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 A RATIONALE FOR CONGLOMERATE MERGERS  
 by  
 NICOLAS S. MAJLUF

Technical Report No. 6

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**MASSACHUSETTS INSTITUTE**  
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STUDY ON MERGERS: A RATIONALE FOR CONGLOMERATE MERGERS

by

NICOLAS S. MAJLUF

Technical Report No. 6

Work Performed Under  
Contract N00014-76-C-1033, Office of Naval Research  
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Alfred P. Sloan School of Management  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139

November 1978

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## FOREWORD

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William F. Pounds  
Dean

## ABSTRACT

Mergers are not a new phenomenon, but rather an on-going process in the business environment. They correspond to the combination of two (or more) firms into a unique business concern. This study is grounded on the notion that looking at mergers from a financial point of view may provide a valid platform for analyzing merger movements.

The fundamental development of this study is an equilibrium model for determining the market value of a firm when the managerial team is assumed to have better information than the market. It is shown that when a firm with superior information does not have sufficient internal resources (financial slack) to undertake a project, the full value of future investment opportunities is not necessarily captured in the market value of the firm. This conclusion is obtained because there are situations in which, by taking the project and bringing in new shareholders, old shareholders lose (from the dilution of their holdings in the firm) more than what they get from the extra value added by the new project.

The dependency of market value from slack availability opens the possibility of justifying mergers via tender offers. In this context, the merger may be understood as a way to inject resources from a "cash rich" to a "cash poor" firm. The expected payoff of this game is positive and equal to the loss in market value due to insufficient slack.

Finally, if it is assumed that the market value of the firm should capture the full value of future investment opportunities, some normative conclusions regarding the behavior of managers may be stated. The most important of these conclusions is that the decision of a firm to issue stock should be unconditional; that is to say, it should be determined only by the value of the investment opportunity and not linked in any way with the superior information that the firm holds. In this setting, mergers may be viewed as a deterrent for managers deviating from this desirable behavior.

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A special paragraph for a very special secretary. Thanks to Deborah Cohen for accepting the madness of this typing with good humor and willingness to help.

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## CHAPTER 1: HISTORICAL ANALYSIS OF MERGER MOVEMENTS

The number of acquisitions reported by Nelson [73] for 1895-1956 coupled with a continuation of this series compiled by the U.S. Federal Trade Commission [92][93], have served as the starting point for this historical analysis of merger movements by most authors. Though Nelson data are taken from sources not directly comparable,\* the time series shows three very distinctive peaks of activity in years 1899, 1929, and 1968 (see Table 1 and Figure 1). These years are normally used as anchors in the distinction of three main waves of merger activity: first, the turn of the century period; second, the late 1920's period; and third, the post-World War II period, beginning in 1945 and with no clear termination date yet. As Steiner [87, pp. 6-7] puts it, "We are not sure if this period already ended after the great boom in the late 60's, or if we are in a long trend of increasing acquisitions which has not yet settled".

The three merger waves are far from being the recurrent manifestation of a unique phenomenon, corresponding instead to situations with a common final response (the increase in merger activity), but a different set of underlying causes. These sets of causes give particular characteristics to each one of the merger cycles.

The first wave is indicated as the one with the most profound and lasting effect on the structure of American industry, because this period saw the emergence of powerful corporations that acquired monopoly control and captured a substantial share of their markets. The most noticeable

---

\* Nelson's own compilation for 1895-1920, and Thorp [90] data for 1919-1939 give substantially different numbers for 1919 and 1920 which are the unique common years.

TABLE 1: Mergers and Acquisitions in Manufacturing and Mining 1895-1968

<u>Nelson Series</u> 1895-1919		<u>Thorp Series</u> 1919-1939		<u>FTC Series</u> 1940-1968	
<u>Year</u>	<u>Annual Total</u>	<u>Year</u>	<u>Annual Total</u>	<u>Year</u>	<u>Annual Total</u>
1895	43	1919	438	1940	140
1896	26	1920	760	1941	111
1897	69	1921	487	1942	118
1898	303	1922	309	1943	213
1899	1,208	1923	311	1944	324
1900	310	1924	368	1945	333
1901	423	1925	554	1946	419
1902	379	1926	856	1947	404
1903	142	1927	870	1948	223
1904	79	1928	1,058	1949	126
1905	226	1929	1,245	1950	219
1906	128	1930	799	1951	235
1907	87	1931	464	1952	288
1908	50	1932	203	1953	295
1909	49	1933	120	1954	387
1910	142	1934	101	1955	683
1911	103	1935	130	1956	673
1912	82	1936	126	1957	585
1913	85	1937	124	1958	589
1914	39	1938	110	1959	835
1915	71	1939	87	1960	844
1916	117			1961	954
1917	195			1962	853
1918	71			1963	861
1919	171			1964	854
1920	206			1965	1,008
				1966	995
				1967	1,496
				1968	2,300

(Sources: Nelson [73], Federal Trade Commission Statistical Compilations on mergers [92][93])

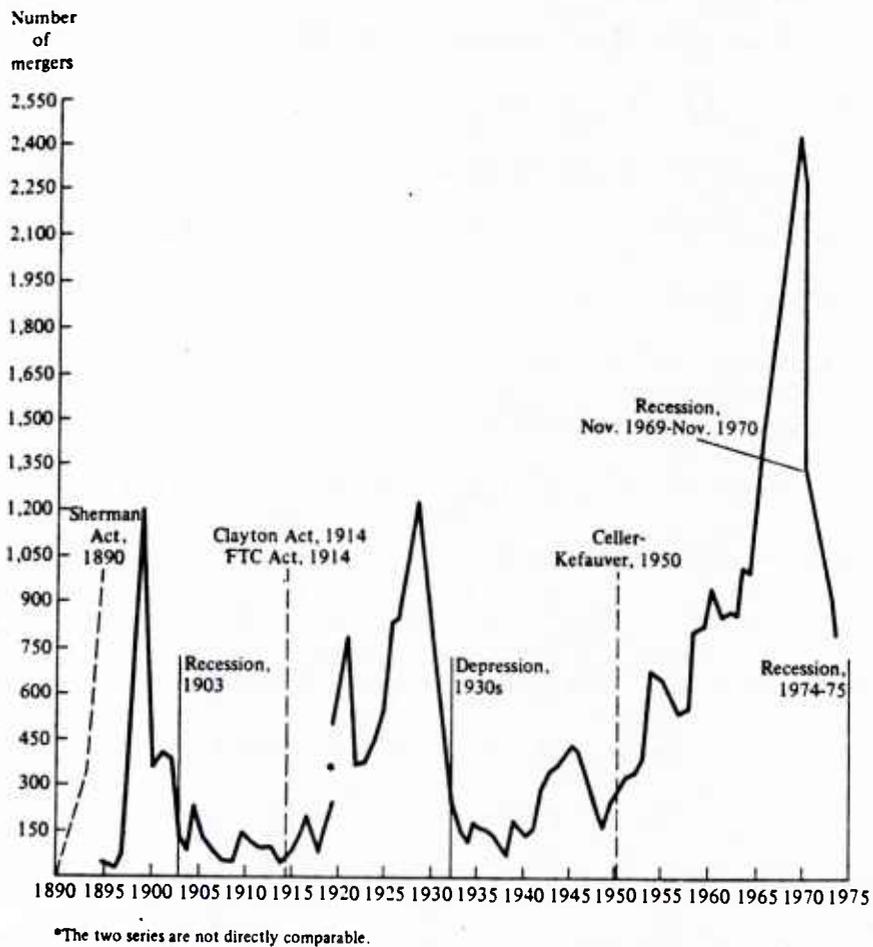


FIGURE 1: Historical series for mergers and acquisitions of manufacturing and mining firms. (Source: Reid [78]).

examples are Standard Oil (which started an aggressive acquisition program as early as 1872 to finally reach 90 percent market share of U.S. refining capacity by the end of the last century), U.S. Steel (65 percent market share), General Electric (virtual duopoly with Westinghouse), American Can (70 percent market share), American Tobacco (90 percent market share), DuPont (85 percent of the market), and many other companies still showing large operations in present times (see Lynch [55], p. 21 and Sherer [84], pp. 103-106). Markham [57, p. 180] has summarized the fundamental change of American industry in this period by stating that "the conversion of approximately 71 important oligopolistic and near-competitive industries into near monopolies by merger between 1890 and 1904 left an imprint in the structure of American economy that fifty years have not yet erased".

No unique factor can be cited as the cause for this first merger movement dampening. Some reasons given are the exhaustion of merger opportunities at that time, the increasing number of failures in merger ventures which concluded with some panic in the securities market, and the recession of 1903-1904. Also, an important role is assigned in the termination of this merger wave to the first successful challenge of a monopoly in a notorious case. In fact, though the Sherman Act had been passed in 1890, it was only in 1904 that the first antimonopoly precedent was set in the Northern Security case."

The second merging wave, that culminated in 1929, "had a much less dramatic effect in concentration" (Sherer [84], p. 107). Stigler [88, p. 31] has described this period as one in which oligopolistic structures are created rather than monopolistic ones. While horizontal concentra-

---

\* U.S. vs. Northern Securities Co., 1904 [94].

tion was largely achieved in the first period,

... casual observation suggests that the wave of the 1920's was characterized by a much higher incidence of vertical integration and conglomerate diversification mergers than its predecessor... Although these conglomerate and product line extension mergers no doubt have some adverse effect on competition, it was an effect qualitatively different from the predominantly horizontal mergers of the 1887-1904 era. (Sherer [84], p. 107).

The lack of documentation on the second merger wave has conspired against the realization of a more careful analysis, and there is some controversy on the real proportions and characteristics it had. Eis [18] has made a study that cast some doubt on Stigler's and Sherer's conclusions when stating that:

- horizontal mergers were more important than vertical and conglomerate mergers;
- the most active acquirers tend to be the large dominant firms, thus suggesting a further increase in monopoly, power;
- antitrust laws were rather ineffective during this period.

Part of the difference between the first and second merger waves is attributed to the legal climate prevailing at that time with regard to anti-trust policy. Two legal bodies which could challenge the legality of mergers had been passed: the already mentioned Sherman Act (1890), and the Clayton and Federal Trade Commission Acts (1914). They probably acted as deterrents for some time, but a loophole was soon recognized in the Clayton Act that made it inoperative until the dictation of the Celler-Kefauver Act in 1950.\* On the other hand, the 1911 case of Standard Oil [86]

\* The Clayton Act is intended to prevent mergers that affect competition, but its initial formulation banned only the acquisition of the stock of another company, but not the direct acquisition of its assets. This is the loophole that the Celler-Kefauver Act corrected.

interpreted the Sherman Act in terms of the rule of reason, which does not prohibit the acquisition of monopoly power, but its exercise against competitors. This tolerant view of monopolies was reversed only in 1945 in the Alcoa case\*, in which it was determined that the mere existence of monopoly power could be unlawful, though it were unexercised.

The Sherman Act is now sufficiently grounded as to make illegal any further increase in market concentration.

As of 1972... if it is an exaggeration to say that there is today a consensus among economists, lawyers, and courts that horizontal acquisitions by leading firms of genuine competitors have a strong presumption of adverse net effect, it is not much of an exaggeration. The Supreme Court has supported the government in virtually every attack on a horizontal merger. To be sure, there is disagreement on details such as how large is a leading company, who is a genuine competitor, what is the relevant market, etc., but these are matters of detail, not of basic principle. (Steiner [87], p. 51).

This view is reinforced with the 1968 dictation of merger guidelines by the Department of Justice, indicating that the legality of any merger that tended to increase concentration beyond well defined limits, would be challenged in the courts. For example: in a highly concentrated market (four-firm concentration ratio is 75 percent or more),

... the Department will ordinarily challenge mergers between firms accounting for, approximately, the following percentages of the market

Acquiring firm	Acquired firm
4%	4% or more
10%	2% or more
15% or more	1% or more

(Thomson and Brady [89], pp. 186-187).

---

\* United States vs. Alcoa, 1945 [95].

The role that antitrust laws are playing in the current merger wave is nicely illustrated by Kraar [48, p. 192] when reporting the General Electric-Utah International merger in Fortune. He indicates that a request was sent to the Justice Department asking for "a review of the merger and assurance that the government did not plan to fight it". This clearing was obtained only after some changes were introduced in the contract between the two merger parties. The final decision to merge was dependent on this favorable opinion of the Justice Department. This Government clearing is a step that has been preventively taken by many firms prior to their decision to merge.

The increased chances of successfully challenging a merger that in some way increases monopoly power, in all likelihood has been a powerful incentive to favor the less discredited conglomerate route during the third merger wave. Table 2 presents the wave of the late 1960's in the perspective of previous and later years. From this table, it may be observed that conglomerates in general went over 80 percent during the late 1960's wave, and unrelated conglomerates have been gaining importance continually, even after the wave's peak.

The premises used by FTC to determine the type of a merger are worthwhile examining in more detail, because they tend to overemphasize the level of conglomeration:

Larger mergers are classified into three basic categories: Horizontal acquisitions involve firms that are direct competitors in the same geographic market.

Vertical mergers link firms that had buyer-seller relationships prior to acquisition.

Conglomerate mergers, essentially, are those mergers that are neither horizontal nor vertical.

The conglomerate category can be subdivided into three classifications:

TABLE 2: Perspective of the Late 1960's Merger Wave (Large Acquisitions\* in Manufacturing and Mining).

TYPE OF ACQUISITION	PRE-WAVE 1948-1962		WAVE 1966-1970		POST-WAVE 1972-1974	
	Number of Firms (%)	Assets (%)	Number of Firms (%)	Assets (%)	Number of Firms (%)	Assets** (%)
	Horizontal	25.2	25.5	7.9	11.4	28.3
Vertical	14.2	16.1	9.1	6.7	10.9	8.2
Conglomerate	60.6	57.4	83.0	81.9	60.9	65.5
- Product extension	38.8	36.0	53.2	40.4	26.1	23.4
- Market extension	6.0	5.5	2.4	6.7	4.9	8.6
- Other (unrelated)	15.8	15.9	27.4	34.7	29.9	33.4
AVERAGES PER YEAR in # of firms & million of dollars	34.7	1111.5	142.2	8607.4	58.3	3076.0
AVERAGE SIZE PER ACQUISITION in million of dollars	32.1		60.5		52.8	

\* Large Acquisitions are those involving assets of \$10 million or more

\*\* Data for 1974 are provisory.

(Sources: FTC, Statistical Report on Mergers and Acquisitions, Bureau of Economics, Report 6-15-22, 1974 and Report 6-15-27, 1975) [26].

Product extension mergers involve companies that are functionally related in production and/or distribution, but sell products which do not compete directly with one another.

Market extension mergers link companies that manufacture the same products, but sell them in different geographic markets.

Unrelated conglomerate mergers involve the consolidation of two essentially unrelated firms.

(Federal Trade Commission [26], p. 15).

By defining as conglomerate any merger in which no clear horizontal or vertical link is found, probably an upward bias in the level of conglomeration is being reported. The most clear example is market extension mergers that must be classified as horizontal when the whole country (instead of a limited geographical region) is considered as the market place.

Nonetheless, even without considering market extension mergers, the level of conglomeration peaked in the late 1960's, and the unrelated conglomerates class has been steadily rising, virtually to double its prewave share of total mergers. To a large extent, this result is being shaped by the enforcement of antitrust laws, because firms move on safer grounds when the merger is of the conglomerate type, mainly when it is a pure conglomerate. Also, horizontal and vertical mergers being consummated do not seem to be affecting competition in any important way. Sherer [84, p. 109] expresses that if the tightened enforcement criterion of the antitrust law continues, "we can confidently expect mergers to contribute virtually nothing to the future growth of concentration within individual markets". This expectation is corroborated by a rather limited study conducted by the FTC [26, pp. 72-74] whose conclusion is that companies in the sample "did not acquire substantial market position in new areas. Individual market shares were less than 1 percent in 53.6 percent of the acquired product classes... in 82 percent of the acquired product classes, the sample con-

glomerates had market shares of less than 5 percent" (market share is defined as the ratio of firm shipments and industry shipments, where industry is a 5-digit SIC sector).

If concentration is not the main concern with conglomeration, "super concentration" is starting to be. This is the accumulation of excessive economic power in a limited number of giant corporations, which is normally measured as the percentage of total assets held by the largest 100 or 200 corporations. The argument is that excessive concentration may endanger the social and political system of the country. Steiner [87, p. 288] citing the then Attorney General of the United States John Mitchell, writes: "I believe that the future vitality of our free economy may be in danger because of the increasing threat of economic concentration by corporate mergers... The danger that this super-concentration poses to our economic, political and social structure cannot be overestimated". Edwards [16, p. 42], in a 1965 testimony before a Subcommittee of the Senate Judiciary Committee expressed this view about the excessive competitive advantages of big firms:

A big firm has advantages over a smaller rival just because it is big. Money is power. A big firm can outbid, outspend, and outlose a small firm. It can advertise more intensively, do more intensive and extensive research, buy up the inventions of others, defend its legal rights or alleged rights more thoroughly, bid higher for scarce resources, acquire the best locations and the best technicians and executives. If it overdoes its expenditures, it would absorb losses that would bankrupt a small rival.

Galbraith [29] has devoted his book The Industrial State to the analysis of big business power.

Despite the inflammatory arguments against big business, the present

antitrust legislation does not appear to be mature enough to handle it. If size per se is considered to be a problem, new legislation and new legal precedents may be required. Size is not necessarily a problem linked with mergers, but legislation of that sort could be a new deterrent for merging. For the time being, superconcentration is only a speculative subject, and does not seem to be playing an important role yet as a deterrent of mergers.

## CHAPTER 2: MOTIVES UNDERLYING MERGERS

A large number of causes have been mentioned in the literature as potential stimuli for merging. Most of these causes have been observed in different merger episodes playing roles of varying degrees of importance, depending on each particular set of circumstances encountered. The multicausality of the merger phenomenon has conspired against the development of a broadly accepted theory for their study, and has confused the discussion around policy issues.

The purpose of this section is to review and briefly comment upon the most commonly mentioned motives underlying mergers.

### 2.1 Acquisition of Monopoly Power

An important piece of the merger activity observed during the turn of the century wave can be explained by the declared interest of firms in acquiring sufficient control of the industry in which they operated. The stabilization of revenues by avoiding price wars, and the securing of a healthy profit margin moved firms into the merging path.

Though the acquisition of monopoly power is still a valid reason to merge, its present importance is not easily discernible, because of the more strict enforcement of antimonopoly laws, and the appearance of conglomerates as the most popular merger route. As opposed to horizontal and vertical mergers, that can affect competition when reaching certain size, conglomerates do not contribute significantly to increasing concentration (Goldberg [30]).

Nonetheless, new and more subtle ways of restraining competition are

available to conglomerates, which are changing the focus of the antitrust debate from pure concentration to bigness, subsidization (cash transfer), reciprocity, exclusive dealings, mutual forbearance (the "live and let live" doctrine among conglomerates that confront each other in many markets), tied sales, discouraging de novo entrants (impact over potential competition), macro-concentration, and economies of scale in public relations (lobbying), litigation, access to capital markets, and research (Steiner [87], Blair [9], Edwards [16]). Lorie and Halpern [54] discuss these and other "allegedly harmful effects on conglomeration" taking a different stand in the debate when concluding that "there is remarkably little evidence that conglomerate mergers do very much harm or very much good for that matter" (p. 165).

## 2.2 Attainment of Real Economies: The Synergistic Effect

Independent of the circumstantial reasons that may lead to the merging of two firms, the presence of real economies will give to this decision a seal of social respectability; because, from society's point of view, the more efficient utilization of resources of the two firms combined will generate a more desirable situation. Economies of scale and complementarities in processes like production, distribution, marketing, administration, and financing, may be sources for the attainment of real economies.

Horizontal and vertical mergers may be explained, in part, as a way of getting scale economies and complementarity of operation, but two reasons stand against this conclusion. First, the present market share of large firms may sustain many plants of optimal scale (see the seminal

article by Bain [4] and his book [5]); and second, when two independent plants are brought under common control, only marginal savings of 1/4 percent to 1 percent may be realized, because the design characteristics of plants in operation are hard to modify. "All of this is very negative, and rightly so, for there simply does not appear to be much opportunity to realize plant scale economies through merger, unless an interaction effect with monopoly elements exists" (Sherer [84], p. 117). Some evidence corroborating this conclusion has been added by many empirical studies that, when looking at the economy from a macro perspective, have found a production technology with constant returns to scale.

Other forms of scale economies have been recently commented upon, in relation with the advantages of big over small business and the impact that this asymmetry may have on competition (lobbying, litigation, access to capital market, and research). Archer and Faerber [3] have found a significant negative relation between the cost of raising external common stock capital and the size of the firm. (This relation is also obtained when using size of the issue instead of size of the firm, because both variables show a high correlation of .87). Jackson [44] finds little support for the defense of mergers based on these kinds of efficiencies, because the welfare loss by the increase in monopoly power is likely to exceed the gain in efficiency.

Aside from scale considerations, complementarities in the production and distribution process are more likely to generate real economies. This assertion is suggested in a study conducted by Rumelt [80, pp. 150-151], in which he concludes that:

... the dominant-constrained and related-constrained groups were unquestionably the best overall performers, and both strategies are based upon the concept of controlled diversity. Neither totally dependent upon a single business nor true multi-industry firms, these companies have strategies of entering only those businesses that build on, draw strength from, and enlarge some central strength or competence. While such firms frequently develop new products and enter new businesses, they are loath to invest in areas that are unfamiliar to management.

This statement is pointing at the superior performance attained by firms that diversify in fields whose production, distribution, or marketing activities are related with the main line of business.

Other forms of complementarity of resources usually mentioned in relation to mergers are the following:

i) The use of managerial skills or advanced managerial technology that a large corporation can make available to a small firm. Steiner [87, pp. 186-187], citing a Brian Hindley study [37, pp. 185-221], advances the conclusion that "an index of the ineffectiveness of the incumbent managements was very much higher for the targets than for the controls, and thus provided the support for the hypothesis that mergers were transactions in the market for corporate control". This conclusion is reinforced with the FTC report on conglomerate mergers performance [26, p. 29], which indicates that the median profit of the acquired firm was 76.5 percent of the corresponding industry profit rate (profit is defined as net income after taxes expressed as percent of stockholders' equity). Mandelker [56, p. 681] also suggests that "our results for the acquired firms are consistent with the hypothesis that mergers are a mechanism by which the market system replaces incompetent management".

ii) The access to sophisticated R&D, patent rights, or other exclusive resource of the acquired firm that becomes accessible to the acquirer through the merger. This scarce resource gains in economic value, because its potential commercialization can be fully realized with the capital and general support that the acquirer makes available to the acquired firm (Sherer [84], p. 118). The validity of this hypothesis is strongly suggested by Mandelker [56, p. 685], when stating that:

There is no evidence to indicate that the acquiring firms overpay and thus lose from mergers, as some studies have previously concluded. However, the stockholders of acquired firms earn abnormal gains from mergers. That is, most of the gains from mergers go to the stockholders of the acquired firms. This result may imply that these stockholders are operating in a market in which they have some unique resources whose potential gains are realized at the time of the merger. Our results are consistent with economic gains associated with mergers and with economic rent for the acquired firms.

Another piece of evidence is given in the FTC study [26, p. 37], when reporting that only 7 out of 99 R&D groups of the acquired companies were affected by the merger. This pattern may be indicating the willingness to make use of the exclusive knowledge, skills, and resources held by the acquired company.

iii) The open flexibility to utilize slack resources of one firm in the other's management or operation. Typical cases are cash transfers between firms (from a cash-rich to a cash-hungry operation), and a more thoroughly utilized debt capacity of the acquired firm. If these transfers in fact generate an increased productivity of the resources, or if they are only redistributational arrangements is something that may be debatable. None-

theless, it is always possible to think of slack resources in one firm being used in the other. Alternative examples are management transfers, and joint utilization of some production, distribution, or marketing facilities.

iv) The obsolescence of the acquiring firm, and the need for a powerful incentive to modernize its operation has been indicated by Levinson [50, p. 66] as a plausible reason for merger when stating:

Inevitable, organizations, like aging people, become more stereotyped in their ways, less adaptable to changing conditions, and less flexible in their efforts to cope with their environments. In a word, they become obsolescent. The executives, too, become obsolescent. One way of obtaining enterprising new blood, they decide, is to buy an enterprising organization.

An implication of this comment is that if a firm has decided to maintain its main line of business, and this business reaches its maturity stage, the firm may become obsolescent even without noticing it. All things may be running smoothly until an aggressive new competitor comes in, the product sold no longer has the favor of the consumer, or any other unforeseen event occurs that catches the firm by surprise. Unless the firm maintains a degree of alertness, it may be thrown out of business, or its market position been seriously damaged. Levinson is suggesting that one way of regaining the enterprising spirit is by merging with a new organization that has it. It may be added that one way of avoiding the organization obsolescence, is to keep a changing atmosphere by adding new lines of business.

### 2.3 Other Economic Motives

Other incentives that two firms may have to merge are the realization of private economies which are more related with an income redistribution, than with the more efficient utilization of resources. Some examples may serve to clarify this point.

i) The hallucination of the capital market with the acquisitive aggressiveness shown by the group called "go-go conglomerates", may have served the purpose of over valuing the expectations of a merge, and increasing the amplitude of the merger wave in the late 1960's. Lynch's [55] core argument in his thesis is the existence of a "feedback relationship" affecting the market valuation due to the immediate impact that a merge may have on earnings per share. This is an argument hard to support in the light of current developments in corporate finance, though it may have played a significant role in the late 1960's. Myers [69, pp. 638-640] calls this result the bootstrapping effect.

ii) Changes in the capital structure of a poorly levered firm may produce an extra revenue stemming from the tax shield of interest payments. In the often mentioned FTC study [26, p. 40], borrowing practices are reported to have changed in 70 out of 99 acquired companies, and partly changed in 4 more. Whenever it was advantageous to borrow from headquarters (mainly for small related acquisitions), this was done.

iii) The increase in size of a firm may allow the attainment of larger quantity discount in input prices, with the consequent cost reduction (which may or may not be partly passed to consumers as a price reduction).

iv) In some cases, the present structure of tax laws can generate powerful incentives to merge. This is usually the case of small firms, closely held by a family group, lightly traded in the market, lacking the required managerial expertise to continue operating it, or simply short of interest to do so. For a capitalized small firm the merger route is more convenient than the dividend route to get the money out, because of the different tax treatment given to capital gains. The application of inheritance laws can give similar incentives to merger (Sherer [84], p. 115).

v) The carry over provision in the corporate tax law, can also provide an incentive to merge under determined circumstances. This provision allows future tax credits on losses incurred during the current accounting period. If the cash flow of a firm can not absorb these credits, they will be irretrievably lost. One way out of this problem is for the firm to merge with another firm showing sufficient slack in its cash flow as to make use of the tax credit.

Textron is often cited as the classic example of use of tax-loss carryovers to finance a great diversification. Between 1952 and 1959 it paid only \$634,000 in corporate income taxes despite an aggregate net income of nearly \$55 million: an effective rate of 1.2 percent. During 1960 and 1961 this rose to a rate of 21 percent, still well below the corporate average.

(Steiner [87], p. 80).

In the very controversial acquisition of Carborundum Co. because of the high premium paid, Kennecot cited a tax deduction as a benefit obtained from the merger that might otherwise have been lost or deferred (The New York Times, Friday, December 9, 1977, p. D11).

vi) Merging with a competitor can produce important savings by the

elimination of self cancelling promotional efforts (in addition to the extra monopoly power that may be gained), but this is a controversial issue.

vii) The public release of accounting reports by a firm has an impact in the capital market, because that information affects the expectations that security holders have with regard to the firm's value. On the other hand, it is a well known fact that firms do have certain leeway for presenting their financial statements. Some accounting practices can generate, at a first glance a more optimistic outlook than the real situation, and produce wrong expectations in the market (at least in the short run). Steiner [87, pp. 96-127] indicates that the accounting practices prevailing in the late 1960's may have fostered the merger wave at that time. See also the testimonies of Mueller, Turner, and Weinberger before the U.S. Congress [91], strongly recommending the modification of existing practices in financial reports.

#### 2.4 Diversification of Risk

A broad spectrum of business will generate a smooth financial operation of the firm, because random fluctuations of cash streams will tend to cancel each other. A diversified portfolio of activities will reduce the overall risk, will make the company less vulnerable to a downturn in one business sector, and will bring added flexibility to mobilize resources among different units of the firm. As Forbes [27, p. 63] puts it: "And that is the continuing appeal of conglomeration: the comforting countercyclicality of a more broadly diversified earning base, and the increased opportunity to shift capital to where the return is".

The stability thus obtained has clear benefits to the managerial team, but the benefits derived by stockholders are not clear, because they can get the same diversification effects by transactions in the capital market. Therefore, financial theory in its current form predicts that diversification per se has a zero market value (see for example Levy and Sarnat [51]).

The fact is that firms do diversify, and that the degree of diversification has been growing (see FTC [26, pp. 63-65]). Berry [8] makes an empirical analysis to determine if large corporations are increasing its degree of diversification, and if the merger activity is affecting (or being affected by) the rate of growth. He detects an increase in diversification between 1960 and 1965, but the relative sizes of the new business lines are small compared with the traditional activity of the firm. The other observation made by Berry is that firms tend to diversify within the 2-digit industry group, thus maintaining some sort of relation with the principal activity. (He suggests that "that kind of diversification is only one small step removed from consolidation of market through horizontal acquisition".)

Spreading the risk of the firm is a contended reason for diversification, as is illustrated in the Kraar [48, pp. 187-188] report in Fortune on the General Electric-Utah International merger:

Littlefield [Utah's chairman], his wife, and their children held shares worth about \$50 million... He belongs to one of 'the families' - the innumerable descendents of Utah International's founders - who still owned 40 percent of the stock... While Utah's mineral interests were highly varied, its disproportionately large investment in Australia greatly concentrated the risks... 90 percent of its earnings came from a single commodity in a single country, metal-

lurgical coal from Australia... Littlefield urgently wanted to diversify... For several years, Littlefield explained to the G.E. Chairman, he had hoped to diversify by acquisition. But Utah's lean management team, superb as it was at mining, knew practically nothing about other industries.

It is apparent from this case that if stockholders do not hold a diversified portfolio, they will benefit from diversification by the firm. This is likely to be the case of managers, who normally own or control a sizeable amount of shares on their own companies (see Table 3).

Lynch [55, p. 79] reports for his sample of acquisitive conglomerates that directors owned an average of 11 percent of the outstanding shares, and chief executives an additional 4 percent. For chief executives the market value of shares represented 17.5 years of average salary.

If trading of these important fractions of shares is not fluid in the market, this will leave the most important decision makers of the firm with an unbalanced portfolio, thus stimulating internal diversification.

Some reasons that may stand against free trading of these shares are:

- Loss of control in a family business (Utah's case).
- Institutional regulations by S.E.C. that forbid the use of internal information. (Rockwell's chairman sold his shares before the decision that turned down the B-1 bomber, and he has been accused under these provisions.)
- The negative signal to the market when a big piece of stock is sold will usually render a lower average price than the current price of shares (Scholes [81]).

Another reason that will prevent stockholders in general from having a perfectly diversified portfolio is the cost of transactions. They may

TABLE 3: Shares Controlled by Chief Executives of Large Corporations

	Average shares owned/controlled by value (1970)	<u>Shares owned/controlled</u> Average salary per year*
Big 10 industrials	16,218,000	48.0
Big 10 commercial banks	2,075,000	10.1
Big 10 retailing co.	1,442,000	6.1
Big 10 transportation co.	188,000	1.1
Big 10 utilities	104,000	0.6
Big 10 conglomerates	5,639,000	18.6

\* Shares owned/controlled expressed as years of average salary.

(Source: Reid [78], p. 53).

provide an incentive for firms to diversify internally in order to eliminate these costs to stockholders.

Other lines of reasoning to justify internal diversification are the increased likelihood of future survivability, and the added flexibility in conducting the firm. Survivability is a primary goal for managers and people working in the firm, though it is not given any extra value in the stock price according to financial theory (unless bankruptcy costs are introduced).

With regard to flexibility in the firm conduction, additional benefits are derived when having a diversity of business with different timing in their cash inflows and outflows, because cash may be transferred from a cash-generating to a cash-hungry division, without recurring to the capital market, which may become very expensive and restrictive (brokerage fees, interest expenses, capital shortages). The company has created a sort of internal capital market operation, with no transaction costs, and with full control over it. This is in itself an approach to strategic planning of business firms, which the interested reader may further explore in Hax and Majluf [35].

#### 2.5 Increase the Size of the Business. The empire building motive.

Managers' objectives do not necessarily coincide with shareholders' objectives, and instead of maximizing shareholders' wealth (equivalent to maximizing profits in the theory of the firm), managers may be pursuing other goals. A lot of theoretical and empirical research has been done in this area, and it is not the purpose of this paper to review that work.

One alternative to profit maximization that has some popularity is the assumption that firms maximize sales subject to a minimum profit constraint (Baumol [6]). This hypothesis corresponds to size maximization, when size is defined as total sales revenues, as it is usually done by business magazines (Fortune, Forbes, for example). As support of this tentative hypothesis, it has been found that managers' compensation is more related to total sales than to total profit, though the evidence is somewhat clouded by the difficulty in getting a stable profit measure. Williamson's work [101, 102, 103] gives credibility to sales maximization as a plausible objective in periods of favorable economic situations, as a way to create a managerial slack that may be disposed of under more stringent conditions. He assumes that managers maximize a personal utility function in which profit and slack variables are present.

A mechanism for continuously maintaining the stockholders' interests within the managers' perspective, is compensating managers with stock options, for making sure that they favor themselves when trying to favor stockholders in general. Nonetheless, managers may pursue an aggressive pattern of growth for getting size related advantages like quantity discounts, and greater negotiation capabilities to confront the government, suppliers, competitors, and others. Also, personal reasons as prestige, other forms of pecuniary compensation, ambition, power, showing own capabilities, and other self-fulfillment needs may justify the managers looking for size rather than for profit maximization.

Mergers are a way for substantially increasing the size of the firm in a short time span, and may be an attractive strategic alternative for accomplishing the growth target of the firm. Moreover, the initial outlay

required for acquiring an ongoing concern may be lower than for raising a new business; and, certainly, merging is faster than building from scratch.

The most acid critic of the managerial motivations behind the merger activity has been Reid [74]. He presents some data for the 500 largest industrial firms in 1961 and concludes that sales growth goes up with the increase in merger activity, but profit growth goes down dramatically. On the other hand, Weston and Mansingka [98] obtain a slightly superior profit performance (measured as return on equity) of conglomerates over a control group, thus contradicting Reid's claims. Some controversy developed around this issue, with Reid's answer [76] and later interchanges (Weston and Mansinghka [99], and Reid [77]; Conn [15], and Weston and Mansinghka [100]. Reid [78] has recently sustained his original position in a new book.). The controversy is not settled, and Steiner [87, p. 195] has argued that the two studies may have provided "complementary rather than conflicting evidence", if assuming a multiple attribute objective function.

A more neutral view of the impact of conglomeration is obtained from a Forbes (January 1976) report indicating that the median return of equity, as well as the median sales growth for conglomerates, are identical to the all industry medians (11.6 percent and 10.9 percent, respectively for the five year period 1971-1975). This suggests that conglomerates behave like a well diversified portfolio of the industries, being neither better nor worse than the average. A similar finding is reported by Melicher and Rush [59], in a comparison of conglomerate and non-conglomerate firms operating in the same basic industries for the 1965-1971 period. Though financial strategies, and risk-levels are different, operating profit is

fully comparable between the two groups.

A prudent conclusion from this contradictory set of evidence is to require a more careful look at the profitability of conglomerates.

## 2.6 Promotional and Speculative Profits

Brokerage fees and the compensation going to the promoters of a merger have been reported to be substantial in the first merger save. In the consolidation of United States Steel Corporation,

... approximately 150 million of the stock of the corporation, nearly one-seventh of the total, was issued, directly or indirectly, to promoters and underwriters. The American Can Co. was formed with an authorized capitalization of \$82 million, and of this amount it is estimated that promoters and underwriters received \$17 million or approximately one fifth. (Lynch [55], p. 23).

It is suggested also by Markham [57, p. 163] that in the second merger wave, as well as in the first one, promotional profits played a fundamental role in the rise and failure of mergers. The crude maneuvers used by that time have been forbidden with the passage of the Securities Act of 1933, and the Securities Exchange Act of 1934, which regulate the disclosure of all information related to a new securities issuing, including the promoters' remuneration.

New and more sophisticated ways of reaping speculative profits can always be invented, and the P-E ratio game of the late 1960's may be one of them. Nevertheless, the feeling is today that people learn, and that promotional profits should not play a major role in the future, at least for explaining a long-run, more stable merger activity.

At present, promoters are acting as arbitrators in merger episodes.

They buy shares at a relatively low price, and have a profit or loss depending on how the merger consolidation develops. Vartan reports in the New York Times (September 20, 1977) [96]

... the biggest pot at the end of the rainhow saw arbitragers recently turn a profit estimated at \$30 million to \$40 million in the bidding duel for Babcock and Wilcox [a successful merger]... computed on the basis of yesterday's closing price for Gerber, potential losses for the arbitrage fraternity could amount to as much as \$10 million to \$15 million, according to several professional arbitragers [unsuccessful merger].

In the same edition of The New York Times (September 20, 1977), Cole [14] comments on the withdrawal of Anderson Clayton tender offer for Gerber, and the immediate deep drop in Gerber's stock price from 34 3/8 to 28 1/4. The Wall Street community, holding "an estimated 500,000 Gerber shares on which they expected to make a profit", filed a suit against Gerber looking for damages compensation. The stockholders charged that Gerber officers were "motivated by interests in preserving their own offices and emoluments, and engaged in 'deceptive, fraudulent, and manipulative' acts in opposing the Anderson Clayton take over."

This episode seems to indicate that brokers can exercise some leverage for successfully completing a merge once it has been announced, because they represent an important group of stockholders at the time of negotiating the merger. A more subtle question is if they can ignite a tender offer for a company after buying a sizeable fraction of its stock at depressed prices.

### CHAPTER 3: MODELS OF MERGER. SOME THEORETICAL AND EMPIRICAL WORK REVIEWED

Many attempts have been made to capture the merger phenomenon in a simple model. A representative sample of the theoretical and empirical work pursuing this end will be reviewed here. Some of these studies have been partly presented already when discussing motives for merger, but they are now repeated within the context of this exposition.

The main objective in this chapter is to uncover the fundamental lines of thought guiding the research on the merger subject, evaluate existing studies under that perspective, and determine in which way they may be complemented or modified to make them more responsive to current thinking in the area.

To accomplish this end, the merger studies will be loosely organized in the following classes:

- empirical determination of mergers profitability;
- mergers as a result of managerial decisions;
- financial explanations for conglomerates;
- studies on merger waves.

#### 3.1 Empirical Determination of Mergers Profitability

The main body of research in mergers is aimed at determining their profitability, either to prove or disprove the existence of net gains, to understand the causes behind that result, and to evaluate the social desirability of an unrestricted merger policy.

Different research lines have been pursued in the profitability studies of the after World War II merger wave, some with more intensity than others.

The basic design utilized in all of them considers the selection of the following components:

- a study period;
- a sample of all mergers in that period;
- different profitability measures; and
- a benchmark to compare these profitability measures against it.

The profitability measures have been selected either from the firm's financial statements, or the capital market parameters. With regard to the benchmark selection, there are many different criteria utilized, but they can be broadly defined as:

- merging firms before vs. after the merger (Segall [82]);
- mergers vs. similar sample of non-merging firms (Kelly [47], Hogarty [41], Gort and Hogarty [32], Weston and Mansinghka [98], Lev and Mandelker [49], Melicher and Rush [59][60]);
- market value of mergers vs. forecasted market value of participants (Shick [83], Halpern [34], Mandelker [56], Ellert [18]).

These works are now reviewed, highlighting those ideas that seem to be the most relevant in them.

### 3.1.1 Merging firms before vs. after the merger

#### The Segall Study [82]

Segall discusses three popular hypotheses for merging: "firms merge to achieve monopoly (or oligopoly) positions, to grow to an optimum size or to reduce risk through diversification" (pp. 17-18). But, he argues, the first two are not sufficient to explain interindustry mergers, and diversification may not be a desirable goal to stockholders, because

"some may prefer the possibility of a very large gain even at the risk of exposure to a very large loss", and also because "stockholders can get all the diversification they want at a relatively low cost through adjustment of their personal portfolios without the intervention of the firm" (p. 18).

Segall elaborates also on other reasons for merging like the notion of idle or excess capacity ("merging may be an effective way of obtaining output from otherwise idle or poorly utilized resources" [p. 18]), the possibility of bargains ("stockholders may benefit because their managers may discover undervalued or perhaps poorly managed firms which may be acquired at bargain prices" [p. 19]), and the exhaustion of opportunities for internal investment (the "firm tries to acquire other firms which do have investment opportunities" [p. 19]).

From examining all these reasons against existing evidence, Segall concludes that "there is no single hypothesis which is both plausible and general and which shows promise of explaining the current merger movement. It may be that there are as many causes of mergers as there are mergers" (p. 19).

Segall measures the profitability of mergers by using a sample of 58 mergers consummated during the 1950 through 1959 period, and such that the assets of the acquired firm were at least ten percent of the assets of the acquiring firm. The index used to measure profitability "is the rate of discount that equates the present value of cash dividends plus the price of the stock at the end of the period in question to the price of the stock at the beginning of the period" (p. 24).

By comparing profitability in different time periods before and after the merging date, Segall concludes that "a policy of merging does not yield substantial profits for the acquiring firms".

### 3.1.2 Mergers vs. a similar sample of non-merging firms

The main problem with Segall's study is that the impact of mergers is confounded with any environmental or internal changes occurring after the merging date. In the group of studies to be exposed, this condition is controlled by choosing as benchmark a sample of non-merging firms similar to the merging ones.

#### The Kelly Study [47] Paired comparison of large merging vs. non-merging firms.

In order to uncover the relative advantages of merger, if any, Kelly makes a paired comparison of 20 "merging", and 20 "non-merging" firms selected from the 500 largest industrial firms and the 50 largest merchandising firms as ranked by sales volume in the period of 1946-1960. A merging firm is defined as one with over 20 percent increase in sales due to merging activities. A non-merging firm has less than 5 percent increase.

The parameters chosen to perform the paired comparisons are:

- market price
- price-earning ratio (as a percentage of Standard and Poor price-earning ratio for 425 industrial corporations)
- earnings per share
- rate of return
- capital turnover
- profit margin.

The conclusions obtained may be summarized as follows:

- (1) The form of investment, external expansions (via mergers) versus internal expansion, does not have a significant impact on profitability, whether judged in terms of market valuation or rate of return.
- (2) Merger-acquired earnings are accorded higher price-earnings ratios by the market than the earnings of nonmerging companies [not a clean conclusion because of sampling problems].
- (3) Merging companies are superior in improving capital turnover.
- (4) The form of expansion does not have a significant impact on the profit margin. That is, there is no evidence of increased economies or diseconomies in the route of growth via mergers. This, however, may not be true for conglomerate merger companies. ([47], pp. 4636-4637).

These conclusions should be tempered by the fact that internal data, based on accounting procedures of firms, are not fully consistent and comparable, and it is hard to get reliable information beyond the published financial statements. Also, Kelly's pairing procedure is somewhat subjective and liable to criticism. Finally, in the analysis of market prices, he doesn't make corrections for different dividend policies being followed.

The Hogarty Study [41] Merging firms vs. corresponding industry average.

The two most immediate objectives in this study are: to determine if acquiring corporations are more or less profitable than non-merging firms, and if there is synergy in mergers.

Hogarty examines the performance of 43 active acquirers drawn from the 1965 edition of Moody's Industrial Manual, and compares it with the average observed for the industry to which the acquirer belongs. Firms included in the sample "have experienced at least a 20 percent growth in

sales and assets due to merger, must have made acquisitions of publicly held firms, and must have completed most of its acquisitions by 1962" ([42], p. 4647), The sample contained firms spread over many different industries and assets from \$2 to \$700 million.

To measure the "actual increase in the wealth of the shareholders", he defines an "investment performance index" including capital gains and cash dividends paid. Comparing these indices with the corresponding average for the 3-digit SIC industry, he concludes that

... only 14 of the 43 acquiring firms had an investment performance superior to that of their respective industries. Clearly, active acquirers are less profitable than ordinary firms, at least in the long run. This result implies that stockholders do not generally benefit from active acquisition programs; in fact, relative to similar opportunities, they lose on the average.  
([42], pp. 4648-4649).

To determine the presence of synergy in a merger formation, Hogarthy compares the profit of the combined firms with the aggregated profits separately forecasted for each of the firms entering the combination. He assumes that profit growth for the individual firms follows the average growth in its particular industry. Profit is measured as net income before taxes.

With this procedure one could say that if actual postmerger profits exceeded predicted postmerger profits, then the combined firm had attained synergy. Similarly, if predicted profits exceeded actual profits, then the merger (series of acquisitions) was unsuccessful... only 20 of the 43 firms obtained synergy from their acquisition programs. This sort of result is hardly indicative of widespread opportunity for synergy. Since these 43 firms were, prior to merger, healthy, typical representatives of their respective industries, it seems fair to conclude that synergy through merger is beyond the reach of the ordinary industrial firm.

([42], pp. 4649-4650).

A similar conclusion is obtained when synergy is measured in terms of sales increases versus expected sales increases.

The Gort and Hogarty Study [32] Distribution of gain and losses between buyers and sellers.

In an extension of the Hogarty study recently discussed, Gort and Hogarty "examine the distribution of gain and losses from merger between buyers and sellers".

They conclude that:

- (1) Mergers, on the average, have an approximately neutral effect on the aggregate worth of firms that participate in them;
  - (2) the owners of acquiring firms lose on the average;  
and
  - (3) the owners of acquired firms gain on the average.
- ([32], p. 175).

The key variable explaining this distribution of costs and benefits is the "premium over the market price that sellers received in the terms of the merger" ([32], p. 177).

From examining these results, Gort and Hogarty suggest three plausible reasons to explain the merger activity. First, mergers are a form of high risk investment that can yield a substantial premium for the buyer. Second, discrepancies in valuation may exist between buyers and sellers, with buyers having the more optimistic expectations. And third, the separation of ownership and managerial control, allows managers to take decisions not always directed toward improving the shareholders' well being.

The Weston and Mansinghka Study [98] Conglomerates vs.  
non-conglomerates.

The main objective of this study is to determine the performance of conglomerate firms. The period covered by the study is from 1958 to 1968. A firm is defined as conglomerate if it satisfies two requirements; first, 20 percent or more of its increase in assets during 1960-1968 is accounted for by external acquisitions, and second, it is broadly diversified (it participates in either 10 or more 3-digit SIC industry categories or 5 or more 2-digit SIC categories).

The performance of conglomerates is compared with two control samples randomly selected from the Fortune 500 Industrial Companies list, and the combined directory of large corporations (500 industrial, 250 non-industrial, published in Fortune 1968) respectively.

They compare growth, earnings, and economic performance of conglomerates and control firms, concluding that:

- (1) The conglomerate firms out-performed samples of other firms or broader groups of firms on all of the growth measures.
- (2) The earnings performance measured by the ratio of net income to net worth is somewhat higher for conglomerate firms, but the difference is not statistically significant. This somewhat higher return on net worth did not result from the differentially higher price/earnings ratio of conglomerate firms in mergers, because on average price/earnings ratios of conglomerate firms were not significantly different from the price/earnings ratio of other samples of firms. The higher return on net worth of the conglomerate firms resulted from the larger and increasing percentage of leverage employed by them during the decade of the 1960's.
- (3) The earnings rates of the conglomerate firms in the late 1950's or the early 1960's were significantly lower than the earnings on total assets or net worth-plus-long term debt for other groups of firms. However, by 1968 no significant differences in earnings performance were

observed. It appears, therefore, that an important economic function of conglomerate firms has been the raising of the profitability of firms with depressed earnings to the average of industry generally... Therefore, the most appropriate test of the earnings performance of conglomerate firms is not superior earnings performance, but whether they were able to achieve at least average earnings performance.

The distinctive conclusion from this study is the suggestion that conglomerate may provide a viable alternative to raise the poorly performing firms to the industry average.

The Lev and Mandelker Study [49] Pairing merging and non-merging firms in the same 4-digit SIC-industry.

Lev and Mandelker try to determine the profit performance of firms engaged in mergers exceeding 10 percent of its size in the period 1952-1963. A sample of 69 acquiring firms has been selected, and each of them has been paired with a firm of similar size in the same 4-digit SIC industry.

Several profitability measures are used for the 5 years prior to the merge and the 5 years following it. They show that acquiring firms were somewhat more profitable than non-merging firms, but the extent of this difference is subject to considerable uncertainty.

The examination of other alledged consequences of mergers is also negative:

... within the limits of our sample size we cannot point to any clear direction of effect of merger on riskiness of the acquiring firm, on its growth rate in the postmerger years, or on the financial structure, percentage of income taxes paid, and liquidity position of the acquiring firms  
([49], p. 102).

This direct pairing technique has the difficulty of being virtually impossible to find a truly matching pair by controlling all variables

except merger activity. For this reason, the results may be distorted by the influence of unnoticed hidden factors.

The Melicher and Rush Studies [59], [60] Conglomerates vs. non-conglomerates.

In these papers, the performance of conglomerates is examined under a wide array of indices related with profitability, leverage, and market characteristics.

A sample of 45 conglomerate firms for which data are available, is selected from the Fortune's list of the 1000 largest industrial firms during 1971. A conglomerate is defined as a broadly diversified firm with a large percentage of growth attained by acquisition.

A comparable sample of 45 non-conglomerate firms is also selected, matching the main field of activity of conglomerates in 1960 and 1971.

By comparing the performance of conglomerate and non-conglomerate samples in the 1965-1971 period, Melicher and Rush conclude that:

- (1) While the conglomerate firms achieved a level of performance comparable to the considered non-conglomerate firms, their performance was not at all outstanding.
- (2) The two groups of firms were found to be highly comparable in terms of their operating profitability characteristics. [However, conglomerates are more highly levered, and have a lower price-earnings ratio.]
- (3) The analysis of market performance characteristics also indicates substantial comparability between the two groups of companies.

The basic conclusion is that "conglomerates were shown to be no better or worse off than those firms that remained in the basic industries that the conglomerates abandoned".

### 3.1.3 Market value of merger vs. forecasted market value of participants

An implicit assumption is being made when comparing the performance of mergers or conglomerates against a group of firms that has not engaged in diversification or acquisition plans. The assumption is that if the firms linked to the merger had continued as independent companies, their performance would be identical (or at least similar) to the firms in the reference group.

The set of studies to be presented next consider the value of the firms in the capital market as an unbiased estimate of their worth. The impact of merging is determined by comparing the market value of the merger against the forecasted market values of the participant firms as independent companies. The time of comparison has to be chosen as to fully capture the reflection of the merger decision in the firms' market values. An important consideration in these studies is the inclusion of risk differences in the valuation of firms.

#### The Shick Study [83] Return to acquiring firms in the capital market.

Shick tries to determine if mergers increase the return on investment to shareholders of acquiring firms. He observes that many studies presented in the literature "yield the general conclusion that mergers do not increase the returns to shareholders" ([83], p. 496), but these studies are distorted by two factors: random fluctuations affecting the valuation of the firm at the time of the merger, and the procedure selected to forecast returns had the merger not occurred.

Shick proposes the following relation to measure the change in

returns produced by a merger:

$$\Delta R_N = \frac{P_N + \sum_{t=0}^N D_t}{P_0} - \frac{P'_N + \sum_{t=0}^N D'_t}{P_0}$$

where:

$\Delta R_N$  = change in return N years after the merger

$P_0$  = price of stock in year 0 (before the merger)

$P_N$  = price of stock in year N (after the merger)

$D_t$  = dividend in year t (after the merger)

$P'_N, D'_t$  = similar meaning to  $P_N, D_t$ , but if the merger had not occurred.

An important assumption in this model is that the capital market registers the impact of the merger after its realization. This implies that in year 0 no information related to the merger is reflected in the stock price.

The following considerations are made for evaluating this change in return:

(1) Prices at any time t may be affected by an unbiased random error:

$$P_t = \hat{P}_t \tilde{\epsilon}_t$$

where:

$P_t$  = actual price of a share

$\hat{P}_t$  = "calculated" price

$\tilde{\epsilon}_t$  = random error with expected value of 1

(2) This "calculated" price is obtained from dividends, growth expecta-

tions, and special investment opportunities according to the following three relations:

$$\hat{P} = \frac{D_0^*}{k-br_L}$$

$$D_0^* = D_0 \beta_1 \left( \frac{1+br_s}{1+br_L} \right)^{\beta_2}$$

$$\frac{1}{k-br_L} = \beta_0 (1+br_L)^{\beta_3} (1+h)^{\beta_4} \left(1 + \frac{\sigma}{A}\right)^{\beta_5} A^{\beta_6}$$

where:

$\hat{P}$  = calculated price

$D_0^*$  = estimated dividends

$D_0$  = actual dividends

$k$  = investors' required rate of return

$br_L$  = dividends growth rate

$r_L$  = return on normal investment opportunities

$r_S$  = return on special investment opportunities

$h$  = leverage

$\sigma/A$  = earnings variability

$A$  = firm size

$\beta_i$  = empirical coefficients.

By applying this proposed methodology to only four cases in the chemical industry, Shick finds two successful and two unsuccessful mergers, but he is unable to generalize his results to all the population of mergers.

The Halpern Study [34] Merger returns vs. capital market

returns. Existence and distribution of premiums in a merger.

The objective in Halpern's study is to determine the premiums obtained by the two parties involved in a merger.

The gain to a company in a merger can be estimated as the change in the total value of the equity between the date the merger was first considered (the base date) and the actual merger date; the measured premium is this gain divided by the total value of the equity at the base date. ([34], p. 554).

There are two problems to get these values; first defining a base date in which no merger related information has affected prices yet, and second, discounting changes in value due to general market influence rather than to the merger.

If the merger date is defined as time 0 and the base date as  $t^*$  months prior to it, the total gain for the merger may be obtained as:

$$\begin{aligned} \text{Total gain} &= \text{Gain to buyer} + \text{Gain to seller} \\ &= (\text{Change in market price of buyer between } t^* \\ &\quad \text{and 0}) + (\text{Change in market price of seller} \\ &\quad \text{between } t^* \text{ and 0}). \end{aligned}$$

In algebraic terms:

$$\begin{aligned} G_T &= G_B + G_S \\ G_B &= S_{B,0} - S_{B,t^*} \\ &= n_B (P_{B,0} - P_{B,t^*}) \\ G_S &= S_{S,0} - S_{S,t^*} \end{aligned}$$

$$\begin{aligned} &= n_S (P_{S,0} - P_{S,t^*}) \\ &= n_S (R_S P_{B,0} - P_{S,t^*}) \end{aligned}$$

where:

$G_T$  = total gain from the merger

$G_B$  = gain to buyers

$G_S$  = gain to sellers

$S_{B,0}; S_{B,t^*}$  = market value of buyers at times 0 and  $t^*$

$S_{S,0}; S_{S,t^*}$  = market value of sellers at times 0 and  $t^*$

$n_B, n_S$  = number of shares of buyers, sellers

$P_{B,0}; P_{B,t^*}$  = price per share of buyers at times 0 and  $t^*$

$P_{S,0}; P_{S,t^*}$  = price per share of sellers at times 0 and  $t^*$

$R_S$  = exchange rate = number of shares of B exchanged for each share of S.

To determine these gains involved in a merger episode, it is necessary to estimate the price changes between 0 and  $t^*$ . But, in order to separate solely the impact of the merger, Halpern discounts price fluctuations explainable by the general movement of prices in the market or in the industry. He defines  $\hat{P}_{B,t^*}$  and  $\hat{P}_{S,t^*}$  at the base date, which are a forecast of the prices for buyers' and sellers' shares respectively, conditioned on the general economic and industrial conditions prevailing at the time of the merger (time 0).

Using the same nomenclature as before, he adjusts the gains defined as follows:

$$\hat{G}_T = \hat{G}_B + \hat{G}_S$$

$$\hat{G}_B = n_B (P_{B,0} - \hat{P}_{B,t^*})$$

$$\hat{G}_S = n_S (R_S P_{B,0} - \hat{P}_{S,t^*})$$

The premiums for buyers and sellers are defined as total gain over total equity at the base date. These premiums are:

$$\hat{\pi}_B = \frac{P_{B,0} - \hat{P}_{B,t^*}}{P_{B,t^*}}$$

$$\hat{\pi}_S = \frac{R_S P_{B,0} - \hat{P}_{S,t^*}}{P_{S,t^*}}$$

The empirical problem is now reduced to find a proper definition of  $t^*$ , and an unbiased estimate of the adjusted prices  $\hat{P}_{B,t^*}$  and  $\hat{P}_{S,t^*}$ . To get  $t^*$ , Halpern uses the "residual technique" of Fama, Fisher, Jensen, and Roll [24], and he finds that around the 7th month prior to the merger, some influence on residuals is detected, therefore  $t^*$  is chosen equal to 8, but a sensitivity analysis is done to determine the influence of this choice in the conclusions. Adjusted prices are obtained through regressions relating the relative change in prices of a given security for a period of k-months with the corresponding changes in the market and industry prices. The parameter k is chosen equal to the base date for each merger case.

Halpern finds a total gain of \$27.35 million for an average merger, distributed in \$14.73 million for the larger company and \$12.62 for the smaller one. The number of situations in which a negative gain is detected is 24 out of 77 (31%), this fraction being higher for larger firms (29

out of 77 or 38%) than for smaller ones (17 out of 77 or 22%). In any case, "the gain to both larger and smaller firms (and the total adjusted gains) appear to be significantly positive. Therefore, as measured by market price behavior, the larger company does not give away everything in a merger" ([34], p. 570).

