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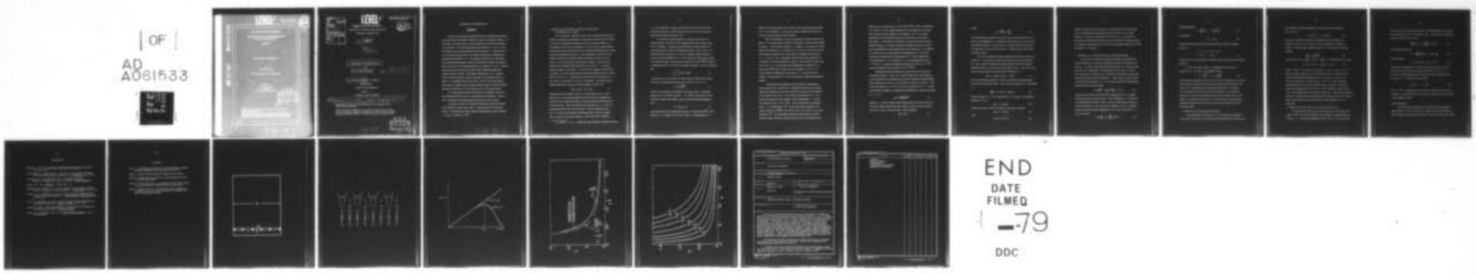

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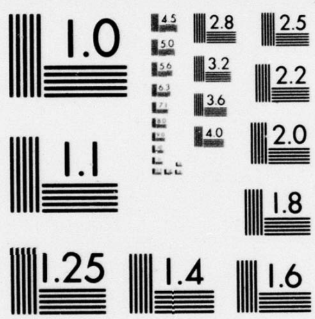

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Department of Earth and Planetary Sciences
Baltimore, Maryland 21218

July 1978

THE DECAY OF TURBULENCE

By
Robert R. Long

Technical Report No. 13 (Series C)

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THE DECAY OF TURBULENCE

ABSTRACT

Solutions are given for the temporal decay of homogeneous turbulence and spatial decay of turbulence induced by a source of energy on a plane $z = 0$. In the latter case there may be a superimposed basic current W perpendicular to the plane. The initial energy spectrum function (at $t = 0$ or $z = 0$) for the idealized source is proportional to the wave number k. With increase of time or distance from the plane, the smaller eddies decay strongly and an energy peak develops at $k = k_s$. The energy in larger waves is permanent in the sense that the energy in a given wave number divided by the energy at that wave number in the original spectrum is a constant for $k \leq k_s$. Thus the energy peak moves to larger and larger eddies containing smaller and smaller amounts of energy. The integral length scale $\ell \propto k_s^{-1}$ increases and the root-mean-square velocity u decreases in accordance with $u\ell = \varkappa$, where $\varkappa$ is a constant characteristic of the "action" of the energy source. This behavior, together with the energy equation, helps to close the problem and leads to solutions for u and ℓ at finite z and t in the various cases.

The results agree with previous theories for turbulence caused by an oscillating grid and by flow in a wind tunnel past a fixed grid. There is also a comparison with experimental observations in the two cases.

An analysis is given for the spatial and temporal decay of the turbulence induced by a plane source after $t = 0$ when the action of the source ceases. The turbulence becomes spatially homogeneous in a layer between $z = 0$ and $z = D$ where $D \propto (\varkappa t)^{\frac{1}{2}}$.

1. Spatial Decay of Turbulence Induced by a Steady Source of Disturbances on a Plane.

In a recent paper, the author (Long, 1978) considered the problem of motion in an infinite, incompressible, homogeneous Newtonian fluid, induced by a steady, horizontally homogeneous source of energy on a plane $z = 0$. The purpose of the present investigation is to gain a deeper understanding of the decay process in this problem and to use this understanding to solve other simple decay problems of interest. In the cited paper, the source of disturbances consists of an infinite number of doublets at points (id, jd, 0) where i, j are all of the positive and negative integers. The axes of the doublets may be aligned along the positive z axis where $i + j$ is even and along the negative z axis when $i + j$ is odd and the strengths μ are all taken to be equal. Use of the energy equation shows that motions at finite z may be finite if $\mu \alpha d^3$ as $d \to 0$. Then all mean quantities must be functions of $\varkappa$, z and ν where $\varkappa$ is the limit of μ/d^3 and ν is viscosity. The results for the integral length scale ℓ and the root-mean-square velocity u are

$$\frac{uz}{\varkappa} = A_1\left(\frac{\varkappa}{\nu}\right), \quad \frac{\ell}{z} = A_2\left(\frac{\varkappa}{\nu}\right) \qquad (1)$$

