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SOUND PROPAGATION IN THE SEA BY RAYS: WAVEFRONT SPREADING: RECI--ETC(U)
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Project M-153

Sound Propagation in the Sea by Rays;

WAVEFRONT SPREADING: RECIPROCITY TRANSFORMATION, AND RELATED INVESTIGATIONS,

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Technical Note ~~One~~ no. 1,

THE RECIPROCITY TRANSFORMATION

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I Introduction

The capability for solving the following two problems, under conditions of speed-of-sound distributions and other factors in the sea, is of well established importance:

Problem (a): Determination of the distribution of sound intensity, throughout the ambient and extended ocean medium, caused by a unit point-source at a given location.

Problem (b): Determination of the distribution of listening sensitivity, throughout the ambient and extended ocean medium, afforded by a receiver at a given location.

A major component in the solution of these problems by ray tracing is the determination of the wavefront spreading factor along the rays. The present technical note establishes the reciprocity transformation, for this factor, between problems (a) and (b).

II The Specific Wavefront Length

We focus our discussion in sound intensity mapping in a vertical plane. For a point source each selected ray is associated with an angle, γ_S, at which the ray emanates from the source; by convention this angle is measured clockwise from the horizontal. The labelling of rays by their source-emission angles, γ_S, defines γ_S wherever the medium is traversed by the rays. It should be noted that γ_S is not necessarily single valued throughout the medium; in regions, two or more different rays may pass through every point; however each ray passing through a point belongs to a particular continuum of γ_S (i.e. family of rays).

For a family of rays, in the vertical plane, the specific wavefront length, denoted by L, is defined at each point by

$$\frac{\mathbf{n}}{L} \equiv \nabla \gamma_S \tag{1}$$

where $\mathbf{n}$ is a unit vector, clockwise normal to the ray at that point. Negative L thus implies γ_S is decreasing in the direction $\mathbf{n}$; sign changes in L are important as indications of foldings of the wavefront as at caustics or by reflections.

The differential equation for integrating L along a ray is developed in Reference [1] as

$$\frac{\partial^2 L}{\partial s^2} = -\frac{L}{C}\left[\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\frac{\partial C}{\partial s}\frac{\partial L}{\partial s}\right] \tag{2}$$

where s is ray path, C is sound speed and n is ray normal. The initial conditions at the source point are

$$L_S = 0 \;, \quad (\partial L/\partial s)_S = 1 \tag{3}$$

The parameter L may be integrated along each ray beginning at the source. The advantages of such direct integration include (1) the preclusion of the problem of measuring <u>specific</u> spreading by geometric measurement over finite distances between diverse rays, (2) the indication, afforded by sign changes in L, of *foldings of the* wavefront caused by caustics or reflections, and (3) indication, afforded by the spatial continuity in L, of the adequacy of the intensity resolution afforded by the selected family of rays.

III Reciprocity

A ray emitted by a source at point 0 and traced to a point N is exactly reversed for a source at point N. Ray traces are reciprocal.

Reference [2] establishes that the parameter L is not reciprocal along a ray trace. Denoting the spreading factor from a source at point 0 to the point N by $L_{0,N}$, and for a source at point N along the reverse ray to point 0 by $L_{N,0}$, then, in general,

$$L_{0,N} \neq L_{N,0} . \tag{4}$$

However reference [2] suggests that the parameter

$$Q \equiv L/C \tag{5}$$

is reciprocal. Reciprocity is shown under conditions where the sound speed, C, is a function of depth only, and for other conditions. We shall show that this reciprocity is general, whereby

$$\frac{L_{0,N}}{C_N} = \frac{L_{N,0}}{C_0} \tag{6}$$

IV Transforming the Governing Equation

We replace the dependent parameter L by the dependent parameter Q, by substituting QC for L, and transform Eq. (2) as follows:

$$C \frac{\partial^2 Q}{\partial s^2} + 2 \frac{\partial C}{\partial s} \frac{\partial Q}{\partial s} + Q \frac{\partial^2 C}{\partial s^2} =$$

$$- Q \left[\frac{\partial^2 C}{\partial n^2} - \frac{1}{QC} \frac{\partial C}{\partial s} \left(Q \frac{\partial C}{\partial s} + C \frac{\partial Q}{\partial s} \right) \right] \tag{7}$$