When dividing these total gains by the size of the companies to get the premiums, large firms appear to receive a substantially smaller premium than small firms. Nonetheless, this difference is only the reflection of size in the almost even distribution of total gains generated by the merger.

Halpern's two suggestions to explain merger gains are: "first, it is possible that both larger and smaller companies have some unique factors, but the rents cannot be captured in a merger with any company", but in a merger of two companies that "fit" together (synergy). "Second, it is possible that the management of one of the companies is very poor and a merger would replace the inferior management" ([34], p. 573).

The Mandelker Study [56] Merger returns vs. capital market

returns. Existence and distribution of premiums in a merger.

As many other empirical studies, Mandelker's constitutes an attempt to determine if abnormal positive or negative gains are associated with mergers, and the way in which they are shared between the acquiring and acquired firm in case they exist. As in the Halpern study recently discussed [34], the fundamental difference in this effort is that the analysis is done from the perspective of capital market theory, thus associating to each return generated in the merger operation its corresponding risk.

Under this framework, a return is considered normal (or fair) if it is in agreement with the return in the capital market for an investment of similar risk.

A second objective in Mandelker's study is to determine if the capital market is efficient with respect to mergers; that is to say, if the information on mergers is reflected immediately in the stock prices of the merging firms.

The study includes all mergers consummated during the November 1941 to August 1962 period, for which data on price of securities were available. More extensive periods were used to estimate the relevant market variables (February 1926 to June 1968). The empirical model used is consistent with the capital-assets pricing model.

The conclusions derived are the following:

- There are economic gains associated with mergers.
- Most of the gains from mergers go to stockholders of acquired firms. They earn, on the average, a total abnormal return of 14% in the seven months prior to the merger consummation.
- Acquiring firm seems to gain normal returns from the merger. There is no indication "that the acquiring firms overpay and thus lose from mergers"; on the contrary, "there is some indication that the stockholders of the acquiring firms may be gaining somewhat from mergers" ([56], p. 685).
- The results observed are consistent with an efficient capital market, because "anticipating price movements preceding the effective date of merger exhaust all valuable information in mergers. Thus, the stock

prices of the constituent firms at the time of merger already reflect all economic gains expected from the acquisition" ([56], pp. 685-686).

The Ellert Study [18] Return of mergers challenged by Government

"This paper examines the risk and return characteristics of 205 large corporations whose merger activities were challenged by the Antitrust Division of the Department of Justice of the Federal Trade Commission over the period 1950-1972". Also, "comparisons are made of the returns realized by stockholders in companies whose merger activity was not challenged under the antitrust law" ([18], p. 715).

The objective in Ellert's study is to determine if the market recognizes some abnormal profits prior to the merger due to expected monopolistic gains. If this is the case, the successful prevention of the merger by the Government should lower the market price of the company; otherwise, the merger is supposed to be relatively benign in its anti-competitive effects.

The methodology used is the analysis of the time series of residuals from a two-factor market model, which is the same one used by Halpern [34], and Mandelker [56], and that has been suggested and applied in Fama, Fisher, Jensen, and Roll [24], and in Fama and MacBeth [23]. Under this methodology, abnormal profit (or losses) correspond to the residual return that cannot be explained by the two-factor asset pricing model. The monthly return should oscillate around 0 if no abnormal profits (or losses) are obtained, or be consistently above (or below) it otherwise. This drift out of 0 is empirically measured as the cumulative average residual, whose statistical significance can be appropriately tested.

The results obtained indicate that firms whose merger activity is challenged have an abnormal return of 23.3 percent in the 100 months (8+ years) prior to the month in which the complaint is filed. The interesting thing is that 16.6 percent of this return already has been obtained 4 years prior to the month of challenge. At the month of complaint there is a significant loss of 1.83 percent. During the period of litigation, which lasts for 34.1 months on average, there is a loss of 1.9 percent, which is not significant. After the settlement, there is a loss of 1.6 percent in four years, which is not significant either. This adjustment takes place in the 5 months after the settlement.

Ellert finds also that, after the month of complaint, there is no significant difference between the returns of firms that were ordered to divest assets, and those that were not. Before the antimerger complaint, companies ordered to divest did experience larger abnormal returns, particularly in the 4 years prior to the complaint filing.

When comparing these results with other companies involved in merger activity during the same period that were not challenged by the government, Ellert concludes that for all samples of acquiring firms (indicted under merger law, not indicted under merger law, and single acquisition firms), we "observe positive and statistically significant abnormal returns, the largest accumulations occurring at least four years before the merger... The timing of these gains suggests that the large positive residuals experienced by Section 7 defendants before the filing of antimerger complaints may reflect something other than capitalization associated with the specific mergers challenged" ([18], p. 727).

Ellert finalizes suggesting that it is

... unlikely that abnormal gains realized in this period are related to specific mergers to be conceived and negotiated at such distant future dates. The abnormal gains of the early period may simply reflect a proven capacity for operational efficiency in the management of assets which motivates merger activity without being an anti-competitive consequence of the specific mergers undertaken later.  
([18], pp. 727-728).

When looking at the returns on acquired companies, Ellert finds that there is an abnormal loss of 9.5 percent between months 100 and 49 prior to the merger (between 8+ and 4 years before the merger). This loss increases by 2.2 percent up to 11.7 percent between months 48 and 8 prior to the merger, and then it is overcompensated by a net gain of 14.6 percent in the 7 months prior to the merger. Ellert suggests that this behavior is consistent with the hypothesis that mergers are a mechanism for replacing inefficient managements in the acquired company.

The reason for reviewing this work with certain detail is because it states empirically two important facts which are consistent with the merger's rationale to be exposed later in this study. First, the acquiring firm gets an abnormal gain about four years prior to the merger realization; that is to say, there is some important internal transformation in the firm which is most likely unrelated with a specific merger episode occurring at a distant future date. And second, the acquired firm suffers an abnormal loss in the 8 years prior to the merger which is quickly compensated for in the 7 months prior to the merger realization. This suggests the existence of a potential source of value in the acquired firm, that regains market recognition with the merger completion.

Therefore, the potential source of value is in the acquired firm, and the recognition of this value is made possible by the participation of the acquiring firm.

#### 3.1.4 Summary of the Empirical Determination of Mergers Profitability

A considerable amount of research has been conducted on the impact of mergers in the firms' profitability. The final conclusion is rather mixed, but the most recent research is reporting some net gains in mergers.

The works reviewed in this section and the main conclusions reached in them are the following:

Segall [82]: Unclear impact of mergers in the profitability of the acquiring firm.

Kelly [47]: There is no significant difference between the profitability of merging and non-merging firms.

Hogarty [41]: Shareholders of acquiring firms do not generally benefit from the acquisition. There is no synergy in the merger.

Lev and Mandelker [49]: There is a neutral impact of mergers on the acquiring firm characteristics.

A different interpretation for this neutrality of mergers is given by Weston-Mansinghka [98] when stating that, in general, conglomerates raise the profitability of firms with depressed earnings to the average of industry. Therefore, the adequate test for conglomerate earnings is not the

achievement of superior, but average performance.

This theory of defensive diversification of conglomerates to get protection for poorly performing firms is reinforced in the conclusions of Melicher and Rush [59][60], who found no difference between the profitability of conglomerates and those firms that remained in the basic industries that conglomerates abandoned.

Gort and Hogarty [32] suggest that, on average, mergers have an approximately neutral effect on the aggregate worth of firms, but there is a redistribution that produces a net gain to the owners of the acquired firm, and a net loss to the owners of the acquiring firm.

The redistributational impact of mergers can be found also in Shick's work [83], who concludes that mergers may or may not be profitable.

A more recent, and more careful measure of the impact of mergers in the market value of firms has been done by Halpern [34]. He detects that, on average, there is a net creation of value that is split almost evenly between the acquired and acquiring firms. Nonetheless, by relating these gains to the size of firms, the average premium for the acquired firm is positive, and for the acquiring firm is not significantly different from zero.

Halpern's conclusions are also obtained in an independent study conducted by Mandelker [56]. He finds economic gains associated with mergers, whose distribution generates a premium for the acquired firm and a fair return for the acquiring firm. That is to say, the acquiring firm investment is producing a return similar to investments in the capital market with a similar risk level.

Ellert's [18] study is similar to Halpern's and Mandelker's, but it spans more than 8 years prior to the merger realization. Though the main concern is with firms whose merger is challenged by the Government due to antitrust considerations, Ellert establishes two important conclusions applicable to mergers in general. First, the acquiring firm does have an abnormal gain about four years prior to the merger realization, and the actual completion of the merger, or its challenge by the Government, do not affect in a substantial way this extra value. And second, the acquired firm has an abnormal loss in the 7 years prior to the merger realization, which is slightly overcompensated in a period of just seven months before the actual materialization of the merger.

Both results are consistent with Mandelker's conclusions (a fair deal to the acquiring firm, and a gain to the acquired firm are induced by the merger); but, by exploring deeper into the past, Ellert is able to find an abnormal gain for the acquiring firm which is most likely unrelated to the merger, and an abnormal loss for the acquired firm which is recouped with the merger completion.

### 3.2 Mergers as a Result of Managerial Decisions

A different line for justifying mergers is based on the separation of managers and owners of firms. The argument is that managers pursue their own objectives independent of owners' preferences. Therefore, goals like growth maximization, size maximization (sales, number of employees, assets), and managers' personal objectives, become important explanatory variables.

The works in this area are classified for the purpose of exposition in terms of:

- Empirical studies on managerial reasons for merger: Reid [74], Fahy [19].
- Managerial models for merger: Williamson [101], Mueller [68], Melnik and Pollatschek [61].

### 3.2.1 Empirical studies on managerial reasons for merger

The Reid Study [74] Managers pursue their own objectives damaging the shareholders' position.

Reid bases his analysis on two propositions made by Berle and Means [7]. In the first place, they "emphasized progressive separation of control from ownership and foresaw a growing independence of management from stockholder influence and classical market constraints. The other proposition related this growing independence to changing managerial behavior and performance" ([74], p. 132).

In more operational terms, Reid states: "The more actively that large, publicly held firms merge, the more they tend to be oriented to furthering managers' interests rather than stockholders' interests" ([74], p. 154). His hypothesis is that "the more actively firms have merged during the period, the larger their relative increases in sales, assets, and number of employees tend to be; similarly, there will be a tendency for their relative increases in market price per share and profits attributable to the original stockholders to be smaller" ([74], p. 156).

To test this hypothesis, he takes a sample of 478 of the 500 largest industrial firms in 1961, and examines their merger activity during the 1951-1961 period. Then he classifies these firms according to their

merging propensity in four groups:

48 firms that did not have any merge in the period,

214 firms with 1 to 5 mergers,

142 firms with 6 to 10 mergers, and

74 firms with 11 or more mergers.

He concludes that "managers' interest variables" (sales growth, assets growth, and number of employees growth) significantly increase during the period; while "stockholders' interest variables" (dividends and capital gains per share, growth in return on assets, growth in profit margin) significantly deteriorate in the same period.

These results make Reid conclude that "the interest of managers and stockholders in these large publicly held firms tended to be more independent or conflicting rather than complementary" (Reid [74], p. 103).

The Fahy Study [19] Controlling the impact of the managerial strategy followed by the firm.

Fahy tries to shed new light on the rather controversial studies contrasting profitability of growth by mergers vis-a-vis internal growth. He adds a new dimension to the problem by including as explanatory variables goals other than profit maximization that the firm may be following. His main caveat is that studies like Kelly's, Reid's, and Hogarty's may not be measuring the impact of mergers on profitability, because the profit level is dependent on the strategy being followed by the firm. Therefore, if most of the growth maximizing firms follow a merging strategy, and the profit maximizing ones do not merge, any difference in profitability detected between them can be explained as coming from the

merging activity or from the objective adopted. The two factors are being confounded in the empirical design.

Consequently with this analysis, Fahey tries to single out the impact of merger activity in the profit performance of a firm by controlling two variables: goals being pursued, and industry to which the firm belongs.

Two different measures of profitability are used: return on the firm's equity, and return on shares when including dividends and price changes.

Merger activity is defined as the amount paid for the merger expressed as a percentage of the firm's total increase in assets over the period 1960-1965.

A dummy variable is defined to signal if a firm is pursuing profit maximization, or if it is deviating from this objective. This variable is subjectively defined in terms of the level of competition in the industry as measured by the height of the industry's barrier to entry, and the degree of managerial control experienced by the firm. A firm is considered to deviate from profit maximization if it is managerially controlled (no external group own 10% of shares), and competition is weak in the industry.

The conclusion obtained is that no difference exists between the profitability of merger vis-a-vis internal expansion.

When testing if the managerially controlled firms are related to the degree of merger activity, Fahy does not find any discernible relationship.

Finally, some indication is found confirming that managerially controlled firms have lower levels of profit than owner controlled firms.

### 3.2.2 Managerial models for mergers

In the exposition to follow, a brief review of some ideas in

Williamson's book have been included [101]. This book is not related to mergers in a specific way, but with the process of managerial decision making in general. This process is characterized in a way that is useful for the model proposed later in this study concerning the explanation of the merger phenomenon.

The Williamson Model [101] The trade-off between profit and managerial slack.

Williamson's book rests in the hypothesis that there are many reasons to think that managers will not follow strictly a profit maximization objective. Based on reasons like the actual separation of management and control, the degree of managerial discretion that may be exercised with imperfectly competitive markets, and certain behavioral considerations, he argues that a better representation of a firm's objective is the maximization of a multi-attribute utility function (in which profit is one of the components), subject to a constraint on minimum reported profit.

Essentially two parameters are included in the utility function and the manager has to define the trade-off among them. These factors are profit and managerial slack. Managerial slack is a proxy measure to account for motivations other than profit being pursued by the managerial team. Williamson suggests three specific alternatives for this slack: staff expenses (roughly speaking, generally administrative and selling expenses), emoluments (the portion of management salaries and prerequisites that is discretionary), and discretionary profit (which is the amount by which earnings exceed a minimum performance constraint).

The value of this model is the formal introduction of the slack

concept, and the added power it has to explain the characteristics of firm's reactions to a sudden adverse change in the environment. For example, in a case study reported by Williamson of a company that was forced to engage in a "cost reduction program", after two years, without changing production, the company attained:

- (1) Return on investment: increased by 125 percent.
  - (2) Breakeven point: reduced from 95 to 74 percent.
  - (3) Total employment: decreased by 25 percent; salaried employees reduced by 32 percent, hourly by 20 percent.
  - (4) Payrolls: reduced by 16 percent or 12 million dollars.
  - (5) Overhead: reduced from 14 million dollars to 12 million dollars and scheduled to go to 8 million dollars.
  - (6) Headquarters employment: reduced from 782 to 462 (with plans to decrease it to 362).
- ([101], p. 95).

Unfortunately, this kind of reaction does not prove that the model is correct, but rather that it is an acceptable explanation. Also, if a strictly profit maximizing behavior is considered, this reaction can be predicted if expectations of the firm with regard to future investments plummet. This is in fact what happened in this case, in which it was made "evident that the condition of excess capacity that had developed in the industry would be a continuing one" ([101], p. 94).

The Mueller Model [68] The growth maximization hypothesis.

Mueller develops three lines of reasoning for explaining mergers in trying to formulate plausible causes for describing the conglomerate phenomenon of the late 1960's.

In the first place he mentions synergistic effects as an important reason for merging when managers are trying to maximize the stockholders'

welfare. Three kinds of synergy are indicated: managerial, financial, and risk reduction through diversification.

A second model is based on separation of ownership and control in the acquired firm, and on different expectations between acquirer and acquired firms with regard to future earnings.

Finally, mergers are justified by assuming growth maximization. Even without synergy, and with the same expectations on future cash flows, growth maximizers will discount cash streams at a lower rate than stockholders' wealth maximizers. Therefore, they will tend to engage in a heavy acquiring activity. Under this hypothesis, all firms are attractive buys at the stock market price, therefore,

... if the desire to grow through acquisitions is to continue to be met during a period of high merger volume, it will have to be satisfied more and more by acquisitions through the stock takeover route. This is precisely what has happened. Between 1962 and 1965 the ratio of stock takeovers to the total number of mergers more than doubled.

The other expectation derived from this model is that "the largest firms in the economy will also be the most mature and, therefore, are likely to have the lowest internal rates of return". Consequently,

... these will be the firms which have to make the greatest resort to outside investment opportunities to achieve growth. Hence John McGowan's finding that the proportion of a firm's growth which stems from mergers is positively related to the size of the firm seems to be consistent with the growth maximization hypothesis.  
([68], pp. 658-659).

A general comment on Mueller's paper is that the model formulation is rather tentative and crude, and the empirical substantiation of the growth hypothesis is rather inconclusive.

The Melnik and Pollatschek Model [61]. Diversification and synergistic effects in the managers' utility function.

Melnik and Pollatschek propose a financial model to explain conglomerate mergers based on synergistic and diversification motives. The model is based on a utility function that managers are trying to maximize, which is increasing with the fraction of shares of another firm controlled by the acquirer (the diversification effect), and it has a jump at a level of shares that allows the full control of the firm (the synergistic effect stemming from increased debt capacity).

The managers' decision problem is to maximize this utility subject to a budget constraint. The fact that conglomerates exist make the authors advance the hypothesis that controlling another firm is an important objective; therefore, the synergistic effects tend to be more important than the diversification effects.

### 3.2.3 Summary of mergers as a result of managerial decisions

The wide latitude that managers have in the conduction of firms is a recognized fact. But it is not clear if they can deviate in a significant way from a profit maximizing behavior, or if there are efficient institutional mechanisms that will prevent strategies not pursuing the maximization of shareholders' wealth.

In this section, studies are reviewed that favor the thesis of a managerially controlled firm. Mergers are justified as a way of attaining managers' rather than stockholders' objectives. The general conclusion from the studies presented are the following:

Reid [74]: The degree of merger activity of a firm is posi-

tively associated with the achievement of managers' objectives, and negatively associated with the achievement of shareholders' objectives.

Fahy [19]: Some firms are managerially controlled and others are owner controlled. There are positive indications that profit is higher in owner controlled firms. The degree of merger activity is not significantly different between these two groups. No difference is detected, either, between the profitability of growth by merger vis-a-vis internal expansion, when the impact of firm control is taken out.

Williamson [101]: The managerial decision making process may be explained in terms of a utility function in which profit and the degree of managerial discretion are traded off.

Mueller [68]: Managers that pursue growth maximization are prone to engaging in merger activity.

Melnik and Pollatschek [61]: Managers engage in mergers because they maximize a utility function that grows with assets diversification, and jumps up with the control of another firm.

### 3.3 Financial Explanations for Mergers and Conglomerates

Generally speaking, the reasonability of conglomerates is not explainable by recurring to arguments like monopoly power acquisition, or synergy creation. There is nothing in the operation of divisions that is being changed, except for the centralization of financial activities at the head-

quarters level. The benefits derived from this form of organization are not clear, and many financial explanations have been explored for sharpening this point. This section will review some works in this area, classifying them, according to the nature of the rationale being pursued, in the following categories:

- Increasing diversification: Levy and Sarnat [51], Smith and Schreiner [85], Westerfield [97], Mason and Goudzwaard [58].
- Increasing debt capacity: Lewellen [52], Higgins and Schall [36].
- Imperfections in the capital market: Lintner [53], Lynch [55], Steiner [87].

### 3.3.1 Increasing diversification

One of the justifications most usually given to explain conglomerates is the achievement of diversification and, through it, superior performance. The works to be presented explore some theoretical and empirical aspects of the problem and cast some doubts on the validity of this argument.

The Levy and Sarnat Study [51] Diversification of conglomerates does not create any economic advantage.

Levy and Sarnat show that in a perfect capital market there is no economic advantage achieved by a purely conglomerate merger because "despite the stabilizing diversification effect, a conglomerate merger per se does not necessarily create opportunities for risk diversification over and beyond what was possible to individual (and institutional) investors prior to the merger" ([51], p. 796).

When turning to an imperfect capital market, by assuming that investors only hold a limited number of securities, the conclusion is more

pessimistic, because investors may lose from the merger due to the reduction of opportunities available for him.

True economic advantages from merging may be derived from economies in capital costs by increasing its size,

... firms have better access to the capital markets, and also enjoy significant cost savings when securing their financing needs... Diversification can be expected to create a true economic gain owing to the fact that the combination of the financial resources of the two firms making up the merger reduces lenders' risk while combining each of the individual shares of the two companies in investors' portfolios does not. ([51], p. 80).

These final considerations are only enunciated but not explored in the Levy and Sarnat paper.

The Smith and Schreiner Study [85], and the Westerfield Reply [97] Conglomerates vs. mutual funds.

Smith and Schreiner produce an ex-ante measure to determine empirically the degree of diversification achieved by conglomerates. They define as diversification index a ratio with the difference between the portfolio expected return and the risk free return in the numerator, and the standard deviation of the portfolio return in the denominator.

Comparing the diversification attained by 19 conglomerates and 8 mutual funds, it is concluded that

... with few exceptions, the mutual funds have attained more efficient diversification than the conglomerates. And they have done so at a relatively lower level of risk as measured by the standard deviation of portfolio return... Conversely, some of the conglomerates appear to have done a commendable job of selecting unrelated industries so as to take advantage of low correlations. ([97], p. 424).

The results of Smith and Schreiner have been challenged by Westerfield

who argues that their measure of diversification is confounding the effects of management as well as diversification. When correcting for this problem, Westerfield finds that "as one would expect, conglomerates are far inferior to mutual funds in effective diversification" ([97], p. 912), although "many have been successful at reducing unsystematic risk between the sample periods July 1954-June 1961 and July 1961-June 1968" ([97], p. 914).

The Mason and Goudzwaard Study [58 ] Conglomerates vs. random portfolio matching their asset diversification.

Mason and Goudzwaard did an empirical study to determine if the asset diversification of conglomerates generate any benefit to the firm or stockholders. They select a sample of 22 conglomerates for which data could be obtained, and compare their performance with an equivalent sample of portfolios. All these portfolios are constructed from randomly selected stocks of undiversified firms, in order to reproduce exactly the asset composition of each one of the conglomerates in the sample. The period covered by this study is 1962 to 1967.

The conclusion of this study is that "randomly selected portfolios offered superior earnings performance and shareholder returns than did the conglomerates in our sample ([ 58 ], p. 39). This conclusion holds "despite the fact that our hypothetical stockholder was forced to incur transaction costs, taxes, and fees associated with buying and selling stocks" ([ 58 ], p. 45).

3.3.2 Increasing debt capacity

There is no satisfactory theoretical model to explain a firm's debt capacity. One line of thought is to assume that at a debt level sufficient-

ly high, lenders deny any additional loan to the firm. This idea is explored by Lewellen to justify mergers, but his conclusions are challenged by Higgins and Schall. These two works are now reviewed.

The Lewellen Model [52] By reducing lenders' risk, mergers can increase debt capacity.

Lewellen tries to determine if mergers with no operating advantages (scale economies, monopoly power, complementarity of resources) can be justified in terms of pure financial reasons (transient errors in market valuation, unused debt capacity, diversification).

A merger is justifiable to "the extent to which corporations can achieve a result by merging that investors cannot achieve for themselves by commensurate manipulations of their personal securities portfolios" ([52], p. 522). This consideration leads Lewellen to disregard three popular reasons to explain mergers. First, transient valuation errors in the market "of the scale and frequency required to explain the level of conglomerate activity during the 1960's would connote a degree of market imperfection, or a pattern of continuing investor perversity, that most investigators nowadays would be unwilling to grant" ([52], p. 523).

Second, "to count very heavily on justifying mergers by attributing incremental benefits to the simple internalization of a process, like diversification, which is so widely practiced externally by investors, does not at this writing have the look of a singularly promising line of inquiry" ([52], p. 524).

And third, if the firm to be acquired is not using its debt capacity in full, a net gain may be obtained by merging and using this latent debt

capacity. But, there is nothing peculiar to the act of merging in the realization of these gains. It is rather "the result of a kind of management ineptitude in capital structure planning that can be undone without the necessity for the joining of two companies". It is enough "to improve their finances independently" ([52], p. 525).

Lewellen suggests that there is a net advantage exclusive to mergers arising from the lower relative variability exhibited by the combination of two cash streams that are not perfectly correlated. This advantage is the product of an increased debt capacity in a world of taxes.

He argues that lenders impose a limit on the borrowing capacity of a corporation by defining the maximum probability of defaulting on the loan. Therefore, by combining the two cash streams and maintaining constant the probability of default required by lenders, the total amount that can be borrowed by the merger goes up, because there are situations in which the cash generated by one firm can be used to compensate for a weak cash flow in the other.

The Higgins and Schall Model [36] Lenders' risk reduction by merger has two counterbalancing effects whose net impact on stockholders is not clear.

Higgins and Schall argue that Alberts [1] and Levy-Sarnat arguments for the lack of impact of a merger in the value of a firm are implicitly assuming that the firm never goes broke; that is to say, bondholders will always receive the amount they are expecting. They show that the total firm value is in fact unaffected by the merger under the assumptions of costless bankruptcy and no taxes; but, "merger reduces shareholders' wealth

and raises bondholder wealth" unless certain conditions are satisfied.

This result is originated in Lewellen's idea of co-insurance between the two merging firms that in some situations allows the use of the excess cash flow of one of the firms to solve the cash needs of the other. Therefore, without a change in leverage, the value additivity principle requires that the total value after merging remains unchanged. On the other hand, bondholders are facing now a lower risk, implying a higher market value for the same bonds. The consequence of these two conditions is that the total value of the stock has to go down.

The conclusion from here is that, in fact, there is a co-insurance effect, but that the premium is paid by shareholders for the benefit of bondholders. There is a counterbalancing effect in the extra debt capacity created (if the Lewellen theory is correct), but the net impact on shareholders will not be clear as it is suggested in the Lewellen paper.

The above conclusions are valid without bankruptcy costs, without transaction costs, and with or without taxes. When adding the impact of bankruptcy cost, the whole situation becomes dependent on the way in which this cost is affected by the merger, making the generalization of any conclusion impossible.

### 3.3.3 Imperfections in the capital market

A clear rationale for mergers under conditions of perfect capital markets has not been found. The case for diversification is yet to be made, and the increase in debt capacity may not be sufficient to compensate stockholders for other losses they incur in the merger. The works to be presented depart from the assumptions of perfect and efficient capital

markets, and build an argument on things like use of internal information, speculative gains, tax laws, accounting manipulation, and capital market expectations based on P/E.

The Lintner Model [53] By increasing the number of holders of a security, its market value goes up.

Lintner observes that market power and cost reduction opportunities are strong incentives to induce merging, even with sizeable promoter's "cuts", and that these elements played an important role in the first two merger waves. But these arguments are not useful for explaining pure conglomerates, because they are not present in that case. The focus is then "on whether the aggregate market value of firms can be raised by merging them, even when there are no gains in combined operating profits anticipated from increments of (product) market power, economies of scale, or other improvements in efficiency" ([53], p. 106).

According to Lintner, under the assumption of purely competitive equilibrium in the security market, the market value of the two firms may increase as a consequence of merger due to one of the following five reasons: (1) tax laws, (2) greater leverage, (3) risk reduction, (4) accounting manipulations, and (5) instantaneous gains in earnings per share.

For risk reduction to take place, it is assumed that all investors act in an optimal way, but with a limited amount of information, since "getting information involves economic costs, and there is an opportunity-value to the time devoted to security appraisal" ([53], p. 109). This practical restriction will stand for investors having but a few securities

in their portfolios. "And under conditions where different securities are held by different subsets of investors, the market price of risk for the  $i^{\text{th}}$  stock varies inversely with the summation of the risk-tolerances (reciprocal of risk aversion coefficients) of investors who have it in their portfolio (long or short)." ([53], p. 108). In following his argument, Lintner shows that by just increasing the number of holders of a security, its market price of risk goes down and its market value goes up. What is yet to be shown, in case this gain really exists, is its significance.

The Lynch Model [55] The "feed-back" relationship between price-earnings ratio, and the rate of growth of earnings per share and dividends per share.

Lynch focuses his research in those conglomerates that in the 1962-1967 period engaged in a rapid growth by acquisition, showed fairly diversified activities, and performed in a superior way for their shareholders. His main objective is to look for the reasons behind this superior performance, and two broad kinds of reasons are suggested: "those which depend on improvement of operations through acquisitions, and those which depend on the various financial characteristics of acquisition". The analysis of these incentives for merging make Lynch draw two main conclusions from his study; first in the category of operating economies through merger

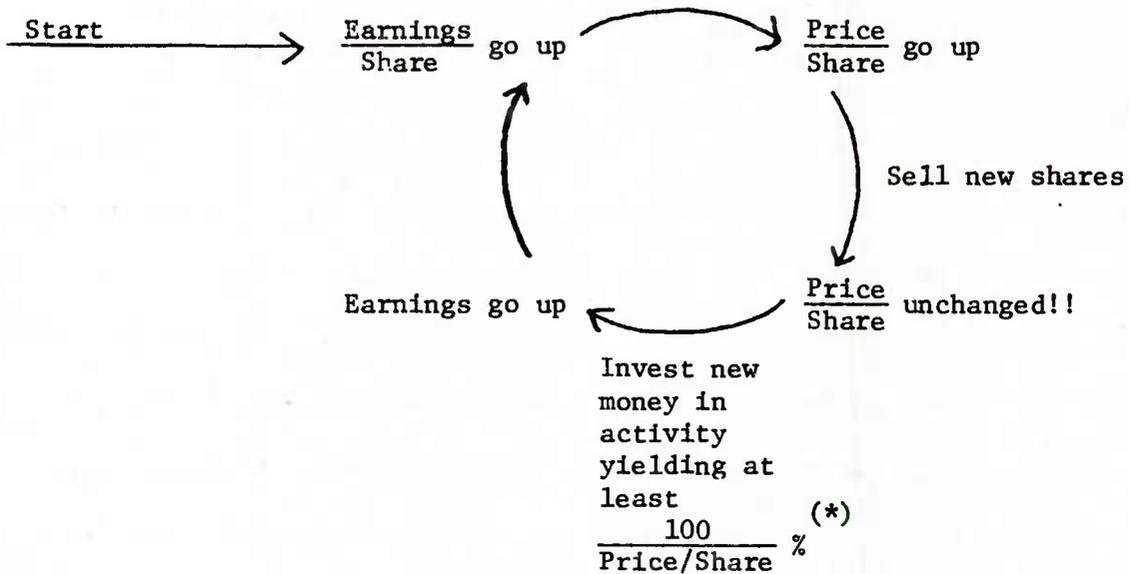
... improved performance is less likely to result from the traditionally discussed economies in the combined use of physical resources than from the more effective use of specialized human resources, specifically managerial and technical expertise. Second, within the category of financial effects of

mergers, the interdependence of market value and shareholder performance appears to have played a major role in the performance achieved by acquisitive conglomerates.

The Lynch study is, by its initial design, biased toward conglomerates showing superior financial performance in 1962-1967 as measured by market price appreciation and dividend yields. In his sample are included corporations like Litton and LTV that, in the 1970's have showed a rather poor performance, and they are at present in deep financial troubles.

Lynch explains the superior financial performance of these conglomerates by means of a "feed-back" relationship between price-earnings ratio, and the rate of growth of earnings per share and dividends per share. He suggests a mechanism of expectations formation such that when the growth in earnings per share (translated into an equivalent growth in dividends per share) goes up, the price per share also rises. This is the result of higher expectations on future firm returns. By selling new shares at this inflated price, and investing the money in activities yielding a return enough to maintain the growth in earnings per share, it is possible to validate the expectations of the firm held in the market. (See figure on next page.)

This elaborate argument can explain a continuously accelerating merger movement, but it is unable to explain the decay stage in a merger cycle. The main flaw is the assumption that all future expectations are formed based exclusively in past performance, without analyzing the real return expected from assets in general, and new investments in particular.



(\*) Note that the minimum yield goes down as price goes up.

The Steiner Model [87] Synergy, insider benefits, and premiums as inducement for merging.

Steiner's purpose is to present a framework that may encompass all plausible reasons for merger, and then determine empirically their relative importance. In his view, "a multiple-cause framework can capture the diversity of probable outcomes", and one may be able "to explain the changing role of different forces and empirically to wonder what attributes of a particular data set led one predictor to appear dominant" ([87], p.32).

The two actors in the deal are the buyer and the seller, and "the merger is assumed to occur when the benefits to each of the parties making the decisions are sufficiently large to outweigh the costs, deterrents, and inertia that may exist" ([87], p. 30). Participating parties "do what promises to advance their own best interests" ([87], p. 32).

The model to be presented is based on rational and unitary behavior

of both buyers and sellers, though later in his book Steiner expands qualitatively his model to include some internal conflict between owners and managers in each participating group ([87], Chap. 6, pp. 128-150).

The basic elements in Steiner's model may be summarized in five points:

- (1) External synergies ( $\alpha$ ), real or pecuniary, may make the stock of a combined company worth more than the sum of its premerger parts. These may embrace among many other possibilities such things as economies of large-scale production or a chance to exploit a monopoly position. Because such synergies benefit both buyers and sellers, they appear symmetrically in the relationships and a high enough synergy, other things equal, can motivate both parties to merge.
- (2) External deterrents ( $\gamma$ ), such as the transaction costs of effecting a merger or the likelihood of anti-trust prosecution, play an opposite role to synergies. They too enter symmetrically and large enough deterrents can cool the ardor of both sides of a potential merger.
- (3) "Insider" benefits ( $A_S$ ) and deterrents ( $D_S$ ) may exist to the stockholders of the selling company. Included here are all the potential sources of gain or loss to the selling company's owners that are not adequately reflected in the stock market prices of the stocks before and after merger. The ratio of benefits to deterrents may be thought of as an insider benefit/cost ratio and may be greater or less than unity according to whether the selling companies' owners see greater or lesser advantages to them in the new company than in retaining the independence of Company S.
- (4) Insider benefits ( $A_B$ ) and deterrents ( $D_B$ ) to the stockholders of the buying company also may exist. While formally similar to those of the seller, insider gains or losses need not be the same for buyers and sellers, and indeed will typically not be the same.
- (5) The premium paid by buyers to sellers to induce them to accept the tender offer ( $k$ ). The premium is the "price" that persuades reluctant sellers to agree to a merger and that thus determines the allocation of advantages accruing from the merger between the parties. In my formulation of the problem, the premium paid takes the form of giving sellers a disproportionate share of the new company by issuing more shares of stock in the new company than the sum of the shares in the buying and selling companies. Buyers pay the premium via the dilution in their holdings.

([87], pp. 42-43).

For the merger to be possible, both buyers and sellers must be benefited. The two equations stating these conditions are:

For sellers:

$$\left(\frac{\alpha}{1+\gamma}\right) \left(\frac{A_S}{D_S}\right) k \left(\frac{n_B+n_S}{n_B+kn_S}\right) > 1$$

For buyers:

$$\left(\frac{\alpha}{1+\gamma}\right) \left(\frac{A_B}{D_B}\right) \left(\frac{n_B+n_S}{n_B+kn_S}\right) > 1$$

where  $n_B$  and  $n_S$  are the number of shares of buyers and sellers, respectively, and the other parameters are defined above (price of shares for buyers and sellers in normalized and assumed = 1). \*

What these equations produce is a range of values of  $k$  that makes the merger profitable to both parties. The only decision variable in the model is  $k$  and it includes the premium that the seller is willing to offer for each share of the buyer.

### 3.3.4 Summary on financial explanations for general mergers and conglomerates

The most popular argument to justify conglomerates is the increased asset diversification. Nonetheless, the works of Levy and Sarnat [51], Smith and Shreiner [85], Westerfield [97], and Mason and Goudzwaard [58], suggest that conglomerates are less efficient than direct diversification in the capital market.

The other argument is the increased debt capacity created through merger, whose plausibility is shown in Lewellen's model [52]. But Higgins

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\* Complete derivation may be seen in Steiner book.

and Schall [36] cast some doubt on the ability of stockholders to reap a net gain from this transaction.

Finally, by considering imperfections in the capital market it is possible to find many arguments for merger. The problem is that the real extent of the incentive provided by these imperfections has not been measured empirically.

Some of the models suggested are:

Lintner [53]: Increasing the number of holders of a security raises its market value.

Lynch [55]: A high P/E leads to increased acquisition propensity.

Steiner [87]: The main elements considered by buyers and sellers in a merger are synergy, external deterrents, insider benefits and deterrents, and the premium paid.

#### 3.4 Studies on Merger Waves

One of the puzzles in the study of merger movements has been the periodical outburst of activity that generated the three merger waves at the turn of the century, the late 1920's, and the late 1960's. This section presents some papers exploring the reasons that may be behind this phenomenon. The theories presented in those papers give proper consideration to the strong positive correlation between merger activity and stock market prices, observed by Nelson in his early empirical study [73].

The Jacoby Theory [45] Merger waves as result of the conjuncture of accumulated technical and social changes, and a buoyant capital market.

In this study, Jacoby advances an hypothesis to explain the uprising of merger movements each 30 to 40 years, arguing that the usual "monopoly", "stock promotion", and "efficiency" explanations given to justify mergers, are insufficient to explain these sudden outbursts of activity.

The conjecture is made that long-term merger waves in the United States are explained by the infrequent conjuncture of two preconditions: (1) an accumulation of perceived and unexploited profit making opportunities for enlarging the scale of enterprises, arising from basic technological and social changes, and (2) a buoyant capital market with strong demand for new securities.  
([45], p. 4950).

This conjecture raises efficiency as well as stock market condition arguments, reasoning that "the predominant motives for mergers are the drive of businessmen to realize larger profits by capitalizing upon newly perceived economies of scale, and the ability of bankers to sell new securities to the public on profitable terms" ([45], p. 4951).

To explain a merger wave, Jacoby makes a distinction between tactical (small, superficial) and strategic (salient, structural) changes in the environment.

Most tactical changes cancel or offset each other through time. A few cumulate into strategic shifts in the structure of technology and society. Not only do strategic changes take many years to accomplish, but there is a time lag between their occurrence and their general perception by people. Many strategic changes create opportunities for profit by enlarging enterprises. In the pervasive optimism of a stock market boom, once overlooked opportunities, or known opportunities previously non financeable, are acted upon. Given the rapidity of communication in financial markets, such perceptions multiply and build up to a climax... Later, the pool of profit-making opportunities for business combinations is drained. Concurrently, financial expectations deteriorate. Merger activity falls of as quickly as it previously mounted.  
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The structural changes listed by Jacoby as potential stimulus of the late 1960's merger wave are: management science and computer evolution, research and development explosion, rise of the service economy, quantum leap in taxation, and doubled price of capital.

The Gort Theory [31] Economic disturbances create uncertainties that make buyers and sellers differ in their valuations.

Gort presents what he calls the "economic disturbance theory of mergers", which is based on the different expectations held by buyers and sellers with regard to the earnings stream of the firm being acquired. Discrepancies in valuations are generated by economic disturbances, the most common of them being rapid changes in the technology and movements in security prices.

The economic disturbance increases the variance in valuations mainly because information about the past becomes less effective in predicting the future. Since the record of the past represents information common to all investors, the common base of assumptions of different investors is narrowed, with the result that the range of alternative predictions increases. ([31], p. 627).

Gort tests his hypothesis using data on acquisitions for the period January 1951 through June 1959, classified according to the 3-digit SIC. He presents regression equations between merger rate and different measures of economic disturbance, concluding that the results "support the valuation discrepancies hypothesis". Alternative explanations like monopoly power gaining, attainment of scale economies, and securities of sellers being more undervalued than securities of buyers (the "bargain theory"), are

not supported by the analysis performed.

The later work by Gort and Hogarty [32] explains merger waves in terms of three elements: First, mergers are a form of high risk investment, with less than even probability of success, but with high prizes when winning. This is an ingredient in speculative activity. Second, due to "economic disturbances", expectations between buyers and sellers differ, buyers being more optimistic. And third, mergers result from decisions pursuing managerial rather than owners' objectives.

#### 3.4.1 Summary on studies on merger waves

Merger waves, as most of the merger phenomenon, are not clearly understood. An early empirical fact stated by Nelson [73] is a strong correlation between merger activity and stock prices. The following two theories reviewed make use of this fact:

Jacoby [45]: Merger waves result from the conjuncture of two preconditions: an accumulation of perceived and unexploited profit making opportunities, and a buoyant capital market.

Gort [31]: A divergence in the valuation of buyers and sellers is created as a consequence of economic disturbances, because these disturbances make past history less relevant in the projection of the future, thus creating uncertainty.

#### CHAPTER 4: CONCISE EXPOSITION OF FINANCIAL THEORY

The review of previous studies on mergers and conglomerates shows the lack of an underlying theory supporting them. Major efforts have been made in measuring the merger phenomenon, assessing profitability of mergers, and formulating and testing plausible hypotheses. But there is still substantial controversy on mergers, indicating that an acceptable paradigm of this phenomenon is not yet available. The objective in this and the following chapters is to unify some of the existing merger knowledge, to produce an analytic model for the study of mergers.

If a merger opens the possibility of exercising monopoly power, realizing production or distribution synergies, or getting important tax savings, then there are clear incentives favoring the merger materialization. In general, if the net cash flow of two firms combined is greater than the sum of the independent cash flows (without deterioration on other dimensions, risk for example), then there is a positive reward in completing the merger.

Net additions to cash flow are sufficient reasons for merging; nonetheless, the bibliographic review displays in a very forceful way the idea that the case for mergers cannot be built on reasons like scale economies, monopoly power, or production synergies. These were important causes underlying the first merger wave, and probably the second one too, but they are not important in the present expression of the merger phenomenon.

The presence of synergies is not denied, but the general attitude is to assume they do not exist when evaluating the decision to merge. If,

at the end, there are some synergies, they are well taken, but the point is that there are no prior expectations of their existence. For example, R. Ames of Textron [2] indicates that the contribution that Textron makes in the deal with the acquired company does not require the existence of synergies or economies of scale, and the decision to merge is independent of them. Another example of the attitude that senior executives of conglomerate firms have on the issue of synergy is presented in a Forbes report on United Technologies' Harry Gray [28]. The report states that "Gray has avoided what Roy Ash used to aim at, synergism. Gray has simply taken in big companies with strong market positions in hard-to-enter industries".

There are other forms of increasing the net cash flow by merger, like changing the incumbent managers in the acquired firm, using tax incentives, or recurring to complementarities of resources (R&D for example). These explanations of merger activity seem to attract less opposition in the literature, but the importance of their role is not yet well understood.

The point is that a model of mergers would not be powerful enough if based solely on the changes expected in cash flows as a consequence of the consolidation. This is especially true when trying to interpret the growing drive towards conglomeration. The standpoint in this study is to assume that sufficient incentives for merging still can exist, even when the cash flows are not affected by the merger.

An immediate conflict with actual financial theory is created by the last assertion; because, broadly speaking, if the cash flows of individual firms are not changed, the total value of the combined cash flow can not exceed the sum of the values of individual cash flows. Therefore, there

is no way to justify the merger.

The usual way out of this dilemma is to revise the strong assumptions underlying financial theory. In the first place, financial models are based on the assumption that managers, in their concern for the well being of shareholders, maximize the value of the firm. Those who attack this premise argue that managers may be after the satisfaction of personal objectives like growth maximization or empire building.

Another controversial basic hypothesis in financial theory is the assumption that capital markets are perfect, efficient, and complete. Many models have been proposed, and reasons given to explain mergers in terms of market imperfections. Insiders' information, transaction costs, taxes, and the malicious use of public information (like financial statements) are some of the issues raised.

Financial scholars' answers to these criticisms follow the following lines, more or less:

First, they recognize the market value maximization as unique objectives is a simplification, but it has given coherence to the theory, it has increased its predictive potential, and there are not yet important reasons impelling its modification. As Miller indicated in his presidential address ([65], p. 21)

Why then do economists keep trying to develop models that assume rational behavior by firms? They are not, I insist, merely hoping to con their business school deans into thinking they are working on problems of business management. Rather they have found from experience - not only in finance, but across the board - that the rational behavior models generally lead to better predictions and descriptions at the level of the industry, the market, and the whole economy than any alternatives available to them. Their experience, at

those levels, moreover, need involve no inconsistency with the heuristic, rule-of-thumb, intuitive kind of decision making they actually observe in firms. It suggests rather that evolutionary mechanisms are at work to give survival value to those heuristics that are compatible with rational market equilibrium, however far from rational they may appear to be when examined up close and in isolation.

Second, they accept that markets may be imperfect, but they have a very strong stand against accepting any major, permanent activity explained by market imperfections. The classic work in this area is the random walk hypothesis of Fama and his comprehensive review study on the efficient market hypothesis [21], [22]. Fama states that market values represent at any moment all available information related to a stock. Bias may exist at any given time, but the probability is the same for positive or negative deviations, the extent of these deviations not being predictable on a priori grounds.\*

And third, they acknowledge that the current theoretical development is far from satisfactory yet, because there are some conclusions that are not corroborated in practice. The most clear example is the debate on the optimal level of debt, whose existence cannot be clearly understood in terms of the Modigliani-Miller Theorems.

As a summary of the ideas exposed, it is possible to give a more complete description now of the characteristics that are guiding the approach to the merger phenomenon in this study.

First, incentives for merging may exist even when the cash flow of the two firms involved remains unchanged after the consolidation.

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\* Fama does not reject the possibility of market imperfections in the strong form tests, when corporate insiders or specialists of major securities exchanges have monopolistic access to information.

Second, this incentive cannot be explained under the current state of financial theory.

Third, to solve this dilemma, some authors challenge some of the assumptions in finance theory (firm value maximization, perfect markets).

And fourth, finance scholars support present assumptions and conclusions derived from them. They are not willing to accept results based on violation of these assumptions, unless strong empirical and theoretical antecedents point to the need to do so.

Consequently, a model has been built within the actual financial framework. The main idea in the construction of this model is to look for a minimum change in the premises or interpretation of financial theory, in order to gain enough explanatory power of the merger phenomenon.

To facilitate the introduction of the model for studying mergers, a summary of the main ideas in financial theory is presented in this chapter.

#### Concise exposition of financial theory

This section is not intended to be a comprehensive review of finance theory topics, but a selective and informal exposition of those issues which are relevant to the formulation of a model for mergers.

The theory that is of interest in this study is centered around the firm and two important groups linked to it: managers and shareholders (the owners). The two groups interact via the capital market.

It is assumed that managers make decisions to maximize the well being of the firm's shareholders. But, there is no need for managers to investigate each shareholder's preferences, because by maximizing the market value of the firm they are assured that shareholders can reach their maxi-

mum benefit. This important property is known as the Separation Theorem, whose implication is that owners never need to go in the nitty-gritty of firm conduct, as long as managers pursue the stated objective. There is a true specialization of functions between owners and managers of a firm. (See for example, Haley and Schall [33, pp. 125-129], and Hirshleifer [38], [39, pp. 421-422]).

The capital market, being the common ground for managers and shareholders, presents a different face to each one of them. For managers, the market is an evaluation mechanism of the decisions they make; it acts as a barometer of the firm's actions based on the cash streams that the firm is expected to distribute to shareholders. On the other hand, for shareholders, the market is an immense pool of alternative investments, and they show their opinion with regard to a firm by bidding for the firm's shares. On the whole, the market must balance continually the total demand and supply of the firms' shares generated by the investors' decisions to buy or sell their holdings.

This description leads to the definition of four basic financial issues that have been schematically represented in Figure 2:

First, the investor's decision problem, which is the object of Portfolio Theory.

Second, capital market equilibrium and pricing mechanism, which are obtained from the Capital Asset Pricing Model.

Third, valuation of cash-streams in the market, which is studied under Valuation Theory.

And fourth, managers' financial decisions, which are covered in Corporate Finance.

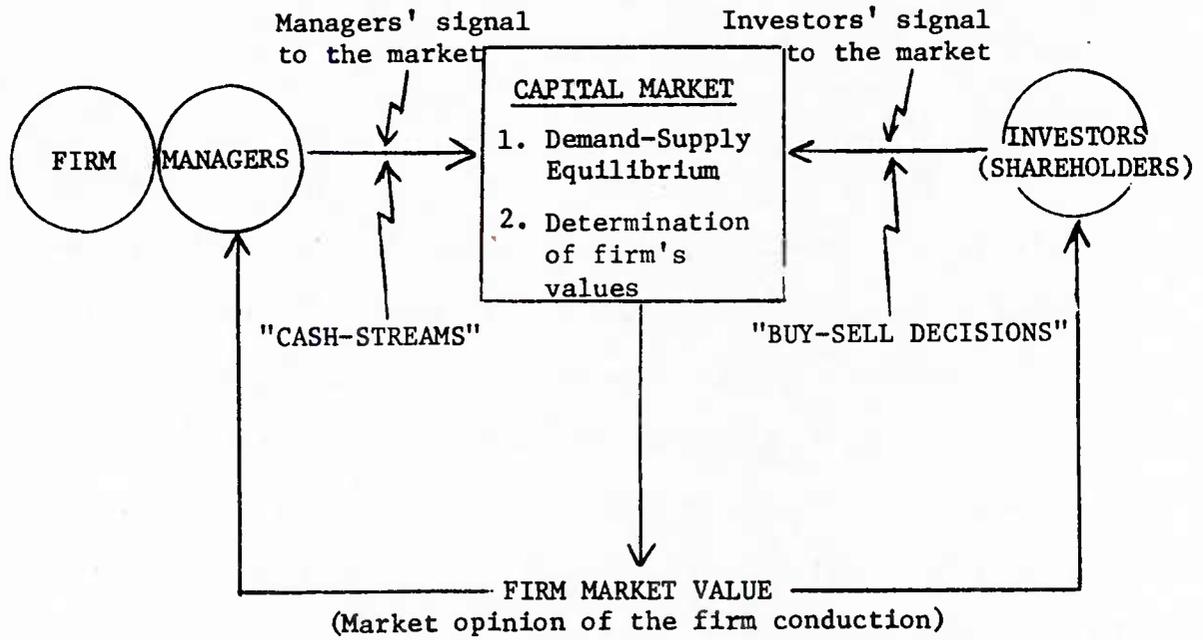


FIGURE 2: Managers-Shareholders Interaction Via the Capital Market

Each one of these subjects is now presented in a compact way, to lay the foundation for a merger model. As general references for the sections to come, see Boudreaux and Long [11], Fama and Miller [25], Haley and Schall [33], Hirshleifer [38],[39], and Merton [62].

#### 4.1 Portfolio Theory

Portfolio theory deals with the problem of an investor confronted with the capital market. The market displays a wide variety of alternatives (securities of shares) for the investor to distribute his budget. The investor is assumed to be a price-taker; that is to say, whatever his decisions are, they will not affect the market price of shares.

Securities in the market may be characterized in terms of the return one period later for \$1 invested today in it. In general, this return will be a random variable (for risky securities), but also, it may be well known in advance (for the risk-free asset). A security return is related to security prices according to the following relation:

$$\tilde{r}_i = \frac{\tilde{p}_i^1}{p_i^0} \quad (4.1)$$

where:

$\tilde{r}_i$  = Random return of \$1 invested in security i

$p_i^0$  = Price of security i today

$\tilde{p}_i^1$  = Random price of security i one period later (includes any cash dividend or other cash distribution).

A portfolio is any combination of securities in the hands of an investor. At the extremes, a portfolio may contain only one security, or a sample of all available securities. A portfolio may be described

in terms of all securities in the market, by indicating the fraction contributed by each of them to \$1 invested in the portfolio. The return on a portfolio can then be found as:

$$\tilde{r}_p = \sum_i w_i \tilde{r}_i \quad (4.2)$$

where:

- $\tilde{r}_p$  = Random return of \$1 invested in portfolio p
- $\tilde{r}_i$  = Random return of \$1 invested in security i
- $w_i$  = Fraction contributed by security i to the portfolio value (dollar value of security i holdings for a portfolio of \$1).

The investor's problem is to find the weights  $w_i$  in order to distribute his budget in an optimal way for him. The investor is assumed to be risk-averse, to prefer more to less, and to have preferences which can be described by a Von Neumann-Morgenstern utility function. Therefore, his problem is one of maximizing a utility subject to a budget constraint.

An important assumption is usually introduced for solving the investor's problem. His utility function is supposed to depend exclusively on the expected value and variance of the portfolio's return. Therefore:

$$\begin{aligned} U &= U(\text{portfolio mean, portfolio variance}) = \\ &= U(\bar{r}_p, B^2\sigma_p^2) \end{aligned} \quad (4.3)$$

where:

- $U$  = Investor's utility function
- $\bar{r}_p, \sigma_p^2$  = Expected value and variance of \$1 invested in the portfolio

B = Total investor's budget

$U_1 = \partial U / \partial \text{mean} > 0$ . Utility goes up if the expected return goes up

$U_2 = \partial U / \partial \text{variance} < 0$ . Utility is reduced with an increase in variance.

If the expected value, variance, and covariances of individual securities can be found, then the portfolio's parameters can be determined as follows:

$$\bar{r}_p = \sum_i w_i \bar{r}_i \quad (4.4)$$

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij} \quad (4.5)$$

where:

$\bar{r}_p, \sigma_p^2$  = Expected return and variance of \$1 invested in the portfolio

$\bar{r}_i$  = Expected return of \$1 invested in security i

$\sigma_{ij}$  = Covariance between  $\tilde{r}_i$  and  $\tilde{r}_j$  (returns to \$1 invested in securities i and j), ( $\sigma_{ii} = \sigma_i^2$  = variance of security i's return)

$w_i$  = Fraction contributed by security i to the portfolio value.

The investor's problem can now be written in terms of the decision variables  $w_i$  by having the expressions for  $\bar{r}_p$  and  $\sigma_p^2$  [Equations (4.4), (4.5)] replaced in the utility function [Equation (4.3)]:

$$\max_{\{w_i\}} U(B \sum_i w_i \bar{r}_i, B^2 \sum_i \sum_j w_i w_j \sigma_{ij})$$

s.t.  $B = \text{constant.}$

$$\sum_i w_i = 1^*$$

The problem is now solvable without any major difficulty for the individual investor, but instead of blindly applying differential calculus to it, some general properties of this solution may be derived by defining the efficient portfolio frontier. This is the set of all portfolios with minimum variance for a given expected value, and with maximum expected value for a given variance. Given that the investor is assumed to prefer a smaller variance for a given expected value, and a larger expected value for a given variance, his optimal choice must lie in this frontier. It may be shown that, when all assets are risky, the efficient portfolio frontier is, in general, the upper branch of a parabola in the plane  $(\sigma_p, \bar{r}_p)$ , as indicated in Figure 3 (Merton [63]).

When one risk-free asset is added to the picture, the efficient portfolio frontier becomes a straight line representing all possible combinations between the risk-free asset and a unique risky-asset called the optimal combination of risky assets (Merton [63]) (see Figure 4). The fraction in which all existing risky securities participate in this optimal risky asset may be derived from the market information according to the following relation:

$$w_i = \frac{d_i}{\sum_j d_j} \tag{4.6}$$

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\* There is no need to restrict  $w_i$  to non-negative values, because a negative  $w_i$  corresponds to a short sale of security  $i$ .

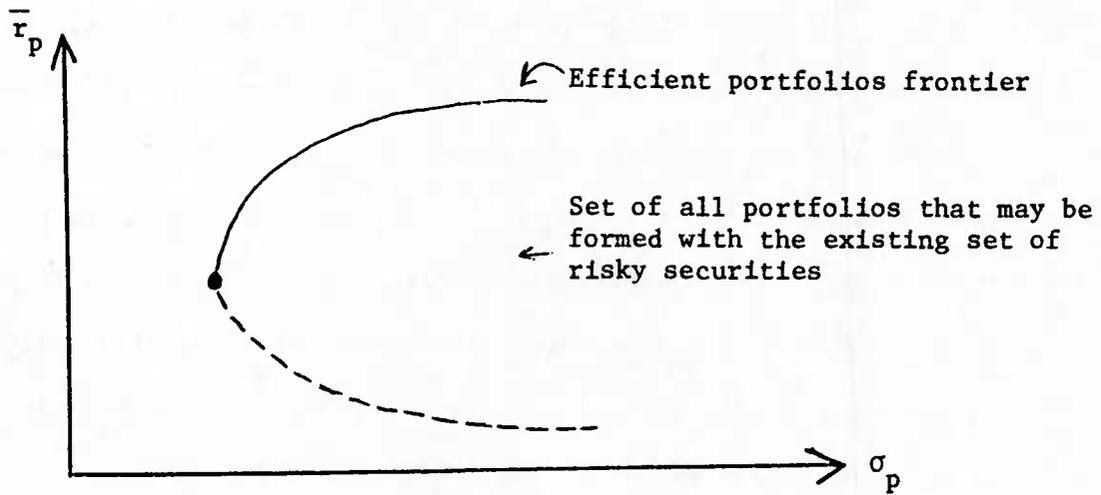


FIGURE 3: Efficient portfolio frontier when all assets are risky

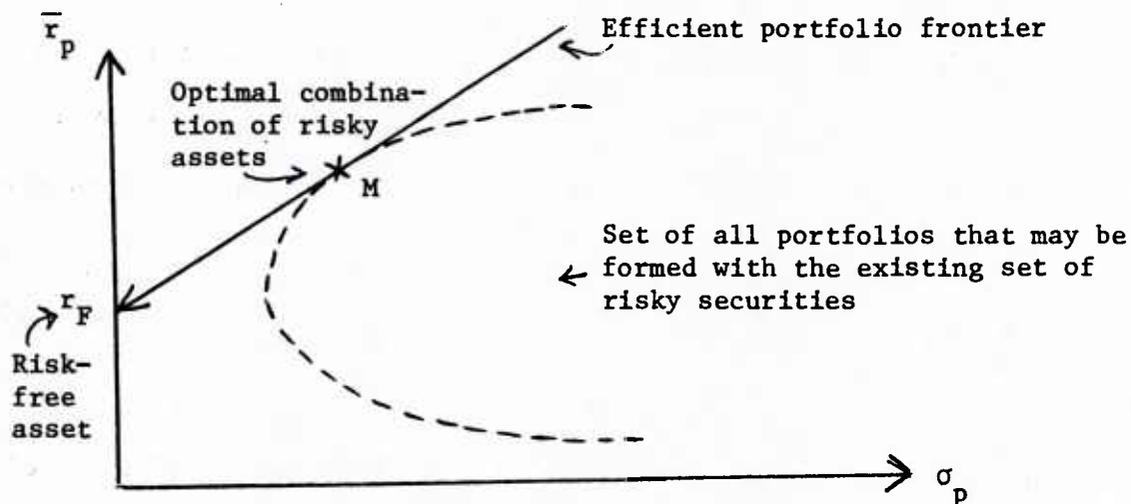


FIGURE 4: Efficient portfolio frontier when there is one risk-free asset and a set of risky assets

where:  $d_i = \sum_j \sigma_{ij}^{-1} (\bar{r}_j - r_F)$   
 $\sigma_{ij}^{-1}$  = (i,j) term in the inverse of the variance covariance matrix  
 $\bar{r}_j$  = Expected return of \$1 invested in security j  
 $r_F$  = Expected return of \$1 invested in the risk-free asset.