Obviously the rms velocities v and w are of the same form as u. The z-behavior in (1) is in rough agreement with observations by Hopfinger and Toly (1976) in an experiment (figure 1) with a horizontal oscillating grid in a vessel of water. Their observations also indicated that u, v, w were all nearly equal except very close to the grid. When the Reynolds number $Re = \varkappa/\nu$ is large, the principle of Reynolds number similarity suggests that A_1 and A_2 are universal constants.[1] If the energy source begins at

[1] A_i, where $i = 1, 2, 3, \ldots$ denote universal constants throughout the paper.

t = 0 in a resting fluid, a layer of turbulence of thickness D(t) develops. The theory predicts $D \propto (\varkappa t)^{\frac{1}{2}}$ and this also agrees with some recent experiments (Dickinson and Long, 1978).

It is useful for present purposes to rederive the results in (1) using an energy source that is more appealing physically, perhaps, than the set of doublets. We begin by considering the classical problem of a steady jet along the z-axis in an infinite fluid (Hinze, 1959, p. 404). We imagine that the jet issues at z = 0 from a long pipe of small diameter d under a pressure gradient force G. The velocity in the pipe and at a point at a distance of order d from the end of the pipe is of order $G^{\frac{1}{2}}d^{\frac{1}{2}}$ if the flow is turbulent and Gd^2/ν if the flow is laminar in the pipe. Also the momentum flux

$$J = \int_0^{2\pi} \int_0^{\infty} (P + w^2)\, r\, dr\, d\theta \qquad (2)$$

is constant for all z and t where P is the pressure divided by the density. The flux J is maintained by the pipe flow so that $J = J(G, d, \nu)$ or

$$J = Gd^3 f\left(\frac{G^{\frac{1}{2}}d^{\frac{3}{2}}}{\nu}\right) \qquad (3)$$

Thus we may maintain a jet with finite velocities at finite z by letting G increase as $d \to 0$ in such a way that $\varkappa = \lim G^{\frac{1}{2}}d^{\frac{3}{2}}$ is finite and non-zero. The mean velocity, for example, may be found by dimensional analysis to be

$$\bar{w} = \frac{\varkappa}{z}\,\varphi\left(\frac{z}{r}, \frac{\varkappa}{\nu}\right) \qquad (4)$$

and observations tend to support this behavior. Notice that if $G^{\frac{1}{2}}d^{\frac{3}{2}}$ is not finite as $d \to 0$, evaluation of J in (2) at a level $z \sim d$ indicates that J is infinite or zero and therefore that velocities must be precisely proportional to d^{-1} for the singularity, in the form of a pipe of infinitesimal diameter d, to produce finite motions at finite distances.

With this background, we may now construct an energy source on the plane $z = 0$ by using an infinite number of long pipes of small diameter filling the space $z < 0$ and ending at the plane $z = 0$ (figure 2). The pipes are evenly spaced with centers on $z = 0$ at points (10 id, 10 jd, 0). Flow, driven by the same pressure gradient force G in each pipe is in opposite directions for $i + j$ even and $i + j$ odd. The spacing 10d has been chosen arbitrarily to yield flows to and from the pipes, as in figure 2, which interfere rather little at the level $z = d$. In accordance with the discussion of a single jet, we require that $G^{\frac{1}{2}}d^{\frac{3}{2}}$ tends to a finite, non-zero quantity $\varkappa$ as $d \to 0$. We thereby obtain an energy source on the plane $z = 0$, characterized by a quantity $\varkappa$ of dimensions L^2T^{-1}, and we may obtain (1) again by dimensional analysis.