The second derivative of C along the curve of the ray trace is developed by vector calculus:

$$\frac{\partial^2 C}{\partial s^2} = \mathbf{t} \cdot \nabla (\mathbf{t} \cdot \nabla C)$$

$$= (\mathbf{t} \cdot \nabla)^2 C + (\mathbf{t} \cdot \nabla \mathbf{t}) \cdot \nabla C \tag{8}$$

where $\mathbf{t}$ is the local ray-tangent unit vector, along increasing ray path, s. The first term on the right hand side of Eq. (8) may be written

$$(\mathbf{t} \cdot \nabla)^2 C = \frac{\partial^2 C}{\partial s_*^2} \tag{9}$$

where s_* is the linear measure along the local ray-tangent axis. The second term on the RHS of Eq. (8) contains the path curvature term which is effected by refraction:

$$\mathbf{t} \cdot \nabla \mathbf{t} = \mathbf{n} \frac{\partial \gamma}{\partial s} = - \frac{\mathbf{n}}{C} \frac{\partial C}{\partial n} \tag{10}$$

where $\mathbf{n}$ is clockwise normal to $\mathbf{t}$ and n is the linear measure along this normal axis. Thus Eq. (8) may be written

$$\frac{\partial^2 C}{\partial s^2} = \frac{\partial^2 C}{\partial s_*^2} - \frac{1}{C} \left(\frac{\partial C}{\partial n}\right)^2 \tag{11}$$

Substitution of Eq. (11) into Eq. (7) and simplifications lead to

$$\frac{\partial^2 Q}{\partial s^2} = - Q \left[\left(\frac{\partial^2 K}{\partial s_*^2} + \frac{\partial^2 K}{\partial n^2} \right) + \frac{1}{Q} \frac{\partial K}{\partial s} \frac{\partial Q}{\partial s} \right] \tag{12}$$

where

$$K \equiv \ell n\, C \qquad (13)$$

At this point we note a remarkable simplification in form: because n and s_* are local orthogonal axes, the terms in parentheses combine to form the Laplacian operator in the vertical plane of the ray. Thus,

$$\frac{\partial^2 Q}{\partial s^2} = -Q \left[\nabla_v{}^2 K + \frac{1}{Q} \frac{\partial K}{\partial s} \frac{\partial Q}{\partial s} \right] \qquad (14)$$

The subscript v denotes that the Laplacian operator is two-dimensional (the circular derivative) in the vertical plane of the ray.

The initial values at a source, point 0, where $s = 0$, are

$$Q_0 = 0 \; ; \quad (\partial Q/\partial s)_0 = 1/C_0 \qquad (15)$$

For further edification we replace the independent ray-path parameter s by the wave travel time, t, related by

$$s = \int_0^t C\, dt \qquad (16)$$

$$\delta s = C\, \delta t$$

Equations (14) and (15) transform into

$$\frac{\partial^2 Q}{\partial t^2} = -C^2 \nabla_v{}^2 K \quad Q \qquad (17)$$

$$Q_0 = 0 \ ; \quad (\partial Q/\partial t)_0 = 1 \tag{18}$$

V Proof of Reciprocity

We write Eq. (17) in the form

$$\frac{\partial^2 Q}{\partial t^2} = - a Q \tag{19}$$

where

$$a \equiv C^2 \nabla_v^2 K \tag{20}$$

In the present section we prove that the parameter Q is reciprocal: Beginning with a source point at either of the two specific end points of a ray path, that is, with

$$Q_0 = 0 \text{ and } (\partial Q/\partial t)_0 = 1 \ , \tag{21}$$

the same value of Q results in integration of Eq. (19) to the other end of the path.

Equation (19) allows a simple and direct finite-difference form;

$$Q_{n+1} + Q_{n-1} - 2 Q_n = - a_n \delta t^2 Q_n \tag{22}$$

where n-1, n, and n+1 are subscripts referring values to three successive positions of a wavefront along the ray path, at fixed time-increments, δt. Equation (22) may be written as a simple recursion formula:

$$Q_{n+1} = A_n Q_n - Q_{n-1} \tag{23}$$

where

$$A_n \equiv 2 - a_n \delta t^2 \tag{24}$$

According to Eq. (21) the initial values are

$$Q_0 = 0 \tag{25}$$

$$Q_1 = \delta t \tag{26}$$

We set

$$Q_1 = 1 \tag{27}$$

by normalizing the amplitude factor. It follows from Eq. (23) that

$$Q_2 = A_1$$

$$Q_3 = A_2 A_1 - 1$$

$$Q_4 = A_3 A_2 A_1 - A_3 - A_1$$

$$Q_5 = A_4 A_3 A_2 A_1 - A_4 A_3 - A_4 A_1 - A_2 A_1 + 1$$

and so on with the numerical integration. Reciprocity is demonstrated for the path $n = 0$ to $n = 5$, for example, by substituting subscripts in reverse order as would result for a wavefront beginning at point 5 as source point and integrated to point 0. It can readily be verified that Eq. (25) is a <u>necessary</u> condition for reciprocity.