This is a very powerful result, because it indicates that all the market information can be summarized in just the following few parameters:

$r_F$  = Return of \$1 invested in the risk-free asset  
 $\bar{r}_M, \sigma_M$  = Mean and standard deviation for the return of \$1 invested in M. (Optimal combination of risky assets).

With these parameters, the efficient portfolio frontier may be written as:

$$\bar{r}_P = r_F + \sigma_P \left( \frac{\bar{r}_M - r_F}{\sigma_M} \right) \quad (4.7)$$

where:

$\bar{r}_P, \sigma_P$  = Mean and standard deviation for the return of \$1 invested in any efficient portfolio P.

The investor's problem can be simply solved now by selecting from this efficient frontier the point with maximum utility (see Figure 5). His choice is just deciding in what proportion he wants to hold the risk-free asset and the optimal portfolio M.

#### 4.2 Capital-Asset Pricing Model\*

Portfolio theory allows the individual investor to solve his budget distribution problem by assuming that prices for securities are given

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\* A good review on the assumptions and limitations of the theory may be found in Jensen [46].

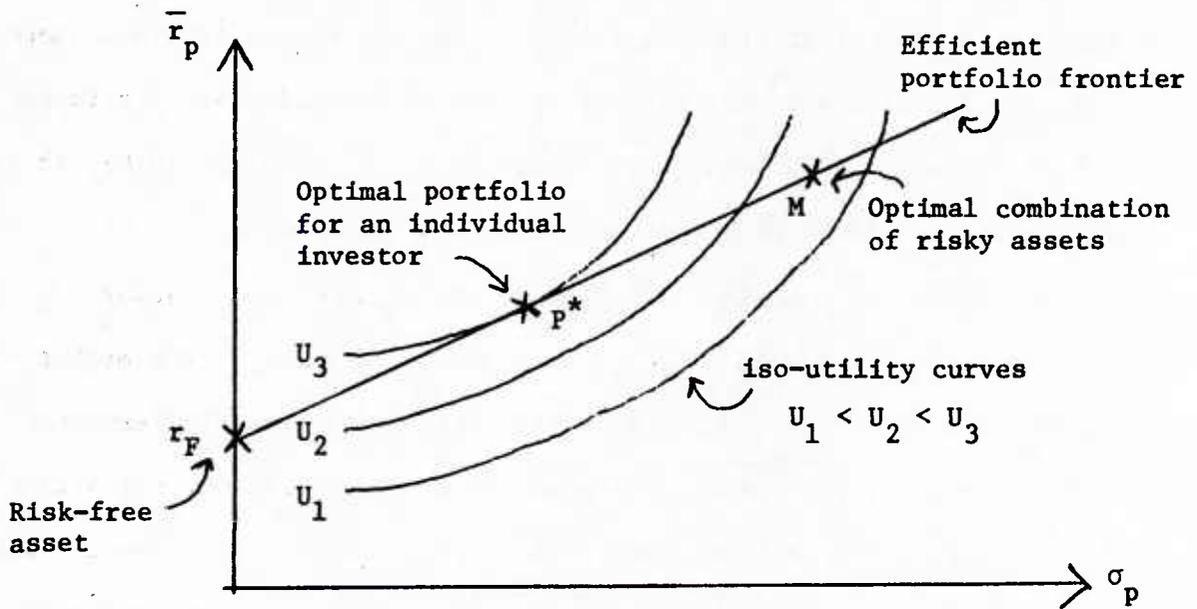


FIGURE 5: Optimal Portfolio for an Individual Investor

in the market, and that his actions will not affect them. But, there is a global equilibrium constraint that must be satisfied for validating the assumed set of prices. It is required that the market clears for all securities at the given prices; that is to say, for all securities demand and supply must be equal.

The capital-asset pricing model states a relation for determining the price of securities under equilibrium. By the nature of its assumptions, it also provides a simple way of finding the efficient portfolio frontier, without going through the computations of relation (4.6), that requires the inversion of a huge variance-covariance matrix.

In addition to assuming market equilibrium, the capital-asset pricing model considers that all investors participating in the market hold homogeneous expectations. That is to say, when assessing expected values, variances, and covariances among securities, all come out with the same numbers. Therefore, the optimal combination of risky assets will be the same for everybody, because all investors will use the same relation and the same parameters.

By requiring the market to be in equilibrium, this unique optimal combination of risky-assets may be shown to be equal to the market portfolio; that is to say, to a portfolio in which all risky securities participate in the same proportion of their value in the market. Consequently, the fraction in which security  $i$  participates in the market portfolio may be found from:

$$w_i^M = \frac{V_i}{\sum_u V_u} \quad (4.8)$$

where:

$w_i^M$  = Fraction contributed by security  $i$  to the market portfolio

$V_i$  = Market value of all risky securities  $i$  (number of securities  $\times$  price).

The efficient portfolio frontier is known in this case as the capital market line, whose equation is still (4.7), but interpreting  $M$  as the market portfolio (Figure 4 is also the same).

The fact that the capital market line holds all efficient portfolios may be used to derive a relation that any individual security must satisfy (see Haley and Schall [33], pp. 146-148). Because of the linear nature of this relation, it is known as the security market line, its equation being:

$$\bar{r}_i = r_F + \sigma_{iM} \left( \frac{\bar{r}_M - r_F}{\sigma_M^2} \right) \quad (4.9)$$

where:

$r_F$  = Return of \$1 invested in the risk-free asset

$\bar{r}_i$  = Expected return of \$1 invested in security  $i$

$\bar{r}_M, \sigma_M$  = Mean and standard deviation for the return of \$1 invested in the market portfolio

$\sigma_{iM}$  = Covariance between  $\tilde{r}_i$  and  $\tilde{r}_M$  (returns to \$1 invested in security  $i$  and the market portfolio).

The constant  $(\bar{r}_M - r_F) / \sigma_M$  is known as the market price of risk and it is designated by  $\lambda$ . By simple substitution, the security market line may be written as:

$$\bar{r}_i = r_F + \lambda \sigma_{iM} \quad (4.10)$$

This relation is a straight line in the plane  $(\sigma_{iM}, \bar{r}_i)$  as shown in Figure 6 .

The market equilibrium captured in the security market line implies that the expected return for \$1 invested in a security should go up when the covariance between the security and the market returns goes up. This is a very interesting conclusion because it contradicts the a priori notion that the return of a security (and as a consequence its price), should be related with the total risk taken for holding that security, which is measured by its variance and not by its covariance with the market.

To further comment on this point, it is convenient to determine what part of the variance is being compensated in the market. With that purpose, consider the following relation that may be derived from the security market line equation:

$$\tilde{r}_i = \bar{r}_i + \sigma_{iM} \frac{\tilde{r}_M - \bar{r}_M}{\sigma_M^2} + \tilde{\epsilon}_i \quad (4.11)$$

where:

- $\tilde{r}_i$  = Random return of \$1 invested in security i
- $\bar{r}_i$  = Expected value of  $\tilde{r}_i$
- $\tilde{r}_M$  = Random return of \$1 invested in the market portfolio
- $\bar{r}_M, \sigma_M$  = Expected value and standard deviation of  $\tilde{r}_M$
- $\sigma_{iM}$  = Covariance between  $\tilde{r}_i$  and  $\tilde{r}_M$

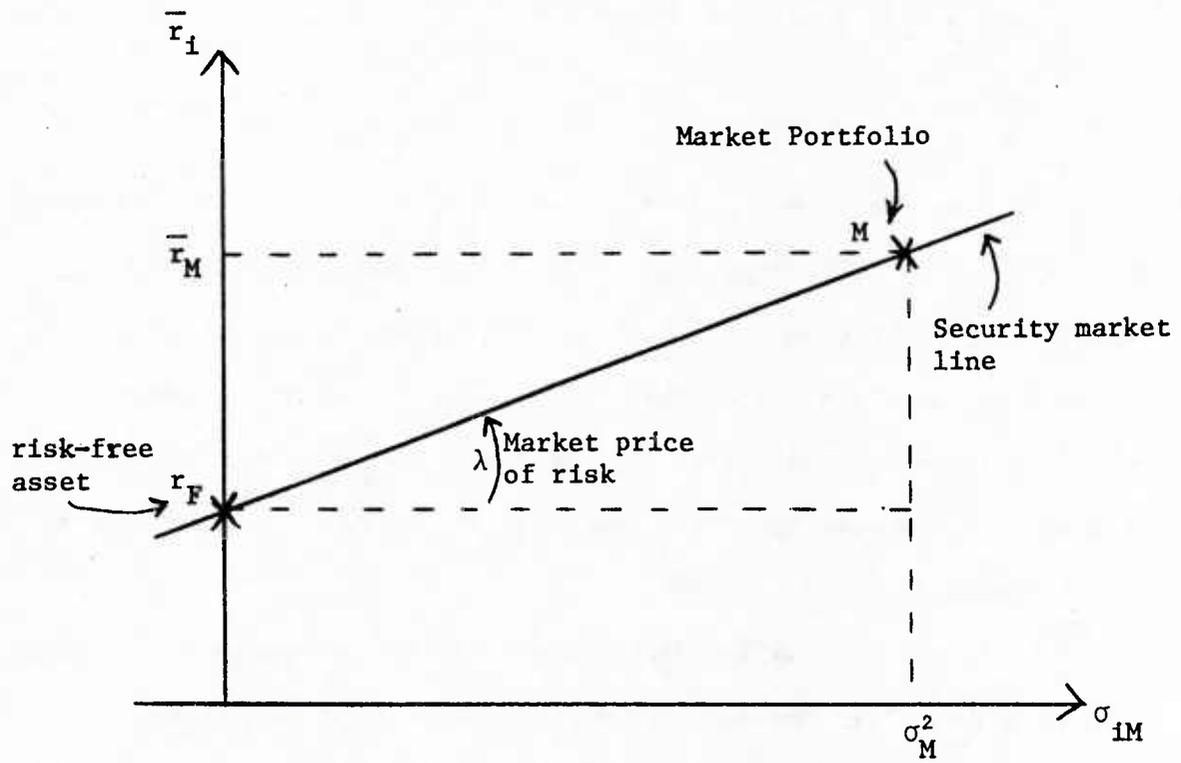


FIGURE 6: The Security Market Line

$\tilde{\epsilon}_1$  = Random fluctuation, with expected value 0, standard deviation  $\sigma_{\epsilon}$ , and uncorrelated with the market.

The variance of  $\tilde{r}_1$ , may then be found as:

$$\sigma_1^2 = \frac{\sigma_{1M}^2}{\sigma_M^2} + \sigma_{\epsilon}^2 \quad (4.12)$$

This equation provides a decoupling of total risk in two components that are usually called non-diversifiable risk and diversifiable risk, respectively. The reason for this is that for any efficient portfolio, the diversifiable risk is 0; that is to say, an investor following an optimal strategy only needs to be concerned with the non-diversifiable part of risk, because the other component is suppressed in the process of diversification.

This is the rationale underlying the pricing mechanism in the market that is reflected in the security market line. According to it, only non-diversifiable risk needs to be compensated because diversifiable risk may be eliminated by holding an efficient portfolio. Also, this is the reason for having the covariance instead of the total variance of a security as a measure of risk in the pricing equation [see relation (4.9)]. Sometimes, this risk is expressed in terms of an adimensional coefficient called beta or volatility of a security, defined as:

$$\beta_1 = \frac{\sigma_{1M}}{\sigma_M^2} \quad (4.13)$$

where:

$\beta_i$  = Volatility of security  $i$  (the volatility of the market portfolio is 1)

$\sigma_{iM}$  = Covariance between  $\tilde{r}_i$  and  $\tilde{r}_M$  (random returns of \$1 invested in security  $i$  and in the market portfolio, respectively).

$\sigma_M$  = Standard deviation of the return of \$1 invested in the market portfolio.

The non-diversifiable risk needs to be compensated because it is beyond the investor's possibilities to control it. This risk is related to uncertainties in the overall economic activities and cannot be avoided by anyone. Examples are economic cycles and general economic policy followed by the government. For an individual investor to take this risk, a premium over the risk-free rate is required. The size of this premium is given by the security market line.

#### 4.3 Valuation Theory

A security is a financial instrument that represents for their holders a promise of future payments to be made to them. The market value of a security is a consensus on the actual value that those future payments may have for investors in the market.

Valuation theory is concerned with determining a fair market value for a cash stream of future payments under conditions of market equilibrium. More formally speaking, valuation theory is focused on the transformation of a cash stream into a unique scalar that represents a generally accepted appraisal for the value of that stream. Figure 7

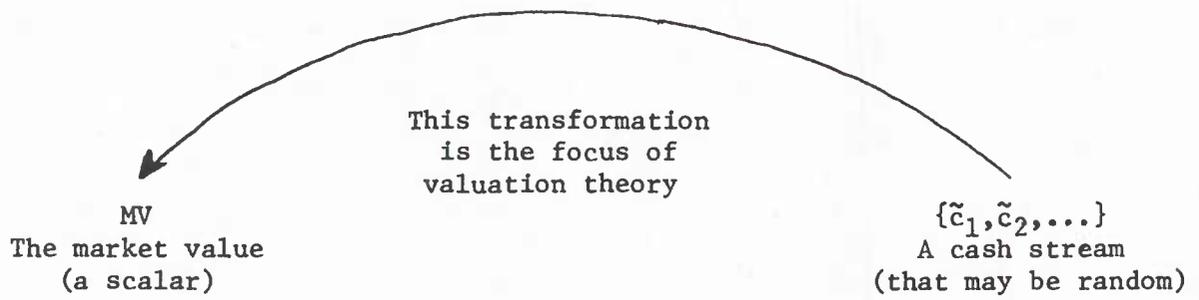


FIGURE 7: Focus of Valuation Theory

illustrates this idea.

When the cash flow is fully deterministic and well known in advance, it is possible to show that its market value must be given by relation (4.14) in order to satisfy the condition of market equilibrium.

$$MV = \sum_{t=1}^{\infty} \frac{c_t}{\prod_{j=1}^t r_{Fj}} \quad (4.14)$$

where:

MV = Market value of the cash stream  $\{c_1, c_2, \dots\}$

$c_t$  = Cash payment to security holders in period  $t$

$r_{Fj}$  = Risk-free rate of return for period  $j$  (an investor is indifferent between \$1 in period  $(j-1)$  and  $r_{Fj}$  dollars for sure in period  $j$ ).

The usual relation employed to get the market value of a security is obtained from here when all risk-free rates are assumed constant through time, and equal to  $r_F = (1+i)$ :

$$MV = \sum_{t=1}^{\infty} \frac{c_t}{r_F^t} = \sum_{t=1}^{\infty} \frac{c_t}{(1+i)^t} \quad (4.15)$$

When future cash flows become random, these relations are no longer applicable. Nevertheless, there is an heuristic extrapolation of formula (4.15) to extend its use to the case of uncertainty. This is given as relation (4.16):

$$MV = \sum_{t=1}^{\infty} \frac{\bar{c}_t}{(1+i+p)^t} \quad (4.16)$$

where:

MV = Market value of the random cash stream  $\{\tilde{c}_1, \tilde{c}_2, \dots\}$

$\bar{c}_t$  = Expected value of  $\tilde{c}_t$  = Payment that security holders forecast for period t

$(1+i)$  = Risk-free rate of return in an investment of \$1, for one period, under conditions of certainty (an investor is indifferent between \$1 in period (t-1) and  $(1+i)$  for sure in period t, for all t)

$(1+i+p)$  = Rate of return in an investment of \$1, for one period, under conditions of uncertainty. It includes the risk-premium p as compensation for the risk involved.

This formula assumes a very peculiar structure of certainty equivalents when used for random cash streams. Robichek-Myers [79] show that this formula implies a ratio between the certainty equivalent and the expected value geometrically decreasing with time. There is no a priori reason to expect this kind of behavior.

Some justification for this heuristic may be found in the work of Myers and Turnbull [72, p.332] when asserting that "... conventional valuation formulas based on discounting expected cash flows give a good approximation to assets values derived from rigorous analysis of equilibrium market values. We have uncovered no evidence that conventional valuation models are unsafe for management consumption." Valuation formulae that can be better justified from a theoretical point of view may be derived

from a multi-period extension of the Capital-Asset Pricing Model (Bogue and Roll [10], Brennan [12], Fama [20], Merton [64]). The problem with those relations is the difficulty in estimating an appropriate set of parameters, and the restrictive nature of the assumptions used in their derivation.

#### 4.4 Corporate Finance

A firm is evaluated in the market according to the characteristics of the cash stream of future payments generated by it. Firms' managers have certain discretion on financial decisions (most importantly in relation to dividends, debt, and investment) that may alter the market value of the firm. This section is intended to review in a succinct way some fundamental propositions in corporate finance, dealing with the way in which the market value of a firm is expected to vary due to dividend payments, changes in the capital structure, and investment decisions.

(A good collection of papers on these issues have been put together by Myers [69].)

##### 4.4.1 Dividend payments

The firm's earnings are partly distributed to stockholders in the form of dividend payments. The firm's managers have to determine how much of earnings they want to distribute, and how much they want to retain for future investments. According to financial theory, this decision should be taken to maximize the shareholders' wealth. But, it has been shown that if shareholders are indifferent between cash payments and capital gains, and capital markets are perfect, then their wealth

is unaffected by the dividend decision (Miller and Modigliani [66]).

To arrive at this conclusion, future operations and investments undertaken by the firm must be kept unchanged, for isolating the exclusive impact of the dividend policy on market value. This independence between the firm's activities and dividend payments, guarantees that all future cash flows received or paid out by the firm remain the same whatever the dividend paid, thus leaving the firm's market value unaffected by dividend policy.

The firm's decision is just the replacement of internal for external funds; instead of retained earnings, they will be using new equity to finance future activities. Old shareholders will receive a cash payment, and give away a share in future cash flows generated by the firm. But due to the hypothesis of perfect markets, the value of the stream they are giving away matches exactly the cash they receive as dividends. Consequently, their wealth is unaffected by the dividend decision taken by the firm.

Most of the arguments running against this conclusion of dividend policy neutrality, are drawn from market imperfections (see Boudreaux and Long [11], pp. 271-279). Among them are the different tax treatments given to dividend payments and capital gains, as well as transaction costs. Both of them favor retention of earnings rather than distribution via dividends. On the other hand, the fact that firms try to pay dividends at whatever cost, expresses a certain tendency favoring dividend payments. Two reasons are usually given to explain the stability of dividend payments: the clientele effect, and the information content of dividends. These reasons do not provide a clue for the selection of a particular payout level.

According to the clientele effect, all firms have a certain composition of shareholders that may be badly damaged if the firm abruptly changes its dividend policy. Therefore, there is a definitive incentive for stabilizing dividends, because the turnover of shareholders provoked by a change in dividend policy, may negatively affect the price of shares for a long time.

The other argument given to explain the stability of dividend policy is the information content of it. Basically, the idea is that a change in dividend policy may be a signal of a change in the firm activities; this having an adverse repercussion on prices.

In summary, some effects on market value are expected as a consequence of dividend policy, though theoretical arguments predict its neutrality.

#### 4.4.2 Capital structure

Capital structure has to do with the fraction of total market value that is contributed by debt and equity. Two fundamental propositions have been presented (Modigliani and Miller [67]):

First, with perfect capital markets, and no taxes, the capital structure does not affect the value of a firm. This is because any equity dollar replaced by debt (or vice-versa) will be equivalent to the cash stream of future payments generated by it. For the firm, neither cash flow nor risk have been changed.

And second, if interest tax deductability is added, the total value of the firm increases with debt, because the net cash flow is augmented by the tax shield generated by interest deductability. Therefore, firms

should increase its debt as much as they can.

It is not clear why firms have a rather stable capital structure, and why they do not tend to go to higher leverage ratios. One hypothesis is that debt-capital is made unavailable, or that the expected value of bankruptcy costs become very important over certain D/E ratio. Another is that future investment opportunities may be lost as a consequence of outstanding debt, a situation which becomes more likely with high leverage (Myers [70]). Still other is the incidence of personal taxes, which may substantially lessen the advantage of debt financing under equilibrium (Miller [65]).

In summary, capital structure does matter with interest tax deductibility, but it is not clear why firms have not used more debt.

#### 4.4.3 Investment decisions

If a manager is presented with an investment opportunity, he has to decide if it is convenient to take it or not. By accepting the commitment of managers to the shareholders' well being, an investment is convenient whenever the net change in the market value of the firm is positive.

This proposition may be formally stated in terms of the market value of a cash stream. Consider a project with an investment  $I_0$  today, producing a cash stream of  $\{\tilde{c}_1, \tilde{c}_2, \dots\}$  in future periods. This project is convenient if

$$-I_0 + MV_0\{\tilde{c}_1, \tilde{c}_2, \dots | \phi_0\} > 0$$

By using an acceptable valuation formula for obtaining the market

value of the random cash stream, this problem is not fundamentally different from the valuation theory previously discussed.

#### Summary of the Main Financial Concepts

There are many concepts in the theory of finance that have evolved in recent years, and that provide a solid foundation for the on-going research in the area. The main purpose in this chapter has been to present some of these basic concepts, in order to establish the theoretical framework for this study. The propositions that are most relevant in the ensuing chapters are summarized in this section.

Managers are assumed to be rational decision makers who pursue the well being of their shareholders. As a consequence of the separation principle, they can assure that this objective is attained by maximizing the market value of the firm.

The market value reflects, at any time, the aggregate value of cash payments to be generated by the firm in the foreseeable future, properly discounted for the time and risk involved. By virtue of the efficient market assumption, each new piece of information is instantly reflected in the market value of the firm; therefore, this value reflects at any time all public expectations regarding the firm's future performance.

A surprising conclusion derived from the theory of valuation is that only systematic risk is compensated in the market. There is no risk premium attached to the undertaking of unsystematic risk, because by holding an adequately diversified portfolio, the total unsystematic risk can be reduced to zero.

In the analysis of the firm's financial decisions, the most important result of the Modigliani-Miller proposition for the firm's capital structure. Briefly, they indicate that the debt-equity ratio does not affect the market value of the firm when corporate taxes are ignored; and that debt is preferred over equity financing when corporate taxes are considered. The puzzling aspect of this proposition is the inability to explain the actual behavior of firms, which use an intermediate mix of debt and equity financing.

## CHAPTER 5: MARKET VALUE OF THE FIRM WITH PURE EQUITY FINANCING\*

The primary purpose in this study is to find a rationale for mergers. When reviewing the perspectives that different authors have employed in the analysis of this phenomenon, a promising area of inquiry appears to be the exploration of financial incentives that may trigger a merger, though the current state of financial theory indicates that no such incentives exist.

It is apparent that the generation of a financial rationale for mergers requires some change in the actual conclusions of financial theory. This chapter and the next present an extension of the theory that opens the possibility of explaining mergers in Chapter 7. The main conclusion to be derived (that contradicts existing notions) is that there are situations in which the market value of an investment opportunity is not fully reflected in the market value of the firm.

### 5.1 Market Value of the Firm: An Extension

Firms are valued in the market according to the expectations of future cash payments to be made to the market participants. The market discounts this cash stream for the timing and risk of these payments, and produces a present value which is equivalent to the cash stream of future payments, conditioned on all publicly known information.

Usual channels employed to convey information to the market are

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\* This chapter and the next are based on a paper that Prof. Myers and myself have written together (Myers and Majluf [71]). I have to acknowledge the fundamental contribution that Prof. Myers made to the ideas to be exposed, and thank his personal dedication to many long discussion sessions.

annual reports, quarterly financial statements, earning figures, and investment prospectuses. More interestingly, some common financial decisions may be interpreted as good or bad signals in the market, and affect the price accordingly. For example, the decision of firms to split their stock has been found to correspond to expectations of increased earnings and dividend payments that firms believe they can maintain in the long run (see Fama, Fisher, Jensen, and Roll [24]). Also, secondary offerings of common stock carry a negative information about earnings, which depresses the stock price between one and two percent on average. This is especially true when corporate executives sell their stock (Scholes [81]).

The cash stream that reaches the market is an aggregation of the net cash generated by the different activities to be developed under the firm's umbrella. Some of these activities may correspond to a very natural projection of the main line of business, while others may depart from it in an important way. Some will be based on existing capacity and equipment, and others on new potential additions to them. Some activities will be using skills and knowledge currently available in the firm, while others will need additional skills and qualifications.

A simple way of interpreting the cash stream reaching the market, is assuming that it subsumes three important pieces of information: First, a random but relatively safe cash stream that the firm will be able to generate as a natural development of its present activities and current situation. The only uncertainties in it are due to the general outlook and movement of the economy as a whole, and to random distur-

bances that all business operations face. Second, a random cash stream contingent on the development of a new investment opportunity, whose undertaking is still to be decided by the firm on the basis of future conditions. And third, an assessment of the firm's willingness to undertake this new investment. In sum, the firm is represented as normally operating an existing line of business, and at the same time owning the exclusive option of taking a new project for a limited time period.

Assuming that the new investment opportunity has a positive net present value, the actual thinking is that there are no contingencies that may prevent the firm from taking the new project. Under these circumstances, the market value of the firm reflects the total value of the new project immediately after the information reaches the market, as shown in relation (5.1).

$$MV\{\text{firm}|\phi\} = MV \left\{ \begin{array}{l} \text{current} \\ \text{line of} \\ \text{business} \end{array} \middle| \phi \right\} + MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| \phi \right\} \quad (5.1)$$

where:

$\phi$  = all public information available.

The extension to be proposed in this study does not support this statement. What is suggested instead is that there are situations in which the current group of shareholders is benefited by rejecting a good investment opportunity. In this case, a fair market value for the firm is given by relation (5.2), which recognizes the contribution of the current line of business, as well as the contribution of the investment opportunity if the firm goes ahead with it.

$$MV\{\text{firm}|\phi\} = MV \left\{ \begin{array}{l} \text{current} \\ \text{line of} \\ \text{business} \end{array} \middle| \phi \right\} + \Pr(M'|\phi) \cdot MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| M', \phi \right\} \quad (5.2)$$

where:

$\phi$  = all public information available

$M'$  = Event representing the firm's decision to go ahead with  
the new investment opportunity

$\Pr(M'|\phi)$  = Probability of going ahead with the new investment oppor-  
tunity.

The argument is built on two basic premises which are totally consist-  
ent with perfect markets and other assumptions in the theory of finance.  
First, managers act in the benefit of current shareholders; and second,  
managers get, prior to the market, some information on the characteristics  
of the new project. With this information, managers can determine if  
current shareholders are better off by undertaking the realization of the  
new project, or by dropping the idea and staying solely with the actual  
line of business.

An important consequence of the difference in information is that  
there is some room for managers to manipulate the public delivery of  
internal information. Throughout this study, the implicit assumption is  
that managers are acting in the best long run interest of shareholders.  
This assumption clearly determines the best strategy for the firm to  
follow, but there is a potential conflict of this objective with a short  
run maximization of the firm's market value. As a consequence of this,  
the separation principle, that guarantees that shareholders are getting  
the most for their shares at all times, need to be reinterpreted. This

is done later in this chapter.

The difference between public and insider's information is a very natural thing to expect when considering that managers are "living" with the business, and that they ought to know more about it. More important than this is the fact that managers are trying to gain some decisive, strategic advantages over competitors, and this normally requires holding from the public some fundamental information until the firm can securely appropriate all abnormal profits generated in a new venture.\*

This difference in information plays an important role in the valuation of the firm only when the firm does not have sufficient slack resources to undertake the investment opportunity. Slack may be interpreted as the holding of resources over and above what is needed for the firm's current operation. In this study, slack is defined as the internal cash that the firm can make readily available for investing in a new project. Clearly, cash on-hand and marketable securities are a form of slack; but also, some form of expenses that are normally incurred but can be readily dropped, may be considered as slack. The essential characteristic of slack resources is that the firm can recur to them without going to the market. Though cash can be made available on very short notice in the form of a new debt or equity issue, that cash cannot be equated with slack even if the firm can issue risk-free debt.

When the internal availability of resources is sufficient to take

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\* Campbell [13] argues very convincingly for the existence of insider-managers with better information than the market as a whole, because information is not free and managers specialize in the acquisition of information related to the firm. He also argues for the strategic value of this information.

the new project, the difference between public and insider's information does not affect the market value of the firm. In fact, given the assumption that the net present value of the new project is positive, and considering that the firm has the money required for the investment, it is apparent that shareholders are better off by going ahead with the new project. Therefore, the market value of the firm is given by (5.1) which fully captures the value of the new project.

A different situation arises when slack is insufficient to undertake the new investment opportunity. In this case, the firm is forced to go to the market if the new project is to be taken. With this decision, the firm involuntarily gives away some of its exclusive internal information; because, under the assumption that managers are acting rationally on behalf of current shareholders, going to the market must imply that this group of shareholders cannot be made worse off.

The informational content of the investment decision is not considered in existing financial theory, because it is taken for granted that current shareholders are always benefited by investing in a project with positive net present value. This notion is shown to be incorrect when the difference between insiders and public information is present in conjunction with insufficient slack. Broadly speaking, if slack is large enough, the firm will be always willing to take an investment opportunity with positive net present value, and the market will recognize its full worth [as indicated in relation (5.1)]. But, if slack is insufficient, it may be in the benefit of current shareholders to reject some good opportunities, this implying that the value of the project is only partly

reflected in the market value of the firm [as indicated in relation (5.2)].

The loss due to insufficient slack is the difference between the full value of the project being considered, and the portion actually reflected in the market value of the firm, as indicated below.

$$\Delta(\phi) = MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| \phi \right\} - \Pr(M'|\phi) \cdot MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| M', \phi \right\}$$

where:

$\Delta(\phi)$  = Market value loss due to insufficient slack resources.

This expression can be simplified by noticing that the market value of the new investment opportunity may be represented in terms of the two mutually exclusive and collectively exhaustive states M and M', where:

M' = Event representing the firm's decision of going ahead with the new investment opportunity

M = Event representing the firm's decision of not going ahead.

$$\begin{aligned} MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| \phi \right\} &= \Pr(M'|\phi) \cdot MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| M', \phi \right\} + \\ &+ \Pr(M|\phi) \cdot MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| M, \phi \right\} \end{aligned} \quad (5.3)$$

From here, it may be seen that the loss in market value due to insufficient slack is equal to the market value of opportunities that are not taken times the probability of being in that situation.

$$\Delta(\phi) = \Pr(M|\phi) \cdot MV \left\{ \begin{array}{l} \text{investment} \\ \text{opportunity} \end{array} \middle| M, \phi \right\} \quad (5.4)$$

This loss is always greater than or equal to zero, because neither terms in that expression can be negative.

This study considers that a firm may raise new funds by issuing new equity or new debt in the market. The following analysis is done in two steps, by considering first that only equity is available to the firm (this chapter), and then that both debt and equity are available (next chapter).

To make the exposition of the subject somewhat easier, two simple examples are developed prior to the more formal analysis in a pure equity situation.

## 5.2 Market Value of the Firm with Pure Equity Financing: Two Simple Examples

This section presents two examples of a firm with insufficient slack, that is confronted with a good investment opportunity. The only source of funds is to raise new equity. In one case, current shareholders are always benefited by undertaking the new project, independent of the prevailing situation. In the other case, there is one state in which it is better to drop the new project, though its net present value is positive. As a consequence of this, the total value of the new opportunity is fully reflected in the first case, but only partly reflected in the second one.

### Example 1. Insufficient slack is inconsequential.

Consider a firm that is operating in a line of business whose market value one period from now is a random variable  $\tilde{A}$ . This firm is suddenly

forced to a new investment opportunity with a net present value  $\tilde{B}$  one period from now. Assume that  $\tilde{A}$  and  $\tilde{B}$  are uncorrelated with the market, the risk-free rate is 0,\* and their joint probability distribution is:

$$(\tilde{A}, \tilde{B}) = \begin{cases} (5, 6) & \text{with probability } 1/2 \\ (15, 10) & \text{with probability } 1/2 \end{cases}$$

Assume also that the investment required to undertake this project is  $I = 20$ , and that the firm has no cash available. Therefore, if managers decide to go ahead with the project, they have to raise new equity totaling  $E = 20$ .

The immediate reaction to this information (which is designated by  $\phi$ ), is that the market value of the firm today has to be equal to the value of current and future opportunities. Therefore:

$$V(\phi) = \bar{A} + \bar{B} = 10 + 8 = 18$$

In this case, this value happens to be right, because even if managers get to know the true outcome prior to the market (which is one of two basic assumptions made in this study), it is always convenient to issue new shares and undertake the project. In fact, the pay-offs for current shareholders under the two possible outcomes, given that shares are not issued, are the following:

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\* The source of randomness that is of concern in this presentation stems from unsystematic risk only. Assuming that the risk-free rate is 0 is only a change in the scale of measurement that simplifies the exposition and it is inconsequential for the analysis.

$$V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B})=(5,6)] = 5$$

$$V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B})=(15,10)] = 15$$

If shares are issued, the corresponding payoffs may be computed as:

$$\left\{ \begin{array}{l} \text{Fraction of the firm} \\ \text{owned by current shareholders} \\ \text{after issuing shares for 20} \end{array} \right\} \times \left\{ \begin{array}{l} \text{The market value of} \\ \text{the firm after the true} \\ \text{outcome is known} \end{array} \right\}$$

Therefore:

$$V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B})=(5,6)] = \frac{18}{18+20} (20+5+6) = 14.68$$

$$V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B})=(15,10)] = \frac{18}{18+20} (20+15+10) = 21.32$$

Arranging these results in a tabular way, it may be appreciated that issuing shares is always a preferred strategy, independent of the outcome, and that the extra value obtained is 8, which is the value of the new opportunity.

Market Value of Old Shareholders' Shares

	Do not issue	Issue
$(\tilde{A}, \tilde{B}) = (5,6)$	5	14.68*
$(\tilde{A}, \tilde{B}) = (15,10)$	15	21.32*
Expected value	10	18.00

\* optimum strategy

Example 2. Insufficient slack is detrimental.

Consider a situation similar to the one described in Example 1, but with the following parameters:

$$(\tilde{A}, \tilde{B}) = \begin{cases} (5,1) \text{ with probability } 1/2 \\ (15,3) \text{ with probability } 1/2 \end{cases}$$

$$E = 20$$

In this case, an equilibrium situation cannot exist with a market value equal to:

$$V(\phi) = \bar{A} + \bar{B} = 10 + 2 = 12$$

To see the reason for this, consider the payoffs obtained by old shareholders when the firm does and does not issue shares after getting information on the exact  $(\tilde{A}, \tilde{B})$  outcome.

$$V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B})=(5,1)] = 5$$

$$V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B})=(15,3)] = 15$$

$$V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B})=(5,1)] = \frac{12}{12+20} (20+5+1) = 9.75$$

$$V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B})=(15,3)] = \frac{12}{12+20} (20+15+3) = 14.25$$

The tabular representation of these results is:

Market Value of Old Shareholders' Shares  
(assuming a market value of 12)

	Do not issue	Issue
(A,B) = (5,1)	5	9.75*
(A,B) = (15,3)	15*	14.25

\* optimum strategy

The fundamental difference shown by these results is that old shareholders are better off by not issuing shares when the outcome is  $(\tilde{A}, \tilde{B}) =$

(15,3), even if they do not take project  $\tilde{B}$  with a positive net present value of 3. The other interesting difference is that the decision to issue shares carries to the market the information that the outcome is  $(\tilde{A}, \tilde{B}) = (5,1)$ . Similarly, not issuing shares implies  $(\tilde{A}, \tilde{B}) = (15,3)$ . Therefore, the value of old shareholders' shares after the firm announces its decision not to issue shares is:

$$V^{\text{old}}(\phi, \text{no-issue}) = V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B}) = (15,3)] = 15$$

If the firm decides to issue shares, the market knows that the outcome is (5,1). Therefore, the market value of the firm must go to  $(5+1) = 6$ . After issuing shares, the fraction owned by old shareholders is:

$$\frac{6}{6+20} (20+5+1) = 6. \text{ This implies that:}$$

$$V^{\text{old}}(\phi, \text{issue}) = V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B}) = (5,1)] = 6$$

The equilibrium market value has to be:

$$\begin{aligned} V^{\text{old}}(\phi) &= V^{\text{old}}(\phi, \text{no-issue}) \cdot \text{Pr}(\text{no-issue}) + V^{\text{old}}(\phi, \text{issue}) \cdot \text{Pr}(\text{issue}) \\ &= 15 \cdot \frac{1}{2} + 6 \cdot \frac{1}{2} = 10.5 \end{aligned}$$

It may be seen that this value falls 1.5 units short of  $\bar{A} + \bar{B}$  (which is 12). This difference corresponds to the loss of a good project when the outcome is  $(\tilde{A}, \tilde{B}) = (15,3)$ . The value of the loss is 3 with probability 1/2; that is to say, its expected value is precisely the 1.5 units lost in the market value of the firm under equilibrium.

The whole problem arises from the inability of managers to communicate in an effective way that their true state is (15,3), when that is

the case. They are impeded from making public all internal information, because that would disclose strategic characteristics of the investment, and would offset the value of the opportunity.

With this course of action closed, managers may turn to a "public image polishing" campaign. They may disclose only partial information, and indicate the outstanding situation of the firm. But, why should the market believe them? They can engage in a similar campaign when the true state is (5,1). Managers are always trying to give an optimistic view of their companies, and it does not cost anything to distort reality by presenting a rosier picture. The proper market reaction is to believe in managers only when they make a decision, rather than when they explain the situation of the firm: "Actions speak louder than words".

Another alternative that managers can think of is to direct the market valuation with two sequential decisions. In the first place, they indicate that the new project is not going to be pursued. The market reaction must drive the price to 15, because the implicit state of the world is (15,3). Then, the firm can reverse its initial decision and issue shares at this high value to undertake the new project. This is perfectly logical if the two sequential decisions narrow down the states of the world to (15,3). But, what prevents the firm from using the same strategy when the true state is (5,1)? If this strategy were to work, it is a better course of action for the firm. Subsequently, by reverting the initial decision that the project is not going to be taken, the information in the market is not narrowed down, but it goes back to its very first state. Two contradictory decisions in sequence

do not reinforce, but cancel, each other.

It is still possible for managers to have a "private line" with stockholders, and communicate to them the true value of the firm without revealing any information to competitors. This is perfectly reasonable when thinking of a private business, in which managers and owners are pretty much the same group of people. But in a public company, giving internal information to shareholders may be forbidden by law. Also, there is no guarantee that secrecy is kept with such a large number of people, thus nullifying the initial reason for having a private communication.

In conclusion, the assumption made in this study is that managers have only one opportunity to indicate if they take the new project, and that the announcement of this decision transfers to the market some internal information. In this simple example, by taking the project the firm makes clear that the state is (5,1), and by not taking it, that it is (15,3).

### 5.3 Market Value of the Firm with Pure Equity Financing: The General Case

Consider a firm that is normally operating its existing line of business. This firm is confronted with a good project, but does not have the amount of resources required for the investment. The only source of funds, in addition to the funds internally available, is raising new equity.

The firm's decision to use this mode of financing has to be dictated by the benefit that current shareholders will derive from it. The firm

is supposed to have better information than the market on the characteristics of the investment option, at the time the decision to go ahead with the project or drop it is taken, and this information cannot be revealed, because that would destroy the nature of the project.

The relation between the firm and the market evolves around three time instants, which are designated by -1, 0, and +1 (the present corresponds to time 0). This relation is now carefully stated, and it is summarized in Figure 8.

At time -1, both the firm and the market have the same information with respect to the firm's value, which is designated by  $\phi$ . This information is that the firm owns an existing line of businesses and a new investment opportunity whose market value at time +1 are represented by the random variables  $(\tilde{A}, \tilde{B})^*$ , with a joint probability distribution  $f(a, b)$ . Also the firm is known to have an amount  $S$  of slack which is less than the total investment  $I$  required by the new project. The market value of the current line of business (which is designated by  $\tilde{A}$ ) is defined excluding the slack available. Therefore, the actual market value of the firm at time +1 (without considering the new project) is  $(S + \tilde{A})$ . The market value of the firm at time -1 is designated by  $V^{\text{old}}(\phi)$ . The word old is added to indicate that all of it is owned by old shareholders.

At time 0, the firm gets to know (prior to the market), the updated value of  $(\tilde{A}, \tilde{B})$ , which is designated by  $(a, b)$ . According to this infor-

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\* These values are discounted to the present (time 0), for time and risk. The only source of randomness in them is assumed to be unsystematic variation.

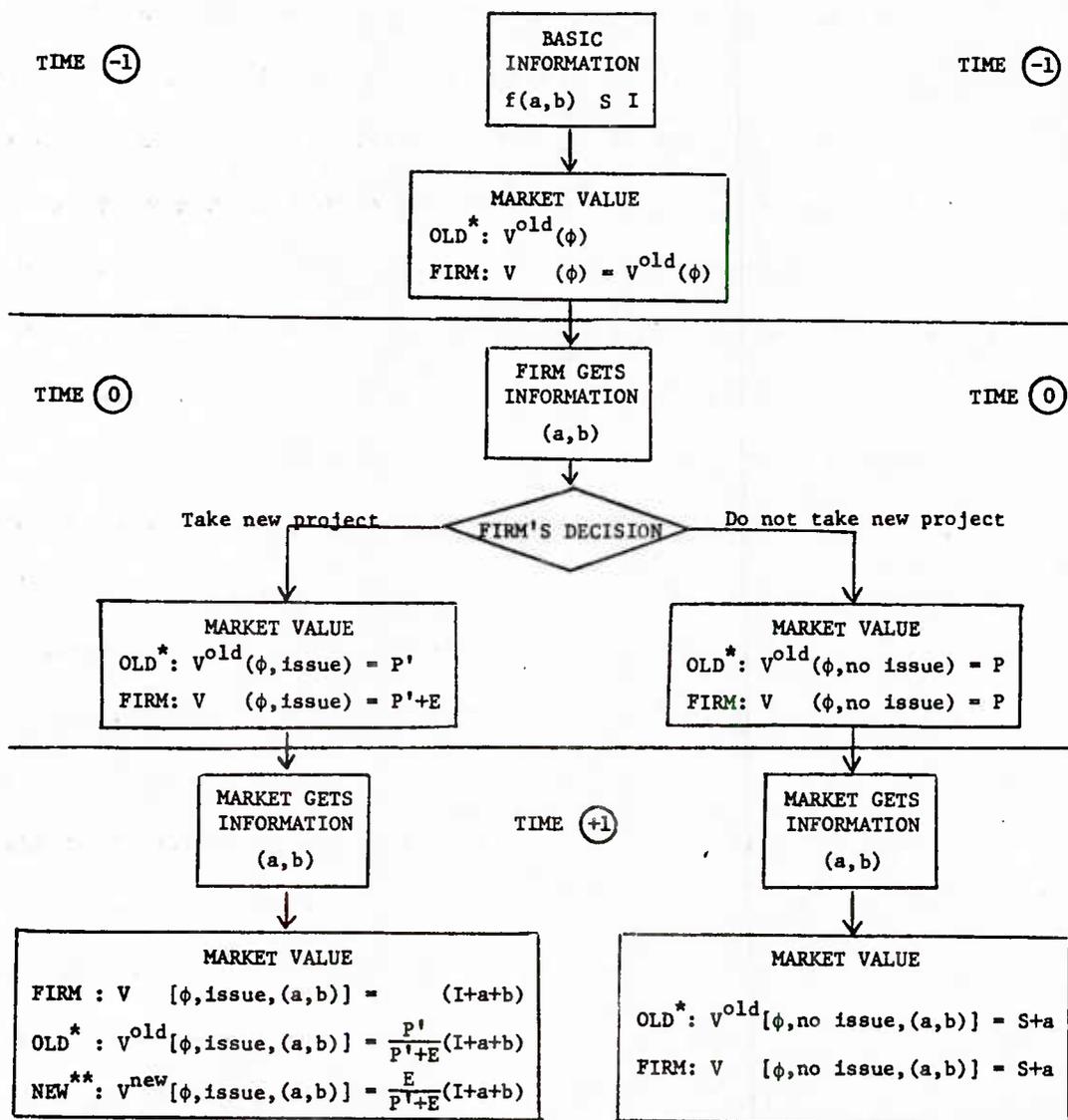


FIGURE 8: Market Value in a Pure Equity Case

\* market value of old shareholders' shares

\*\* market value of new shareholders' shares.

mation, the managers of the firm may decide to forget the project, or to issue an amount  $E$  of new equity and undertake it (with  $E = I - S$ ).

The market, without knowing the exact outcome  $(a, b)$ , will react to the firm's decision by adjusting the value of the firm. If the firm does not take the project, its market value becomes  $V^{\text{old}}(\phi, \text{no-issue})$ ; while, if the project is taken, this value is  $V^{\text{old}}(\phi, \text{issue})$ . For simplicity, these market values are designated by  $P$  and  $P'$ , respectively. Then:

$$P \equiv V^{\text{old}}(\phi, \text{no-issue}) = \begin{cases} \text{market value of old stockholders' shares} \\ \text{at time 0, when new shares are not issued} \end{cases} \quad (5.5)$$

$$P' \equiv V^{\text{old}}(\phi, \text{issue}) = \begin{cases} \text{market value of old stockholders' shares} \\ \text{at time 0, when new shares for a total} \\ \text{value } E \text{ are issued.} \end{cases} \quad (5.6)$$

These adjustments in market value are fully borne by current shareholders, and they are a consequence of the information contained in the firm's decision. When adding the contribution of new shareholders to the market value of the firm, if stock is issued, the firm value at time 0 becomes:

$$V(\phi, \text{no-issue}) = V^{\text{old}}(\phi, \text{no-issue}) = P \quad (5.7)$$

$$V(\phi, \text{issue}) = V^{\text{old}}(\phi, \text{issue}) + E = P' + E \quad (5.8)$$

When new shares are issued, the claims that old and new shareholders have over the market value of the firm, are the following:

Old shareholders own a fraction  $\frac{P'}{P'+E}$  of the firm

New shareholders own a fraction  $\frac{E}{P'+E}$  of the firm.

At time +1, the market gets full information on the updated value  $(a, b)$ , and the market value adjusts accordingly. If the project is not

taken, the market value of the firm becomes  $V[\phi, \text{no-issue}, (a, b)]$ , while if the project is taken, this value is  $V[\phi, \text{issue}, (a, b)]$ . The part of these totals that is taken by current shareholders is designated by  $V^{\text{old}}[\phi, \text{no-issue}, (a, b)]$  and  $V^{\text{old}}[\phi, \text{issue}, (a, b)]$ , respectively. Similarly,  $V^{\text{new}}[\phi, \text{issue}, (a, b)]$  may be defined for new shareholders when shares are issued.

It is clear that if no equity is issued, the full value of the firm is taken by current shareholders, and that this value is equal to the existing line of business with the slack included. Then:

$$V[\phi, \text{no-issue}, (a, b)] = V^{\text{old}}[\phi, \text{no-issue}, (a, b)] = S + a \quad (5.9)$$

On the other hand, if the project is taken, the total value of the firm (which is  $I+a+b$ ) must be subdivided between old and new shareholders, in proportion to the value of their claims [which are  $P'/(P'+E)$  and  $E/(P'+E)$ , respectively]. Then:

$$\begin{aligned} V[\phi, \text{issue}, (a, b)] &= V^{\text{old}}[\phi, \text{issue}, (a, b)] + V^{\text{new}}[\phi, \text{issue}, (a, b)] = \\ &= I + a + b \end{aligned} \quad (5.10)$$

with:

$$V^{\text{old}}[\phi, \text{issue}, (a, b)] = \frac{P'}{P'+E} (I+a+b) \quad (5.11)$$

$$V^{\text{new}}[\phi, \text{issue}, (a, b)] = \frac{E}{P'+E} (I+a+b) \quad (5.12)$$

#### Equilibrium conditions in a pure equity situation

The possibility of managers to get, prior to the market, information on the firm's current business and future opportunities, allows them to

know in advance the market reaction at time +1, under each one of the two alternative settings: when the project is taken, or when it is not taken. Managers must act on the basis of this information to decide whether shares should be issued for undertaking the project, or if it is better to put the project aside.

If managers decide not to issue shares, old shareholders get  $V^{\text{old}}[\phi, \text{no issue}, (a, b)]$  given by relation (5.9). If they issue shares instead, old shareholders obtain  $V^{\text{old}}[\phi, \text{issue}, (a, b)]$  given by (5.11). Old shareholders will be better off by not issuing shares, whenever the following relation is true:

$$V^{\text{old}}[\phi, \text{issue}, (a, b)] < V^{\text{old}}[\phi, \text{no issue}, (a, b)]$$

By substituting the expressions given by (5.9) and (5.11) for these two quantities, the following relation may be stated:

$$\frac{P'}{P'+E} (I + a + b) < (S + a)$$

By recalling that I must be equal to (E+S), this relation may be rewritten as:

$$\frac{P'}{P'+E} (E+b) < \frac{E}{P'+E} (S+a) \tag{5.13}$$

The term on the left is old shareholders' share in the new project, and the term on the right is what they give up of the existing business to new shareholders. The condition implied by (5.13) is that the new project should not be taken whenever the benefit derived by old shareholders is less than the cost of the decision.

A simpler way to write condition (5.13) for not issuing stock is as indicated below:

$$E + b < \frac{E}{P^*} (S+a) \quad (5.14)$$

Consider a plane  $(a,b)$  of all possible outcomes of the process  $(\tilde{A}, \tilde{B})$ .

Equation (5.14) defines a region to the right of line

$$E + b = \frac{E}{P^*} (S+a)$$

In Figure 9, this region is designated by  $M$  and its complement by  $M'$ ,

Formally stated:

$$M = \{(a,b) \geq 0 \mid (E+b) < \frac{E}{P^*} (S+a)\} \quad (5.15)$$

$$M' = \{(a,b) \geq 0 \mid (E+b) \geq \frac{E}{P^*} (S+a)\} \quad (5.16)$$

Notice that  $a$  and  $b$  have been restricted to be positive, because if  $a$  is negative, the firm is better off by liquidating the business, and if  $b$  is negative, the new project is not attractive. Therefore, even if there is a positive likelihood for  $\tilde{A}$ ,  $\tilde{B}$ , or both to become negative, the manager's reaction to this situation will prevent this event from happening.\*

After updating the information on the values  $(a,b)$  that existing and new businesses will take in period 1, and having considered the impact that the decision to issue or not to issue shares will render to old

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\* It is possible to find situations in which non-positive values for  $(a,b)$  make sense; for example, when the new project realization requires the continued operation of the current business. These cases are not included in this study.

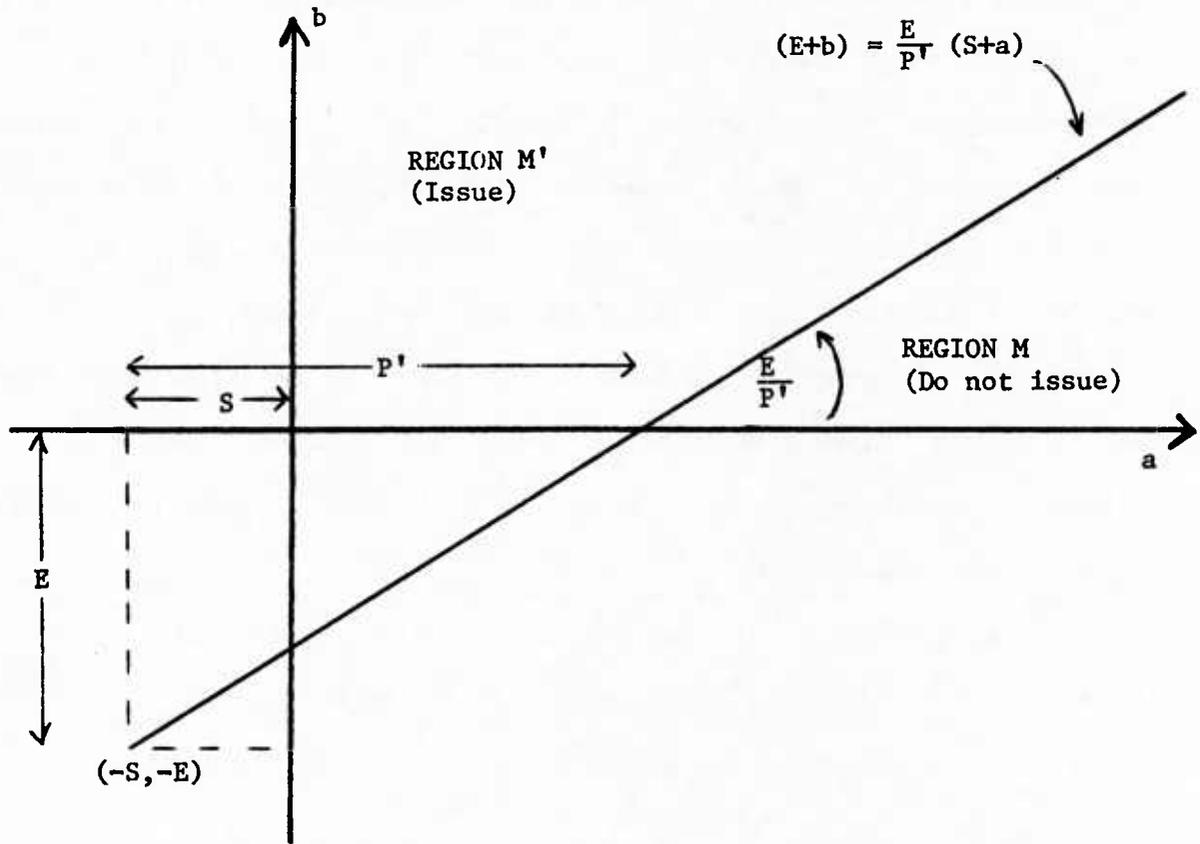


FIGURE 9: Regions M and M' in the Pure Equity Case

shareholders, managers will adopt the following decision rule when acting on behalf of their shareholders. If the pair (a,b) belongs in M, it is disadvantageous to issue shares and undertake the project. If the pair (a,b) belongs to M', shares should be issued and the project undertaken.

The decision to issue or not to issue shares conveys to the market the information that (a,b) is in M' or in M, respectively, and this information must be reflected in the market value of the firm. Therefore, after the firm's announcement of its decision, the market value will adjust to the new expectations, which are contingent upon the firm issuing or not issuing shares.

If the firm does not issue shares, the value of old shareholders' shares must go to  $V^{\text{old}}(\phi, \text{no-issue})$ , which is the expectation of the market value at time +1, contingent on  $(\tilde{A}, \tilde{B})$  being in M. Consequently:

$$V^{\text{old}}(\phi, \text{no-issue}) = \text{Ex} * \{V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B})] | (\tilde{A}, \tilde{B}) \text{ in } M\}$$

By using relation (5.9) for the case in which (a,b) is known to be in M, it is possible to write this expression as:

$$V^{\text{old}}(\phi, \text{no-issue}) = S + \bar{A}(M) \tag{5.17}$$

where:

$$\bar{A}(M) = \text{Expected value of } \tilde{A} \text{ contingent upon } (\tilde{A}, \tilde{B}) \text{ being in } M.$$

On the other hand, if the firm does issue shares, the value of old shareholders' shares must go to  $V^{\text{old}}(\phi, \text{issue})$ , which is the expectation of the market value of time +1, contingent on  $(\tilde{A}, \tilde{B})$  being in M'. Conse-

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\* Ex is used for Expected value.

quently:

$$V^{\text{old}}(\phi, \text{issue}) = \text{Ex} \{V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B})] | (\tilde{A}, \tilde{B}) \text{ in } M'\}$$

The use of relation (5.11) for the case in which (a,b) is known to be in  $M'$ , allows writing the expression above as:

$$V^{\text{old}}(\phi, \text{issue}) = \frac{P'}{P'+E} [I + \bar{A}(M') + \bar{B}(M')]$$

where:

$\bar{A}(M')$  = Expected value of  $\tilde{A}$  contingent upon  $(\tilde{A}, \tilde{B})$  being in  $M'$

$\bar{B}(M')$  = Similar but for  $\tilde{B}$ .

The expression above can be simplified to (5.18) by recalling that  $V^{\text{old}}(\phi, \text{issue})$  has been defined as  $P'$ , and that  $I$  is equal to  $(E+S)$ :

$$V^{\text{old}}(\phi, \text{issue}) = P' = S + \bar{A}(M') + \bar{B}(M') \quad (5.18)$$

The importance of this formula is that it provides an equation for computing the value of  $P'$  under equilibrium. In general, it will not be a simple equation to solve, because  $P'$  participates in the definition of regions  $M-M'$ , making  $\bar{A}(M')$  and  $\bar{B}(M')$  dependent on  $P'$ . But this equation can always be solved when the joint probability distribution of  $(\tilde{A}, \tilde{B})$  lies entirely in the positive quadrant, as is the case here. (The proof of this statement is given in Appendix 1.)