A useful physical interpretation of the problem of constructing a plane energy source is that there is a competition between the very large energy flux issuing from the source and the very large viscous dissipation in the vicinity of the source. If the velocity is proportional to d^{-1}, the source is characterized by a quantity $\varkappa$ with the dimensions of ν and the only length unit near $z = d$ is d itself. Then the energy flux crossing the plane $z = d$ is proportional to d^{-3}. On the other hand the integral of the energy dissipation, $\nu\overline{(\nabla u'_a)^2}$, over the layer from d to a finite z is also proportional to d^{-3}. The small (finite) difference between these two large quantities equals the finite energy flux at the level z corresponding to the finite, non-zero motions there. If the velocity $G^{\frac{1}{2}}d^{\frac{1}{2}}$ or Gd^2/ν is proportional to d^{-n} where $n > 1$, the energy dissipation is weak compared to the energy flux divergence and the motions at z are infinite. If $n < 1$, the energy dissipation is too large and the motion dies away before it can reach a finite level z. If the motion is turbulent as it issues from the pipe and the Reynolds number κ/ν is large, the energy dissipation may occur in eddies that are quantitatively much smaller that the pipe diameter d. Nevertheless our estimate of the velocity derivative as proportional to d^{-2} is correct because all velocity scales are proportional to d^{-1} and all length scales are proportional to d. For example at $z = d$, the dissipation function is $\epsilon = \kappa^3 d^{-4} f(\kappa/\nu)$, so that the Kolmogoroff length scale $\nu^{\frac{3}{4}}/\epsilon^{\frac{1}{4}}$ is $d(\nu/\kappa)^{\frac{3}{4}}/f^{\frac{1}{4}}$ and the Kolmogoroff velocity scale $(\nu\epsilon)^{\frac{1}{4}}$ is $d^{-1}\kappa^{\frac{3}{4}}\nu^{\frac{1}{4}}f^{\frac{1}{4}}$.

Dimensional analysis permits the argument to be very simple but one problem remains. Thus, as $d \to 0$ and the pipes move closer together, it seems possible at first glance that the inflow and outflow through alternate pipes might cancel in such a way as to yield zero velocities at finite z. This would correspond to $A_1 = 0$ in (1) or to a zero energy flux as $d \to 0$. To show that this does not happen, we calculate the energy flux for a finite set of pipes at $z = d$. It is

$$EF \propto \frac{(S_0 w_0^3 - S_1 w_1^3)}{S} \qquad (5)$$

where w_0, w_1 are the average vertical speeds upward in the jets and downward in the region outside of the jets, where S_0 and S_1 are the respective areas and $S = S_0 + S_1$ is the total area. From continuity

$$S_0 w_0 = S_1 w_1 \qquad (6)$$

so that

$$EF \propto \frac{w_0^3 S_0}{S}\left(1 - \frac{S_0^2}{S_t^2}\right) \qquad (7)$$

The non-dimensional quantities S_0/S and S_0/S_t can only be functions of κ/ν and so are independent of d. According to the basic nature of viscous flows, as portrayed in figure 2, the ratio S_0/S_t is certainly less than one so that the energy flux is not zero in the limit as $d \to 0$ and the source yields finite velocities at finite z.

Let us now consider the energy distribution among the various wave numbers k. At any level z the energy comes from below with energy flux EF. Some is used to maintain the energy spectrum at that level against dissipation and the rest is passed along to higher levels. At the level z and near the source plane, the energy spectrum functions are

$$E(k, z) = \kappa^2 k f(kz), \quad E(k, 0) = \kappa^2 k f(0) \qquad (8)$$

These are illustrated in figure 3, where we recognize that energy losses occur first in the smaller eddies. The peak of the spectrum at z corresponds to

$$\frac{\partial E}{\partial k} = 0 = \kappa^2 f(kz) + \kappa^2 kz f'(kz) \qquad (9)$$

Thus at the peak, $k_s z = b$ is a constant and $f(b)$ = constant. The spectrum function at $k = k_s$ is

$$E(k_s, z) = \kappa^2 k_s f(b) \qquad (10)$$

whereas the energy at that wave number at the place of origin is

$$E(k_s, 0) = \kappa^2 k_s f(0) \qquad (11)$$

Thus

$$E(k_s, z) \propto E(k_s, 0) \qquad (12)$$

so that the energy at the wave number k_s of the peak (at the integral length scale) is proportional to the energy at that wave number in the original spectrum. It is easy to see that this is true for all $k \leq k_s$ and this is in accordance with the classical concept of the "permanence" of the large eddies. This, together with the linearity in k of the original spectrum function, explains the behavior $u\ell$ = constant.

