, R_1 = 1\text{ km} \quad (\text{A42})$$

If more than one mode were contributing, Eq. (A8) should be used. A comparison of Eq. (A42) and their data are shown on Fig. A4.

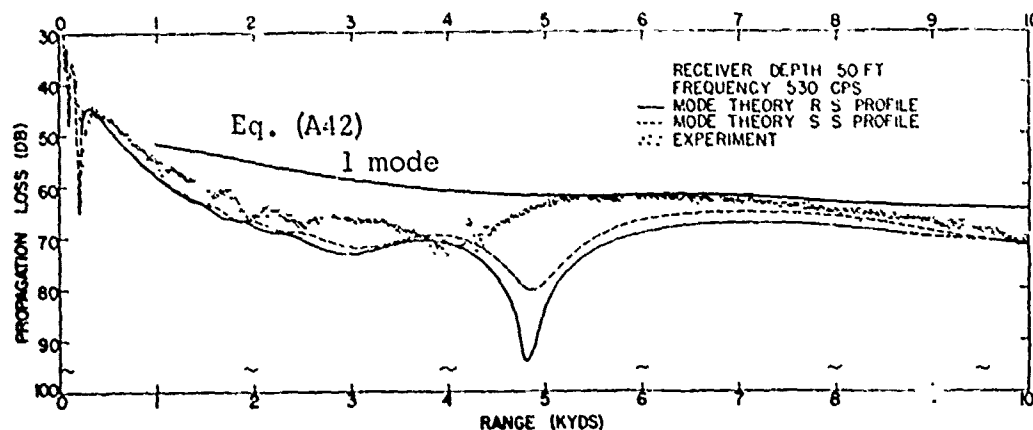


FIG. 5. Propagation losses for 50-ft receiver, 55-ft source.

Fig. A4 From Pedersen and Gordon, 1965.

In conclusion, we suggest the next steps. At high frequencies, the number of trapped modes is high and correspondingly, Eq. (A41) should not be used. Brekhovskikh has studied this and has suggested the use of "average decay laws", pp. 415-421 (1960). He averaged p^2 over the depth and then replaced the summation of modes by an integration. For mode attenuations proportional to m^2 [Eq. (A36)], the average decay of p^2 is proportional to $r^{-3/2}$.