Given that the market can determine the value that the firm should attain after the announcement of issuing or not issuing shares, the market value of the firm at time  $-1$ , prior to that announcement, must

be the expected value of these two outcomes:

$$V^{\text{old}}(\phi) = V^{\text{old}}(\phi, \text{no-issue}) \cdot \text{Pr}(\text{no-issue}) + V^{\text{old}}(\phi, \text{issue}) \cdot \text{Pr}(\text{issue})$$

Replacing the values recently obtained in (5.17) and (5.18) for

$V^{\text{old}}(\phi, \text{no-issue})$ , and  $V^{\text{old}}(\phi, \text{issue})$ , the market value of the firm becomes:

$$V^{\text{old}}(\phi) = [S + \bar{A}(M)] \cdot \text{Pr}(M) + [S + \bar{A}(M') + \bar{B}(M')] \cdot \text{Pr}(M')$$

where:

$$\text{Pr}(M) = \text{Pr}(\text{no-issue}) = \text{Pr}\{(\tilde{A}, \tilde{B}) \text{ belongs in } M\}$$

$$\text{Pr}(M') = \text{Pr}(\text{issue}) = \text{Pr}\{(\tilde{A}, \tilde{B}) \text{ belongs in } M'\}.$$

This relation can be further simplified by noticing that:

$$\text{Pr}(M) + \text{Pr}(M') = 1$$

and

$$\bar{A}(M) \cdot \text{Pr}(M) + \bar{A}(M') \cdot \text{Pr}(M') = \bar{A}$$

where:

$\bar{A}$  = Unconditional expected value of  $\tilde{A}$  (the market value of the current line of business).

Using these two equalities, it is possible to express the market value of the firm prior to any announcement as:

$$V^{\text{old}}(\phi) = (S + \bar{A}) + \bar{B}(M') \cdot \text{Pr}(M') \tag{5.19}$$

This expression is identical to relation (5.2), in which the market value of the firm corresponds to the contribution of the current line of business plus the contribution of the investment opportunity when the

firm undertakes it (that is to say, when (a,b) belongs in M').

Another useful way of expressing the market value of the firm at time -1 may be derived by using the following identity:

$$\bar{B}(M) \cdot \text{Pr}(M) + \bar{B}(M') \cdot \text{Pr}(M') = \bar{B}$$

where:

$\bar{B}$  = Unconditional expected value of  $\tilde{B}$  (the market value of the investment opportunity).

Replacing in (5.19) the value for  $\bar{B}(M') \cdot \text{Pr}(M')$  given by this relation allows writing  $V^{\text{old}}(\phi)$  as:

$$V^{\text{old}}(\phi) = S + \bar{A} + \bar{B} - \bar{B}(M) \cdot \text{Pr}(M) \quad (5.20)$$

This formula shows that the amount  $\bar{B}(M) \cdot \text{Pr}(M)$  is being subtracted from the total market value of both the current line of business and the investment opportunity. This amount corresponds to the loss in market value due to insufficient slack, and it is defined as:

$$\Delta(\phi) = \bar{B}(M) \cdot \text{Pr}(M) \quad (5.21)$$

This loss is exactly equal to relation (5.4), and it is greater than zero whenever a project with a positive net present value must be disregarded, because when going to the market for the equity needed, old shareholders lose from the dilution more than what they get from the new project. (The outcome (a,b) is in region M.)

A final way of writing the market value of the firm at time -1 provides a new interpretation of this value. Relation (5.19) may also

be expressed as:

$$V^{\text{old}}(\phi) = (S + \bar{A}) + \pi \bar{B} \quad (5.22)$$

where:

$$\pi = \frac{\bar{B}(M) \cdot \text{Pr}(M')}{\bar{B}} = \frac{\bar{B}(M') \cdot \text{Pr}(M')}{\bar{B}(M) \cdot \text{Pr}(M) + \bar{B}(M') \cdot \text{Pr}(M')} \quad (5.23)$$

In these two expressions the market value of the firm is interpreted as capturing the full value of the current line of business plus a fraction of the value of the new project which goes to 100 percent when slack available approaches the total investment required.

A summary of the equilibrium conditions derived for a pure equity situation is provided in Figure 10.

#### 5.4 General Implications of Market Equilibrium in the Pure Equity Case

Many general conclusions can be advanced from the relations derived under market equilibrium in the pure equity case. Some of them are presented in this section to dig into the qualitative behavior of the model proposed.

##### 5.4.1 Issuing shares always drives down the market value of the firm

Before the firm announcement of issuing or not issuing shares, the market value of the firm corresponds to the weighted average of the values under each one of those two situations:

$$V^{\text{old}}(\phi) = V^{\text{old}}(\phi, \text{issue}) \cdot \text{Pr}(\text{issue}) + V^{\text{old}}(\phi, \text{no-issue}) \cdot \text{Pr}(\text{no-issue})$$

TIME (+1)

FIRM TAKES NEW PROJECT

FIRM DOES NOT TAKE NEW PROJECT

$$v^{\text{old}}[\phi, \text{issue}, (a, b)] = \frac{P'}{P'+E}(I+a+b) \dots (5.11)$$

$$v^{\text{old}}[\phi, \text{no-issue}, (a, b)] = S+a \dots (5.9)$$

TIME (0)

Definition of Region M'  $\equiv$  ISSUE

Definition of Region M  $\equiv$  DO NOT ISSUE

$$M' = \{(a, b) \geq 0 \mid (E+b) \geq \frac{E}{P'}(S+a)\} \dots (5.16)$$

$$M = \{(a, b) \geq 0 \mid (E+b) < \frac{E}{P'}(S+a)\} \dots (5.15)$$

$$v^{\text{old}}(\phi, \text{issue}) = P' = S+\bar{A}(M')+\bar{B}(M') \dots (5.18)$$

$$v^{\text{old}}(\phi, \text{no-issue}) = P = S+\bar{A}(M) \dots (5.17)$$

TIME (-1)

$$v^{\text{old}}(\phi) = (S+\bar{A}) + \bar{B}(M') \cdot \text{Pr}(M') \dots (5.19)$$

$$= S+\bar{A}+\bar{B}-\bar{B}(M) \cdot \text{Pr}(M) \dots (5.20)$$

$$= (S+\bar{A}) + \pi \bar{B} \dots (5.22)$$

$$\text{where } \pi = \frac{\bar{B}(M') \cdot \text{Pr}(M')}{\bar{B}(M) \cdot \text{Pr}(M) + \bar{B}(M') \cdot \text{Pr}(M')} \quad (5.23)$$

Loss due to insufficient slack

$$\Delta(\phi) = \bar{B}(M) \cdot \text{Pr}(M) \dots (5.21)$$

FIGURE 10: Conditions for Equilibrium in a Pure Equity Case

If shares are issued, the market value becomes  $V^{\text{old}}(\phi, \text{issue}) = P'$ ; while if shares are not issued, the market value goes to  $V^{\text{old}}(\phi, \text{no-issue})$ , which is greater than  $P'$  [because  $S+a < P'$  for all  $(a,b)$  in  $M$ ]. Therefore, the following relation has to be true given that  $V^{\text{old}}(\phi)$  is a weighted average of these two numbers:

$$V^{\text{old}}(\phi, \text{issue}) < V^{\text{old}}(\phi) < V^{\text{old}}(\phi, \text{no-issue})$$

If the loss for insufficient slack is zero,  $V^{\text{old}}(\phi)$  would be equal to  $V^{\text{old}}(\phi, \text{issue})$ . Then, the general relation between these two market values is:

$$V^{\text{old}}(\phi, \text{issue}) \leq V^{\text{old}}(\phi) \tag{5.24}$$

It may be concluded that the firm decision to issue shares cannot be interpreted as "good news" in the market. This may explain, at least in part, the reluctance to raise funds via new equity issues.

The unexpected conclusion from this analysis is that the positive action of taking the new project drives the market value of the firm downwards, while the negative action of not taking it, drives this value upwards. There is a clear incentive for managers "to lie". By not taking the project, they are making shareholders better off in the short run (at time 0). The problem is that, in the long run (at time +1), when the updated information is received in the market, the market value of current shares goes below the value they could have with the new project.

This opportunity to fool the market appears because the firm deviates from its presupposed behavior. The model presented in this chapter considers that the firm pursue in a consistent way the policy of maximizing the

long run market value of the firm, though at time 0 this policy may depress the market value. Alternative equilibrium models may be constructed for different policies. The important conclusions from these observations is that the separation theorem needs a more careful statement. The current formulation only says that market value of the firm must be maximized; but, the consideration of internal information exceeding public information leads to the need to specify the time span.

5.4.2 The market value of the firm goes down when investment required goes up (for a fixed amount of slack)

The money required by the firm to undertake the new project is  $I$ . The firm has an amount  $S$  of slack available ( $S < I$ ), and it has to get the remaining  $E = (I-S)$  in the market. The claim being made is that if the investment required goes up, the market value of the firm goes down. The proposition is proven in Appendix 2. This is an expected result, because for a smaller firm reliance on external equity financing, the share of the current business that has to be given up to new shareholders goes down, and the overall situation becomes more attractive for the undertaking of the new project.

This is reflected in a greater market value of old shareholders' shares (a smaller loss due to insufficient slack).

5.4.3 The market value of the firm goes up when slack available goes up (for a fixed investment)

The arguments behind this statement are similar to the previous case. Basically, a greater slack availability reduces the firm's reliance on external equity financing. This improves the position of old shareholders,

because what they have to give up of the current business goes down.\*

#### 5.4.4 Cases in which slack unavailability does not hurt

There are some extreme situations in which slack unavailability is not detrimental for the firm's market value. In the first place, assume that the market values of current business and future opportunity at time +1 can be known by both the firm and the market at time 0 [ $(\tilde{A}, \tilde{B}) = (\bar{A}, \bar{B})$ ]; that is to say, there is no difference between public and insider information.

In this case, the loss due to insufficient slack is 0, independent of the slack available. This conclusion is perfectly consistent with actual propositions in finance theory, showing that the loss in market value results from the joint impact of slack unavailability, and difference of information.

To prove that the loss due to insufficient slack is 0, it is sufficient to show that  $P' = (S + \bar{A} + \bar{B})$  is always an equilibrium solution, and the probability of being in region M is 0 (see Figure 11). In fact, given that all mass is concentrated in region M', the following relations are satisfied:

$$\bar{A}(M') = \bar{A}$$

$$\bar{B}(M') = \bar{B}$$

$$P' = S + \bar{A}(M') + \bar{B}(M') = S + \bar{A} + \bar{B}$$

$$\text{Pr}(M) = 0$$

$$\Delta = \text{Pr}(M) \cdot \bar{B}(M) = 0$$

---

\* A formal proof of this statement is not included, because the arguments are similar to the ones given in Appendix 2. In addition, this case can be reduced to a case in which slack is the same and the equity required is smaller (as in Appendix 2), allowing the extension of all conclusions drawn in there to this case.

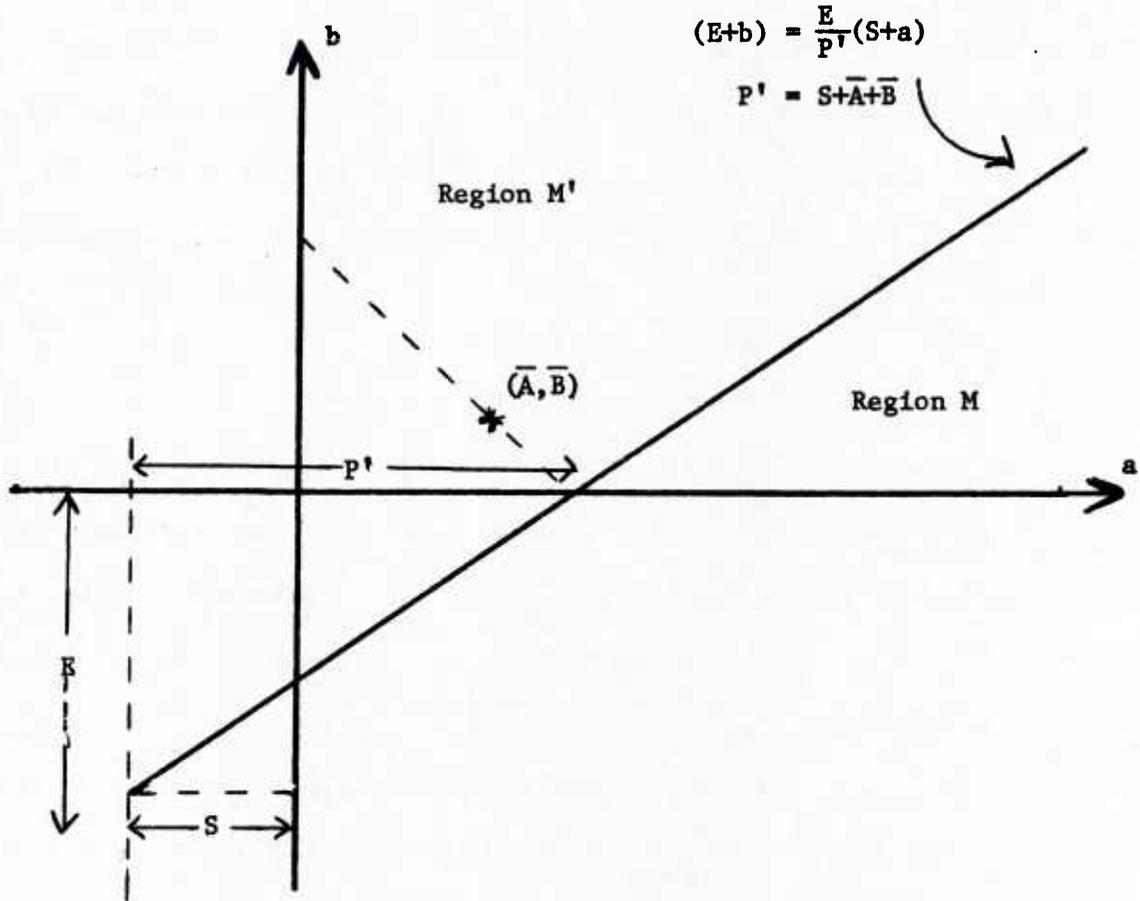


FIGURE 11: Equilibrium Situation with No Difference in Information

A similar conclusion can be drawn if there is some uncertainty, but all mass is concentrated in a "small" neighborhood of  $(\bar{A}, \bar{B})$ . The idea is that the entire probability distribution lies in region  $M'$  when  $P' = (S + \bar{A} + \bar{B})$ .

An interesting situation in which this condition is satisfied is when only  $\tilde{B}$  is a random variable,  $\tilde{A}$  being always equal to  $\bar{A}$  (see Figure 12). This means that the firm is always willing to issue shares when the existing business is a very stable one, and the amount of this business they give up to new shareholders is a constant independent of the outcome  $\tilde{B}$ .

[It is  $\frac{E}{I + \bar{A} + \bar{B}} (S + \bar{A})$ .]

5.4.5 Cases in which slack unavailability does hurt

In general, when the distribution of  $(\tilde{A}, \tilde{B})$  is spread all over the positive quadrant of space  $(a, b)$ , there is a positive loss for insufficient slack that depends on all the parameters of the problem. This case is analyzed in some detail in the next section for some special probability distributions.

This section presents a particular case in which slack is valuable. Assume that the market value of the new project is well known in advance (and equal to  $\bar{B}$ ), while the current business has a random market value (see Figure 13).

It may be appreciated that  $P' = (S + \bar{A} + \bar{B})$  cannot be an equilibrium solution in this case, because:

$$\begin{array}{r} \bar{A}(M') < \bar{A} \\ \bar{B}(M') = \bar{B} \\ \hline \text{Then: } S + \bar{A}(M') + \bar{B}(M') < S + \bar{A} + \bar{B} \\ \text{or } S + \bar{A}(M') + \bar{B}(M') < P' \end{array}$$

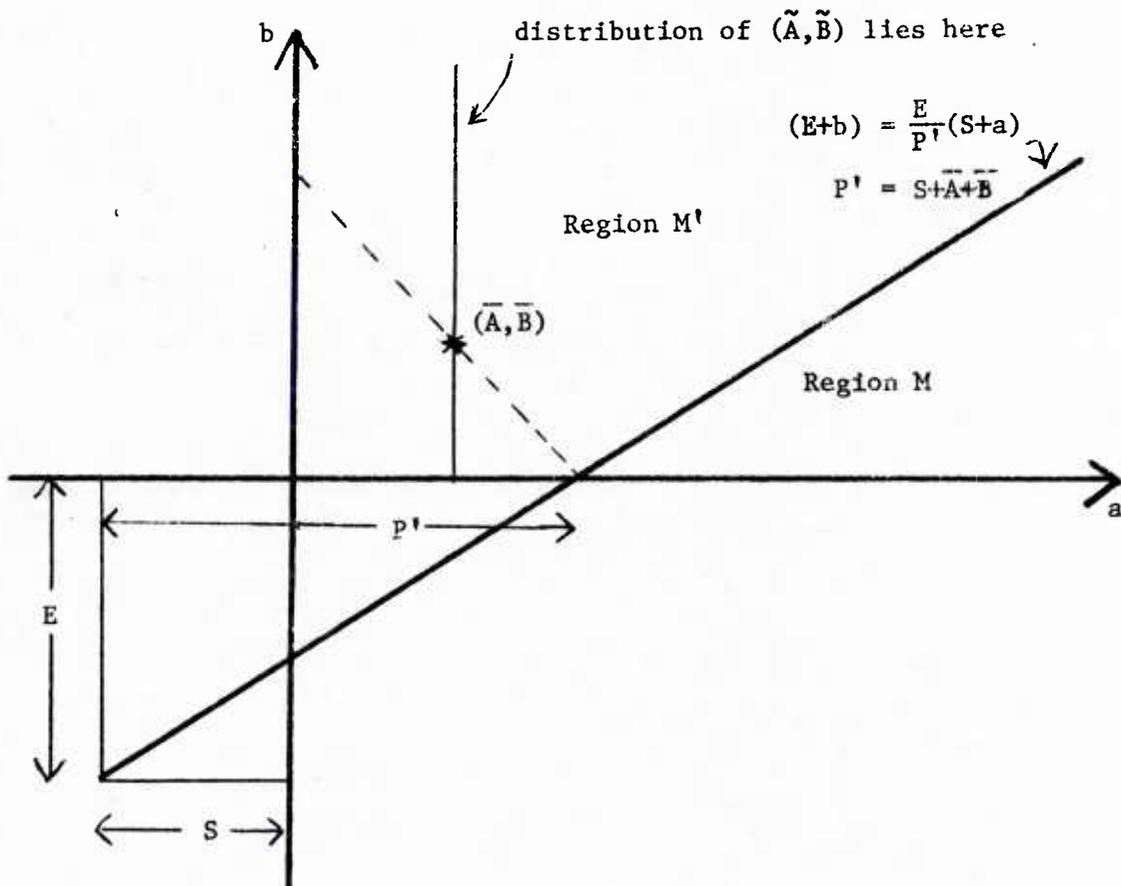


FIGURE 12: Equilibrium Situation When the Market Value on the Current Business is Known

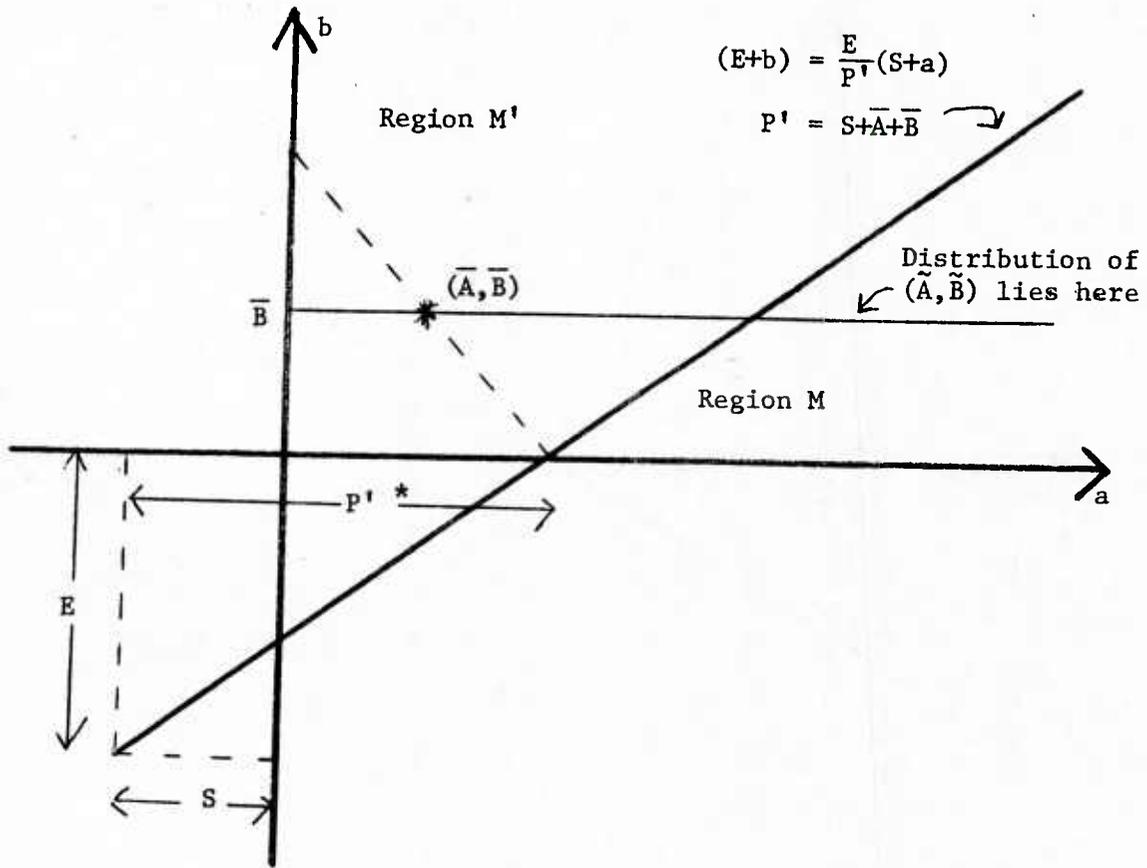


FIGURE 13: Equilibrium Situation When the Market Value of the New Project is Known

\*  $P' = (S+\bar{A}+\bar{B})$  is not an equilibrium solution.

The actual  $P'$  of equilibrium is less than  $(S + \bar{A} + \bar{B})$ , and there is always a positive loss due to insufficient slack, because region M is nonempty, and the expected value of  $\tilde{B}$  in region M is equal to  $\bar{B} > 0$ . The reason for this is that when the existing business has a very high market value, old shareholders prefer to discard the new project rather than sharing the existing business with new shareholders.

If the net present value of the new project goes to 0 ( $\bar{B} = 0$ ), the equilibrium value for  $P'$  is

$$P' = S + a_{\min}$$

where:

$$a_{\min} = \text{Minimum value of } \tilde{A}.$$

This is because for  $P' = S + a^\circ$  with  $a^\circ > a_{\min}$ , the expected value of  $[S + \bar{A}(M') + \bar{B}(M')]$  is less than  $P'$ ; therefore, it cannot be an equilibrium solution. In fact, take  $P' = S + a^\circ$ ; then, the following relations follow:

$$\bar{B}(M') = 0 \quad (b = \bar{B} = 0 \text{ by assumption})$$

$$\frac{\bar{A}(M') < a^\circ}{\text{(because no value } a > a^\circ \text{ can be in } M', \text{ and at least } a_{\min} < a^\circ \text{ is in } M').}$$

Then:  $S + \bar{A}(M') + \bar{B}(M') < S + a^\circ$

or  $S + \bar{A}(M') + \bar{B}(M') < P'$

The conclusion from this result is that when a firm in a risky business ( $\tilde{A}$  is random) is confronted with a fair project ( $b = \bar{B} = 0$  for sure), this project can be taken only in the worst possible scenario for the existing business ( $\tilde{A} = a_{\min}$ ). If the probability distribution of  $\tilde{A}$  is assumed to be continuous, the probability of this event is 0; that is to say, the new project should never be taken.

#### 5.4.6 General case of perfect correlation between $(\tilde{A}, \tilde{B})$

Consider that A and B are perfectly correlated; that is to say, their probability distribution lies in a straight line. In this case, the following relation is satisfied:

$$\tilde{B} - \bar{B} = m(\tilde{A} - \bar{A})$$

where:

$$m = \rho_{AB} \frac{\sigma_B}{\sigma_A}$$
$$\rho_{AB} = \pm 1$$

It may be seen graphically in Figure 14 that  $P^1 = (S + \bar{A} + \bar{B})$  is an equilibrium value (slack is not valuable), when  $m \leq -1$  or  $m \geq E / (S + \bar{A} + \bar{B})$ .

In the range  $-1 < m < E / (S + \bar{A} + \bar{B})$  there is a positive loss for slack unavailability. Under this analysis, a positive correlation [ $m \geq E / (S + \bar{A} + \bar{B})$ ] is better than a negative one in some cases ( $-1 < m < 0$ ), because the loss due to insufficient slack is zero in the first case and positive in the second one. This contradicts the notion that countercyclicality per se is a good thing to have.

#### 5.4.7 Uniqueness of the equilibrium solution

There are situations in which more than one stable equilibrium solution exists. An example of one of those cases is given in Figures 15 and 16.

There are two equilibrium situations for  $P^1 = 30$  and 40, respectively. If  $P^1 = 30$ , the firm is unable to take the new project when the outcome

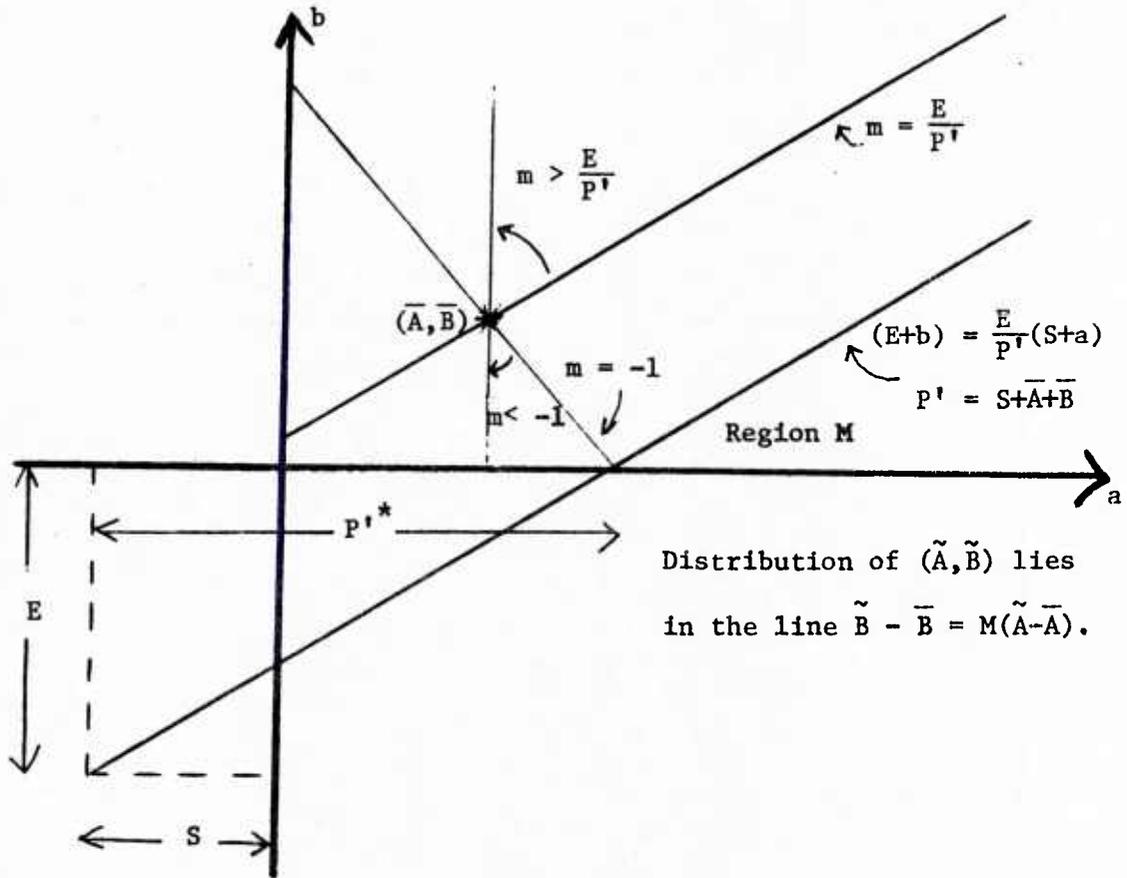


FIGURE 14: Perfect Correlation Between Market Value of the Firm and New Project

\*  $P' = (S+A+B)$  is an equilibrium value only for  $m < -1$  or  $m > \frac{E}{P'}$ .

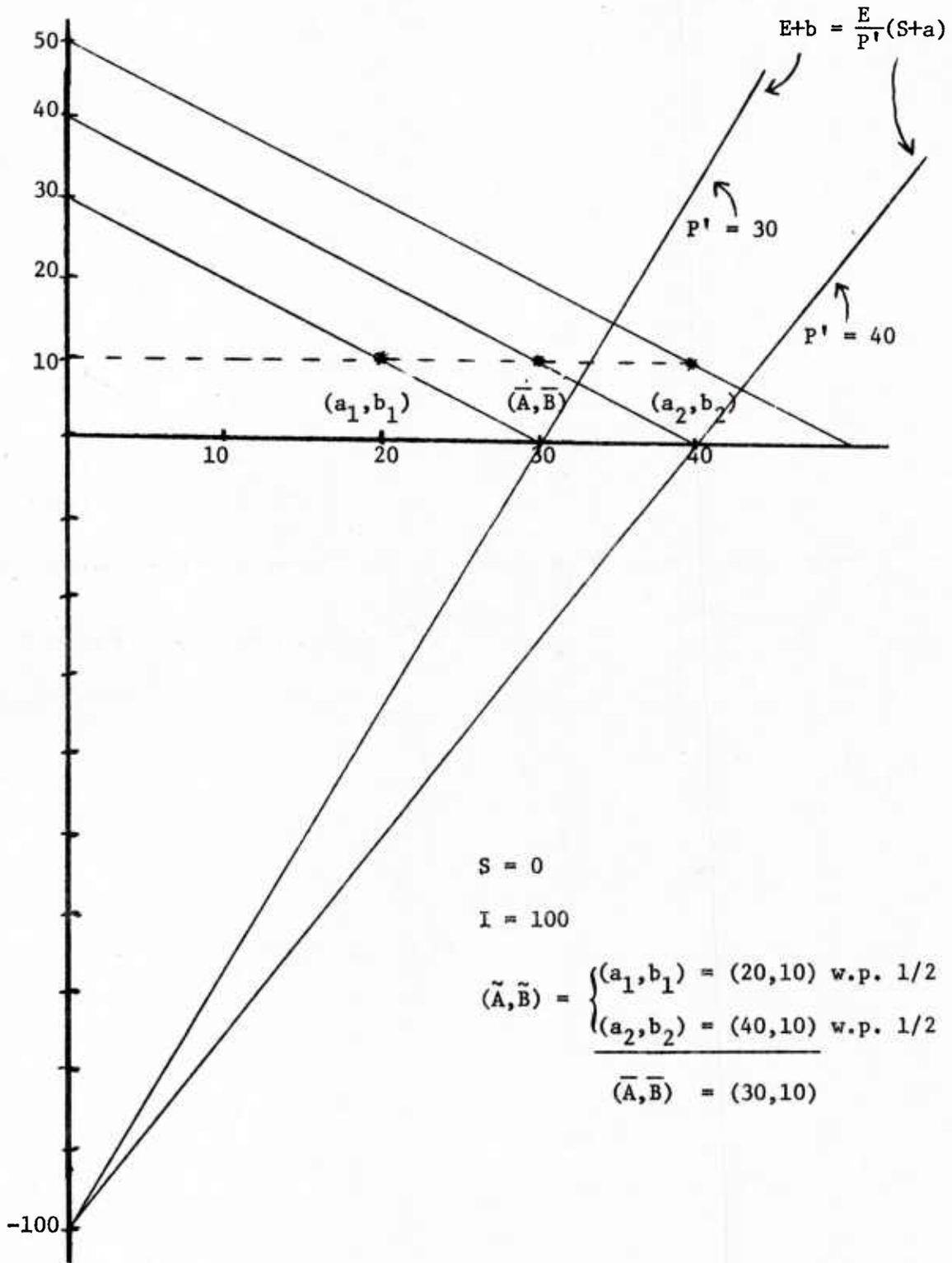


FIGURE 15: Example of a Situation with More than One Stable Equilibrium Solution

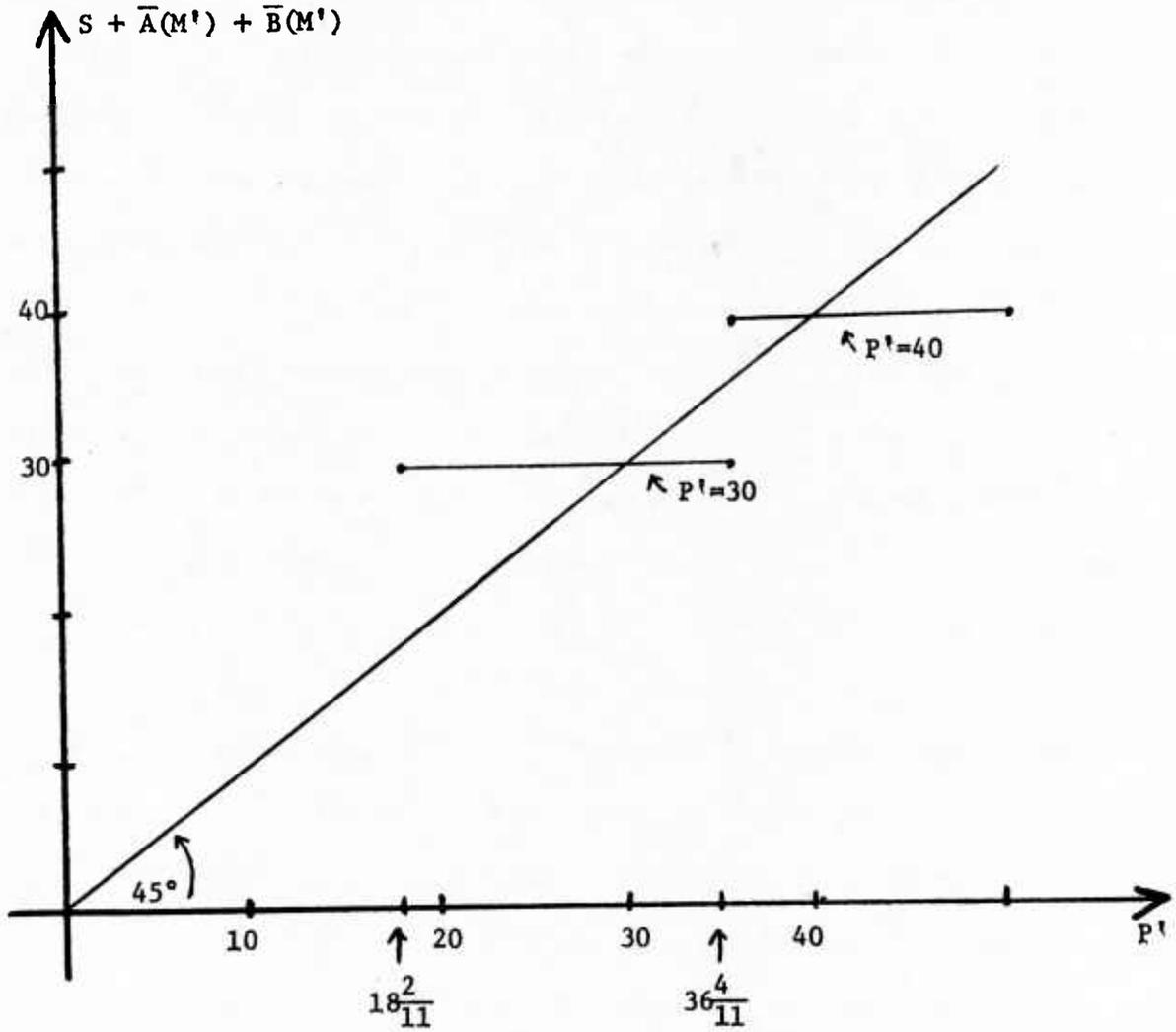


FIGURE 16: Example of a Situation with More Than One Stable Equilibrium  
Solution: The  $P'$  vs.  $S + \bar{A}(M') + \bar{B}(M')$  Graph.

is  $(a_2, b_2)$ , this implying a net loss due to insufficient slack of  $\frac{1}{2} b_2 = 5$ . On the other hand, if  $P' = 40$ , the firm can always take the new project, and the loss due to insufficient slack is zero.

The market value of the firm goes up when the value of  $P'$  goes up; therefore, for shareholders, it is always better to have the maximum value of  $P'$ . But the choice of the equilibrium  $P'$  in the model presented in this study is an exogenous decision. There is no argument to justify the selection of one or other stable equilibrium; it is a genuine "degree of freedom".

The way in which the market reaches a consensus when more than one stable equilibrium is possible, is an open question in this study. A tentative explanation is that it depends on the "mood" of the market; when the level of optimism is high, and the market is in an upswing, the choice goes to the maximum  $P'$ ; while, if the opposite is true, another  $P'$  is chosen. Another explanation is that  $P'$  is selected in accordance to the "image of the firm"; if the firm appears as a "solid" and "serious" organization, the market may pick the high  $P'$ , while a firm with a history of troubles will be assigned a low  $P'$ , this aggravating the already difficult situation of the firm.

For the purposes of this study, the selection of  $P'$  under these circumstances is arbitrarily taken as the maximum equilibrium value. No attempt has been made in this study to determine the frequency of this multiple equilibrium situation, but some isolated numerical explorations done with a well behaved continuous distribution showed a unique value for  $P'$ .

### 5.5 Sensitivity Analysis in the Pure Equity Case

This section presents an algorithm for finding the equilibrium market value of a firm when the joint probability distribution  $f(a,b)$  is specified in fairly general terms. The algorithm is applied to a bivariate log-normal distribution and to some cases of a truncated bivariate normal distribution.\*

#### 5.5.1 Algorithm for finding $P'$

The value of  $P'$  corresponds to the solution of equation (5.18), which is:

$$P' = S + \bar{A}(M') + \bar{B}(M')$$

where:

$$M' = \{(a,b) \geq 0 \mid (E+b) \geq \frac{E}{P'} (S+a)\}$$

The parameters considered fixed in this equation are  $S$ ,  $E$  (with  $S+E = I$ ), and the probability distribution  $f(a,b)$ . This distribution is assumed to be discrete, and it is fully specified by the parameters  $\{p_{ij}, a_i, b_j\}$ , where:

$$p_{ij} = \Pr\{\tilde{A} = a_i, \tilde{B} = b_j\} \quad (i=1, \dots, N_i), (j=1, \dots, N_j).$$

The algorithm for finding  $P'$  is a common procedure for solving an equation. It is summarized in Figure 17, and it is described below:

---

\* The probability of having  $a < 0$  or  $b < 0$  is concentrated in  $a = 0$  and  $b = 0$ , respectively, where the bivariate normal distribution is used. The probability distribution thus obtained is mixed discrete-continuous, with a non-zero probability of having  $a$  or  $b$  equal 0.

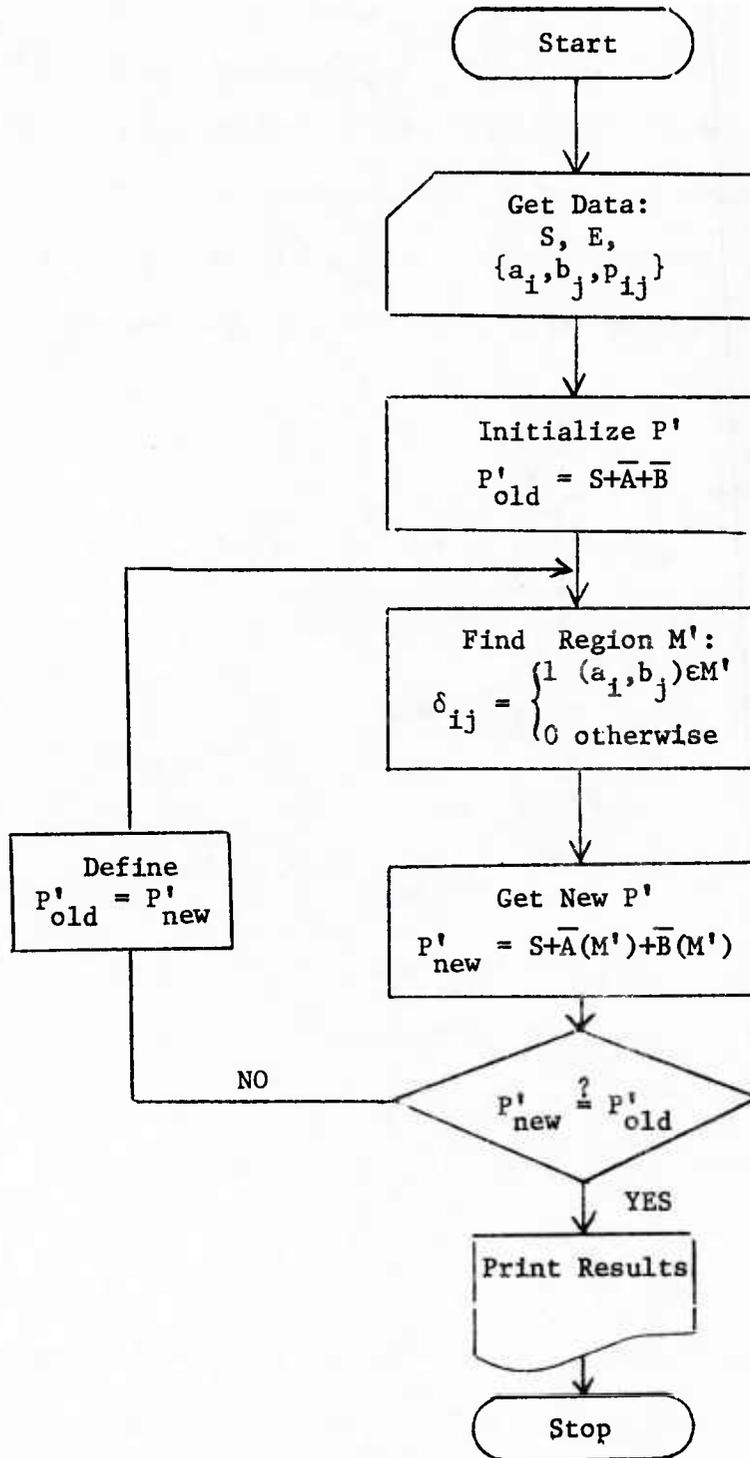


FIGURE 17: Algorithm for Finding P' in the Pure Equity Case

Step 1: Get Data:

S, E, Distribution  $f(a,b) : \{a_i, b_j, p_{ij}\}$ .

Step 2: Initialize the value of  $P'$  as:

$$\begin{aligned} P'_{old} &= S + \bar{A} + \bar{B} \\ &= S + \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} (a_i + b_j) p_{ij} \end{aligned}$$

Step 3: Find region  $M'$  defined by  $P'_{old}$ .

Define:

$$\begin{aligned} \delta_{ij} &= 1 \text{ if } (a_i, b_j) \text{ belongs in } M'; \text{ that is to say:} \\ &\quad (E + b_j) \geq \frac{E}{P'}(S + a_i) \end{aligned}$$

$$\delta_{ij} = 0 \text{ otherwise.}$$

Step 4: Get the new value for  $P'$ :

$$\begin{aligned} P'_{new} &= S + \bar{A}(M') + \bar{B}(M') \\ &= S + \left[ \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \delta_{ij} (a_i + b_j) p_{ij} \right] / \left[ \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \delta_{ij} p_{ij} \right] \end{aligned}$$

Step 5: Check if  $P'_{old}$  and  $P'_{new}$  are equal:

- If  $P'_{new} = P'_{old}$ , then PRINT results and STOP.

- If  $P'_{new} \neq P'_{old}$ , then define:

$$P'_{old} = P'_{new}$$

and go back to Step 3.

There are two technical points about this algorithm that are discussed in Appendix 3: Convergence and uniqueness. The algorithm is proven to converge always, and to give the highest solution for  $P'$  in case more than one exists.

### 5.5.2 Generation of a truncated bivariate normal distribution

To generate the truncated distribution and, later, the bivariate log-normal, it is necessary to have a bivariate normal distribution. Consider that  $(\tilde{A}, \tilde{B})$  follows a bivariate normal distribution with means  $(\bar{A}, \bar{B})$ , standard deviations  $(\sigma_A, \sigma_B)$ , and correlation coefficient  $\rho_{ab}$ . This section presents an algorithm for generating this probability distribution with a standard table for the normal distribution. This is important for computer efficiency in terms of time for data input and cost of running each trial. The algorithm is based on the generation of  $(\tilde{A}, \tilde{B})$  from two independent, identically distributed standardized normal random variables  $(\tilde{u}, \tilde{v})$ . (Expected values of  $\tilde{u}$  and  $\tilde{v}$  are 0, and their standard deviation is 1.)

Define  $(\tilde{A}, \tilde{B})$  as follows:

$$\tilde{A} = \bar{A} + \sigma_u \cdot \cos\psi \tilde{u} - \sigma_v \sin\psi \tilde{v} \quad (5.25)$$

$$\tilde{B} = \bar{B} + \sigma_u \cdot \sin\psi \tilde{u} + \sigma_v \cos\psi \tilde{v} \quad (5.26)$$

$\tilde{A}$  and  $\tilde{B}$  are the linear combination of two independent random variables.

Therefore, they are also normal. Their parameters are:

$$\text{Ex}(\tilde{A}) = \bar{A}, \quad \text{Ex}(\tilde{B}) = \bar{B} \quad (5.27)$$

$$\text{Var}(\tilde{A}) = \sigma_u^2 \cos^2\psi + \sigma_v^2 \sin^2\psi \quad (5.28)$$

$$\text{Var}(\tilde{B}) = \sigma_u^2 \sin^2\psi + \sigma_v^2 \cos^2\psi \quad (5.29)$$

$$\text{Cov}(\tilde{A}, \tilde{B}) = (\sigma_u^2 - \sigma_v^2) \sin\psi \cdot \cos\psi \quad (5.39)$$

Selecting  $\sigma_u$ ,  $\sigma_v$ , and  $\psi$  to make  $\text{Var}(\tilde{A}) = \sigma_A^2$ ,  $\text{Var}(\tilde{B}) = \sigma_B^2$ , and

$\text{Cov}(\tilde{A}, \tilde{B}) = \rho_{AB} \sigma_A \sigma_B$ , allows having  $(\tilde{A}, \tilde{B})$  expressed in terms of  $(\tilde{u}, \tilde{v})$ .

These three equations are equivalent to:

$$\begin{aligned}\sigma_u^2 + \sigma_v^2 &= \sigma_A^2 + \sigma_B^2 \\ (\sigma_u^2 - \sigma_v^2) \cos 2\psi &= \sigma_A^2 - \sigma_B^2 \\ (\sigma_u^2 - \sigma_v^2) \sin 2\psi &= 2\rho_{AB}\sigma_A\sigma_B\end{aligned}$$

The general solution of this system (obtained squaring the last two equations and adding them) is:\*

$$\sigma_u = \sqrt{\frac{1}{2}(\sigma_A^2 + \sigma_B^2) + \frac{1}{2} \sqrt{(\sigma_A^2 - \sigma_B^2)^2 + 4\rho_{AB}^2 \sigma_A^2 \sigma_B^2}} \quad (5.31)$$

$$\sigma_v = \sqrt{\frac{1}{2}(\sigma_A^2 + \sigma_B^2) - \frac{1}{2} \sqrt{(\sigma_A^2 - \sigma_B^2)^2 + 4\rho_{AB}^2 \sigma_A^2 \sigma_B^2}} \quad (5.32)$$

$$\psi = \frac{1}{2} \arccos \frac{\sigma_A^2 - \sigma_B^2}{\sqrt{(\sigma_A^2 - \sigma_B^2)^2 + 4\rho_{AB}^2 \sigma_A^2 \sigma_B^2}} \quad (5.33)$$

\* An equivalent way of expressing this solution is the following one:

For  $\sigma_A^2 \neq \sigma_B^2$ :

$$\begin{aligned}\psi &= \frac{1}{2} \operatorname{arctg} \frac{2\rho_{AB}\sigma_A\sigma_B}{\sigma_A^2 - \sigma_B^2} \\ \sigma_u^2 &= \frac{1}{2}(\sigma_A^2 + \sigma_B^2) + \frac{1}{2} \frac{\sigma_A^2 - \sigma_B^2}{\cos 2\psi} \\ \sigma_v^2 &= \frac{1}{2}(\sigma_A^2 + \sigma_B^2) - \frac{1}{2} \frac{\sigma_A^2 - \sigma_B^2}{\cos 2\psi}\end{aligned}$$

For  $\sigma_A^2 = \sigma_B^2 = \sigma^2$

$$\psi = \pi/4$$

$$\sigma_u^2 = \sigma^2(1+\rho)$$

$$\sigma_v^2 = \sigma^2(1-\rho)$$

The algorithm to generate the distribution of (A,B) is the following (see Figure 18):

Step 1: Get parameters of distribution f(a,b):

$$\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB}$$

Step 2: Get table for standard normal random variable  $\tilde{Z}$ :

$$p_i = \Pr\{\tilde{Z} = Z_i\}^* \quad i=1, \dots, N_Z$$

Step 3: Compute values of  $\sigma_u, \sigma_v, \psi$  from relations (5.31), (5.32), and (5.33), respectively.

Step 4: Generate values  $(a_i, b_j)$  with relations (5.25) and (5.26) applied to a pair  $(u_i, v_j)$  (see below) and  $p_{ij} = p_i \cdot p_j$  for all combinations  $i, j = 1, \dots, N_Z$ .

$$a_i = \bar{A} + \sigma_u \cos\psi u_i - \sigma_v \sin\psi v_j \quad (5.34)$$

$$b_j = \bar{B} + \sigma_u \sin\psi u_i + \sigma_v \cos\psi v_j \quad (5.35)$$

Step 5: When any one of  $\tilde{A}$  and  $\tilde{B}$  goes negative, the firm is supposed to react for making this value 0 (liquidation of  $\tilde{A} < 0$ , or not taking the project if  $\tilde{B} < 0$ ). Therefore, the actual distribution of  $(\tilde{A}, \tilde{B})$  is truncated at 0:

For all  $(a_i, b_j)$  redefine:

$$a_i = \max(0, a_i)$$

$$b_j = \max(0, b_j)$$

- 
- \* 1) Only positive values need to be given, because distribution is symmetric.  
 2) The selection of values  $Z_i$  (their number and distance) is determinant of the closeness that this discrete distribution has with the normal.

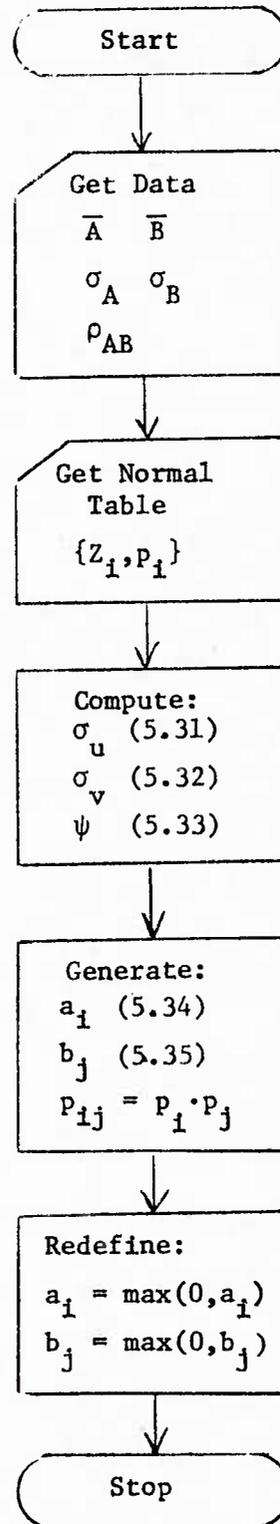


FIGURE 18: Algorithm for Generating a Bivariate Normal Distribution Truncated at 0

### 5.5.3 Generation of a bivariate log normal distribution

Assume for this case that  $(\tilde{A}, \tilde{B})$  follows a bivariate lognormal distribution; that is to say,  $(\ln \tilde{A}, \ln \tilde{B})$  follows a bivariate normal one.

Define the following terms:

$$\tilde{X} = \ln \tilde{A}$$

$$\tilde{Y} = \ln \tilde{B}$$

$$(\bar{A}, \bar{B}) = \text{Expected values of } (\tilde{A}, \tilde{B})$$

$$(\bar{X}, \bar{Y}) = \text{Expected values of } (\tilde{X}, \tilde{Y})$$

$$(\sigma_A, \sigma_B) = \text{Standard deviations of } (\tilde{A}, \tilde{B})$$

$$(\sigma_X, \sigma_Y) = \text{Standard deviations of } (\tilde{X}, \tilde{Y})$$

$$\rho_{AB} = \text{Correlation coefficient of } (\tilde{A}, \tilde{B})$$

$$\rho_{XY} = \text{Correlation coefficient of } (\tilde{X}, \tilde{Y})$$

The problem is finding an algorithm for generating the joint distribution of  $(\tilde{A}, \tilde{B})$  with a standard table for the normal distribution. This is done in two steps: first, finding the equivalence between the two sets of parameters  $(\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB})$  and  $(\bar{X}, \bar{Y}, \sigma_X, \sigma_Y, \rho_{XY})$ ; and second, having the random variables  $(\tilde{X}, \tilde{Y})$  expressed in terms of two independent, standard, normal random variables  $(\tilde{u}, \tilde{v})$ . This second step corresponds to the previous algorithm, so the attention is focused in passing from  $(\tilde{A}, \tilde{B})$  to  $(\tilde{X}, \tilde{Y})$ .

The distribution of  $(\tilde{A}, \tilde{B})$  is completely determined by the parameters  $(\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB})$ . Assuming that all these parameters are known, the problem is to determine  $(\bar{X}, \bar{Y}, \sigma_X, \sigma_Y, \rho_{XY})$ . It may be proven that the following system of equations links these two sets of parameters (see

Appendix 4):

$$\bar{A} = e^{\frac{\bar{x} + \frac{1}{2}\sigma_x^2}{x}} \quad (5.36)$$

$$\bar{B} = e^{\frac{\bar{y} + \frac{1}{2}\sigma_y^2}{y}} \quad (5.37)$$

$$\sigma_A^2 = e^{\frac{2\bar{x} + \sigma_x^2}{x}} \left( e^{\frac{\sigma_x^2}{x}} - 1 \right) \quad (5.38)$$

$$\sigma_B^2 = e^{\frac{2\bar{y} + \sigma_y^2}{y}} \left( e^{\frac{\sigma_y^2}{y}} - 1 \right) \quad (5.39)$$

$$\rho_{AB} = \frac{e^{\frac{\rho_{xy}\sigma_x\sigma_y}{x} - 1}}{\sqrt{\left( e^{\frac{\sigma_x^2}{x}} - 1 \right) \left( e^{\frac{\sigma_y^2}{y}} - 1 \right)}} \quad (5.40)$$

Solving for  $(\bar{X}, \bar{Y}, \sigma_X, \sigma_Y, \rho_{XY})$ , the following relations are obtained:

$$\bar{X} = \ln \frac{\bar{A}}{\sqrt{1 + cv_a^2}} \quad (5.41)$$

$$\bar{Y} = \ln \frac{\bar{B}}{\sqrt{1 + cv_b^2}} \quad (5.42)$$

$$\sigma_X = \sqrt{\ln(1 + cv_a^2)} \quad (5.43)$$

$$\sigma_Y = \sqrt{\ln(1 + cv_b^2)} \quad (5.44)$$

$$\rho_{XY} = \frac{\ln(1 + \rho_{AB} cv_a cv_b)}{\sqrt{\ln(1 + cv_a^2)} \sqrt{\ln(1 + cv_b^2)}} \quad *$$

\* If parameters  $(\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB})$  are picked arbitrarily, it has to be checked that  $-1 \leq \rho_{XY} \leq 1$  to test their consistency. If this relation is not satisfied, a lognormal distribution with those parameters cannot exist.

where:

$$cv_a = \frac{\bar{A}}{\sigma_A} = \text{coefficient of variation of } \tilde{A} \quad (5.46)$$

$$cv_b = \frac{\bar{B}}{\sigma_B} = \text{coefficient of variation of } \tilde{B} \quad (5.47)$$

The algorithm for generating the bivariate log normal distribution, can be summarized in the following steps (see Figure 19):

Step 1: Get parameters of distribution  $f(a,b)$ :

$$\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB}$$

Step 2: Get table for standard normal random variable  $\tilde{Z}$ :

$$p_i = \Pr\{\tilde{Z} = Z_i\} \quad i=1, \dots, N_Z$$

Step 3: Compute  $\bar{X}, \bar{Y}, \sigma_X, \sigma_Y, \rho_{XY}$  with relations (5.41) through (5.45).