2. Turbulence in a Current Moving Past a Plane Energy Source.

We are now in a position to consider the problem of high Reynolds number turbulence in a current moving through the plane energy source of Section 1. For large z this should resemble turbulence in a stream moving past a grid in a wind tunnel. We suppose that a mean velocity W issues from the pipes at the energy source in addition to the agitation caused by the alternate up and down motions of Section 1. Since the mean current should not affect the relationships $u \sim v \sim w$, the energy equation for high Reynolds number turbulence may be written

$$A_3 W \frac{\partial u^2}{\partial z} - A_4 \frac{\partial}{\partial z}\left(K \frac{\partial u^2}{\partial z}\right) + \frac{u^3}{\ell} = 0 \qquad (13)$$

where the second term is the energy flux divergence and K is a quantity with the dimensions of eddy viscosity. The relationship $u\ell \propto \varkappa$ should also be uninfluenced by the basic current, as we may infer from the discussion at the end of Section 1, and we use this to eliminate ℓ in (13). In addition the eddy viscosity should be independent of W so that we may take $K \propto \varkappa$. Eq. (13) becomes

$$A_5 W \frac{\partial u^2}{\partial z} - A_3 \varkappa \frac{\partial^2 u^2}{\partial z^2} + \frac{u^4}{\varkappa} = 0 \qquad (14)$$

With the definitions

$$u^2 = \frac{A_5^2}{A_6} W^2 T \;, \quad \zeta = \frac{A_5}{A_6} \frac{Wz}{\kappa} \tag{15}$$

(14) becomes

$$T'' = T' + T^2 \tag{16}$$

This differential equation was solved numerically, using the asymptotic behaviors,

$$T \to \zeta^{-1} + 2\zeta^{-2} \ln\zeta, \quad \text{as } \zeta \to \infty$$
$$T \to 6\zeta^{-2}, \quad \text{as } \zeta \to 0 \tag{17}$$

and the solution is shown in figure 4 together with the curves of the asymptotic solutions.

At small ζ the basic current is unimportant and we obtain the behavior in (1). For large ζ the solutions become

$$u = A_7 \left(\frac{W\kappa}{z}\right)^{\frac{1}{2}} \;, \quad \ell = A_8 \left(\frac{\kappa z}{W}\right)^{\frac{1}{2}} \tag{18}$$

These results correspond to one of the two classes of solutions to the equation of Karman and Howarth (1938) assuming "self-preservation" (Korneyev and Sedov, 1976). Experiments in a wind tunnel downstream of a "passive" grid (Comte-Bellot and Corrsin, 1966) or downstream of a mechanically agitated grid similar to the present theoretical model (Ling and Wan, 1972) suggest an approach to the behavior in (18) as the effect of the mechanical agitation becomes relatively large and overcomes the effect of the linear momentum wakes.

3. Temporal Decay of Homogeneous Turbulence.

In the flow behind a grid where W >> u, the space rate of change of energy flux is small and in a coordinate system moving with the mean speed W the turbulence is nearly homogeneous in space but decaying with time. Eq. (18) yields

$$u = A_9 \left(\frac{\varkappa}{t}\right)^{\frac{1}{2}} , \quad \ell = A_{10} (\varkappa t)^{\frac{1}{2}} \tag{19}$$

We may also obtain (19) directly by imagining doublets of strength μ at points (id, jd, kd) for $t < 0$, where i, j, k are all the positive and negative integers. Let the doublets suddenly cease at $t = 0$ and integrate the energy equation,

$$\frac{\partial u^2}{\partial t} = - A_{11} \nu \overline{(\nabla u_n')^2} \tag{20}$$

from time $(d^2/\varkappa)$ to t, where $\varkappa = \mu/d^2$. Using $u^2 = (\varkappa^2/d^2)\varphi(t\varkappa/d^2, \varkappa/\nu)$, we get

$$\varphi(\tau, \frac{\varkappa}{\nu}) - \varphi(1, \frac{\varkappa}{\nu}) = -\frac{\nu}{\varkappa} \int_1^{\tau} \chi(\tau, \frac{\varkappa}{\nu}) d\tau \tag{21}$$

where $\tau = t\varkappa/d^2$. As $d \to 0$ with t and $\varkappa$ fixed, $\varphi(\tau, \varkappa/\nu) \to \varphi(\infty, \varkappa/\nu) \to 0$, on physical grounds, whereas $\varphi(1, \varkappa/\nu)$ is finite because $d^2/\varkappa$ is of the order of an eddy time just after the doublets cease. χ also goes to zero as $d \to 0$ with t and $\varkappa$ fixed but is of order one for $\tau \sim 1$ as $d \to 0$. Thus the energy equation permits finite motions at finite times if, ultimately, the distance between the doublets tends to zero and $\varkappa = \mu/d^2$ is held fixed. Then at time t, u and ℓ must depend only on $\varkappa$ and t and we obtain (19). Notice that a discussion similar to the discussion of the energy spectrum function at the end of Section 1 again reveals that the eddies larger than the integral length scale ℓ are permanent.