The general proof lies in showing that the general solution for a wavefront emanating from a source point 0 to a point N, which we shall denote by $Q_{0,N}$ is symmetric in the position subscripts on the A's. That is, the solution is unchanged by replacing

$$n \text{ by } N-m;$$

that

$$Q_{0,N} = Q_{N,0} \; .$$

At this point (and before turning to the next page) those readers who are stimulated by mathematical problems are invited to pause and consider formulating the general term $Q_{0,N}$, for completion of the reciprocity proof.

Although the solution is simple in appropriate formalism the present writer admits that it was not immediately apparent. The general solution is expressed by the determinant which has the elements A_{N-1}, A_{N-2}, ... A_1, in the main diagonal, all ones in the two adjoining diagonals, and all other elements zero:

$$Q_N = \begin{vmatrix} A_{N-1} & 1 & & & & & \text{(all zeroes)} \\ 1 & A_{N-2} & 1 & & & & \\ & 1 & A_{N-3} & 1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & 1 & A_n & 1 & \\ & & & & \ddots & \ddots & \ddots \\ \text{(all zeroes)} & & & & 1 & A_2 & 1 \\ & & & & & 1 & A_1 \end{vmatrix} \tag{28}$$

That this solution is symmetric in subscripts is immediately apparent. That this is the solution is shown by expansion of the determinant in terms of the minors of the first column, which yields

$$Q_N = A_{N-1} Q_{N-1} - Q_{N-2}, \tag{29}$$

the second term on the right resulting from further expansion as the only minor of the first row.

Thus, in general,

$$Q_{N,0} = Q_{0,N} \tag{30}$$

and

$$\frac{L_{N,0}}{C_0} = \frac{L_{0,N}}{C_N} \tag{31}$$

VI The Specific Wavefront Area and Reciprocity Transformation

A wavefront is a surface. Wavefront spreading may be expressed in terms of two orthogonal dimensions. The specific wavefront area may be denoted by E:

$$E = L\,H \tag{32}$$

where L has been defined as the specific wavefront length per unit radian of emission angle at the source, measured normally to the ray, in the vertical plane of the ray, that is, along $\mathbb{n}$, and H is the specific wavefront length per unit radian of emission angle at the source, measured normally to the ray, in the horizontal, that is, along $\mathbb{m}$. The relevant orthogonal triple of unit vectors is

$$\mathbb{m} = \mathbb{t} \times \mathbb{n} \tag{33}$$

In reference [2] the governing equations for the spreading of L and H along a ray are given as

$$\frac{\partial^2 L}{\partial s^2} = -\frac{L}{C}\left[\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\frac{\partial C}{\partial s}\frac{\partial L}{\partial s}\right] \tag{34}$$

$$\frac{\partial^2 H}{\partial s^2} = -\frac{H}{C}\left[\frac{\partial^2 C}{\partial h^2} - \frac{1}{H}\frac{\partial C}{\partial s}\frac{\partial H}{\partial s}\right] \tag{35}$$

where h is linear coordinate along the horizontal ray-lateral axis defined by unit vector $\mathbf{m}$. These equations encompass the general treatment which permits refraction in the horizontal as well as the vertical, and the ray traces are not confined to planes.

In preceding sections we have replaced the dependent parameter L by Q:

$$Q \equiv L/C \tag{36}$$

We have transformed the governing equation into the form

$$\frac{\partial^2 Q}{\partial t^2} = -C^2 \left(\frac{\partial^2 K}{\partial s_*^2} + \frac{\partial^2 K}{\partial n^2} \right) Q \tag{37}$$

where s_* is a linear measure along the local ray tangent axis, and we have proven that Q is reciprocal, as defined. In analogous fashion we replace the dependent parameter H by P:

$$P \equiv H/C \tag{38}$$

The governing equation transforms into

$$\frac{\partial^2 P}{\partial t^2} = -C^2 \left(\frac{\partial^2 K}{\partial s_*^2} + \frac{\partial^2 K}{\partial h^2} \right) P \tag{39}$$

and the proof that P is reciprocal is similar to that for Q.

The reciprocal parameter for the specific wavefront area, E, may be denoted by W:

$$W = E/C^2 \tag{40}$$

The reciprocity transformation for specific wavefront area is

$$E_{N,0} = \frac{C_0^{\,2}}{C_N^{\,2}} E_{0,N} \tag{41}$$

VII References

[1] "The wavefront-divergence factor in ray-intensity integration", M. M. Holl, Meteorology International Incorporated, Project M-140; Technical Note Two, Contract No. N62271-67-M-2000, April 1967.

[2] "Analytic verification of wavefront spreading formulations, non-reciprocity in ray reversal, and FNWF computer program", M. M. Holl, Meteorology International Incorporated, Project M-148: Technical Note One, Contract No. N62271-68-M-0791, December 1967.