Step 4: Compute  $\sigma_u, \sigma_v, \psi$  with relations (5.31), (5.32), (5.33)

(replace  $\sigma_A$  for  $\sigma_X$ ,  $\sigma_B$  for  $\sigma_Y$ , and  $\rho_{AB}$  for  $\rho_{XY}$ ), as indicated below:

$$\sigma_u = \sqrt{\frac{1}{2}(\sigma_X^2 + \sigma_Y^2) + \frac{1}{2}\sqrt{(\sigma_X^2 - \sigma_Y^2)^2 + 4\rho_{XY}^2\sigma_X^2\sigma_Y^2}} \quad (5.48)$$

$$\sigma_v = \sqrt{\frac{1}{2}(\sigma_X^2 + \sigma_Y^2) - \frac{1}{2}\sqrt{(\sigma_X^2 - \sigma_Y^2)^2 + 4\rho_{XY}^2\sigma_X^2\sigma_Y^2}} \quad (5.49)$$

$$\psi = \frac{1}{2} \arccos \frac{\sigma_X^2 - \sigma_Y^2}{\sqrt{(\sigma_X^2 - \sigma_Y^2)^2 + 4\rho_{XY}^2\sigma_X^2\sigma_Y^2}} \quad (5.50)$$

Step 5: Generate values of  $(a_i, b_j)$  and  $p_{ij}$ , with the following relations:

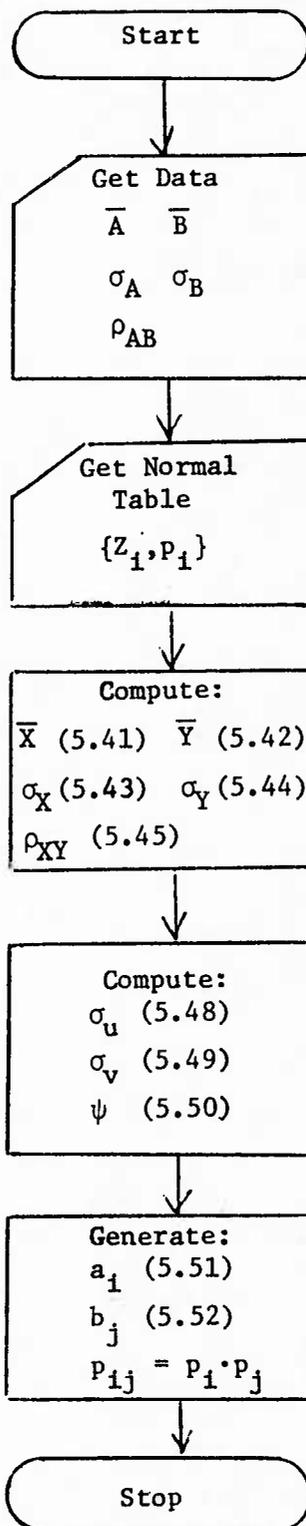


FIGURE 19: Algorithm for Generating a Bivariate Log-Normal Distribution

$$a_i = e^{x_i} = e^{\bar{x} + \sigma_u \cos\psi u_i - \sigma_v \sin\psi v_j} \quad (5.51)$$

$$b_j = e^{y_j} = e^{\bar{y} + \sigma_u \sin\psi u_i + \sigma_v \cos\psi v_j} \quad (5.52)$$

$$P_{ij} = P_i \cdot P_j$$

#### 5.5.4 Results obtained

Previous sections have presented well defined algorithms to generate the bivariate log-normal and truncated normal probability distribution, and to find the equilibrium value  $P^*$ . It should be stressed that these algorithms are in no way unique, and that different procedures can be devised to accomplish these same ends.

The numerical analysis done in the pure equity case is intended to explore the importance of slack availability under many different combinations of the situational parameters. Most of the analysis is done for  $(\tilde{A}, \tilde{B})$  assumed to be bivariate log-normal, but some conclusions from this analysis, that may appear controversial, have been established also with the truncated normal.

The set of parameters that fully describe the situation is:  $I, S$  ( $S \leq I, E = I - S$ ),  $\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB}$ . To determine the importance of slack under rather extreme situations, the problem has been solved for the following combinations of parameters:\*

$\bar{A} = 100$  (taken as a reference value)

$I = 10$  and  $100$

$S = 0\%$  of  $I, 50\%$  of  $I, 100\%$  of  $I$

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\* Computer program has been written in APL language and a copy is presented in Appendix 5.

$$\bar{B} = 1\% \text{ of } I, 10\% \text{ of } I$$

$$\sigma_A = 10\% \text{ of } \bar{A}, 100\% \text{ of } \bar{A}$$

$$\sigma_B = 10\% \text{ of } \bar{B}, 100\% \text{ of } \bar{B}$$

$$\rho_{AB} = \text{minimum negative value allowed, } 0, \text{ maximum negative value allowed.}^*$$

The results of the analysis are presented in Tables 4 through 15.

The following conclusions may be advanced from the tables presented:

- i. Increasing the slack (S) always reduces the loss in market value ( $\Delta/\bar{B}$ ) [and increases the probability of undertaking the new project:  $P(M')$ ].
- ii. Increasing the expected return on the new project ( $\bar{B}/I$ ) always reduces the loss in market value ( $\Delta/\bar{B}$ ).
- iii. Increasing the investment required (I) always increases the loss in market value ( $\Delta/\bar{B}$ ) when the expected net addition to the market value brought in by the new project is constant ( $\bar{B}$  constant) (compare  $I = 10, \bar{B}/I = 10\%$  with  $I = 100, \bar{B}/I = 1\%$ ).
- iv. Reducing the variance of the existing business (equivalent to reducing the coefficient of variation while other parameters are constant) always reduces the loss in market value ( $\Delta/\bar{B}$ ) (compare cases in which  $cv_a = 1, cv_b = 1$  with  $cv_a = .1, cv_b = 1$ , and  $cv_a = 1, cv_b = .1$  with  $cv_a = .1, cv_b = .1$ ).
- v. Reducing the variance of the new business ( $\sigma_b = \bar{B} \cdot cv_b$ ) has an

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\* The maximum and minimum value of  $\rho_{AB}$  are obtained by making  $\rho_{XY} = +1$  and  $-1$  respectively [relation (6.22)].

TABLE 4: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = 0 \quad (\rho_{XY} = 0)$$

$$cv_a = 1$$

$$cv_b = 1$$

$\frac{S}{I}$ (%)	$\frac{\bar{B}}{I} = 1\%$	I = 10		I = 100	
		$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 10\%$
0	100-	97.8	$\Delta/\bar{B}$	99.9	68.8
	0+	1.6	$\text{Pr}(M')$	0.1	28.0
50	100-	84.4	$\Delta/\bar{B}$	97.1	39.4
	0+	11.2	$\text{Pr}(M')$	2.1	51.7
90	97.0	18.7	$\Delta/\bar{B}$	65.0	5.1
	1.9	70.5	$\text{Pr}(M')$	25.9	89.6
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

TABLE 5: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = 0 \quad (\rho_{XY} = 0)$$

$$cv_a = .1$$

$$cv_b = .1$$

$\frac{S}{I}$ (%)	$\frac{\bar{B}}{I} = 1\%$	I = 10		I = 100	
		$\frac{\bar{B}}{I} = 10\%$		$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$
0	100-	25.0	$\Delta/\bar{B}$	99.9	3.0
	0+	74.7	$\text{Pr}(M')$	0.1	97.0
50	100-	0.2	$\Delta/\bar{B}$	91.0	0+
	0+	97.6	$\text{Pr}(M')$	8.9	100-
90	25.8	0	$\Delta/\bar{B}$	3.6	0
	73.9	100	$\text{Pr}(M')$	96.2	100
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

TABLE 6: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = 0 \quad (\rho_{XY} = 0)$$

$$cv_a = .1$$

$$cv_b = 1$$

$\frac{S}{I}$ (%)	I = 10		I = 100	
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$
0	99.8	17.8	$\Delta/\bar{B}$	98.5
	0.1	68.4	$\text{Pr}(M')$	1.2
50	94.1	5.1	$\Delta/\bar{B}$	68.7
	3.2	87.0	$\text{Pr}(M')$	21.7
90	19.9	0.1	$\Delta/\bar{B}$	5.7
	65.2	99.5	$\text{Pr}(M')$	85.8
100	0	0	$\Delta/\bar{B}$	0
	100	100	$\text{Pr}(M')$	100

TABLE 7: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:  
 Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

		$\bar{A} = 100$ (Reference value)			
		$\rho_{ab} = 0$ ( $\rho_{XY} = 0$ )			
		$cv_a = 1$			
		$cv_b = .1$			
		$I = 10$		$I = 100$	
$\frac{S}{I}$ (%)	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$		$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$
0	100- 0+	98.8	$\Delta/\bar{B}$	99.5	72.6
		1.2	$\text{Pr}(M')$	0.5	27.4
50	99.9 0.1	92.0	$\Delta/\bar{B}$	98.8	49.9
		8.0	$\text{Pr}(M')$	1.2	50.0
90	98.8 1.2	21.9	$\Delta/\bar{B}$	78.7	5.1
		77.9	$\text{Pr}(M')$	21.3	94.8
100	0 100	0	$\Delta/\bar{B}$	0	0
		100	$\text{Pr}(M')$	100	100

TABLE 8: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = 1 \quad (\rho_{XY} = 1)$$

$$cv_a = 1$$

$$cv_b = 1$$

$\frac{S}{I}$ (%)	$\frac{\bar{B}}{I} = 1\%$	I = 10		I = 100	
		$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 10\%$
0	100- 0+	100- 0+	$\Delta/\bar{B}$ $\text{Pr}(M')$	100- 0+	100- 0+
50	100- 0+	100- 0+	$\Delta/\bar{B}$ $\text{Pr}(M')$	100- 0+	99.6 3.6
90	100- 0+	0 100	$\Delta/\bar{B}$ $\text{Pr}(M')$	100- 0+	0 100
100	0 100	0 100	$\Delta/\bar{B}$ $\text{Pr}(M')$	0 100	0 100

TABLE 9: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:  
 Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = 1 \quad (\rho_{XY} = 1)$$

$$cv_a = .1$$

$$cv_b = .1$$

$\frac{S}{I}$ (%)	I = 10		I = 100	
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$
0	99.9	24.2	$\Delta/\bar{B}$	99.6
	0.1	78.8	$\text{Pr}(M')$	0.5
50	99.6	0.6	$\Delta/\bar{B}$	93.3
	0.5	99.5	$\text{Pr}(M')$	8.1
90	24.2	0	$\Delta/\bar{B}$	1.6
	78.8	100	$\text{Pr}(M')$	98.8
100	0	0	$\Delta/\bar{B}$	0
	100	100	$\text{Pr}(M')$	100

TABLE 10: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \text{ (Reference value)}$$

$$\rho_{ab} = 0.86594 \quad (\rho_{XY} = 1)$$

$$cv_a = .1$$

$$cv_b = 1$$

$\frac{S}{I}$ (%)	I = 10		I = 100		
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	
0	100-	0	$\Delta/\bar{B}$	100-	0
	0+	100	$\text{Pr}(M')$	0+	100
50	100-	0	$\Delta/\bar{B}$	100-	0
	0+	100	$\text{Pr}(M')$	0+	100
90	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

TABLE 11: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:  
 Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \text{ (Reference value)}$$

$$\rho_{ab} = 0.86594 \text{ } (\rho_{XY} = 1)$$

$$cv_a = 1$$

$$cv_b = .1$$

$\frac{S}{I}$ (%)	I = 10				I = 100	
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$
0	100-	99.1	$\Delta/\bar{B}$	99.6	75.8	
	0+	1.2	$\text{Pr}(M')$	0.5	27.4	
50	100-	95.5	$\Delta/\bar{B}$	99.1	54.0	
	0+	5.5	$\text{Pr}(M')$	1.2	50.0	
90	99.1	18.4	$\Delta/\bar{B}$	81.6	4.4	
	1.2	84.1	$\text{Pr}(M')$	21.2	96.4	
100	0	0	$\Delta/\bar{B}$	0	0	
	100	100	$\text{Pr}(M')$	100	100	

TABLE 12: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \quad (\text{Reference value})$$

$$\rho_{ab} = -.5 \quad (\rho_{XY} = -1)$$

$$cv_a = 1$$

$$cv_b = 1$$

$\frac{S}{I}$ (%)	I = 10 :				I = 100	
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$		$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	
0	96.1	64.3	$\Delta/\bar{B}$	77.8	26.3	
	0.5	11.5	$\text{Pr}(M')$	5.5	42.1	
50	92.1	48.6	$\Delta/\bar{B}$	64.3	20.2	
	1.2	21.2	$\text{Pr}(M')$	11.5	50.0	
90	64.3	15.1	$\Delta/\bar{B}$	33.2	5.2	
	11.5	57.9	$\text{Pr}(M')$	34.5	78.8	
100	0	0	$\Delta/\bar{B}$	0	0	
	100	100	$\text{Pr}(M')$	100	100	

TABLE 13: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \text{ (Reference value)}$$

$$\rho_{ab} = -.9901 \text{ } (\rho_{XY} = -1)$$

$$cv_a = .1$$

$$cv_b = .1$$

$\frac{S}{I}$ (%)	I = 10		I = 100		
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	
0	99.3	24.2	$\Delta/\bar{B}$	97.1	2.9
	0.5	72.6	$\text{Pr}(M')$	2.3	96.4
50	97.1	4.4	$\Delta/\bar{B}$	75.8	0.1
	2.3	94.5	$\text{Pr}(M')$	21.2	99.9
90	24.2	0	$\Delta/\bar{B}$	4.4	0
	72.6	100	$\text{Pr}(M')$	94.5	100
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

TABLE 14: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:  
 Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\begin{aligned} \bar{A} &= 100 \quad (\text{Reference value}) \\ \rho_{ab} &= -.79693 \quad (\rho_{XY} = -1) \\ cv_a &= .1 \\ cv_b &= 1 \end{aligned}$$

$\frac{S}{I}$ (%)	I = 10		I = 100		
	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	
0	77.8	15.1	$\Delta/\bar{B}$	56.6	21.0
	5.5	57.9	$\text{Pr}(M')$	15.9	88.5
50	64.3	7.6	$\Delta/\bar{B}$	40.7	0.7
	11.5	72.6	$\text{Pr}(M')$	27.4	94.5
90	20.2	0.4	$\Delta/\bar{B}$	7.6	0+
	50.0	96.4	$\text{Pr}(M')$	72.6	99.5
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

TABLE 15: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:  
 Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\bar{A} = 100 \text{ (Reference value)}$$

$$\rho_{ab} = -.79693 \text{ } (\rho_{XY} = -1)$$

$$cv_a = 1$$

$$cv_b = .1$$

$\frac{S}{I}$ (%)	$\frac{\bar{B}}{I} = 1\%$	I = 10		I = 100	
		$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 1\%$	$\frac{\bar{B}}{I} = 10\%$	$\frac{\bar{B}}{I} = 10\%$
0	100-	97.1	$\Delta/\bar{B}$	98.4	61.8
	0+	2.3	$\text{Pr}(M')$	1.2	34.5
50	99.8	86.4	$\Delta/\bar{B}$	97.1	38.2
	0.1	11.5	$\text{Pr}(M')$	2.3	57.9
90	97.1	18.4	$\Delta/\bar{B}$	69.1	4.4
	2.3	78.8	$\text{Pr}(M')$	27.4	94.5
100	0	0	$\Delta/\bar{B}$	0	0
	100	100	$\text{Pr}(M')$	100	100

ambiguous effect on the loss in market value ( $\Delta/\bar{B}$ ). Depending on the set of parameters for each specific situation, the loss may increase or decrease.

- vi. The correlation coefficient between the market values of existing and new business ( $\rho_{ab}$ ) also has an ambiguous effect on the loss in market value ( $\Delta/\bar{B}$ ). But, in general, when slack is high, a positive correlation is more favorable, because it tends to reduce this loss; while if slack is low, a negative correlation tends to be more favorable.

In conclusion, the policy that seems to emerge from this numerical analysis reinforces the ideas that firms should try to increase their slack, reduce the variance of the existing business, and look for new investment opportunities which have both a high return on investment and a low investment requirement. What is somewhat surprising is that there are no clear recommendations for the variance of the new business<sup>\*</sup> and its correlation with the existing business.

To check the robustness of the conclusions that the standard deviation of  $\tilde{B}$  and the correlation coefficient between  $\tilde{A}$  and  $\tilde{B}$  have ambiguous effects over the market value, a limited exploration is conducted with a truncated normal distribution. The results presented in Tables 16 and 17 confirm the conclusions obtained with the log-normal. The first one of these tables shows that decreasing the variance of B improves the situation for 0% slack and deteriorates it for 90% slack. The second

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\* Remember that this variance and correlation coefficient are referred to the non-systematic variations in cash-flows.

TABLE 16: Pure Equity Case

Sensitivity Analysis with the bivariate truncated normal distribution: Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\begin{aligned} \bar{A} &= 100^* \quad (\text{Reference value}) \\ \bar{B} &= 10^* \\ \rho_{ab} &= 1^* \quad (\rho_{XY} = 1) \\ I &= 100 \end{aligned}$$

$\frac{S}{I}$ (%)	$cv_a = 1^*$	$cv_b = 1^*$	$cv_a = 1^*$	$cv_b = .1^*$
0	100		$\Delta/\bar{B}$	81.7
	0		$\text{Pr}(M')$	21.2
90	0		$\Delta/\bar{B}$	0.2
	100		$\text{Pr}(M')$	99.9

\* These are the parameters of the distribution before truncation.

TABLE 17: Pure Equity Case

Sensitivity Analysis with the bivariate truncated normal distribution: Values for  $(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%)

$$\begin{aligned} \bar{A} &= 100^* \quad (\text{Reference value}) \\ \bar{B} &= 10^* \\ \text{cv}_a &= 0.5^* \\ \text{cv}_b &= 0.5^* \\ I &= 40 \end{aligned}$$

$\frac{S}{I}$ (%)	$\rho_{ab} = 0^*$		$\rho_{ab} = 0.7^*$	
	$\Delta/\bar{B}$	$\text{Pr}(M')$	$\Delta/\bar{B}$	$\text{Pr}(M')$
0	92.4	7.7	99.9	2.3
90	0.1	98.4	0+	100-

\* These are the parameters of the distribution before truncation.

table shows the different impact that an increase in the correlation coefficient has over the loss due to insufficient slack.

A final numerical analysis performed in the pure equity case is obtaining the loss due to insufficient slack under a more intermediate combination of parameters, to assess an order of magnitude for this phenomenon in a more standard situation. The conditions and results of the experiment are presented in Table 18. The value of  $\bar{A}$  is arbitrarily fixed in 100 as a reference level. To estimate the other parameters, it is necessary to have a notion of the real time elapsing between instants 0 and +1 in the model. This lapse has been assumed to be around 4 years. It is not unusual to find firms growing from 40 to 50 percent in this period of time, so the investment requirement is fixed at 40, and the net present value at 25 percent of the investment, or  $\bar{B} = 10$ . The new project is assumed to be very much related to the existing business, as represented by a correlation coefficient of 0.7. Finally, the standard deviation of the market value has been estimated at 50 percent of the expected values  $\bar{A}$  and  $\bar{B}$ .

The conclusion from this analysis is that slack availability plays a very important role in the market value of the firm, because it may reach 63.2 percent of the net present value of the new project if slack is zero. In general, for 0 and 25 percent slack availability the loss is high; for 75 and 100 percent, it is low, and around 50 percent it is not insignificant.

TABLE 18: Pure Equity Case

Sensitivity Analysis with the bivariate log-normal distribution:

$(\Delta/\bar{B})$  (%) and  $\text{Pr}(M')$  (%) for intermediate values of the parameters

$$\begin{aligned} \bar{A} &= 100 & \bar{B} &= 10 \\ cv_a &= 0.5 & cv_b &= 0.5 \\ \rho_{ab} &= 0.7 \\ I &= 40 \end{aligned}$$

$\frac{S}{I}$ (%)	$\Delta/\bar{B}$ (%)	$\text{Pr}(M')$ (%)
0	63.2	48.0
25	29.7	78.0
50	7.1	95.0
75	0.2	99.8
100	0.0	100.0

## 5.6 Some Notes on the Proposed Model\*

Many valid questions have been raised on the assumptions and implications of this model. The purpose of this section is to try to answer some of the issues that seem to be the most controversial.

- 1) If slack is important, why not issue shares just to get slack, rather than for project investment?

There is no gain for current stockholders when following this strategy. This case is equivalent to a situation where the net market value of the investment opportunity is always zero (see case  $b=0$  in section 5.4.5). It was shown in there that the firm never issues shares unless the implicit value of existing assets is at its minimum. If the probability distribution for the value of this asset is assumed to be continuous, this event has probability zero.

- 2) Why not issue shares at time  $-1$ , when everyone has the same information?

As stated in section 5.4.4, if the firm and the public have the same amount of information, there is no loss for insufficient slack. This condition is assumed to exist at time  $-1$ , so the question is, why not issue shares at that moment and avoid the signaling impact of the decision?

In the context of this model, time 0 is defined by the decision of the firm to issue or not to issue shares. So, the way in which the question is presented is not an appropriate one. We are concerned with the case in which there is a difference in information between the firm and the public at the time the decision to issue or not to issue shares is taken.

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\* We have to thank Prof. Fischer Black for his careful reading and valuable commentaries.

An important assumption made in the presentation of this model is that it is a normal thing to have this difference in information. In the special case of identical information in the public and the firm, the loss due to insufficient slack goes away.

- 3) A strategy always available to a firm is selling part of its existing assets and repurchasing its own shares. Does introduction of this alternative change our analysis?

This question forces us to clarify an assumption that has not been explicitly stated in the formulation of the model; namely, that the existing asset is illiquid. The concern expressed is if by relaxing this assumption signaling effects go away.

What in fact happens is that the signal transmitted to the market by the decision of the firm is a more complex one, because it incorporates both the investment decision, and the decision related to the sale of the existing assets.

To properly address this problem, it is convenient to analyze first the situation of a firm with no investment opportunity and holding two assets. One of these assets can be sold for an amount  $R$ , while the other is not sellable. Let:

$\tilde{A}_1$  = Market value of the non-sellable business at time  $t+1$

$\tilde{A}_2$  = Same for the sellable business

$R$  = Sale value of the second business (value of its assets).

There are two alternative decisions for the firm: do nothing, or sell

the second business. The question is what to do with the money in case the second business is sold. Two situations will be explored; first, the money is used to repurchase shares; and second, the money is kept by the firm as cash.

In the first place, suppose that whenever the second business is sold, the money collected must be used to repurchase shares. In this case, it is possible to derive the equilibrium conditions for each one of the two alternative decisions of the firm (do nothing, and sale-repurchase). Without going into the details of the derivation, Table 19 presents the close resemblance of this case, with the model presented in this chapter.

The alternative scenario for this analysis is assuming that the decision to sell an asset is not linked with the repurchase of shares. That is to say, the firm may keep the money obtained from the sale. Table 20 shows the comparison of this situation with the model in Chapter 5, when issue is not linked to the investment decision. The important conclusion from this comparison is that most of the symmetry observed in the previous case, when cash could not be retained by the firm, is lost in this more realistic case. Briefly explained, a good sale opportunity for a firm is invariably linked to the retention of the cash obtained, and not with the repurchase of shares (there is no "negative" stock issue). A good project undertaken by a firm with insufficient slack is always linked with the issue of new shares and the investment of the cash obtained in the project (there is a "positive" stock issue).

What this analysis shows is that restructuring the pool of existing

TABLE 19: Equilibrium conditions when the sale of an asset is linked with the repurchase of shares. Comparison with the model in Chapter 5 when issue is linked to the investment decision. (Cash retention is not allowed.)

Firm with two existing businesses,  
one being sellable  
(Sale-Repurchase/Do Nothing)

---

Firm with one existing business  
and one investment opportunity  
(Issue-Invest/Do Nothing)

---

Case

Case

$a_2 \leq R$  An interior equilibrium is always attained. Repurchasing shares cannot reduce the market value.

$b \geq 0$  An interior equilibrium is always attained. Issuing shares cannot increase the market value.

$a_2 > R$  Negative sum game for the firm. No equilibrium solution exists.

$b < 0$  Negative sum game for the firm. No equilibrium solution exists.

$a_2 \begin{matrix} > \\ \approx \\ < \end{matrix} R$  Interior equilibrium solution is possible but not guaranteed.

$b \begin{matrix} > \\ \approx \\ < \end{matrix} 0$  Interior equilibrium solution is possible but not guaranteed.

TABLE 20: Equilibrium conditions when the sale of an asset is not linked with the repurchase of shares. Comparison with the model in Chapter 5 when issue is not linked to the investment decision.

(Cash retention is allowed.)

Firm with two existing businesses,  
one being sellable  
(Sale-Repurchase/Sale-Keep Cash/  
Do Nothing)

Firm with one existing business  
and one investment opportunity  
(Issue-Investment/Issue-Keep Cash/  
Do Nothing)

Case

Case

$a_2 \leq R$  Sell and keep cash is always an optimum strategy. Repurchase is justifiable only if  $a_2$  is at its maximum possible value, event occurring with probability zero for continuous distributions.

$b \geq 0$  An interior equilibrium is always attained. If the firm issues shares, it always invests when  $b > 0$  and it is indifferent between investing and keeping cash when  $b = 0$ .

$a_2 > R$  Negative sum game for the firm. No equilibrium solution exists.

$b < 0$  Negative sum game for the firm.

$a_2 \begin{matrix} > \\ < \end{matrix} R$  Sell and keep cash when  $a_2 < R$ . Do not sell when  $a_2 > R$ .

$b \begin{matrix} > \\ < \end{matrix} 0$  Equivalent to  $b \geq 0$ . When issuing with  $b \leq 0$ , the firm keeps cash.

assets via sales and repurchases of shares is a strategy that can be pursued only under certain very special conditions. This strategy cannot be viewed as a general mechanism to absorb the variability in the market value of all existing assets. Therefore, it cannot eliminate the signaling impact of a future investment opportunity.

To properly analyze the problem of signaling in the presence of both existing assets that can be sold and a future investment opportunity, a more complex model is required to represent the market value of the firm. This model should include:

- $\tilde{A}_1$  = Market value of the non-sellable business at time +1
- $\tilde{A}_2$  = Same for the sellable business
- $\tilde{B}$  = Market value of the investment opportunity at time +1.

The set of decisions available to the firm is not the simple issue/no-issue dichotomy. In this case, it is necessary to distinguish the following alternatives:

- Sell the asset - Keep the cash - Issue (if needed) and invest
- Sell the asset - Keep the cash - Do not invest
- Sell the asset - Repurchase - Issue and invest
- Sell the asset - Repurchase - Do not invest
- Do not sell the asset - Issue and invest
- Do not sell the asset - Do not invest
- Do nothing

Some of these alternatives may be eliminated at the outset if they are dominated by others, but the important point is that more than one signal may be implicit in the decision of the firm. Therefore, the

analytical and computational effort required to derive the equilibrium solution increases accordingly.

4) Another way of getting cash is by selling an existing asset.

As shown in the previous case, this strategy is easily justifiable when the value of the assets if sold is larger than the market value of the business. If the sales value is equal to the going concern value this is just like having extra slack. In other circumstances, the situation is not at all clear, because the signal generated by the decision of the firm is a complex one.

5) Investment and financing decisions should be considered separately.

If the firm does not have sufficient slack, there is no way to separate these decisions under the assumptions of this model. Some of the alternatives that come to mind to disengage the investment and financing decisions are discussed below.

- Issue shares just to create slack. (This has been discussed in point 1) above).
- Issue stock when the firm and the market have the same information. (Discussed in point 2) above.)
- Sell an existing asset. (Discussed in points 3) and 4) above.)
- Borrow money. (Discussed in Chapter 6.)
- Have a private line with shareholders. (Discussed at the end of section 5.2.)
- Spin off the new project into a separate corporation. If there are no technological constraints, this is a viable course of action. But most investment opportunities are not costlessly separable.

- Delay the investment. If internal cash can be generated, this may help in the solution of the problem. But, there may be costs associated with the delay; most important, the competitive advantage and the whole project may be lost.

If no cash is generated, no problem is solved with the delay, because the difference in information between the firm and the market is understood as a "normal" situation in the context of this model.

- Issue rights. If the signal implied by the rights issue is equivalent to a private line with shareholders, the problem is solved. If it is equivalent to a normal issue of shares, the signaling impact of the investment decision does not disappear, so it cannot be viewed as independent of the financing decision.

6) How does the dividend decision affect the signaling impact of the investment decision?

The full study of the dividends question is beyond the scope of this thesis, and it is proposed in the last chapter as a fruitful line of future research. Nevertheless, it is possible to advance some conclusions regarding the impact of a dividend policy.

Consider the description of a firm with an existing business and an investment opportunity in terms of the parameters defined many times in Chapter 5:  $(\tilde{A}, \tilde{B})$ ,  $S$ ,  $I$ ,  $E = I - S$ , and  $f(a, b)$ . The firm has an option to issue or not to issue shares. Suppose that if shares are not issued, the firm pays an amount  $u$  of dividends, while if shares are issued, this payment is  $v$ .

The net worth of current shareholders in terms of the market value

at time +1 is:

- If shares are not issued:

$$\text{Dividends} + \text{Market value of the firm} = u + (S+a-u) = S + a$$

- If shares are issued:

$$\text{Dividends} + \text{Market value of the firm} = v + \frac{P'}{P'+E} (I+a+b-v).$$

It may be seen that shares should be issued whenever:

$$v + \frac{P'}{P'+E} (I+a+b-v) \geq S + a$$

This relation is equivalent to:

$$E + b \geq \frac{E}{P'} (S + a - v)$$

If  $v$ , the dividend payment when shares are issued, is a constant known to the market, the payment of dividends can be understood as a net reduction of the firm's slack from  $S$  to  $(S-v)$ , this having a negative impact over the market value. If, on the other hand,  $v$  is positively correlated to  $a$ , the impact of dividends over market value is ambiguous, because the negative impact originated in the reduction of slack is opposed by a positive impact of a smaller variance in the market value of existing assets.

This simple analysis made of the dividend decision ignores the possibility of a change in the a priori distribution of  $(a,b)$  produced by the announcement of dividends. This effect can be incorporated in a more complete analysis. To put the model presented in Chapter 5 under a proper perspective,  $\tilde{A}$  and  $\tilde{B}$  must be interpreted as the market values after the dividend decision is made,  $f(a,b)$  their a posteriori joint probability distribution, and  $S$  the slack net of dividend payments.

CHAPTER 6: MARKET VALUE OF THE FIRM WITH DEBT-EQUITY FINANCING

The previous chapter shows that, under certain circumstances, rejecting a good investment opportunity may favor current shareholders of a firm. This proposition has been proven to hold in a perfect market when public and insiders information are different, the firm has insufficient slack resources to cover the investment needs, and the only external source of funds is equity issues. This chapter extends the validity of this proposition when two external sources of funds are available: debt and equity.

The interaction of slack insufficiency, and difference in public and insiders information, generates a signal jointly with the investment decision of the firm. Inescapably, managers disclose to the market part of their internal information with the decision to invest, because a rational behavior on their part presupposes that current shareholders are not being hurt. Consequently, the market reaction to the announcement of a new investment, narrows down the set of states of the world to a subset in which current shareholders are not worse off with the undertaking of the new project.

The same basic signaling effect is also present when the sources of external financing are debt and equity instead of pure equity. But, as might be expected, this signal grows in complexity when debt is added as an alternative source of financing. A general treatment of this signaling effect in a debt-equity case is not included in this study. What is given instead, is a sequence of progressively more complete representations of the problem, to illustrate the characteristics of this signal, gain some intuitive understanding, and show how an optimal capital structure may be derived from it.

In a debt-equity situation, the public announcement of the firm's plans seems to convey two signals: the investment decision (the firm indicates if the project is or is not taken, just as in the pure equity case), and the financing decision (debt-equity composition in case the project is taken). In this chapter, the financing composition is assumed to have no informational content whatsoever. The market is supposed to know in advance the financial policy to be pursued by the firm in case the project is taken.

Some fundamental differences remain between the debt-equity and the pure equity cases, notwithstanding the apparent similarity between them when the financing composition is assumed to have no informational content. A most important difference is the addition of a new equilibrium condition required by the introduction of debt. Also, debtholders' claims depart in some crucial ways from shareholders' claims; namely, their claim has priority, their compensation cannot exceed the face value of debt, and there is a deadline for debt repayment.

When putting all these conditions together, the choice between debt and equity financing is not clear cut. There are some situations in which debt seems to be preferred (most notably when risk-free debt can be issued), and others in which equity seems to be better. The signaling impact of debt varies from situation to situation, and it is not always simple to gain an intuitive understanding of a case, until the equilibrium conditions are visualized.

In the analysis of the impact of debt over a longer planning horizon, two opposing effects emerge. In general, debt financing in the short run has a positive effect as a result of the reduced reliance on new equity;

while in the long run, the higher level of debt outstanding may damage subsequent investment opportunities of the firm. An optimum capital structure may be explained from the trade-off between these two effects; but, as in the previous case, the signals under different situations are not simple to visualize at a first glance.

This chapter presents the situation of a firm confronted with one or two sequential projects. An effort is made to separate the impact of "old" debt (previously outstanding debt) and newly issued debt. Also included are some qualitative properties of debt financing and a limited numerical analysis.

#### 6.1 One Project, New Debt Only

The case presented in this section assumes that a firm is confronted with only one investment opportunity. The model is developed within the same time framework used in the pure equity case, which is designated by the instants  $-1, 0, +1$ . The evaluation of information through time is indicated in Figure 20, and it is explained in what follows.

At time  $-1$ , the market and the firm have the same information. This information is designated by  $\phi$ , and may be summarized in the following set of parameters:

- The firm owns an existing line of businesses and a new investment opportunity whose market values at time  $+1$  are represented by the random variables  $(\tilde{A}, \tilde{B})$ .\*
- The investment required to undertake the new opportunity is  $I$ .
- The slack available is  $S$ , and it is less than  $I$ . (Slack is not included

\* These values are discounted to the present for time and risk.

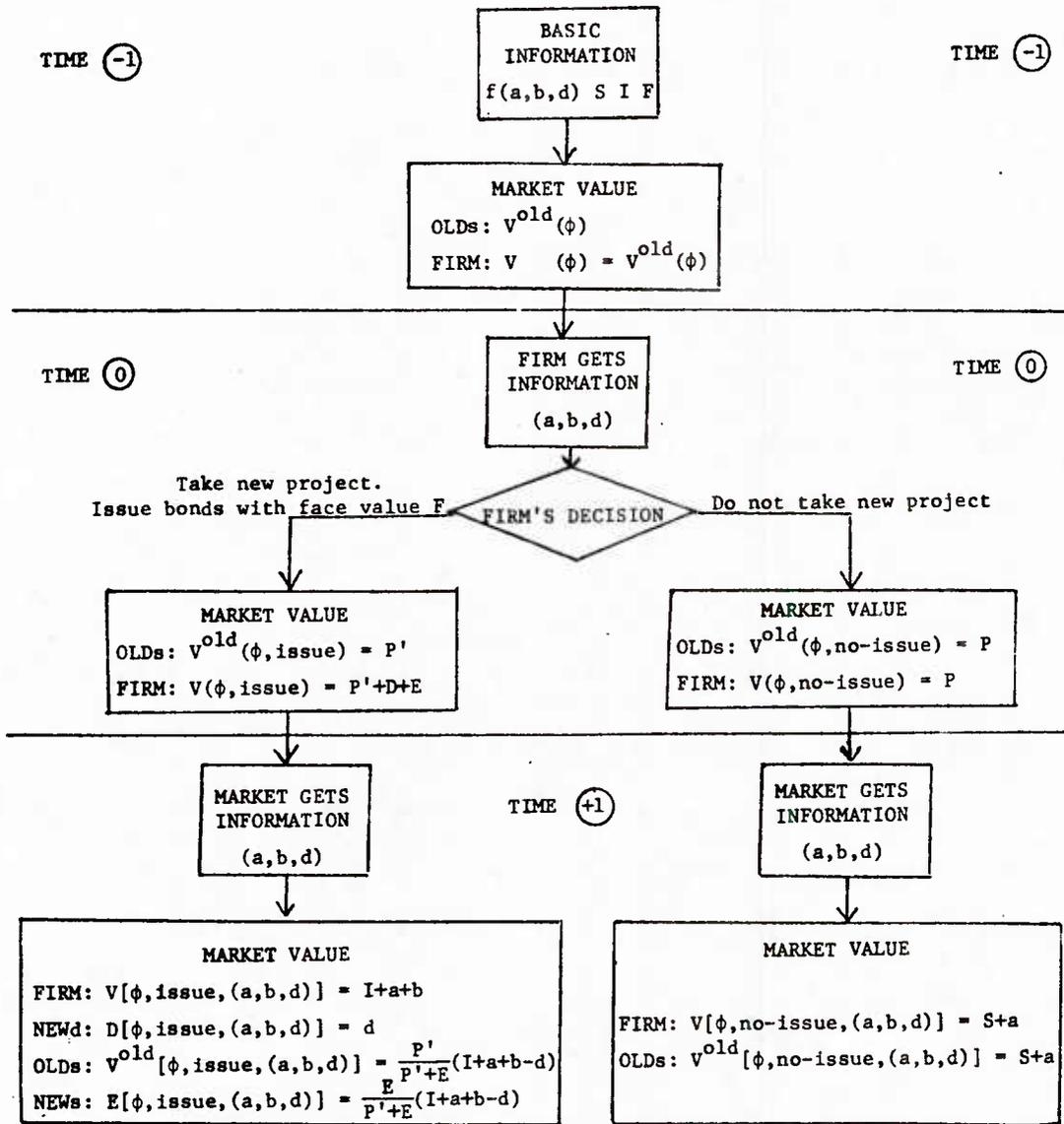


FIGURE 20: Market Value in a Debt Equity Case

in the market value  $\tilde{A}$ ).

- If the firm goes ahead with the project, new debt with a face value  $F$  will be issued. In this case, the market value of debt at time  $+1$  is a random variable which is designated by  $\tilde{D}$ .\*
- The random variable  $\tilde{D}$  is dependent on  $(\tilde{A}, \tilde{B})$  and  $F$ , but for the purposes of the presentation, it is easier to assume that the triplet  $(\tilde{A}, \tilde{B}, \tilde{D})$  has a joint probability distribution  $f(a, b, d)$  defined over positive values of  $(a, b, d)$  only. The values  $(a, b, d)$  are restricted to positive values only, because if  $a$  is negative, shareholders are better off by liquidating the current business; if  $b$  is negative, the new project is not attractive, and it is dominated by the strategy of putting money in the bank; finally,  $d$  cannot be negative because debt has limited liability.
- The market value of the firm at time  $-1$  is  $V^{\text{old}}(\phi)$ , and it is owned exclusively by the current group of shareholders.

At time 0, managers of the firm get the updated values of  $(\tilde{A}, \tilde{B}, \tilde{D})$ , which are designated by  $(a, b, d)$ . On the basis of this information, they decide on the strategy that best serves the interest of current shareholders.

The market reacts to the announcement of this decision by driving the market value of current shares to  $V^{\text{old}}(\phi, \text{issue})$  or  $V^{\text{old}}(\phi, \text{no-issue})$ , depending on the project being or not being taken, respectively. As in the pure equity case, the market value of old shareholders' claims at time 0, after the firm announces that the project is taken, is designated by  $P'$ :

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\* These values are discounted to the present for the time and risk involved.

$$P' \equiv V^{\text{old}}(\phi, \text{issue}) \quad (6.1)$$

If the project is taken, the firm issues debt with a face value  $F$ , and its corresponding market value is designated by  $D$ . In general, new equity must also be issued to fill the gap between the investment requirement and the funds available. New debt, new equity, and slack must add up to the total investment, as indicated in (6.2):

$$D + E + S = I \quad (6.2)$$

Adding the contribution of new debt and equity holders, the market value of the firm at time 0 becomes

$$\begin{aligned} V(\phi, \text{issue}) &= V^{\text{old}}(\phi, \text{issue}) + D + E \\ &= P' + D + E \end{aligned} \quad (6.3)$$

Debt holders have the first priority claim over this market value, and old and new equity holders must share the residual amount in proportions  $(P'/P'+E)$  and  $(E/P'+E)$ , respectively.

At time +1, the updated information  $(a, b, d)$  reaches the market, and the value of the firm must adjust to it. If the new project is not taken, the market value of the firm changes to  $V[\phi, \text{no-issue}, (a, b, d)]$ . This value is equal to old shareholders' claims, which is designated by  $V^{\text{old}}[\phi, \text{no-issue}, (a, b, d)]$ , and must satisfy relation (6.4):

$$V[\phi, \text{no-issue}, (a, b, d)] = V^{\text{old}}[\phi, \text{no-issue}, (a, b, d)] = S + a \quad (6.4)$$

If the new project is taken, the market value of the firm adjusts to  $V[\phi, \text{issue}, (a, b, d)]$ . The claims over this value are:

$V^{\text{old}}[\phi, \text{issue}, (a, b, d)]$  = old shareholders' claim

$D[\phi, \text{issue}, (a, b, d)]$  = new debtholders' claim

$E[\phi, \text{issue}, (a, b, d)]$  = new shareholders' claim.

Relation (6.5) must be satisfied, because all claims must add up to the total market value.

$$\begin{aligned} V[\phi, \text{issue}, (a, b, d)] &= V^{\text{old}}[\phi, \text{issue}, (a, b, d)] + D[\phi, \text{issue}, (a, b, d)] + \\ &+ E[\phi, \text{issue}, (a, b, d)] = I + a + b \end{aligned} \quad (6.5)$$

By definition, the claim of debtholders at time +1 is equal to  $d$ . Therefore, (6.6) holds:

$$D[\phi, \text{issue}, (a, b, d)] = d \quad (6.6)$$

The residual value of the firm after subtracting this claim, is shared by old and new equity holders as indicated in relations (6.7) and (6.8):

$$V^{\text{old}}[\phi, \text{issue}, (a, b, d)] = \frac{P'}{P'+E} (I+a+b-d) \quad (6.7)$$

$$E[\phi, \text{issue}, (a, b, d)] = \frac{E}{P'+E} (I+a+b-d) \quad (6.8)$$

### Equilibrium Conditions

The impact of the investment decision over the current group of shareholders is summarized in the value of their holdings under each one of the two alternative settings. These values are given by  $V^{\text{old}}[\phi, \text{issue}, (a, b, d)]$  [relation (6.7)], and  $V^{\text{old}}[\phi, \text{no-issue}, (a, b, d)]$  [relation (6.4)], depending on the project being or not being taken, respectively. Old shareholders are better off by not taking the project, whenever the following relation is true:

$$v^{\text{old}}[\phi, \text{issue}, (a, b, d)] < v^{\text{old}}[\phi, \text{no-issue}, (a, b, d)]$$

By substituting the expressions recently derived for these two quantities [relations (6.4) and (6.7)], the condition obtained is:

$$\frac{P'}{P'+E} (I+a+b-d) < S + a$$

Recalling that  $I = D+E+S$  [relation (6.2)], and collecting the terms  $(S+a)$  at the right of the inequality, this condition may be restated as:

$$\frac{P'}{P'+E} [E+b-(d-D)] < \frac{E}{P'+E} (S+a) \quad (6.9)$$

The term  $[b-(d-D)]$  represents the contribution of the new project once the capital gain or loss to debtholders is net out, and it is designated by  $b_{\text{net}}$  as indicated in (6.10):

$$b_{\text{net}} = b - (d-D) \quad (6.10)$$

Replacing this term in condition (6.9) for not taking the new project produces a more familiar relation:

$$\frac{P'}{P'+E} (E+b_{\text{net}}) < \frac{E}{P'+E} (S+a) \quad (6.11)$$

This expression is identical to the one obtained in the pure equity case (5.13), and its interpretation is also the same: the new project should not be taken when the benefits derived by old shareholders (expression on the left-hand side) are less than the cost of giving up part of the existing business to new shareholders (expression on the right-hand side).

The condition for not taking the new project may be rewritten as:

$$E + b_{\text{net}} < \frac{E}{P'} (S+a) \quad (6.12)$$

In the space  $(a, b_{\text{net}})$ , this condition is identical to (5.14) in the pure equity case. In Figure 21, two regions M and M' are defined by the line

$$E + b_{\text{net}} = \frac{E}{P'} (S+a)$$

An outcome  $(a, b_{\text{net}})$  in region M' signals a favorable project, while one in region M indicates an unfavorable one. Formally, these regions are defined as:

$$M = \{(a, b_{\text{net}}) \mid a \geq 0, E + b_{\text{net}} < \frac{E}{P'} (S+a)\} \quad (6.13)$$

$$M' = \{(a, b_{\text{net}}) \mid a \geq 0, E + b_{\text{net}} \geq \frac{E}{P'} (S+a)\} \quad (6.14)$$

The definition of regions M and M' in this case is not strictly equal to the corresponding definitions in the pure equity case. The difference is that  $b_{\text{net}}$  is not restricted to non-negative values. This implies that there are situations in which old shareholders are better off by taking a project that, after subtracting the net capital gain or loss of debt-holders, has a negative net present value. In those particular situations, the group of new shareholders will take the burden of the loss. This is not unfair to new shareholders, because in other cases they will be getting more than their contribution to net out all a priori expectations of a capital gain or loss.

In a priori terms (when looking from time -1 perspective), it seems to be an odd behavior to have in the set of alternatives a project with

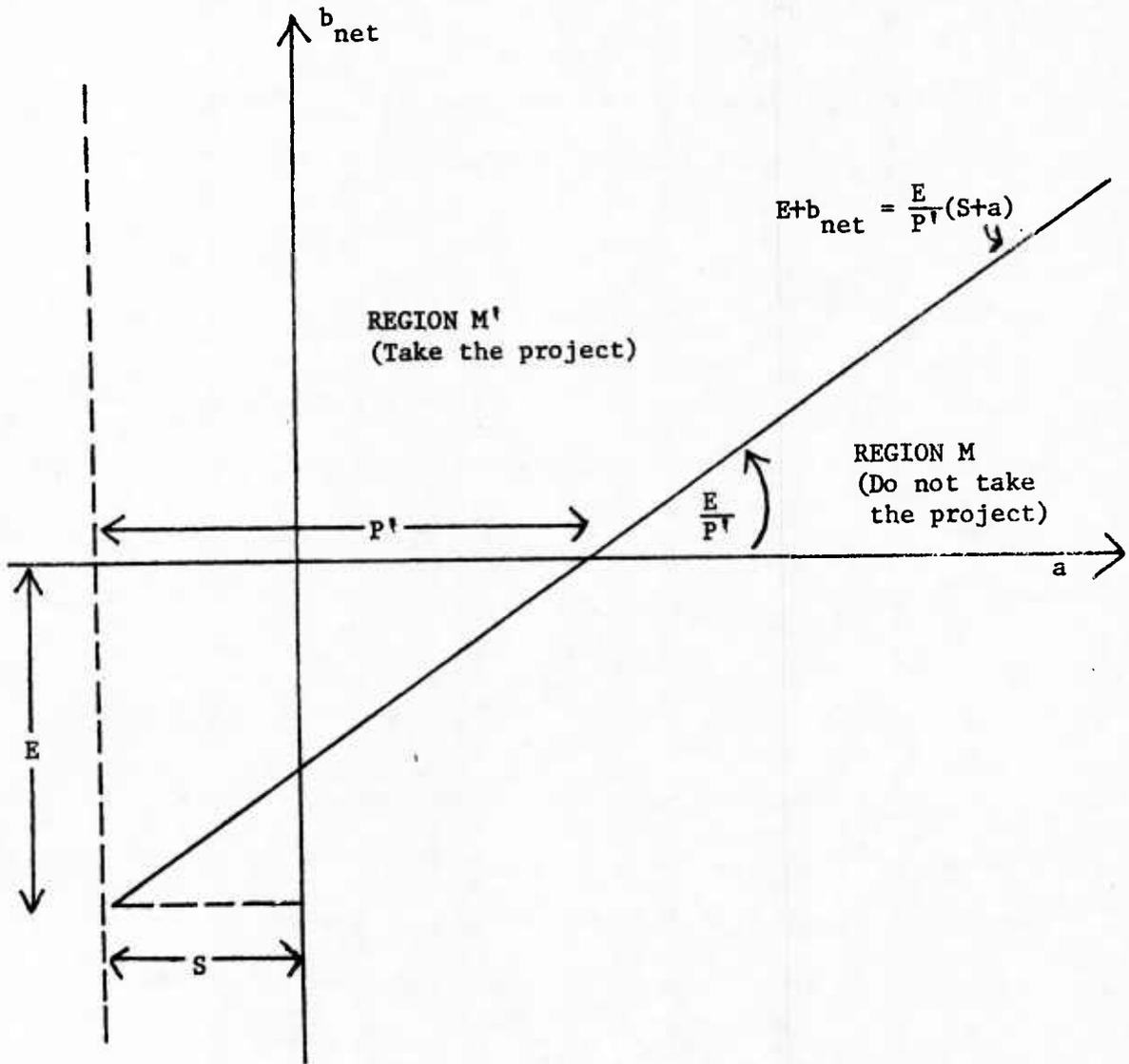


FIGURE 21: Regions M and M' in the Debt-Equity Case

negative  $b_{net}$ , because the market value of old shareholders' shares at that time is penalized by that inclusion. The firm may be more than willing to declare its intention not to take such a course of action, and the price of shares should go up immediately. The problem is that if later on the firm really faces one of those alternatives with  $b_{net} < 0$ ,  $[(a, b_{net})$  is in  $M'$ ], the economic incentives are acting in favor of undertaking the new project. Therefore, the market determines the value of the firm according to the assumption that managers will pursue the maximum long run benefit for old shareholders\* and this means taking projects with a negative  $b_{net}$  on some occasions.

Condition (6.12) for not taking a new project may be expressed in terms of the original variables  $(a, b, d)$ , by replacing the values of  $b_{net}$  for  $[b - (d - D)]$ , as given by (6.10). The result of this transformation is:

$$(D+E) + (b-d) < \frac{E}{P^1} (S+a) \quad (6.15)$$

The same two regions  $M$  and  $M'$  may be defined in the three-dimensional space  $(a, b, d)$  or in the two-dimensional space  $(a, b-d)$  (see Figure 22).

Formally stated:

$$M = \{(a, b, d) \geq 0 \mid (D+E) + (b-d) < \frac{E}{P^1} (S+a)\} \quad (6.16)$$

$$M' = \{(a, b, d) \geq 0 \mid (D+E) + (b-d) \geq \frac{E}{P^1} (S+a)\} \quad (6.17)$$

These definitions do not give any new economic insight but are helpful from a computational point of view, because the original values of  $(a, b, d)$  are fixed, while the value of  $b_{net}$  depends on the unknown parameter  $D$ .

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\* In the framework of this model, the firm maximizes the value of current shareholders' shares at time +1. It is important to notice that among the group of shareholders, the actual managerial team is most likely included.

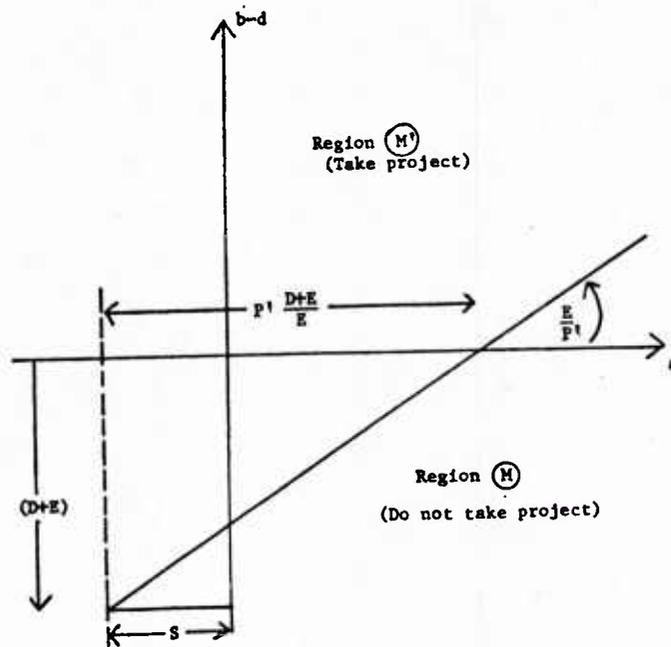
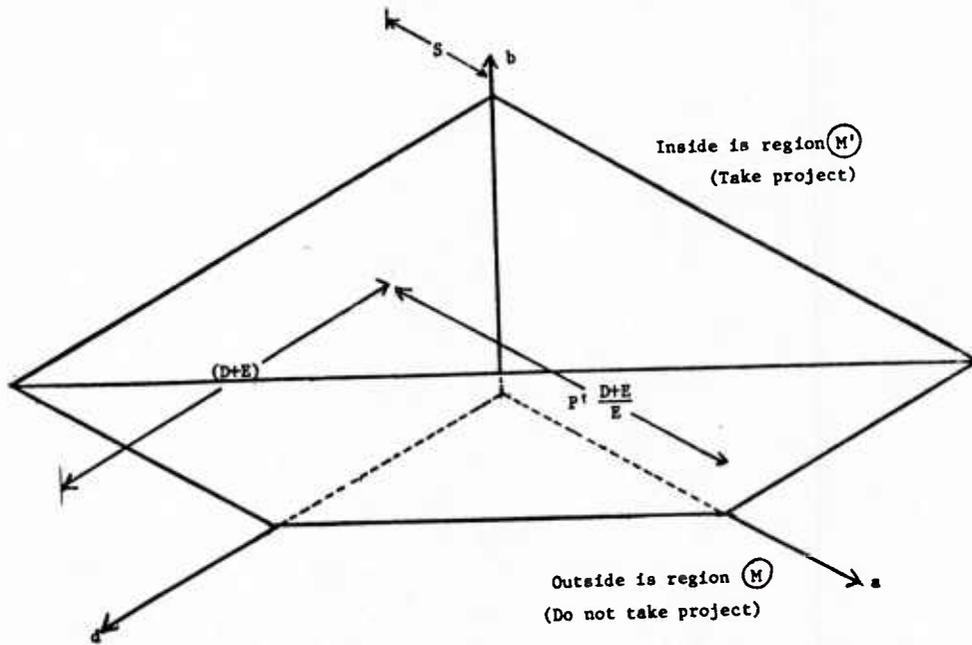


FIGURE 22: Alternative Definitions of Regions M and M' (Debt-Equity)

The decision to take or not to take the project, conveys to the market the information that (a,b,d) is on M' or M, respectively, and the market value of the firm adjusts accordingly. If the new project is not taken, old shareholders' shares go to  $V^{\text{old}}(\phi, \text{no-issue})$ , which is equal to the expectation indicated below:

$$V^{\text{old}}(\phi, \text{no-issue}) = \text{Ex}\{V^{\text{old}}[\phi, \text{no-issue}, (\tilde{A}, \tilde{B}, \tilde{D})] | (\tilde{A}, \tilde{B}, \tilde{D}) \text{ in } M\}$$

Using relation (6.4), this expression is reduced to:

$$V^{\text{old}}(\phi, \text{no-issue}) = S + \bar{A}(M) \quad (6.18)$$

where:

$$\bar{A}(M) = \text{Expected value of } \tilde{A} \text{ contingent upon } (\tilde{A}, \tilde{B}, \tilde{D}) \text{ being in } M.$$

Consider now that the new project is taken. For market equilibrium, two conditions must be satisfied. First, the market value of debt must be equated with the expected value of debt repayment:

$$D = \text{Ex}\{\tilde{D} | (\tilde{A}, \tilde{B}, \tilde{D}) \text{ in } M'\} = \bar{D}(M') \quad (6.19)$$

Second, the market value of old shareholders' shares must fulfill the following expectation:

$$V^{\text{old}}(\phi, \text{issue}) = \text{Ex}\{V^{\text{old}}[\phi, \text{issue}, (\tilde{A}, \tilde{B}, \tilde{D})] | (\tilde{A}, \tilde{B}, \tilde{D}) \text{ in } M'\}$$

Using relation (6.7), this second condition may be expressed as:

$$V^{\text{old}}(\phi, \text{issue}) = \frac{P'}{P'+E} [I + \bar{A}(M') + \bar{B}(M') - \bar{D}(M')]$$

where:

$\bar{A}(M'), \bar{B}(M'), \bar{D}(M')$  = Expected values of  $\tilde{A}, \tilde{B},$  and  $\tilde{D}$ , respectively,  
conditioned upon  $(\tilde{A}, \tilde{B}, \tilde{D})$  being in  $M'$ .

Finally, notice that  $P'$  is identical to  $V^{\text{old}}(\phi, \text{issue})$  [relation (6.1)],  $I$  is equal to  $D+E+S$  [relation (6.2)], and  $D$  is equal to  $\bar{D}(M')$  [relation (6.19)]. Therefore, the market value of old shareholders' shares under equilibrium, when the new project is taken, must satisfy the following relation:

$$V^{\text{old}}(\phi, \text{issue}) \equiv P' = S + \bar{A}(M') + \bar{B}(M') \quad (6.20)$$

Relations (6.19) and (6.20) are a set of two equations in the unknowns  $P'$  and  $D$ . As in the pure equity case, this is not a simple system to solve, because both  $P'$  and  $D$  participate in the definition of region  $M'$ . Nonetheless, the existence of a solution can be proven to exist for joint probability distributions continuous in  $(a, b)$ . (The proof of this statement is given in Appendix 6.)

Once the value of  $P'$  is found, the market value of old shareholders' shares at time  $-1$  is simply determined as a weighted average under the alternatives of taking or not taking the project:

$$V^{\text{old}}(\phi) = V^{\text{old}}(\phi, \text{no-issue}) \cdot \text{Pr}(\text{no-issue}) + V^{\text{old}}(\phi, \text{issue}) \cdot \text{Pr}(\text{issue})$$

Algebraic manipulation of this formula produces the following equivalent relations for  $V^{\text{old}}(\phi)$ . (Algebra is similar to the pure equity case, so it is not repeated in here.)

$$V^{\text{old}}(\phi) = S + \bar{A} + \bar{B}(M') \text{Pr}(M') \quad (6.21)$$

$$V^{\text{old}}(\phi) = S + \bar{A} + \bar{B} - \bar{B}(M) \text{Pr}(M) \quad (6.22)$$

These relations show that the market value of the firm, prior to the announcement of the investment decision, fully reflects the current line of business, and partly reflects the value of the opportunity. The total loss due to insufficient slack is equal to:

$$\Delta(\phi) = \bar{B}(M) \cdot \text{Pr}(M) \quad (6.23)$$

A summary of the equilibrium conditions is given in Figure 23.