4. Spatial and Temporal Decay Above a Plane Energy Source.

The final problem of this paper concerns the decay of turbulence with respect to both z and t when the plane source of Section 1 suddenly ceases its output at $t = 0$. According to Hopfinger and Toly (1976), u, v, w are all equal except very near the grid when the grid is oscillating steadily, and we may safely assume this is true in the present case. Using the same assumptions as used for equation (14), we get

$$\frac{\partial}{\partial t}(u^2/2) - A_{12}\varkappa\frac{\partial^2 u^2}{\partial z^2} + A_{13}\frac{u^4}{\varkappa} = 0 \qquad (22)$$

The solution has the form

$$u^2 = \frac{2A_{12}}{A_{13}}\frac{\varkappa^2}{z^2}T(\eta), \quad \eta = 4A_{12}\frac{\varkappa t}{z^2} \qquad (23)$$

and (22) becomes

$$T' - 3T - 7T'\eta - 2T''\eta^2 + T^2 = 0 \qquad (24)$$

The numerical integration of (24) differs from that in Section 2 in that a universal constant is unknown, namely the limit of $T\eta = a_1$ as $\eta \to \infty$. The limit corresponds physically to homogeneous decaying turbulence in a region $-D < z < D$ about the origin in which

$$u \propto \left(\frac{\varkappa}{t}\right)^{\frac{1}{2}}, \quad \ell \propto (\varkappa t)^{\frac{1}{2}} \qquad (25)$$

and $D \propto (\varkappa t)^{\frac{1}{2}}$. At smaller times (or larger z), T is given as a function of η in figure 5 in which the choice of curves (or a_1) must be made on the basis of experimental evidence.

Acknowledgements.

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LEGENDS

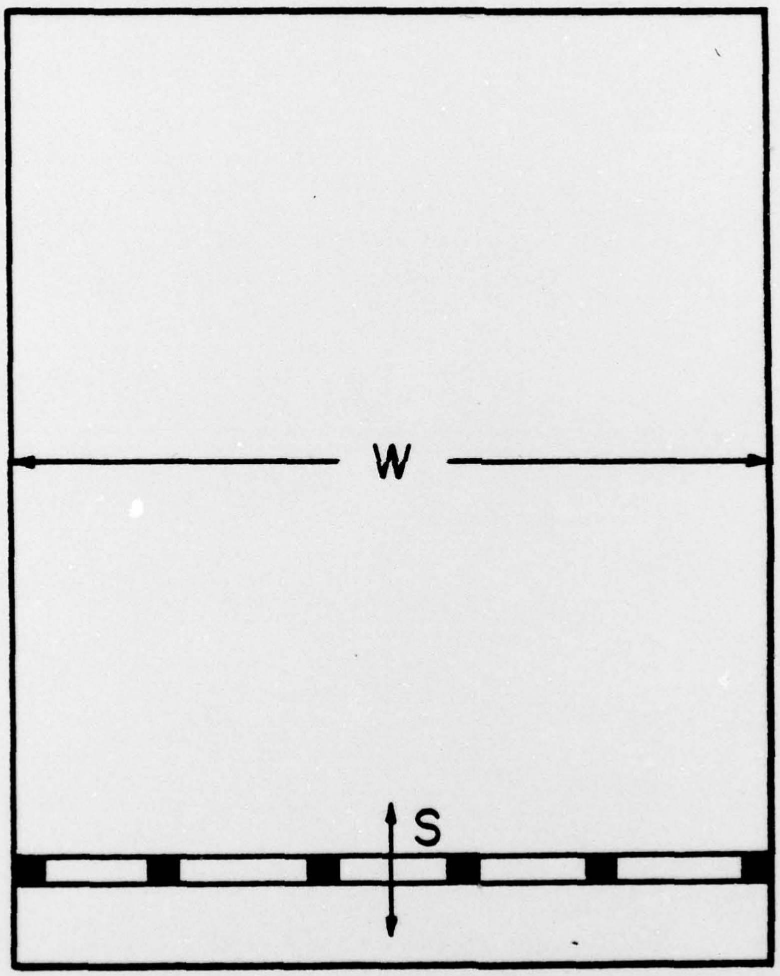
Figure 1. Oscillating grid experiment. The grid oscillates vertically producing turbulence in the homogeneous fluid above.

Figure 2. System of pipes producing an energy source at a plane.

Figure 3. Energy spectrum functions at a plane energy source and at level z above the source.

Figure 4. Solution of Eq. (16). T is proportional to the kinetic energy and ζ is proportional to distance from the energy source.

Figure 5. Solution of Eq. (28). The various curves correspond to a parameter a_1 which is a universal constant to be chosen on the basis of experimental evidence.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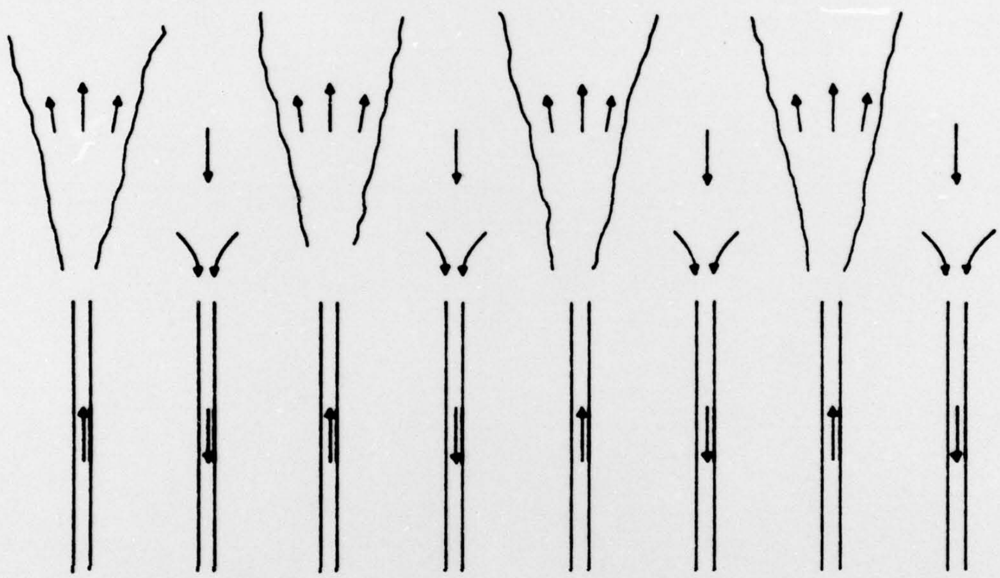