## 6.2 General Implications of Market Equilibrium in the Debt-Equity Case

Getting a more intuitive understanding of the equilibrium relations in the debt-equity case is not an easy thing. Even the simple model in this study displays sufficient variety to make the overall picture not a simple one. In this section, an effort is made to gain a better understanding of some general implications stemming from the equilibrium relations.

### 6.2.1 The case of risk free debt

A risk free debt is defined as one paying an amount equal to the face value of the bond for all possible combinations of (a,b,d); therefore, the market value of debt is also equal to this face value:

$$d = D = F$$

Region M' to take a new project [relation (6.17)] is defined by:

$$M' = \{(a,b,d) \geq 0 \mid E+b \geq \frac{E}{P^*} (S+a)\}$$

This definition is identical to region M' in a pure equity situation [relation (5.16)], except for the equity requirement which goes down to

TIME (+1)

FIRM TAKES NEW PROJECT

$$v^{\text{old}}[\phi, \text{issue}, (a, b, d)] = \frac{P'}{P'+E}(I+a+b-d) \dots (6.7)$$

FIRM DOES NOT TAKE NEW PROJECT

$$v^{\text{old}}[\phi, \text{no-issue}, (a, b, d)] = S+a \dots (6.4)$$

TIME (0)

Definition of Region M'  $\equiv$  ISSUE

$$M' = \{(a, b, d) \geq 0 \mid (D+E)+(b-d) \geq \frac{E}{P'}(S+a)\} \dots (6.17)$$

$$\text{with: } D = \bar{D}(M') \dots (6.19)$$

$$E = I-S-D \dots (6.2)$$

$$v^{\text{old}}(\phi, \text{issue}) \equiv P' = S+\bar{A}(M')+\bar{B}(M') \dots (6.20)$$

Definition of Region M  $\equiv$  DO NOT ISSUE

$$M = \{(a, b, d) \geq 0 \mid (D+E)+(b-d) < \frac{E}{P'}(S+a)\} \dots (6.16)$$

$$v^{\text{old}}(\phi, \text{no-issue}) = S+\bar{A}(M) \dots (6.18)$$

TIME (-1)

$$v^{\text{old}}(\phi) = S+\bar{A}+\bar{B}(M') \cdot \text{Pr}(M') \dots (6.21)$$

$$= S+\bar{A}+\bar{B}-\bar{B}(M) \cdot \text{Pr}(M) \dots (6.22)$$

Loss due to insufficient slack

$$\Delta(\phi) = \bar{B}(M) \cdot \text{Pr}(M) \dots (6.23)$$

FIGURE 23: Conditions for Equilibrium in a Debt Equity Case

(I-S-D) from (I-S). This smaller requirement makes the combination of risk-free debt and equity more attractive than pure equity financing, because the loss in market value due to insufficient slack is lower for a reduced reliance on external equity (see Theorem 2 in Appendix 2). This situation improves with increasing debt financing.

Another interesting conclusion is that slack cannot be equated with risk-free debt, because the total claim of shareholders is different for these two situations. Consider two identical firms, one with an amount  $S$  of slack and no risk-free debt, and the other with an amount  $S$  of risk-free debt and 0 slack. The first one of these two firms is more valuable to shareholders, because they own more of it. This is no more than a scale problem, however. The economics of the two cases are basically similar. The conclusions in this section may be extended to a situation in which the debt payment in all states of the world is a well known constant, independent of the face value of debt.

### 6.2.2 Local approximation for risky debt

A more realistic representation for the value of risky debt at time  $+1$ , is to assume that it moves jointly with the market value of the firm. The simpler approximation for debt value is to consider that, between times 0 and  $+1$ , the change in debt value is a fraction of the change of the overall firm's value. Formally:

$$\left(\frac{d-D}{D}\right) = \gamma \left\{ \frac{V[\phi, \text{issue}, (a, b, d)] - V(\phi, \text{issue})}{V(\phi, \text{issue})} \right\}$$

with  $\gamma$  constant in  $(0,1)$ .

Using relations (6.2), (6.3), and (6.5), this condition is rewritten as:

$$\left(\frac{d-D}{D}\right) = \gamma \left[ \frac{(D+E+S) + a + b - (P'+D+E)}{P'+D+E} \right]$$

or

$$(d-D) = \frac{\gamma D}{P'+D+E} (S+a+b-P')$$

Replacing this difference in the definition of region M' [relation (6.17)] produces the set of values (a,b,d) for which the project is advantageous under these assumptions:

$$M' = \{(a,b,d) \geq 0 \mid E^{\circ} + b \geq \frac{E^{\circ}}{P'} (S+a)\} \quad (6.24)$$

where:

$$E^{\circ} = \frac{E + \alpha P'}{1 - \alpha} \quad (6.25)$$

$$E = (I-S) - D \quad (6.26)$$

$$\alpha = \frac{\gamma D}{P'+D+E} = \frac{\gamma D}{P'+(I-S)} \quad (6.27)$$

This problem is equivalent to a pure equity situation in which the amount of equity required is  $E^{\circ}$ , as may be seen in (6.25). The special condition imposed by the presence of debt, is that  $E^{\circ}$  must satisfy relation (6.25), which is a function of the market value of debt (D). Consequently, the equilibrium solution must be found by solving simultaneously for  $P'$  and  $D$ . One way of doing that is presented in Figure 24.

By solving the "pure equity" problem as a function of  $E^{\circ}$ , it is possible to find the equilibrium value  $P'(E^{\circ})$ , which is a downward sloping curve in the figure, because  $\frac{\partial P'}{\partial E^{\circ}} < 0$  (Lemma 1, Appendix 2). On the other hand, formula (6.25) relating  $P'$  and  $E^{\circ}$  must be satisfied too. This equation is the branch of an equilateral hyperbole asymptotically approaching the axis  $E^{\circ} = \gamma D + E$ , and  $P' = [(1-\gamma)D + E]$ , as shown in the

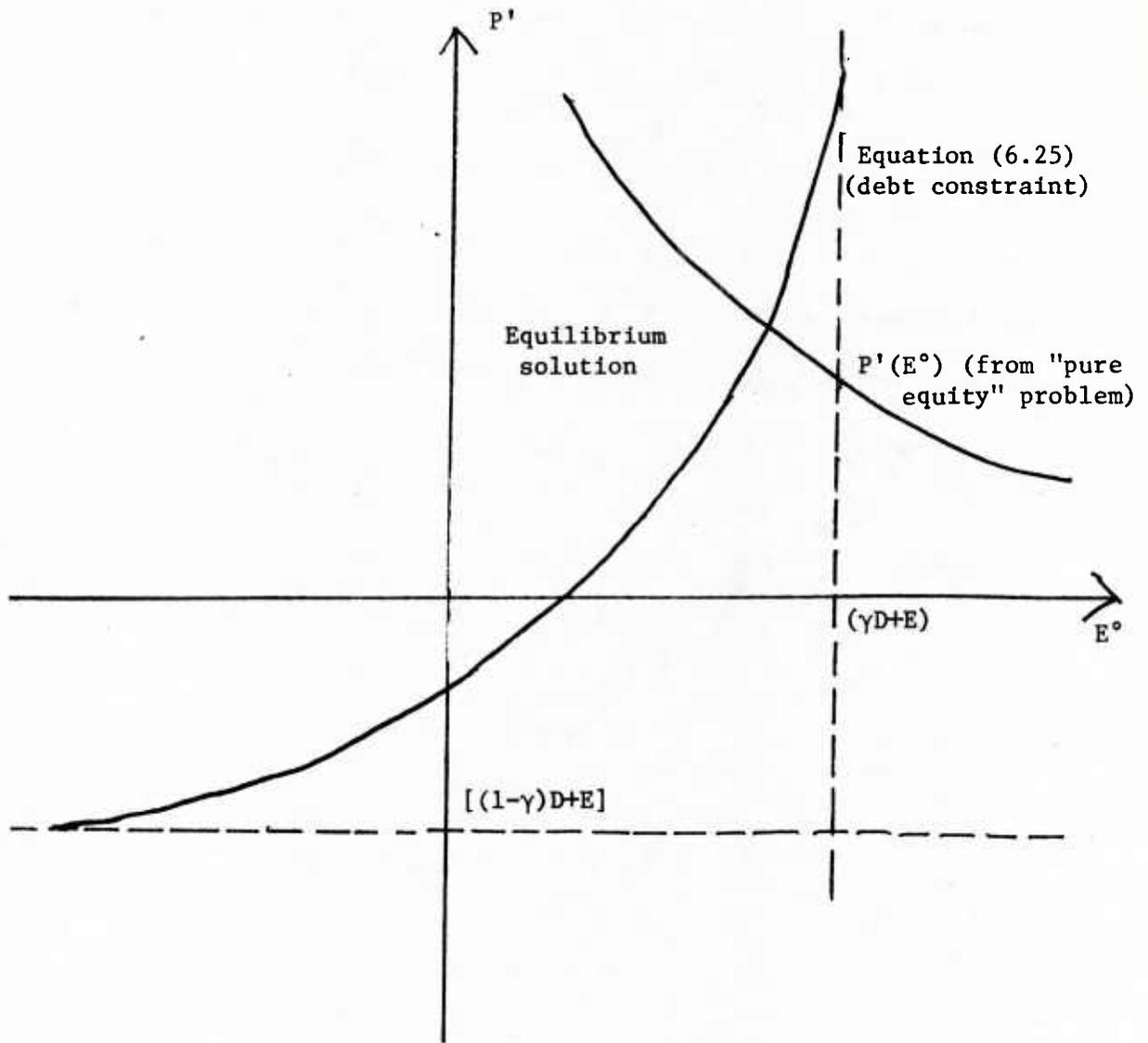


FIGURE 24: Equilibrium solution for a debt-equity situation when a local approximation of risky debt is used

figure.

The equation of this hyperbole may be obtained by replacing (6.27) in (6.25). This equation is:

$$[(\gamma D + E) - E^{\circ}] \cdot \{P' + [(1-\gamma)D + E]\} = \gamma(1-\gamma)D^2$$

From here it may be found that there is an advantage in debt financing, because the hyperbole in the figure moves up when debt is increased, as may be seen from the differential:

$$\left(\frac{\partial P'}{\partial D}\right)_{E^{\circ}} = \frac{(1-\gamma)[P' + (1-\gamma)D + E] + [\gamma D + E - E^{\circ}]\gamma + 2D\gamma(1-\gamma)}{(\gamma D + E - E^{\circ})}$$

Both numerator and denominator are positive, therefore,

$$\left(\frac{\partial P'}{\partial D}\right)_{E^{\circ}} > 0$$

When  $\gamma$  is allowed to go up with  $D$ , this result does not necessarily hold, and the possibility exists for having an intermediate combination of debt and equity being preferable to pure debt or pure equity financing. This is due only to the signaling impact of the investment decision in the presence of debt. No treatment is made in this section of the variable  $\gamma$  case. In that situation, a full model seems to be more appropriate than this local approximation for debt payment. A more general analysis of the impact of debt is made in the ensuing sections of this chapter.

But still, there is no mention of the long-range impact of debt. Nothing is said of the difference between the face and market value of debt. The introduction of the long-range impact of debt is done in two steps. First, the model with only one new investment opportunity is

expanded to allow for old debt outstanding; and second, a new model with two projects in sequence is considered.

### 6.3 One Project, New and Old Debt

The main objective in this section is to present a model in which the combined effect of both old outstanding debt, and new debt issues affect the firm's investment decision. By adhering closely to the steps followed in section 6.1, it may be shown that, after some parameters are redefined, this case is equivalent to a situation in which only new debt is considered.

The algebraic detail is omitted because it may be reconstructed from the previous case. Only the formulation of the problem and the conclusions are included.

The problem may be stated in terms of the following information:

- The firm owns an existing line of business and a new investment opportunity whose market values at time +1 are represented by the random variables  $(\tilde{A}, \tilde{B})$ .\*
- The investment required to undertake the new opportunity is I.
- The slack available is S, and it is less than I (slack is not included in the market value  $\tilde{A}$ ).
- There is an old debt outstanding with a face value of  $F^{\text{old}}$ .\* The market value of old debt at time +1, when the new project is not taken, is a random variable which is designated by  $\tilde{D}_0^{\text{old}}$ .\* If the new project is taken, this random variable is  $\tilde{D}^{\text{old}}$ .\*
- Also, when the new project is taken, new debt with a face value  $F^*$  is

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\* These values are discounted to the present for time and systematic risk.

going to be issued. The market value of this new debt at time +1 is designated by the random variable  $\tilde{D}$ .\*

- The distribution of the random variables  $(\tilde{A}, \tilde{B}, \tilde{D}_0^{\text{old}}, \tilde{D}^{\text{old}}, \tilde{D})$  is assumed to be known.

This problem is identical to the situation with only new debt, if the following variables are replaced for  $(\tilde{A}, \tilde{B})$  in section 6.1:

$$\tilde{A}^\circ = \tilde{A} - \tilde{D}_0^{\text{old}} \quad (6.28)$$

$$\tilde{B}^\circ = \tilde{B} - (\tilde{D}^{\text{old}} - \tilde{D}_0^{\text{old}}) \quad (6.29)$$

$\tilde{A}^\circ$  and  $\tilde{B}^\circ$  correspond to the net market value of the existing business and the new investment opportunity, when the payment already committed to old debt outstanding is discounted from them. In this way, the market value of the existing business (which is  $\tilde{A}$ ), is reduced by the payment committed to old debtholders when the new project is not taken (which is  $\tilde{D}_0^{\text{old}}$ ). On the other hand, if the new project is taken, old debt will change its value from  $\tilde{D}_0^{\text{old}}$  to  $\tilde{D}^{\text{old}}$ . This change is subtracted from the net present value of the new project (which is  $\tilde{B}$ ).

The only difference between this case and the situation with only new equity, is that  $\tilde{A}^\circ$  and  $\tilde{B}^\circ$  are not restricted to non-negative values. If  $\tilde{A}^\circ$  were negative, it is better for current shareholders to liquidate the old business before engaging in the new project, if that is a permissible reaction. In this case, a more proper definition of  $\tilde{A}^\circ$  is:

$$\tilde{A}^\circ = \max[0, \tilde{A} - \tilde{D}_0^{\text{old}}] \quad (6.30)$$

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\* These values are discounted to the present for time and systematic risk.

A negative  $\tilde{B}^{\circ}$  implies that a formerly attractive project has been transformed into a project with negative net present value after payments to old debt outstanding are discounted. This is a long-range impact of debt that becomes more important when the face value of old debt exceeds its market value by a wide margin.

A conclusion from this analysis is that old debt outstanding may impact the market value of the firm in two main ways: by changing the signaling effect of the investment decision ( $\tilde{A}^{\circ}$ ,  $\tilde{B}^{\circ}$  depend on old debt), and by forcing the rejection of good projects with negative  $B^{\circ}$ . Some numerical analyses are performed later on the relevance of these impacts.

#### 6.4. Two Sequential Projects, Only New Debt. The Optimal Capital Structure Problem.

In this study, a firm has been characterized as having an existing line of business and owning the option for a good investment opportunity. At a certain point in time, the firm must announce its decision to go ahead or discard this opportunity. If the decision is to go ahead with the investment, the firm must indicate also the financing plans in terms of new debt and new equity to be raised. In the context of the model presented, the sources of financing are fully specified by the parameter  $F$ , which is the face value for new debt being issued.

The assumption made in developing the equilibrium relations in the debt-equity case is that  $F$  is well-known prior to the firm's announcement. In other words, when  $F$  is announced, it carries no information content: the value given could have been guessed at time  $-1$ .

To justify this assumption it is necessary to observe that the firm

is a going concern, and that the selection of F will have an impact on future investment decisions considered by the firm.

When a new bond series is issued, the market must determine its fair value. This value depends on the firm's current business and future investment decisions. But those future decisions are going to be conditioned by all debt outstanding at that time, including the future issue of still more new bonds.

As a consequence, the value of a new bond is dependent on the policy that the firm follows with regard to its capital structure. If the firm were to have a very unpredictable capital structure, the market value of new bonds should be properly discounted, because managers are pursuing the well being of shareholders, and this objective may be in conflict with debtholders' own objectives. If, on the contrary, the firm announces its capital structure policy, this extra variability would be eliminated and the bonds could reach a higher market value. This digression may justify the managers' preference for a stable capital structure, as a way to communicate in a simple and very forceful manner the policy being pursued by the firm.

Assuming that the capital structure policy has to be declared by the firm at time -1, what is a sensible selection for F? A proper objective for this problem is the maximization of shareholders' claims at that time. To get an adequate representation of the value of these claims it is necessary to expand the model presented in the debt-equity situation. That model fails to capture the delayed impact of F over investment decisions taken after the immediate one.

A complete analysis of the capital structure problem falls beyond

the scope of this study; therefore, what is presented in the rest of this section is a more limited exploration of the problem to illustrate that the selection of  $F$  is a trade-off between the short-term advantages obtained by the increased financing of the immediate investment, and the long-term problems created by the higher level of debt outstanding.

Consider a firm owning an actual line of business, and two sequential investment opportunities. The two opportunities may be understood as a short term and a long term investment, respectively. As a first step, the firm must announce its decision regarding the immediate project. Later in the future, a similar decision has to be made with the other project.

Both opportunities are assumed to have positive net present values; therefore, if the firm has sufficient slack at the time the decisions are made, both projects would be taken. The model to be presented assumes that the firm does not have sufficient slack when the decision on the short term investment is made. Therefore, if the project is taken, the firm must select its financing composition.

The slack availability at the time the second decision is taken is an endogenous variable. To fully specify the model, it is necessary to introduce some additional assumptions in order to determine slack at that time. The assumption used here is that slack at the time the second decision is taken does not matter. Whether the firm is sure to have sufficient slack later in the future, or it can issue a risk free debt, or raise new debt and new equity in some proportion to avoid the signaling effect, the assumption made is that there is no situation in which the

firm will discard the second investment decision due to insufficient slack.

In this way, a model can be formulated to capture, in an exclusive way, the problem of slack availability with the first investment decision, and the problem of debt outstanding with the second one. The model is formally developed around three time instants which are designated by -1, 0, and +1. As before, the information in the firm and the market evolves at a different pace, but the important problem now is the selection of F, which is assumed to occur at time -1.

The model is expressed now by closely following the steps in Section 6.1, for a debt-equity situation. Special emphasis will be given only to those features which are characteristic of this problem, mainly the selection of F.

At time -1, the information available to both the market and the firm is designated by  $\phi$ , and it is the following:

- The firm owns an existing line of businesses and two sequential investment opportunities whose market values at time +1 are designated by  $(\tilde{A}, \tilde{B})$  and  $c$ , respectively.\* The market value of the second investment opportunity is assumed to be a well-known number. (It is not random.)
- The investment required to undertake the first opportunity is  $I$ .
- The slack currently available is  $S$ , and it is less than  $I$ . (Slack is not included in the market value  $\tilde{A}$ .)
- There is no old debt outstanding.\*\*
- If the firm goes ahead with the immediate project, new debt with a face

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\* These values are discounted to the present for time and systematic risk.

\*\* This is not an important assumption, because by redefining variables  $(\tilde{A}, \tilde{B})$  it is possible to transform a problem with old debt outstanding to one without it (see section 6.3).

value  $F^*$  is going to be issued. The market value of this debt at time +1, when the second project is not taken, is a random variable which is designated by  $\tilde{D}$ .<sup>\*</sup> If the second project is taken, this debt is assumed to become risk-free, and the payment at time +1 is equal to the face value.

- The triplet  $(\tilde{A}, \tilde{B}, \tilde{D})$  has a known probability distribution.
- The market value of the firm at time -1 is  $V^{\text{old}}(\phi)$ , and it is equal to the claims of old shareholders.
- The firm must choose  $F$  at this time.

At time 0, the firm gets to know  $(a, b, d)$ , and it must decide if the first project is taken or not. In case the project is taken, the firm issues a new debt with face value  $F$ .

If the project is not taken, the market value of old shareholders' claims becomes  $V^{\text{old}}(\phi, \text{no-issue})$ , while if it is taken, these claims go to  $P' = V^{\text{old}}(\phi, \text{issue})$ . In this last case, the market value of the firm is:

$$V(\phi, \text{issue}) = P' + D + E \tag{6.31}$$

where:

$$D + E + S = I \tag{6.32}$$

and

$D$  = Market value of the new debt issue with a face value  $F$ .

$E$  = Market value of the new share issue.

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<sup>\*</sup> These values are discounted to the present for time and systematic risk.

The claims over this market value are assumed to be as usual: the first priority is given to debtholders, and the residual is shared between old and new shareholders in the fractions  $P'/(P'+E)$  and  $E/(P'+E)$ , respectively.

At time +1, the market gets the updated information (a,b,d) and the market value must adjust to it. If the project is not taken, the market value is identical to old shareholders' claims, and it is equal to:

$$V^{\text{old}}[\phi, \text{no-issue}, (a, b, d)] = S+a \quad (6.33)$$

If the project is taken, the market value aggregates the claims of old shareholders, new debtholders, and new shareholders, and it is equal to:

$$V[\phi, \text{issue}, (a, b, d)] = I+a+b \quad (6.34)$$

At the same time, the firm announces if the second project is taken or not, and the market value of the firm should also reflect this new piece of information. Consequently, the real market values at time +1 are the following ones for each specific situation:

Neither the first nor the second projects are taken:

$$V^{\text{old}}[\phi, \text{no-issue}, (a, b, d), \text{no-take 2}] = S+a \quad (6.35)$$

First is not taken, and second is taken:

$$V^{\text{old}}[\phi, \text{no-issue}, (a, b, d), \text{take 2}] = S+a+c \quad (6.36)$$

First is taken, and second is not taken:

$$V[\phi, \text{issue}, (a, b, d), \text{no-take 2}] = I+a+b \quad (6.37)$$

Both first and second are taken:

$$V[\phi, \text{issue}, (a, b, d), \text{take 2}] = I+a+b+c \quad (6.38)$$

When the first project is taken, the claims of old shareholders depend on the decision with the second project.

If the second project is not taken:

$$V^{\text{old}}[\phi, \text{issue}, (a, b, d), \text{no-take 2}] = \frac{P^f}{P^f+E}(I+a+b-d) \quad (6.39)$$

If the second project is taken, new debt is supposed to become risk free, paying in all states of nature the face value of the new debt (F). Therefore, old shareholders have the following claim over the market value.

$$V^{\text{old}}[\phi, \text{issue}, (a, b, d), \text{take 2}] = \frac{P^f}{P^f+E}(I+a+b+c-F) \quad (6.40)$$

### Equilibrium Conditions

As in all previous cases, managers of the firm use the updated information (a,b,d) to the benefit of current shareholders. This gives a signal to the market that affects the market value of the firm, requiring an adjustment to the new situation. It is in the benefit of current shareholders not to take the second project when the first one is taken, whenever the following condition is satisfied:

$$V^{\text{old}}[\phi, \text{issue}, (a, b, d), \text{take 2}] < V^{\text{old}}[\phi, \text{issue}, (a, b, d), \text{no-take 2}]$$

Using (6.39) and (6.40) for the two sides of this inequality, this condition is reduced to:

$$c < F-d \quad (6.41)$$

This implies that the second project should not be taken whenever the gains to debtholders are greater than the gains from the new project. This is exactly the rationale given by Myers when discussing corporate borrowing [70]. His observation is that, without taxes, the value of the firm decreases with the use of debt, because, higher values of  $F$  imply a larger fraction of situations in which the gain to debtholders derived from the decision to take the project outweighs completely its total net value.

When the first project is not taken, the decision with regard to the second project is to take it under all circumstances. This is because the second project has a positive net present value ( $c > 0$ ), and the assumption is made that slack availability is not a problem when the decision is announced. In fact, comparing the pay-off when the second project is not taken [ $S+a$ , given by (6.35)], and when it is taken [ $S+a+c$ , given by (6.36)], it may be observed that the second strategy dominates the first one.

In summary, the equilibrium conditions imply that the market value of old shareholders' claims at time  $+1$  has the following values:

If the first project is not taken:

$$V^{\text{old}}[\phi, \text{no-issue}, (a, b, d)] = S+a+c \quad (6.42)$$

If the first project is taken:

$$v^{\text{old}}[\phi, \text{issue}, (a, b, d)] = \begin{cases} \frac{P'}{P'+E} (I+a+b-d) & \text{if } c < F-d \\ & \text{(do not take 2)} \\ \frac{P'}{P'+E} (I+a+b+c-F) & \text{if } c \geq F-d \\ & \text{(take 2)} \end{cases} \quad (6.43)$$

At this point, the problem is not different from the debt-equity situation analyzed in section 6.1. All relations obtained in there apply also to this case by redefining the variables (a,b,d) as indicated below.

$$a^{\circ} = a + c \quad (6.44)$$

$$b^{\circ} = \begin{cases} b - c & \text{if } c < F-d \text{ (do not take 2 if 1 is taken)} \\ b & \text{if } c \geq F-d \text{ (take 2)} \end{cases}$$

$$d^{\circ} = \begin{cases} d & \text{if } c < F-d \text{ (do not take 2 if 1 is taken)} \\ F & \text{if } c \geq F-d \text{ (take 2)} \end{cases} \quad (6.46)$$

An immediate conclusion to be derived from relation (6.21) is that the market value of the firm is equal to:

$$v^{\text{old}}(\phi) = S + \bar{A}^{\circ} + \bar{B}^{\circ}(M') \cdot \text{Pr}(M') \quad (6.47)$$

Replacing the values of  $\bar{A}^{\circ}$  and  $\bar{B}^{\circ}(M')$  that may be derived from (6.44) and (6.45), the market value of the firm may be expressed as:

$$v^{\text{old}}(\phi) = S + (\bar{A}+c) + [\bar{B}(M') - c \cdot \text{Pr}(c < F-d | M')] \cdot \text{Pr}(M')$$

Substituting  $\bar{B}(M') \cdot \text{Pr}(M')$  for  $[\bar{B} - \bar{B}(M) \cdot \text{Pr}(M)]$ , this relation is finally reduced to:

$$V^{\text{old}}(\phi) = S + \bar{A} + \bar{B} + c - [\bar{B}(M) \cdot \text{Pr}(M) + c \cdot \text{Pr}(c < F - d | M') \cdot \text{Pr}(M')] \quad (6.48)$$

The term in brackets is the loss in market value due to insufficient slack, and it is equal to:

$$\Delta(\phi) = \bar{B}(M) \cdot \text{Pr}(M) + c \cdot \text{Pr}(c < F - d | M') \cdot \text{Pr}(M') \quad (6.49)$$

The first term captures the loss in the immediate project due to insufficient slack, and the second one represents the loss in the future project due to the level of debt outstanding. In general, the impact of insufficient slack is reduced for large values of  $F$ , because the project is better financed. At the same time, the impact of debt outstanding is increased for large values of  $F$ , because it is harder to exceed the gap between face and market value of debt [which is  $(F-d)$  in the relation above]. The optimal selection of  $F$  is the one that minimizes the total loss, and this selection is a compromise between these two opposing effects.

In this way, it is possible to define an optimal capital structure as one that balances the short-run negative effects of not using sufficient debt ("the signaling problem"), with the long-run negative effects of using too much debt ("the moral hazard problem").\*

\* These two names were suggested by Prof. Myers in the course of our discussions, and they capture very nicely the issue of the two problems. Too little debt in the short run implies that the firm cannot avoid giving a signal when announcing its decision with regard to the first investment. (This is the signaling problem.) Too much debt in the long-run implies that there are situations in which a good project will not be taken, because too much of the benefits accrue to bondholders. If these situations could be avoided by some sort of contract, shareholders would be benefited; so, in a priori terms, they are willing to sign it. But, if a situation like that arises, managers will lack the incentive to satisfy their commitment, and the contract could hardly be enforced (see Myers [70]). This is the moral hazard problem.

## 6.5 Sensitivity Analysis in the Debt-Equity Case

This section presents a limited numerical analysis of the debt-equity case. This analysis is focused on issues which are peculiar to the presence of debt. In the first term, the joint probability density function for all random variables is specified, and then an algorithm is proposed for solving the system of two equations in the unknowns  $D$  (market value of new debt at time 0), and  $P'$  (market value of old shareholders' shares, after the firm announces its decision to take the project).

### 6.5.1 Joint probability density function, and the market value of debt at time +1.

In the case presented in section 6.1, a firm with insufficient slack is faced with only one investment opportunity. If this opportunity is taken, the firm issues new debt with a face value  $F$ . From today's perspective, the market value at time +1 of the existing business, the new project, and the new debt, are designated by the random variables  $(\tilde{A}, \tilde{B}, \tilde{D})$ . The probability density function of this triplet is  $f(a,b,d)$ .

In this study, the value  $d$  is supposed to be a well-known function of the pair  $(a,b)$ . The reason for this assumption is that  $(a,b)$  determines the market value of the firm at time +1, and that the market value of debt can be computed with this information and the face value of debt ( $F$ ), under fairly general assumptions. Formally stated, the specification of the joint probability function is done in terms of:

$f(a,b)$  = Joint probability density function for  $(\tilde{A}, \tilde{B})$

$d(a,b)$  = Market value of debt contingent upon the updated information  $(a,b)$ .

For the numerical analysis,  $f(a,b)$  is assumed to be a bivariate log-normal distribution, and  $d(a,b)$  is the following function:

$$d(a,b) = \text{Ex}\{\min(\tilde{V}, F)\} \quad (6.50)$$

where  $\tilde{V}$  is log normal, with mean  $\bar{V} = (I+a+b)$ , and coefficient of variation  $cv_d$ .

In Appendix 7 it is shown that this debt value may be computed as:

$$d = \bar{V} \Phi\left(\frac{\ln F - \bar{u}}{\sigma_u} - \sigma_u\right) + F \left[1 - \Phi\left(\frac{\ln F - \bar{u}}{\sigma_u}\right)\right] \quad (6.51)$$

where:

$$\bar{u} = \ln \frac{\bar{V}}{\sqrt{1+cv_d^2}} \quad (6.52)$$

$$\sigma_u^2 = \ln(1+cv_d^2) \quad (6.53)$$

$\Phi(Z)$  = Distribution function for a standardized normal random variable =  $\Pr(\tilde{Z} \leq Z)$

In section 6.3, when old and new debt are considered, similar assumptions are made for the numerical analysis. For the pair  $(\tilde{A}, \tilde{B})$ , a bivariate log-normal is also assumed. The market values of old and new debt are computed with relation (6.51) for the following values of the parameters  $\bar{V}$  and  $F$  (the coefficient of variation  $cv_d$  is assumed constant):

$d_o^{\text{old}}$  = Market value of old debt if the new project is not taken  
 = Relation (6.51) with  $\bar{V} = (S+a)$ , and  $F = F^{\text{old}}$ .

$d^{\text{old}}$  = Market value of old debt if new project is taken  
 = Relation (6.51) with  $\bar{V} = (I+a+b)$ , and  $F = F^{\text{old}}$ .

$d^{\text{new}}$  = Market value of new debt (new project is taken). It is computed jointly with  $d^{\text{old}}$ ,

$d^{\text{old}} + d^{\text{new}}$  = Relation (6.51) with  $\bar{V} = (I+a+b)$ , and  $F = (F^{\text{old}} + F^{\text{new}})$ .

Finally, in section 6.4, when two sequential projects are considered, the market value of debt when the first project is taken and the second one is not, is computed also with relation (6.51).

#### 6.5.2 Algorithm for finding the equilibrium solution.

A general algorithm for finding the equilibrium conditions in a debt-equity case, can be derived on the basis of relations expressed in terms of  $(a^{\circ}, b^{\circ})$  (the market values net of old debt payments). The basic steps in this algorithm are get started by assuming an initial configuration of region  $M'$  (new project is taken) to find a first estimate of  $D$ , then solve the "pure-equity" problem obtained when  $D$  is known, and finally check if the assumptions done with regard to  $M'$  are correct or need to be modified.

In more detail, the steps of the algorithm are described below and summarized in Figure 25.

Step 1: Get Data:

$I, S, F^{\text{old}}, F^{\text{new}}$ , Distribution of  $f(a, b) : \{a_i, b_j, p_{ij}\}$ ,  
and parameter  $cv_d$

Step 2: Compute  $(a^{\circ}, b^{\circ})$  and  $d$  for all pairs  $(a, b)$

$$a^{\circ} = a - d^{\text{old}}$$

[Relation (6.28)]

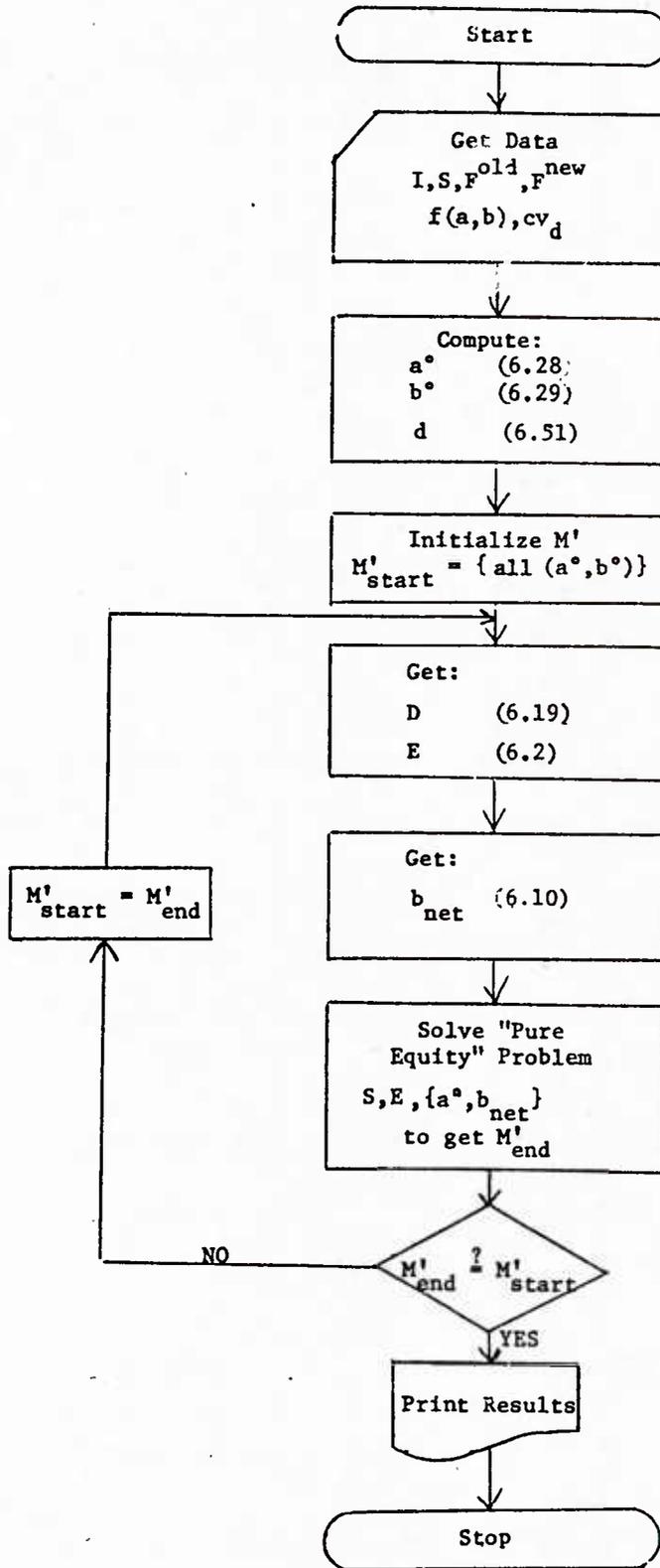


FIGURE 25: Algorithm for Finding M' and P' in the Debt-Equity Case

$$b^{\circ} = b - [d^{\text{old}} - d_0^{\text{old}}] \quad [\text{Relation (6.29)}]$$

$$d = [\text{relation (6.51) with } \bar{v} = (I+a+b), F = (F^{\text{old}} + F^{\text{new}})] - d^{\text{old}}$$

Step 3: Initialize region  $M'$  as

$$M'_{\text{start}} = \text{All pairs } (a^{\circ}, b^{\circ})$$

Step 4: Get D, E as:

$$D = \text{Ex}\{\tilde{D} | M'_{\text{start}}\} \quad [\text{Relation (6.19)}]$$

$$E = I - S - D \quad [\text{Relation (6.2)}]$$

Step 5: Get  $b_{\text{net}}$  for all pairs  $(a^{\circ}, b^{\circ})$ :

$$b_{\text{net}} = b^{\circ} - (d - D) \quad [\text{Relation (6.10)}]$$

Step 6: Solve the "pure equity" problem with parameters S, E, Distribution of  $\{a^{\circ}, b_{\text{net}}\}$ . In this step a new  $M'$  (named  $M'_{\text{end}}$ ) and the corresponding  $P'$  are obtained.

Step 7: Check if  $M'_{\text{start}}$  and  $M'_{\text{end}}$  are equal.

- If  $M'_{\text{end}} = M'_{\text{start}}$ , then PRINT results and STOP

- If  $M'_{\text{end}} \neq M'_{\text{start}}$ , then define

$$M'_{\text{end}} = M'_{\text{start}}$$

and go back to step 4.

The convergence properties of this algorithm are discussed in Appendix 8.

### 6.5.3 Results obtained

The first issue examined in this numerical analysis is observing the total loss in market value when debt is made available in a situation with an intermediate set of parameters, and comparing those results with the pure equity situation. The set of parameters chosen and the results obtained are presented in Table 21. The conclusion is that debt-equity financing is dominated by pure equity financing in the two examples presented.

The observation above cannot be generalized to conclude that, in general, pure equity is preferred to debt-equity financing.\* In fact, the second issue examined is determining the total loss in market value for different combinations of old and new debt. The conditions and results of this experiment are present in Table 22. Three conclusions may be derived from here: first, the impact of debt in the long-run appears to be detrimental for the market value of the firm. (Observe in Case 1 that increasing old debt enlarges the loss in market value.) Second, the impact of debt in the short-run appears to be favorable for the market value of the firm, (Observe in Case 2 that increasing new debt reduces the loss in market value.) Third, a stable debt-equity financing policy may be better or worse than pure equity financing. (Observe in Case 3 that moving toward debt financing increases the loss in market value in the first two examples and reduces it in the last one.)

The first two conclusions may be explained in terms of the "moral

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\* Remember that no tax shield advantages are included for debt.

TABLE 21: Debt-Equity Case

Market Value Loss for Intermediate Value of Parameters

$(\tilde{A}, \tilde{B})$  log-normally distributed with parameter

$$\bar{A} = 100 \qquad \bar{B} = 10$$

$$cv_a = .5 \qquad cv_b = .5$$

$$\rho_{ab} = .7$$

$$I = 40$$

$$\text{Debt policy: } \frac{F^{\text{old}}}{(S+\bar{A})} = \frac{F^{\text{new}}}{(I-S)} = .5$$

$$cv_d = .5$$

$\frac{S}{I}$ (%)	Debt-equity situation		Pure equity situation	
	$\Delta/\bar{B}$ (%) *	Pr(M') (%)	$\Delta/\bar{B}$ (%)	Pr(M') (%)
0	100	0	63.2	48.0
50	14.2	89.5	7.1	95.0

\*  $\Delta$  represents the loss in market value of the firm and it is computed as  $\bar{B}(M') \cdot \text{Pr}(M')$ .

TABLE 22: Debt-Equity Case

Market Value Loss for Different Combinations of Old and New Debt

$(\tilde{A}, \tilde{B})$  log-normally distributed

$\bar{A} = 100$                        $\rho_{ab} = 0$

$S = 0$

$cv_d = 0$

Case 1:  $F^{new} = 100\%$  of I

$cv_a$	$cv_b$	I	$\bar{B}$	$F^{old}$ (% of $\bar{A}$ )	$F^{new}$ (% of I)	$\Delta/\bar{B}$ (%)	Pr(M') (%)
1	1	100	10	0	100	0	100
				10	100	0.03	99.9
				50	100	18.2	75.1
1	.1	10	10	0	100	0	100
				25	100	7.4	96.9
				75	100	45.7	54.0

Case 2:  $F^{old} = 10\%$  of  $\bar{A}$

$cv_a$	$cv_b$	I	$\bar{B}$	$F^{old}$ (% of $\bar{A}$ )	$F^{new}$ (% of I)	$\Delta/\bar{B}$ (%)	Pr(M') (%)
1	1	100	10	10	10	70.8	26.4
				10	90	18.2	72.4
				10	100	0.03	99.9
1	1	10	10	10	10	15.4	75.8
				10	100	0.03	99.9

Case 3:  $F^{old}/\bar{A} = F^{new}/I$

$cv_a$	$cv_b$	I	$\bar{B}$	$F^{old}$ (% of $\bar{A}$ ) = $F^{new}$ (% of I)	$\Delta/\bar{B}$ (%)	Pr(M') (%)
.5	.5	100	10	0	66.3	32.0
				10	67.6	30.6
				20	70.1	28.4
				25	71.9	26.9
.3	.3	100	10	0	48.3	50.5
				10	48.3	50.5
				50	53.8	45.2
.1	.1	100	10	0	3.0	97.0
				10	2.9	97.0
				50	2.4	97.6
				90	0.4	99.5

hazard problem" created by old debt outstanding, and the "signaling problem" generated by the insufficient slack availability.\* Nonetheless, this model is inadequate to explain the existence of an optimum capital structure, because it considers only the long run effect of old debt with the short-run effect of new debt.

The last set of experiments have been designed to gain some further insight into the problem of a firm capital structure. In the first place, Table 23 presents some numerical analysis to measure the level of magnitude of the moral hazard and signaling losses, for different combinations of slack availability and old debt outstanding. The moral hazard problem alone is important only at very high levels of old debt, especially when slack is low. This is the loss obtained when the expectations for the future are that new projects will be 100% debt financed without any problem. If on the contrary, projects in the future must be 100% equity financed, the total loss in market value increases in a very significant way when old debt is outstanding for all levels of slack. Therefore, though raising debt is helpful in reducing the equity required for the immediate project, its future impact may go between "midly negative" (if forthcoming projects are pretty much financed), to "very negative" (if those projects can only be financed via new equity issues). The important conclusion for the purposes of this thesis is that slack availability is an important determinant of the loss in market value.

The final exploration of the capital structure problem is done with the very simple dynamic model introduced in section 6.4. The main

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\* See section 6.4 for the description of these two problems.

TABLE 23: Debt-Equity Case

A Measure of the Long and Short Run Impacts of Debt

$(\tilde{A}, \tilde{B})$  log-normally distributed with:

$$\begin{aligned} \bar{A} &= 100 & \bar{B} &= 10 \\ cv_a &= .5 & cv_b &= .5 \\ \rho_{ab} &= .7 \\ I &= 40 \\ cv_d &= 0 \end{aligned}$$

The long run impact of debt outstanding (the moral hazard loss):  
 $F^{new} = 100\%$  of  $(S-I)$

$\frac{S}{I}$ (%)	Loss in market value as percentage of $\bar{B}$ *				
	$F^{old}/\bar{A}$ (%)				
	0	25	50	75	100
0	0	0	3.2	16.5	36.7
25	0	0	1.0	10.0	28.0
50	0	0	0.1	5.1	20.2
75	0	0	0	1.9	13.5
100	0	0	0	0.4	7.4

The short run impact of debt unavailability (the signaling loss):  
 $F^{old} = F^{new} = 0$  (pure equity situation)

$\frac{S}{I}$ (%)	Loss in market value as percentage of $\bar{B}$	
0	63.2	
25	29.7	
50	7.1	
75	0.2	
100	0.0	

The long run impact of debt (both signaling and moral hazard loss): \*\*  
 $F^{new} = 0$

$\frac{S}{I}$ (%)	Loss in market value as percentage of $\bar{B}$				
	$F^{old}/\bar{A}$ (%)				
	0	25	50	75	100
0	63.2	97.1	100	100	100
25	29.7	57.7	100	100	100
50	7.1	14.8	41.8	100	100
75	0.2	0.6	2.3	9.1	31.0
100	0.0	0.0	0.0	0.4	7.4

\* This loss is computed as the reduction in market value obtained when the new project is not taken because its net present value is less than the increase in the value of old debt.

\*\* In general, when  $F^{old} \neq 0$  and  $F^{new} \neq 100\%$ , the signaling and moral hazard losses cannot be separated.

feature of that model is considering a short-term and a long-term investments decision, to capture the impact of new debt in its immediate and delayed effects. The conditions of the analysis and the results of the experiments are presented in Table 24.

The results obtained for a coefficient of variation of  $.5 (cv_d = .5)$  follow exactly the a priori expectations for the total loss in market value; namely, when  $F^{new}$  is increased, the loss in market value in the immediate project goes down (because the project is better financed), and the loss in the future project goes up (because the level of debt outstanding increases). In the examples presented, the total loss has an interior minimum.

The results obtained for a coefficient of variation of 1 ( $cv = 1$ ) are somewhat surprising but not unexpected. The short-run impact of new debt completely dominates the situation. This loss goes down initially (as expected), but then it turns around and climbs up to 10 (which is 100% of the value of the immediate project). That is to say, in the signal transmitted for very high values of new debt, the clean statement of extra funds availability is overshadowed by the heavy loss incurred in the future. The market conclusion is that the immediate project cannot be taken at that high level of debt. This example shows that the distortion of the signal introduced by the presence of debt, may escape our intuition of the problem. Further analysis is certainly required to gain a deeper insight on the market signals generated by the combination of debt and equity in an on-going business.

TABLE 24: Debt-Equity Case  
 The Optimal Capital Structure for a Firm with Two Sequential Investments

$(\tilde{A}, \tilde{B})$  log-normally distributed with:  
 $\bar{A} = 100$        $\bar{B} = 10$   
 $cv_a = .5$        $cv_b = .5$   
 $\rho_{ab} = .7$   
 Net present value of second project:  $c = 10$   
 $S = 0$   
 $I = 100$

Debt parameters and policy			Loss in Market Value		TOTAL
$cv_d$	$F^{old}$	$F^{new}$	Short-run $\bar{B}(M) \cdot Pr(M)$	Long-run $c \cdot Pr(c < F-d   M') \cdot Pr(M')$	
.5	50	75	3.79	0.000	3.79
	50	90	0.725	0.225	0.95
	50	99	0.045	2.235	2.28
.5	75	75	3.06	0.000	3.06
	75	90	2.515	2.315	4.83
1	50	50	6.44	0.00*	6.44
	50	60	5.70	0.00*	5.70
	50	65	6.59	0.66	7.25
	50	75	10.00	0.00**	10.00
	50	90	10.00	0.00**	10.00
1	60	60	5.34	0.00*	5.34
	60	70	10.00	0.00**	10.00

\* The total loss in the long run project is zero, because that project can always be taken at the level of debt outstanding shown.

\*\* The total loss in the long run project is zero, because the immediate project cannot be taken at the level of debt shown. Therefore, by the assumptions in the model, the second project is always taken.

CHAPTER 7: A MODEL FOR THE STUDY OF MERGERS: A MERGERS RATIONALE

The theoretical development presented in this study opens the possibility of specifying conditions favoring merger. As was stated at the beginning of Chapter 4, the basic idea has been to find a way of explaining the merger phenomenon without violating fundamental assumptions in the theory of finance; namely, market equilibrium and rational maximization of shareholders' well being pursued by the managers of the firm.

The mechanism suggested rests on the idea of financial slack. The concept of slack and the way in which slack availability may affect the market value of the firm has been presented in Chapters 5 and 6. The fundamental conclusion is that the market value of a firm with insufficient slack may not capture the total value of future opportunities.

When the full value of future opportunities is not recognized in the market value of a firm, there is room for an abnormal gain by injecting resources into the firm. If the drop in market value is  $\Delta$  (the loss due to insufficient slack), and the amount of resources injected is  $(I-S)$  (to insure that the investment requirement  $I$  is available), then the market value of the firm has to jump in  $(I-S)+\Delta$ . There is a net capital gain by the amount  $\Delta$  in this operation.

This mechanism is a very powerful incentive for merging. In general, if a "cash rich" firm (a firm with excess slack) merges with a "cash poor" firm (a firm with insufficient slack), the consolidation automatically generates a capital gain when the excess slack in the cash rich firm can

fully satisfy the need for slack in the other firm. This gain stems from the market recognition of the full value of future opportunities owned by the cash poor firm, and it is equal to  $\Delta$ .

Mergers understood as a procedure for transferring cash from cash-rich to cash-poor firms can explain many mergers episodes. The actual mechanism by which this transfer of resources may occur is the tender-offer route. According to the assumptions used in the development of this model, there is sufficient public information to know if all potential opportunities that a firm has are being represented in the market value. When slack is insufficient, it is clear for everybody that there is an expected positive gain by buying the firm at its market value at time 0, before the firm declares its intention regarding the new project. Consequently, the unexpected tender offer is a form of merging that can be justified within the scope of this model. What may require some further analysis is the negotiated merger. This is done in section 7.5 at the conclusion of this chapter.

The transfer of resources to fully cover the cash needs of the acquired firm is not the only situation under which a merger may be potentially advantageous. The methodology for determining the market value of slack exposed along this study, can be used also to explore many different situations in which mergers can make sense. The procedure is to determine the market value of the firms as independent entities, and as a combined business concern, and choose the strategy with the highest value. Notice that the arguments are identical to justify the merger of two firms, or the separation of them. The choice depends on the condition

that renders the best value, as indicated in the following decision table:

<u>Current State</u>	<u>Market Values</u>	
	<u>Independent &gt; Merged</u>	<u>Independent &lt; Merged</u>
Firms are Independent	Do nothing	Merger
Firms are Merged	Divest	Do nothing

In general, nothing can be said about the best course of action in a particular situation. It all depends on the conditions prevailing on the firms participating in the deal at the time the decision to merge (or divest) is being considered.

This section develops a methodology for determining the equilibrium market value of two firms that merge, in cases where the total slack is insufficient to cover for the needs of the two firms. (The analysis corresponds to the case of one firm with two investment opportunities instead of one.) Three cases must be distinguished: first, total slack is insufficient to cover the investment requirements for any one of the two firms separately, but not for both of them; second, slack can cover only the needs of the firm with lower requirement; and third, slack cannot cover the need for any one of the firms.

The slack availability and the investment requirements are designated by  $S_1$  and  $S_2$ , and  $I_1$  and  $I_2$ , for firms 1 and 2, respectively (with  $I_1 \geq I_2$ ). The formal definition of the three cases above is:

$$\text{Case 1: } I_2 \leq I_1 \leq S_1 + S_2 < I_1 + I_2$$

Case 2:  $I_2 \leq S_1 + S_2 < I_1$  \*

Case 3:  $S_1 + S_2 < I_2 \leq I_1$

The three cases are analyzed when only equity is available as an external source of cash.

7.1 Case 1 (Slack is sufficient to cover any one of two investment requirements):  $I_2 \leq I_1 \leq S_1 + S_2 < I_1 + I_2$

A firm in this situation has the choice of going ahead with either of the two projects. Also, if it appears advantageous, it can raise in the market an amount  $E = (I_1 + I_2) - (S_1 + S_2)$  of new equity and go ahead with the two projects. It is always advantageous to invest in at least one project, because their net present values are non-negative.

The nomenclature and methodology used is similar to the pure equity case. Firm 1 has market values at time +1  $\{\tilde{A}_1, \tilde{B}_1\}$  for current business and future opportunity. For firm 2, the corresponding market values are  $\{\tilde{A}_2, \tilde{B}_2\}$ . These market values have a joint probability distribution  $f(a_1, b_1, a_2, b_2)$ , and the actual outcomes are designated by  $(a_1, b_1)$  and  $(a_2, b_2)$ , respectively.

The firm gets to know  $(a_1, b_1, a_2, b_2)$  at time 0, and announces its decision, which may be:

- Take only project 1
- Take only project 2
- Take both projects.

---

\* Case 2 is defined for  $I_2 < I_1$  only.

The market response in these three situations is to drive market values to the following levels, respectively:

$$V^{\text{old}}(\phi, \text{take-1}) = P_1 \quad (7.1)$$

$$V^{\text{old}}(\phi, \text{take-2}) = P_2 \quad (7.2)$$

$$V^{\text{old}}(\phi, \text{take-both}) = P_{12} \quad (7.3)$$

The market values of old shareholders' shares at time +1, when  $(a_1, b_1, a_2, b_2)$  become known in the market are, respectively:

$$V^{\text{old}}[\phi, \text{take-1}, (a_1, b_1, a_2, b_2)] = S_1 + S_2 + a_1 + a_2 + b_1 \quad (7.4)$$

$$V^{\text{old}}[\phi, \text{take-2}, (a_1, b_1, a_2, b_2)] = S_1 + S_2 + a_1 + a_2 + b_2 \quad (7.5)$$

$$V^{\text{old}}[\phi, \text{take-both}, (a_1, a_2, b_1, b_2)] = \frac{P_{12}}{E + P_{12}} (E + S_1 + S_2 + a_1 + a_2 + b_1 + b_2) \quad (7.6)$$

Three regions are defined in space  $(a_1, b_1, a_2, b_2)$ :  $M_1$ ,  $M_2$ , and  $M_{12}$ . In the first region, the best strategy is to take project 1 only; in the second, take project 2 only; and in the third, take both projects. The three regions are mutually exclusive and collectively exhaustive. Their formal definition is the following:

For each 4-tuple  $(a_1, b_1, a_2, b_2)$  define the following term:

$$\begin{aligned} \text{mx} = \text{mx}(a_1, b_1, a_2, b_2) = \max [ & S_1 + S_2 + a_1 + a_2 + b_1; S_1 + S_2 + a_1 + a_2 + b_2; \\ & \frac{P_{12}}{E + P_{12}} (S_1 + S_2 + a_1 + a_2 + b_1 + b_2) ] \end{aligned} \quad (7.7)$$

Then:\*

$$M_1 = \{(a_1, b_1, a_2, b_2) \mid mx = S_1 + S_2 + a_1 + a_2 + b_1\} \quad (7.8)$$

$$M_1 = \{(a_1, b_1, a_2, b_2) \mid mx = S_1 + S_2 + a_1 + a_2 + b_2\} \quad (7.9)$$

$$M_{22} = \{(a_1, b_1, a_2, b_2) \mid mx = \frac{P_{12}}{E+P_{12}} (S_1 + S_2 + a_1 + a_2 + b_1 + b_2)\} \quad (7.10)$$

The equilibrium conditions that may be derived from here are the following:

If project 1 is taken:

$$\begin{aligned} P_1 &= \text{Ex} \{V^{\text{old}}[\phi, \text{take-1}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2)] \mid M_1\} \\ &= S_1 + S_2 + \overline{A_1}(M_1) + \overline{A_2}(M_1) + \overline{B_1}(M_1) \end{aligned} \quad (7.11)$$

If project 2 is taken:

$$\begin{aligned} P_2 &= \text{Ex} \{V^{\text{old}}[\phi, \text{take-2}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2)] \mid M_2\} \\ &= S_1 + S_2 + \overline{A_1}(M_2) + \overline{A_2}(M_2) + \overline{B_2}(M_2) \end{aligned} \quad (7.12)$$

If both projects are taken:

$$\begin{aligned} P_{12} &= \text{Ex} \{V^{\text{old}}[\phi, \text{take-both}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2)] \mid M_{12}\} \\ &= \frac{P_{12}}{E+P_{12}} [E + S_1 + S_2 + \overline{A_1}(M_{12}) + \overline{A_2}(M_{12}) + \overline{B_1}(M_{12}) + \overline{B_2}(M_{12})] \\ &= S_1 + S_2 + \overline{A_1}(M_{12}) + \overline{A_2}(M_{12}) + \overline{B_1}(M_{12}) + \overline{B_2}(M_{12}) \end{aligned} \quad (7.13)$$

This case looks more complicated than the pure equity case with one

\* If a 4-tuple  $(a_1, b_1, a_2, b_2)$  belongs to more than one of the  $M_1, M_2, M_{12}$ , it has to be assigned to only one of them in any arbitrary way.

investment opportunity, but in fact it can be reduced to it without any problem. This is possible because there is only one degree of freedom in the system, and this is the variable  $P_{12}$ . Once this variable is found, all three sets ( $M_1$ ,  $M_2$ , and  $M_{12}$ ) are determined, and the other two equilibrium market values ( $P_1$  and  $P_2$ ) are also determined.

The one project situation equivalent to this case may be defined in terms of the following parameters:

$$\begin{aligned} S^* &= S_1 + S_2 \\ I^* &= I_1 + I_2 \\ E^* &= I^* - S^* \\ a^* &= a_1 + a_2 + \max(b_1, b_2) \\ b^* &= \min(b_1, b_2) \end{aligned}$$

$P_{12}$  is the equilibrium market value when project  $b^*$  is taken.

Without going into the details of the derivation, it is not hard to see that the loss for insufficient slack becomes:

$$\Delta_{12}(\phi) = \overline{B}_1(M_2) \cdot \Pr(M_2) + \overline{B}_2(M_1) \cdot \Pr(M_1) \quad (7.14)$$

This loss should be compared with the sum of the two losses when  
 \*\*  
 firms are independent:

$$\Delta_S(\phi) = \Delta_1(\phi) + \Delta_2(\phi) = \overline{B}_1(M_1^o) \cdot \Pr(M_1^o) + \overline{B}_2(M_2^o) \cdot \Pr(M_2^o) \quad (7.15)$$

No general statement can be made from comparing these two expressions. The loss  $\Delta_{12}(\phi)$  in the merger may be larger or smaller than the loss  $\Delta_S(\phi)$  of the two individual firms aggregated. For example, consider

\*\*  
 The superscript o has been added to indicate that these regions are defined for each firm considered separately.

that firm 1 is a firm in a mature industry, in which all new investment opportunities have a net present value close to zero ( $b_1$  small). On the other hand, firm 2 is in a rapidly growing market with the net present value of investment opportunities very high ( $b_2$  large). The convenience of merging or not depends on where slack is initially located. If most of the slack is in the aging firm, the merger is good, while if the situation is the other way around, individual firms look more attractive.