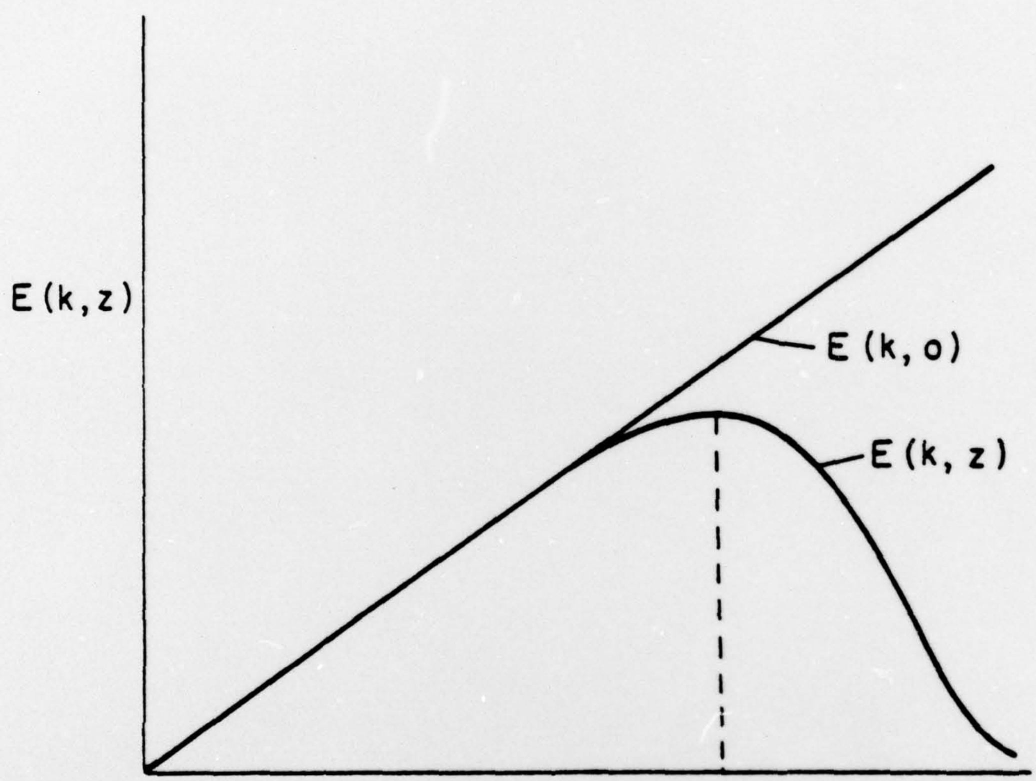

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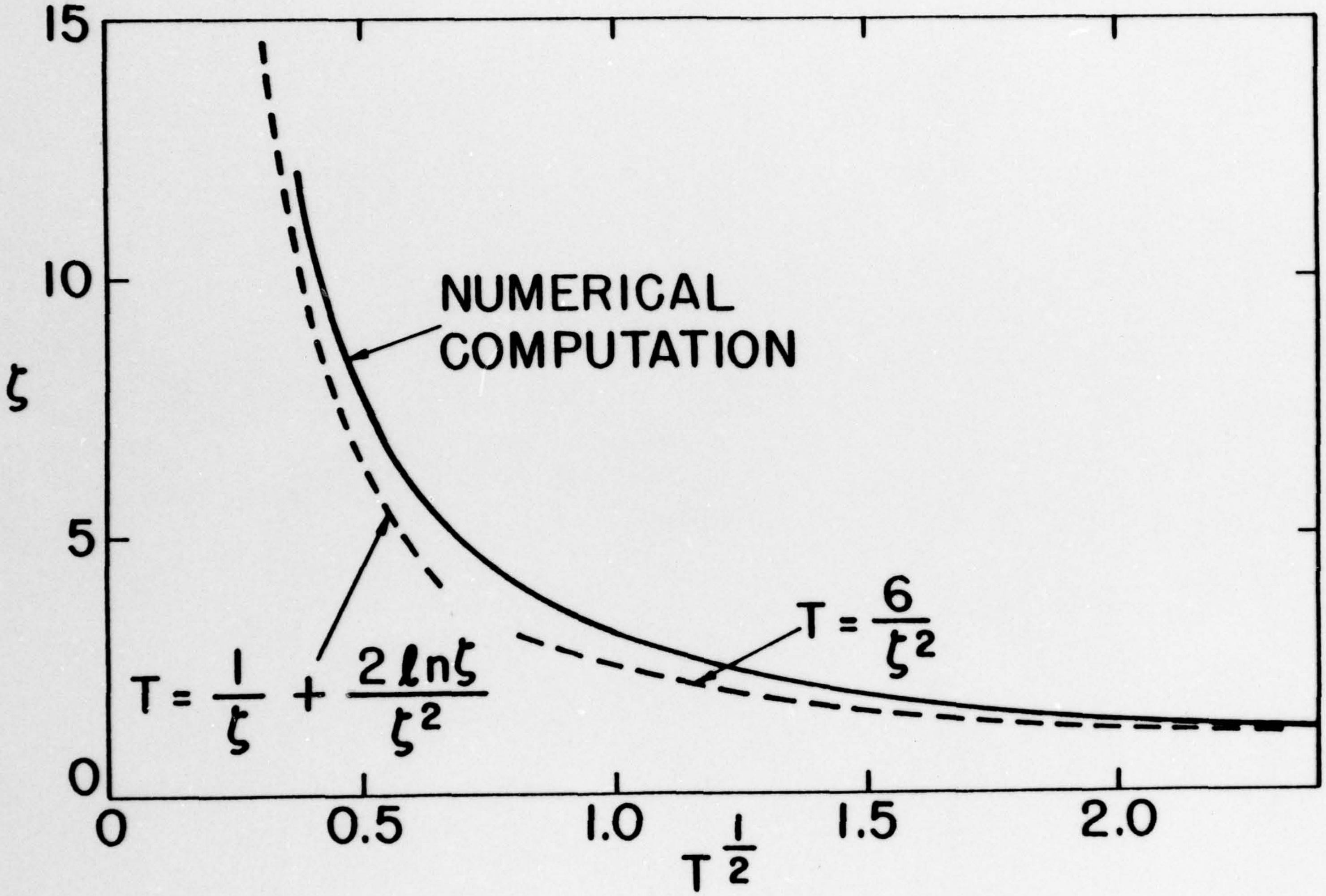