7.2 Case 2 (Slack is sufficient to cover only the lowest of two investment requirements):  $I_2 \leq S_1 + S_2 < I_1$

In this case, the firm always has sufficient slack to take project 2, but if project 1 is taken, it must raise  $E_1 = I_1 - (S_1 + S_2)$  in new equity, while if both projects are taken, the need for new equity is

$$E = (I_1 + I_2) - (S_1 + S_2).$$

The parameters  $P_1$ ,  $P_2$ , and  $P_{12}$  are defined just as in the previous case, as the market values at time 0, after the firm announces it is going to take project 1 only, project 2 only, or both projects, respectively [see (7.1), (7.2), (7.3)].

At time +1, when  $(a_1, b_1, a_2, b_2)$  becomes known to the market, the values of old shareholders' shares are the following under each one of these alternative decisions:

$$V^{\text{old}}[\phi, \text{take-1}, (a_1, b_1, a_2, b_2)] = \frac{P_1}{E_1 + P_1} (E_1 + S_1 + S_2 + a_1 + a_2 + b_1) \quad (7.16)$$

$$V^{\text{old}}[\phi, \text{take-2}, (a_1, b_1, a_2, b_2)] = S_1 + S_2 + a_1 + a_2 + b_2 \quad (7.17)$$

$$V^{\text{old}}[\phi, \text{take-both}, (a_1, b_1, a_2, b_2)] = \frac{P_{12}}{E+P_{12}} (E+S_1+S_2+a_1+a_2+b_1+b_2) \quad (7.18)$$

As before, three regions may be defined in space  $(a_1, b_1, a_2, b_2)$ , and they are called  $M_1$ ,  $M_2$ , and  $M_{12}$ . The best strategy is to take project 1 only in the first region, two only in the second, and both projects in the third region. These regions are mutually exclusive and collectively exhaustive, and their formal definitions are:

For each 4-tuple  $(a_1, b_1, a_2, b_2)$  define the term:

$$\begin{aligned} \text{mx} = \text{mx}(a_1, b_1, a_2, b_2) &= \max \left[ \frac{P_1}{E_1+P_1} (E_1+S_1+S_2+a_1+a_2+b_1); S_1+S_2+a_1+a_2+b_2; \right. \\ &\left. \frac{P_{12}}{E+P_{12}} (E+S_1+S_2+a_1+a_2+b_1+b_2) \right] \end{aligned} \quad (7.19)$$

Then:\*

$$M_1 = \{(a_1, b_1, a_2, b_2) \mid \text{mx} = \frac{P_1}{E_1+P_1} (E_1+S_1+S_2+a_1+a_2+b_1)\} \quad (7.20)$$

$$M_2 = \{(a_1, b_1, a_2, b_2) \mid \text{mx} = S_1+S_2+a_1+a_2+b_2\} \quad (7.21)$$

$$M_{12} = \{(a_1, b_1, a_2, b_2) \mid \text{mx} = \frac{P_{12}}{E+P_{12}} (E+S_1+S_2+a_1+a_2+b_1+b_2)\} \quad (7.22)$$

The equilibrium conditions imply the following relations in this case:

If project one is taken:

$$\begin{aligned} P_1 &= \text{Ex} \{V^{\text{old}}(\phi, \text{take-1}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2) \mid M_1)\} \\ &= \frac{P_1}{E_1+P_1} [E_1+S_1+S_2+\overline{A_1}(M_1)+\overline{A_2}(M_1)+\overline{B_1}(M_1)] \end{aligned}$$

\* If  $(a_1, b_1, a_2, b_2)$  belongs in more than one region, the tie is broken arbitrarily.

$$P_1 = S_1 + S_2 + \overline{A_1}(M_1) + \overline{A_2}(M_1) + \overline{B_1}(M_1) \quad (7.23)$$

If project 2 is taken:

$$\begin{aligned} P_2 &= \text{Ex} \{V^{\text{old}}[\phi, \text{take-2}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2) | M_2] \} \\ &= S_1 + S_2 + \overline{A_1}(M_2) + \overline{A_2}(M_2) + \overline{B_2}(M_2) \end{aligned} \quad (7.24)$$

If both projects are taken:

$$\begin{aligned} P_{12} &= \text{Ex} \{V^{\text{old}}[\phi, \text{take-both}, (\tilde{A}_1, \tilde{B}_1, \tilde{A}_2, \tilde{B}_2) | M_{12}] \} \\ &= \frac{P_{12}}{E+P_{12}} [E+S_1+S_2+\overline{A_1}(M_{12})+\overline{A_2}(M_{12})+\overline{B_1}(M_{12})+\overline{B_2}(M_{12})] \\ &= S_1 + S_2 + \overline{A_1}(M_{12}) + \overline{A_2}(M_{12}) + \overline{B_1}(M_{12}) + \overline{B_2}(M_{12}) \end{aligned} \quad (7.25)$$

Solving for  $P_1$ ,  $P_2$ , and  $P_{12}$  is more laborious than in the previous case, because there are two degrees of freedom instead of one. It is necessary to solve simultaneously for  $P_1$  and  $P_{12}$ , the value of  $P_2$  being dependent on the other two.

The loss for insufficient slack may be expressed just as in relation (7.14):

$$\Delta_{12}(\phi) = \overline{B_1}(M_2) \cdot \text{Pr}(M_2) + \overline{B_2}(M_1) \cdot \text{Pr}(M_1)$$

This value may bear any relation with the sum of individual losses when firms are independent. Therefore, no general statement can be made on the convenience or inconvenience of mergers.

7.3 Case 3 (Slack is insufficient to cover any one of two investment requirements):  $S_1+S_2 < I_2 < I_1$

This final situation finds the firm unable to undertake any of the two projects with its own resources. If a project has to be taken, the firm must turn to the market, and the amount of equity required in each one of the three alternative situations is:

To take project 1 only:

$$E_1 = I_1 - (S_1+S_2)$$

To take project 2 only:

$$E_2 = I_2 - (S_1+S_2)$$

To take both projects:

$$E = (I_1+I_2) - (S_1+S_2)$$

In this case, the firm also has the option of not taking any project. This option was not considered before, because the firm could always take project 1 or 2, and that decision strictly dominates the decision of not taking any project (net present values of new projects are assumed non-negative).

Consequently, the market value of the firm after it announces its decision may have the following values:  $P_1$  if only project 1 is taken [relation (7.1)],  $P_2$  if only project 2 is taken [relation (7.2)],  $P_{12}$  if both projects are taken [relation (7.3)], or  $P_0$  if no project is taken, where:

$$P_0 = V^{\text{old}}(\phi, \text{no take}) \quad (7.26)$$

At time +1, when  $(a_1, b_1, a_2, b_2)$  becomes known, the values in the market are:

$$V^{\text{old}}[\phi, \text{no-take}, (a_1, b_1, a_2, b_2)] = S_1 + S_2 + a_1 + a_2 \quad (7.27)$$

$$V^{\text{old}}[\phi, \text{take-1}, (a_1, b_1, a_2, b_2)] = \frac{P_1}{E_1 + P_1} (E_1 + S_1 + S_2 + a_1 + a_2 + b_1) \quad (7.28)$$

$$V^{\text{old}}[\phi, \text{take-2}, (a_1, b_1, a_2, b_2)] = \frac{P_2}{E_2 + P_2} (E_2 + S_1 + S_2 + a_1 + a_2 + b_2) \quad (7.29)$$

$$V^{\text{old}}[\phi, \text{take-both}, (a_1, b_1, a_2, b_2)] = \frac{P_{12}}{E + P_{12}} (E + S_1 + S_2 + a_1 + a_2 + b_1 + b_2) \quad (7.30)$$

Depending on which one of these four values is higher, it is possible to define four mutually exclusive and collectively exhaustive regions in the space  $(a_1, b_1, a_2, b_2)$ . These regions are designated by  $M_0$ ,  $M_1$ ,  $M_2$ , and  $M_{12}$ , and they represent the set of values of  $(a_1, b_1, a_2, b_2)$  for which the optimal strategy is to take no project, take project 1 only, take project 2 only, and take both projects, respectively. Their formal definition is:

For each 4-tuple  $(a_1, b_1, a_2, b_2)$  define the term:

$$\begin{aligned} mx = mx(a_1, b_1, a_2, b_2) = \max & [S_1 + S_2 + a_1 + a_2; \frac{P_1}{E_1 + P_1} (E_1 + S_1 + S_2 + a_1 + a_2 + b_1); \\ & \frac{P_2}{E_2 + P_2} (E_2 + S_1 + S_2 + a_1 + a_2 + b_2); \frac{P_{12}}{E + P_{12}} (E + S_1 + S_2 + a_1 + a_2 + b_1 + b_2)] \end{aligned} \quad (7.31)$$

Then: \*

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\* If  $(a_1, b_1, a_2, b_2)$  belongs in more than one region, the tie is broken arbitrarily.

$$M_0 = \{(a_1, b_1, a_2, b_2) \mid mx = S_1 + S_2 + a_1 + a_2\} \quad (7.32)$$

$$M_1 = \{(a_1, b_1, a_2, b_2) \mid mx = \frac{P_1}{E_1 + P_1} (E_1 + S_1 + S_2 + a_1 + a_2 + b_1)\} \quad (7.33)$$

$$M_2 = \{(a_1, b_1, a_2, b_2) \mid mx = \frac{P_2}{E_2 + P_2} (E_2 + S_1 + S_2 + a_1 + a_2 + b_2)\} \quad (7.34)$$

$$M_{12} = \{(a_1, b_1, a_2, b_2) \mid mx = \frac{P_{12}}{E + P_{12}} (E + S_1 + S_2 + a_1 + a_2 + b_1 + b_2)\} \quad (7.35)$$

The equilibrium conditions, in this case, can be shown to be:

$$P_0 = S_1 + S_2 + \overline{A_1}(M) + \overline{A_2}(M) \quad (7.36)$$

$$P_1 = S_1 + S_2 + \overline{A_1}(M_1) + \overline{A_2}(M_1) + \overline{B_1}(M_1) \quad (7.37)$$

$$P_2 = S_1 + S_2 + \overline{A_1}(M_2) + \overline{A_2}(M_2) + \overline{B_2}(M_2) \quad (7.38)$$

$$P_{12} = S_1 + S_2 + \overline{A_1}(M_{12}) + \overline{A_2}(M_{12}) + \overline{B_1}(M_{12}) + \overline{B_2}(M_{12}) \quad (7.39)$$

To find these equilibrium values, it is necessary to solve simultaneously for  $P_1$ ,  $P_2$ , and  $P_{12}$ . The value of  $P_0$  is dependent on these three equilibrium values.

The loss for insufficient slack can be expressed as:

$$\begin{aligned} \Delta_{12}(\phi) &= \overline{B_1}(M_0) \cdot \Pr(M_0) + \overline{B_1}(M_2) \cdot \Pr(M_2) + \overline{B_2}(M_0) \cdot \Pr(M_0) + \overline{B_2}(M_1) \cdot \Pr(M_1) \\ &= \overline{B_1}(M_0 \text{ or } M_2) \cdot \Pr(M_0 \text{ or } M_2) + \overline{B_2}(M_0 \text{ or } M_1) \cdot \Pr(M_0 \text{ or } M_1) \end{aligned} \quad (7.40)$$

No general statement can be made from comparing this expression with the losses for insufficient slack for the two individual firms when added together. Under all circumstances, these losses follow the same basic pattern; namely, they aggregate the expected market value of projects 1 and 2, for all cases in which they cannot be taken, times the corresponding

probabilities of those events. Nevertheless, comparisons are hard, because the set of  $(a_1, b_1, a_2, b_2)$  values for which projects 1 and 2 should not be taken are defined by different rules. Also, they depend on the values of parameters for slack and investment, and on the joint probability distribution of  $(a_1, b_1, a_2, b_2)$ .

#### 7.4 Some Numerical Examples

To illustrate the models presented in this chapter, two examples of merger analysis are presented in Tables 25 and 26. The two examples correspond to a case 1 situation, in which the combined slack of the two firms is supposed to be sufficient for taking any one of the two projects. The conclusion is that when at least one of the firms does not have sufficient slack, the possibility exists of deriving a net capital gain from the realization of the merger.

The two examples address the merger of firms of different size. Consider example 1, in which firm 1 represents the "large" one, and firm 2 the "small" one (1/5 the size of the other firm approximately). Firm 1 has an investment opportunity with a net present value of only 5% of the investment (a "mature" firm). The investment in firm 2 has a net present value of 12.5% (a "growing" firm). If the total slack is equal to 40, the optimum strategy is merging always, because the loss due to insufficient slack for the merger is always less than the losses added for individual firms for all combinations of slack in each firm. This conclusion is not true for some higher values of slack. For example, if total slack is 50, it is better not to merge when the slack in firm 1 is 40, and the slack in firm 2 is 10 (the total loss is 1.926 for the two individual

TABLE 25: Changes in Market Value When Two Firms Merge. Example 1.

<u>Description of Firm 1</u>		<u>Description of Firm 2</u>	
$(\tilde{A}_1, \tilde{B}_1)$ approximately truncated normal		$(\tilde{A}_2, \tilde{B}_2)$ approximately truncated normal	
$\bar{A}_1 = 100.4$	$\bar{B}_1 = 2.1$	$\bar{A}_2 = 20$	$\bar{B}_2 = 5.0$
$\sigma_{a1} = 49.1$	$\sigma_{b1} = 1.54$	$\sigma_{a2} = 9.36$	$\sigma_{b2} = 2.18$
$\rho_{ab1} = 0.697$		$\rho_{ab2} = 0.365$	
$I_1 = 40$		$I_2 = 40$	
<u>Description of the merger</u> (only when total slack $\geq 40$ )			
$\bar{A} = 125.5$	$\bar{B} = 2.0$		
$\sigma_a = 58.55$	$\sigma_b = 1.55$		
$\rho_{ab} = 0.751$			

1) Total Slack = 40

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 2.007

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
0	40	2.098	0.000	2.098	Merge
10	30	2.098	0.000	2.098	Merge
20	20	2.098	0.016	2.114	Merge
30	10	2.096	1.926	4.022	Merge
40	0	2.000	3.410	3.410	Merge

2) Total Slack = 50

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 2.007

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
0	50	2.098	0.000	2.098	Merge
10	40	2.098	0.000	2.098	Merge
20	30	2.098	0.000	2.098	Merge
30	20	2.096	0.016	2.112	Merge
40	10	0.000	1.926	1.926	No Merge
50	0	0.000	3.410	3.410	Merge

TABLE 25: Continued.

3) Total Slack = 60

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 2.007

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
0	60	2.098	0.000	2.098	Merge
10	50	2.098	0.000	2.098	Merge
20	40	2.098	0.000	2.098	Merge
30	30	2.096	0.000	2.096	Merge
40	20	0.000	0.016	0.016	No Merge
50	10	0.000	1.926	1.926	No Merge
60	0	0.000	3.410	3.410	Merge

4) Total Slack = 70

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 2.007

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
0	70	2.098	0.000	2.098	Merge
10	60	2.098	0.000	2.098	Merge
20	50	2.098	0.000	2.098	Merge
30	40	2.096	0.000	2.096	Merge
40	30	0.000	0.000	0.000	No Merge
50	20	0.000	0.016	0.016	No Merge
60	10	0.000	1.926	1.926	No Merge
70	0	0.000	3.410	3.410	Merge

TABLE 25: Continued.

5) Total Slack = 80

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 0

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
0	80	2.098	0.000	2.098	Merge
10	70	2.098	0.000	2.098	Merge
20	60	2.098	0.000	2.098	Merge
30	50	2.096	0.000	2.096	Merge
40	40	0.000	0.000	0.000	Indifferent
50	30	0.000	0.000+	0.000+	Merge
60	20	0.000	0.016	0.016	Merge
70	10	0.000	1.926	1.926	Merge
80	0	0.000	3.410	3.410	Merge

6) Total Slack Between 70 and 80

DETAILED ANALYSIS FOR THE CASE SLACK IN FIRM 2 IS 40

(LOSS FOR INSUFFICIENT SLACK IN FIRM 2 IS 0)

Slack in Firm 1	Total Slack	Losses Added*	Loss for Merger	Optimum Decision
30	70	2.096	2.007	Merge
32	72	2.095	1.398	Merge
34	74	0.390	0.083	Merge
36	76	0.035	0.011	Merge
38	78	0.002	0.001	Merge
40	80	0.000	0.000	Indifferent

\* This is equivalent to the loss in Firm 1.

TABLE 26: Change in Market Value When Two Firms Merge. Example 2.

<u>Description of Firm 1</u>		<u>Description of Firm 2</u>	
$(\bar{A}_1, \bar{B}_1)$ approximately truncated normal		$(\bar{A}_2, \bar{B}_2)$ approximately truncated normal	
$\bar{A}_1 = 100.01$	$\bar{B}_1 = 10.02$	$\bar{A}_2 = 10.03$	$\bar{B}_2 = 2.11$
$\sigma_{a1} = 35.7$	$\sigma_{b1} = 4.53$	$\sigma_{a2} = 4.72$	$\sigma_{b2} = 1.40$
$\rho_{ab1} = 0.274$		$\rho_{ab2} = 0.144$	
$I_1 = 100$		$I_2 = 20$	
<u>Description of the merger (only when total slack <math>\geq 100</math>)</u>			
$\bar{A} = 120.07$	$\bar{B} = 2.09$		
$\sigma_a = 38.8$	$\sigma_b = 1.39$		
$\rho_{ab} = 0.204$			

1) Total Slack = 100

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 1.649

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
70	30	0.225	0.000	0.225	No Merge
80	20	0.051	0.000	0.051	No Merge
90	10	0.002	0.157	0.159	No Merge
100	0	0.000	1.432	1.432	No Merge

2) Total Slack = 110

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 0.199

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
70	40	0.225	0.000	0.225	Merge
80	30	0.051	0.000	0.051	No Merge
90	20	0.002	0.000	0.002	No Merge
100	10	0.000	0.157	0.157	No Merge
110	0	0.000	1.432	1.432	Merge

TABLE 26: Continued.

3) Total Slack = 120

LOSS DUE TO INSUFFICIENT SLACK FOR THE MERGER = 0

LOSS DUE TO INSUFFICIENT SLACK FOR INDIVIDUAL FIRMS:

Slack Firm 1	Slack Firm 2	Loss in Firm 1	Loss in Firm 2	Losses Added	Optimum Decision
70	50	0.225	0.000	0.225	Merge
80	40	0.051	0.000	0.051	Merge
90	30	0.002	0.000	0.002	Merge
100	20	0.000	0.000	0.000	Indifferent
110	10	0.000	0.157	0.157	Merge
120	0	0.000	1.432	1.432	Merge

firms, vis-a-vis, 2.007 for the merger). Similar cases arise for slack of 60, and 70. When total slack is 80 or more, the merger is convenient again always, because the total slack allows the undertaking of both projects, without recurring to the market (there is no signaling effect).

Mergers appear as a mechanism for transferring slack from one firm to another. In general, if this transfer could be completed without merger, opportunities for capital gains should be available until the marginal value of one extra unit of slack is identical for all firms. This clean transfer of funds does not occur with a merger, because at the same time the slack resources are pooled together, the interest of the two firms are brought under a unique group of shareholders, and this changes the information content of the investment decisions taken by the merged firms. The combined effect of slack resources transfers and information changes may or may not be in favor of the merger completion, this depending on the parameters defining each particular case.

This analysis presented in the pure equity case can be generalized to a debt-equity situation, and to the case in which any finite number of projects is available for the firm to take. These generalizations are not presented in the context of this thesis, because they do not add a great deal to the basic conclusion that the amount of slack held by firms affects their market value, and that this mechanism opens the possibility of justifying mergers in terms of the capital gain that they can generate when part or all the value of future opportunities that was not being valued in the market is recognized in the joint value of participating firms.

### 7.5 Negotiated Mergers

All previous sections in this chapter have explored the conditions under which a tender offer may be an attractive strategy for a firm to follow. Broadly speaking, whenever there is a positive expected payoff from the merger, there are clear incentives for immediate take over at the prevailing market price or at a premium up to the amount  $\Delta$  (the loss in market value due to insufficient slack).

This section analyzes the negotiated merger under the assumptions of this study; that is to say, superior internal information and insufficient slack in the firm being acquired. The conditions in Chapter 5 are assumed to be descriptive of the prevailing situation: the firm has an existing business, a good investment opportunity, and the only source of external funds, aside from merger, is a new equity issue.

Assume that this firm is approached by a prospective buyer, and it is offered an amount  $Q'$  for the whole firm. The expected payoffs for current shareholders depend on the decision of the firm, as indicated below:

If the firm accepts the merger (and it goes ahead with the project):

$$V^{\text{old}}[\phi, \text{merger}, (a, b)] = Q' \quad (7.41)$$

If the firm does not accept the merger (and it does not go ahead with the project):

$$V^{\text{old}}[\phi, \text{no-merger}, (a, b)] = S + a \quad (7.42)$$

The merger route appears to be convenient whenever  $Q'$  exceeds  $(S+a)$ . This condition defines a region  $N'$  in which the merger is convenient. Formally stated:

$$N' = \{(a,b) \geq 0 \mid Q' \geq (S+a)\} \quad (7.43)$$

The equilibrium value of  $Q'$  must satisfy the condition:

$$Q' = S + \bar{A}(M') + \bar{B}(N') \quad (7.44)$$

It is not hard to show that this merger offer is totally unsatisfactory for the firm, because it can always do better by going directly to the market and issuing stock. In fact, by methods similar to the ones used in Appendix 1, it may be shown that this equilibrium value  $Q'$  is a lower limit for the value  $P'$  which the firm would obtain by issuing stock. The intuitive reason for this result is that the decision to issue stock carries some negative information with respect to the current business. This makes new shareholders discount the market value of the firm by a certain amount, in order to make the expected payoff identical to their contribution. In this case, current shareholders still have some fraction of ownership; but, if the firm is sold, they are completely disengaged from the firm. That makes the prospective buyer discount the market value more heavily for having the expected payoff equal to the price they offer.

Given that this one shot negotiation procedure is not available as an alternative, the only way to understand a negotiated merger under the premises of this study is by assuming that there is a partial or total disclosure of the internal information during the negotiation process. The firm being acquired may get a full recognition of its intrinsic value in this process; or, at least, it may get more value than through the normal channels in the market. In this way, there is a potential gain for the seller, because it can exceed the valuation performed in the market

with less updated information. There is also a potential gain for the buyer, if it can force the other firm to accept less than its intrinsic value. The actual equilibrium price for the merger would depend on the supply and demand equilibrium in the market for the acquisition of firms.

A more complete analysis of the negotiated route for mergers is not included in this study, and further exploration of the subject is certainly necessary. Nonetheless, it is clear that accepting the notion of superior information inside the firm, the merger negotiation may be seen as the transfer of this information, and the agreement in a merger price between two firms. What is not sufficiently clear are the rules driving this negotiation process.

## CHAPTER 8: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

Mergers are not a new phenomenon, but rather an on-going process in the business environment. They correspond to the combination of two (or more) firms into a unique business concern. This study presents a review of some interesting issues in the study of mergers, and proposes a financial rationale to explain mergers in general and conglomerates in particular.

As an overview of the overall study, each one of the previous chapters is summarized in this section, and the most important conclusions are restated.

Chapter 1 looks at mergers from an historical perspective, and analyzes the characteristics of the three waves in merger activity: those peaking at the turn of the century (1899), the late twenties (1929), and the late sixties (1968). Important elements in this analysis are the evolution and enforcement of antitrust legislation, and the new concern for superconcentration (which is the accumulation of economic power in a few hands), for the political implications it may have.

Chapter 2 discusses some of the most popular arguments given to explain mergers. Among them may be mentioned: acquisition of monopoly power, the synergistic effect, tax and other economic incentives, diversification of risk, the empire building motive, and the search for promotional and speculative profits. It is a widespread belief that most of these economic incentives played a fundamental role in the first and second waves, but their importance is doubtful in the last wave.

Chapter 3 presents the results of a bibliographic search on the sub-

ject of mergers. The studies reviewed are classified in four main groups: Empirical Determination of Merger Profitability, Mergers as Result of Managerial Decisions, Financial Explanations for Mergers and Conglomerates, and Studies on Merger Waves.

Studies that measure directly the profitability of mergers are very abundant, but their conclusions on the profitability for the acquiring firm are mixed. In general, all authors conclude that the acquired firm gains on average; but, for the acquiring firm, older papers tend to report a loss, while more recent papers find a neutral or even positive impact.

Studies looking at mergers as a result of managerial decisions assume that managers are pursuing their own aims rather than the satisfaction of shareholders' objectives. Financial explanations for mergers explore the issues of diversification, increase in debt capacity, and imperfections in the capital markets as potential inducement for mergers. Finally, the section on merger waves presents some insights on this periodical outburst of merger activity. The main conclusion from this chapter is that many alternative avenues have been explored to explain the merger phenomenon, but an acceptable paradigm has not been proposed yet.

Chapter 4 introduces the notion that looking at mergers from a financial point of view may provide a valid platform for analyzing the current expression of the merger movement. The main objective in the study is to build a model to explain the merger phenomenon within the existing financial framework. This chapter presents a brief review of some important issues in finance.

Chapter 5 develops a model to obtain the market value of a firm with insufficient financial slack to undertake an investment opportunity.

The only external source of funds assumed in this chapter is new equity financing. The conclusion obtained is that, in general, the full value of that opportunity is not captured in the market value of the firm, because there are situations in which, by taking the project and bringing in new shareholders, old shareholders lose (from the dilution of their holdings in the firm) more than what they get from the extra value added by the new project.

The fundamental assumption that gives value to the availability of financial slack is the difference between public information and internal information held by managers of the firm. When the decision to raise new funds for a project is taken, the firm is sending to the market the signal that old shareholders are not worse off under this investment/financing choice, and this piece of data must change the value of the firm to maintain the market equilibrium.

A sensitivity analysis performed to determine the impact of different combinations of the situational parameters over market value, shows that firms should try to increase their slack, reduce the variance of the existing business, and look for new investment opportunities which have both a high return on investment, and a low investment requirement. What is somewhat surprising is that no clear recommendations for the variance of the new business and its correlation with the existing business seem to emerge from the analysis.

The impact of slack for an intermediate situation, under roughly realistic assumptions, is determined in a separate numerical analysis. The loss in market value due to insufficient slack is 63.2 percent of the

net present value of a new project when slack is 0. This percentage goes to 29.7 percent when slack is equal to 25 percent of the total investment, to 7.1 percent for 50 percent slack, to 0.2 percent for 75 percent slack, and to 0 percent for 100 percent slack.

Chapter 6 develops a similar model to determine the market value of the firm with a good investment opportunity but insufficient slack to undertake it. It is also assumed that the firm holds information that has not reached the market at the time the decision regarding the new project is taken. The important addition in this chapter is to assume that both debt and equity are available as external sources of financing.

The extension of the pure-equity model to a debt-equity situation is done step by step, from very simple to more general situations. When debt is assumed to be risk free, there is a clear advantage in the use of debt. This advantage is maintained when assuming that debt is risky, as long as no more than one period is considered in the analysis.

Numerical analysis performed under these conditions conform exactly with these conclusions.

Going somewhat beyond the scope of this study, a tentative exploration on the optimal capital structure of a firm is also presented. This is done extending the time horizon of the model, in order to include two sequential investment decisions instead of one. The idea developed is that there are two counterbalancing effects in the acquisition of debt that must be traded-off to derive a capital structure for a firm. In general, debt has a short run effect which is beneficial, because it augments the funds available to undertake a new project. On the other hand, in the long run, debt has a negative impact on market value, because when the

level of debt outstanding is increased, the possibility to reject new projects is also increased (all benefits derived from the project are channeled to debtholders).

A limited numerical exploration of a two-period model detects in a clear way that an optimal capital structure may be derived from the trade-off between short and long run effects of debt. What is somewhat surprising, is that there are situations in which the signaling problem by itself can justify the existence of an optimal capital structure. In fact, in experiments with a "high" future uncertainty, the immediate project is initially benefited by the better financing provided by an increasing level of new debt. But, after a certain point, this positive signal is reversed because the future loss is so high if the immediate project is taken, that the actual signal received in the market is that this project must be abandoned. This signal is opposing the initial expectation that better financing availability is always good for the immediate project.

One sure conclusion to be derived from the various debt-equity models presented, and from many numerical analyses, is that our intuition of the signaling impact under this setting is, at the moment of this writing, very limited.

Chapter 7 uses the dependency of market value on the availability of slack to show that when a firm with excess slack can satisfy the need for cash of a firm with insufficient slack, the merger has a value over and above the sum of market values of the two individual firms. The difference corresponds to the loss in the value of the second firm due to insufficient slack. Under the assumptions in this study, this profit opportunity can be

exploited by the tender-offer route.

Expanding the analysis to a more general situation, it may be shown that tender offers may also be justified when the combined slack of the two firms is insufficient to fully cover their investment needs. In this case, the merger profitability is dependent on each particular set of circumstances. A model to determine the market value of a merger is obtained as an extension of the model developed in Chapter 5 when only equity financing is considered. In this case, there is an extra degree of complexity, because the signals given by a merger with its investment decisions do not have a strict correspondence with the signals given by individual firms.

The numerical exploration of merger cases is very limited, but it shows that merging may or may not be advantageous. An interesting observation is that the profitability of a merger depends, among other things, on the distribution of slack between individual firms. Therefore, the merger of the same two firms can make sense when one of those firms holds most of the slack, but not vice versa.

The negotiated merger is harder to justify under the assumptions in this study. To understand them, it is necessary to assume that part or all of the internal information is being disclosed in the negotiation process, and that the equilibrium price depends on the overall equilibrium in the market for the transaction of firms.

#### Recommendations for Further Research

The study presented seems to open some new alternatives for exploring important topics in finance theory in general, and in the merger phenomenon

in particular.

The basic observation made in this study is that the investment decision taken by a firm with insufficient slack has an informational content that the market must consider in the valuation of that firm. This result is obtained with a simple model in which only one future investment opportunity is being considered. It seems to be interesting to extend this analysis when more than one opportunity is available. The case in which two opportunities are available has been presented in Chapter 7, and the same technique can be used with a firm owning a finite number of discrete projects.

An alternative approach for studying the informational content in an investment decision is assuming that the firm has a continuum of projects, and that the choice is determining the amount to invest, rather than taking or discarding an opportunity.

The information content of the financing decision has been addressed in a very limited way; in fact, the financing decision is taken as given (or without information content) in developing a model for finding the market value when debt and equity financing are available. This assumption is partly relaxed in a two period model, but much more can be done.

As a first thing, a more precise statement of the investment options in a multiperiod setting is required. A complex problem that needs to be addressed is the availability of slack over time (note that the dividend decision is crucial in this setting).

The other important point is the timing of the financing decision, and its relation to the evolution of internal and public information. If

internal information can have an important impact on the selection of the financing strategy, the problem becomes much more complex, because the market would be receiving the investment and the financing signals at the same time.

Another line of analysis is the selection of a capital structure, which should come out as a consequence of studying the signaling effect of financing decisions indicated in the previous paragraph. Gaining a more thorough intuition of these signals in a debt-equity context, seems to be a most urgent need to better understand some behavior observed in the financial policies of firms.

A subject which has been touched very lightly in the comments at the end of Chapter 5, and that must be addressed in a more general model, is the selection of a dividend policy. There is a clear link with market value via the impact of dividends on slack availability and on the variance in the value of existing assets, but another kind of signal given by dividends is not out of the question.

Empirical studies could try to determine if slack is, in fact, an important explanatory variable of market value. The empirical analysis of mergers under this new perspective may yield some clues to this issue, and at the same time test the strength of a rationale given to explain mergers in terms of slack availability.

If the difference between internal and public information in the presence of insufficient financial slack appears to be an important element in the valuation of firms, it may be necessary to review some public policies regarding the disclosure of information by firms. It is clear that

firms tend to maintain the secrecy of their strategic information, but this may penalize the market value of a firm with insufficient slack. In this context, mergers appear as a viable mechanism to fully exploit the investment opportunities of a firm not having enough resources on its own. The possibility of further regulating the transfer of information to the market, or the transfer of funds to firms having need of it, are open questions of public policy analysis.

#### Closing Comments

When looking in retrospect at the basic conclusions of this study, the notion of having situations in which firms may be unwilling to take an investment opportunity with positive net present value, is a very disturbing one. The immediate consequence is that the normal operation of the capital market does not guarantee a socially optimal policy. Under this setting, mergers may be understood as a corrective mechanism for this socially undesirable distortion.

The conclusions in this study may be reinterpreted in a more encouraging way, however.\* Suppose that the capital market actually captures the full value of current business and future investment opportunities. This assumption has some implications on the presupposed behavior of managers of the firm. It is necessary that managers adhere to at least one of the three courses of action indicated below:

- Stock issues should be unconditional;
- The firm should not fight take-overs; or

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\* I have to thank Prof. Fischer Black for suggesting this reformulation of the conclusions of this study.

- If risk-free debt is available, this should be preferred over equity financing.

The assumptions of insufficient slack and difference in information between the public and the firm are maintained. But, if the total value of current business and future opportunities are captured in the market value of the firm, the managers are not following a strategy that depends on its superior information. This strategy must be known in advance, and it must be determined by institutional constraints or informal rules. One such strategy is the unconditional stock issuing. This means that stock should be issued whenever it is needed, regardless of the extra information held by managers. Therefore, the first proposition is "Stock issues should respond only to investment opportunities". Managers deviating from this policy would be penalized by a reduction in the market value of the firm (for an amount equal to the loss due to insufficient slack).

An important deterrent for managers is the threat of a merger. If a firm deliberately chooses to deviate from the rule of issuing unconditional stock only, it may still have the full value of the current business and its future investment opportunities represented in its market value, by having a policy of never fighting take-overs. This leads to the second proposition: "A firm which issues stock contingent upon its superior information should not fight take-overs".

The third proposition is obtained from the one period model with both risk-free debt and equity as sources of external financing. "A firm which does not follow any of the two previous rules should use one hundred percent debt financing".

It may be observed that the merger threat, institutional constraints, or informal rules, may be sufficient inducement for managers to act in a value maximizing way, regardless of the incentives to use their superior information. But there is no reason to neglect the possibility that a perfectly rational behavior is the use of all internal information in the firm, and this may imply a deviation from the value maximizing strategy. This closing section opens an intriguing question regarding the actual behavior of managers. Do they follow an issuing policy contingent upon their superior internal information, or they neglect the incentives to use their special information and follow an unconditional policy?

There is no sufficient information to answer this question. It can only be added that the empirical studies of mergers performed by Halpern [34], Mandelker [56], and mainly Ellert [18], strongly suggest that there is a net gain in mergers. There is no synergy or special economies, but the quick recovery of a value that could be fully detected in the market value eight years prior to the merger, then gradually eroded, and finally was regained in only seven or eight months prior to the merger. These empirical observations strongly suggest that the market value of a firm may, under certain circumstances, not be fully representative of the actual potential in the firm. This observation has been crucial in the selection of the point of view chosen in this study to present the model for the market value of a firm with superior information. This model has been intended to be descriptive of an observed situation. The reformulation of the conclusions in this final section points more to the normative implications of the model: What managers ought to be doing.

REFERENCES

1. Alberts, W. W., "The Profitability of Growth Through Merger", in W. W. Alberts and J. E. Segall, The Corporate Merger, Chicago, Chicago University Press, 1966.
2. Ames, R. S., "Return on Investment and Internal Growth: Basic Objectives of a Multimarket Company", in A. C. Hax and Z. S. Zannetos (ed.), "The Conglomerate Firm", Technical Report No. 5, Sloan School of Management, M.I.T., Cambridge, 1978.
3. Archer, S. H., and L. G. Faerber, "Firm Size and the Cost of Externally Secured Equity Capital", Journal of Finance, Vol. 21, No. 1, March 1966, pp. 69-83.
4. Bain, J. S., "Economies of Scale, Concentration, and the Condition of Entry in Twenty Manufacturing Industries", American Economic Review, Vol. 54, No. 1, March 1954, pp. 15-39.
5. Bain, J. S., Barriers to New Competition, Their Character and Consequences in Manufacturing Industries, Cambridge, Harvard University Press, 1956.
6. Baumol, W. J., Business Behavior, Value and Growth, Second edition, New York, Macmillan, 1967.
7. Berle, A. A., and G. C. Means, The Modern Corporation and Private Property, New York, Macmillan, 1932.
8. Berry, C. H., "Corporate Growth and Diversification", The Journal of Law and Economics, Vol. 14, No. 2, October 1971, pp. 371-383.
9. Blair, J. M., Economic Concentration: Structure, Behavior, and Public Policy, New York, Harcourt Brace Jovanovich, 1972.
10. Bogue, M. C., and R. Roll, "Capital Budgeting of Risky Projects with Imperfect Markets for Physical Capital", Journal of Finance, Vol. 29, No. 2, May 1974, pp. 601-613.
11. Boudreaux, K. J., and H. W. Long, The Basic Theory of Corporate Finance, Englewood Cliffs, NJ, Prentice Hall, 1977.
12. Brennan, M. J., "An Approach to the Valuation of Uncertain Income Streams", Journal of Finance, Vol. 28, No. 3, June 1973, pp. 661-674.
13. Campbell, T. S., "Optimal Investment Financing Decisions When Information is Costly", unpublished paper, Department of Finance, University of Utah, January 1978.

14. Cole, R. J., "Gerber Triumphant But Investors Lose As Suitor Gives Up", The New York Times, Tuesday, September 20, 1977, pp. 57-58.
15. Conn, R. L., "Performance of Conglomerate Firms: Comment", Journal of Finance, Vol. 28, No. 3, June 23, pp. 754-758.
16. Edwards, C., "Economic Concentration", Hearings before the subcommittee on Antitrust and Monopoly of the Committee on the Judiciary, United States Senate, 88th Congress, Part 1, Overall and Conglomerate Aspects.  
Partly reprinted in "The Large Conglomerate Firm: A Critical Appraisal", in E. Mansfield (ed.), Monopoly Power and Economic Performance, Third edition, New York, Norton, 1974.
17. Eis, C., "The 1919-1930 Merger Movement in American Industry", The Journal of Law and Economics, Vol. 12, No. 2, October 1969, pp. 267-296.
18. Ellert, J. C., "Mergers, Antitrust Law Enforcement and Stockholder Returns", Journal of Finance, Vol. 31, No. 2, May 1976, pp. 715-732.
19. Fahy, P. R., "Mergers and Their Effect on Profitability", unpublished Ph.D. dissertation, The University of Connecticut, Economics, Commerce, Business, 1976.
20. Fama, E. F., "Risk-Adjusted Discount Rates and Capital Budgeting Under Uncertainty", Journal of Financial Economics, Vol. 5, No. 1, August 1977, pp. 3-24.
21. Fama, E. F., "The Behavior of Stock Market Prices", Journal of Business, Vol. 38, No. 1, January 1965.
22. Fama, E. F., "Efficient Capital Markets: A Review of Theory and Empirical Work", Journal of Finance, Vol. 25, No. 2, May 1970, pp. 383-423.
23. Fama E. F., and D. MacBeth, "Risk Return and Equilibrium: Empirical Tests", Journal of Political Economy, Vol. 81, No. 3, 1973, pp. 607-636.
24. Fama, E. F., L. Fisher, M. Jensen, and R. Roll, "The Adjustment of Stock Prices to New Information", International Economic Review, February 1969, pp. 1-21.
25. Fama, E. F., and M. M. Miller, The Theory of Finance, New York, Holt Rinehart and Winston, 1972.

26. Federal Trade Commission, "Economic Report, Conglomerate Merger Performance: An Empirical Analysis of Nine Corporations", Staff Report to the Federal Trade Commission, Washington, November 1972. See also references [92] and [93].
27. Forbes, "Multicompanies", Forbes, Vol. 117, No. 1, January 1976. pp. 62-66.
28. Forbes, "Where Will Harry Gray Strike Next?", Forbes, Vol. 119, No. 3, January 23, 1978.
29. Galbraith, J. K., The New Industrial State, Boston, Houghton Mifflin, 1967.
30. Goldberg, L. G., "The Effect of Conglomerate Mergers on Competition", The Journal of Law and Economics, Vol. 16, No. 1, April 1973, pp. 137-158.
31. Gort, M., "An Economic Disturbance Theory of Mergers", Quarterly Journal of Economics, Vol. 83, No. 4, November 1969.
32. Gort, M., and T. F. Hogarty, "New Evidence on Mergers", Journal of Law and Economics, Vol. 13, No. 1, April 1970, pp. 167-184.
33. Haley, C. W., and L. D. Schall, The Theory of Financial Decisions, New York, McGraw-Hill Series in Finance, 1973.
34. Halpern, P. J., "Empirical Estimates of the Amount and Distribution of Gains to Companies in Mergers", Journal of Business, Vol. 46, 1973, pp. 554-575.
35. Hax, A. C., and N. S. Majluf, "A Methodology for the Development of Strategic Planning in Diversified Corporations", Technical Report No. 3, Sloan School of Management, M.I.T., Cambridge, August 1977.
36. Higgins, R. C., and L. D. Schall, "Corporate Bankruptcy and Conglomerate Merger", Journal of Finance, Vol. 30, No. 1, March 1975.
37. Hindley, B., "Separation of Ownership and Control in the Modern Corporation", Journal of Law and Economics, Vol. 13, No. 1, April 1970.
38. Hirshleifer, J., Investment, Interest, and Capital, Englewood Cliffs, NJ, Prentice-Hall, 1970.
39. Hirshleifer, J., Price Theory and Applications, Englewood Cliffs, NJ, Prentice-Hall, 1975.

40. Hogarty, T. F., "The Success of Industrial Mergers", unpublished Ph.D. dissertation, State University of New York at Buffalo, June 1969.
41. Hogarty, T. F., "The Profitability of Corporate Mergers", Journal of Business, Vol. 43, No. 3, July 1970, pp. 317-327.
42. Hogarty, T. F., "Economic Concentration", Hearings before the Subcommittee on Antitrust and Monopoly, 91st Congress, Part 8: The Conglomerate Merger Problem, 1969-1970.
43. IBM, Annual Report, 1976.
44. Jackson, R., "The Consideration of Economies in Merger Cases", The Journal of Business, Vol. 43, 1970, pp. 439-447.
45. Jacoby, N. H., "The Conglomerate Corporation", The Center Magazine, Vol. , No. , July 1969.  
Reprinted in "Economic Concentration", Hearings before the Subcommittee on Antitrust and Monopoly, 91st Congress, Part 8: The Conglomerate Merger Problem, 1969-1970, pp. 4948-4960.
46. Jensen, M. C., "Capital Markets: Theory and Evidence", The Bell Journal of Economics and Management Science, Vol. 3, No. 2, Autumn 1972, pp. 357-398.
47. Kelly, E. M., Profitability of Growth Through Mergers, University Park, The Pennsylvania State University, 1967.  
See also his testimony in: "Economic Concentration", Hearings before the Subcommittee on Antitrust and Monopoly, 91st Congress, Part 8: The Conglomerate Merger Problem, 1969-1970.
48. Kraar, L., "General Electric's Very Personal Merger", Fortune, Vol. 96, No. 2, August 1977.
49. Lev, B., and G. Mandelker, "The Microeconomic Consequences of Corporate Mergers", Journal of Business, Vol. 45, No. 1, January 1972, pp. 85-104.
50. Levinson, H., "A Psychologist Diagnoses Merger Failures", Harvard Business Review, March-April 1970. Reprinted in J. M. Samuels (ed.), Readings on Mergers and Takeovers, New York, St. Martin's Press, 1972.
51. Levy, H., and M. Sarnat, "Diversification, Portfolio Analysis and the Uneasy Case for Conglomerate Mergers", The Journal of Finance, Vol. 25, No. 4, September 1970, pp. 795-802.

52. Lewellen, W. G., "A Pure Financial Rationale for the Conglomerate Merger", The Journal of Finance, Vol. 28, No. 2, May 1973, pp. 521-537.
53. Lintner, J., "Expectations, Mergers and Equilibrium in Purely Competitive Security Markets", American Economic Review. Papers and Proceedings, Vol. 61, No. 2, May 1971, pp. 101-111.
54. Lorie, H. H., and P. Halpern, "Conglomerates: The Rhetoric and the Evidence", The Journal of Law and Economics, Vol. 13, No. 1, April 1970, pp. 149-166.
55. Lynch, H. H., Financial Performance of Conglomerates, Division of Research, Graduate School of Business Administration, Harvard University, Boston, 1971.
56. Mandelker, G., "Risk and Return: The Case of Merging Firms", Journal of Financial Economics, Vol. 1, No. 4, December 1974. Reprinted in: Myers, S. C., Modern Developments in Financial Management, New York, Praeger, 1976, pp. 660-695.
57. Markham, J. W., "Survey of the Evidence and Findings on Mergers", in Business Concentration and Price Policy, National Bureau of Economic Research, Special Conference Series 5, Princeton University Press, Princeton, 1955.
58. Mason, R. H., and M. B. Goudzwaard, "Performance of Conglomerate Firms: A Portfolio Approach", The Journal of Finance, Vol. 31, No. 1, March 1976, pp. 39-48.
59. Melicher, R. W., and D. F. Rush, "The Performance of Conglomerate Firms: Recent Risk and Return Experience", The Journal of Finance, Vol. 28, No. 2, May 1973, pp. 381-388.
60. Melicher, R. W., and D. F. Rush, "Evidence on the Acquisition-Related Performance of Conglomerate Firms", The Journal of Finance, Vol. 29, No. 1, March 1974, pp. 141-149.
61. Melnik, A., and M. A. Pollatschek, "Debt Capacity, Diversification and Conglomerate Mergers", The Journal of Finance, Vol. 28, No. 5, December 1973.
62. Merton, R. C., unpublished class notes for 15.415: Finance Theory I, Sloan School of Management, M.I.T., 1975.
63. Merton, R. C., "An Analytical Derivation of the Efficient Portfolio Frontier", Journal of Financial and Quantitative Analysis, Vol. 7, No. 4, September 1972. pp. 1851-1872.

64. Merton, R. C., "Capital Budgeting in the Capital Asset Pricing Model", Unpublished note, Sloan School of Management, M.I.T., 1973.
65. Miller, M. H., "Debt and Taxes", Presidential Address to be delivered at the Annual Meeting of the American Finance Association in Atlantic City, NJ, September 17, 1976.  
Published in Journal of Finance, Vol. 32, No. 2, May 1977.
66. Miller, M. H., and F. Modigliani, "Dividend Policy, Growth, and the Valuation of Shares", Journal of Business, Vol. 34, No. 4, October 1961, pp. 411-433.
67. Modigliani, F., and M. H. Miller, "Reply to Heins and Sprinkle", American Economic Review, Vol. 59, No. 4, September 1969, pp. 592-595.
68. Mueller, D. C., "A Theory of Conglomerate Merges", Quarterly Journal of Economics, Vol. 83, No. 4, November 1969.
69. Myers, S. C., Modern Developments in Financial Management, New York, Praeger, 1976.
70. Myers, S. C., "Determinants of Corporate Borrowing", Journal of Financial Economics, Vol. 5, No. 2, November 1977, pp. 147-175.
71. Myers, S. C., and N. S. Majluf, "Why Firms Need Financial Slack", Working Paper, Sloan School of Management, M.I.T., 1978.
72. Myers, S. C., and S. M. Turnbull, "Capital Budgeting and the Capital Assets Pricing Model: Good News and Bad News", The Journal of Finance, Vol. 32, No. 2, May 1977, pp. 321-333.
73. Nelson, R. L., Merger Movements in American Industry, 1895-1956, Princeton, Princeton University Press, 1959.
74. Reid, S. R., Mergers, Managers and the Economy, New York, McGraw-Hill, 1968.
75. Reid, S. R., "Economic Concentration", Hearings before the Subcommittee on Antitrust and Monopoly, 91st Congress, Part 8: The Conglomerate Merger Problem, 1969-1970.
76. Reid, S. R., "A Reply to the Weston-Mansinghka Criticisms Dealing with Conglomerate Mergers", Journal of Finance, Vol. 26, No. 4, September 1971, pp. 937-946.
77. Reid, S. R., "Conglomerate Performance Measurement: Reply", The Journal of Finance, Vol. 29, No. 3, June 1974, pp. 1013-1015.

78. Reid, S. R., The New Industrial Order: Concentration, Regulation and Public Policy, New York, McGraw-Hill, 1976.
79. Robichek, A. A., and S. C. Myers, "Conceptual Problems in the Use of Risk-Adjusted Discount Rates, Journal of Finance, Vol. 21, No. 4, December 1966, pp. 727-730.
80. Rumelt, R. P., Strategy, Structure, and Economic Performance, Division of Research, Graduate School of Business Administration, Harvard University, Boston, 1974.
81. Scholes, M., "A Test on the Competitive Hypothesis: The Market for New Issues and Secondary Offerings", unpublished Ph.D. thesis, Graduate School of Business, University of Chicago, 1969.
82. Segall, J., "Merging for Fun and Profit", Industrial Management Review, Vol. 9, No. 2, Winter 1968, pp. 17-29.
83. Shick, R. A., "The Analysis of Mergers and Acquisitions", The Journal of Finance, Vol. 27, No. 2, May 1972, pp. 495-502.
84. Sherer, R. M., Industrial Market Structure and Economic Performance, Chicago, Rand McNally, 1970.
85. Smith, K. V., and J. C. Schreiner, "A Portfolio Analysis of Conglomerate Diversification", The Journal of Finance, Vol. 24, No. 3, June 1969.
86. Standard Oil Co. vs. United States, 1911.
87. Steiner, P. O., Mergers: Motives, Effects, Policies, Ann Arbor, University of Michigan Press, 1975.
88. Stigler, G. J., "Monopoly and Oligopoly by Merger", American Economic Review Papers and Proceedings, Vol. 40, May 1970.
89. Thomson, G. C., and G. P. Brady, Antitrust Fundamentals, Second edition, St. Paul, Minn., 1974.
90. Thorp, W. L., and W. Crowder, "The Merger Movement", in The Structure of Industry, Temporary National Economic Committee Monograph 27, 1941.
91. U. S. Congress Senate, Committee on the Judiciary, "Economic Concentration", Hearings before the Subcommittee on Antitrust and Monopoly, 91st Congress, Part 8: The Conglomerate Merger Problem, 1969-1970.

92. U. S. Federal Trade Commission, Economic Report on Corporate Mergers, Staff Report to the Federal Trade Commission, 1969.
93. U. S. Federal Trade Commission, Statistical Report on Mergers and Acquisitions.  
See also: Current Trends in Merger Activity.
94. United States vs. Northern Securities Co., 1904.
95. United States vs. Aluminium Company of America, 1945.
96. Vartan, V. G., "Market Place: Gerber Leaves Egg on Arbitragers' Bibs", The New York Times, Tuesday, September 20, 1977, p. C-58.
97. Westerfield, R., "A Note on the Measurement of Conglomerate Diversification", The Journal of Finance, Vol. 25, No. 4, September 1970, pp. 904-914.
98. Weston, J. F., and S. K. Mansinghka, "Test of the Efficiency Performance of Conglomerate Firms", Journal of Finance, Vol. 26, No. 4, September 1971, pp. 919-936.  
Reprinted in J. M. Samuels (ed.), Readings on Mergers and Takeovers, New York, St. Martin's Press, 1972, pp. 25-39.
99. Weston, J. F., and S. K. Mansinghka, "Reply", The Journal of Finance, Vol. 28, No. 3, June 1973, p. 857.
100. Weston, J. F., and S. K. Mansinghka, "Conglomerate Performance Measurement: Comment", The Journal of Finance, Vol. 29, No. 3, June 1974, pp. 1011-1012.
101. Williamson, O. E., The Economies of Discretionary Behavior: Managerial Objectives in a Theory of the Firm, Englewood Cliffs, NJ, Prentice-Hall, 1964.
102. Williamson, O. E., Corporate Control and Business Behavior: An Inquiry into the Effects of Organization Form on Enterprise Behavior, Englewood Cliffs, NJ, Prentice Hall International Series in Management, 1970.
103. Williamson, O. E., Markets and Hierarchies Analysis and Antitrust Implications: A Study in the Economics of Internal Organization, New York, Free Press, 1975.

APPENDIX 1: EXISTENCE OF EQUILIBRIUM IN A PURE EQUITY CASE

This appendix shows that an equilibrium value can always be found if the firm decides to issue shares for undertaking a project. This value corresponds to the solution of equation (5.18):

$$P' = S + \bar{A}(M') + \bar{B}(M')$$

The solution of this equation is shown to be in the interval  $S \leq P' \leq S + \bar{A} + \bar{B}$ . The only assumption being made is that  $(\bar{A}, \bar{B}) \geq 0$ ; that is to say, the market value of individual business cannot go negative.

Lemma 1

Consider partition  $M'-M$  of the positive quadrant in space  $(a,b)$ . If  $M$  is non-empty, all pairs  $(a,b)$  in  $M$  satisfy the following relation:

$$S + a + b > P'$$

Proof:

$M$  is defined as:

$$M = \{(a,b) \geq 0 \mid (b+E) < \frac{E}{P'}(S+a)\}$$

Therefore, for all  $(a,b)$  in  $M$

$$(S+a) > P'$$

$$\underline{b \geq 0}$$

$$S+a+b > P'$$

Lemma 2

Consider partition  $M'-M$  of the positive quadrant in space  $(a,b)$ . If  $M$  is non-empty, the expected value of  $(S+\bar{A}+\bar{B})$  in  $M$  is greater than  $P'$ ; that is to say:

$$S + \bar{A}(M) + \bar{B}(M) > P'$$

Proof:

This is a direct consequence of lemma 1, because if all  $(a,b)$  in  $M$  satisfy this relation, their expected value must satisfy it too.

Theorem 1

The market value of the firm when shares are issued, must be in the following interval under market equilibrium

$$S \leq P' \leq S + \bar{A} + \bar{B}$$

Proof:

For the value  $P'$  to be an equilibrium value, it must satisfy the following relation:

$$P' = S + \bar{A}(M') + \bar{B}(M')$$

We also know that the expected value  $(S+\bar{A}+\bar{B})$  may be expressed in terms of the partition  $M'-M$  as:

$$(S+\bar{A}+\bar{B}) = [S+\bar{A}(M')+\bar{B}(M')]Pr(M') + [S+\bar{A}(M)+\bar{B}(M)]Pr(M)$$

or:

$$S + \bar{A} + \bar{B} = P' \cdot \Pr(M') + [S + \bar{A}(M) + \bar{B}(M)] \cdot \Pr(M)$$

By lemma 2, we know that  $[S + \bar{A}(M) + \bar{B}(M)] > P'$ . Replacing this in the above formulation, we get:

$$S + \bar{A} + \bar{B} > P' [\Pr(M') + \Pr(M)]$$

or:

$$P' < S + \bar{A} + \bar{B}$$

If region M is empty, we will have  $P' = S + \bar{A} + \bar{B}$ , therefore, in general, we can write:

$$P' \leq S + \bar{A} + \bar{B}$$

This relation establishes the upper limit for  $P'$ .

The lower limit may be established in a more direct way. We know that a and b are greater or equal to 0. Therefore:

$$\bar{A}(M') + \bar{B}(M') \geq 0$$

and

$$S + \bar{A}(M') + \bar{B}(M') \geq S$$

or:

$$P' \geq S$$

This statement completes the proof of the theorem.

Theorem 2 (Existence)

The equation for  $P'$  indicated below always has at least one solution in the interval  $S \leq P' \leq S + \bar{A} + \bar{B}$ :

$$P' = S + \bar{A}(M') + \bar{B}(M')$$

Proof:

Consider the way in which the right-hand side varies with  $P'$ , and call this function:

$$p(P') = S + \bar{A}(M') + \bar{B}(M')$$

In graphical terms, the solution being sought corresponds to the intersection of  $p(P')$  with the  $45^\circ$  line in Figure A1.

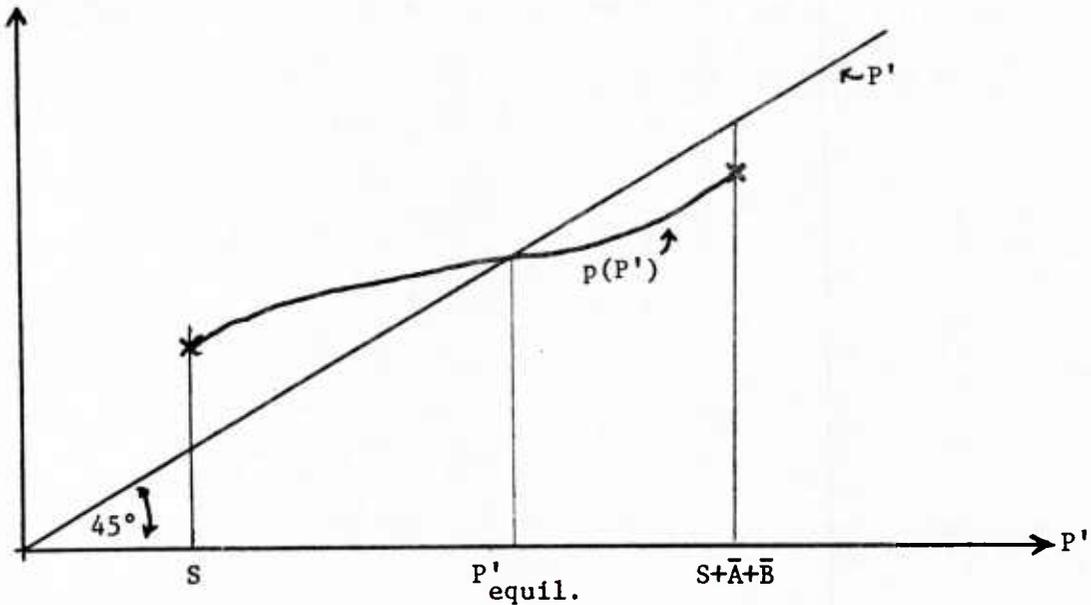


FIGURE A1: The function  $p(P') = S + \bar{A}(M') + \bar{B}(M')$

The proof that this intersection always exists is done by showing first that for  $P' = S$  we have  $p(P') \geq P'$ ; and for  $P' = S + \bar{A} + \bar{B}$  we have  $p(P') \leq P'$ . In the first place, take  $P' = S$ . By the definition of  $p(P')$  we know that  $p(P') \geq S$  because  $\bar{A}(M') + \bar{B}(M') \geq 0$  always. This implies that  $p(P') \geq P'$  for  $P' = S$ .