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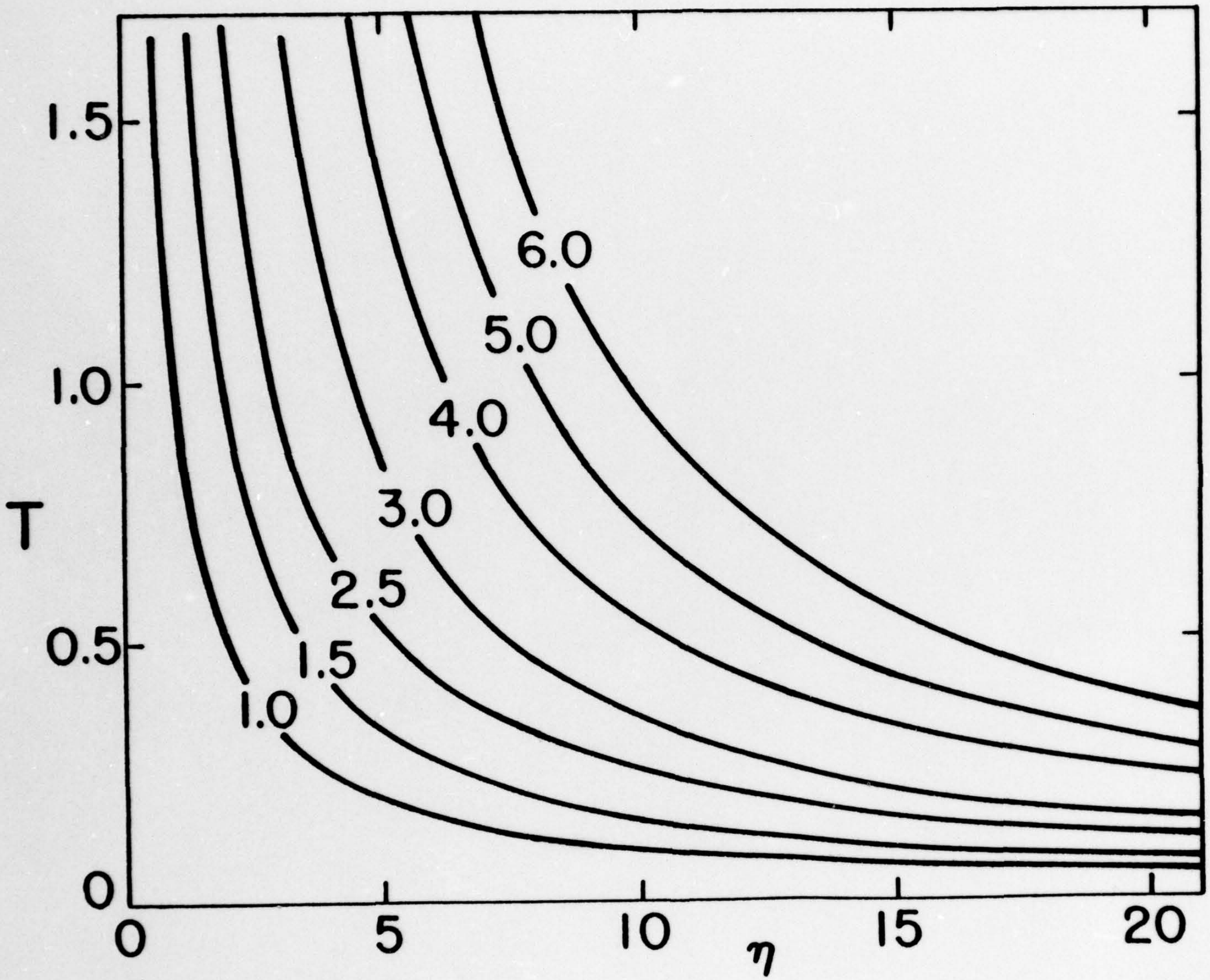
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