Now consider  $P' = S + \bar{A} + \bar{B}$ . We know that, in general, we can express the expected value  $(S + \bar{A} + \bar{B})$  in terms of the partition  $M' - M$  as:

$$S + \bar{A} + \bar{B} = [S + \bar{A}(M') + \bar{B}(M')]Pr(M') + [S + \bar{A}(M) + \bar{B}(M)]Pr(M)$$

In this case, this relation may be rewritten as:

$$P' = p(P')Pr(M') + [S + \bar{A}(M) + \bar{B}(M)]Pr(M)$$

But, by lemma 2, we know that  $[S + \bar{A}(M) + \bar{B}(M)] > P'$ . Replacing this in the above formula we get:

$$P' > p(P')Pr(M') + P' Pr(M)$$

or

$$P'[1 - Pr(M)] > p(P')Pr(M')$$

or

$$P' > p(P') \quad *$$

By noticing that if  $M$  is empty for  $P = S + \bar{A} + \bar{B}$  we have  $P' = p(P')$ , the general relation will be  $P' \geq p(P')$  for  $P' = S + \bar{A} + \bar{B}$ .

In this way we have bounded the solution to the region indicated in

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\* When  $P' = S + \bar{A} + \bar{B}$ , we must have  $Pr(M) < 1$ , because if  $Pr(M) = 1$  we would have  $S + a + b > S + \bar{A} + \bar{B}$  for all  $(a, b)$  (lemma 2). This is a contradiction with the fact that the expected value of  $(S + A + B)$  is  $(S + \bar{A} + \bar{B})$ .

Theorem 1, and we can ascertain that:

$$\begin{array}{ll} \text{for } P' = S & p(P') \geq P' \\ P' = S + \bar{A} + \bar{B} & p(P') \leq P' \end{array}$$

It is clear that if  $p(P')$  is a continuous function [the probability density function (a,b) is continuous], then there is at least one solution to the equation  $P' = p(P')$  in the range  $S \leq P' \leq S + \bar{A} + \bar{B}$ .

For a discrete probability distribution,  $p(P')$  is a discontinuing function as indicated in Figure A2.\*

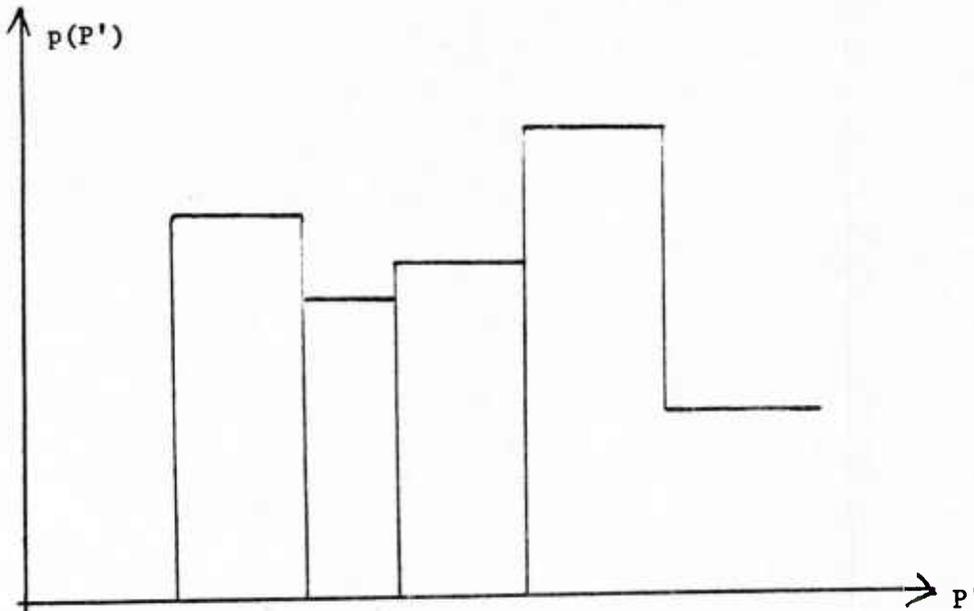


FIGURE A2: The function  $p(P') = S + \bar{A}(M') + \bar{B}(M')$  when  $(\bar{A}, \bar{B})$  are discrete variables

One could think that the  $45^\circ$  line might never intercept this function if it goes without crossing the horizontal segments. This would imply that

\* For some special discrete probability distribution, the function  $p(P')$  may be nowhere continuous. The proof to be given is not applicable in those cases.

the equation  $P' = p(P')$  has no solution. In fact, it is shown below that always there is at least one intersection.

The proof is done by showing that if  $p(P')$  is above the  $45^\circ$  line and it jumps (up or down), it cannot go below this line. Given that we know that  $p(P')$  is above the line for  $P' = S$  and below it for  $P' = S + \bar{A} + \bar{B}$ , the only way to go from one side to the other is for a horizontal segment to intercept the  $45^\circ$  line.

Consider  $p(P' - \epsilon) > (P' - \epsilon)$  for some  $(P' - \epsilon)$  in the interval  $S \leq (P' - \epsilon) < S + \bar{A} + \bar{B}$ . Suppose that if  $(P' - \epsilon)$  is increased to  $P'$ , a new outcome  $(\tilde{A}, \tilde{B})$  is included in region  $M'$ . This will make the value of  $p(P')$  different from  $p(P' - \epsilon)$ . The relation between these two quantities may be expressed as:

$$p(P') = \frac{p(P' - \epsilon) \cdot \text{Pr}(M') + v \cdot \text{Pr}(\epsilon)}{\text{Pr}(M') + \text{Pr}(\epsilon)}$$

where:

$\text{Pr}(M')$  = Probability of being in region  $M'$  defined by  $(P' - \epsilon)$

$v = (S + a + b)$  for the new outcome included in region  $M'$  when  $(P' - \epsilon)$  is increased to  $P'$

$\text{Pr}(\epsilon)$  = Probability of getting this new outcome.

We have assumed that  $p(P' - \epsilon) > P' - \epsilon$  and we also know that  $v > P' - \epsilon$  (by lemma 1). Therefore, we can state that

$$p(P') > (P' - \epsilon)$$

By letting  $\epsilon$  go to 0 we can conclude that  $p(P') \geq P'$ . This means that the function  $p(P')$  remains over the  $45^\circ$  line, or that  $P'$  is an equilibrium solution.

APPENDIX 2: CHANGES IN THE MARKET VALUE OF THE FIRM WITH DIFFERENT  
INVESTMENT REQUIREMENTS (ONLY EQUITY FINANCING)

This Appendix shows that for a fixed amount of slack, the market value of the firm deteriorates when the investment required goes up, because a larger new equity issue is required to fill the need for funds (remember that  $E = I - S$ ).

The proposition is shown by stating that the loss due to insufficient slack has to go up when the equity required goes up.

Lemma 1

For  $S$  fixed, the market value of old shareholders' shares after the announcement of a new issue goes up when the equity required goes down ( $P'$  goes up when  $E$  goes down).

Proof:

Consider two equity requirements  $E_1$  and  $E_2$  such that  $E_1 > E_2$ . Call  $P'_1$  and  $P'_2$  the market value when a share issue is announced for each one of these two cases. What we want to show is that  $P'_1 \leq P'_2$ ; that is to say, that  $P'_1$  is a lower bound for  $P'_2$ .

The market value  $P'_2$  is the solution of the equation:

$$P' = p(P')$$

where:

$$p(P') = S + A(M') + B(M')$$

and

$$M' = M'(P') = \{(a, b) > 0 | (E_2 + b) > \frac{E_2}{P'} (S + a)\}$$

To prove that  $P'_1 \leq P'_2$ , it is sufficient to state that  $P'_1 \leq p(P'_1)$ , because this will imply that a solution of equation  $P' = p(P')$  can be found in the range  $(P'_1, S + \bar{A} + \bar{B})$ . (See arguments presented in theorem 2, Appendix 1.)

Consider regions  $M'_1$  and  $M'_2$  in Figure A3.

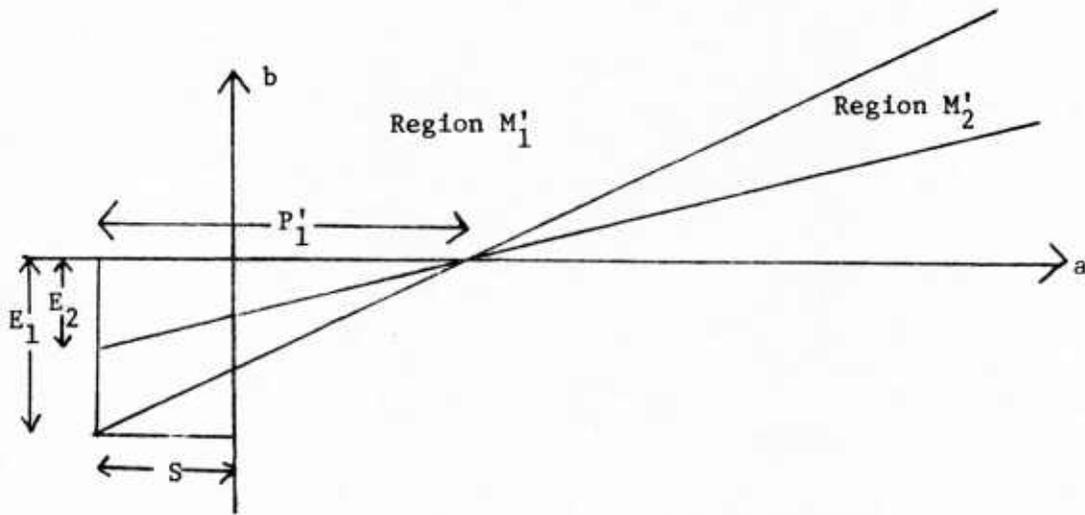


FIGURE A3: Definition of regions  $M'_1$  and  $M'_2$

Region  $M'_1$  corresponds to the equilibrium situation for  $E = E_1$ . Therefore:

$$P'_1 = S + \bar{A}(M'_1) + \bar{B}(M'_1)$$

When  $E_1$  is changed to  $E_2$  (with  $E_2 < E_1$ ), and  $P'$  is maintained equal to  $P'_1$ , the new definition of region  $M'$  becomes the union of  $M'_1$  and  $M'_2$ . The value of the function  $p(P'_1)$  under these circumstances may be written as a weighted average of  $[S + \bar{A}(M'_1) + \bar{B}(M'_1)]$  and  $[S + \bar{A}(M'_2) + \bar{B}(M'_2)]$ , as indicated below:

$$p(P'_1) = S + \bar{A}(M') + \bar{B}(M') = \frac{[S + \bar{A}(M'_1) + \bar{B}(M'_1)] \Pr(M'_1) + [S + \bar{A}(M'_2) + \bar{B}(M'_2)] \Pr(M'_2)}{\Pr(M'_1) + \Pr(M'_2)}$$

The term in the first bracket is  $P'_1$ , while the term in the second bracket is greater or equal to  $P'_1$  (see lemma 2 in Appendix 1). Therefore,

$$p(P'_1) \geq P'_1$$

Lemma 2

Consider the equilibrium values  $P'_1$  and  $P'_2$  defined by the equity requirements  $E_1$  and  $E_2$ , respectively, where  $E_1 > E_2$ . Region  $M$  defined by the pair  $(E_1, P'_1)$  (call it  $M_1$ ) fully contains the equivalent region defined by the pair  $(E_2, P'_2)$  (call it  $M_2$ ); that is to say  $M_1 \supset M_2$ .

Proof: (see Figure A4 for a graphical proof)

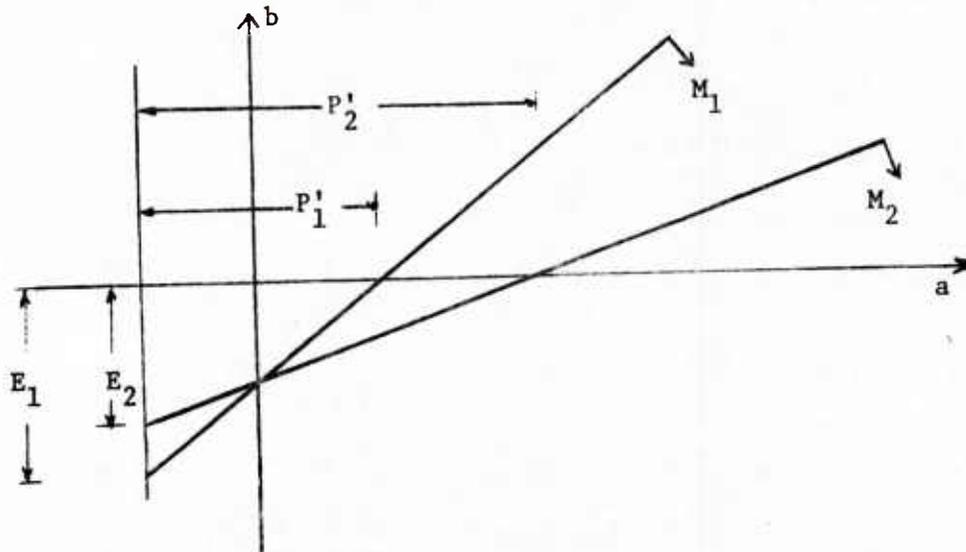


FIGURE A4: Graphical Proof that  $M_1 \supset M_2$  when  $E_1 > E_2$

The definition of region  $M$  is:

$$M = \{(a, b) \geq 0 \mid (E+b) < \frac{E}{P'} (S+a)\}$$

or

$$M = \{(a,b) \geq 0 \mid P' < \frac{S+a}{1+\frac{b}{E}}\}$$

For  $(a,b)$  in  $M_2$  we have

$$P'_2 < \frac{S+a}{1+\frac{b}{E_2}}$$

By lemma 1,  $P'_1 \leq P'_2$ . Then:

$$P'_1 \leq P'_2 < \frac{S+a}{1+\frac{b}{E_2}}$$

Also  $E_1 > E_2$ . Therefore:

$$P'_1 \leq P'_2 < \frac{S+a}{1+\frac{b}{E_2}} < \frac{S+a}{1+\frac{b}{E_1}}$$

Consequently, if  $(a,b)$  belongs to  $M_2$ , it also belongs to  $M_1$  ( $M_1$  contains  $M_2$ ).

Theorem 1

For  $S$  fixed, the loss due to insufficient slack (defined below), goes up when the equity required goes up:

$$\Delta = \bar{B}(M) \cdot \Pr(M) = \sum_{(a,b) \text{ in } M} b \cdot \Pr(\tilde{B}=b)$$

Proof:

Consider two equity requirements  $E_1, E_2$  with  $E_1 > E_2$ . Call  $M_1$  and  $M_2$  the corresponding regions  $M$  over which the loss for insufficient slack is defined. By Lemma 2 we know that  $M_1 \supset M_2$ .

The loss  $\Delta$  when  $E = E_1$  may be expressed as:

$$\Delta_1 = \sum_{(a,b) \text{ in } M_1} b \cdot \Pr(\tilde{B}=b)$$

The summation may be broken down into the two regions  $M_2$  and  $(M_1 - M_2)$  as follows:

$$\Delta_1 = \sum_{(a,b) \text{ in } M_2} b \cdot \Pr(\tilde{B}=b) + \sum_{(a,b) \text{ in } (M_1 - M_2)} b \cdot \Pr(\tilde{B}=b)$$

The first term after the equal sign is the loss due to insufficient slack when  $E = E_2$ , which is designated by  $\Delta_2$ . Then:

$$\Delta_1 = \Delta_2 + \sum_{(a,b) \text{ in } (M_2 - M_1)} b \cdot \Pr(\tilde{B}=b)$$

The sum to the right of the equal sign cannot be negative. Therefore:

$$\Delta_1 \geq \Delta_2$$

### Theorem 2

For a fixed amount of slack, the market value of the firm goes down when the investment required goes up; that is to say, when the new equity required goes up ( $E = I - S$ ).

### Proof:

According to relations (5.20) and (5.21), the market value of the firm may be expressed as:

$$V = S + \bar{A} + \bar{B} - \Delta$$

According to Theorem 1, for  $E_1 > E_2$  we have  $\Delta_1 \geq \Delta_2$ . This implies that

$$V_1 \leq V_2.$$

APPENDIX 3: ALGORITHM FOR FINDING THE EQUILIBRIUM SOLUTION IN A PURE-EQUITY CASE

This Appendix shows that the algorithm defined in Figure 17, for finding  $P'$  in a pure-equity case converges monotonically to the equilibrium value of  $P'$ . The proof is built on the idea that starting with an upper limit for  $P'$ , the new value of  $P'$  generated by the algorithm has to be between the equilibrium solution being sought, and the value for  $P'$  in the previous iteration. Formally:

let  $P'_0$  = Initial value of  $P'$   
 =  $S + \bar{A} + \bar{B}$  (it is always an upper limit by Theorem 1, Appendix 1).

for iteration  $h$ , define  $P'_h$  as:

$$P'_h = S + \bar{A}(M'_h) + \bar{B}(M'_h)$$

where:

$$M'_h = \{(a_i, b_j) > 0 \mid (E + b_j) > \frac{E}{P'_{h-1}} (S + a_i)\}$$

We have to prove that  $P'_* \leq P'_h \leq P'_{h-1}$ , where  $P'_*$  is the equilibrium solution being sought.

The proof is done in two steps: first, prove that  $P'_* \leq P'_h$  ( $P'_h$  is an upper limit for  $P'_*$ ); and second, prove that  $P'_h \leq P'_{h-1}$  ( $P'_h$  is a non increasing function).

Step 1: Prove that  $P'_* \leq P'_h$ .

We know that  $P'_* \leq P'_0$ . Assume that, for some  $h$ , the following relation is satisfied:  $P'_* \leq P'_{h-1}$  ( $P'_{h-1}$  is an upper limit for  $P'_*$ ). Then, the following conditions must prevail (see Figure A5):

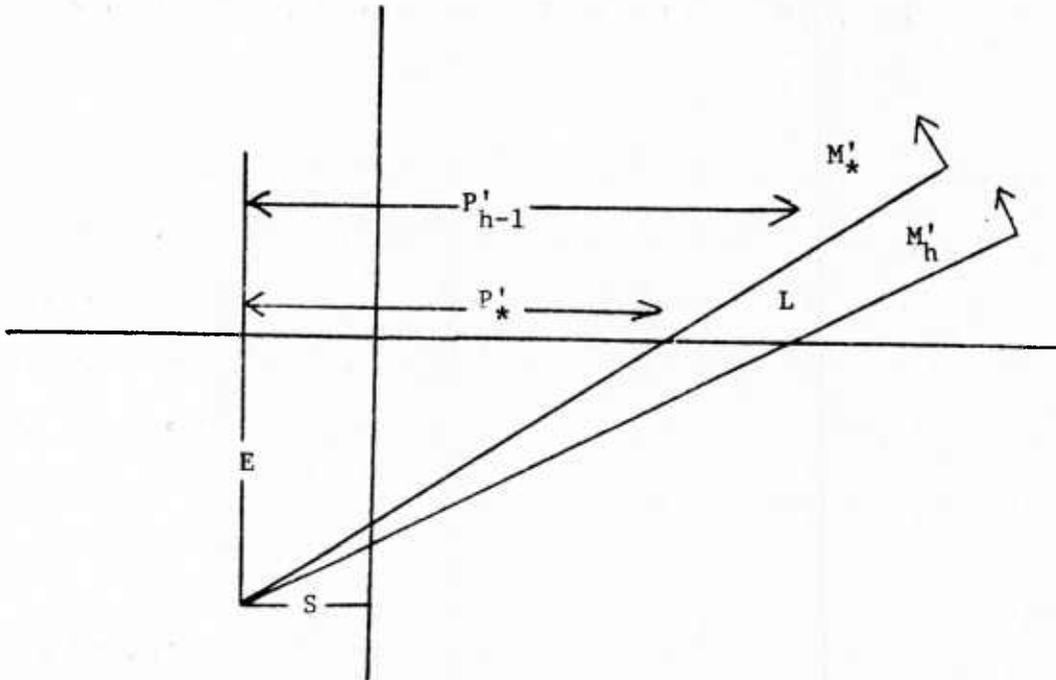


FIGURE A5: Relation between region  $M'_h$  (defined for  $P'_{h-1}$ ) and the equilibrium situation (defined for  $P'_*$ )

$$M'_* \subset M'_h$$

where:  $M'_*$  is region  $M'$  defined for  $P' = P'_*$ .

Moreover:

$$M'_h = M'_* + L$$

Consider the definition of  $P'_h$ :

$$P'_h = \frac{\left[ \sum_{(a_i, b_j) \text{ in } M'_h} (S+a_i+b_j)p_{ij} \right]}{\left[ \sum_{(a_i, b_j) \text{ in } M'_h} p_{ij} \right]}$$

The numerator in this expression may be decomposed in two sums:

$$\begin{aligned} \left[ \sum_{(a_i, b_j) \text{ in } M'_h} (S+a_i+b_j)p_{ij} \right] &= \\ &= \left[ \sum_{(a_i, b_j) \text{ in } M'_*} (S+a_i+b_j)p_{ij} \right] + \left[ \sum_{(a_i, b_j) \text{ in } L} (S+a_i+b_j)p_{ij} \right] \end{aligned}$$

The first sum to the right of the equal sign is equivalent to the following expression:

$$P'_* \left[ \sum_{(a_i, b_j) \text{ in } M'_*} P_{ij} \right] \quad (\text{by definition of } P'_*)$$

The second sum is greater than the expression below (unless L is empty):

$$P'_* \left[ \sum_{(a_i, b_j) \text{ in } L} P_{ij} \right] \quad (\text{by Lemma 2, Appendix 1})$$

The conclusion is that  $P'_h > P'_*$  (if L is non-empty) and  $P'_h = P'_*$  (if L is empty; which implies  $M'_h = M'_*$ ). In general,  $P'_h \geq P'_*$ .

Step 2: Prove that  $P'_h \leq P'_{h-1}$ .

Suppose that  $P'_1 < P'_0$  (otherwise  $P'_0 = P'_1 = P'_*$ ). Take now any three values of the sequence generated by the iteration procedure, and call them;

$P'_h, P'_{h-1}, P'_{h-2}$ . Assume that  $P'_{h-1} < P'_{h-2}$ , and show that  $P'_h \leq P'_{h-1}$ .

Consider the sets  $M'_h$  and  $M'_{h-1}$  that define the values  $P'_h$  and  $P'_{h-1}$ , respectively.

$$M'_{h-1} = \{(a, b) \geq 0 \mid (E+b) \geq \frac{E}{P'_{h-2}} (S+a)\}$$

$$M'_h = \{(a, b) \geq 0 \mid (E+b) \geq \frac{E}{P'_{h-1}} (S+a)\}$$

Consider also the definitions of  $P'_{h-1}$  and  $P'_h$ :

$$P'_{h-1} = \text{Ex}\{S+\tilde{A}+\tilde{B} \mid M'_{h-1}\}^*$$

$$\text{Pr}(M'_{h-1}) = \text{Probability of } (\tilde{A}, \tilde{B}) \text{ being in } M'_{h-1}.$$

Similarly:

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\* Ex = Expected value.

$$P'_h = \text{Ex}\{S+\tilde{A}+\tilde{B}|M'_h\}$$

Given that  $P'_{h-1} < P'_{h-2}$ , it must be true that  $M'_h \subset M'_{h-1}$ . Therefore,  $P'_{h-1}$  may also be expressed as:

$$P'_{h-1} = \frac{\text{Ex}\{S+\tilde{A}+\tilde{B}|M'_h\} \cdot \text{Pr}(M'_h) + \text{Ex}\{(S+\tilde{A}+\tilde{B})|M'_{h-1}-M'_h\} \cdot \text{Pr}(M'_{h-1}-M'_h)}{\text{Pr}(M'_h) + \text{Pr}(M'_{h-1}-M'_h)}$$

This is equivalent to:

$$P'_{h-1} = \frac{P'_h \text{Pr}(M'_h) + \text{Ex}\{S+\tilde{A}+\tilde{B}|M'_{h-1}-M'_h\} \cdot \text{Pr}(M'_{h-1}-M'_h)}{\text{Pr}(M'_h) + \text{Pr}(M'_{h-1}-M'_h)}$$

If  $(M'_{h-1}-M'_h)$  is empty, then  $P'_{h-1} = P'_h$  because  $\text{Pr}(M'_{h-1}-M'_h) = 0$ .

If  $(M'_{h-1}-M'_h)$  is non-empty, then:  $\text{Ex}\{S+\tilde{A}+\tilde{B}|M'_{h-1}-M'_h\} > P'_{h-1}$  (Lemma 2,

Appendix 1). Also:

$$\text{Pr}(M'_{h-1}-M'_h) > 0 \text{ and } P'_{h-1} > P'_h.$$

The general conclusion is that  $P'_{h-1} \geq P'_h$ .

### Conclusions:

If  $P'_h$  is always decreasing, and bounded below by  $P'_*$ , it has to converge to an equilibrium solution in a finite number of steps, because the number of  $(a_i, b_j)$  pairs is finite ( $P'_h$  decreasing means that at least one pair  $(a_i, b_j)$  is shaved out of region  $M'_h$  in each iteration).

The other conclusion is that when more than one solution exists for  $P'$ , this algorithm picks the largest one, because it starts from an upper limit. ( $P'_0 = S+\bar{A}+\bar{B}$ ).

In this case,  $P'_*$  used in this proof may be any valid equilibrium solution.

APPENDIX 4: PARAMETERS IN THE BIVARIATE LOG NORMAL DISTRIBUTION

This Appendix shows the relations between the parameters of the bivariate log normal distribution of  $(\tilde{A}, \tilde{B})$ , and the bivariate normal distribution of  $(\tilde{X}, \tilde{Y})$ , where  $\tilde{X} = \ln \tilde{A}$  and  $\tilde{Y} = \ln \tilde{B}$ . More specifically, if  $(\bar{X}, \bar{Y}, \sigma_x, \sigma_y, \rho_{xy})$  are assumed known, the problem is to find  $(\bar{A}, \bar{B}, \sigma_A, \sigma_B, \rho_{AB})$ .

The relations between  $(\tilde{A}, \tilde{B})$  and  $(\tilde{X}, \tilde{Y})$  may be inverted to:  $\tilde{A} = e^{\tilde{X}}$  and  $\tilde{B} = e^{\tilde{Y}}$ . Therefore, to find the parameters of the distribution of  $(\tilde{A}, \tilde{B})$  it is necessary to determine the expectation for the following functions of  $(\tilde{X}, \tilde{Y})$ :

$$\begin{aligned} \bar{A} &= \text{Ex}\{e^{\tilde{X}}\} & \bar{B} &= \text{Ex}\{e^{\tilde{Y}}\} \\ \bar{A}^2 &= \text{Ex}\{e^{2\tilde{X}}\} & \bar{B}^2 &= \text{Ex}\{e^{2\tilde{Y}}\} \\ \overline{AB} &= \text{Ex}\{e^{\tilde{X}+\tilde{Y}}\} \end{aligned}$$

Then, we can compute:

$$\begin{aligned} \sigma_A^2 &= \bar{A}^2 - \bar{A}^2 & \sigma_B^2 &= \bar{B}^2 - \bar{B}^2 \\ \text{Cov}(\tilde{A}, \tilde{B}) &= \overline{AB} - \bar{A} \bar{B} \end{aligned}$$

There is no problem in computing all the expected values above, from the density function for  $(\tilde{X}, \tilde{Y})$  which may be expressed as follows in terms of the standardized random variables  $(\tilde{U}, \tilde{V})$ :

$$f(u, v) = \frac{1}{(2\pi)\sqrt{1-\rho_{xy}^2}} \exp \left\{ -\frac{1}{2(1-\rho_{xy}^2)} (u^2 + v^2 - 2\rho_{xy} uv) \right\} \quad (\text{A4.1})$$

where:

$$\tilde{U} = \frac{\tilde{X} - \bar{X}}{\sigma_x} \quad \tilde{V} = \frac{\tilde{Y} - \bar{Y}}{\sigma_y}$$

or:  $\tilde{X} = \bar{X} + \sigma_x \tilde{U}$      $\tilde{Y} = \bar{Y} + \sigma_y \tilde{V}$

Therefore:

$$\begin{aligned} \bar{A} &= \text{Ex}\{e^{\tilde{X}}\} = \text{Ex}\{e^{\bar{X} + \sigma_x \tilde{U}}\} = e^{\bar{X}} \text{Ex}\{e^{\sigma_x \tilde{U}}\} \\ &= e^{\bar{X}} \iint e^{\sigma_x u} f(u,v) du dv \end{aligned}$$

This expression is equal to:

$$\bar{A} = e^{\bar{X} + \frac{1}{2}\sigma_x^2} \tag{A4.2}$$

Similarly

$$\bar{B} = e^{\bar{Y} + \frac{1}{2}\sigma_y^2} \tag{A4.3}$$

To compute  $\bar{A}^2$ , a similar procedure is followed:

$$\begin{aligned} \bar{A}^2 &= \text{Ex}\{e^{2\tilde{X}}\} = \text{Ex}\{e^{2(\bar{X} + \sigma_x \tilde{U})}\} = e^{2\bar{X}} \text{Ex}\{e^{2\sigma_x \tilde{U}}\} \\ &= e^{2\bar{X}} \iint e^{2\sigma_x u} f(u,v) du dv \end{aligned}$$

Solving the integral we get:

$$\bar{A}^2 = e^{2\bar{X} + 2\sigma_x^2} \tag{A4.4}$$

Similarly:

$$\bar{B}^2 = e^{2\bar{Y} + 2\sigma_y^2} \tag{A4.5}$$

From these results it is possible to find the variances for  $\tilde{A}$  and  $\tilde{B}$ :

$$\sigma_A^2 = \bar{A}^2 - \bar{A}^2 = e^{2\bar{X} + \sigma_x^2} (e^{\sigma_x^2} - 1) \tag{A4.6}$$

$$\sigma_B^2 = \bar{B}^2 - \bar{B}^2 = e^{2\bar{Y} + \sigma_y^2} (e^{\sigma_y^2} - 1) \tag{A4.7}$$

Finally, the expected value of the product  $A B$  has to be obtained for getting their covariance:

$$\begin{aligned} \overline{AB} &= \text{Ex}\{e^{\tilde{X}+\tilde{Y}}\} = \text{Ex}\{e^{(\bar{X}+\sigma_x \tilde{U})+(\bar{Y}+\sigma_y \tilde{V})}\} = e^{(\bar{X}+\bar{Y})} \text{Ex}\{e^{\sigma_x \tilde{U}+\sigma_y \tilde{V}}\} \\ &= e^{\bar{X}+\bar{Y}} \iint e^{\sigma_x u+\sigma_y v} f(u,v) du dv \end{aligned}$$

Solving the integral we find:

$$\overline{AB} = e^{\bar{X}+\bar{Y}+\frac{1}{2}(\sigma_x^2+\sigma_y^2+2\rho_{xy}\sigma_x\sigma_y)} \quad (\text{A4.8})$$

Therefore:

$$\begin{aligned} \text{Cov}(\tilde{A}, \tilde{B}) &= \overline{AB} - \bar{A} \bar{B} \\ &= e^{(\bar{X}+\bar{Y})+\frac{1}{2}(\sigma_x^2+\sigma_y^2)} (e^{\rho_{xy}\sigma_x\sigma_y} - 1) \end{aligned} \quad (\text{A4.9})$$

The correlation coefficient is:

$$\begin{aligned} \rho(\tilde{A}, \tilde{B}) &= \frac{\text{Cov}(\tilde{A}, \tilde{B})}{\sigma_A \sigma_B} \\ &= \frac{e^{\rho_{xy}\sigma_x\sigma_y} - 1}{\sqrt{(e^{\sigma_x^2} - 1)(e^{\sigma_y^2} - 1)}} \end{aligned} \quad (\text{A4.10})$$

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VPAPAM[Γ]
▽ PAPAΓ;AX1;AX2;AX3;COS;STN;PPI2;POXY;STGXY;STGXY2;SIGUV;STGUV2;ΣΣ;CA;CB;CAB;SL;ABE;VP;PE;F
[1] +1+(I26)+1≥|Γ+POXY+(1+*/SIGXY+(STGXY2+*1+CVAB*2)*0.5)**1+POAB**/GVAR+0*XY+*ABE+(1+CVAB*2)*0.5
[2] +0.0pΓ+'PAPAΓ'PPI2 FOR (A,B) DISTRIBUTION APP YPCONSISTENT. CHANGE ROAB = ':(+*/CVAB)*-1+* 1 -1 **/(*1+CVAB*
2)*0.5
[3] +(0=-/STGXY2)/SKVP+0*Γ+INV-SL+INV*STN+100
[4] +COMP.0pSTGUV2+0.5*(+/SIGXY2)+ 1 -1 *(-/STGXY2)+2OPPI2+3o(2*POXY**/STGXY)+-/STGXY2
[5] SKVP:STGUV2+STGXY2[1]*1+ 1 -1 *POXY+0*PPI2+(00.5)
[6] COMP:STGUV+SIGUV2*0.5+0*STN+(*PPI2)*00COS+2OPPI2+2+0*ABE+SL+*/ABE
[7] CA+SL+,*XY[1]+(SIGUV[1]*COS*AX1)0.-SIGUV[2]*STN*AX1+(-φSG),SG
[8] CAB+CA+CB+,*XY[2]+(SIGUV[1]*STN*AX1)0.+SIGUV[2]*COS*AX1,0pAX2+*/X20.*AX2+(φPG),PG
[9] CHECK CA-SL
[10] +(VP*Γ+ABE+(AX3+.*GAB)+PE+*/AX3+AX2*GB≥(-F)+(E+VP+ABE)*GA)/I26
[11] 'ABE = ':ABE;' GVAB = ':GVAB;' ROAB = ':ROAB;' INV = ':INV;' SL = ':SL
[12] 'ABE = ':ABE-SL;' VP = ':VP;' PE = ':PE;' Δ = ':(AX2-AX3)+.*GB

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▽

APPENDIX 6: EXISTENCE OF EQUILIBRIUM IN A DEBT-EQUITY CASE

In Appendix 1, the existence of market equilibrium in a pure-equity situation is shown to exist always for any joint probability distribution  $f(a,b)$ , as long as  $(\tilde{A}, \tilde{B})$  is defined in the positive quadrant. To establish the existence of equilibrium in a debt-equity situation, it is necessary to relax the assumption of non-negative B, because in the range of permissible values for  $(\tilde{A}, \tilde{B}_{\text{net}})$ , negative values of  $\tilde{B}_{\text{net}}$  are allowed.

The proof that an equilibrium exists in a debt-equity situation is done in this Appendix for all joint probability distributions which are continuous in the half-space  $\{a>0\}$ . ( $b_{\text{net}}$  can take any value.)\*

The proof is done in two steps: first, the existence of a solution for the equilibrium equation below is established (similar to "pure equity", but allowing negative values for b).

$$P' = S + \bar{A}(M') + \bar{B}(M') \quad [\text{Relation (6.20)}]$$

Second, this equation is shown to have a solution also when the equilibrium constraint for the new debt issuing is imposed:

$$D = \text{Ex}\{\tilde{D}|M'\} \quad [\text{Relation (6.19)}]$$

Theorem 1

The equation below has at least one equilibrium solution in the range  $S \leq P' < \infty$ , when the pair  $(\tilde{A}, \tilde{B})$  has a joint probability distribution

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\* The subscript net will not be carried in the rest of this Appendix, unless there is some possibility of ambiguity.

defined in the half-space  $\{(a,b) | a \geq 0\}$  which is continuous for  $a > 0$ :

$$P' = S + \bar{A}(M') + \bar{B}(M')$$

$$M' = \{(a,b) | a \geq 0; E+b \geq \frac{E}{P'}(S+a)\}$$

Proof:

The basic idea in the proof is identical to the proof of Theorem 2 in Appendix 1.

Given a value of  $P'$ , it is possible to find  $M'$  and the corresponding  $\bar{A}(M')$  and  $\bar{B}(M')$ . For all values of  $P'$ , define the function  $p(P')$  as:

$$p(P') = S + \bar{A}(M') + \bar{B}(M')$$

In graphical terms, the value of  $P'$  being sought corresponds to the intersection of  $p(P')$  with the  $45^\circ$  line in Figure A6.

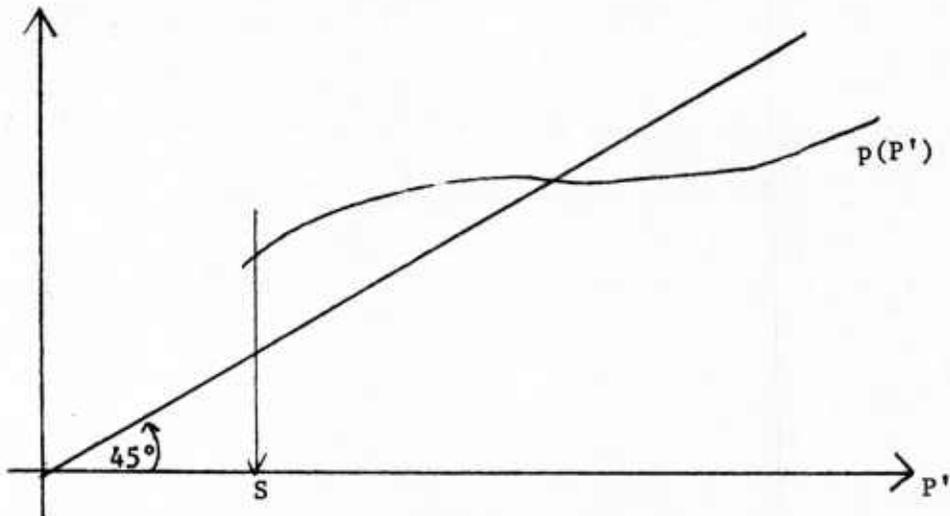


FIGURE A6: Function  $p(P')$  in a debt-equity situation.

The proof that this intersection exists is done by showing that for  $P' = S$ , the value of  $p(P')$  is above the  $45^\circ$  line, and that for  $P'$  sufficiently

large,  $p(P')$  is below this line.

First, take  $P' = S$ . It is clear from Figure A7 that region  $M'$  only contains non-negative values of  $(a,b)$ . Therefore:  $\bar{A}(M') + \bar{B}(M') \geq 0$  and  $S + \bar{A}(M') + \bar{B}(M') \geq S$  or  $p(P') \geq P'$ .

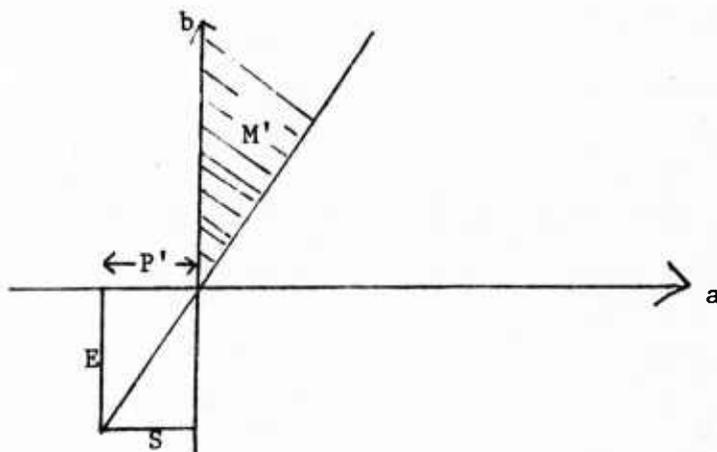


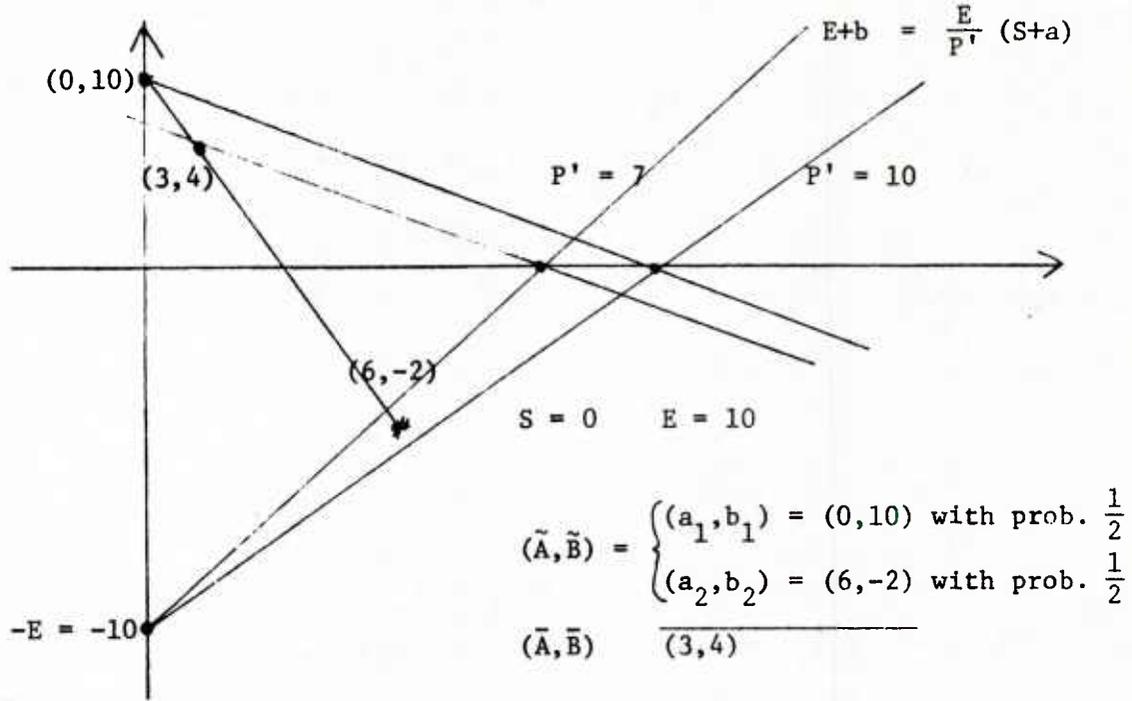
FIGURE A7: Definition of region  $M'$  when  $P' = S$

On the other hand, if  $P'$  is sufficient large,  $S + \bar{A}(M') + \bar{B}(M') < P'$  (or  $p(P') < P'$ ). because the three terms in the left-hand side are bounded (there are no real situations with expected net present values being infinity).

Consequently, at least one value of  $P'$  must exist for which the condition  $p(P') = P'$  is satisfied.

Note

If the joint probability distribution is not strictly continuous in the half-space  $a > 0$ , the existence of this equilibrium solution cannot be guaranteed. Figure A8 offers an example in which no equilibrium solution can be found.



Note that for  $P' = S = 0$        $p(P') = 10 > P'$   
 $P' = 10$                        $p(P') = 7 < P'$

But there is no situation for which  $p(P') = P'$ .

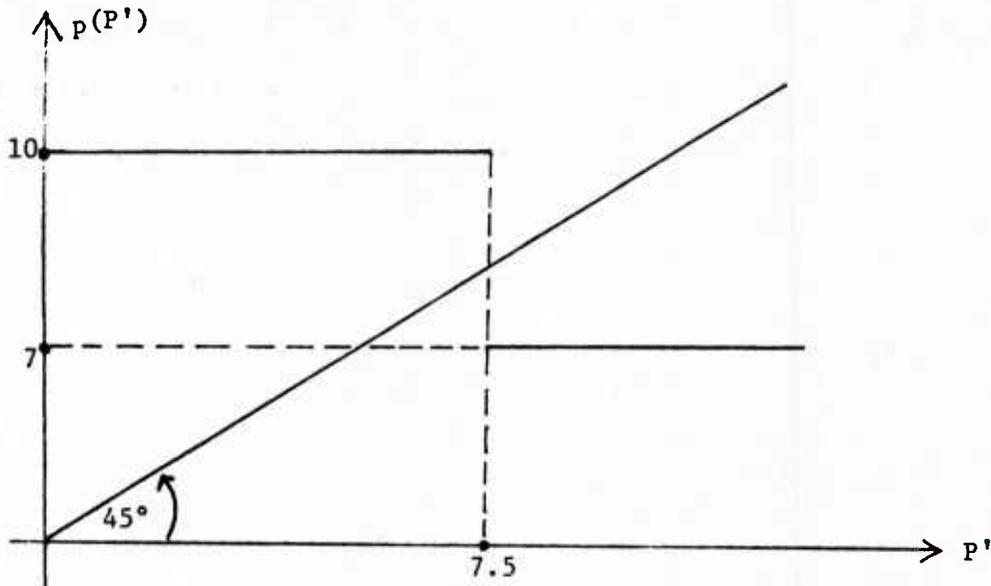


FIGURE A8: Example of a situation with no equilibrium solution  
 (negative values of  $b$  are allowed)

Theorem 2

For a fixed face value of debt (F constant), the system of equations below has at least one equilibrium solution in the range  $S < P' < \infty$ ,  $0 < D < F$ , when the pair  $(\tilde{A}, \tilde{B}_{\text{net}})$  has a joint probability distribution defined in the half space  $\{(a, b_{\text{net}}) | a \geq 0\}$  which is continuous for  $a > 0$ .

$$P' = S + \bar{A}(M') + \bar{B}_{\text{net}}(M')$$

$$D = \text{Ex}\{\tilde{D} | M'\}$$

$$M' = \{(a, b_{\text{net}}) | a \geq 0; E + b_{\text{net}} \geq \frac{E}{P'} (S+a)\}$$

with:

$$b_{\text{net}} = b - (d-D) \quad [\text{Relation (6.10)}]$$

$$E = I - S - D \quad [\text{Relation (6.2)}]$$

$$0 \leq d \leq F$$

Proof:

Given that the value of F is assumed fixed, the value of  $b_{\text{net}}$  may be expressed as  $b_{\text{net}} = (b-d)+D$  where the expression in parentheses is a known quantity.

If the market value of the debt issued is assumed to be equal  $D^\circ$ , the full value of  $b_{\text{net}}$  becomes known, and it is possible to solve for the equilibrium conditions in the pure-equity problem defined by the following parameters: Distribution of  $\{a, b_{\text{net}}\}$ ,  $S$ ,  $E = I - S - D^\circ$ . Theorem 1 guarantees the existence of this solution, and a value of  $P'(D^\circ)$  and a region  $M'(D^\circ)$  can be obtained from the exercise. For this solution to be valid in the debt-equity situation, it is necessary to satisfy the addi-

tional relation:

$$D^{\circ} = \text{Ex}\{\tilde{D}|M'(D^{\circ})\}$$

To prove that this relation can always be satisfied, it is shown that for  $D^{\circ} = 0$ ,  $\text{Ex}\{\tilde{D}|M'(D^{\circ})\} \geq D^{\circ}$ , while for a sufficiently large  $D^{\circ}$ ,  $\text{Ex}\{\tilde{D}|M'(D^{\circ})\} \leq D^{\circ}$ . Given that the distribution of (a,b) is continuous, a value of  $d$  must exist for which the equality between  $D^{\circ}$  and  $\text{Ex}\{\tilde{D}|M'(D^{\circ})\}$  is achieved.

First, if  $D^{\circ} = 0$ , and the region  $M'(D^{\circ})$  is non-empty, (which is always the case with continuous distributions), then  $\text{Ex}\{\tilde{D}|M'(D^{\circ})\} \geq 0$  because  $d \geq 0$  always.

Take now  $D^{\circ} = F$ . If the equilibrium region  $M'(D^{\circ})$  is non-empty, then  $\text{Ex}\{\tilde{D}|M'(D^{\circ})\} \leq F$  because  $d \leq F$  always.\*

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\* This argument should be somewhat refined, because  $E(D^{\circ}) = (I-S-D^{\circ})$  should not be allowed to go negative. This is always the case for  $F > (I-S)$ .

APPENDIX 7: MARKET VALUE OF DEBT WHEN THE MARKET VALUE OF THE FIRM IS  
LOG-NORMAL

This section develops a relation to compute the market value of debt when the market value of the firm at the time the debt is due is a random variable  $\tilde{V}$ , with a log-normal distribution, expected value  $\bar{V}$  and coefficient of variation CV.

$\tilde{V}$  being log-normal implies that  $\tilde{U} = \ln \tilde{V}$  is normal with expected values and variance given by (see Appendix 4):

$$\bar{U} = \ln \frac{\bar{V}}{\sqrt{1+CV^2}}$$

$$\sigma_u^2 = \ln (1+CV^2)$$

Also,  $\tilde{Z}$  defined below has a standardized normal distribution:

$$\tilde{Z} = \frac{\tilde{U} - \bar{U}}{\sigma_u}$$

The market value of debt is defined in terms of its face value (F) as follows:

$$D = \text{Ex}\{\min(\tilde{V}, F)\}$$

Replacing  $\tilde{V} = e^{\tilde{U}} = e^{\bar{U} + \sigma_u \tilde{Z}}$ , this relation is equivalent to:

$$D = \text{Ex}\{\min(e^{\bar{U} + \sigma_u \tilde{Z}}, F)\}$$

$$D = \int_{-\infty}^{\frac{\ln F - \bar{U}}{\sigma_u}} e^{\bar{U} + \sigma_u z} \phi(z) dz + F \int_{\frac{\ln F - \bar{U}}{\sigma_u}}^{\infty} \phi(z) dz$$

where:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

This debt may be rewritten as follows, after some algebraic transformations are performed with the first term.

$$D = e^{\bar{U} + \frac{1}{2}\sigma_u^2} \int_{-\infty}^{\frac{\ln F - \bar{U}}{\sigma_u}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z - \sigma_u)^2} dz + F \frac{\ln F - \bar{U}}{\sigma_u} \int_{\frac{\ln F - \bar{U}}{\sigma_u}}^{\infty} \phi(z) dz$$

Define  $\Phi(Z) = \Pr(\tilde{Z} < Z)$  = cumulative probability distribution for a standardized normal random variable.

Note also that  $e^{\bar{U} + \frac{1}{2}\sigma_u^2} = \bar{V}$ .

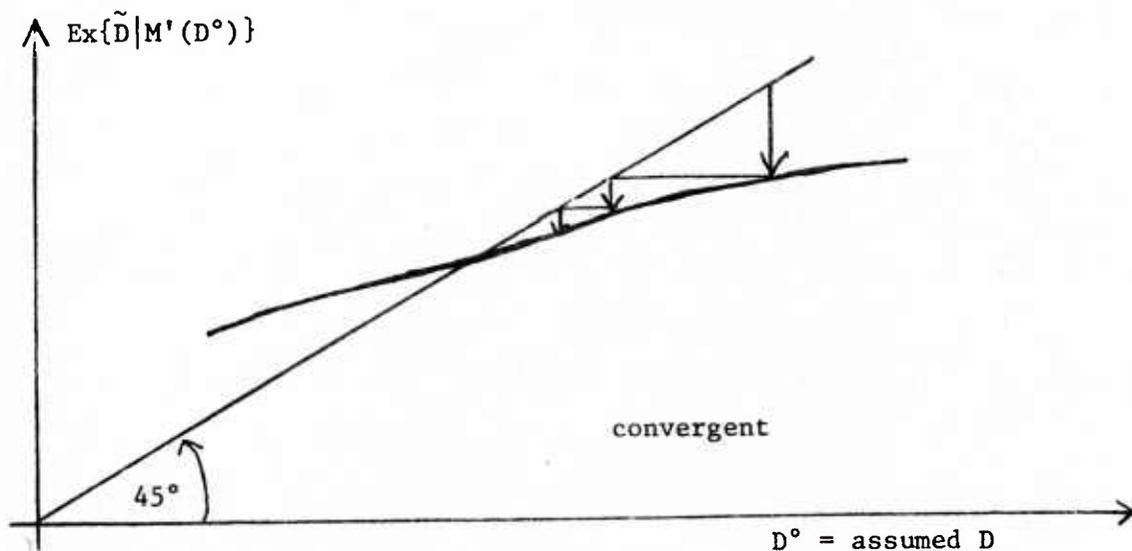
Then:

$$D = \bar{V} \Phi\left(\frac{\ln F - \bar{U}}{\sigma_u} - \sigma_u\right) + F \{1 - \Phi\left(\frac{\ln F - \bar{U}}{\sigma_u}\right)\}$$

APPENDIX 8: ALGORITHM FOR FINDING THE EQUILIBRIUM SOLUTION IN A DEBT-EQUITY CASE

This Appendix informally shows that the algorithm defined in Figure 24 for finding  $P'$  and  $D$  in the debt-equity case converges monotonically to the equilibrium values under some circumstances. The existence of an equilibrium is proven in Appendix 7 for continuous distributions of  $(\tilde{A}, \tilde{B})$  in the semi-space  $\{a > 0\}$ . This assumption is maintained here.

Given that convergence cannot be fully proven without making some additional assumptions with regard to debt, no formal treatment of this subject is included in here. If the algorithm were to fail in being convergent, it is always possible to turn to a direct search of the solution by progressively narrowing the set of possible values of  $D$ . In all numerical cases the algorithm was convergent, so the use of this direct search was not required.



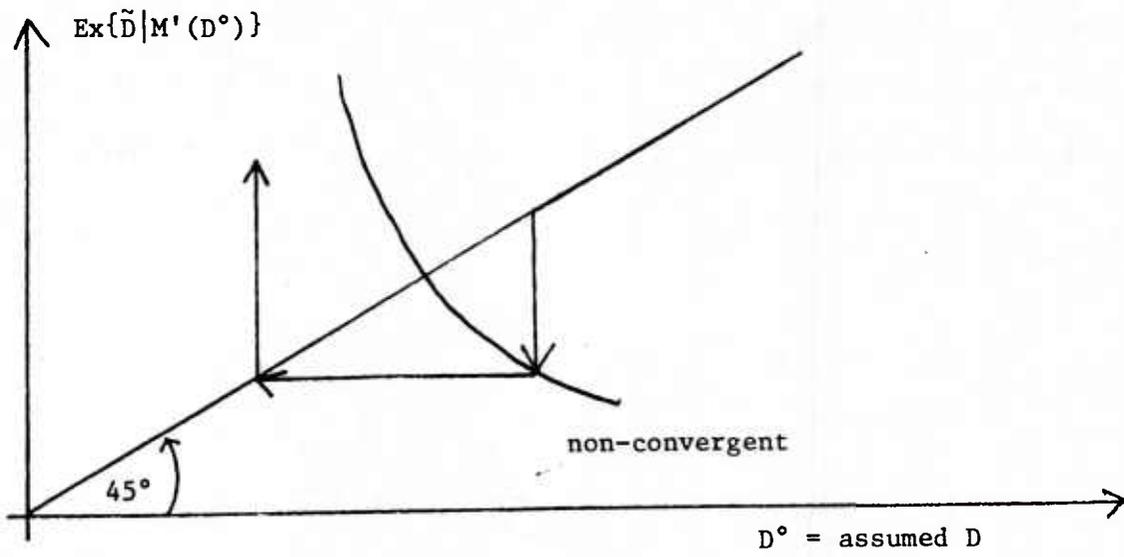
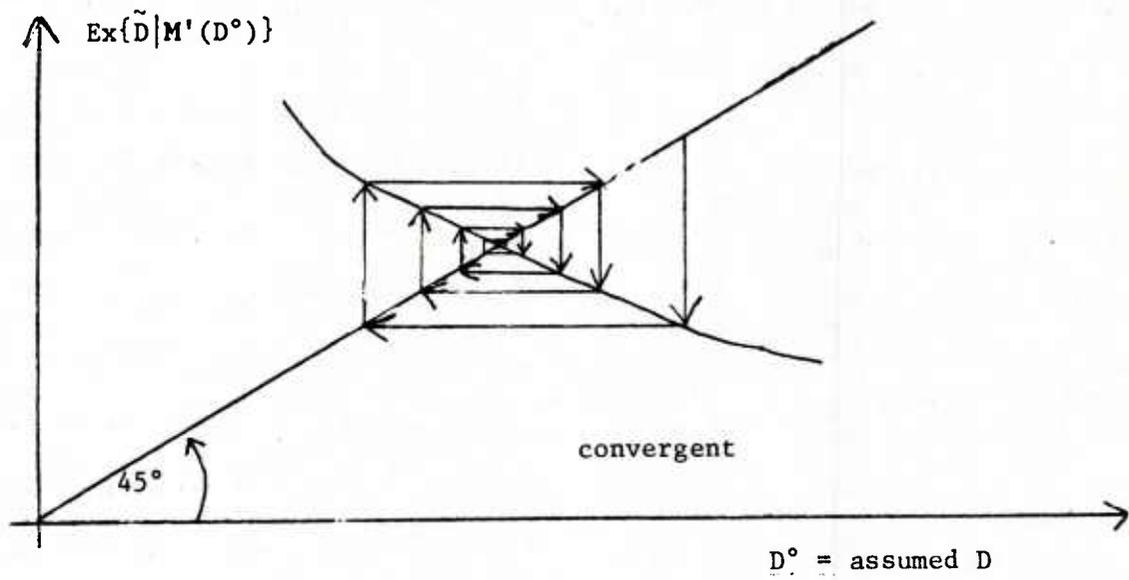


FIGURE A9: The function  $Ex\{\tilde{D}|M'(D^o)\}$  vs.  $D^o$ . Some examples.



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